

Article

Two-Sided Beneficial Value-Added Service Investment and Pricing Strategies in Asymmetric/Symmetric Investment Scenarios

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Abstract: We explore media platforms' investment strategies for two-sided beneficial value-added services, which can directly benefit both consumers and advertisers, and ad pricing strategies by using game theory. We consider an asymmetric investment scenario, scenario A, and a symmetric investment scenario, scenario S, and develop game models under each scenario. First, we obtain the equilibrium investments, prices and profits and analyze the influence of the three important parameters—marginal investment costs, positive consumer effects and negative advertising effects, on the equilibrium outcomes in each scenario. Then we compare these equilibrium outcomes between both scenarios. Finally, we conduct numerical simulations to verify the conclusions obtained in both scenarios. We show that in scenario A, the value-added service levels and ad prices of the investment platforms remain constant and then decrease with marginal investment costs. The ad prices and profits of the investment platforms increase (decrease) with positive consumer effects (negative advertising effects). The same change is true for the value-added service levels only under certain conditions. In scenario S, the value-added service levels of the investment platforms change with positive consumer effects or negative advertising effects only when marginal investment costs are high. The ad prices of the platforms always increase with positive consumer effects but increase with negative advertising effects only when marginal costs are low. The profits of the platforms vary monotonically with negative advertising effects, but not necessarily with positive consumer effects. Compared to scenario S, the ad prices of the investment platforms in scenario A are higher, but that is not always true for the value-added service levels.



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Keywords: two-sided beneficial value-added service; platform; investment strategy; pricing strategy; asymmetry/symmetry; game approach

1. Introduction

With the development of mobile internet and informatization, various media platforms, e.g., video platforms, are booming [1–4]. Media platforms provide content for consumers and ad space for advertisers and profit mostly from charging advertisers [5]. To engage more consumers and advertisers, media platforms have introduced a category of value-added services that can directly enhance the experience of both consumers and advertisers, referred to as two-sided beneficial value-added services. For example, video platforms such as “Iqiyi” and “Youku” allow advertisers to embed links in ads, and if consumers are interested in the products in the ads, they can quickly find the purchase pages by clicking on the links. This value-added service can not only satisfy consumers' demand for shopping while watching videos but also help advertisers quickly obtain marketing benefits. It is clear that such value-added service investment can increase the utility that can be obtained from the platform by viewers and advertisers. However, more investment in value-added services is not always better for two major reasons. First, in some cases, a platform's overinvestment in value-added services may be detrimental to its ability to

attract consumers and advertisers. This is easy to understand. On the one hand, the high-level value-added services provided by the investment platform can directly bring consumers more utility, which improves the consumer demand for the platform. On the other hand, the high-level value-added services also can directly bring advertisers more utility, which improves the advertiser demand for the platform; however, this will reduce the consumer demand for the platform with the impact of negative cross-network effects from advertisers to consumers (i.e., negative advertising effects) [6]. Clearly, the high-level value-added services have two opposite influences on the consumer demand for the investment platform. If the latter influence is dominant, the consumer demand for the platform may decrease. Furthermore, the advertiser demand for the platform also decreases due to positive cross-network effects from consumers to advertisers (i.e., positive consumer effects) [6]. Briefly, in some conditions, with the impact of positive consumer effects and negative advertising effects, overinvestment instead suppresses consumer and advertiser demands for the platform. Second, overinvestment will cause the platform to generate more investment costs. For the above two reasons, we know that only by formulating an appropriate value-added service investment strategy can the platform effectively attract consumers and advertisers and avoid excessive costs. Note that platform pricing, as well as value-added service investments, is also closely related to the utilities and demands of consumers and advertisers. When the consumer demand or advertiser demand for a platform changes due to value-added service investments, the platform should adjust its price accordingly to maximize profits. In summary, only appropriate two-sided beneficial value-added service investment and pricing strategies can help media platforms to effectively carry out the services, avoid more costs and achieve more profits. This will promote the sustainable development of media platforms.

This paper aims to explore the optimal two-sided beneficial value-added service investment and pricing strategies considering three important parameters—marginal investment costs, positive consumer effects and negative advertising effects. This paper mainly answers the following questions:

1. In an asymmetric investment scenario, i.e., a platform invests in two-sided beneficial services and its competitor does not, how should the platform decide the value-added service level and the corresponding ad price? How are the value-added service level and ad price influenced by marginal investment costs, positive consumer effects and negative advertising effects?
2. In a symmetric investment scenario, i.e., a media platform and its competitor both invest in two-sided beneficial value-added services, how should the platform decide the value-added service level and ad price? How do marginal investment costs, positive consumer effects and negative advertising effects impact the value-added service level and ad price of the platform?
3. What are the differences in the value-added service level and ad price of a platform that has invested between the asymmetric investment scenario and symmetric investment scenario?

To address the above problems, this paper investigates duopoly media platforms' two-sided beneficial value-added service investment strategies and corresponding ad pricing strategies by using game theory. We develop duopoly game models under different investment scenarios. These include an asymmetric investment scenario where a single platform invests in two-sided beneficial value-added services, scenario A, and a symmetric investment scenario where two platforms both invest in such services, scenario S. First, we obtain the equilibrium value-added service levels, ad prices and profits of investment platforms and analyze the impact of marginal investment costs and opposite cross-network effects between consumers and advertisers (i.e., positive consumer effects and negative advertising effects) on these equilibrium outcomes in each scenario. Second, we compare the differences in equilibrium outcomes of investment platforms between scenario A and scenario S. Finally, we conduct numerical simulations to verify the conclusions obtained in both scenarios.

The main contributions of this paper are embodied in the following aspects.

1. This paper focuses on a category of value-added services that can directly benefit users on two sides of investment platforms, which directly improve the utilities of users on two sides of the platforms. Previous research on service investment strategies of platforms has investigated primarily the value-added services that can directly benefit users on one side of the platforms. Therefore, in their research, only users on one side can obtain the direct utility brought by the value-added service investments. However, such research has neglected the value-added services that can directly benefit users on two sides, which can directly improve the utilities of users on two sides. We bridge this gap by focusing on two-sided beneficial value-added services that directly benefit users on two sides of media platforms, i.e., consumers and advertisers.
2. This paper develops duopoly game models to explore the value-added service investment strategies of platforms in asymmetric and symmetric investment scenarios. Unlike previous research that analyzes value-added service investment strategies of platforms by developing monopoly game models, we investigate such strategies by developing duopoly game models. In addition, we differentiate duopoly game models in asymmetric and symmetric investment scenarios.
3. This paper investigates the impact of value-added service investments on media platform pricing. Unlike most research that examines the impact of mergers and entries, advertising regulations and privacy concerns on media platform pricing, we investigate the impact of value-added service investments on such pricing by comparing the differences in equilibrium prices of media platforms under asymmetric and symmetric investment scenarios.
4. We analyze the impact of important parameters—marginal investment costs, positive consumer effects and negative advertising effects, on the equilibrium two-sided beneficial value-added service levels, ad prices and profits of media platforms. This provides management implications for media platforms when they determine the investments in two-sided beneficial value-added services and ad prices.

The remainder of the paper is organized as follows. In the next section, we review the related literature. Section 3 describes the model and basic assumptions. Section 4 conducts the equilibrium analysis for scenario A and scenario S. Section 5 compares the differences in equilibrium outcomes between scenario A and scenario S. Section 6 reports the numerical simulations. Section 7 presents the conclusions, managerial insights and possible future research directions.

2. Literature Review

This paper is related to two streams of literature. One is associated with service investment strategies of platforms, and the other is related to media platform pricing strategies.

2.1. Service Investment Strategies of Platforms

There is an emerging body of literature on the service investment strategies of platforms [7–13]. Focusing on a category of value-added services that directly benefit users on only one side, e.g., buyers, Dou et al. [7] investigated the value-added service investment strategy of a monopoly e-commerce platform. They found that as marginal investment costs increased, the value-added service level set by the platform for buyers first remained constant and then decreased. Furthermore, Dou et al. [8] focused on a value-added service that directly benefits only sellers and investigated how negative network effects among sellers affect the value-added service investment strategy of a monopoly e-commerce platform. They demonstrated that when marginal investment costs were high, the value-added service level of the platform decreased with network effects among sellers. Xu et al. [9] concentrated on a value-added service that directly benefits only sellers and studied the value-added service investment strategy set by the government. They found that the impact of social welfare loss coefficients on the investment was associated with the maximum marginal social welfare generated per unit of the value-added service. Zhu et al. [10]

developed monopoly models under two scenarios of an e-commerce platform cooperating and not cooperating with a social media platform and investigated the value-added service investments of the e-commerce platform to buyers. The study showed that the value-added service level was higher in the scenario in which the e-commerce platform was cooperating with the social media platform than in the scenario in which the e-commerce platform was not cooperating with the social media platform. Liu et al. [11] developed a monopoly model and studied the value-added service investment strategies of O2O platforms. They considered the value-added services that directly benefit only consumers. They found that as platform capital increased, the value-added service level of the platform first increased and then remained constant. Sun et al. [12] constructed an evolutionary game model between governments and ridesharing platforms under social media participation and analyzed the impact of social media exposure on the platforms' service investment strategies in customer safety. They found that social media exposure to safety issues in ridesharing promoted platforms' service investments in customer safety. Zhang and He [13] focused on a category of services that could reduce transaction costs for both buyers and sellers and studied the service investment strategy of a monopoly e-commerce platform. They showed that the service level of the e-commerce platform was related to the investment efficiency coefficients of buyers and sellers.

In the above literature on the service investment strategies of platforms, scholars mainly studied the value-added service investment strategies of platforms [7–11]. The existing studies on the value-added service investment strategies of platforms have focused mostly on value-added services that directly benefit users on only one side [7,8,10,11], ignoring those value-added services that directly benefit users on two sides. Therefore, in existing studies, users on only one side can obtain direct utility from value-added service investments. Different from the existing studies, this paper focused on a category of value-added services that can directly benefit users on two sides of media platforms, i.e., consumers and advertisers. Consequently, both consumers and advertisers can obtain direct utilities brought by value-added service investments. In addition, most studies on the value-added service investment strategies of platforms have analyzed such strategies by developing monopoly game models [10,11]; however, fewer studies have explored such strategies based on duopoly game models. To fill this gap, this paper examines the value-added service investment strategies of platforms by developing duopoly game models and further distinguishes two duopoly investment scenarios, i.e., an asymmetric investment scenario, scenario A, and a symmetric investment scenario, scenario S.

2.2. Media Platform Pricing Strategies

There is extensive literature on media platform pricing strategies. Several scholars have studied the impact of mergers on media platform pricing [14,15]. Anderson et al. [14] developed duopoly game models and investigated the impact of platform mergers on ad prices in a scenario with multi-homing consumers. They found that platform mergers can lead to higher ad prices. Furthermore, Anderson et al. [15] analyzed how consumer behavior affects the role of platform mergers in media platform pricing. They found that if consumers were single-homing, platform mergers could not change ad prices, and if consumers were multi-homing, platform mergers should increase ad prices.

Kerkhof and Munster [16] and Henriques [17] explored the impact of advertising regulations on media platform pricing. Kerkhof and Munster [16] developed monopoly game models and studied the impact of advertising regulations on the pricing of media platforms. They found that advertising regulations led to an increase in the ad prices of the platforms. Henriques [17] also examined the impact of advertising regulations on media platform pricing, similar to Kerkhof and Munster [16]. They also found that advertising regulations incentivized media platforms to increase their ad prices.

Scholars have also studied how other factors, such as privacy concerns, ad-targeting capabilities and social influences, affect media platform pricing [18–20]. Duan et al. [18] studied the impact of privacy concerns on the pricing of a monopoly media platform. They

showed that the ad price first decreased, and then increased slightly as privacy concerns rose. Gong et al. [19] developed a competitive model containing one platform with high ad-targeting capabilities and another with low ad-targeting capabilities and examined the effect of ad-targeting capabilities on ad prices. They found that as the ad-targeting abilities of the platform with high ad-targeting abilities increased, the ad price of the platform with high ad-targeting abilities either increased or decreased. Wang and Guo [20] also investigated the role of social influences on the pricing of a monopoly video platform under a trial-viewing model. They found that the role of social influences on consumer prices was related to whether the social influences were positive or negative.

The literature on media platform pricing strategies has mainly explored impact factors such as mergers [14,15], advertising regulations [16,17], privacy concerns [18], ad-targeting abilities [19] and social influences [20] on media platform pricing, ignoring the impact of value-added service investments. In this study, we fill the gap and address the impact of value-added service investments on media platform pricing.

3. Models

This section develops a model consisting of two media platforms and two groups of users, namely, consumers and advertisers, as shown in Figure 1. The two media platforms provide content for consumers and ad space for advertisers and profit mostly from charging advertisers [5]. To attract more consumers and advertisers, some media platforms have attempted to invest in value-added services that can directly benefit consumers and advertisers, referred to as two-sided beneficial value-added services. For example, video platforms such as AAuto Quicker and TikTok allow advertisers to embed purchase links in the ads, and if consumers are interested in the products in the ads, they can quickly find the purchase pages by clicking on the links. This can not only help advertisers gain marketing revenue quickly but also help consumers buy products that they are interested in. Obviously, such value-added services can directly enhance the utilities of both consumers and advertisers. In this paper, we consider two value-added services investment scenarios: (i) an asymmetric investment scenario where a single platform (platform 1) invests in value-added services, hereafter referred to as “scenario A”, as shown in Figure 1a. (ii) a symmetric investment scenario where the two platforms (platform 1 and platform 2) invest in value-added services, hereafter referred to as “scenario S”, as shown in Figure 1b.

