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Review

Conformal Theory of Gravitation and Cosmic Expansion

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Abstract: The postulate of universal Weyl conformal symmetry for all elementary physical fields introduces nonclassical gravitational effects in both conformal gravitation (CG) and the conformal Higgs model (CHM). The resulting theory is found to explain major observed phenomena, including excessive galactic rotation velocities and accelerating Hubble expansion, without invoking dark matter (DM). The recent history of this development is surveyed here. The argument is confined to implications of classical field theory, which include galactic baryonic Tully–Fisher relationships and dark galactic haloes of a definite large radius. Cosmological CHM parameters exclude a massive Higgs boson but are consistent with a novel alternative particle of the observed mass.

Keywords: conformal gravity theory; conformal Higgs model; depleted galactic halo model

PACS: 04.20.Cv; 98.80.-k; 98.62.Gq

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Citation: Nesbet, R.K. Conformal Theory of Gravitation and Cosmic Expansion. *Symmetry* **2024**, *16*, 3. https://doi.org/10.3390/sym16010003

Academic Editors: Jose Antonio de Freitas Pacheco and Stefano Profumo

Received: 3 November 2023
Revised: 12 December 2023
Accepted: 13 December 2023
Published: 19 December 2023

1. Introduction

Conformal gravity (CG), reviewed in detail [1,2] by Mannheim, replaces the Einstein–Hilbert Lagrangian density by a quadratic contraction of the conformal Weyl tensor [3,4], invariant under Weyl scaling by an arbitrary scalar field. The implied Schwarzschild equation, in spherical geometry [5,6], fitted to observed galactic rotation [1], determines parameters with no need for dark matter (DM).

Conformal theory is limited here to symmetry defined by invariance of the coupled field action integral under Weyl scaling, defined below, without altering the postulated Riemannian geometry of Einstein–Hilbert general relativity. The further development of Weyl geometry [7], possibly relevant to the intense gravitation of black holes and the Big Bang model, is not considered here.

The Weyl tensor vanishes identically in the isotropic uniform geometry postulated to describe cosmic Hubble expansion, precluding explanation by CG. However, if the Higgs scalar field of particle theory Φ is postulated to have Weyl scaling symmetry, its Lagrangian density acquires a gravitational term [1]. The resulting conformal Higgs model (CHM) implies a modified Friedmann equation consistent with observed Hubble expansion [8].

In spherical geometry, the CHM Friedmann equation determines luminosity distance as a function of redshift $z \leq 1$ for observed supernovae as accurately as any alternative theory [8,9]. The Higgs symmetry-breaking mechanism produces the gravitational effect of a dark-energy-source cosmological constant [8]. Centrifugal cosmic acceleration persists in the present epoch $z \leq 1$. CG and CHM together have been found to determine the basic parameters of the Higgs model from observed galactic and cosmological data. These results are reviewed here.

2. Qualitative Regularities of Cosmological Data

Radial centripetal acceleration a implies $v^2 = ra$ for the velocity of circular geodesics. When it became possible to observe and measure velocities of matter in stable circular galactic orbits, v^2 was found to exceed Newtonian ra_N of observed baryonic matter by quite large factors and to remain nearly constant (flat) as r increases. This was well before the observation of galactic haloes with implied fixed cutoff boundaries. This motivated the MOND model [10,11]. MOND implies flat $v(r)$ for large r. For $a_N \ll a_0$, MOND $v^4 = a^2r^2 \to GMa_0$, the empirical baryonic Tully–Fisher relationship [12–15].

It has recently been shown [16] for 153 disc galaxies that observed acceleration a is effectively a universal function of Newtonian acceleration a_N, computed for the observed baryonic distribution. The empirical function has negligible observed scatter. Such a radial acceleration relationship (RAR) is a postulate of the empirical MOND model [10,11,17], which does not invoke dark matter (DM). It implies some very simple natural law.

Observed deviations from Newton/Einstein galactic gravitation have been modeled by distributed but unobserved dark matter. Typical DM halo models imply centripetal radial acceleration $a = a_N + \Delta a$, a function of radius in an assumed spherical galactic halo. DM Δa is added to baryonic Newtonian a_N. DM fits galactic rotation (orbital velocity vs. circular orbit radius) depending on model parameter central density ρ_0 and core radius r_0 for a DM halo distribution. Observed data imply that the surface density product $\rho_0 r_0 \simeq 100 M_\odot pc^{-2}$ is nearly constant for a large range of galaxies [18–20].

It will be shown here that the conformal postulate for gravitation and the Higgs scalar field implies all these regularities and derives values of the required parameters without dark matter.

3. Formalism for Unified Conformal Theory

Gravitational phenomena that cannot be explained by general relativity as formulated by Einstein are attributed to cold dark matter in the currently accepted ΛCDM paradigm for cosmology. Dark energy Λ remains without an explanation. The search for tangible dark matter has continued for many years with no conclusive results [21].

Consideration of an alternative paradigm is motivated by this situation. The postulate of universal conformal symmetry, requiring local Weyl scaling covariance [3] for all massless elementary physical fields [9], is a falsifiable alternative. Conformal symmetry, already valid for fermion and gauge boson fields [22], is extended to both the metric tensor field of general relativity and the Higgs scalar field of elementary-particle theory [23]. This postulate is satisfied by conformal gravity [1,2,5,6] and by the conformal Higgs model [8,9], eliminating the need for dark matter, with no novel elementary fields. The Higgs symmetry-breaking mechanism produces an effective cosmological constant driving Hubble expansion.

