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Adaptive Virtual Inertia Control Strategy for a Grid-Connected Converter of DC Microgrid Based on an Improved Model Prediction

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Abstract: Aiming at the problem that the bus voltage in a low-inertia DC microgrid is prone to be affected by internal power fluctuations, an adaptive virtual inertia control strategy for a grid-connected converter of a DC microgrid based on an improved model prediction is proposed. Firstly, the adaptive analog virtual synchronous generator (AVSG) is introduced into the voltage outer loop by combining the inertial parameters with the voltage change rate, and the flexible adjustment of the inertial parameters is realized. Secondly, the improved model predictive control is introduced into the current inner loop to realize the fast-tracking of the given current value and improve the dynamic characteristics of the control system. Finally, a system model is established based on Matlab/Simulink for simulation. The results show that compared with the traditional virtual inertia control strategy, the proposed control strategy has smaller bus voltage fluctuation amplitude and better dynamic performance; when a 10 kW load mutation occurs, the magnitude of bus voltage drop is reduced by 60%, and the voltage recovery time is shortened by 30%. The proposed control strategy can effectively improve the stability of DC bus voltage and the operation ability of the system under asymmetric conditions.

Keywords: DC microgrid; bidirectional grid-connected converter; adaptive virtual inertia control; model predictive control; dynamic performance

1. Introduction

In order to cope with the increasing depletion of traditional fossil energy sources and the resulting global environmental pollution, new energy sources represented by solar and wind power have been utilized on a large scale, and the proportion of renewable energy sources in the distribution network has been increasing [1–3]. With the development of power electronics technology, the converter-based DC microgrid has received widespread attention due to the high efficiency of system operation and the absence of frequency and rotor angle stability problems compared to the AC microgrid [4,5]. The DC microgrid is a converter-driven low-inertia system; DC bus voltage is very sensitive to power fluctuations of an intermittent power supply and load; when a disturbance caused by power fluctuation of the power supply and load occurs in the network, it will cause the fluctuation of the DC bus voltage, which is harmful to the stable operation of the DC microgrid [6,7]. The DC microgrid is connected to the grid through a bidirectional grid-connected converter (BGC). The BGC controls the energy exchange between the DC microgrid and the AC grid and plays a key role in stabilizing the DC bus voltage [8,9].

To enhance the inertia of the DC microgrid, some scholars have proposed virtual inertia control strategies applicable to the DC microgrid. At present, the virtual inertia control strategies for the DC microgrid are mainly divided into the following three main categories: variable droop coefficient control, additional differential inertia control, and AVSG control [10–15]. In literature [16], a variable droop coefficient control method is proposed. The droop coefficient is changed according to the voltage change rate of the DC...
bus, which changes the inertia margin of the microgrid. However, the introduction of the voltage differential term may lead to a decrease in system stability. In literature [17], by adding an additional virtual inertia control loop in the control loop, the released power of the controllable power supply is adjusted to provide inertia support for the microgrid. However, the introduction of a high-pass filter in the control loop will bring high-frequency interference. Literature [18] proposed AVSG control for a DC microgrid by analogy with a virtual synchronous generator (VSG) in an AC microgrid, and this control strategy can make the BGC provide additional power quickly when a power difference occurs in the DC microgrid, effectively suppressing the fluctuation of the DC bus voltage and enhancing the inertia of the DC microgrid. However, the inertia parameters in the control strategy are constant and cannot be adaptively adjusted according to the system dynamics.

In the virtual inertial control strategies described above, the current inner loop still uses the traditional PI control, but for nonlinear systems, it is difficult to obtain good dynamic performance by using PI control. In order to obtain better dynamic performance, nonlinear algorithms can be considered in the virtual inertial control strategy. Literature [19] introduced passivity-based control in the virtual inertial control strategy, verified the passivity of the BGC, and designed the current inner loop passive controller from the perspective of system energy to achieve good tracking of the expected current value, but the tracking effect of passive control is not good when the system model and parameters change due to interference. Literature [20] proposed a second-order sliding mode control of a grid-connected power converter. The controller adopts a cascade structure composed of two controllers. The current inner loop uses a sliding mode algorithm to track the actual current value to the expected value. Sliding mode control is not easily affected by system model and parameter changes and, at the same time, can effectively improve the dynamic performance of the DC microgrid system. However, the introduction of the sliding mode algorithm may lead to the problem of sliding mode chattering.

With the development of computer control technology, model predictive control (MPC) was proposed and applied in industrial practice. MPC can predict and calculate the output of the system at the next moment according to the state variables of the current system. The actual output of the system will track the given reference value with good transient steady-state characteristics. Literature [21] proposed a virtual inertial control strategy based on model prediction for DC microgrid battery systems, which can provide inertial support during the transient period and enhance the dynamic characteristics of DC bus voltage. Literature [22] proposed a VSG control strategy based on MPC for isolated island microgrids. The MPC control strategy replaces the traditional voltage-current double loop control, which eliminates the parameter setting and improves the dynamic response of the system. However, the proposed control strategy is aimed at the AC microgrid and cannot be applied to the DC microgrid.

Based on the shortcomings of the above studies, in order to improve the dynamic performance of the system and enhance the inertia of the DC microgrid, this paper proposes an adaptive virtual inertia control strategy for the DC microgrid grid-connected converter based on an improved model prediction. The main contributions are as follows:

1. The traditional droop control is a non-inertial control method, which cannot provide inertial support for the DC microgrid. In order to solve this problem, an adaptive AVSG control is introduced into the voltage outer loop of the control strategy. The inertia parameter can be adjusted according to the voltage change rate, which improves the stability of DC bus voltage and the operation ability of the system under asymmetric conditions.

