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Soft Faint Continuity and Soft Faint Theta Omega Continuity between Soft Topological Spaces

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Abstract: The concepts of soft faint continuity as a weaker form of soft weak continuity and soft faint θ_ω -continuity as a weaker form of soft weak θ_ω -continuity are introduced. Numerous characterizations of them are given. We further demonstrate that, under soft restrictions, they are retained. Moreover, we show that a soft function is soft faintly continuous (respectively, soft faintly θ_ω -continuous) if its soft graph function is soft faintly continuous (respectively, soft faintly θ_ω -continuous). In addition, we show that a soft function with a soft almost regular (respectively, soft extremally disconnected) co-domain is soft faintly continuous iff it is soft almost continuous (respectively, soft δ -continuous). Furthermore, we show that soft faintly continuous surjective functions are soft set-connected functions, and as a corollary, we demonstrate how soft faintly continuous functions sustain soft connectivity. Finally, we studied the symmetry between our new notions and their topological counterparts.

Keywords: soft θ -openness; soft θ_ω -openness; soft connectedness; faint continuity; generated soft topology



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1. Introduction and Preliminaries

Mathematical modeling of uncertainty is vital for solving complex issues in fields such as economics, environmental science, engineering, medicine, and the social sciences. Other theories, such as fuzzy set theory [1], rough set theory [2], and probability theory, are effective for dealing with uncertainty and ambiguity, but each has limitations. A common shortcoming among these mathematical strategies is a lack of parametrization equipment.

Molodtsov [3] created the soft set theory in 1999 to address issues with the aforementioned uncertainty management approaches. Soft sets, or parameterized universe possibilities, were postulated. The interpretation of sets for modeling uncertainty was described in [4] and further refined in [5]. The relationship between parameter sets and soft sets provides a standardized framework for modeling uncertain data. This has led to rapid advancements in soft set theory and related topics, as well as practical applications (see [6–12]).

Shabir and Naz [13] created a soft topology over a family of soft sets to build one over a certain set of parameters. Shabir and Naz demonstrated that soft topological notions are similar to those in classical topology, inspiring more study in this area. Since the inception of soft topology, there have been several contributions to the study of topological ideas in soft settings ([14–19] are some of the recent). Majumdar and Samanta [20] discussed mappings on soft sets and their use in medical diagnostics. Kharal and Ahmed [21] introduced the concept of soft mapping with attributes, and [22] proposed soft continuity for soft mappings.

The literature reviews in [23–31] include numerous research papers on soft continuity and its characterizations.

Soft topology is an extension of classical topology that has various benefits. Soft topology has several advantages over traditional topology, including the following:

(1) Compared to traditional topology, soft topology allows for greater flexibility in defining open sets. In classical topology, open sets are defined as subsets of a given set that meet specific requirements. In soft topology, open sets are defined by a collection of parameters or functions that give each point a certain amount of openness. This variety allows inquiry into a greater range of topological structures. (2) The soft topology structure provides better accuracy in identifying open sets. Classical topology often works with binary concepts such as open and closed sets, but soft topology allows for the introduction of intermediate degrees of openness. This provides more detailed knowledge about topological spaces and their properties. (3) Soft topology has a broad domain, which includes classical topology as an example. Soft topology expands the spectrum of topological structures by reducing the stringent limitations of classical topology, making it a great instrument for comprehending more complicated and diverse situations. (4) Researchers have employed soft topology in various domains, including computer science, image processing, fuzzy logic, and decision making, since it can best express uncertainty and imprecision. (5) As an effective mathematical modeling technique for dealing with uncertain or ambiguous data, soft topology plays an important role in encoding and analyzing imprecise data, which are frequently encountered in real-world problems. This confirms its usefulness for decision making and problem solving in a variety of disciplines.

Soft topology and other branches of mathematics have conducted substantial studies on soft continuity. Soft continuity is widely used in several fields, including soft topological models, data modeling, engineering, science, economics, and business. Scientists have paid close attention to this area. This motivated us to write this paper.

In this paper, the concepts of soft faint continuity as a weaker form of soft weak continuity and soft faint θ_ω -continuity as a weaker form of soft weak θ_ω -continuity are introduced. Numerous characterizations of them are given. We further demonstrate that, under soft restrictions, they are retained. Moreover, we show that a soft function is soft faintly continuous (respectively, soft faintly θ_ω -continuous) if its soft graph function is soft faintly continuous (respectively, soft faintly θ_ω -continuous). In addition, we show that a soft function with a soft almost regular (respectively, soft extremally disconnected) co-domain is soft faintly continuous iff it is soft almost continuous (respectively, soft δ -continuous). Furthermore, we show that soft faintly continuous surjective functions are soft set-connected functions, and as a corollary, we demonstrate how soft faintly continuous functions sustain soft connectivity. Finally, we examine the relationships between our soft topological concepts and their general topological counterparts.

Let M be a set of parameters and R be an initial universe. A soft set over R relative to M is a function $K : M \rightarrow \mathcal{P}(R)$, where $\mathcal{P}(R)$ is the power set of R . The collection of soft sets over R relative to M is denoted by $SS(R, M)$. Let $G, H \in SS(R, M)$. If there exist $x \in R$ and $a \in M$ such that $G(a) = \{x\}$ and $G(b) = \emptyset$ for all $b \in M - \{a\}$, then G is called a soft point over R relative to M and denoted by a_x . The collection of all soft points over R relative to M is denoted by $SP(R, M)$. G is called a soft subset of H (notation: $G \subseteq H$) if $G(a) \subseteq H(a)$ for all $a \in M$. If $G(a) = \emptyset$ for every $a \in M$, then G is called the null soft set over R relative to M and denoted by 0_M . If $G(a) = R$ for all $a \in M$, then G is called the absolute soft set over R relative to M and denoted by 1_M . If for some $a \in M$ and $X \subseteq R$, $G(a) = X$ and $G(b) = \emptyset$ for all $b \in M - \{a\}$, then G will be denoted by a_X . If for some $X \subseteq R$, $G(a) = X$ for all $a \in M$, then G is denoted by C_X . If $a_x \in SP(R, M)$, then a_x is said to belong to G (notation: $a_x \tilde{\in} G$) if $x \in G(a)$.

Soft topological spaces were defined in [13] as follows: A triplet (R, Φ, M) , where $\Phi \subseteq SS(R, M)$, is called a soft topological space if $0_M, 1_M \in \Phi$, and Φ is closed under finite soft intersections and arbitrary soft unions.

Let (R, Φ, M) be a soft topological space, and let $G \in SS(R, M)$. Then G is called a soft open set in (R, Φ, M) if $G \in \Phi$. The soft complements of the members of Φ are called soft closed sets in (R, Φ, M) . The family of all soft closed sets in (R, Φ, M) is denoted by Φ^c . G is called a soft clopen set in (R, Φ, M) if $G \in \Phi \cap \Phi^c$. The family of all soft clopen

sets in (R, Φ, M) is denoted by $CO(R, \Phi, M)$. The soft interior and the soft closure of G in (R, Φ, M) are denoted by $Int_{\Phi}(G)$ and $Cl_{\Phi}(G)$, respectively.

Let (R, λ) be a TS, and let $U \subseteq R$. The family of all closed sets in (R, λ) is denoted by λ^c . U is called a soft clopen set in (R, λ) if $U \in \lambda \cap \lambda^c$. The family of all clopen sets in (R, λ) is denoted by $CO(R, \lambda)$. The soft interior and the soft closure of U in (R, λ) are denoted by $Int_{\lambda}(U)$ and $Cl_{\lambda}(U)$, respectively.