For fixed coordinates x^μ, local Weyl scaling is defined by $g_{\mu\nu}(x) \to g_{\mu\nu}(x)\Omega^2(x)$ [3] for arbitrary real differentiable $\Omega(x)$. Conformal symmetry is defined by invariant action integral $I = \int d^4x\sqrt{-g}\mathcal{L}$. For any Riemannian tensor $T(x)$, $T(x) \to \Omega^d(x)T(x) + \mathcal{R}(x)$ defines weight $d[T]$ and residue $\mathcal{R}[T]$. For a scalar field, $\Phi(x) \to \Phi(x)\Omega^{-1}(x)$, so that $d[\Phi] = -1$. Conformal Lagrangian density $\mathcal{L}$ must have weight $d[\mathcal{L}] = -4$ and residue $\mathcal{R}[\mathcal{L}] = 0$ up to a 4-divergence [1].

Variational theory for fields in general relativity is a straightforward generalization of classical field theory [24]. Given scalar Lagrangian density $\mathcal{L} = \sum_a \mathcal{L}_a$, action integral $I = \int d^4x\sqrt{-g}\mathcal{L}$ is required to be stationary for all differentiable field variations, subject to appropriate boundary conditions. g here is the determinant of metric tensor $g_{\mu\nu}$. Standard conservation laws follow from the variational principle.

Gravitational field equations are determined by metric functional derivative $X^{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta I}{\delta g_{\mu\nu}}$. Any scalar $\mathcal{L}_a$ determines energy-momentum tensor $\Theta_a^{\mu\nu} = -2X_a^{\mu\nu}$, evaluated for a solution of the coupled field equations. Generalized Einstein equation $\sum_a X_a^{\mu\nu} = 0$ is expressed as $X_g^{\mu\nu} = \frac{1}{2}\sum_{a\neq g}\Theta_a^{\mu\nu}$. Hence summed trace $\sum_a g_{\mu\nu}\Theta_a^{\mu\nu}$ vanishes for exact field solutions. Given $\delta\mathcal{L} = x^{\mu\nu}\delta g_{\mu\nu}$, metric functional derivative $\frac{1}{\sqrt{-g}}\frac{\delta I}{\delta g_{\mu\nu}}$ is $X^{\mu\nu} = x^{\mu\nu} + \frac{1}{2}\mathcal{L}g^{\mu\nu}$, evaluated for a solution of the field equations. Tensor $\Theta^{\mu\nu} = -2X^{\mu\nu}$ is symmetric.

Conformal gravity was introduced by Weyl [3,4] with the prospect of uniting electromagnetic theory and gravitation. This was shown to fail by Einstein, which sidetracked CG for many years. It was reconsidered by Mannheim as a possible source of observed deviations from general relativity [1,2]. The formidable Lagrangian density, a quadratic contraction of the Weyl tensor, simplified by elimination of a 4-divergence [25], reduces to two terms, quadratic, respectively, in the Ricci tensor and Ricci scalar [1]: $\mathcal{L}_g = -2\alpha_g(R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2)$. Conformal symmetry fixes the relative coefficient of the two quadratic terms.

In the conformal Higgs model, the unique Lagrangian density for scalar field Φ adds a gravitational term [1,8,26] to Higgs $-V$ [23]. The conformal Lagrangian density is $(\partial_\mu\Phi)^\dagger\partial^\mu\Phi - \frac{1}{6}R\Phi^\dagger\Phi$, where Ricci scalar $R = g_{\mu\nu}R^{\mu\nu}$ is the trace of gravitational Ricci tensor $R^{\mu\nu}$. This is augmented by Higgs $\Delta\mathcal{L}_\Phi = (w^2 - \lambda\Phi^\dagger\Phi)\Phi^\dagger\Phi$.

Metric tensor $g_{\mu\nu}$ is determined by conformal field equations derived from $\mathcal{L}_g + \mathcal{L}_\Phi$ [27], driven by energy-momentum tensor $\Theta_m^{\mu\nu}$, where subscript m refers to conventional matter and radiation. The gravitational field equation within halo radius r_H is

$$X_g^{\mu\nu} + X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}. \tag{1}$$

Defining mean galactic source density $\bar{\rho}_G$ and residual density $\hat{\rho}_G = \rho_G - \bar{\rho}_G$ and assuming $\Theta_m^{\mu\nu}(\rho) \simeq \Theta_m^{\mu\nu}(\bar{\rho}) + \Theta_m^{\mu\nu}(\hat{\rho})$, solutions for $r \leq r_G$ of the two equations

$$\begin{aligned} X_\Phi^{\mu\nu} &= \frac{1}{2}\Theta_m^{\mu\nu}(\bar{\rho}_G) \\ X_g^{\mu\nu} &= \frac{1}{2}\Theta_m^{\mu\nu}(\hat{\rho}_G) \end{aligned} \tag{2}$$

imply a solution to the full equation. The X_Φ equation has an exact solution for source density $\bar{\rho}$ in uniform, isotropic geometry (FLRW metric tensor) [8,9]. The two field equations are made compatible by imposing appropriate boundary conditions. Fitting constants of integration of source-free X_g at r_G [5] extends a combined solution out to halo radius r_H.

Conformal gravity is described in Schwarzschild metric [5]

$$ds_{ES}^2 = -B(r)c^2dt^2 + \frac{dr^2}{B(r)} + r^2d\omega^2, \tag{3}$$

which defines Schwarzschild potential $B(r)$, assuming spherical symmetry. The conformal Higgs model is described in the FLRW metric

$$ds_\Phi^2 = -c^2dt^2 + a^2(t)(\frac{dr^2}{1-kr^2} + r^2d\omega^2), \tag{4}$$

which defines Friedmann scaling factor $a(t)$. The Higgs model breaks both gauge and conformal symmetry. Compatibility requires a hybrid metric with $k = 0$, such as

$$ds^2 = -B(r)c^2dt^2 + a^2(t)(\frac{dr^2}{B(r)} + r^2d\omega^2), \tag{5}$$

4. Depleted Galactic Halo Model

In the ΛCDM model, an isolated galaxy is considered to be surrounded by a much larger spherical dark matter halo. What is actually observed is a halo of the gravitational field that deflects photons in gravitational lensing and increases the velocity of orbiting mass particles. Conformal theory [1,2,9], which modifies both Einstein–Hilbert general relativity and the Higgs scalar field model, supports an alternative interpretation of lensing and anomalous rotation as gravitational effects due to depletion of the cosmic background by concentration of diffuse primordial mass into an observed galaxy [27].