2. In order to improve the dynamic response of the control system, an improved model predictive control is introduced into the current inner loop of the control strategy, which eliminates the parameter setting, eliminates the traditional PI controller and PWM regulator, realizes the fast-tracking of the given current value, and improves the dynamic characteristics of the system.
The rest of this paper is organized as follows. The topology of the DC microgrid and its traditional control strategy are presented in Section 2. The adaptive virtual inertia control strategy based on model prediction is introduced in Section 3. Section 4 analyzes the stability of the proposed control strategy. The proposed strategy is validated in Section 5, and the conclusion is given in Section 6.

2. DC Microgrid Topology and Control Strategy

2.1. DC Microgrid Topology

The topology of the DC microgrid studied in this paper is shown in Figure 1, including the PV unit, energy storage unit, AC main network, DC constant power load, and corresponding power electronic converter. BGC is responsible for balancing the power in the grid when the DC microgrid is in the grid-connected mode. The energy storage unit is connected to the DC bus through a bidirectional DC–DC converter (BDC) to realize bidirectional energy flow with the DC microgrid. The PV is connected to the DC bus through a boost converter, using maximum power point tracking control. The DC constant power load is connected to the DC bus through a buck converter, using maximum power point tracking control.

![Figure 1. DC microgrid topology.](image)

The focus of this paper is to analyze the operation control strategy of BGC in the grid-connected mode of the DC microgrid, so the DC microgrid is simplified into an equivalent topology containing BGC, as shown in Figure 2. In the equivalent topology, the PV unit, the energy storage unit, and their corresponding power electronic converters can be equivalent to current sources [18]. In Figure 2, $V_1 \sim V_6$ are the six IGBTs, $u_k(k=a,b,c)$ is the phase voltage of the three-phase AC supply, $e_k(k=a,b,c)$ is the AC side voltage of the grid-connected converter, $i_k(k=a,b,c)$ is the three-phase line current, $u_{bus}$ is the DC bus voltage, $i_{bus}$ is the DC side current, $i_0$ is the DC side output current, $i_L$ is the current flowing through the load, $L$ is the inductance of the filter reactor, $R_L$ is the resistance of the filter reactor, $C$ is the DC side capacitor, and $R$ is the constant power load.

![Figure 2. Equivalent topology of DC microgrid with BGC.](image)
2.2. Traditional Control Strategy of DC Microgrid

Among many coordinated control methods of the DC microgrid system, one of the more commonly used traditional control strategies is droop control, and the typical voltage-current droop control equation is as follows:

$$u_{dc}^* = U_0 - ki_0$$  \hspace{1cm} (1)

where $u_{dc}^*$ is the reference value of the DC bus voltage. $U_0$ is the reference value of the DC side voltage when BGC is not loaded; $k$ is the droop coefficient, which measures the relationship between voltage and current.

The specific control block diagram is shown in Figure 3. The voltage reference value $u_{dc}^*$ is calculated by $U_0$ and $i_0$ using the droop curve. Then, the voltage outer loop control is applied to obtain the current reference value, which is further utilized in the current inner loop control to generate the control signals for the converter. In practical applications, the traditional droop control has defects such as long transient response time, large response overshoot, and large output voltage deviation. In addition, the traditional droop control is an inertia-free control method, which cannot provide inertia and damping for the DC microgrid to improve the system stability.

![Diagram of droop control.](image)

3. Adaptive Virtual Inertia Control Strategy

3.1. Voltage Outer Loop

In an AC microgrid, the VSG control technique enables the inverter to simulate inertia, droop, and damping characteristics similar to the synchronous generator by introducing virtual inertia and damping. Assuming that the number of poles of the synchronous generator is 1, the mechanical equations of VSG can be expressed as:

$$P_{set} - P_e - D_p(\omega - \omega_n) = J\omega \frac{d\omega}{dt} \approx J\omega_n \frac{d\omega}{dt}$$  \hspace{1cm} (2)

where: $P_{set}$ and $P_e$ are the active power given and electromagnetic power respectively; $D_p$ is the frequency damping factor, which describes the change in active power output of a VSG in response to a unit change in frequency; $\omega$ and $\omega_n$ are the angular frequency of VSG and the rated angular frequency of the grid respectively; $J$ is the virtual rotational inertia.

For an AC power grid, its inertia can be reflected in both voltage stability and frequency stability. Reactive power control is a commonly used method for voltage control, while VSG control simulates the behavior of a traditional synchronous generator to achieve frequency regulation; when the frequency of the AC grid changes abruptly, the grid exhibits a large inertia due to the existence of $J$, and the VSG is able to quickly adjust the active output to achieve grid frequency support. As for the DC microgrid, the inertia is expressed as the ability of the system to hinder the sudden change of DC bus voltage. In the DC microgrid, the electric energy stored by the DC bus capacitor can be used to hinder the sudden change of the DC bus voltage. The stored electric energy can be expressed as:

$$W_c = \frac{1}{2} C u_{bus}^2$$  \hspace{1cm} (3)

However, the DC microgrid is a small inertia system connected by a power electronic converter. When disturbances caused by intermittent power supply and load power fluctuations occur in the network, the DC bus voltage will fluctuate, which will adversely
affect the safe and reliable operation. The power stored in the DC bus capacitor alone cannot suppress this fluctuation.

The AC microgrid and DC microgrid have many control strategy variables that can correspond to each other, as shown in Table A1. Therefore, according to the correspondence of variables, the AVSG control equation suitable for the DC microgrid can be obtained by analogy reasoning:

\[
i_{\text{set}} - i_0 - D_v(u_{\text{bus}}^{*} - u_n) = C_v u_{\text{bus}}^{*} \frac{du_{\text{bus}}^{*}}{dt} \approx C_v u_n \frac{du_{\text{bus}}^{*}}{dt}
\]

where: \(i_{\text{set}}\) is the given value of BGC output current; \(D_v\) is the voltage damping coefficient, which characterizes the ability of BGC to dampen the oscillations in the DC bus voltage; \(u_n\) is the DC side voltage rating value; \(u_{\text{bus}}^{*}\) is the DC voltage reference value of the virtual inertia outer loop output; \(C_v\) is the virtual capacitance value.