In this research, the terminologies and concepts utilized in [25,32,33] are applied, and topological space and soft topological space will be referred to as TS and STS, respectively.

We will now go over some of the notions that will be used in the remainder of this paper.

Definition 1. Let (R, λ) be a TS and let $U \subseteq R$. Then

(a) [34] U is called a θ -open set in (R, λ) if for every $r \in U$, there exists $V \in \lambda$ such that $r \in V \subseteq Cl_{\lambda}(V) \subseteq U$. The family of all θ -open sets in (R, λ) is denoted by λ_{θ} .

(b) [35] (R, λ) is called set-connected between its subsets V and W if there is no $U \in CO(R, \lambda)$ such that $V \subseteq U$ and $U \cap W = \emptyset$.

It is well known that $\lambda_{\theta} \subseteq \lambda$ and $\lambda_{\theta} \neq \lambda$ in general.

Definition 2. A function $p : (R, \mu) \rightarrow (L, \phi)$ is called

(a) [36] weakly continuous if for each $r \in R$ and $V \in \phi$ such that $p(r) \in V$, there exists $U \in \mu$ such that $r \in U$ and $p(U) \subseteq Cl_{\phi}(V)$;

(b) [37] faintly continuous (FC, for short) if for each $r \in R$ and $V \in \phi_{\theta}$ such that $p(r) \in V$, there exists $U \in \mu$ such that $r \in U$ and $p(U) \subseteq V$;

(c) [38] faintly θ_{ω} -continuous ($F\text{-}\theta_{\omega}\text{-C}$, for short) if for each $r \in R$ and $V \in \phi_{\theta}$ such that $p(r) \in V$, there exists $U \in \mu$ such that $r \in U$ and $p(U) \subseteq V$.

Definition 3 ([39]). A soft set G of an STS (R, Ψ, M) is called a soft regular open set in (R, Ψ, M) if $G = Int_{\Psi}(Cl_{\Psi}(G))$. The collection of all soft regular open sets in an STS (R, Ψ, M) will be denoted by $RO(R, \Psi, M)$.

Ψ_s will denote the soft topology having $RO(R, \Psi, M)$ as a soft base.

Definition 4. An STS (R, Ψ, M) is called

(a) [40] soft connected if $CO(R, \Psi, M) = \{0_M, 1_M\}$;

(b) [41] soft extremally disconnected if $Cl_{\Psi}(G) \in \Psi$ for all $G \in \Psi$;

(c) [42] soft set-connected between its soft subsets G and H if there is no $K \in CO(R, \Psi, M)$ such that $G \subseteq K$ and $K \cap H = \emptyset$;

(d) [43] soft almost regular if for every $m_r \in SP(R, M)$ and every $G \in RO(R, \Psi, M)$ such that $m_r \tilde{\in} G$, there exists $K \in \Psi$ such that $m_r \tilde{\in} K \subseteq Cl_{\Psi}(K) \subseteq G$.

Definition 5. A soft function $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is called

(a) [44] soft weakly continuous if for each $m_r \in SP(R, M)$ and $H \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} H$, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(G) \subseteq Cl_{\Phi}(H)$;

(b) [45] soft almost continuous if for each $m_r \in SP(R, M)$ and $H \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} H$, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(G) \subseteq Int_{\Phi}(Cl_{\Phi}(H))$;

(c) [46] soft θ -continuous if for each $m_r \in SP(R, M)$ and $H \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} H$, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(Cl_{\Psi}(G)) \subseteq Cl_{\Phi}(H)$;

(d) [47] soft δ -continuous if for each $m_r \in SP(R, M)$ and $H \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} H$, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(Int_{\Psi}(Cl_{\Psi}(G))) \subseteq Int_{\Phi}(Cl_{\Phi}(H))$;

(e) [48] soft weakly θ_{ω} -continuous ($W\text{-}\theta_{\omega}\text{-C}$) if for each $m_r \in SP(R, M)$ and $H \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} H$, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(G) \subseteq Cl_{\Phi_{\omega}}(H)$.

Definition 6 ([46]). Let (R, Ψ, M) be an STS and let $G \in SS(R, M)$. Then G is called a soft θ -open set in (R, Ψ, M) if for every $m_r \in G$, there exists $H \in \Psi$ such that $m_r \in H \subseteq Cl_\Psi(H) \subseteq G$. The family of all soft θ -open sets in (R, Ψ, M) is denoted by Ψ_θ .

It is well known that $\Psi_\theta \subseteq \Psi$ and $\Psi_\theta \neq \Psi$ in general.

2. Soft Faint Continuity

Definition 7. A soft function $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is called soft faintly continuous (soft FC, for short) if for each $m_r \in SP(R, M)$ and $H \in \Phi_\theta$ such that $f_{pu}(m_r) \in H$, there exists $G \in \Psi$ such that $m_r \in G$ and $f_{pu}(G) \subseteq H$.

Theorem 1. For a soft function $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$, T.F.A.E.:

- (a) f_{pu} is soft FC.
- (b) $f_{pu}^{-1}(H) \in \Psi$ for every $H \in \Phi_\theta$.
- (c) $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi_\theta, N)$ is soft continuous.
- (d) $f_{pu}^{-1}(W) \in \Psi^c$ for each $W \in (\Phi_\theta)^c$.
- (e) $Cl_\Psi(f_{pu}^{-1}(G)) \subseteq f_{pu}^{-1}(Cl_{\Phi_\theta}(G))$ for every $G \in SS(L, N)$.
- (f) $f_{pu}^{-1}(Int_{\Phi_\theta}(G)) \subseteq Int_\Psi(f_{pu}^{-1}(G))$ for every $G \in SS(L, N)$.

Proof. (a) $\rightarrow$ (b): Let $H \in \Phi_\theta$ and let $m_r \in f_{pu}^{-1}(H)$. Then $f_{pu}(m_r) \in H$, and by (a), there exists $G \in \Psi$ such that $m_r \in G$ and $f_{pu}(G) \subseteq H$. Hence, $m_r \in G \subseteq f_{pu}^{-1}(f_{pu}(G)) \subseteq f_{pu}^{-1}(H)$. Therefore, $f_{pu}^{-1}(H) \in \Psi$.

(b) $\rightarrow$ (c): Obvious.

(c) $\rightarrow$ (d): Let $W \in (\Phi_\theta)^c$. Then $1_N - W \in \Phi_\theta$, and by (c), $f_{pu}^{-1}(1_N - W) = 1_M - f_{pu}^{-1}(W) \in \Psi$. Hence, $f_{pu}^{-1}(W) \in \Psi^c$.

(d) $\rightarrow$ (e): Let $G \in SS(L, N)$. Then $Cl_{\Phi_\theta}(G) \in (\Phi_\theta)^c$ and by (d), $f_{pu}^{-1}(Cl_{\Phi_\theta}(G)) \in \Psi^c$ and so $f_{pu}^{-1}(Cl_{\Phi_\theta}(G)) = Cl_\Psi(f_{pu}^{-1}(Cl_{\Phi_\theta}(G)))$. Since $f_{pu}^{-1}(G) \subseteq f_{pu}^{-1}(Cl_{\Phi_\theta}(G))$, $Cl_\Psi(f_{pu}^{-1}(G)) \subseteq Cl_\Psi(f_{pu}^{-1}(Cl_{\Phi_\theta}(G))) = f_{pu}^{-1}(Cl_{\Phi_\theta}(G))$.