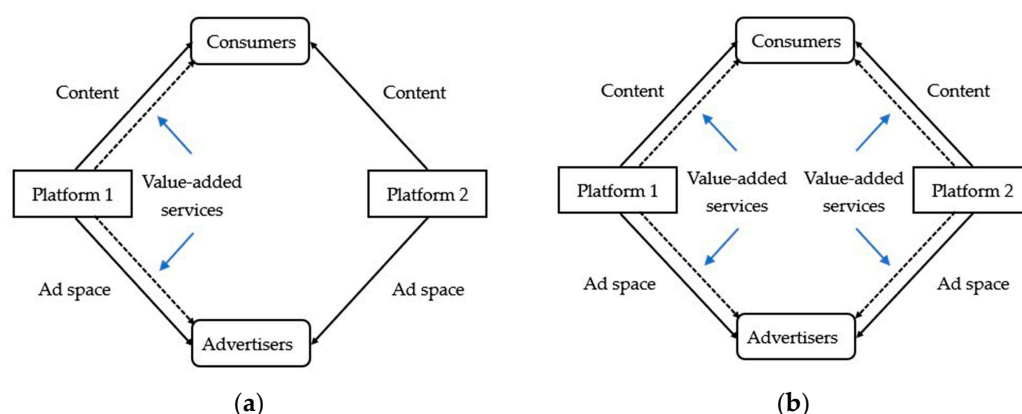


Figure 1. Research model: (a) scenario A; (b) scenario S.

Before introducing the three types of agents (i.e., consumers, advertisers and platforms) as well as the game stage, we first present some notations that will be used later, as shown in Table 1.

Below, we introduce the three types of agents and the game stage.

Table 1. Summary of notations.

Notations	Descriptions
i	index for platforms, $i \in \{1, 2\}$. $i = 1$ represents platform 1, and $i = 2$ represents platform 2.
j	index for scenarios, $j \in \{A, S\}$. $j = A$ represents scenario A, and $j = S$ represents scenario S.
x_i	the location of platform i .
v_i	the basic utility that consumers can obtain from the content provided by platform i .
Q_{ij}	the value-added service level of platform i in scenario j .
P_{ij}	the ad price of platform i in scenario j .
π_{ij}	the profit function of platform i in scenario j .
u_{ij}	the total utility that a consumer derives from platform i in scenario j .
U_{ij}	the total utility that an advertiser obtains from platform i in scenario j .
N_{ij}	the consumer demand of platform i in scenario j .
M_{ij}	the advertiser demand of platform i in scenario j .
d_{ij}	the state variable representing whether platform i invests in value-added services in scenario j . $d_{ij} = 0$ means that platform i does not invest in value-added services in scenario j , and $d_{ij} = 1$ means that platform i invests in value-added services in scenario j .
x	the location of consumers.
t	the unit transport cost.
f	the cost of designing and producing an ad.
a	the negative cross-network effect that each advertiser has on a consumer, $a \in (0, 1]$. It measures the disutility that a consumer suffers from each advertiser, hereafter “the negative advertising effect”.
r	the positive cross-network effect that each consumer brings to an advertiser, $r \in (0, 1]$. It measures the benefit that an advertiser receives from each consumer, hereafter “the positive consumer effect”.
c	the marginal investment cost that the platform invests in value-added services, hereafter “the marginal investment cost”.

3.1. Consumers

There is a mass of 1 of consumers [21]. Consumers are distributed uniformly over a unit interval [22]. Two competing platforms are located at opposite ends of the unit interval, i.e., platform 1 is located at $x_1 = 0$, and platform 2 is located at $x_2 = 1$ [23,24]. Consumers located in $x \in [0, 1]$ can only receive content from one platform, i.e., consumers are single-homing. Consumers can obtain the basic utility v_i from the content provided by platform i ($i = 1, 2$). Following the literature [25,26], we assume that $v_1 = v_2 = v_0$. We also assume that v_0 is sufficiently large that all potential consumers join at least one platform, that is, we have full market coverage [27–29]. If the platform that consumers join can offer value-added services, then consumers can also receive additional utility. Specifically, in scenario A, consumers can only receive additional utility from the value-added service on platform 1, and the corresponding utility is $Q_{1A} \cdot 1$. Q_{1A} is the value-added service level of platform 1, and “1” is the additional utility that consumers derive from one unit of value-added services. In scenario S, consumers can receive additional utility from the value-added services on platform 1 or platform 2, with the corresponding additional utility being $Q_{1S} \cdot 1$ or $Q_{2S} \cdot 1$. Consumers are annoyed by the presence of ads [6,18,30]. The corresponding utility loss is aM_{ij} ($i = 1, 2$ and $j = A, S$). To join the platform, each consumer also incurs a transport cost of $t|x - x_i|$ [31–34]. Without loss of generality, let $t = 1$ [35]. Based on the above description, the total utility that a consumer derives from platform i can be expressed as $u_{ij} = v_0 + d_{ij}Q_{ij} - aM_{ij} - |x - x_i|$, $i = 1, 2$.

3.2. Advertisers

There is a mass of 1 of advertisers. We assume that each advertiser can place at most one ad on each media platform [27,36]. Advertisers incur the cost f for designing and producing an ad. As in previous studies [36], advertisers are assumed to be heterogeneous with respect to f , where f is uniformly distributed in a unit interval. Advertisers can benefit from consumers who join the same platform, and thus, the utility that advertisers obtain from platform i can be increased by rN_{ij} ($i = 1, 2$) [37,38]. If the platform that advertisers join can offer value-added services, then advertisers also gain additional utility on the

platform due to the value-added service. Specifically, in scenario A, the advertiser can obtain an additional utility of $Q_{1A} \cdot 1$ on platform 1, where Q_{1A} is the value-added service level set by platform 1 and “1” is the unit utility that an advertiser obtains on the platform due to the value-added service. In scenario S, advertisers can gain additional utility on platform 1 or platform 2, and the corresponding utility is $Q_{1S} \cdot 1$ or $Q_{2S} \cdot 1$, respectively. Each advertiser joining platform i needs to pay a lump-sum advertising fee P_{ij} to the platform. Based on the previous description, the total utility that each advertiser obtains from platform i can be expressed as $U_{ij} = rN_{ij} + d_{ij}Q_{ij} - P_{ij} - f$.

3.3. Media Platforms

There are two competing media platforms. The two platforms are located at the 0 and 1 ends of the unit interval [23]. They provide content for consumers and space for advertisers and generate their revenues purely from advertisers. The advertising revenue of platform i is $P_{ij}M_{ij}$, where P_{ij} is the ad price and M_{ij} is the advertiser demand. To attract more consumers and advertisers, platform i can choose to invest in two-sided beneficial value-added services that directly enhance the utilities of both consumers and advertisers. There are two investment scenarios. The first scenario is scenario A, an asymmetric investment scenario in which a single platform (i.e., platform 1) invests in value-added services. The second is scenario S, a symmetric investment scenario in which two platforms (i.e., platform 1 and platform 2) invest in value-added services. As in previous studies [7,8], we assume that platforms investing in value-added services incur costs. Thus, in scenario A, platform 1 has an investment cost of $cQ_{1A}^2/2$ [39]. Similarly, in scenario S, platform 1 has an investment cost of $cQ_{1S}^2/2$, and platform 2 has an investment cost of $cQ_{2S}^2/2$. Based on the above description, the profit function of platform i can be expressed as $\pi_{ij} = P_{ij}M_{ij} - cd_{ij}Q_{ij}^2/2$. The former and latter terms of the function are the advertising revenue and investment cost, respectively.

3.4. The Three-Stage Game

Our model can be represented by the following three-stage game:

Stage 1: If platform i invests in two-sided beneficial value-added services, then it determines both the value-added service level and the ad price. Otherwise, if platform i does not invest in value-added services, it determines the ad price only. Specifically, in scenario A, platform 1 determines the value-added service level, Q_{1A} , as well as the ad price, P_{1A} , and platform 2 only determines the ad price, P_{2A} . In scenario S, platform 1 determines the value-added service level, Q_{1S} , and the ad price, P_{1S} , and platform 2 determines the value-added service level, Q_{2S} , and the ad price, P_{2S} .

Stage 2: Advertisers decide whether to advertise on each platform.

Stage 3: Consumers decide from which platform to obtain their content.

4. Equilibrium Analysis

4.1. Scenario A

In scenario A, platform 1 provides basic and value-added services for its users, while platform 2 provides only basic services for its users. According to the assumptions in Section 3, the consumer utilities for platform 1 and platform 2 can be represented as $u_{1A} = v_0 + Q_{1A} - aM_{1A} - x$, $u_{2A} = v_0 - aM_{2A} - (1 - x)$; the advertiser utilities for platform 1 and platform 2 can be represented as $U_{1A} = rN_{1A} + Q_{1A} - P_{1A} - f$, $U_{2A} = rN_{2A} - P_{2A} - f$; and the profit functions of the two platforms can be expressed as $\pi_{1A} = P_{1A}M_{1A} - cQ_{1A}^2/2$, $\pi_{2A} = P_{2A}M_{2A}$.

If a consumer obtaining content from platforms 1 and 2 receives equal utilities, i.e., $u_{1A} = u_{2A}$, then the consumer will choose to source content from either platform. Let $u_{1A}(x) = u_{2A}(x)$, and we find $x = -(M_{1A}a - Q_{1A} - M_{2A}a - 1)/2$. If $x \leq x$, i.e., $u_{1A}(x) \geq u_{2A}(x)$, then the consumer prefers to obtain content from platform 1 rather than from platform 2. If $x > x$, the consumer prefers to obtain content from platform 2 than

from platform 1. Thus, the consumer demands of platform 1 and platform 2, N_{1A} and N_{2A} , are given by

$$\begin{cases} N_{1A} = \frac{-(M_{1A}a - Q_{1A} - M_{2A}a - 1)}{2}, \\ N_{2A} = \frac{M_{1A}a - Q_{1A} - M_{2A}a + 1}{2}. \end{cases} \quad (1)$$

An advertiser will advertise on platform 1 only if the utility he obtains from the platform is nonnegative, i.e., if $U_{1A} = rN_{1A} + Q_{1A} - P_{1A} - f \geq 0$. Let $U_{1A}(f_{1A}) = 0$, and we find that $f_{1A} = rN_{1A} + Q_{1A} - P_{1A}$. If $f \leq f_{1A}$, then the advertiser will choose to advertise on platform 1; otherwise, the advertiser will choose not to advertise on platform 1. Similarly, if $f \leq f_{2A}$, where $f_{2A} = rN_{2A} - P_{2A}$, the advertiser will choose to advertise on platform 2; otherwise, the advertiser will choose not to advertise on platform 2. Hence, we can obtain that the advertiser demands of the two platforms, M_{1A} and M_{2A} , are given by

$$\begin{cases} M_{1A} = f_{1A} = rN_{1A} + Q_{1A} - P_{1A}, \\ M_{2A} = f_{2A} = rN_{2A} - P_{2A}. \end{cases} \quad (2)$$

Substituting N_{1A} and N_{2A} from Equation (1) into Equation (2), M_{1A} and M_{2A} are rewritten as

$$\begin{cases} M_{1A} = \frac{-(2P_{1A} - r - 2Q_{1A} - rQ_{1A} + M_{1A}ar - M_{2A}ar)}{2}, \\ M_{2A} = \frac{r - 2P_{2A} - rQ_{1A} + M_{1A}ar - M_{2A}ar}{2}. \end{cases} \quad (3)$$

Solving for M_{1A} and M_{2A} in Equation (3), we obtain that M_{1A} and M_{2A} are expressed with respect to P_{1A} , Q_{1A} and P_{2A} as

$$\begin{cases} M_{1A} = \frac{r - 2P_{1A} + 2Q_{1A} + rQ_{1A} + ar^2 - aP_{1A}r - aP_{2A}r + arQ_{1A}}{2(ar+1)}, \\ M_{2A} = \frac{-(2P_{2A} - r + rQ_{1A} - ar^2 + aP_{1A}r + aP_{2A}r - arQ_{1A})}{2(ar+1)}. \end{cases} \quad (4)$$

Substituting M_{1A} and M_{2A} from Equation (4) into Equation (1), we obtain that N_{1A} and N_{2A} are expressed with respect to Q_{1A} , P_{1A} and P_{2A} as

$$\begin{cases} N_{1A} = \frac{Q_{1A} + aP_{1A} - aP_{2A} + ar - aQ_{1A} + 1}{2(ar+1)}, \\ N_{2A} = \frac{-Q_{1A} - aP_{1A} + aP_{2A} + ar + aQ_{1A} + 1}{2(ar+1)}. \end{cases} \quad (5)$$

The decision problems for platform 1 and platform 2 can be expressed as follows:

$$\begin{cases} \max \pi_{1A}(P_{1A}, Q_{1A}) = \frac{P_{1A}(r - 2P_{1A} + 2Q_{1A} + rQ_{1A} + ar^2 - aP_{1A}r - aP_{2A}r + arQ_{1A})}{2(ar+1)} - \frac{cQ_{1A}^2}{2}, \\ \max \pi_{2A}(P_{2A}) = \frac{-P_{2A}(2P_{2A} - r + rQ_{1A} - ar^2 + aP_{1A}r + aP_{2A}r - arQ_{1A})}{2(ar+1)}. \end{cases} \quad (6)$$

By Equation (6), we can obtain the equilibrium value-added service level and ad price of platform 1, Q_{1A}^* and P_{1A}^* , and the equilibrium ad price of platform 2, P_{2A}^* . Substituting Q_{1A}^* , P_{1A}^* and P_{2A}^* into Equations (4)–(6) yields the equilibrium advertiser demand, consumer demand and profit of platform 1, M_{1A}^* , N_{1A}^* and π_{1A}^* , and the equilibrium advertiser demand, consumer demand and profit of platform 2, M_{2A}^* , N_{2A}^* and π_{2A}^* . The above equilibrium outcomes are summarized in Proposition 1.