In standard theory, Poisson's equation determines a distribution of dark matter for any unexplained gravitational field. If dark matter interacts only through gravity, the concept of dark matter provides a compact description of observed phenomena, not a falsifiable explanation. Postulated universal conformal symmetry [9] promises a falsifiable alternative free of novel elementary fields.

As a galaxy forms, background matter density ρ_m condenses into observed galactic density ρ_g. Conservation of mass and energy requires total galactic mass M to be missing from a depleted background. Since the primordial density is uniform and isotropic, the depleted background can be modeled by an empty sphere of radius r_H, such that $4\pi\rho_m r_H^3/3 = M$. In particular, the integral of $\rho_g - \rho_m$ must vanish. The resulting gravitational effect, derived from conformal theory, describes a dark-depleted halo. This halo model accounts for the otherwise remarkable fact that galaxies of all shapes are embedded in essentially spherical haloes. Conformal theory relates the acceleration parameter of anomalous galactic rotation to the observed acceleration of Hubble expansion.

Assuming that galactic mass is concentrated within an average radius r_g, the ratio of radii r_H/r_g should be large, the cube root of the mass–density ratio ρ_g/ρ_m. Thus, if the latter ratio is $8^3 = 512$, a galaxy of radius 15 kpc would be accompanied by a halo of radius $8 \times 15 = 120$ kpc. Equivalence of galactic and deleted halo mass resolves the paradox for ΛCDM such that, despite any interaction other than gravity, the amount of dark matter inferred for a galactic halo is strongly correlated with the galactic luminosity or baryonic mass [28,29].

5. Conformal Gravity

Given spherical mass-energy density enclosed within $r \leq \bar{r}$, the X_g field equation in the ES metric is [5,6]

$$\partial_r^4(rB(r)) = rf(r), \tag{6}$$

where $f(r)$ is determined by the source energy-momentum tensor.

An exact solution of the tensorial X_g equation, for source-free $r \geq \bar{r}$, derived by [1,5,6], is

$$y_0(r) = rB(r) = -2\beta + \alpha r + \gamma r^2 - \kappa r^3. \tag{7}$$

This adds two constants of integration to the classical external potential: nonclassical radial acceleration γ and halo cutoff parameter κ [1,27]. Derivative functions $y_i(r) = \partial_r^i(rB(r)), 0 \leq i \leq 3$ satisfy differential equations

$$\begin{aligned} \partial_r y_i = y_{i+1}, 0 \leq i \leq 2, \\ \partial_r y_3 = rf(r). \end{aligned} \tag{8}$$

Independent constants $a_i = y_i(0)$ determine coefficients $\beta, \alpha, \gamma, \kappa$ such that at endpoint $\bar{r}$

$$\begin{aligned} y_0(\bar{r}) &= -2\beta + \alpha\bar{r} + \gamma\bar{r}^2 - \kappa\bar{r}^3, \\ y_1(\bar{r}) &= \alpha + 2\gamma\bar{r} - 3\kappa\bar{r}^2, \\ y_2(\bar{r}) &= 2\gamma - 6\kappa\bar{r}, \\ y_3(\bar{r}) &= -6\kappa. \end{aligned} \tag{9}$$

To avoid a singularity at the origin, $a_0 = 0$. γ and κ must be determined consistently with the X_Φ equation. Specified values can be fitted by adjusting a_1, a_2, a_3, subject to $a_0 = 0, \alpha = 1$ [1]. The solution for $B(r)$ remains regular if $\gamma/\beta \simeq 0$. If Ricci scalar $R(r \to 0)$ is unconstrained, a particular solution for $r \leq \bar{r}$ is given with specified constants $a_0 = 0, \alpha, \gamma, \kappa$ [30]:

$$\begin{aligned} rB(r) = y_0(r) = \alpha r - \frac{1}{6}\int_0^r q^4 f dq - \frac{1}{2} r \int_r^{\bar{r}} q^3 f dq \\ + \gamma r^2 + \frac{1}{2} r^2 \int_r^{\bar{r}} q^2 f dq - \kappa r^3 - \frac{1}{6} r^3 \int_r^{\bar{r}} q f dq. \end{aligned} \tag{10}$$

Integrated parameter $2\beta = \frac{1}{6}\int_0^{\bar{r}} q^4 f dq$. This particular solution differs from [1,5], replacing $-\frac{1}{2}r^2 \int_0^r q^2 f dq$ by $\gamma r^2 + \frac{1}{2}r^2 \int_r^{\bar{r}} q^2 f dq$ [30]. Constant γ is determined independently of the CG equation. At the galactic center, $r \to 0$ implies regular $B(r)$ but singular Ricci $R(r)$ for this solution. This is consistent with the development of a massive black hole, recently observed for many galaxies.

For uniform density $\bar{\rho}$ the Weyl tensor vanishes, so that $X_g^{\mu\nu} \equiv 0$ for a uniform, isotropic cosmos. Schwarzschild potential,

$$B(r) = -2\beta/r + \alpha + \gamma r - \kappa r^2, \quad (11)$$

is exact for a source-free region outside a spherical mass-energy source density [5,6], in particular, within an empty galactic halo.

For a test particle in a stable exterior circular orbit with velocity v, the centripetal acceleration is $a = v^2(r)/r = \frac{1}{2}B'(r)c^2$. Newtonian $B(r) = 1 - 2\beta/r$, where $\beta = GM/c^2$, so that classical acceleration $a_N = \beta c^2/r^2 = GM/r^2$. CG adds nonclassical $\Delta a = \frac{1}{2}c^2\gamma - c^2\kappa r$, a universal constant when halo cutoff $2\kappa r/\gamma \simeq r/r_H$ can be neglected [30]. Orbital velocity squared is the sum of $v^2(a_N; r)$ and $v^2(\Delta a; r)$, which cross with equal and opposite slope at some $r = r_{TF}$, if $2\kappa r/\gamma$ can be neglected. This defines a flat range of $v(r)$ centered at stationary point r_{TF}, without constraining behavior at large r.