(e) $\rightarrow$ (f): Let $G \in SS(L, N)$. Then, by (e), $Cl_\Psi(f_{pu}^{-1}(1_N - G)) \subseteq f_{pu}^{-1}(Cl_{\Phi_\theta}(1_N - G))$. But,

$$\begin{aligned} Cl_\Psi(f_{pu}^{-1}(1_N - G)) &= Cl_\Psi(1_M - f_{pu}^{-1}(G)) \\ &= 1_M - Int_\Psi(f_{pu}^{-1}(G)) \end{aligned}$$

$$\begin{aligned} \text{and} \\ f_{pu}^{-1}(Cl_{\Phi_\theta}(1_N - G)) &= f_{pu}^{-1}(1_N - Int_{\Phi_\theta}(G)) \\ &= 1_M - f_{pu}^{-1}(Int_{\Phi_\theta}(G)). \end{aligned}$$

Thus, $1_M - Int_\Psi(f_{pu}^{-1}(G)) \subseteq 1_M - f_{pu}^{-1}(Int_{\Phi_\theta}(G))$ and hence, $f_{pu}^{-1}(Int_{\Phi_\theta}(G)) \subseteq Int_\Psi(f_{pu}^{-1}(G))$.

(f) $\rightarrow$ (a): Let $m_r \in SP(R, M)$ and $H \in \Phi_\theta$ such that $f_{pu}(m_r) \in H$. Then $Int_{\Phi_\theta}(H) = H$. Thus, by (f), $f_{pu}^{-1}(H) = f_{pu}^{-1}(Int_{\Phi_\theta}(H)) \subseteq Int_\Psi(f_{pu}^{-1}(G))$ and thus, $f_{pu}^{-1}(H) \subseteq Int_\Psi(f_{pu}^{-1}(H))$. Therefore, $f_{pu}^{-1}(H) = Int_\Psi(f_{pu}^{-1}(H))$, and hence, $f_{pu}^{-1}(H) \in \Psi$. Put $G = f_{pu}^{-1}(H)$. Then $G \in \Psi$ such that $m_r \in G$ and $f_{pu}(G) = f_{pu}(f_{pu}^{-1}(H)) \subseteq H$. Therefore, f_{pu} is soft FC. $\square$

Theorem 2. Let $\{(X, \Psi_m) : m \in M\}$ and $\{(Y, \Phi_n) : n \in N\}$ be two collections of TSs. Let $p : X \rightarrow Y$ and $u : M \rightarrow N$ be two functions, with u being a bijection. Then $f_{pu} : (X, \oplus_{m \in M} \Psi_m, M) \rightarrow (Y, \oplus_{n \in N} \Phi_n, N)$ is soft FC iff $p : (X, \Psi_m) \rightarrow (Y, \Phi_{u(m)})$ is FC for all $m \in M$.

Proof. *Necessity.* Let $f_{pu} : (X, \oplus_{m \in M} \Psi_m, M) \rightarrow (Y, \oplus_{n \in N} \Phi_n, N)$ be soft FC. Let $m \in M$. Let $V \in (\Phi_{u(m)})_\theta$. Then, according to Theorem 2.21 of [25], $(u(m))_V \in (\oplus_{n \in N} \Phi_n)_\theta$. Thus, $f_{pu}^{-1}((u(m))_V) \in \oplus_{m \in M} \Psi_m$. Since $u : M \rightarrow N$ is injective, $f_{pu}^{-1}((u(m))_V) = m_{p^{-1}(V)}$. Therefore, $(m_{p^{-1}(V)}) (m) = p^{-1}(V) \in \Psi_m$. This implies that $p : (X, \Psi_m) \rightarrow (Y, \Phi_{u(m)})$ is FC.

Sufficiency. Let $p : (X, \Psi_m) \rightarrow (Y, \Phi_{u(m)})$ be FC for all $m \in M$. Let $H \in (\oplus_{n \in N} \Phi_n)_\theta$. Then, by Theorem 2.21 of [25], $H(n) \in (\Phi_n)_\theta$ for all $n \in N$. For every $n \in N$, $p : (X, \Psi_{u^{-1}(n)}) \rightarrow (Y, \Phi_n)$ is FC, and thus $p^{-1}(H(n)) \in \Psi_{u^{-1}(n)}$. Hence, for each $m \in M$, $(f_{pu}^{-1}(H))(m) = p^{-1}(H(u(m))) \in \Psi_{u^{-1}(u(m))} = \Psi_m$. Therefore, $f_{pu}^{-1}(H) \in \oplus_{m \in M} \Psi_m$. This implies that $f_{pu} : (X, \oplus_{m \in M} \Psi_m, M) \rightarrow (Y, \oplus_{n \in N} \Phi_n, N)$ is soft FC. $\square$

Corollary 1. Let $p : (X, \mu) \rightarrow (Y, \phi)$ be a function between two TSSs, and let $u : M \rightarrow N$ be a bijective function. Then $p : (X, \mu) \rightarrow (Y, \phi)$ is FC iff $f_{pu} : (X, \tau(\mu), M) \rightarrow (Y, \tau(\phi), N)$ is soft FC.

Proof. For each $m \in M$ and $n \in N$, put $\Psi_m = \mu$ and $\Phi_n = \phi$. Then $\tau(\mu) = \oplus_{m \in M} \Psi_m$ and $\tau(\phi) = \oplus_{n \in N} \Phi_n$. Thus, Theorem 2.3 yields the result that is desired. $\square$

Theorem 3. Every soft weakly continuous function is soft FC.

Proof. Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ be soft weakly continuous. Let $m_r \in SP(R, M)$ and $H \in \Phi_\theta$ such that $f_{pu}(m_r) \tilde{\in} H$. Then there exists $K \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} K \subseteq Cl_\Phi(K) \subseteq H$. Since $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft weakly continuous, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(G) \subseteq Cl_\Phi(K) \subseteq H$. Consequently, $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft FC. $\square$

The opposite of Theorem 3 is not always true.

Example 1. Let $R = \{a, b\}$, $L = \{1, 2, 3\}$, $M = \mathbb{Z}$, $\mu = \{\emptyset, R, \{b\}\}$, and $\phi = \{\emptyset, L, \{1\}, \{2\}, \{1, 2\}\}$. Define $p : R \rightarrow L$ and $u : M \rightarrow M$ as follows:

$p(a) = 1$, $p(b) = 2$, and $u(m) = m$ for all $m \in M$.

Note that $p(a) = 1 \in \{1\} \in \phi$. Suppose that p is weakly continuous. Then we find $U \in \mu$ such that $a \in U$ and $p(U) \subseteq Cl_\phi(\{1\}) = \{1, 3\}$. Therefore, $U = R$ and $p(U) = \{1, 2\} \subseteq \{1, 3\}$. Hence, p is not weakly continuous. On the other hand, it is not difficult to check that $\phi_\theta = \{\emptyset, L\}$ and thus p is FC.

Therefore, by Corollary 2.4 and Corollary 3.4 of [49], $f_{pu} : (R, \tau(\mu), M) \rightarrow (L, \tau(\phi), M)$ is soft FC but not soft weakly continuous.