When the DC bus voltage changes abruptly, the DC microgrid exhibits great inertia due to the existence of \(C_v\), and the AVSG can quickly regulate the DC side output current, thus suppressing the DC bus voltage fluctuations. After adopting the AVSG control strategy, BGC virtualizes a virtual capacitor on the DC side that is larger than the actual capacitance value, thereby providing inertial support for the DC microgrid.

In the traditional AVSG control, the virtual capacitance and virtual voltage damping parameters are fixed values. The system can obtain a large inertia, but its inertia parameters are constant and cannot be adaptively adjusted according to the system dynamics. For a DC microgrid with AVSG control, when a large power disturbance of intermittent power or load occurs, it will cause a large bus voltage change rate, and then the system is expected to generate a large virtual inertia to reduce the voltage fluctuation. When the large disturbance is over, the system power reaches balance, and the voltage change rate is small, then the system is expected to generate a small virtual inertia to achieve a fast recovery of voltage.

To meet the above requirements, the adaptive virtual inertia control strategy is considered to combine the inertia coefficient in AVSG with the voltage variation rate to achieve flexible and adjustable inertia parameters. The virtual inertia expression designed in this paper is:

\[
C_v = \begin{cases} 
C_v^0, & |\frac{dU_{\text{bus}}}{dt}| < M_0 \\
C_v^0 + k_1 |\frac{dU_{\text{bus}}}{dt}|, & M_0 \leq |\frac{dU_{\text{bus}}}{dt}| < M_1 \\
C_v^0 + k_2 |\frac{dU_{\text{bus}}}{dt}|, & |\frac{dU_{\text{bus}}}{dt}| \geq M_1 
\end{cases}
\]

where: \(C_v^0\) is the virtual capacitance value under the stable state of system voltage; \(dU_{\text{dc}}/dt\) is the voltage change rate; \(M_0\), \(M_1\) is the voltage threshold when the virtual capacitance value changes; \(k_1, k_2, k_3\) are related control parameters for flexible adjustment of virtual inertia.

When \(|dU_{\text{dc}}/dt| < M_0\), \(C_v\) is equal to the fixed value \(C_v^0\), which avoids frequent switching of virtual capacitance and maintains the normal operation of the system. When \(|dU_{\text{dc}}/dt| > M_0\), \(C_v\) is an expression containing \(dU_{\text{dc}}/dt\). In this case, \(C_v\) is changed to adjust the BGC output current, and then the inertia of the DC microgrid is adjusted, which prevents the sudden rise or fall of the DC bus voltage when the DC microgrid is affected by impulse disturbance.

In the selection of parameter \(k_1, k_2, k_3\) is the adjustment coefficient when the capacitance value changes linearly. In this case, the voltage change rate is not large, and the virtual inertia is expected to be small, so the smaller \(k_1\) can be selected. In the selection of parameters \(k_2\) and \(k_3\), when the system is subjected to large disturbance, the voltage change rate is large, and the virtual inertia is expected to be large. Therefore, the virtual capacitance value should be increased by increasing the value of \(k_3\) on the premise of ensuring the stability of the system. Otherwise, the value of \(k_3\) can be appropriately reduced. After selecting the appropriate value of \(k_3, k_2\) are selected according to the stability requirements of the system. Figure 4 shows the influence of the change of \(k_2\) and \(k_3\) on the value of virtual capacitance. As can be seen from Figure 5, an increase of \(k_2\) increases the value of \(C_v\) during the transient.

\[
\text{Figure 4 shows the influence of the change of } k_2 \text{ and } k_3 \text{ on the value of virtual capacitance.}
\]
process. For $k_3$, when the DC voltage change rate $|dU_{dc}/dt| < 1$, the value of $C_3$ in the transient process will gradually decrease with the increase of $k_3$. When $|dU_{dc}/dt| > 1$, the value of $C_3$ gradually increases with the increase of $k_3$. With the increase of exponential coefficient $k_3$, the change rate and range of $C_3$ along with $|dU_{dc}/dt|$ also increase, indicating that $k_3$ mainly affects the change rate of $C_3$.

Figure 4. The influence of the change of $k_2$ and $k_3$ on the value of virtual capacitance.

After introducing adaptive virtual inertia, the outer loop control block diagram shown in Figure 5 can be obtained.

3.2. Current Inner Loop

(1) Principle of current model predictive control

BGC often adopts the voltage and current double closed-loop control structure based on PI control. However, the traditional PI-based current inner loop control uses the measured current value for hysteresis regulation, which makes it difficult to obtain better dynamic performance. Therefore, this paper designs an adaptive virtual inertia strategy for a grid-connected converter based on improved model prediction.

For the topology of a two-level, three-phase BGC, the value of each switch is either 0 or 1. In order to avoid direct conduction of each bridge arm switch, it is common to design each bridge arm with two complementary switches, resulting in a total of $2^3 = 8$ effective switch combinations for the inverter. The system structure diagram of the Finite Control Set Model Predictive Control (FCS-MPC) is shown in Figure 6.

Figure 5. The outer loop control block diagram.

After introducing adaptive virtual inertia, the outer loop control block diagram shown in Figure 5 can be obtained.

Figure 6. Structure diagram of FCS-MPC.
In Figure 6, \( x_{ref}(k) \) represents the reference output signal, \( S(k) \) denotes the switching sequence triggered by the inverter during the interval from \( k \) to \( k + 1 \), and \( x(k + 1) \) corresponds to the output signal of the predictive model, which represents the alternative predicted output signal. The specific algorithmic process of FCS-MPC can be described as follows: at time \( k \), the sampling module measures the output state of the inverter and feeds the measured electrical quantity information into the predictive model. Based on the acquired electrical quantities, the predictive model calculates all the alternative predicted output signals \( x_i(k + 1) \) at time \( k + 1 \) \((i = 0, 1, 2, 3, 4, 5, 6, 7)\). The value function compares each alternative predicted output signal with the reference signal and selects the switching sequence corresponding to the alternative predicted output signal that minimizes the value function for triggering.