MOND [10,11,21] modifies the Newtonian force law for acceleration below an empirical scale a_0. Using $y = a_N/a_0$ as an independent variable, for assumed universal constant $a_0 \simeq 10^{-10}$ m/s^2, MOND postulates an interpolation function $\nu(y)$ such that observed radial acceleration $a = f(a_N) = a_N\nu(y)$. A flat velocity range approached asymptotically requires $a^2 \to a_N a_0$ as $a_N \to 0$. For $a_N \ll a_0$, MOND $v^4 = a^2r^2 \to GMa_0$, the empirical baryonic Tully–Fisher relationship [12–15].

In conformal gravity (CG), centripetal acceleration $a = v^2/r$ determines the exterior orbital velocity $v^2/c^2 = ra/c^2 = \beta/r + \frac{1}{2}\gamma r - \kappa r^2$, compared with asymptotic $ra_N/c^2 = \beta/r$. Assuming Newtonian constant $\beta = \beta_N$ and neglecting $2\kappa r/\gamma$, the slope of $v^2(r)$ vanishes at $r_{TF}^2 = 2\beta/\gamma$. This implies that $v^4(r_{TF})/c^4 = (\beta/r_{TF} + \frac{1}{2}\gamma r_{TF})^2 = 2\beta\gamma$ [15,31]. This is the Tully–Fisher relationship, exactly at stationary point r_{TF} of the $v(r)$ function. The implied MOND acceleration constant is $a_0 = 2\gamma c^2 = 1.14 \times 10^{-10}$ m/s^2 [30].

6. The Conformal Higgs Model

The Newton-Einstein model of galactic growth and interaction is substantially altered by conformal theory [8,27]. A gravitational halo is treated as a natural consequence of galaxy formation. Condensation of matter from the primordial uniform mass-energy distribution leaves a depleted sphere that has an explicit gravitational effect. The implied subtracted mass, which integrates into the total galactic mass, cannot be ignored. If this mass were simply removed, the analogy to vacancy scattering in solids implies a lensing effect. A background geodesic is no longer a geodesic in the empty sphere [27].

The conformal gravitational field equation is

$$X_g^{\mu\nu} + X_\Phi^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}, \quad (12)$$

where index m refers to matter and radiation. An exact solution inside the depleted halo radius is given by

$$X_g^{\mu\nu} = \frac{1}{2}\Theta_m^{\mu\nu}(\rho_g), X_\Phi^{\mu\nu} = 0, r \le r_H. \quad (13)$$

The exact source-free solution [5] of the X_g equation is valid in the external halo, $r_g \le r \le r_H$, because ρ_g vanishes. Constants of integration fitted at r_g and r_H.

In uniform, isotropic geometry with uniform mass/energy density $\bar{\rho}$, $\mathcal{L}_\Phi$ implies a modified Friedmann equation [8,9,32] for cosmic distance scale factor $a(t)$, in FLRW metric

$$ds_\Phi^2 = -c^2dt^2 + a^2(t)(\frac{dr^2}{1-kr^2} + r^2d\omega^2), \quad (14)$$

which determines Friedmann scaling factor $a(t)$. With $a(t_0) = 1$ at present time t_0:

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\ddot{a}}{a} = \frac{2}{3}(\bar{\Lambda} + \bar{\tau}c^2\bar{\rho}(t)). \tag{15}$$

Consistency with the X_g equation requires $k = 0$. $\bar{\Lambda} = \frac{3}{2}w^2 \geq 0$ and $\bar{\tau} \sim -3/\phi_0^2 \leq 0$ are determined by parameters of the Higgs model [8]. For uniform $\rho = 0$ within the halo radius, this solves the second of Equation (13).

Dividing the modified equation by $(\dot{a}/a)^2$ determines dimensionless sum rule $\Omega_m + \Omega_\Lambda + \Omega_k + \Omega_q = 1$. Scale factor $a(t)$ determines dimensionless Friedmann acceleration weight $\Omega_q(t) = \frac{\ddot{a}a}{\dot{a}^2}$ [8]. Radiation energy density is included in Ω_m here. The dimensionless weights are $\Omega_m(t)$ for mass–density, $\Omega_k(t)$ for curvature, and $\Omega_\Lambda(t)$ for dark energy, augmented by acceleration weight $\Omega_q(t)$. Galactic rotation velocities, observed at relatively small redshifts, determine radial acceleration values, which can be compared with acceleration weights inferred from Hubble expansion data. Omitting Ω_m completely, with $k = 0$, conformal sum rule $\Omega_\Lambda(t) + \Omega_q(t) = 1$ fits observed data accurately for redshifts $z \leq 1$ (7.33 Gyr) [8,26,30]. This eliminates any need for dark matter to explain Hubble expansion.

The X_Φ equation includes dark energy, present regardless of density ρ. This produces a centrifugal acceleration of background Hubble expansion [8,9], which must be subtracted to compute observable radial acceleration. Observed geodesics, whose bending determines the extragalactic centripetal acceleration responsible for lensing and anomalous orbital rotation velocities, are defined relative to the cosmic background.