Theorem 4. Let (R, Ψ, M) be an STS. If $G \in \Psi_\theta$, then there exists $S \in RO(R, \Psi, M)$ such that $m_r \tilde{\in} S \subseteq Cl_\Psi(S) \subseteq G$.

Proof. Let $G \in \Psi_\theta$ and let $m_r \tilde{\in} G$. Then there exists $H \in \Psi$ such that $m_r \tilde{\in} H \subseteq Cl_\Psi(H) \subseteq G$, and so $m_r \tilde{\in} H = Int_\Psi(H) \subseteq Int_\Psi(Cl_\Psi(H)) \subseteq Cl_\Psi(H) \subseteq G$. Put $S = Int_\Psi(Cl_\Psi(H))$. Then $S \in RO(R, \Psi, M)$ and $m_r \tilde{\in} S \subseteq Cl_\Psi(S) = Cl_\Psi(Int_\Psi(Cl_\Psi(H))) \subseteq Cl_\Psi(Cl_\Psi(H)) = Cl_\Psi(H) \subseteq G$. $\square$

Corollary 2. For any STS (R, Ψ, M) , $\Psi_\theta \subseteq \Psi_s$.

Theorem 5. If (R, Ψ, M) is soft almost regular, then $RO(R, \Psi, M) \subseteq \Psi_\theta$.

Proof. Suppose that (R, Ψ, M) is soft almost regular. Let $G \in RO(R, \Psi, M)$ and let $m_r \tilde{\in} G$. Since (R, Ψ, M) is soft almost regular, by Theorem 3.4 (ii) of [43], there exists $H \in \Psi$ such that $m_r \tilde{\in} H \subseteq Cl_\Psi(H) \subseteq G$. Hence, $G \in \Psi_\theta$. $\square$

Corollary 3. If (R, Ψ, M) is soft almost regular, then $\Psi_s \subseteq \Psi_\theta$.

Corollary 4. If (R, Ψ, M) is soft almost regular, then $\Psi_\theta = \Psi_s$.

Proof. The proof follows from Corollaries 2.8 and 2.10. $\square$

Theorem 6. If $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is a soft FC function where (L, Φ, N) is soft almost regular, then f_{pu} is soft almost continuous.

Proof. Let $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ be soft FC where (L, Φ, N) is soft almost regular. Since (L, Φ, N) is soft almost regular, by Corollary 2.11, $\Psi_\theta = \Psi_s$. Thus, by Theorem 1 (b) and Theorem 3.8 (b) of [45], it follows that f_{pu} is soft almost continuous. $\square$

Lemma 1. For any soft topological space (R, Ψ, M) and any $H \in \Psi$, $1_M - Cl_\Psi(H) \in RO(R, \Psi, M)$.

Proof. Let $H \in \Psi$. Then $1_M - Cl_\Psi(H) \in \Psi$ and so $1_M - Cl_\Psi(H) \subseteq Int_\Psi(Cl_\Psi(1_M - Cl_\Psi(H)))$. To show that $Int_\Psi(Cl_\Psi(1_M - Cl_\Psi(H))) \subseteq 1_M - Cl_\Psi(H)$, suppose to the contrary that there exists $a_x \in Int_\Psi(Cl_\Psi(1_M - Cl_\Psi(H))) \cap Cl_\Psi(H)$. Since $Int_\Psi(Cl_\Psi(1_M - Cl_\Psi(H))) \in \Psi$, $Int_\Psi(Cl_\Psi(1_M - Cl_\Psi(H))) \cap H \neq 0_M$. Choose $b_y \in Int_\Psi(Cl_\Psi(1_M - Cl_\Psi(H))) \cap H \subseteq Cl_\Psi(1_M - Cl_\Psi(H)) \cap H$. Since $H \in \Psi$, then $(1_M - Cl_\Psi(H)) \cap H \neq 0_M$, which is a contradiction. $\square$

Theorem 7. Every soft almost continuous function is soft θ -continuous.

Proof. Let $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ be soft almost continuous. Let $a_x \in SP(R, M)$ and let $G \in \Phi$ such that $f_{pv}(a_x) \subseteq G$. Then there exists $K \in \Psi$ such that $a_x \subseteq K$ and $f_{pv}(K) \subseteq Int_\Phi(Cl_\Phi(G))$. $\square$

Claim 1. $f_{pu}(Cl_\Psi(K)) \subseteq Cl_\Phi(G)$.

Proof. Suppose, to the contrary, that there exists $b_y \in Cl_\Psi(K)$ such that $f_{pu}(b_y) \not\subseteq 1_N - Cl_\Phi(G) \in \Phi$. Since f_{pu} is soft almost continuous, there exists $H \in \Psi$ such that $b_y \subseteq H$ and $f_{pu}(H) \subseteq Int_\Phi(Cl_\Phi(1_N - Cl_\Phi(G)))$. Since, by Lemma 1, $Int_\Phi(Cl_\Phi(1_N - Cl_\Phi(G))) = 1_N - Cl_\Phi(G)$, then $f_{pu}(H) \subseteq 1_N - Cl_\Phi(G)$. Since $b_y \subseteq H \cap Cl_\Psi(K)$, then $H \cap K \neq 0_M$. Choose $c_z \in H \cap K$. Since $c_z \in H$, then $f_{pu}(c_z) \subseteq f_{pu}(H) \subseteq 1_N - Cl_\Phi(G)$. On the other hand, since $c_z \in K$, then $f_{pu}(c_z) \subseteq f_{pu}(K) \subseteq Int_\Phi(Cl_\Phi(G)) \subseteq Cl_\Phi(G)$. Therefore, $f_{pu}(c_z) \subseteq (1_N - Cl_\Phi(G)) \cap Cl_\Phi(G)$, which is a contradiction. $\square$

The above Claim ends the proof of the theorem.

Theorem 8. Let $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ be a soft function. Then

- (a) If f_{pu} is soft δ -continuous, then f_{pu} is soft almost continuous.
- (b) If f_{pu} is soft θ -continuous, then f_{pu} is soft weakly continuous.

Proof. (a) Let f_{pu} be soft δ -continuous. Let $a_x \in P(R, M)$ and let $G \in \Phi$ such that $f_{pv}(a_x) \subseteq G$. Then there exists $K \in \Psi$ such that $a_x \subseteq K$ and $f_{pv}(K) \subseteq f_{pv}(Int_\Psi(Cl_\Psi(K))) \subseteq Int_\Phi(Cl_\Phi(G))$. Thus, f_{pu} is soft almost continuous.

(b) Let f_{pu} be soft θ -continuous. Let $a_x \in SP(R, M)$ and let $G \in \Phi$ such that $f_{pv}(a_x) \subseteq G$. Then there exists $K \in \Psi$ such that $a_x \subseteq K$ and $f_{pv}(K) \subseteq f_{pv}(Cl_\Psi(K)) \subseteq Cl_\Phi(G)$. Thus, f_{pu} is soft weakly continuous. $\square$

Corollary 5. Let $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ be a soft function where (L, Φ, N) is soft almost regular. Then T.F.A.E.:

- (a) f_{pu} is soft almost continuous.
- (b) f_{pu} is soft θ -continuous.
- (c) f_{pu} is soft weakly continuous.
- (d) f_{pu} is soft FC.