(2) Prediction model of BGC

Assume that the three-phase power supply on the grid side is symmetrical, the switching devices are ideal devices, and define the switching function as:

\[
S_k = \begin{cases} 
1, & \text{the upper bridge arm is open} \\
0, & \text{the upper bridge arm is closed} 
\end{cases}
\] (6)

where \( k = a, b, c \). According to the topology diagram of BGC in Figure 2, the mathematical model of BGC under three-phase can be obtained as:

\[
\begin{align*}
L \frac{di}{dt} &= e_\alpha - R_L i_\alpha - u_a \\
L \frac{di}{dt} &= e_\beta - R_L i_\beta - u_b \\
L \frac{di}{dt} &= e_c - R_L i_c - u_c \\
C \frac{du_{bus}}{dt} &= S_{i_a} i_a + S_{i_b} i_b + S_{i_c} i_c - i_0 
\end{align*}
\] (7)

After the \( abc/\alpha\beta \) coordinate transformation, the two-phase decoupling expression of BGC is:

\[
\begin{align*}
L \frac{d\alpha}{dt} &= e_\alpha - u_\alpha - R_L i_\alpha \\
L \frac{d\beta}{dt} &= e_\beta - u_\beta - R_L i_\beta \\
C \frac{du_{bus}}{dt} &= S_{i_\alpha} i_\alpha + S_{i_\beta} i_\beta - i_0 
\end{align*}
\] (8)

where, \( e_\alpha \) and \( e_\beta \) are the voltage components of three-phase grid voltages \( e_a \), \( e_b \) and \( e_c \) in axis \( \alpha\beta \), \( i_\alpha \) and \( i_\beta \) are the current components of three-phase grid-connected currents \( i_a \) and \( i_c \) in axis \( \alpha\beta \), \( u_\alpha \) and \( u_\beta \) are the voltage components of AC side voltages \( u_a \) and \( u_b \) in axis \( \alpha\beta \), \( S_\alpha \) and \( S_\beta \) are the voltage components of the switching function in the axis \( \alpha\beta \). In order to establish the prediction model of BGC, the continuous model shown in Equation (7) is discretized at time \( k \) and \( k + 1 \), and collated as follows:

\[
\begin{align*}
i_\alpha(k + 1) &= \frac{T_s}{L} (u_\alpha(k) - R_L i_\alpha(k)) + i_\alpha(k) \\
i_\beta(k + 1) &= \frac{T_s}{L} (u_\beta(k) - R_L i_\beta(k)) + i_\beta(k) 
\end{align*}
\] (9)

The voltage relationship between the AC and DC sides of BGC is:

\[
\begin{align*}
u_\alpha &= \sqrt{\frac{2}{3}} (S_a - \frac{1}{2} S_b - \frac{1}{2} S_c) u_{dc} \\
u_\beta &= \sqrt{\frac{2}{3}} (S_b - S_c) u_{dc}
\end{align*}
\] (10)

where \( T_s \) is the sampling period. Since there are eight switching states of BGC, the values of \( u_\alpha \) and \( u_\beta \) for each state can be obtained according to Equation (10), as shown in Table A2. By substituting the values of \( u_\alpha \) and \( u_\beta \) from Table A2 into Equation (9), the values of \( i_\alpha(k + 1) \) and \( i_\beta(k + 1) \) at moment \( k + 1 \) can be obtained. Taking the input current of the converter as the control object, the expression of the value function is:
where $i_α^*$ and $i_β^*$ are the reference currents. By comparing the 8 groups of function values obtained by Equation (11) and taking the minimum value $f_{\text{min}}$, a group of switch sequences $S_α$, $S_β$ and $S_γ$ satisfying the minimum value of the value function are applied to the converter. The current inner loop control block diagram can be obtained as shown in Figure 7.

\[
\begin{align*}
  f_{11} &= (i_α^* - i_α1(k+1))^2 + (i_β^* - i_β1(k+1))^2 \\
  f_{12} &= (i_α^* - i_α2(k+1))^2 + (i_β^* - i_β2(k+1))^2 \\
  \vdots \\
  f_{18} &= (i_α^* - i_α8(k+1))^2 + (i_β^* - i_β8(k+1))^2 \\
\end{align*}
\]

(11)

Figure 7. The current inner loop control block diagram.

(3) Improved model predictive control

Control delay is a major issue faced by predictive control methods. In practical applications, predictive control algorithms require extensive computations, which inevitably introduce time delays between inputs and optimization drives. The control delay can result in errors in the prediction of the next time step. When considering the delay issue without compensation, the operational flowchart of the FCS-MPC algorithm is depicted in Figure 8a. At time $k$, the system samples and performs calculations to predict the output value at time $k+1$. Based on the value function, a switching sequence is selected, and the system outputs switch signals to drive the inverter at time $k'$. Similarly, at time $k+1$, the system samples and performs calculations to predict the output value at time $k+2$. The switching sequence is selected based on the value function, and the system outputs switch signals to drive the inverter at time $k+1'$. After the above analysis, the FCS-MPC algorithm continues to employ the switch sequence implemented in the previous control cycle before the output switch sequence. For instance, during the period from $k+1$ to $k+1'$, the switch sequence $S_2$ used is the optimal switch sequence employed during the period from $k$ to $k+1$, and so forth. Consequently, this leads to deviations of the inverter’s output voltage from the reference voltage, resulting in a deterioration of the quality of the inverter’s output power.