Defining $\frac{\dot{a}}{a} = h(t)H_0$, for Hubble constant H_0, it is convenient to use Hubble units, such that $c = \hbar = 1$, and dimensionless $a(t_0) = 1, h(t_0) = 1$. The units for frequency, energy, and acceleration are $H_0, \hbar H_0, cH_0$, respectively. Setting $\Omega_k = \Omega_m = 0$, for $\alpha = w^2 = \Omega_\Lambda(t_0)$, the CHM Friedmann equation [8,9] is

$$\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} = \frac{2}{3}\bar{\Lambda} = \alpha. \tag{16}$$

In Hubble units, Equation (16) reduces to $\frac{d}{dt}h(t) = -\alpha$. The explicit solution for $t \leq t_0$ is

$$\begin{aligned} h(t) = \frac{\dot{a}}{a}(t) = \frac{d}{dt}\ln a(t) = 1 + \alpha(t_0 - t), \\ \ln a(t) = -(t_0 - t) - \frac{1}{2}\alpha(t_0 - t)^2, \\ a(t) = \exp[-(t_0 - t) - \frac{1}{2}\alpha(t_0 - t)^2]. \end{aligned} \tag{17}$$

From the definition of redshift $z(t)$:
$1 + z(t) = \frac{1}{a(t)} = \exp[(t_0 - t) + \frac{1}{2}\alpha(t_0 - t)^2]$,
$(t_0 - t)^2 + \frac{2}{\alpha}(t_0 - t) = \frac{2}{\alpha}\ln(1 + z)$.
In Hubble units with $\frac{\dot{a}}{a}(t_0) = 1$, this implies

$$\begin{aligned} t_0 - t(z) = (\sqrt{2\alpha\ln(1+z) + 1} - 1)/\alpha, \\ \frac{dt}{dz} = \frac{-1}{(1+z)\sqrt{2\alpha\ln(1+z) + 1}}. \end{aligned} \tag{18}$$

If $\alpha = 0.732$ and $z = 1$, $(t_0 - t)/H_0 = 7.33Gyr$, for $H_0 = 67.8 \pm 0.9$ km/s/Mpc $= 2.197 \times 10^{-18}$/s [33].

Neglecting both curvature weight Ω_k and cosmic mass/energy weight Ω_m, conformal sum rule $\Omega_\Lambda + \Omega_q = 1$ holds for acceleration weight $\Omega_q = \frac{\ddot{a}a}{\dot{a}^2}$. For $\Omega_k = 0$, luminosity distance $d_L(z) = (1+z)\chi(z)$ [26], where

$$\chi(z) = \int_{t(z)}^{t_0} \frac{dt}{a(t)} = \int_z^0 d\bar{z}(1+\bar{z})\frac{dt}{dz}(\bar{z}) = \int_0^z \frac{d\bar{z}}{\sqrt{2\alpha\ln(1+\bar{z})+1}}. \quad (19)$$

Evaluated for $\alpha = \Omega_\Lambda(t_0) = 0.732$, the fit to scaled luminosity distances $H_0 d_L/c$ from Hubble expansion data [8,34] is given in Table 1.

Table 1. Scaled luminosity distance fit to Hubble data.

z	Ω_Λ	Ω_q	Theory $H_0 d_L/c$ Equation (19)	Observed $H_0 d_L/c$ [34]
0.0	0.732	0.268	0.0000	0.0000
0.2	0.578	0.422	0.2254	0.2265
0.4	0.490	0.510	0.5013	0.5039
0.6	0.434	0.566	0.8267	0.8297
0.8	0.393	0.607	1.2003	1.2026
1.0	0.363	0.637	1.6209	1.6216

Observed redshifts have been fitted to an analytic function [34] with statistical accuracy comparable to the best standard ΛCDM fit, with $\Omega_m = 0$. Table 1 compares CHM $d_L(z)$ to this Mannheim function. Because $\bar{\tau}$ is negative [1,8], cosmic acceleration Ω_q remains positive (centrifugal) back to the earliest time [8].

Conformal Friedmann Equation (15) [8,26] determines cosmic acceleration weight Ω_q. With both weight parameters Ω_k and Ω_m set to zero, Equation (15) fits scaled Hubble function $h(t) = H(t)/H_0$ for redshifts $z \leq 1$, Table 1, as accurately as standard ΛCDM, with only one free constant. This determines Friedmann weights, at the present time t_0, $\Omega_\Lambda = 0.732, \Omega_q = 0.268$ [8]. Hubble constant $H(t_0) = H_0 = 2.197 \times 10^{-18}/s$ [33] is independent of these data. For uniform primordial energy-momentum source density ρ_m, the dimensionless sum rule [8] with $\Omega_k = 0$ determines $\Omega_q(\rho_m) = 1 - \Omega_\Lambda - \Omega_m$ in the cosmic background, and $\Omega_q(0) = 1 - \Omega_\Lambda$ in the depleted halo [27].

Schwarzschild $B(r)$ parameter $\gamma > 0$, independent of galactic mass and structure, implies centripetal acceleration due to an isotropic cosmological source [31]. The parametrized gravitational field forms a spherical halo [9]. The depleted halo model removes a particular conceptual problem in fitting $B(r)$ parameters γ, κ to galactic rotation data [1,31,35]. In empirical parameter $\gamma = \gamma^* N^* + \gamma_0$, γ_0 does not depend on galactic mass, so it must be due to the surrounding cosmos [31]. Since the interior term, coefficient $\gamma^* N^*$, is centripetal, one might expect the term in γ_0 to be centrifugal, describing an attraction to exterior sources. However, coefficient γ, derived here for a depleted halo, determines net centripetal acceleration, in agreement with observation [30]. Recent galactic rotation results for galaxies with independently measured mass [16] imply that γ cannot depend on galactic mass [36].

The conformal Friedmann cosmic evolution equation implies dimensionless cosmic acceleration parameters $\Omega_q(\rho)$ [27] which are locally constant but differ across the halo boundary r_H. Smooth evolution of the cosmos implies observable particle acceleration γ within r_H proportional to $\Omega_q(in) - \Omega_q(out) = \Omega_q(0) - \Omega_q(\rho_m)$. Uniform cosmological ρ_m implies constant γ for $r \leq r_H$, independent of galactic mass [30]. This result is consistent with recent observations of galactic rotational velocities for galaxies with directly measured mass [16,36].