Proof. The proof follows from Theorems 3 and 6–8. $\square$

Theorem 9. If $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is a soft FC function, then for any nonempty subset $X \subseteq R$, $(f_{pu})|_{C_X} : (X, \Psi_X, M) \longrightarrow (L, \Phi, N)$ is soft FC.

Proof. Let $H \in \Phi_\theta$. Since $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is soft FC, by Theorem 1 (b), $f_{pu}^{-1}(H) \in \Psi$ and so $\left((f_{pu})|_{C_X}\right)^{-1}(H) = C_X \tilde{\cap} f_{pu}^{-1}(H) \in \Psi_X$. Hence, again by Theorem 1 (b), $(f_{pu})|_{C_X} : (X, \Psi_X, M) \longrightarrow (L, \Phi, N)$ is soft FC. For any function $h : X \longrightarrow Y$, the function $w : X \longrightarrow X \times Y$ defined by $w(x) = (x, w(x))$ will be denoted by $h^\#$. $\square$

Lemma 2. If $f_{pu} : SS(R, M) \longrightarrow SS(L, N)$ and $f_{p^\#u^\#} : SS(R, M) \longrightarrow SS(R \times L, M \times N)$, then for any $T \in SS(R, M)$ and $S \in SS(L, N)$, $f_{p^\#u^\#}^{-1}(T \times S) = T \tilde{\cap} f_{pu}^{-1}(S)$.

Proof. $a_x \tilde{\in} f_{p^\#u^\#}^{-1}(T \times S)$ iff $f_{p^\#u^\#}(a_x) = ((u^\#)(a))_{((p^\#)(x))} = (a, u(a))_{(x, p(x))} = a_x \times (u(a)_{p(x)}) \tilde{\in} T \times S$ iff $a_x \tilde{\in} T$ and $u(a)_{p(x)} = f_{pu}(a_x) \in S$ iff $a_x \tilde{\in} T \tilde{\cap} f_{pu}^{-1}(S)$. $\square$

Theorem 10. Let (R, Ψ, M) and (L, Φ, N) be two STSs, and let $(R \times L, pr(\Psi \times \Phi), M \times N)$ be the soft product soft topology between them. Then for any $G \in \Psi_\theta$ and $H \in \Phi_\theta$, we have $G \times H \in (pr(\Psi \times \Phi))_\theta$.

Proof. Let $G \in \Psi_\theta$ and $H \in \Phi_\theta$, and let $(a, b)_{(x, y)} \tilde{\in} G \times H$. Since $(a, b)_{(x, y)} = a_x \times b_y$, $a_x \tilde{\in} G$ and $b_y \tilde{\in} H$. Thus, there exist $U \in \Psi$ and $V \in \Phi$ such that $a_x \tilde{\in} U \tilde{\subseteq} Cl_\Psi(U) \tilde{\subseteq} G$ and $b_y \tilde{\in} V \tilde{\subseteq} Cl_\Phi(V) \tilde{\subseteq} H$. Therefore, we have $U \times V \in pr(\Psi \times \Phi)$ and $(a, b)_{(x, y)} = a_x \times b_y \tilde{\in} U \times V \tilde{\subseteq} Cl_{pr(\Psi \times \Phi)}(U \times V) = Cl_{pr(\Psi \times \Phi)}(U \times V) \tilde{\subseteq} G \times H$. Hence, $G \times H \in (pr(\Psi \times \Phi))_\theta$. $\square$

Theorem 11. Let $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ be a soft function. If $f_{p^\#u^\#} : (R, \Psi, M) \longrightarrow (R \times L, pr(\Psi \times \Phi), M \times N)$ is soft FC, then $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is soft FC.

Proof. Let $H \in \Phi_\theta$. Since $1_M \in \Psi_\theta$, by Theorem 2.19, $1_M \times H \in (pr(\Psi \times \Phi))_\theta$. Since $f_{p^\#u^\#} : (R, \Psi, M) \longrightarrow (R \times L, pr(\Psi \times \Phi), M \times N)$ is soft FC, by Theorem 1 (b), $f_{p^\#u^\#}^{-1}(1_M \times H) \in \Psi$. By Lemma 2, $f_{p^\#u^\#}^{-1}(1_M \times H) = 1_M \tilde{\cap} f_{pu}^{-1}(H) = f_{pu}^{-1}(H)$, and hence $f_{pu}^{-1}(H) \in \Psi$. Thus, again by Theorem 1 (b), $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is soft FC. $\square$

Theorem 12. If $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is a soft weakly continuous function, then $f_{p^\#u^\#} : (R, \Psi, M) \longrightarrow (R \times L, pr(\Psi \times \Phi), M \times N)$ is soft FC.

Proof. Suppose that $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is a soft weakly continuous function. Let $m_r \in SP(R, M)$ and let $G \in (pr(\Psi \times \Phi))_\theta$ such that $(f_{p^\#u^\#}(m_r)) = ((u^\#)(m))_{(p^\#)(r)} = (m, u(m))_{(r, p(r))} \tilde{\in} G$. Then there exists $K \in pr(\Psi \times \Phi)$, such that $(m, u(m))_{(r, p(r))} \tilde{\in} K \tilde{\subseteq} Cl_{pr(\Psi \times \Phi)}(K) \tilde{\subseteq} G$. Choose $U \in \Psi$ and $V \in \Phi$ such that $(m, u(m))_{(r, p(r))} = m_r \times (u(m))_{p(r)} \tilde{\in} U \times V \tilde{\subseteq} Cl_{pr(\Psi \times \Phi)}(U \times V) = Cl_\Psi(U) \times Cl_\Phi(V) \tilde{\subseteq} Cl_{pr(\Psi \times \Phi)}(K) \tilde{\subseteq} G$. Since $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is soft weakly continuous and

$(u(m))_{p(r)} = f_{pu}(m_r) \in V$, there exists $W \in \Psi$ such that $m_r \in W$ and $f_{pu}(W) \subseteq Cl_\Phi(V)$. Consequently, $f_{p^\#u^\#}(W) \subseteq Cl_\Psi(U) \times Cl_\Phi(V) \subseteq G$. It follows that $f_{p^\#u^\#} : (R, \Psi, M) \rightarrow (R \times L, pr(\Psi \times \Phi), M \times N)$ is soft FC. $\square$

Theorem 13. If $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft almost continuous, then $f_{pu} : (R, \Psi_\theta, M) \rightarrow (L, \Phi_\theta, N)$ is soft continuous.

Proof. Suppose that $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft almost continuous. Let $G \in \Phi_\theta$ and let $m_r \in f_{pu}^{-1}(G)$. Then $f_{pu}(m_r) \in G$. Thus, by Theorem 4, there exists $S \in RO(L, \Phi, N)$ such that $f_{pu}(m_r) \in S \subseteq Cl_\Phi(S) \subseteq G$. Thus, we have $m_r \in f_{pu}^{-1}(S) \subseteq f_{pu}^{-1}(Cl_\Phi(S)) \subseteq f_{pu}^{-1}(G)$. Since $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft almost continuous, by Theorem 3.8 of [45], $f_{pu}^{-1}(S) \in \Psi$ and $f_{pu}^{-1}(Cl_\Phi(S)) \in \Psi^c$. Hence, we have $f_{pu}^{-1}(S) \in \Psi$ and

$$m_r \in f_{pu}^{-1}(S) \subseteq Cl_\Psi(f_{pu}^{-1}(S)) \subseteq f_{pu}^{-1}(Cl_\Phi(S)) \subseteq f_{pu}^{-1}(G).$$

This shows that $f_{pu}^{-1}(G) \in \Psi_\theta$. $\square$

Corollary 6. If $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft continuous, then $f_{pu} : (R, \Psi_\theta, M) \rightarrow (L, \Phi_\theta, N)$ is soft continuous.