In order to address the aforementioned issues, this study improves the traditional algorithm based on the characteristic of multi-step prediction in MPC. The improved algorithm modifies the logical sequence of algorithm implementation by triggering the optimal switching sequence at the beginning of each control cycle. The specific algorithm process is illustrated in Figure 8b.
The specific algorithm process is as follows:

Step 1: Implement the optimal switching sequence $S_2$ obtained from the previous control cycle calculation at time $k$.

Step 2: Measure the state vector and disturbance vector of the inverter at time $k$ and predict the optimal output vector $x_2(k+1)$ at time $k+1$ based on the input vector corresponding to the switching sequence $S_2$.

Step 3: Starting from $x_2(k+1)$, predict the eight sets of candidate output vectors for time $k+2$ based on the predictive model. Evaluate the candidate output vectors using the cost function and select the switching sequence $S_1$ that minimizes the cost function.

Step 4: Implement the optimal switching sequence $S_1$ at time $k+1$.

From the above algorithmic process, it can be observed that by changing the triggering logic of the optimal switching sequence, the switching sequence implemented from $k+1$ to $k+2$ is computed at $k$ to $k+1$. This alignment of the switching sequence’s action period with the prediction period of the predictive model reduces the issue of poor-quality inverter output power caused by delay problems. In order to visually illustrate the operation process of the delay compensation algorithm, this paper presents a flowchart of the FCS-MPC algorithm considering delay compensation, as shown in Figure 9.

**Figure 9.** Flowchart of the FCS-MPC algorithm considering delay compensation.
3.3. Inertial Control Strategy under DC Inter-Pole Fault

One of the most severe faults in a DC system is the inter-pole short circuit fault in the DC line, as shown in Figure 10. When an inter-pole short circuit fault occurs in the DC line, the DC-side capacitors discharge rapidly, and the fault current reaches its peak within a few milliseconds. When the voltage across the fault-side capacitor drops to zero and starts to reverse charge, it can easily lead to the damage of reverse-parallel freewheeling diodes and capacitors. If virtual inertia control is considered and the inertia of the DC system is increased, the time for the capacitor voltage to drop to zero during a fault is delayed, and the peak fault current is reduced. This provides more response time for fault detection, protection actions, and reduces the impact of faults on grid operation.

![Figure 10. Equivalent circuit of inter-pole short circuit.](image)

According to the circuit response characteristics of the inter-pole short circuit fault in the DC line, the fault process can be divided into three stages: the DC-side capacitor discharge stage, the uncontrolled rectifier initial stage, and the uncontrolled rectifier steady-state stage. Figure 11 shows the equivalent circuit diagrams for each stage.

![Figure 11. The equivalent circuit diagrams for each stage. (a) shows equivalent circuit of capacitor discharge stage; (b) shows DC side equivalent circuit of uncontrolled rectifier in initial stage; (c) shows AC side equivalent circuit of uncontrolled rectifier in initial stage; (d) shows equivalent circuit of uncontrolled rectifier in steady state stage.](image)

After the occurrence of a fault, the short-circuit current provided by the DC side is significantly larger than that of the AC side. Neglecting the AC side’s continuation of current flow, the short-circuit loop can be approximated as a second-order RLC discharge circuit composed of resistance, inductance, and DC-side capacitance, as shown in Figure 11a. As the capacitor continues to discharge, the capacitor voltage drops below the AC-side voltage, and the DC side inductance discharges towards the fault point through the circuit formed by the freewheeling diodes of the converter. When the DC voltage is smaller than
the peak value of the AC line voltage, the fault circuit enters the uncontrolled rectifier stage, and both the AC side and the capacitor discharge towards the fault point. When the short-circuit impedance is small, the capacitor continues to discharge until the voltage reaches zero. At this moment, the DC side short-circuit reactance accumulates a large amount of energy, and the counter electromotive force on it causes the freewheeling diodes to conduct simultaneously as the capacitor voltage drops to zero, forming an RL first-order free discharge circuit on the DC side. At the same time, the capacitor voltage is clamped by the diodes and remains at zero; the AC side can be considered as experiencing a three-phase short circuit. The AC and DC sides can be decomposed into two relatively independent circuits, as shown in Figure 11b,c. Under the action of the power source, the system gradually reaches a steady state. In the steady state stage, the DC voltage stabilizes at a fixed value, and the short-circuit current remains nearly constant, as shown in Figure 11d.

As the first stage is the main rising phase of the fault current, it is possible to control the fault current of the converter in this stage through control methods. Therefore, the primary focus is on analyzing the impact of the virtual inertia control strategy on the first stage of the fault. According to Figure 11a, assuming the instantaneous DC voltage is $U_0$ and current is $I_0$ after the fault occurrence, there is:

$$CL \frac{d^2u_{dc}}{dt^2} + CR \frac{du_{dc}}{dt} + u_{dc} = 0 \quad (12)$$

When $R > 2\sqrt{\frac{1}{L(C+C_v)}}$, the fault circuit is in an over-damped state, and the capacitor voltage will not cross zero. When $R < 2\sqrt{\frac{1}{L(C+C_v)}}$, the fault circuit is in an under-damped state. However, due to the small equivalent resistance $R$ of the DC network, the fault current in this stage is essentially the capacitor output current. The rate of change of the capacitor current is high, leading to a rapid decrease in the DC bus voltage due to the fast discharge of the capacitor.

If the virtual capacitance parameter is introduced after the fault occurrence, the following can be obtained:

$$(C + C_v)L \frac{d^2u_{dc}}{dt^2} + (C + C_v)R \frac{du_{dc}}{dt} + u_{dc} = 0 \quad (13)$$

When $R < 2\sqrt{\frac{1}{L(C+C_v)}}$, it corresponds to a second-order underdamped oscillation. Solving this system of equations, there is:

$$\left\{ \begin{array}{l}
U_{dc}(t) = e^{-at}\left(\frac{U_0}{\omega_0}\sin(\omega t + \beta) - \frac{l_0}{\omega_0(C+C_v)} \sin(\omega t)\right)

I_c(t) = e^{-at}\left(-\frac{U_0}{\omega}\sin(\omega t - \beta) + \frac{l_0}{\omega} \sin(\omega t)\right)
\end{array} \right. \quad (14)$$

where $\omega = \sqrt{1/L(C+C_v) - (R/2L)^2}$, $\omega_0 = \sqrt{\omega^2 + a^2}$, $a = R/2L$, $\beta = \arctan(\omega/a)$.