From the CHM, observed nonclassical gravitational acceleration $\frac{1}{2}\gamma c^2$ in the halo is proportional to $\Delta\Omega_q = \Omega_q(0) - \Omega_q(\rho_m) = \Omega_m(\rho_m)$ [27], where, given ρ_m and H_0, $\Omega_m(\rho_m) = \frac{2}{3}\frac{\bar{\tau}c^2\rho_m}{H_0^2}$ [8]. Positive ρ_m implies $\Omega_m < 0$ because coefficient $\bar{\tau} < 0$ [1,8]. Therefore, the depleted halo model determines constant γ from uniform, universal cosmic baryonic mass–density ρ_m/c^2, which includes radiation energy density here.

$\Omega_m < 0$ implies centripetal acceleration, converted from Hubble units, $\frac{1}{2}\gamma c^2 = -cH_0\Omega_m(\rho_m)$ [27]. Hence, $\Delta\Omega_q = \Omega_m < 0$ is consistent with nonclassical centripetal acceleration, confirmed by inward deflection of photon geodesics observed in gravitational lensing [27]. This logic is equivalent to requiring radial acceleration to be continuous across halo boundary r_H:

$$\frac{1}{2}\gamma c^2 - cH_0\Omega_q(0) = -cH_0\Omega_q(\rho_m). \tag{20}$$

Signs here follow from the definition of Ω_q as centrifugal acceleration weight.

For a single spherical solar mass isolated in a galactic halo, mean internal mass–density $\bar{\rho}_\odot$ within $r_\odot$ determines an exact solution of the conformal Higgs gravitational equation, giving internal acceleration $\Omega_q(\bar{\rho}_\odot)$. Given γ outside $r_\odot$, continuous acceleration across boundary $r_\odot$,

$$\frac{1}{2}\gamma_{\odot,in}c^2 - cH_0\Omega_q(\bar{\rho}_\odot) = \frac{1}{2}\gamma c^2 - cH_0\Omega_q(0), \tag{21}$$

determines constant $\gamma_{\odot,in}$ valid inside $r_\odot$. $\gamma_{\odot,in}$ is determined by local mean source density $\bar{\rho}_\odot$. γ in the halo is not changed. Its value is a constant of integration that cannot vary in the source-free halo [27,30]. Hence, there is no way to determine a mass-dependent increment to γ. This replaces the usually assumed $\gamma_0 + N^*\gamma^*$ by γ_H determined at halo boundary r_H.

7. Values of Relevant Parameters

Anomalous rotation velocities for 138 galaxies are fitted using only four universal CG parameters $\beta^*, \gamma^*, \gamma_0, \kappa$ [1,2,35,37,38] such that $\beta = N^*\beta^* = GM/c^2, \gamma = \gamma_0 + N^*\gamma^*$. N^* is galactic baryonic mass M in solar mass units. Inferred parameter values [1,37],

$$\beta^* = 1.475 \times 10^3\text{m}, \gamma_0 = 3.06 \times 10^{-28}/\text{m},$$
$$\gamma^* = 5.42 \times 10^{-39}/\text{m}, \kappa = 9.54 \times 10^{-50}/\text{m}^2, \tag{22}$$

fit conformal gravity to galactic rotation velocities. The observed radial acceleration relationship (RAR) [16] is consistent with the CHM conclusion that γ^* must vanish. Then γ is a universal constant [36]. Using data for the Milky Way galaxy, $\gamma = \gamma_0 + N^*\gamma^* = 6.35 \times 10^{-28}/\text{m}$ [38,39].

$\zeta > 0$ for computed $R(t)$ [8] implies $\lambda < 0$. $\hbar\phi(t_0) = 174$ GeV [40]= $1.203 \times 10^{44}\hbar H_0$ in Hubble units. For $\Omega_m = 0$, $\zeta(t_0) = 2\Omega_q(t_0) = 0.536$. For $\lambda(t) = \zeta/(-2\phi^2)$ and $\phi(t_0) = \phi_0$, dimensionless $\lambda(t_0) = -0.185 \times 10^{-88}$.

It is widely assumed that negative Higgs λ would imply an unstable physical vacuum, but the present analysis does not support this conclusion. The conformal Higgs scalar field does not have a well-defined mass, instead inducing dynamical w^2, which acts as a cosmological constant determining dark energy. Nearly constant $\dot{\phi}/\phi$ for redshifts $z \leq 1$ is shown here to imply Higgs $\lambda < 0$, consistent with a dynamical origin. Negative empirical λ implies finite but 'tachyonic' mass for a scalar field fluctuation. This does not support the conventional concept of a massive Higgs particle [41].

Scalar field $ZZ = g_{\mu\nu}Z^{\mu*}Z^\nu$ interacts with neutral scalar field $WW = g_{\mu\nu}W_-^\mu W_+^\nu$ through the exchange of quarks and leptons. There is no contradiction in treating resulting scalar diboson W_2 as an independent field or particle, in analogy to atoms, molecules, and nuclei. Neutral W_2 dresses scalar field Φ to produce biquadratic Lagrangian term $\lambda(\Phi^\dagger\Phi)^2$ [41]. If the mass of W_2 is 125 GeV, and the conformal Higgs λ assumes its empirical value. This implies an alternative identification of the recently observed LHC resonance [42–44]. The interacting bare fields produce relatively stable $W_2 = WW\cos\theta_x + ZZ\sin\theta_x$ and complementary resonance $Z_2 = -WW\sin\theta_x + ZZ\cos\theta_x$ [41]. W_2 can dress the bare Φ field while maintaining charge neutrality. Assumed W_2 mass 125 GeV implies $\lambda = -0.455 \times 10^{-88}$ [41], consistent with empirical value $\lambda(t_0) \simeq -10^{-88}$.