Proposition 1. In scenario A, the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform i ($i = 1, 2$) are as follows:

$$(a) \text{ If } 0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)},$$

$$Q_{1A}^* = 1, P_{1A}^* = \frac{(r+1)(a^2r^2+8ar+8)}{(ar+4)(3ar+4)}, P_{2A}^* = \frac{ar(ar+2)(r+1)}{(ar+4)(3ar+4)},$$

$$\begin{aligned}
N_{1A}^* &= \frac{a^2r^2 - a^2r + 8ar - 2a + 8}{2(ar+1)(ar+4)}, N_{2A}^* = \frac{a(r+1)(ar+2)}{2(ar+1)(ar+4)}, \\
M_{1A}^* &= \frac{(ar+2)(r+1)(a^2r^2 + 8ar + 8)}{2(ar+1)(ar+4)(3ar+4)}, M_{2A}^* = \frac{ar(ar+2)^2(r+1)}{2(ar+1)(ar+4)(3ar+4)}, \\
\pi_{1A}^* &= \frac{(r+1)^2(a^2r^2 + 8ar + 8)(a^3r^3 + 10a^2r^2 + 24ar + 16)}{(ar+4)(2ar+2)(3ar+4)(3a^2r^2 + 16ar + 16)} - \frac{c}{2}, \pi_{2A}^* = \frac{a^2r^2(ar+2)^3(r+1)^2}{2(ar+1)(3a^2r^2 + 16ar + 16)^2}. \\
(b) \text{ If } &\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1, \\
Q_{1A}^* &= \frac{(r+ar+2)r(ar+1)(ar+4)}{H}, P_{1A}^* = \frac{2cr(ar+1)^2(ar+4)}{H}, \\
P_{2A}^* &= \frac{2r(ar+1)(4c-3r-ar-ar^2-r^2+a^2cr^2+5acr-2)}{H}, \\
N_{1A}^* &= \frac{(ar+1)(ar+4)(4c-r-ar+3acr-2)}{H}, N_{2A}^* = \frac{(3ar+4)(4c-3r-ar-ar^2-r^2+a^2cr^2+5acr-2)}{H}, \\
M_{1A}^* &= \frac{ca^3r^4 + 7ca^2r^3 + 14car^2 + 8cr}{H}, M_{2A}^* = \frac{r(ar+2)(4c-3r-ar-ar^2-r^2+a^2cr^2+5acr-2)}{H}, \\
\pi_{1A}^* &= \frac{cr^2(a^2r^2 + 5ar + 4)^2(8c-4r-4ar-2ar^2-r^2-a^2r^2+4a^2cr^2+12acr-4)}{2H^2}, \\
\pi_{2A}^* &= \frac{2r^2(ar+1)(ar+2)(3r-4c+ar+ar^2+r^2-a^2cr^2-5acr+2)^2}{H^2}.
\end{aligned}$$

where $H = (6a^3c - 3a - 4a^2 - a^3)r^3 + (38a^2c - 18a - 10a^2 - 4)r^2 + (64ac - 24a - 16)r + 32c - 16$.

Proof. See Appendix A.

Based on Proposition 1, the impacts of marginal investment costs c on the equilibrium outcomes in scenario A are analyzed, and then Corollary 1 is obtained.

Corollary 1. In scenario A, the impacts of marginal investment costs c on the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform 1 are as follows:

$$\begin{aligned}
(a) \text{ If } 0 < c &\leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}, \\
\frac{\partial Q_{1A}^*}{\partial c} &= 0, \frac{\partial P_{1A}^*}{\partial c} = 0, \frac{\partial N_{1A}^*}{\partial c} = 0, \frac{\partial M_{1A}^*}{\partial c} = 0, \frac{\partial \pi_{1A}^*}{\partial c} < 0. \\
(b) \text{ If } &\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1, \\
\frac{\partial Q_{1A}^*}{\partial c} &< 0, \frac{\partial P_{1A}^*}{\partial c} < 0, \frac{\partial N_{1A}^*}{\partial c} < 0, \frac{\partial M_{1A}^*}{\partial c} < 0, \frac{\partial \pi_{1A}^*}{\partial c} < 0.
\end{aligned}$$

Proof. See Appendix A.

When $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$, platform 1 investing in value-added services that can directly benefit consumers and advertisers faces low costs, and therefore, it is optimal for the platform to maintain maximum investment in value-added services, that is, $Q_{1A}^* = 1$. With constant investment, consumers can derive an invariant utility, and thus their demand for the platform remains constant, which implies that the cross-network utility that advertisers receive from consumers remains the same. In addition to cross-network utility, advertisers can also receive a constant benefit directly from the value-added service investment. Due to the same cross-network utility and benefit, advertiser utility and demand for platform 1 remain constant, and naturally, platform 1 does not need to change its ad price. Clearly, the ad revenue of the platform remains unchanged. However, the total investment cost of platform 1 increases with the marginal investment cost, and thus, the profit of the platform decreases with the marginal investment cost.

When $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$, a platform investing in value-added services that can directly benefit consumers and advertisers faces higher costs, and thus, the platform reduces the value-added service level as the marginal investment cost increases. The reduction in the value-added service investment decreases consumer utility and demand for the platform. Furthermore, this decreases the cross-network utility of advertisers from consumers due to the impact of positive consumer effects. Note that the reduction in the value-added service investment can also directly adversely affect the experience that advertisers receive from the platform, which reduces advertiser utility and demand for platform 1. To prevent a further decrease in advertiser demand, platform 1 lowers its ad price.

According to Proposition 1, we analyze the impacts of positive consumer effects r on the equilibrium outcomes in scenario A and obtain Corollary 2.

Corollary 2. *In scenario A, the impacts of positive consumer effects r on the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform 1 are as follows:*

$$(a) \text{ If } 0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)},$$

$$\frac{\partial Q_{1A}^*}{\partial r} = 0, \frac{\partial P_{1A}^*}{\partial r} > 0, \frac{\partial N_{1A}^*}{\partial r} < 0, \frac{\partial M_{1A}^*}{\partial r} > 0, \frac{\partial \pi_{1A}^*}{\partial r} > 0.$$

$$(b) \text{ If } \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1,$$

$$\frac{\partial Q_{1A}^*}{\partial r} > 0, \frac{\partial P_{1A}^*}{\partial r} > 0, \frac{\partial N_{1A}^*}{\partial r} > 0, \frac{\partial M_{1A}^*}{\partial r} > 0, \frac{\partial \pi_{1A}^*}{\partial r} > 0.$$

Proof. See Appendix A.

Corollary 2a states that if $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$, the value-added service level of platform 1 does not change with the positive consumer effect. The corollary also indicates that if the marginal investment cost is low, the advertiser demand and ad price of platform 1 increase with the positive consumer effect, and the consumer demand of the platform decreases with the positive consumer effect. The intuition behind this result is as follows: For a given consumer demand, as the positive consumer effect increases, advertisers can gain more cross-network utility on platform 1. Thus, advertiser demand for platform 1 is improved, and the platform can charge advertisers more. However, the increased advertiser demand for platform 1 in turn creates more disruption for consumers due to the negative advertising effect, leading to some consumer utility loss. Thus, the consumer demand for platform 1 decreases.

Corollary 2b shows that if $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$, the value-added service level of platform 1 increases with the positive consumer effect. The corollary also indicates that the advertiser demand and ad price of platform 1 increase with the positive consumer effect. To understand the intuition behind this result, first, we analyze the threefold impact of the positive consumer effect on advertiser utility. First, the increasing positive consumer effect improves the value-added service level of platform 1; as a result, consumer utility and demand for the platform can be improved accordingly. Indirectly, the utility that advertisers derive from the platform can also be enhanced. Second, for given consumer demand, as the positive consumer effect increases, advertisers can obtain more cross-network utility from consumers on the platform due to the positive consumer effect. Third, with the improved value-added service due to a higher positive consumer effect, advertisers can gain more direct utility on the platform. Obviously, the above three impacts significantly enhance advertiser utility for platform 1; thus, advertiser demand for the platform can be enlarged, and the platform can charge advertisers more.

Following Proposition 1, we also analyze the impacts of negative advertising effects a on the equilibrium outcomes and obtain Corollary 3.

Corollary 3. *In scenario A, the impacts of negative advertising effects a on the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform 1 are as follows:*

$$(a) \text{ If } 0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)},$$

$$\frac{\partial Q_{1A}^*}{\partial a} = 0, \frac{\partial P_{1A}^*}{\partial a} < 0, \frac{\partial N_{1A}^*}{\partial a} < 0, \frac{\partial M_{1A}^*}{\partial a} < 0, \frac{\partial \pi_{1A}^*}{\partial a} < 0.$$

$$(b) \text{ If } \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1,$$

$$\frac{\partial Q_{1A}^*}{\partial a} < 0, \frac{\partial P_{1A}^*}{\partial a} < 0, \frac{\partial N_{1A}^*}{\partial a} < 0, \frac{\partial M_{1A}^*}{\partial a} < 0, \frac{\partial \pi_{1A}^*}{\partial a} < 0.$$

Proof. See Appendix A.

Corollary 3a shows that if the marginal investment cost is low, i.e., $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$, the value-added service level of platform 1 is independent of the negative advertising effect. This result implies that under low marginal investment costs when the disutility that a consumer suffers from each advertiser increases, platform 1 does not need to adjust its value-added service level accordingly. Corollary 3a also shows that the ad price, consumer demand and advertiser demand and profit of platform 1 decrease with the negative advertising effect.

Corollary 3b reveals that if the marginal investment cost is high, i.e., $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$, the value-added service level of platform 1 decreases with the negative advertising effect. The result indicates that under high marginal investment costs when the disutility that a consumer suffers from each advertiser increases, platform 1 should reduce its value-added service level. Corollary 3b also states that the consumer demand, advertiser demand, ad price and profit of platform 1 decrease with the negative advertising effect. To understand the intuition behind this finding, we first analyze how the reduced value-added service due to an increase in the negative advertising effect influences the utilities that consumers and advertisers derive from platform 1. Consumers receive less benefit from the lowered value-added service, thus reducing their utility and demand for platform 1. For advertisers, the impact of the lowered value-added service on their demand for platform 1 is twofold. First, with the reduced consumer demand due to the lowered value-added service, advertisers obtain less cross-network utility from consumers on the platform. Second, advertisers receive less direct benefit from the reduced value-added

service on the platform. Clearly, the reduced value-added service can significantly diminish advertisers' demand and willingness to pay for platform 1, which further decreases the profit of the platform.

4.2. Scenario S

In scenario S, the two platforms offer basic and value-added services to their users. Based on the assumptions in Section 3, the consumer utilities for platform 1 and platform 2 can be expressed as $u_{1S} = v_0 + Q_{1S} - aM_{1S} - x$, $u_{2S} = v_0 + Q_{2S} - aM_{2S} - (1 - x)$; the advertiser utilities for platform 1 and platform 2 can be represented as $U_{1S} = rN_{1S} + Q_{1S} - P_{1S} - f$, $U_{2S} = rN_{2S} + Q_{2S} - P_{2S} - f$; and the profit functions of the two platforms can be expressed as $\pi_{1S} = P_{1S}M_{1S} - cQ_{1S}^2/2$, $\pi_{2S} = P_{2S}M_{2S} - cQ_{2S}^2/2$.

Let $u_{1S} = u_{2S}$; we find that the marginal consumer who is indifferent between obtaining content from platform 1 and platform 2 is located at $x = -(Q_{2S} - Q_{1S} + M_{1S}a - M_{2S}a - 1)/2$. Hence, the consumer demands of platform 1 and platform 2, N_{1S} and N_{2S} , respectively, are given by

$$\begin{cases} N_{1S} = \frac{-(Q_{2S} - Q_{1S} + M_{1S}a - M_{2S}a - 1)}{2}, \\ N_{2S} = \frac{Q_{2S} - Q_{1S} + M_{1S}a - M_{2S}a + 1}{2}. \end{cases} \quad (7)$$

Let $U_{1S}(f_{1S}) = 0$; we find that the marginal advertiser who is indifferent between advertising on platform 1 and not is characterized by $f_{1S} = rN_{1S} + Q_{1S} - P_{1S}$. Similarly, we also find that the marginal advertiser who is indifferent between advertising on platform 2 and not is given by $f_{2S} = rN_{2S} + Q_{2S} - P_{2S}$. Thus, we can obtain the advertiser demands of platform 1 and platform 2 as $M_{1S} = f_{1S} = rN_{1S} + Q_{1S} - P_{1S}$ and $M_{2S} = f_{2S} = rN_{2S} + Q_{2S} - P_{2S}$, respectively. Substituting N_{1T} and N_{2T} from Equation (7) into $M_{1S} = rN_{1S} + Q_{1S} - P_{1S}$ and $M_{2S} = rN_{2S} + Q_{2S} - P_{2S}$, we can rewrite M_{1S} and M_{2S} as

$$\begin{cases} M_{1S} = \frac{-(2P_{1S} - r - 2Q_{1S} - rQ_{1S} + rQ_{2S} + M_{1S}ar - M_{2S}ar)}{2}, \\ M_{2S} = \frac{r - 2P_{2S} + 2Q_{2S} - rQ_{1S} + rQ_{2S} + M_{1S}ar - M_{2S}ar}{2}. \end{cases} \quad (8)$$

Solving for M_{1S} and M_{2S} in Equation (8), we obtain that M_{1S} and M_{2S} are expressed with respect to P_{1S} , Q_{1S} and P_{2S} as

$$\begin{cases} M_{1S} = \frac{r - 2P_{1S} + 2Q_{1S} + rQ_{1S} - rQ_{2S} + ar^2 - aP_{1S}r - aP_{2S}r + arQ_{1S} + arQ_{2S}}{2(ar+1)}, \\ M_{2S} = \frac{r - 2P_{2S} + 2Q_{2S} - rQ_{1S} + rQ_{2S} + ar^2 - aP_{1S}r - aP_{2S}r + arQ_{1S} + arQ_{2S}}{2(ar+1)}. \end{cases} \quad (9)$$

Substituting M_{1S} and M_{2S} in Equation (9) into Equation (7), we obtain N_{1S} and N_{2S} with respect to P_{1S} , P_{2S} , Q_{1S} and Q_{2S} as

$$\begin{cases} N_{1S} = \frac{Q_{1S} - Q_{2S} + aP_{1S} - aP_{2S} + ar - aQ_{1S} + aQ_{2S} + 1}{2(ar+1)}, \\ N_{2S} = \frac{-(Q_{1S} - Q_{2S} + aP_{1S} - aP_{2S} - ar - aQ_{1S} + aQ_{2S} - 1)}{2(ar+1)}. \end{cases} \quad (10)$$

The profit maximization problems for platform 1 and platform 2 are formulated as

$$\begin{cases} \max \pi_{1S}(P_{1S}, Q_{1S}) = \frac{P_{1S}(r - 2P_{1S} + 2Q_{1S} + rQ_{1S} - rQ_{2S} + ar^2 - aP_{1S}r - aP_{2S}r + arQ_{1S} + arQ_{2S})}{2(ar+1)} - \frac{cQ_{1S}^2}{2}, \\ \max \pi_{2S}(P_{2S}, Q_{2S}) = \frac{P_{2S}(r - 2P_{2S} + 2Q_{2S} - rQ_{1S} + rQ_{2S} + ar^2 - aP_{1S}r - aP_{2S}r + arQ_{1S} + arQ_{2S})}{2(ar+1)} - \frac{cQ_{2S}^2}{2}. \end{cases} \quad (11)$$

Solving Equation (11) yields the equilibrium value-added service levels, Q_{1S}^* and Q_{2S}^* , and ad prices, P_{1S}^* and P_{2S}^* . Furthermore, substituting Q_{1S}^* , Q_{2S}^* , P_{1S}^* and P_{2S}^* into Equations (9)–(11) yields the equilibrium advertiser demands, M_{1T}^* and M_{2T}^* , consumer demands, N_{1S}^* and N_{2S}^* , and profits π_{1S}^* and π_{2S}^* . The above equilibrium outcomes are shown in Proposition 2.