Theorem 14. A soft function $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft FC iff $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi_s, N)$ is soft FC.

Proof. *Necessity.* Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ be soft FC. Let $G \in (\Phi_s)_\theta$. Let $q : L \rightarrow L$ and $v : N \rightarrow N$ be the identities. Since $\Phi_s \subseteq \Phi$, $f_{qv} : (L, \Phi, N) \rightarrow (L, \Phi_s, N)$ is soft continuous. Thus, by Corollary 6, $f_{qv}^{-1}(G) \in \Phi_\theta$. Since $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft FC, $f_{pu}^{-1}(f_{qv}^{-1}(G)) = f_{pu}^{-1}(G) \in \Psi$. This ends the proof.

Sufficiency. Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi_s, N)$ be soft FC. Let $G \in \Phi_\theta$. Let $q : L \rightarrow L$ and $v : N \rightarrow N$ be the identities. Then, by Theorem 3.8 of [45], $f_{qv} : (L, \Phi_s, N) \rightarrow (L, \Phi, N)$ is soft almost continuous. Thus, by Theorem 13, $f_{qv}^{-1}(G) \in (\Phi_s)_\theta$. Since $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi_s, N)$ is soft FC, $f_{pu}^{-1}(f_{qv}^{-1}(G)) = f_{pu}^{-1}(G) \in \Psi$. This shows that $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft FC. $\square$

3. Soft Set-Connected Functions

Theorem 15. Let (Z, λ) be a TS. Then, for any set M , (Z, λ) is set-connected between V and W iff $(Z, \tau(\lambda), M)$ is soft set-connected between C_V and C_W .

Proof. *Necessity.* Let (Z, λ) be set-connected between V and W . Suppose, on the contrary, there is $H \in CO(Z, \tau(\lambda), M)$ such that $C_V \subseteq H \subseteq 1_M - C_W$. Pick $m \in M$. Then we obtain $H(m) \in CO(Z, \lambda)$ such that $V \subseteq H(m) \subseteq R - W$. Therefore, (Z, λ) is not set-connected between V and W . This is a contradiction.

Sufficiency. Let $(Z, \tau(\lambda), M)$ be soft set-connected between C_V and C_W . Suppose, on the contrary, there is $U \in CO(Z, \lambda)$ such that $V \subseteq U \subseteq W$. Then we have $C_U \in CO(Z, \tau(\lambda), M)$ such that $C_V \subseteq C_U \subseteq 1_M - C_W$. Therefore, $(Z, \tau(\lambda), M)$ is not soft set-connected between C_V and C_W . This is a contradiction. $\square$

Theorem 16. Let $p : (R, \lambda) \rightarrow (L, \phi)$ and $u : \{m\} \rightarrow \{n\}$ be surjective functions. Then $p : (R, \lambda) \rightarrow (L, \phi)$ is set-connected iff $f_{pu} : (R, \tau(\lambda), \{m\}) \rightarrow (L, \tau(\phi), \{n\})$ is soft set-connected.

Proof. *Necessity.* Let $p : (R, \lambda) \rightarrow (L, \phi)$ be set-connected. Let $(R, \tau(\lambda), \{m\})$ be soft set-connected between $S = C_{S(m)}$ and $T = C_{T(m)}$. Then, by Theorem 15, (R, λ) is set-connected between $S(m)$ and $T(m)$. Since $p : (R, \lambda) \rightarrow (L, \phi)$ is set-connected, (L, ϕ) is set-connected between $p(S(m))$ and $p(T(m))$. Hence, again by Theorem 15, $(L, \tau(\phi), \{n\})$

is soft set-connected between $n_{p(S(m))} = f_{pu}(S)$ and $n_{p(T(m))} = f_{pu}(T)$. Therefore, $f_{pu} : (R, \tau(\lambda), \{m\}) \rightarrow (L, \tau(\phi), \{n\})$ is soft set-connected.

Sufficiency. Let $f_{pu} : (R, \tau(\lambda), \{m\}) \rightarrow (L, \tau(\phi), \{n\})$ be soft set-connected. Let (R, λ) be set-connected between U and V . Then, by Theorem 15, $(R, \tau(\lambda), \{m\})$ is soft set-connected between m_U and m_V . Since $f_{pu} : (R, \tau(\lambda), \{m\}) \rightarrow (L, \tau(\phi), \{n\})$ is soft set-connected, $(L, \tau(\phi), \{n\})$ is soft set-connected between $f_{pu}(m_U) = n_{p(U)}$ and $f_{pu}(m_V) = n_{p(V)}$. Hence, again by Theorem 15, (L, ϕ) is set-connected between $p(U)$ and $p(V)$. Therefore, $p : (R, \lambda) \rightarrow (L, \phi)$ is set-connected. $\square$

Theorem 17. Every surjective soft FC function is soft set-connected.

Proof. Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ be surjective and soft FC. Let $G \in CO(L, \Phi, N)$. Then $G \in \Phi_\theta \cap (\Phi_\theta)^c$. Since f_{pu} is soft FC, $f_{pu}^{-1}(G) \in \Psi \cap (\Psi)^c = CO(R, \Psi, M)$. Thus, by Theorem 4.1 of [42], f_{pu} is soft set-connected. $\square$

Corollary 7. Soft connectedness is preserved under soft FC surjections.

Proof. The proof follows from Theorems 3.3 and 4.5 of [42].

Every soft set-connected surjection is not always soft FC, even though the range is a soft regular STS, as the following example shows. $\square$

Example 2. Let $R = [0, 1]$, $M = \{m\}$, $\lambda = \{\emptyset\} \cup \{U \subseteq R : R - U \text{ is countable}\}$, and ϕ be the usual topology on R . Consider the identities $p : R \rightarrow R$ and $u : M \rightarrow M$. Example 3.6 of [50] shows that $p : (R, \lambda) \rightarrow (R, \phi)$ is set-connected but not FC. According to Corollary 1 and Theorem 16, $f_{pu} : (R, \tau(\lambda), M) \rightarrow (R, \tau(\phi), M)$ is soft set-connected but not soft FC. However, it is clear that f_{pu} is surjective and $(R, \tau(\phi), M)$ is soft regular.

Theorem 18. If $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft set-connected and (L, Φ, N) is soft extremally disconnected, then f_{pu} is soft δ -continuous.