Parameters w^2 and λ are defined as positive constants in Higgs [23,45]

$$\Delta\mathcal{L}_\Phi = -V(\Phi^\dagger\Phi) = w^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2. \tag{23}$$

Stationary action implies constant finite $\Phi^\dagger\Phi = \phi_0^2 = w^2/2\lambda$, where $\hbar\Phi$ and $\hbar w$ are energies. The conformal Higgs model [8] acquires an additional term in $\Delta\mathcal{L}_\Phi$: $-\frac{1}{6}R\Phi^\dagger\Phi$ [1] for gravitational Ricci scalar $R = g_{\mu\nu}R^{\mu\nu}$. The modified Lagrangian, defined for neutral gauge field Z_μ [26], determines w^2, which becomes a cosmological constant in the conformal Friedmann cosmic evolution equation [8]. Lagrangian term $w^2\Phi^\dagger\Phi$ is due to induced neutral Z_μ field amplitude [26], which dresses the bare scalar field. Finite w^2 breaks conformal symmetry but does not determine the Higgs mass.

The conformal scalar field equation including parametrized $\Delta\mathcal{L}_\Phi$ is [1,8]

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\Phi) = (-\frac{1}{6}R + w^2 - 2\lambda\Phi^\dagger\Phi)\Phi. \tag{24}$$

Ricci scalar R introduces gravitational effects. Only real-valued solution $\phi(t)$ is relevant in uniform, isotropic geometry. Defining $V(\phi) = \frac{1}{2}(\zeta + \lambda\phi^2)\phi^2$, where $\zeta(t) = \frac{1}{6}R - w^2$, the field equation is

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} = -(\zeta(t) + 2\lambda\phi^2). \tag{25}$$

Omitting R and assuming constant $\lambda > 0$ and w^2, Higgs solution $\phi_0^2 = w^2/2\lambda$ [45] is exact. All-time derivatives drop out. In the conformal scalar field equation, the cosmological time dependence of Ricci scalar $R(t)$, determined by the CHM Friedmann cosmic evolution equation introduces nonvanishing time derivatives and implies $\lambda < 0$ [8]. An exact solution for $\phi(t)$ should include the time derivative terms in $\zeta(t)$. Neglecting them here limits the argument to redshift $z \leq 1$.

Ricci scalar $R(t)$ drops out of the scalar field equations if time derivatives are neglected. This justifies an estimate of Higgs parameter w^2 [8]. Dressing scalar field Φ by neutral gauge field Z^μ produces Lagrangian term $w^2\Phi^\dagger\Phi$. Defining $\Delta I_\Phi = \int d^4x\sqrt{-g}\Delta\mathcal{L}_\Phi$ from Equation (23), the parametrized effective potential term in the scalar field equation is given by

$$\frac{1}{\sqrt{-g}}\frac{\delta\Delta I_\Phi}{\delta\Phi^\dagger} = (w^2 - 2\lambda\Phi^\dagger\Phi)\Phi. \tag{26}$$

Gauge invariance replaces bare derivative ∂_μ by gauge covariant derivative [23]

$$D_\mu = \partial_\mu - \frac{i}{2}g_z Z_\mu. \tag{27}$$

This retains $\mathcal{L}_Z$ in terms of $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ and augments conformal

$$\mathcal{L}_\Phi^0 = (\partial_\mu\Phi)^\dagger\partial^\mu\Phi - \frac{1}{6}R\Phi^\dagger\Phi \tag{28}$$

by coupling term

$$\Delta\mathcal{L} = (D_\mu\Phi)^\dagger D^\mu\Phi - (\partial_\mu\Phi)^\dagger\partial^\mu\Phi =$$
$$\frac{i}{2}g_z Z_\mu^*\Phi^\dagger\partial^\mu\Phi - \frac{i}{2}g_z Z^\mu(\partial_\mu\Phi)^\dagger\Phi + \frac{1}{4}g_z^2\Phi^\dagger Z_\mu^* Z^\mu\Phi. \tag{29}$$

Parametrized for a generic complex vector field [23],

$$\Delta\mathcal{L}_Z = \frac{1}{2}m_Z^2 Z_\mu^* Z^\mu - \frac{1}{2}(Z_\mu^* J_Z^\mu + Z^\mu J_{Z\mu}^*), \tag{30}$$

given mass parameter m_Z and source current density J_Z^μ. The field equation for parametrized Z^μ is [23]

$$\partial_\nu Z^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta \Delta I_Z}{\delta Z_\mu^*} = m_Z^2 Z^\mu - J_Z^\mu. \tag{31}$$

$\Delta\mathcal{L}$ determines parameters for field Z^μ:

$$\frac{2}{\sqrt{-g}}\frac{\delta \Delta I}{\delta Z_\mu^*} = \frac{1}{2}g_z^2 \Phi^\dagger \Phi Z^\mu + i g_z \Phi^\dagger \partial^\mu \Phi. \tag{32}$$

This implies Higgs mass formula $m_Z^2 = \frac{1}{2}g_z^2 \Phi^\dagger \Phi$ and field source density $J_Z^\mu = -i g_z \Phi^\dagger \partial^\mu \Phi$. Using $\Delta\mathcal{L}$ derived from the covariant derivative,

$$\frac{1}{\sqrt{-g}}\frac{\delta \Delta I_\Phi}{\delta \Phi^\dagger} = \frac{1}{4}g_z^2 Z_\mu^* Z^\mu \Phi + \frac{i}{2}g_z (Z_\mu^* + Z_\mu)\partial^\mu \Phi. \tag{33}$$

Comparison with Equation (26) implies $w^2 = \frac{1}{4}g_z^2 Z_\mu^* Z^\mu$. Neglecting derivatives of Z^μ, Equation (31) reduces to

$$Z^\mu = J_Z^\mu / m_Z^2 = -\frac{2i}{g_z}\Phi^\dagger \partial^\mu \Phi / \Phi^\dagger \Phi. \tag{34}$$

Then $|Z^0|^2 = \frac{4}{g_z^2}(\frac{\dot\phi}{\phi})^2$, so that the scalar field equation implies nonvanishing $w^2 = \frac{1}{4}g_z^2 |Z^0|^2 = (\frac{\dot\phi}{\phi})^2$. Implied pure imaginary Z^μ does not affect parameter λ.