Proposition 2. In scenario S, the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform i ($i = 1, 2$) are as follows:

(a) If $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$,

$$Q_{iS}^* = 1, P_{iS}^* = \frac{(ar+1)(r+2)}{3ar+4},$$

$$N_{iS}^* = \frac{1}{2}, M_{iS}^* = \frac{(r+2)(ar+2)}{2(3ar+4)},$$

$$\pi_{iS}^* = \frac{(8r-16c+12ar+12ar^2+3ar^3+2r^2+4a^2r^2+4a^2r^3+a^2r^4-9a^2cr^2-24acr+8)}{2(3ar+4)^2}.$$

(b) If $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$,

$$Q_{iS}^* = \frac{r(r+ar+2)}{2(4c-r-ar+3acr-2)}, P_{iS}^* = \frac{r(c+acr)}{4c-r-ar+3acr-2},$$

$$N_{iS}^* = \frac{1}{2}, M_{iS}^* = \frac{cr(ar+2)}{2(4c-r-ar+3acr-2)},$$

$$\pi_{iS}^* = \frac{cr^2(8c-4r-4ar-2ar^2-r^2-a^2r^2+4a^2cr^2+12acr-4)}{8(r-4c+ar-3acr+2)^2}.$$

Proof. See Appendix A.

Based on Proposition 2, the impacts of marginal investment costs c on the equilibrium outcomes of the two platforms in scenario S are analyzed to obtain Corollary 4.

Corollary 4. In scenario S, the impacts of marginal investment costs c on the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform i ($i = 1, 2$) are shown in Table 2.

Table 2. Impacts of marginal investment costs on equilibrium outcomes in scenario S.

Cross-Network Effects	Marginal Investment Costs	Impacts of Marginal Investment Costs
$a < \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$	$0 < c \leq c_1$	$\frac{\partial Q_{iS}^*}{\partial c} = 0, \frac{\partial P_{iS}^*}{\partial c} = 0, \frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} = 0, \frac{\partial \pi_{iS}^*}{\partial c} < 0.$
	$c_1 < c \leq c_2$	$\frac{\partial Q_{iS}^*}{\partial c} < 0, \frac{\partial P_{iS}^*}{\partial c} < 0, \frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} < 0, \frac{\partial \pi_{iS}^*}{\partial c} \geq 0.$
	$c_2 < c \leq 1$	$\frac{\partial Q_{iS}^*}{\partial c} < 0, \frac{\partial P_{iS}^*}{\partial c} < 0, \frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} < 0, \frac{\partial \pi_{iS}^*}{\partial c} < 0.$
$a \geq \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$	$0 < c \leq c_1$	$\frac{\partial Q_{iS}^*}{\partial c} = 0, \frac{\partial P_{iS}^*}{\partial c} = 0, \frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} = 0, \frac{\partial \pi_{iS}^*}{\partial c} < 0.$
	$c_1 < c \leq 1$	$\frac{\partial Q_{iS}^*}{\partial c} < 0, \frac{\partial P_{iS}^*}{\partial c} < 0, \frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} < 0, \frac{\partial \pi_{iS}^*}{\partial c} < 0.$

* where $c_1 = \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$, $c_2 = \frac{(r+ar+2)^2}{5a^2r^2-3ar^2+14ar-4r+8}$.

Proof. See Appendix A.

We observe from Corollary 4 that in conditions of $a < \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$ and $a \geq \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$, the value-added service level and ad price of platform i , $i = 1, 2$, remain constant and then decrease with the marginal investment cost.

From Corollary 4, we can also see that if $a < \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$, the profit of platform i varies in three stages with the marginal investment cost; that is, as the marginal investment cost increases, the profit first decreases, then increases, and finally

decreases. If $a \geq \frac{\sqrt{9r^4+68r^3+180r^2+160r+64}+3r^2-2r-8}{2r(5r+4)}$, the profit of platform i always decreases with the marginal investment cost.

Following Proposition 2, the impacts of positive consumer effects r on the equilibrium outcomes are analyzed, and Corollary 5 is obtained.

Corollary 5. *In scenario S, the impacts of positive consumer effects r on the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform i ($i = 1, 2$) are as follows:*

$$(a) \text{ If } 0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)},$$

$$\frac{\partial Q_{iS}^*}{\partial r} = 0; \frac{\partial P_{iS}^*}{\partial r} > 0; \frac{\partial N_{iS}^*}{\partial r} = 0; \frac{\partial M_{iS}^*}{\partial r} > 0; \frac{\partial \pi_{iS}^*}{\partial r} > 0.$$

$$(b) \text{ If } \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1,$$

$$\frac{\partial Q_{iS}^*}{\partial r} > 0; \frac{\partial P_{iS}^*}{\partial r} > 0; \frac{\partial N_{iS}^*}{\partial r} = 0; \frac{\partial M_{iS}^*}{\partial r} > 0;$$

$$\text{when } 0 < r < r^*, \frac{\partial \pi_{iS}^*}{\partial r} > 0, \text{ and when } r^* \leq r \leq 1, \frac{\partial \pi_{iS}^*}{\partial r} \leq 0,$$

where $f(r^*) = L_1 r^{*4} + L_2 r^{*3} + L_3 r^{*2} + L_4 r^* = 0$, $L_1 = -c(a - 3ac + 1)(2a - 4a^2c + a^2 + 1)$, $L_2 = -c(50a^2c^2 - 36a^2c + 6a^2 - 28ac + 12a - 8c + 6)$, $L_3 = 12c(2c - 1)(a - 3ac + 1)$, and $L_4 = -8c(2c - 1)^2$.

Proof. See Appendix A.

Corollary 5a shows that if the marginal investment cost is low, i.e., $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$, the optimal value-added service level and the consumer demand of platform i are not affected by the positive consumer effect, and the ad price of platform i increases with the positive consumer effect.

Corollary 5b states that if $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$, the value-added service level of platform i increases with the positive consumer effect. The intuition for this result is as follows. As the positive consumer effect increases, each consumer can bring more cross-network utility for the advertisers joining platform i , and thus, advertiser demand for the platform is expanded, and the platform can generate more revenue from advertisers. In other words, the higher the positive consumer effect is, the more revenue each viewer can indirectly contribute to the platform, which implies that consumers are more important to the platform. Therefore, platform i has a strong incentive to attract consumers by investing more in value-added services. Corollary 5b also suggests that the ad price of platform i increases with the positive consumer effect. To obtain an intuition for this finding, first, note the twofold impacts of the positive consumer effect on the utility advertisers receive from platform i . First, as the positive consumer effect increases, advertisers can gain more cross-network utility from the consumers on platform i , and thus, they are willing to pay for the platform. Second, advertisers can obtain more direct benefits from the value-added service that is enhanced with the increase in the positive consumer effect, and therefore, advertisers are willing to pay the platform more. Obviously, with the twofold impacts, platform i can charge a higher ad price when the positive consumer effect increases.

According to Proposition 2, we analyze the impacts of negative advertising effects a on the equilibrium outcomes, which leads to Corollary 6.

Corollary 6. *In scenario S, the impacts of negative advertising effects a on the equilibrium value-added service level, ad price, consumer demand, advertiser demand and profit of platform i ($i = 1, 2$) are as follows:*

$$(a) \text{ If } 0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)},$$

$$\frac{\partial Q_{iS}^*}{\partial a} = 0, \frac{\partial P_{iS}^*}{\partial a} > 0, \frac{\partial N_{iS}^*}{\partial a} = 0, \frac{\partial M_{iS}^*}{\partial a} < 0, \frac{\partial \pi_{iS}^*}{\partial a} < 0.$$

$$(b) \text{ If } \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1,$$

$$\frac{\partial Q_{iS}^*}{\partial a} < 0, \frac{\partial P_{iS}^*}{\partial a} < 0, \frac{\partial N_{iS}^*}{\partial a} = 0, \frac{\partial M_{iS}^*}{\partial a} < 0, \frac{\partial \pi_{iS}^*}{\partial a} > 0.$$

Proof. See Appendix A.

Corollary 6a shows that if $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$, the value-added service level and the consumer demand of platform i , $i = 1, 2$, do not change with the negative advertising effect, the ad price of the platform increases with the effect, and the advertiser demand and profit of the platform decreases with the effect. Corollary 6b shows that if $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$, the value-added service level, ad price and advertiser demand of platform i decrease with the negative advertising effect; the consumer demand of platform i do not change with the effect, and the profit of platform i increase with the effect.

5. Comparison of Equilibrium Outcomes

Based on the equilibrium outcomes in Sections 4.1 and 4.2, we compare the differences in equilibrium outcomes between scenario A and scenario S and obtain Corollary 7.

Corollary 7. *The differences in value-added service level, ad price, consumer demand, advertiser demand and profit of platform 1 in scenarios A and S are as follows:*

$$(a) \text{ If } 0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)},$$

$$Q_{1A}^* = Q_{1S}^*, P_{1A}^* > P_{1S}^*, N_{1A}^* > N_{1S}^*, M_{1A}^* > M_{1S}^*, \pi_{1A}^* > \pi_{1S}^*.$$

$$(b) \text{ If } \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1,$$

$$Q_{1A}^* > Q_{1S}^*, P_{1A}^* > P_{1S}^*, N_{1A}^* > N_{1S}^*, M_{1A}^* > M_{1S}^*, \pi_{1A}^* > \pi_{1S}^*.$$

Proof. See Appendix A.

Corollary 7 indicates that if $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$, the value-added service levels of platform 1 in scenario A and scenario S are equal; if $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$, the value-added service level of the platform is higher in scenario A than in scenario S.

Corollary 7 also suggests that compared to scenario S, the ad price of platform 1 is higher in scenario A. The difference in the ad price of platform 1 between the two scenarios can be understood as follows. In scenario A, advertisers can derive more direct benefits and cross-network utility from the higher value-added service level exclusively invested in by platform 1, and thus, a higher ad price can be charged. In scenario S, advertisers only obtain less direct benefit and cross-network utility from the lower value-added service level non-exclusively invested in by platform 1, and thus, a lower price can be charged to advertisers.

Following Corollary 7, we further discuss the impacts of marginal investment costs c on the differences in the value-added service level and ad price of platform 1 (investment platform) between scenarios A and S and obtain Corollary 8.

Corollary 8. *The impacts of marginal investment costs c on the differences in value-added service level and ad price of platform 1 between scenarios A and S are as follows:*

- (a) If $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$, $\frac{\partial(Q_{1A}^*-Q_{1S}^*)}{\partial c} = 0$, $\frac{\partial(P_{1A}^*-P_{1S}^*)}{\partial c} = 0$.
- (b) If $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$, $\frac{\partial(Q_{1A}^*-Q_{1S}^*)}{\partial c} > 0$, $\frac{\partial(P_{1A}^*-P_{1S}^*)}{\partial c} > 0$.
- (c) If $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$, $\frac{\partial(Q_{1A}^*-Q_{1S}^*)}{\partial c} < 0$, $\frac{\partial(P_{1A}^*-P_{1S}^*)}{\partial c} < 0$.

Proof. See Appendix A.

Corollary 8 shows that the differences in the value-added service level and ad price of platform 1 between the two scenarios do not change with the marginal investment cost when $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$, increase with the marginal investment cost when $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$, and decrease with the marginal investment cost when $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$.

6. Numerical Simulation

To further verify the conclusions obtained in the previous sections and analyze how the key parameters affect the equilibrium outcomes (such as value-added service levels, ad prices and profits) under different scenarios, this section conducts numerical simulations by using MATLAB [40,41]. Section 6.1 analyzes the impacts of key parameters—marginal investment costs, positive consumer effects and negative advertising effects—on the equilibrium outcomes of the investment platform under scenario A. Similarly, Section 6.2 analyzes the corresponding impacts under scenario S.

6.1. Scenario A

(1) Impacts of marginal investment costs

First, we performed a simulation to analyze the impact of marginal investment costs c on the equilibrium outcomes of the investment platform (i.e., platform 1) in scenario A. The parameters are set as $r = 0.60$, $a = 0.70$. The simulation results are shown in Figure 2.

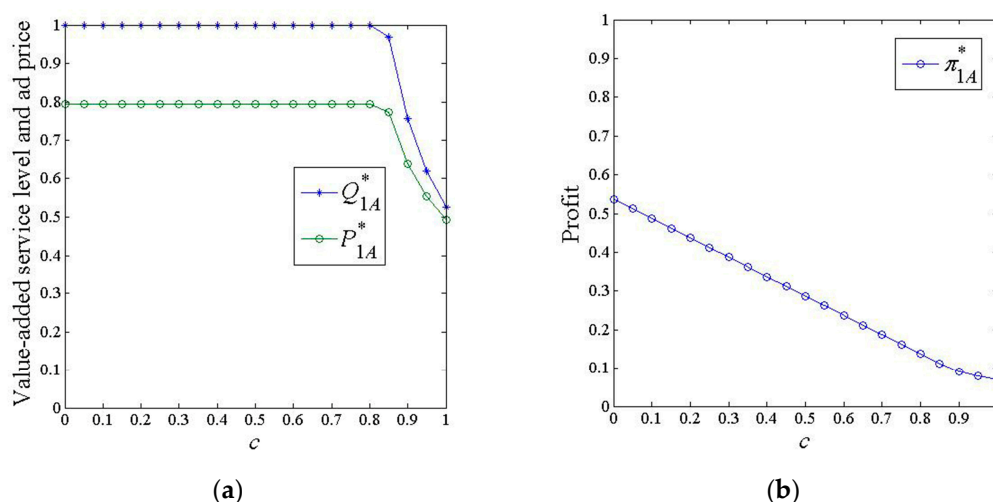


Figure 2. Impacts of marginal investment costs c in scenario A: (a) impact of c on value-added service level and ad price; (b) impact of c on profit.