Proof. Let $m_r \in SP(R, M)$ and $G \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} G$. Since (L, Φ, N) is soft extremally disconnected, $Cl_\Phi(G) \in CO(L, \Phi, N)$. Since f_{pu} is soft set-connected, by Theorem 4.1 of [42], $f_{pu}^{-1}(Cl_\Phi(G)) \in CO(R, \Psi, M)$. Set $H = f_{pu}^{-1}(Cl_\Phi(G))$. Then $m_r \tilde{\in} H \in \Psi$ and

$$\begin{aligned} f_{pu}(Int_\Psi(Cl_\Psi(H))) &= f_{pu}\left(Int_\Psi\left(Cl_\Psi\left(f_{pu}^{-1}(Cl_\Phi(G))\right)\right)\right) \\ &= f_{pu}\left(Int_\Psi\left(f_{pu}^{-1}(Cl_\Phi(G))\right)\right) \\ &= f_{pu}\left(f_{pu}^{-1}(Cl_\Phi(G))\right) \\ &\subseteq Cl_\Phi(G) \\ &= Int_\Phi(Cl_\Phi(G)). \end{aligned}$$

This shows that f_{pu} is soft δ -continuous. $\square$

Corollary 8. Let (L, Φ, N) is soft extremally disconnected. Then, for a soft surjection $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$, T.F.A.E.:

- f_{pu} is soft δ -continuous.
- f_{pu} is soft almost continuous.
- f_{pu} is soft weakly continuous.
- f_{pu} is soft FC.
- f_{pu} is soft set-connected.

Proof. (a) $\rightarrow$ (b) and (b) $\rightarrow$ (c) follow from Theorems 7 and 8.

(c) $\rightarrow$ (d): The proof follows from Theorem 3.

(d) $\rightarrow$ (e): The proof follows from Theorem 17.

(e) $\rightarrow$ (a): The proof follows from Theorem 18. $\square$

4. Soft Faint θ_ω -Continuity

Definition 8. A soft function $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$ is called soft faintly θ_ω -C (soft F - θ_ω -C, for short) if for each $m_r \in SP(R, M)$ and $H \in \Phi_{\theta_\omega}$ such that $f_{pu}(m_r) \tilde{\subseteq} H$, there exists $G \in \Psi$ such that $m_r \tilde{\subseteq} G$ and $f_{pu}(G) \tilde{\subseteq} H$.

Theorem 19. For a soft function $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi, N)$, T.F.A.E.:

- (a) f_{pu} is soft F - θ_ω -C.
- (b) $f_{pu}^{-1}(H) \in \Psi$ for every $H \in \Phi_{\theta_\omega}$.
- (c) $f_{pu} : (R, \Psi, M) \longrightarrow (L, \Phi_{\theta_\omega}, N)$ is soft continuous.
- (d) $f_{pu}^{-1}(W) \in \Psi^c$ for every $W \in (\Phi_{\theta_\omega})^c$.
- (e) $Cl_\Psi(f_{pu}^{-1}(G)) \tilde{\subseteq} f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G))$ for every $G \in SS(L, N)$.
- (f) $f_{pu}^{-1}(Int_{\Phi_{\theta_\omega}}(G)) \tilde{\subseteq} Int_\Psi(f_{pu}^{-1}(G))$ for every $G \in SS(L, N)$.

Proof. (a) $\longrightarrow$ (b): Let $H \in \Phi_{\theta_\omega}$ and let $m_r \tilde{\subseteq} f_{pu}^{-1}(H)$. Then $f_{pu}(m_r) \tilde{\subseteq} H$, and by (a), there exists $G \in \Psi$ such that $m_r \tilde{\subseteq} G$ and $f_{pu}(G) \tilde{\subseteq} H$. Hence, $m_r \tilde{\subseteq} G \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(G)) \tilde{\subseteq} f_{pu}^{-1}(H)$. Therefore, $f_{pu}^{-1}(H) \in \Psi$.

(b) $\longrightarrow$ (c): Obvious.

(c) $\longrightarrow$ (d): Let $W \in (\Phi_{\theta_\omega})^c$. Then $1_N - W \in \Phi_{\theta_\omega}$, and by (c), $f_{pu}^{-1}(1_N - W) = 1_M - f_{pu}^{-1}(W) \in \Psi$. Hence, $f_{pu}^{-1}(W) \in \Psi^c$.

(d) $\longrightarrow$ (e): Let $G \in SS(L, N)$. Then $Cl_{\Phi_{\theta_\omega}}(G) \in (\Phi_{\theta_\omega})^c$ and by (d), $f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G)) \in \Psi^c$ and so $f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G)) = Cl_\Psi(f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G)))$. Since $f_{pu}^{-1}(G) \tilde{\subseteq} f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G))$, $Cl_\Psi(f_{pu}^{-1}(G)) \tilde{\subseteq} Cl_\Psi(f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G))) = f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(G))$.

(e) $\longrightarrow$ (f): Let $G \in SS(L, N)$. Then, by (e), $Cl_\Psi(f_{pu}^{-1}(1_N - G)) \tilde{\subseteq} f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(1_N - G))$. But,

$$\begin{aligned} Cl_\Psi(f_{pu}^{-1}(1_N - G)) &= Cl_\Psi(1_M - f_{pu}^{-1}(G)) \\ &= 1_M - Int_\Psi(f_{pu}^{-1}(G)) \end{aligned}$$

$$\begin{aligned} \text{and} \\ f_{pu}^{-1}(Cl_{\Phi_{\theta_\omega}}(1_N - G)) &= f_{pu}^{-1}(1_N - Int_{\Phi_{\theta_\omega}}(G)) \\ &= 1_M - f_{pu}^{-1}(Int_{\Phi_{\theta_\omega}}(G)). \end{aligned}$$

Thus, $1_M - Int_\Psi(f_{pu}^{-1}(G)) \tilde{\subseteq} 1_M - f_{pu}^{-1}(Int_{\Phi_{\theta_\omega}}(G))$ and hence, $f_{pu}^{-1}(Int_{\Phi_{\theta_\omega}}(G)) \tilde{\subseteq} Int_\Psi(f_{pu}^{-1}(G))$.

(f) $\longrightarrow$ (a): Let $m_r \in SP(R, M)$ and $H \in \Phi_{\theta_\omega}$ such that $f_{pu}(m_r) \tilde{\subseteq} H$. Then $Int_{\Phi_{\theta_\omega}}(H) = H$. Thus, by (f), $f_{pu}^{-1}(H) = f_{pu}^{-1}(Int_{\Phi_{\theta_\omega}}(H)) \tilde{\subseteq} Int_\Psi(f_{pu}^{-1}(H))$, and thus, $f_{pu}^{-1}(H) \tilde{\subseteq} Int_\Psi(f_{pu}^{-1}(H))$. Therefore, $f_{pu}^{-1}(H) = Int_\Psi(f_{pu}^{-1}(H))$ and hence, $f_{pu}^{-1}(H) \in \Psi$. Put $G = f_{pu}^{-1}(H)$. Then $G \in \Psi$ such that $m_r \tilde{\subseteq} G$ and $f_{pu}(G) = f_{pu}(f_{pu}^{-1}(H)) \tilde{\subseteq} H$. Therefore, f_{pu} is soft F - θ_ω -C. $\square$

Theorem 20. Let $\{(X, \Psi_m) : m \in M\}$ and $\{(Y, \Phi_n) : n \in N\}$ be two collections of TSs. Let $p : X \longrightarrow Y$ and $u : M \longrightarrow N$ be two functions where u is a bijection. Then $f_{pu} : (X, \oplus_{m \in M} \Psi_m, M) \longrightarrow (Y, \oplus_{n \in N} \Phi_n, N)$ is soft F - θ_ω -C iff $p : (X, \Psi_m) \longrightarrow (Y, \Phi_{u(m)})$ is F - θ_ω -C for all $m \in M$.