From Equation (14), it can be observed that if the virtual capacitance parameter is maximized within the allowable range, it can effectively limit the magnitude of the fault current. When the virtual capacitance is sufficiently large, it is possible to change the relationship between $R$ and $C$ from under-damped to over-damped, which significantly limits the growth of the fault current, reduces the rate of decrease in the DC bus voltage, and lowers the peak fault current. This provides sufficient time for protective actions to be taken.

4. Stability Analysis

According to Equations (4) and (8), the block diagram of the BGC system control strategy can be obtained, as shown in Figure 12.
In order to study the stability of the BGC system after adopting the control strategy proposed in this paper, small signal modeling and analysis of the BGC system are carried out. By writing the state variable in Equation (4) as the sum of steady state values and small disturbance, that is, \( i_0 = i_d + \Delta i_0, u_{bus}^* = u_{bus} + \Delta u_{bus} \), the small signal equation of the virtual inertia control equation can be obtained as:

\[
-\Delta i_0 - D_v \Delta u_{bus}^* = C_v u_n \frac{d\Delta u_{bus}}{dt} \tag{15}
\]

The BGC is set to operate at a unit power factor and does not transmit reactive power to the grid, that is, \( i_q = 0 \). According to the power balance on both sides of the BGC, there is:

\[
\begin{align*}
\frac{2}{3} u_d i_d &= u_{bus}^* i_d \\
i_d &= i_0 + C \frac{d i_{bus}}{dt}
\end{align*} \tag{16}
\]

The state variable in Equation (16) is written as the sum of steady state value and small disturbance, that is, \( i_d = I_d + \Delta I_d, u_{bus} = U_{bus} + \Delta u_{bus}, i_0 = I_0 + \Delta I_0 \), ignoring the disturbance term of power grid voltage and the secondary disturbance term, the small signal equation of Equation (16) is:

\[
\frac{3}{2} U_d \Delta I_d = U_{bus} (C \frac{d\Delta u_{bus}}{dt} + \Delta I_0) + \Delta u_{bus} I_0 \tag{17}
\]

According to the superposition theorem, ignoring the perturbation term \( \Delta I_d \) and applying Laplace changes to Equation (17), the relationship between \( \Delta u_{bus} \) and \( \Delta I_0 \) is:

\[
\frac{\Delta u_{bus}(s)}{\Delta I_0(s)} = -\frac{U_{bus}}{sC U_{bus} + I_0} = G_1(s) \tag{18}
\]

Similarly, ignoring the perturbation term \( \Delta I_0 \) and applying Laplace changes to Equation (17), the relationship between \( \Delta u_{bus} \) and \( \Delta I_d \) is:

\[
\frac{\Delta u_{bus}(s)}{\Delta I_d(s)} = \frac{3U_d}{2(sC U_{bus} + I_0)} = G_2(s) \tag{19}
\]
Thus, the small signal model of the BGC system can be obtained as shown in Figure 13, where $G_c(s) = k_p + k_i/s$, $k_p$ and $k_i$ are proportional constants and integral constants of the PI controller, and the parameters are adjusted by a typical second-order system parameter tuning method.

\[
\begin{align*}
\Delta i_n & \rightarrow D_s \rightarrow \Delta u_{bus}^* \\
& \rightarrow 1/C_{in} \rightarrow \frac{1}{s} \rightarrow G_i(s) \rightarrow \Delta i'_n \\
& \rightarrow 1/C_{in} \rightarrow \frac{1}{s} \rightarrow G_i(s) \rightarrow \Delta i'_n \\
& \rightarrow \Delta i' \rightarrow G_i(s) \rightarrow \Delta u_{bus}^* \\
& \rightarrow 1/C_{in} \rightarrow \frac{1}{s} \rightarrow G_i(s) \rightarrow \Delta i'_n \\
& \rightarrow \Delta i_n \rightarrow D_s \rightarrow \Delta u_{bus}^*
\end{align*}
\]

Figure 13. The small signal model of the BGC system.

Since the model prediction algorithm is adopted in the inner loop, considering the tracking accuracy and rapidity of the algorithm, and time delay compensation is introduced, it can be considered that the output current of the converter is equal to the reference current value in real-time [23], that is, the simplified small signal model can be obtained as shown in Figure 14.

\[
\begin{align*}
\Delta i_n & \rightarrow \frac{1}{C_{in}} \rightarrow \Delta u_{bus}^* \\
& \rightarrow \frac{1}{s} \rightarrow G_i(s) \rightarrow \Delta i'_n \\
& \rightarrow \Delta i_n \rightarrow D_s \rightarrow \Delta u_{bus}^*
\end{align*}
\]

Figure 14. Simplified small signal model of the BGC system.

According to Figure 14, the transfer function between $\Delta u_{bus}$ and $\Delta i_0$ can be obtained:

\[
G(s) = \frac{-\Delta u_{bus}(s)}{\Delta i_0(s)} = \frac{a_2s^2 + a_1s + a_0}{b_3s^3 + b_2s^2 + b_1s + b_0} \tag{20}
\]

where

\[
\begin{align*}
\left\{ 
\begin{array}{l}
a_2 = 2C_vU_nU_{bus} \\
a_1 = 2D_vU_{bus} + 3U_ik_p \\
a_0 = 3U_ik_i \\
b_3 = 2CC_vU_nU_{bus} \\
b_2 = 2C_vU_nI_0 + 2CD_vU_{bus} + 3U_ik_vU_{bus} \\
b_1 = 2D_vI_0 + 3U_ik_Dk_p + 3U_ik_vk_i \\
b_0 = 3U_ik_Dk_p
\end{array}
\right.
\tag{21}
\]

The zero-pole diagram of the BGC control system can be drawn according to Equation (21). Figure 15 shows the dominant pole distribution of the BGC system when different virtual capacitance values $C_v$ are taken under the given voltage damping coefficient $D_v$.