Figure 2 shows the impact of the marginal investment cost on the equilibrium outcomes of platform 1 in scenario A. As Figure 2 illustrates, when the marginal investment cost is less than 0.84, the value-added service level and ad price of platform 1 do not change with the marginal investment cost, while the profit of the platform decreases with the marginal investment cost. When the marginal investment cost is greater than 0.84, the

value-added service level, ad price and profit of platform 1 all decrease with the marginal investment cost.

(2) Impacts of positive consumer effects

Second, we simulate the impact of positive consumer effects r on the equilibrium outcomes of the investment platform in scenario A. According to Corollary 2, we set $c = 0.30$ and $a = 0.80$ for the case of $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$ (case 1) and $c = 0.80$ and $a = 0.80$ for the case of $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$ (case 2). The simulation results are shown in Figure 3.

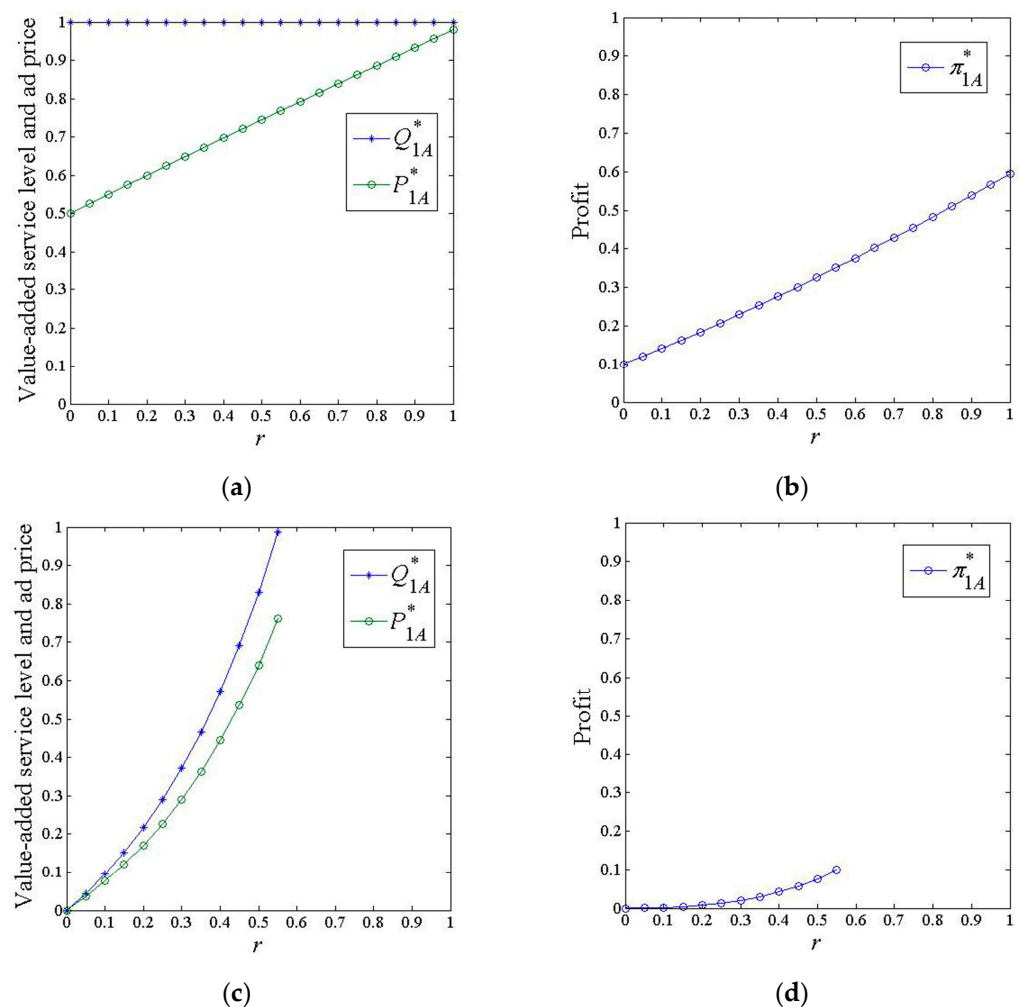


Figure 3. Impacts of positive consumer effects r in scenario A: (a) impact of r on value-added service level and ad price for case 1; (b) impact of r on profit for case 1, (c) impact of r on value-added service level and ad price for case 2; (d) impact of r on profit for case 2. * case 1: $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$; case 2: $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$.

Figure 3 illustrates the impact of the positive consumer effect on the equilibrium outcomes of platform 1 in scenario A. Comparing Figure 3a,c, we find that as the positive consumer effect r increases, the ad price of platform 1 increases, and the value-added service level of the platform either does not change or increases, depending on the marginal investment cost c . Specifically, for case 1, the value-added service level of platform 1 does not change with the positive consumer effect; for case 2, the value-added service level of the platform increases with the positive consumer effect. From Figure 3b,d, we can see

that the profit of platform 1 increases with the positive consumer effect for both case 1 and case 2.

(3) Impacts of negative advertising effects

Let $c = 0.30$, $r = 0.40$ (applied to Figure 4a,b) for the case of $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$ (case 1); $c = 0.90$, $r = 0.40$ (applied to Figure 4c,d) for the case of $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$ (case 2). Then, we draw the impact of negative advertising effects a on the equilibrium outcomes of the investment platform, i.e., platform 1, under scenario A, as shown in Figure 4.

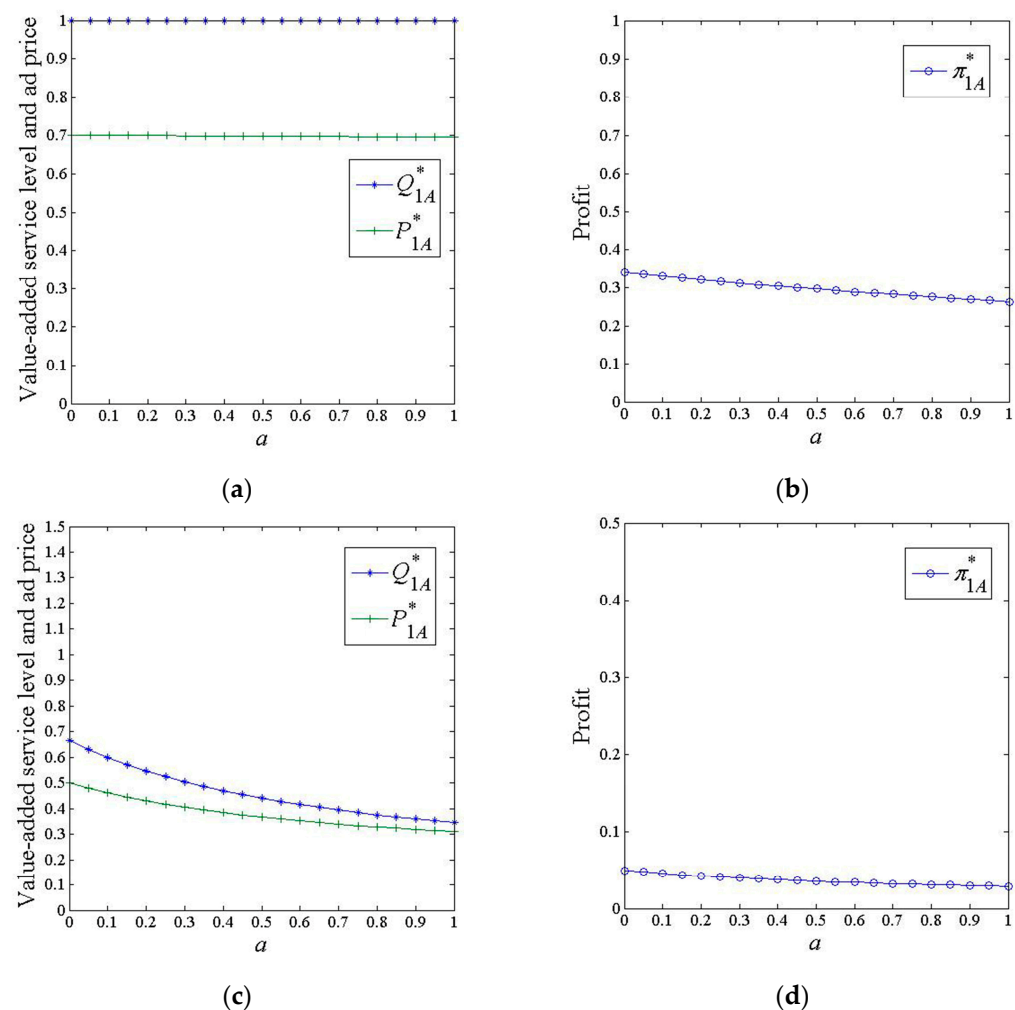


Figure 4. Impacts of negative advertising effects a in scenario A: (a) impact of a on value-added service level and ad price for case 1; (b) impact of a on profit for case 1; (c) impact of a on value-added service level and ad price for case 2; (d) impact of a on profit for case 2. * case 1: $0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)}$; case 2: $\frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1$.

Figure 4 illustrates the impact of the negative advertising effect on the equilibrium outcomes of platform 1 in scenario A. Combining Figure 4a with c, we know that the ad price of platform 1 decreases with the negative advertising effect, while the value-added service level of the platform either does not change with or decreases with the effect, depending on the range of the marginal investment cost. From Figure 4b,d, we find that the profit of platform 1 decreases with the negative advertising effect.

6.2. Scenario S

(1) Impacts of marginal investment costs

First, we conduct a simulation to analyze how marginal investment costs c affect the equilibrium outcomes of the two investment platforms, platforms 1 and 2, in scenario S. Based on Corollary 4, the values of the parameters are set as $r = 0.60$ and $a = 0.30$ for the case of $a < \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$ (case 1) and $r = 0.60$ and $a = 1.00$ for the case of $a \geq \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$ (case 2). The detailed simulation results are displayed in Figure 5.

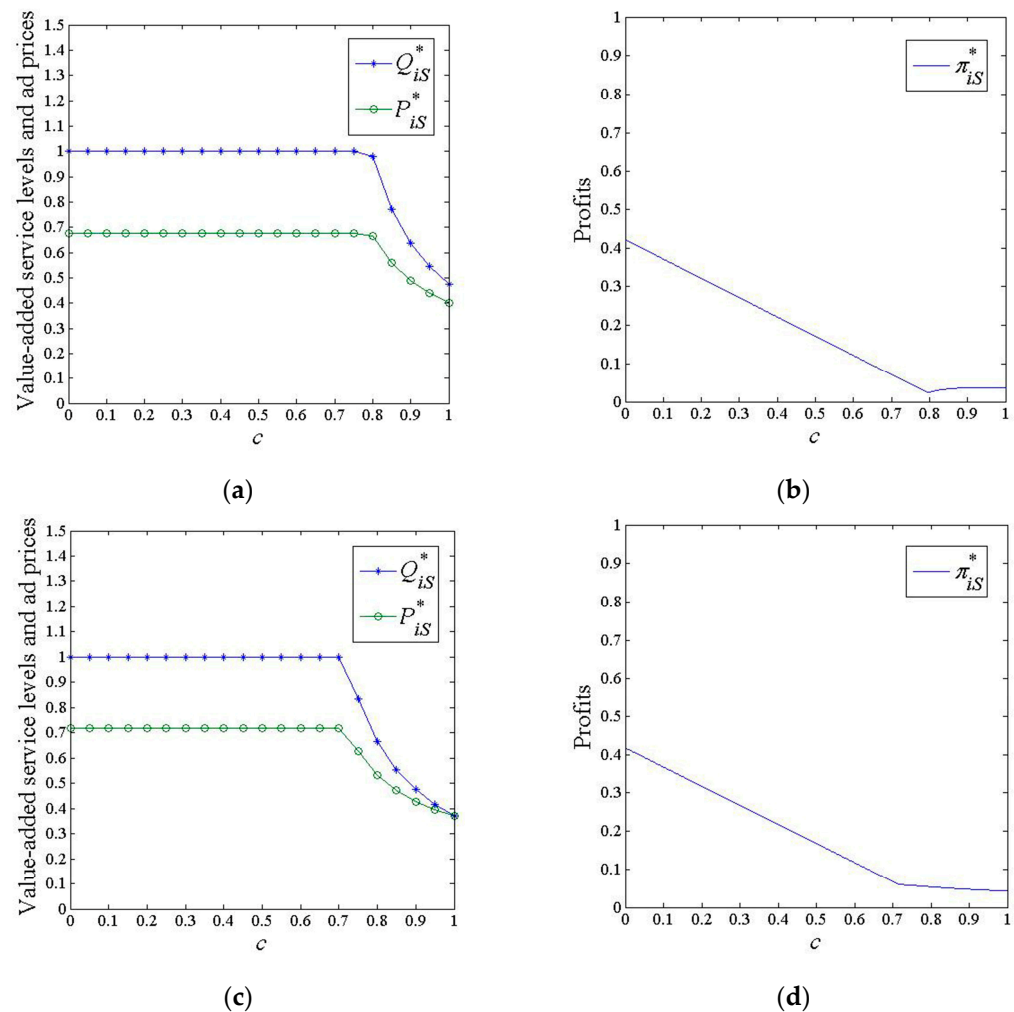


Figure 5. Impacts of marginal investment costs c in scenario S: (a) impact of c on value-added service levels and ad prices for case 1; (b) impact of c on profits for case 1; (c) impact of c on value-added service levels and ad prices for case 2; (d) impact of c on profits for case 2. * case 1: $a < \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$; case 2: $a \geq \frac{\sqrt{9r^4+68r^3+180r^2+160r+64+3r^2-2r-8}}{2r(5r+4)}$.

Figure 5 shows the impact of the marginal investment cost on the equilibrium outcomes of the two platforms in scenario S. From Figure 5a,c, we find that for both cases 1 and 2, the value-added service levels and ad prices of the two platforms first remain unchanged and then decline as the marginal investment cost increases. From Figure 5b,d, we observe that for case 1, the profits of the two platforms first decrease with, then increase with and finally decrease with the marginal investment cost; for case 2, the profits of the two platforms always decrease with the cost.