8. Alternative Models of Observed Data

Numerous dark matter studies of galactic halo gravitation depend on models with core radius r_0 and central density ρ_0. Central surface density product $\rho_0 r_0$ is found to be nearly a universal constant of order $100 M_\odot pc^{-2}$ for a large range of galaxies [18–20]. Conformal theory implies nonclassical centripetal acceleration Δa, for Newtonian a_N due to observable baryonic matter. Neglecting the halo cutoff for $r \ll r_H$, conformal Δa is constant over the halo, and $a = a_N + \Delta a$ is a universal function, consistent with a recent study of galaxies with independently measured mass [16]. An equivalent dark matter source would be a pure cusp distribution with a cutoff parameter determined by the halo boundary radius. This is shown here to imply universal central surface density for any dark matter core model.

CG implies $\Delta a = \frac{1}{2}\gamma c^2 (1 - r/r_H)$ in a depleted halo. For $r \ll r_H$, CG acceleration constant γ [1,36] predicts $\Delta a = \frac{1}{2}\gamma c^2$ [27,36] for all DM models.

Uniform constant Δa puts a severe constraint on any DM model. The source density must be of the form ξ/r, a pure radial cusp [30,36], where constant $\xi = \Delta a / 2\pi G = 0.06797\ \mathrm{kg/m^2} = 32.535 M_\odot / pc^2$ is determined by CG parameter γ, with CODATA $G = 6.67384 \times 10^{-11}\ \mathrm{m^3 s^{-2} kg^{-1}}$ [46].

A DM galactic model equivalent to conformal theory would imply uniform DM radial acceleration $\Delta a = 2\pi G \xi$, attributed to radial DM density ξ/r for universal constant ξ, modified at large galactic radius by a halo cutoff function. Enclosed mass $M_r = 2\pi \xi r^2$ implies $r\Delta a / c^2 = G M_r / r$. DM models avoid a distribution cusp by assuming finite central core density.

The distribution of source matter within a sphere cannot affect asymptotic gravitational acceleration. For arbitrary r_0, asymptotic radial acceleration is unchanged if mass within r_0

is redistributed to uniform density $\rho(r)$ within a sphere of this radius. Conformal density ξ/r implies mass $M_0 = 2\pi\xi r_0^2$ in volume $V_0 = \frac{4\pi}{3}r_0^3$. For a DM spherical model core that replaces a central cusp density, conformal theory implies constant $\rho(r_0)r_0 = r_0 M_0/V_0 = 3\xi/2$. For assumed PI core DM density [18] $\rho(r) = \rho_0 r_0^2/(r^2 + r_0^2)$, central $\rho_0 = 2\rho(r_0)$. Hence, for a PI core, $\rho_0 r_0 = 3\xi = \frac{3\Delta a}{2\pi G} = 0.204 \times 10^{-2}$ kg/m^2 $= 97.6 M_\odot pc^{-2}$, independent of r_0.

A radial acceleration relationship RAR [16] is postulated by MOND [10]. Paradigms ΛCDM and CG/CHM both represent galactic radial acceleration by $a_N + \Delta a$, for baryonic Newtonian a_N. The RAR requires Δa to be a universal constant [36], for $r \ll r_H$. Given a_N, the three paradigms differ only in the behavior of Δa for large galactic radius.

Milky Way Tully–Fisher radius $r_{TF} = 17.2$ kpc. Halo radius $r_H = 107.8$ kpc [27,30], for $r_G \simeq 15.0$ kpc. Implied MOND constant $a_0 = 2\gamma c^2 = 1.14 \times 10^{-10}$ m/s^2. Outside r_G, $a_N \simeq \beta c^2/r^2$. Then a(CDM)$= a_N + \frac{1}{2}\gamma c^2$, using empirical CG Δa but omitting halo cutoff; a(CG) $= a_N + \frac{1}{2}\gamma c^2(1 - r/r_H)$, including halo cutoff; and a(MOND [16])$\simeq a_N/(1 - e^{-\sqrt{a_N/a_0}})$, just MOND with a particular interpolation function and $a_0 = 1.20 \times 10^{-10}$ m/s^2. $a(CDM)$ is generic for any model with universal constant Δa.

Table 2 [36] compares detailed predictions for the implied external orbital velocity curve of the Milky Way galaxy. The CDM curve rises gradually, and the CG curve remains remarkably flat, while the MOND [16] curve falls gradually toward asymptotic velocity.

Table 2. Milky Way: radial acceleration a (10^{-10}m/s^2).

r kpc	a_N	CDM a	CDM $10^3\frac{v}{c}$	CG a	CG $10^3\frac{v}{c}$	MOND a	MOND $10^3\frac{v}{c}$
15	0.376	0.661	0.584	0.621	0.566	0.877	0.672
20	0.212	0.497	0.584	0.444	0.552	0.617	0.650
25	0.135	0.420	0.601	0.354	0.551	0.475	0.638
30	0.094	0.379	0.625	0.300	0.556	0.385	0.630
35	0.069	0.354	0.652	0.261	0.560	0.324	0.624
40	0.053	0.338	0.682	0.232	0.565	0.279	0.619
45	0.042	0.327	0.717	0.208	0.567	0.246	0.616
50	0.034	0.319	0.740	0.187	0.566	0.219	0.613

9. Conclusions

The postulate of universal conformal symmetry is shown to be in good agreement with gravitational phenomena wherever tested. Implied values of Higgs scalar field parameters are also found to be valid. The resulting gravitational paradigm can be considered a valid alternative to ΛCDM and MOND, justifying wider study and application. Selection among theoretical models depends on the improvement of the precision of observations of galactic rotation at large galactic radius, which is currently scarce and of limited accuracy.

Funding: This research received no external funding.

Data Availability Statement: All data is from cited publications.

Conflicts of Interest: The author declares no conflict of interest.

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