It can be seen from Figure 15 that under the proposed control strategy, the real part of all changing poles of the BGC system is less than zero, and the poles will gradually approach the imaginary axis but do not cross the imaginary axis into the right half plane. Therefore, when $C_v$ changes within a certain range, the change of $C_v$ will not affect the stability of the BGC system.
5. Simulation Analysis

In order to verify the effectiveness of the control strategy proposed in this paper, the DC microgrid simulation model shown in Figure 1 is built on the Matlab/Simulink simulation platform, and the simulation parameters are shown in Table A3.

5.1. Unit Step Response of Different Virtual Inertia Parameter

Figure 16a shows the simulation results of the unit step response when the damping coefficient $D_v$ is given and the virtual capacitance value $C_v$ is different. It can be seen from Figure 16a that as the value of $C_v$ increases, the magnitude of the DC bus voltage drop gradually decreases, indicating that the inertia of the DC microgrid is enhanced. In addition, the larger the value of $C_v$, the smaller the amplitude of DC bus voltage drop, the more gentle the change of the DC bus voltage, and the stronger the inertia of the DC microgrid. This is consistent with the previous theoretical analysis. However, too large value of $C_v$ will lead to too long voltage recovery time.

Figure 16b shows the simulation results of the unit step response when the virtual capacitance value $C_v$ is given and the damping coefficient $D_v$ is different. As can be seen from Figure 16b, with the increase of value $D_v$, the steady-state error of DC bus voltage gradually decreases. It can be seen that $D_v$ mainly affects the steady-state value of DC bus voltage.
5.2. Simulation Comparison of Load Mutation

Figure 17 shows the dynamic response diagram of DC bus when the constant power load surges or drops under three control strategies, namely PI-based parameter fixed AVSG, MPC-based parameter fixed AVSG, and MPC-based adaptive AVSG, respectively. Tables A4 and A5 show the corresponding DC bus dynamic response index.

![Figure 17](image)

*Figure 17. The dynamic response diagram of the DC bus. (a) shows the dynamic response diagram of the DC bus when load suddenly increases; (b) shows the dynamic response diagram of the DC bus when load suddenly decreases.*

During the simulation, the constant power load suddenly increased by 10 kW at $t = 2$ s and then decreased by 10 kW at $t = 4$ s. As can be seen from Tables A4 and A5, under the three control strategies: PI-based parameter fixed AVSG, MPC-based parameter fixed AVSG, and MPC-based adaptive AVSG, when the load suddenly increased, the amplitude of voltage fluctuation is 8.2 V, 5.2 V, and 3.4 V, respectively, and the voltage recovery time is 0.21 s, 0.19 s, and 0.14 s, respectively; when the load suddenly decreased, the amplitude of voltage fluctuation is 9.8 V, 5.9 V, and 3.7 V, respectively, and the voltage recovery time is 0.22 s, 0.19 s, and 0.16 s, respectively.

It can be seen from Figure 17 that among the three control strategies, the adaptive AVSG control based on MPC also has the smallest voltage fluctuation amplitude, the shortest voltage recovery time, and the best dynamic performance.

The above results show that the proposed adaptive AVSG control strategy based on MPC can effectively suppress the DC bus voltage fluctuation when the BGC system is in the network power disturbance. The introduction of adaptive virtual inertia can reduce the voltage fluctuation amplitude, enhance the inertia of the DC microgrid, and smooth the bus voltage change, thus improving the stability of DC bus voltage.

5.3. Grid-Side Power Quality Analysis

Figure 18 shows the current and voltage waveforms at the grid side, as well as the Total Harmonic Distortion (THD) analysis of the grid-side current under three control strategies.

It can be observed that the grid-side voltage and current exhibit good sinusoidal waveforms with minimal distortion. From the THD analysis, it is found in Table A6 that under the three control strategies, the THD is 5.32% for PI-based parameter fixed AVSG control, 3.88% for MPC-based parameter fixed AVSG control, and 2.98% for MPC-based adaptive AVSG control. The MPC-based adaptive AVSG control strategy yields the lowest total harmonic distortion of the grid-side current, indicating the best electrical energy quality at the grid side under this control strategy.
Figure 18. Grid side voltage and current waveform and THD analysis. (a) shows the network side waveform and THD analysis under PI+AVSG control strategy; (b) shows the network side waveform and THD analysis under MPC + AVSG control strategy; (c) shows the network side waveform and THD analysis under MPC + adaptive AVSG control strategy.

5.4. Inter-Pole Short Circuit Fault

When a BGC system experiences an inter-pole fault, the DC bus voltage rapidly drops to zero within a few milliseconds. Under this condition, there is little difference in the performance between the adaptive parameter AVSG and fixed parameter AVSG control strategies. Therefore, for simulation purposes, the fixed parameter approach is sufficient. Figure 19 illustrates the dynamic response comparison of the DC voltage and current before and after applying AVSG control when an inter-pole fault occurs in the BGC system.

From Figure 19a, it is evident that when the BGC system is not utilizing AVSG control, the DC voltage rapidly decreases after a fault occurrence due to the quick discharge of the DC-side capacitor. Within 5.75 ms, the voltage drops to zero. However, when AVSG control is applied, the rate of DC bus voltage decrease slows down, and it takes approximately 8 ms to reach zero. Compared to the scenario without AVSG control, the descent time has been extended by 39.13%. This extended time is beneficial for fault detection in the system, allowing more time for protective actions to be taken.