(2) Impacts of positive consumer effects

We conduct a simulation to analyze how positive consumer effects r influence the equilibrium outcomes of the two investment platforms, platforms 1 and 2, in scenario S. The parameters are set to $c = 0.30$, $a = 0.30$ (applied to Figure 6a,b) for the case of $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$ (case 1) and $c = 1.00$, $a = 0.30$ (applied to Figure 6c,d) for the case of $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$ (case 2). The simulation results are shown in Figure 6.

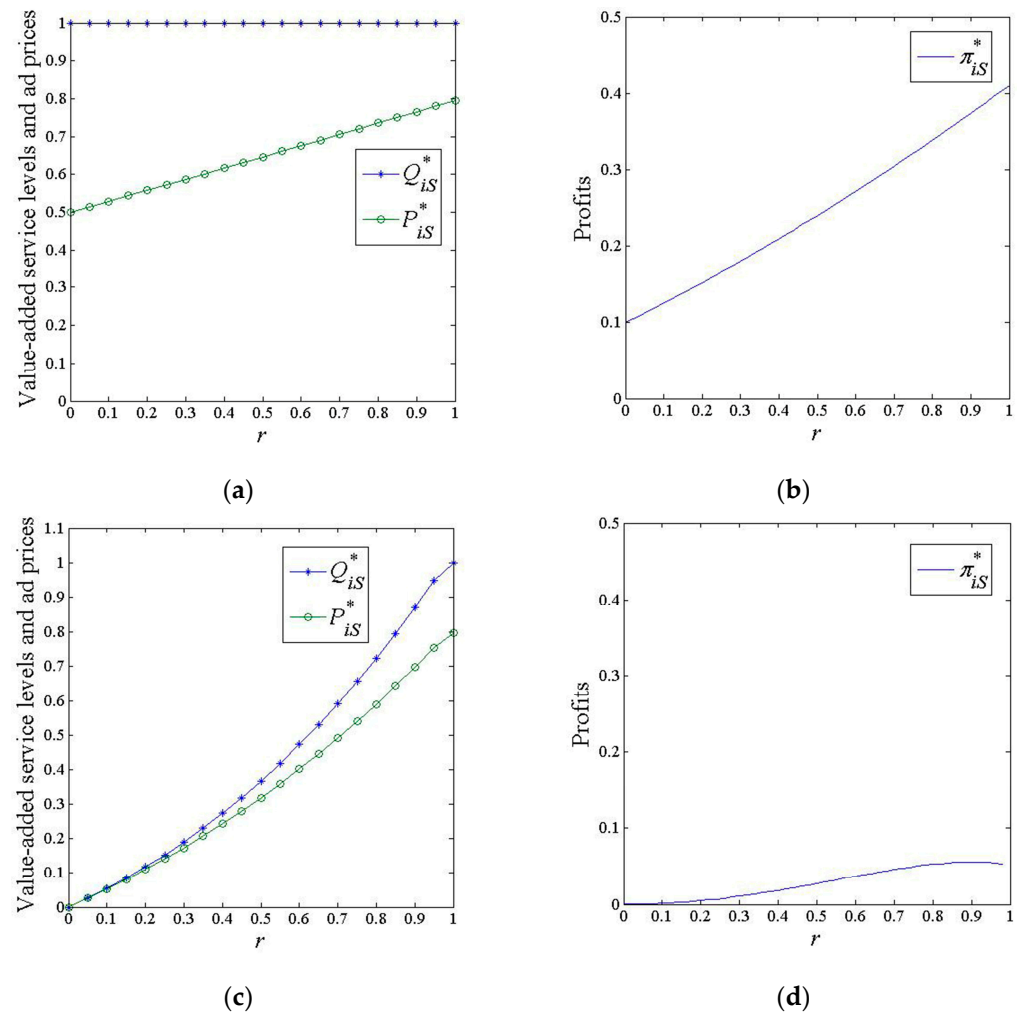


Figure 6. Impacts of positive consumer effects r in scenario S: (a) impact of r on value-added service levels and ad prices for case 1; (b) impact of r on profits for case 1; (c) impact of r on value-added service levels and ad prices for case 2; (d) impact of r on profits for case 2. * case 1: $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$; case 2: $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$.

Figure 6 reflects the impact of the positive consumer effect on the equilibrium outcomes of the two platforms in scenario S. Figure 6a,c indicates that as the positive consumer effect increases, the platforms do not need to change value-added service levels in case 1 and should improve value-added service levels in case 2, and the platforms should increase ad prices in both cases 1 and 2. Comparing Figure 6b,d, we see that the profits of the platforms increase with the positive consumer effect in case 1 and first increase and then decrease with the positive consumer effect in case 2.

(3) Impacts of negative advertising effects

Let $c = 0.50$ and $r = 0.40$ (applied to Figure 7a,b) for the case with $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$ (case 1) and $c = 0.85$ and $a = 0.40$ (applied to Figure 7c,d) for the

case with $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$ (case 2). Then, we simulate the impact of negative advertising effects on the equilibrium outcomes of the two investment platforms in scenario S. The simulation results are shown in Figure 7.

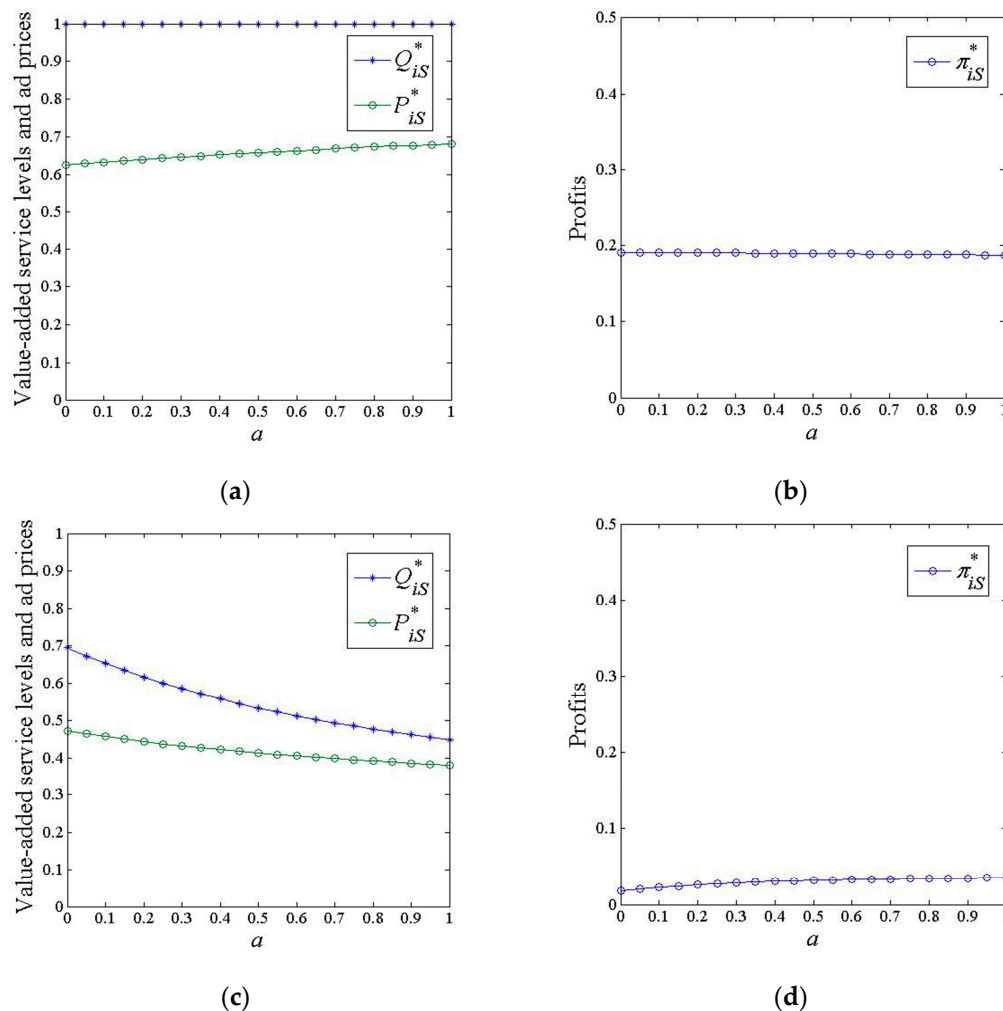


Figure 7. Impacts of negative advertising effects a in scenario S: (a) impact of a on value-added service levels and ad prices for case 1; (b) impact of a on profits for case 1; (c) impact of a on value-added service levels and ad prices for case 2; (d) impact of a on profits for case 2. * case 1: $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$; case 2: $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$.

Figure 7 reflects the impact of the negative advertising effect on the equilibrium outcomes of the two investment platforms in scenario S. Figure 7a indicates that in case 1, with the negative advertising effect increasing, the platforms should increase their ad prices but need not adjust their value-added service levels. Figure 7c shows that in case 2, the platforms should reduce both their ad prices and value-added service levels as the negative advertising effect increases. By observing Figure 7b,d, we derive that in case 1, the platforms will obtain less profit with a stronger negative advertising effect; in case 2, the platforms can earn more profit through a stronger negative advertising effect.

7. Conclusions

With the development of mobile internet technology and its wide application in the digital age, more and more people meet their entertainment needs through media platforms, which provides favorable opportunities for the development of media platforms. To ensure sustainable development, the platforms need to formulate appropriate two-sided beneficial value-added service investment and pricing strategies. This paper develops

duopoly models of two competing media platforms that connect two sides of users, i.e., consumers and advertisers, and explores the value-added service investment and pricing strategies of the platforms, by using game theory. The paper considers two investment scenarios: scenario A, an asymmetric investment scenario in which a single platform invests in the two-sided beneficial value-added services and scenario S, a symmetric investment scenario in which two platforms both invest in such value-added services. First, we derive the equilibrium value-added service levels, ad prices and profits of the investment platforms in each scenario and further analyze the impacts of marginal investment costs, positive consumer effects and negative advertising effects on these equilibrium outcomes. Second, we compare the differences in equilibrium outcomes of investment platforms under scenarios A and S; then, we analyze how these differences are affected by marginal investment costs. Finally, we conduct numerical simulations to verify the conclusions obtained in the above equilibrium analysis.

The conclusions of this paper are as follows. In scenario A, (i) the value-added service levels and ad prices of the investment platforms first remain constant and then decrease with marginal investment costs, while the profits of the platforms decrease monotonically with the costs. (ii) The ad prices and profits of the investment platforms increase (decrease) with positive consumer effects (negative advertising effects); however, the value-added service levels of the investment platforms change as such only when marginal investment costs are high.

In scenario S, (i) the value-added service levels and ad prices of the investment platforms will first remain constant and then decrease with marginal investment costs, and the profits of the platforms either vary in three stages with the costs or monotonically decrease with the costs, depending on negative advertising effects. (ii) The influence of positive consumer effects and negative advertising effects on the equilibrium outcomes of the investment platforms correlates with marginal investment costs. Specifically, if marginal investment costs are low, the value-added service levels of the platforms do not change with positive consumer effects or negative advertising effects; the ad prices of the platforms increase with the two effects; and the profits of the platforms increase with positive consumer effects while decreasing with negative advertising effects. If marginal investment costs are high, then the value-added service levels and ad prices of the investment platforms increase with positive consumer effects but decrease with negative advertising effects; the profits of the platforms first increase and then decrease with positive consumer effects and always increase with negative advertising effects.

Comparatively, the ad prices and profits of the investment platforms in scenario A are higher than those in scenario S, and the value-added service levels of the investment platforms in scenario A are higher than those in scenario S only when marginal investment costs are high.

The above conclusions provide some practical implications for duopoly media platforms in determining the investments in two-sided beneficial value-added services and the corresponding prices. In an asymmetric scenario, the platforms should develop investments and ad pricing strategies as follows. As marginal investment costs increase, the platforms should first keep the value-added service levels and ad prices constant and then decrease them. As positive consumer effects and negative advertising effects increase, the platforms should increase and reduce the ad prices, respectively; however, the platforms only need to adjust the value-added service levels when marginal investment costs are high. In a symmetric scenario, the platforms should develop value-added service investments and ad pricing strategies as follows. As marginal investment costs increase, the platforms first need not change the value-added service levels and ad prices, and then they should reduce the levels and prices. As positive consumer effects and negative advertising effects increase, how the platforms consequently adjust their value-added service levels and ad prices will also account for the role of marginal investment costs. If marginal investment costs are low, as positive consumer effects or negative advertising effects increase, the platforms should increase the ad prices, while they need not change the value-added service levels; if

marginal investment costs are high, the platforms should increase the value-added service levels and ad prices as positive consumer effects increase but reduce the levels and the prices as negative advertising effects increase.

This paper presents several limitations, which can be improved upon in future research. First, the paper considers only the network effects between users on different sides, i.e., consumers and advertisers. In reality, network effects also exist between users on the same side, i.e., among consumers and advertisers. Future research should also consider the network effects among users on the same side. Second, this paper assumes that consumers are single-homing. However, in reality, consumers may join more than one platform; that is, consumers are multi-homing. In the future, we will explore the two-sided beneficial value-added service investment and pricing strategies of media platforms in the context of multi-homing consumers.

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Appendix A

Proof of Proposition 1. Taking the first-order derivatives of π_{1A} in Equation (6) with respect to P_{1A} and Q_{1A} , and the first-order derivatives of π_{2A} with respect to P_{2A} , we can get the following:

$$\begin{cases} \frac{\partial \pi_{1A}}{\partial P_{1A}} = \frac{-(4P_{1A} - r - 2Q_{1A} - rQ_{1A} - ar^2 + 2aP_{1A}r + aP_{2A}r - arQ_{1A})}{2(ar+1)}, \\ \frac{\partial \pi_{1A}}{\partial Q_{1A}} = \frac{-(2cQ_{1A} - 2P_{1A} - P_{1A}r - aP_{1A}r + 2acrQ_{1A})}{2(ar+1)}, \\ \frac{\partial \pi_{2A}}{\partial P_{2A}} = \frac{-(4P_{2A} - r + rQ_{1A} - ar^2 + aP_{1A}r + 2aP_{2A}r - arQ_{1A})}{2(ar+1)}. \end{cases} \quad (A1)$$

Furthermore, we can obtain the Hessian matrix **A** as follows:

$$\mathbf{A} = \begin{bmatrix} \frac{-(ar+2)}{ar+1} & \frac{r+ar+2}{2(ar+1)} \\ \frac{r+ar+2}{2(ar+1)} & -c \end{bmatrix} \quad (A2)$$

The first-order sequential principal minor of the Hessian matrix **A** in Equation (A2) is $-(ar+2)/(ar+1)$, which is negative; the second-order sequential principal minor of **A** is $-(4r-8c+4ar+2ar^2+r^2+a^2r^2-4a^2cr^2-12acr+4)/4(ar+1)^2$, which is either positive or negative. The second derivative of π_{2A} to P_{2A} is $\pi_{2A}/\partial(P_{2A})^2 = -(ar+2)/(ar+1)$, which is negative. As the second-order sequential principal minor of the **A** is ambiguous, the following two cases are considered.

(1) When $(r+ar+2)^2/4(ar+1)(ar+2) < c \leq 1$, $|\mathbf{A}| > 0$, the Hessian matrix in Equation (A2) is a negative definite. Thus, $\partial \pi_{1A}/\partial P_{1A} = 0$, $\partial \pi_{1A}/\partial Q_{1A} = 0$ and $\partial \pi_{2A}/\partial P_{2A} = 0$ in Equation (A1) gives Q_{1A}^* , P_{1A}^* and P_{2A}^* , which can be expressed by the following:

$$\begin{cases} Q_{1A}^* = \frac{(r+ar+2)r(ar+1)(ar+4)}{H}, \\ P_{1A}^* = \frac{2cr(ar+1)^2(ar+4)}{H}, \\ P_{2A}^* = \frac{2r(ar+1)(4c-3r-ar-ar^2-r^2+a^2cr^2+5acr-2)}{H}. \end{cases} \quad (A3)$$

From Equation (A3), we obtain $(r + ar + 2)r(ar + 1)(ar + 4)/H < 1$ when $(r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)/2(ar + 1)(ar + 4)(3ar + 4) < c \leq 1$; and $(r + ar + 2)r(ar + 1)(ar + 4)/H > 1$ when $(r + ar + 2)^2/4(ar + 1)(ar + 2) < c \leq (r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)/2(ar + 1)(ar + 4)(3ar + 4)$. As $Q_{1A} \in [0, 1]$, the optimal value-added service level of platform 1 is $Q_{1A}^* = (r + ar + 2)r(ar + 1)(ar + 4)/H$ when $(r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)/2(ar + 1)(ar + 4)(3ar + 4) < c \leq 1$ and $Q_{1A}^* = 1$ when $(r + ar + 2)^2/4(ar + 1)(ar + 2) < c \leq (r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)/2(ar + 1)(ar + 4)(3ar + 4)$. Furthermore, depending on the range of c , the other equilibrium outcomes can be discussed in the following two subcases:

(i) the case of $(r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)/2(ar + 1)(ar + 4)(3ar + 4) < c \leq 1$.

Inserting Q_{1A}^* , P_{1A}^* and P_{2A}^* from Equation (A3) into Equations (4)–(6), we obtain the following:

$$M_{1A}^* = \frac{ca^3r^4 + 7ca^2r^3 + 14car^2 + 8cr}{H}, M_{2A}^* = \frac{r(ar + 2)(4c - 3r - ar - ar^2 - r^2 + a^2cr^2 + 5acr - 2)}{H},$$

$$N_{1A}^* = \frac{(ar + 1)(ar + 4)(4c - r - ar + 3acr - 2)}{H}, N_{2A}^* = \frac{(3ar + 4)(4c - 3r - ar - ar^2 - r^2 + a^2cr^2 + 5acr - 2)}{H},$$

$$\pi_{1A}^* = \frac{cr^2(a^2r^2 + 5ar + 4)^2(8c - 4r - 4ar - 2ar^2 - r^2 - a^2r^2 + 4a^2cr^2 + 12acr - 4)}{2H^2},$$

$$\pi_{2A}^* = \frac{2r^2(ar + 1)(ar + 2)(3r - 4c + ar + ar^2 + r^2 - a^2cr^2 - 5acr + 2)^2}{H^2}.$$

where $H = (6a^3c - 3a - 4a^2 - a^3)r^3 + (38a^2c - 18a - 10a^2 - 4)r^2 + (64ac - 24a - 16)r + 32c - 16$.

(ii) the case of $(r + ar + 2)^2/4(ar + 1)(ar + 2) < c \leq (r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)/2(ar + 1)(ar + 4)(3ar + 4)$.

By inserting $Q_{1A}^* = 1$ into Equation (6) and solving, we obtain $P_{1A}^* = (r + 1)(a^2r^2 + 8ar + 8)/(ar + 4)(3ar + 4)$ and $P_{2A}^* = ar(ar + 2)(r + 1)/(ar + 4)(3ar + 4)$. Furthermore, by inserting $Q_{1A}^* = 1$, $P_{1A}^* = (r + 1)(a^2r^2 + 8ar + 8)/(ar + 4)(3ar + 4)$ and $P_{2A}^* = ar(ar + 2)(r + 1)/(ar + 4)(3ar + 4)$ into Equations (4)–(6), we obtain the following:

$$M_{1A}^* = \frac{(ar + 2)(r + 1)(a^2r^2 + 8ar + 8)}{2(ar + 1)(ar + 4)(3ar + 4)}, M_{2A}^* = \frac{ar(ar + 2)^2(r + 1)}{2(ar + 1)(ar + 4)(3ar + 4)},$$

$$N_{1A}^* = \frac{a^2r^2 - a^2r + 8ar - 2a + 8}{2(ar + 1)(ar + 4)}, N_{2A}^* = \frac{a(r + 1)(ar + 2)}{2(ar + 1)(ar + 4)}, \quad (A4)$$

$$\pi_{1A}^* = \frac{(r + 1)^2(a^2r^2 + 8ar + 8)(a^3r^3 + 10a^2r^2 + 24ar + 16)}{(ar + 4)(2ar + 2)(3ar + 4)(3a^2r^2 + 16ar + 16)} - \frac{c}{2}, \pi_{2A}^* = \frac{a^2r^2(ar + 2)^3(r + 1)^2}{2(ar + 1)(3a^2r^2 + 16ar + 16)^2}.$$

(2) When $0 < c \leq (r + ar + 2)^2/4(ar + 1)(ar + 2)$, $|A| \leq 0$, the Hessian matrix in Equation (A2) is neither positive nor negative definite. As the profit function is continuous and bounded, and its stationary point is unique, the optimal solution is obtained at $Q_{1A} = 0$ or $Q_{1A} = 1$.

When $Q_{1A} = 0$, $\pi_{1A} = \pi_{2A} = r^2(ar + 1)(ar + 2)/2(3ar + 4)^2$, and when $Q_{1A} = 1$, $\pi_{1A} = ((r + 1)^2(a^2r^2 + 8ar + 8)(a^3r^3 + 10a^2r^2 + 24ar + 16))/(2(ar + 4)(ar + 1)(3ar + 4)(3a^2r^2 + 16ar + 16)) - c/2$ and $\pi_{2A} = a^2r^2(ar + 2)^3(r + 1)^2/2(ar + 1)(3a^2r^2 + 16ar + 16)^2$. Comparing the difference in π_{1S} when $Q_{1A} = 1$ and when $Q_{1A} = 0$, we find the difference is positive when $0 < c \leq (r + ar + 2)^2/4(ar + 1)(ar + 2)$. Thus, the optimal value-added service level of platform 1 is $Q_{1A}^* = 1$. Furthermore, we obtain the corresponding equilibrium ad prices, advertiser demands, consumer demands and profits, which are the same as in Equation (A4).

Based on the above analysis, the equilibrium outcomes in scenario A can be obtained, as given in Proposition 1. $\square$

Proof of Corollary 1.

$$1. \quad \text{If } 0 < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)},$$

$$\frac{\partial Q_{1A}^*}{\partial c} = 0, \quad \frac{\partial P_{1A}^*}{\partial c} = 0, \quad \frac{\partial N_{1A}^*}{\partial c} = 0, \quad \frac{\partial M_{1A}^*}{\partial c} = 0, \quad \frac{\partial \pi_{1A}^*}{\partial c} = \frac{-1}{2} < 0.$$

$$2. \quad \text{If } \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1,$$

$$\frac{\partial Q_{1A}^*}{\partial c} = \frac{-2r(ar+1)^2(ar+4)^2(r+ar+2)(3ar+4)}{H^2} < 0,$$

$$\frac{\partial P_{1A}^*}{\partial c} = \frac{-2r(ar+4)(r+ar+2)(a^2r^2+3ar^2+8ar+4r+8)(ar+1)^2}{H^2} < 0,$$

$$\frac{\partial N_{1A}^*}{\partial c} = \frac{r(ar+4)(r+ar+2)(2a-3ar+a^2r-4)(3ar+4)(ar+1)}{H^2} < 0,$$

$$\frac{\partial M_{1A}^*}{\partial c} = \frac{-r(ar+4)(r+ar+2)(ar+2)(ar+1)(a^2r^2+3ar^2+8ar+4r+8)}{H^2} < 0,$$

$$\frac{\partial \pi_{1A}^*}{\partial c} = (r^2(r+ar+2)(a^2r^2+5ar+4)^2((r+ar+2)^2(a^2r^2+3ar^2+8ar+4r+8) - 2(ar+1)(a^3r^3+9a^2r^3+18a^2r^2+24ar^2+48ar+16r+32)c))/2H^3 < 0$$

where $H = (6a^3c - 3a - 4a^2 - a^3)r^3 + (38a^2c - 18a - 10a^2 - 4)r^2 + (64ac - 24a - 16)r + 32c - 16$.

Corollary 1 can be obtained based on this analysis. $\square$

Proof of Corollary 2. This proof is analogous to Corollary 1. $\square$

Proof of Corollary 3. This proof is analogous to Corollary 1. $\square$

Proof of Proposition 2. Solving the first-order derivatives of π_{1S} in Equation (11) with respect to P_{1S} and Q_{1S} , and the first-order derivatives of π_{2S} with respect to P_{2S} and Q_{2S} , we can obtain:

$$\begin{cases} \frac{\partial \pi_{1S}}{\partial P_{1S}} = \frac{r-4P_{1S}+2Q_{1S}+rQ_{1S}-rQ_{2S}+ar^2-2aP_{1S}r-aP_{2S}r+arQ_{1S}+arQ_{2S}}{2(ar+1)}, \\ \frac{\partial \pi_{1S}}{\partial Q_{1S}} = \frac{P_{1S}(r+ar+2)}{2(ar+1)} - cQ_{1S}, \\ \frac{\partial \pi_{2S}}{\partial P_{2S}} = \frac{r-4P_{2S}+2Q_{2S}-rQ_{1S}+rQ_{2S}+ar^2-aP_{1S}r-2aP_{2S}r+arQ_{1S}+arQ_{2S}}{2(ar+1)}, \\ \frac{\partial \pi_{2S}}{\partial Q_{2S}} = \frac{P_{2S}(r+ar+2)}{2(ar+1)} - cQ_{2S}. \end{cases} \quad (A5)$$

Furthermore, we can obtain the Hessian matrix $\mathbf{A}_i$, $i = 1, 2$, as follows:

$$\mathbf{A}_i = \begin{bmatrix} \frac{-(ar+2)}{ar+1} & \frac{r+ar+2}{2(ar+1)} \\ \frac{r+ar+2}{2(ar+1)} & -c \end{bmatrix}, \quad (A6)$$

The first-order sequential principal minor of the Hessian matrix $\mathbf{A}_i$ in Equation (A6) is $-(ar+2)/(ar+1)$, which is negative; the second-order sequential principal minor of $\mathbf{A}_i$ is $-(4r-8c+4ar+2ar^2+r^2+a^2r^2-4a^2cr^2-12acr+4)/4(ar+1)^2$, which is either positive or negative. As the second-order sequential principal minor of $\mathbf{A}_i$ is ambiguous, the following two cases are considered.

(1) When $(r+ar+2)^2/4(ar+1)(ar+2) < c \leq 1$, $|\mathbf{A}| > 0$, the Hessian matrix $\mathbf{A}_i$ in Equation (A6) is a negative definite. Solving $\pi_{1S}/\partial P_{1S} = 0$, $\partial \pi_{1S}/\partial Q_{1S} = 0$, $\partial \pi_{2S}/\partial P_{2S} = 0$ and $\partial \pi_{2S}/\partial Q_{2S} = 0$ yields the following:

$$Q_{iS}^* = \frac{r(r+ar+2)}{2(4c-r-ar+3acr-2)}, P_{iS}^* = \frac{r(c+acr)}{4c-r-ar+3acr-2}.$$

As the value-added service level of platform i , $Q_{iS} \in [0, 1]$, $i = 1, 2$, we can find that the optimal value-added service level of platform i ($i = 1, 2$) is $Q_{iS}^* = r(r+ar+2)/2(4c-r-ar+3acr-2)$ when $(2r+2ar+r(r+ar+2)+4)/2(3ar+4) < c \leq 1$ and $Q_{iS}^* = 1$ when $(r+ar+2)^2/4(ar+1)(ar+2) < c \leq (2r+2ar+r(r+ar+2)+4)/2(3ar+4)$. Furthermore, we can obtain the other equilibrium outcomes as follows:

(i) When $(2r+2ar+r(r+ar+2)+4)/2(3ar+4) < c \leq 1$,

$$P_{iS}^* = \frac{r(c+acr)}{4c-r-ar+3acr-2}, M_{iS}^* = \frac{cr(ar+2)}{2(4c-r-ar+3acr-2)}, N_{iS}^* = \frac{1}{2},$$

$$\pi_{iS}^* = \frac{cr^2(8c-4r-4ar-2ar^2-r^2-a^2r^2+4a^2cr^2+12acr-4)}{8(r-4c+ar-3acr+2)^2}.$$

(ii) When $(r+ar+2)^2/4(ar+1)(ar+2) < c \leq (2r+2ar+r(r+ar+2)+4)/2(3ar+4)$,

$$P_{iS}^* = \frac{(ar+1)(r+2)}{3ar+4}, M_{iS}^* = \frac{(r+2)(ar+2)}{2(3ar+4)}, N_{iS}^* = \frac{1}{2},$$

$$\pi_{iS}^* = \frac{(8r-16c+12ar+12ar^2+3ar^3+2r^2+4a^2r^2+4a^2r^3+a^2r^4-9a^2cr^2-24acr+8)}{2(3ar+4)^2}.$$