From Figure 19b, it can be observed that when the BGC system does not employ AVSG control, the fault current rapidly increases after a fault occurrence due to the quick discharge of the DC-side capacitor. The peak fault current reaches 962.55 A. However, when AVSG control is applied, the rate of fault current increase slows down, and the peak current is reduced to 734.74 A. Compared to the scenario without AVSG control, the peak fault current is reduced by 227.81 A. This helps to avoid excessive short-circuit current impacts and prevents device burnout.
Figure 19. The dynamic response comparison of the DC voltage and current. (a) shows the fault current waveform under different control strategies; (b) shows the fault voltage waveform under different control strategies.

6. Conclusions

In order to improve the dynamic performance of DC bus voltage, enhance the inertia of DC microgrid, and suppress the drastic fluctuation of DC bus voltage under the power disturbance in the network, this paper improves the traditional virtual inertia control strategy and proposes an adaptive virtual inertia control strategy based on model predictive control, and the following conclusions are obtained:

1. The model predictive control is used in the inner loop, and the two-step predictive delay compensation is used to realize the fast-tracking of the given current value, eliminating the traditional PI controller and PWM regulator and improving the dynamic performance of the control system.

2. The adaptive AVSG control is introduced in the outer loop. By combining the inertia coefficient in AVSG with the voltage change rate, the flexible adjustment of the inertia parameters is realized. The BGC system using this control strategy can quickly provide additional power when the power difference occurs in the DC microgrid, thereby enhancing the inertia of the DC microgrid and effectively improving the stability of DC bus voltage and the operation ability of the system under asymmetric conditions.

3. This study focuses exclusively on the developed four-terminal DC microgrid. When multiple grid-connected units or distributed energy sources are incorporated into the grid, complex systems impose stricter requirements on system stability and coordination among individual units. Further research is still needed in order to address these more stringent demands.

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Conflicts of Interest: The authors declare no conflict of interest.
Appendix A

Table A1. AC and DC microgrid control variable analogy.

<table>
<thead>
<tr>
<th>Analogy Term</th>
<th>VSG</th>
<th>AVSG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Droop equation</td>
<td>( \omega - P )</td>
<td>( u - i )</td>
</tr>
<tr>
<td>Control objective</td>
<td>( \omega )</td>
<td>( u )</td>
</tr>
<tr>
<td>Output</td>
<td>( P )</td>
<td>( i )</td>
</tr>
<tr>
<td>Inertia</td>
<td>( J )</td>
<td>( C )</td>
</tr>
<tr>
<td>Storage capacity</td>
<td>( \frac{1}{2}J\omega^2 )</td>
<td>( \frac{1}{2}Cv^2 )</td>
</tr>
</tbody>
</table>

Table A2. The relationship between the switching state and the output voltage component.

<table>
<thead>
<tr>
<th>Switch Status</th>
<th>( S_a )</th>
<th>( S_b )</th>
<th>( S_c )</th>
<th>( u_a )</th>
<th>( u_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.8165 Udc</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.4083 Udc</td>
<td>0.7071 Udc</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.4083 Udc</td>
<td>0.7071 Udc</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.8165 Udc</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.4083 Udc</td>
<td>0.7071 Udc</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.4083 Udc</td>
<td>0.7071 Udc</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A3. System simulation parameters.

<table>
<thead>
<tr>
<th>Simulation Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three – phase grid phase voltage ( u_{a,b,c} / V )</td>
<td>220</td>
</tr>
<tr>
<td>Rated voltage value ( U_a / V )</td>
<td>800</td>
</tr>
<tr>
<td>Filter inductance ( L/mH )</td>
<td>3</td>
</tr>
<tr>
<td>Filter resistors ( R_L / \Omega )</td>
<td>0.05</td>
</tr>
<tr>
<td>DC side capacitance ( C/ \mu F )</td>
<td>5000</td>
</tr>
<tr>
<td>Constant power load ( R/kW )</td>
<td>10</td>
</tr>
<tr>
<td>Virtual capacitance value ( C_v / \mu F )</td>
<td>1500</td>
</tr>
<tr>
<td>Voltage damping factor ( D_v )</td>
<td>5</td>
</tr>
<tr>
<td>Proportional factor ( k_p )</td>
<td>10</td>
</tr>
<tr>
<td>Integral factor ( k_i )</td>
<td>120</td>
</tr>
</tbody>
</table>

Table A4. DC bus dynamic response index when constant power load suddenly increases.

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Voltage Fluctuations</th>
<th>Voltage Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude /V</td>
<td>Time /s</td>
</tr>
<tr>
<td>PI + Fixed AVSG</td>
<td>8.2</td>
<td>0.21</td>
</tr>
<tr>
<td>MPC + Fixed AVSG</td>
<td>5.2</td>
<td>0.19</td>
</tr>
<tr>
<td>MPC + Adaptive AVSG</td>
<td>3.4</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Table A5. DC bus dynamic response index when constant power load suddenly decreases.

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Voltage Fluctuations Magnitude/V</th>
<th>Voltage Recovery Time t/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI + Fixed AVSG</td>
<td>9.8</td>
<td>0.22</td>
</tr>
<tr>
<td>MPC + Fixed AVSG</td>
<td>5.9</td>
<td>0.19</td>
</tr>
<tr>
<td>MPC + Adaptive AVSG</td>
<td>3.7</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table A6. THD analysis of the grid-side current under three control strategies.

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>THD/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI + Fixed AVSG</td>
<td>5.32</td>
</tr>
<tr>
<td>MPC + Fixed AVSG</td>
<td>3.88</td>
</tr>
<tr>
<td>MPC + Adaptive AVSG</td>
<td>2.98</td>
</tr>
</tbody>
</table>

References


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