(2) When $0 < c \leq (r+ar+2)^2/4(ar+1)(ar+2)$, $|\mathbf{A}_i| \leq 0$, the Hessian matrix $\mathbf{A}_i$ in Equation (A6) is neither positive nor negative definite. As the profit function is continuous and bounded and its stationary point is unique, the optimal solution is obtained at $Q_{iS} = 0$ or $Q_{iS} = 1$. There are four combinations of value-added service strategies for the two platforms: (I) $Q_{1S} = 0$ and $Q_{2S} = 0$; (II) $Q_{1S} = 0$ and $Q_{2S} = 1$; (III) $Q_{1S} = 1$ and $Q_{2S} = 0$; and (IV) $Q_{1S} = 1$ and $Q_{2S} = 1$. The profits for both platforms under the four combinations are as follows:

(I) $Q_{1S} = 0$ and $Q_{2S} = 0$

$$\pi_{1S}^I = \frac{r^2(ar+1)(ar+2)}{2(3ar+4)^2}, \pi_{2S}^I = \frac{r^2(ar+1)(ar+2)}{2(3ar+4)^2}.$$

(II) $Q_{1S} = 0$ and $Q_{2S} = 1$

$$\pi_{1S}^{II} = \frac{a^2r^2(ar+2)^3(r+1)^2}{2(ar+1)(3a^2r^2+16ar+16)^2}, \pi_{2S}^{II} = \frac{(r+1)^2(a^2r^2+8ar+8)(a^3r^3+10a^2r^2+24ar+16)}{(ar+4)(2ar+2)(3ar+4)(3a^2r^2+16ar+16)} - \frac{c}{2}.$$

(III) $Q_{1S} = 1$ and $Q_{2S} = 0$

$$\pi_{1S}^{III} = \frac{(r+1)^2(a^2r^2+8ar+8)(a^3r^3+10a^2r^2+24ar+16)}{(ar+4)(2ar+2)(3ar+4)(3a^2r^2+16ar+16)} - \frac{c}{2}, \pi_{2S}^{III} = \frac{a^2r^2(ar+2)^3(r+1)^2}{2(ar+1)(3a^2r^2+16ar+16)^2}.$$

(IV) $Q_{1S} = 1$ and $Q_{2S} = 1$

$$\pi_{1S}^{IV} = \frac{(8r-16c+12ar+12ar^2+3ar^3+2r^2+4a^2r^2+4a^2r^3+a^2r^4-9a^2cr^2-24acr+8)}{2(3ar+4)^2},$$

$$\pi_{2S}^{IV} = \frac{(8r-16c+12ar+12ar^2+3ar^3+2r^2+4a^2r^2+4a^2r^3+a^2r^4-9a^2cr^2-24acr+8)}{2(3ar+4)^2}.$$

From (I)–(IV), we know that $\pi_{1S}^{III} > \pi_{1S}^I$, $\pi_{1S}^{IV} > \pi_{1S}^{II}$, $\pi_{2S}^{II} > \pi_{2S}^I$ and $\pi_{2S}^{IV} > \pi_{2S}^{III}$. Thus, the dominant strategies for platform 1 and platform 2 are $Q_{1S} = 1$, $Q_{2S} = 1$, respectively.

Based on this analysis, we can obtain Proposition 2. $\square$

Proof of Corollary 4.

1. If $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$,

$$\frac{\partial Q_{iS}^*}{\partial c} = 0, \frac{\partial P_{iS}^*}{\partial c} = 0, \frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} = 0, \frac{\partial \pi_{iS}^*}{\partial c} = \frac{-1}{2} < 0.$$

2. If $\frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1$,

$$\frac{\partial Q_{iS}^*}{\partial c} = \frac{-r(6ar+8)(r+ar+2)}{(2r-8c+2ar-6acr+4)^2} < 0, \frac{\partial P_{iS}^*}{\partial c} = \frac{-r(ar+1)(r+ar+2)}{(r+ar-c(3ar+4)+2)^2} < 0,$$

$$\frac{\partial N_{iS}^*}{\partial c} = 0, \frac{\partial M_{iS}^*}{\partial c} = \frac{-r(ar+2)(r+ar+2)}{2(r+ar-c(3ar+4)+2)^2} < 0,$$

$$\frac{\partial \pi_{iS}^*}{\partial c} = \frac{(r^2(r+ar+2)(5a^2r^2-3ar^2+14ar-4r+8))c-r^2(r+ar+2)(a^2r^2+2ar^2+4ar+r^2+4r+4)}{8(r-4c+ar-3acr+2)^3}.$$

Clearly, $\partial \pi_{iS}^* / \partial c \geq 0$ when $c \leq c_2$, and $\partial \pi_{iS}^* / \partial c < 0$ when $c > c_2$, where $c_2 = (r+ar+2)^2 / (5a^2r^2-3ar^2+14ar-4r+8)$. If $a < (\sqrt{9r^4+68r^3+180r^2+160r+64}+3r^2-2r-8)/2r(5r+4)$, $(2r+2ar+r(r+ar+2)+4)/2(3ar+4) < c_2$; if $a \geq (\sqrt{9r^4+68r^3+180r^2+160r+64}+3r^2-2r-8)/2r(5r+4)$, $(2r+2ar+r(r+ar+2)+4)/2(3ar+4) \geq c_2$. Thus, if $a < (\sqrt{9r^4+68r^3+180r^2+160r+64}+3r^2-2r-8)/2r(5r+4)$, then $\partial \pi_{iS}^* / \partial c \geq 0$ when $(2r+2ar+r(r+ar+2)+4)/2(3ar+4) < c \leq c_2$, and $\partial \pi_{iS}^* / \partial c < 0$ when $c_2 < c \leq 1$; if $a \geq (\sqrt{9r^4+68r^3+180r^2+160r+64}+3r^2-2r-8)/2r(5r+4)$, $\partial \pi_{iS}^* / \partial c < 0$ when $(2r+2ar+r(r+ar+2)+4)/2(3ar+4) < c \leq 1$.

Corollary 4 can be obtained based on this analysis. $\square$

Proof of Corollary 5.

1. If $0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)}$,

$$\frac{\partial Q_{iS}^*}{\partial r} = 0, \frac{\partial P_{iS}^*}{\partial r} = \frac{3a^2r^2+8ar+2a+4}{(3ar+4)^2} > 0, \frac{\partial N_{iS}^*}{\partial r} = 0, \frac{\partial M_{iS}^*}{\partial r} = \frac{3a^2r^2+8ar-4a+8}{2(3ar+4)^2} > 0,$$

$$\frac{\partial \pi_{iS}^*}{\partial r} = \frac{(r+2)(6a^3r^3+25a^2r^2-2a^2r+36ar+16)}{2(3ar+4)^3} < 0.$$

$$2. \quad \text{If } \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq 1,$$

$$\frac{\partial Q_{iS}^*}{\partial r} = \frac{(3a^2r^2 + 3ar^2 + 8ar + 8r + 8)c - (r + ar + 2)^2}{2(4c + 3acr - ar - r - 2)^2} > 0,$$

$$\frac{\partial P_{iS}^*}{\partial r} = \frac{(3a^2r^2 + 8ar + 4)c^2 - (a^2r^2 + ar^2 + 4ar + 2)c}{(4c + 3acr - ar - r - 2)^2} > 0,$$

$$\frac{\partial N_{iS}^*}{\partial r} = 0, \frac{\partial M_{iS}^*}{\partial r} = \frac{(3a^2r^2 + 8ar + 8)c^2 - (a^2r^2 + ar^2 + 4ar + 4)c}{2(4c + 3acr - ar - r - 2)^2} > 0,$$

$$\frac{\partial \pi_{iS}^*}{\partial r} = \frac{L_1r^4 + L_2r^3 + L_3r^2 + L_4r}{4(4c + 3acr - ar - r - 2)^3}.$$

Clearly, $\partial \pi_{iS}^* / \partial r > 0$ when $0 < r < r^*$, and $\partial \pi_{iS}^* / \partial r \leq 0$ when $r^* \leq r \leq 1$, where $f(r^*) = L_1r^{*4} + L_2r^{*3} + L_3r^{*2} + L_4r^* = 0$, $L_1 = -c(a - 3ac + 1)(2a - 4a^2c + a^2 + 1)$, $L_2 = -c(50a^2c^2 - 36a^2c + 6a^2 - 28ac + 12a - 8c + 6)$, $L_3 = 12c(2c - 1)(a - 3ac + 1)$, and $L_4 = -8c(2c - 1)^2$.

Corollary 5 can be obtained based on this analysis. $\square$

Proof of Corollary 6. This proof is analogous to Corollary 5. $\square$

Proof of Corollary 7.

$$1. \quad \text{If } 0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)},$$

$$Q_{1A}^* - Q_{1S}^* = 0, P_{1A}^* - P_{1S}^* = \frac{r(3ar - 2a - a^2r + 4)}{(3ar + 4)(ar + 4)},$$

$$N_{1A}^* - N_{1S}^* = \frac{3ar - 2a - a^2r + 4}{2(ar + 1)(ar + 4)}, M_{1A}^* - M_{1S}^* = \frac{r(ar + 2)(3ar - 2a - a^2r + 4)}{2(ar + 1)(3ar + 4)(ar + 4)},$$

$$\pi_{1A}^* - \pi_{1S}^* = \frac{r(ar + 2)(3ar - 2a - a^2r + 4)(2a^2r^3 + 3a^2r^2 + 13ar^2 + 18ar + 12r + 16)}{2(ar + 1)(3ar + 4)^2(ar + 4)^2}.$$

$$2. \quad \text{If } \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)},$$

$$Q_{1A}^* - Q_{1S}^* = \frac{8c - 4r - 2ar - ar^2 - r^2 + 6acr - 4}{2(4c - r - ar + 3acr - 2)},$$

$$P_{1A}^* - P_{1S}^* = \frac{(3ar + 4)(a^2r^2 + 3ar^2 + 8ar + 4r + 8)c - (r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)}{(3ar + 4)(ar + 4)(4c - r - ar + 3acr - 2)},$$

$$N_{1A}^* - N_{1S}^* = \frac{3ar - 2a - a^2r + 4}{2(ar + 1)(ar + 4)},$$

$$M_{1A}^* - M_{1S}^* = \frac{(3ar + 4)(a^2r^2 + 3ar^2 + 8ar + 4r + 8)c - (r + 1)(r + ar + 2)(a^2r^2 + 8ar + 8)}{2(3ar + 4)(ar + 4)(ar + 1)(4c - r - ar + 3acr - 2)},$$

$$\pi_{1A}^* - \pi_{1S}^* = \frac{cr^2(4r-8c+4ar+2ar^2+r^2+a^2r^2-4a^2cr^2-12acr+4)}{8(r-4c+ar-3acr+2)^2} - \frac{c}{2} + \frac{(r+1)^2(a^2r^2+8ar+8)(a^3r^3+10a^2r^2+24ar+16)}{(ar+4)(2ar+2)(3ar+4)(3a^2r^2+16ar+16)}.$$

$$3. \quad \text{If } \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1,$$

$$Q_{1A}^* - Q_{1S}^* = \frac{r^2(3ar-2a-a^2r+4)(r+ar+2)^2}{2H(4c-r-ar+3acr-2)},$$

$$P_{1A}^* - P_{1S}^* = \frac{cr^2(3ar-2a-a^2r+4)(r+ar+2)(ar+1)}{H(4c-r-ar+3acr-2)},$$

$$N_{1A}^* - N_{1S}^* = \frac{r(r+ar+2)(3ar-2a-a^2r+4)}{2H},$$

$$M_{1A}^* - M_{1S}^* = \frac{cr^2(3ar-2a-a^2r+4)(r+ar+2)(ar+2)}{2H(4c-r-ar+3acr-2)},$$

$$\pi_{1A}^* - \pi_{1S}^* = \frac{cr^2(4r-8c+4ar+2ar^2+r^2+a^2r^2-4a^2cr^2-12acr+4)}{8(r-4c+ar-3acr+2)^2} - \frac{cr^2(a^2r^2+5ar+4)^2(4r-8c+4ar+2ar^2+r^2+a^2r^2-4a^2cr^2-12acr+4)}{2H^2}.$$

Corollary 7 can be obtained based on this analysis. $\square$

Proof of Corollary 8.

$$1. \quad \text{If } 0 < c \leq \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)},$$

$$\frac{\partial(Q_{1A}^* - Q_{1S}^*)}{\partial c} = 0, \quad \frac{\partial(P_{1A}^* - P_{1S}^*)}{\partial c} = 0.$$

$$2. \quad \text{If } \frac{2r+2ar+r(r+ar+2)+4}{2(3ar+4)} < c \leq \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)},$$

$$\frac{\partial(Q_{1A}^* - Q_{1S}^*)}{\partial c} = \frac{r(3ar+4)(r+ar+2)}{2(r+ar-c(3ar+4)+2)^2}, \quad \frac{\partial(P_{1A}^* - P_{1S}^*)}{\partial c} = \frac{r(ar+1)(r+ar+2)}{(r+ar-c(3ar+4)+2)^2}.$$

$$3. \quad \text{If } \frac{(r+1)(r+ar+2)(a^2r^2+8ar+8)}{2(ar+1)(ar+4)(3ar+4)} < c \leq 1,$$

$$\frac{\partial(Q_{1A}^* - Q_{1S}^*)}{\partial c} = \frac{r^2(3ar+4)(2a-3ar+a^2r-4)(r+ar+2)^2(4(ar+1)(ar+4)(3ar+4)c - (r+ar+2)(3a^2r^2+3ar^2+18ar+4r+16))}{2H^2(r-4c+ar-3acr+2)^2},$$

$$\frac{\partial(P_{1A}^* - P_{1S}^*)}{\partial c} = \frac{r^2(r+ar+2)(ar+1)(2a-3ar+a^2r-4)(2(ar+1)(ar+4)(3ar+4)^2c^2 - (r+ar+2)^2(a^2r^2+3ar^2+8ar+4r+8))}{H^2(r-4c+ar-3acr+2)^2}.$$

Corollary 8 can be obtained based on this analysis. $\square$

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