Proof. Necessity. Let $f_{pu} : (X, \oplus_{m \in M} \Psi_m, M) \longrightarrow (Y, \oplus_{n \in N} \Phi_n, N)$ be soft F - θ_ω -C. Let $m \in M$. Let $V \in (\Phi_{u(m)})_{\theta_\omega}$. Then, by Theorem 2.24 of [25], $(u(m))_V \in (\oplus_{n \in N} \Phi_n)_{\theta_\omega}$. Therefore, $f_{pu}^{-1}((u(m))_V) \in \oplus_{m \in M} \Psi_m$. Since $u : M \longrightarrow N$ is injective, $f_{pu}^{-1}((u(m))_V) = m_{p^{-1}(V)}$. Thus, $(m_{p^{-1}(V)})(m) = p^{-1}(V) \in \Psi_m$. This implies that $p : (X, \Psi_m) \longrightarrow (Y, \Phi_{u(m)})$ is F - θ_ω -C.

Sufficiency. Let $p : (X, \Psi_m) \rightarrow (Y, \Phi_{u(m)})$ be $F\text{-}\theta_\omega\text{-C}$ for all $m \in M$. Let $H \in (\oplus_{n \in N} \Phi_n)_{\theta_\omega}$. Then, according to Theorem 2.24 of [25], $H(n) \in (\Phi_n)_{\theta_\omega}$ for all $n \in N$. For every $n \in N$, $p : (X, \Psi_{u^{-1}(n)}) \rightarrow (Y, \Phi_n)$ is $F\text{-}\theta_\omega\text{-C}$, and hence $p^{-1}(H(n)) \in \Psi_{u^{-1}(n)}$. Thus, for each $m \in M$, $(f_{pu}^{-1}(H))(m) = p^{-1}(H(u(m))) \in \Psi_{u^{-1}(u(m))} = \Psi_m$. Therefore, $f_{pu}^{-1}(H) \in \oplus_{m \in M} \Psi_m$. It follows that $f_{pu} : (X, \oplus_{m \in M} \Psi_m, M) \rightarrow (Y, \oplus_{n \in N} \Phi_n, N)$ is soft $F\text{-}\theta_\omega\text{-C}$. $\square$

Corollary 9. Let $p : (X, \mu) \rightarrow (Y, \phi)$ be a function between two TSs, and let $u : M \rightarrow N$ be a bijection. Then $p : (X, \mu) \rightarrow (Y, \phi)$ is $F\text{-}\theta_\omega\text{-C}$ iff $f_{pu} : (X, \tau(\mu), M) \rightarrow (Y, \tau(\phi), N)$ is soft $F\text{-}\theta_\omega\text{-C}$.

Proof. For each $m \in M$ and $n \in N$, put $\Psi_m = \mu$ and $\Phi_n = \phi$. Then $\tau(\mu) = \oplus_{m \in M} \Psi_m$ and $\tau(\phi) = \oplus_{n \in N} \Phi_n$. Thus, Theorem 4.3 yields the result that is desired. $\square$

Theorem 21. Every soft $W\text{-}\theta_\omega\text{-C}$ function is soft $F\text{-}\theta_\omega\text{-C}$.

Proof. Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ be soft $W\text{-}\theta_\omega\text{-C}$. Let $m_r \in SP(R, M)$ and $H \in \Phi_{\theta_\omega}$ such that $f_{pu}(m_r) \tilde{\in} H$. Then there exists $K \in \Phi$ such that $f_{pu}(m_r) \tilde{\in} K \subseteq Cl_{\Phi_\omega}(K) \subseteq H$. Since $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft $W\text{-}\theta_\omega\text{-C}$, there exists $G \in \Psi$ such that $m_r \tilde{\in} G$ and $f_{pu}(G) \subseteq Cl_\Phi(K) \subseteq H$. Consequently, $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft $F\text{-}\theta_\omega\text{-C}$. $\square$

The opposite of Theorem 21 is not always true.

Example 3. Let $\mu = \{\emptyset, \mathbb{R}, \mathbb{N}\}$ and ϕ be the standard topology on $\mathbb{R}$. Let $p : \mathbb{R} \rightarrow \mathbb{R}$ and $u : \mathbb{Q} \rightarrow \mathbb{Q}$ be the identities. Example 2.38 of [38] shows that $p : (\mathbb{R}, \mu) \rightarrow (\mathbb{R}, \phi)$ is $F\text{-}\theta_\omega\text{-C}$ but not $W\text{-}\theta_\omega\text{-C}$. According to Corollaries 4.4 and 4.3 of [48], $f : (\mathbb{R}, \tau(\mu), \mathbb{Q}) \rightarrow (\mathbb{R}, \tau(\phi), \mathbb{Q})$ is soft $F\text{-}\theta_\omega\text{-C}$ but not soft $W\text{-}\theta_\omega\text{-C}$.

Theorem 22. Every soft $F\text{-}\theta_\omega\text{-C}$ function is soft FC.

Proof. Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ be soft $F\text{-}\theta_\omega\text{-C}$. Let $H \in \Phi_\theta$. Since by Theorem 2.9 of [25], $\Phi_\theta \subseteq \Phi_{\theta_\omega}$, $H \in \Phi_{\theta_\omega}$. Since f_{pu} is soft $F\text{-}\theta_\omega\text{-C}$, by Theorem 19 (b), $f_{pu}^{-1}(H) \in \Psi$. Hence, by Theorem 1 (b), f_{pu} is soft faintly θ -continuous. $\square$

The opposite of Theorem 22 is not always true.

Example 4. Let $\mu = \{\emptyset, \mathbb{R}\}$ and $\phi = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$. Let $p : \mathbb{R} \rightarrow \mathbb{R}$ and $u : \mathbb{Q} \rightarrow \mathbb{Q}$ be the identities. Example 2.38 of [38] shows that $p : (\mathbb{R}, \mu) \rightarrow (\mathbb{R}, \phi)$ is FC but not $F\text{-}\theta_\omega\text{-C}$. According to Corollaries 2.4 and 4.4, $f : (\mathbb{R}, \tau(\mu), \mathbb{Q}) \rightarrow (\mathbb{R}, \tau(\phi), \mathbb{Q})$ is soft FC but not soft $F\text{-}\theta_\omega\text{-C}$.

Theorem 23. If $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is a soft $F\text{-}\theta_\omega\text{-C}$ function, then for any nonempty subset $X \subseteq R$, $(f_{pu})|_{C_X} : (X, \Psi_X, M) \rightarrow (L, \Phi, N)$ is soft $F\text{-}\theta_\omega\text{-C}$.

Proof. Let $H \in \Phi_{\theta_\omega}$. Since $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft $F\text{-}\theta_\omega\text{-C}$, by Theorem 19 (b), $f_{pu}^{-1}(H) \in \Psi$ and so $((f_{pu})|_{C_X})^{-1}(H) = C_X \cap f_{pu}^{-1}(H) \in \Psi_X$. Hence, again by Theorem 19 (b), $(f_{pu})|_{C_X} : (X, \Psi_X, M) \rightarrow (L, \Phi, N)$ is soft $F\text{-}\theta_\omega\text{-C}$. $\square$

Theorem 24. Let (R, Ψ, M) and (L, Φ, N) , and let $(R \times L, pr(\Psi \times \Phi), M \times N)$ be the soft product soft topology between them. Then for any $G \in \Psi_{\theta_\omega}$ and $H \in \Phi_{\theta_\omega}$, we have $G \times H \in (pr(\Psi \times \Phi))_{\theta_\omega}$.

Proof. Let $G \in \Psi_\theta$ and $H \in \Phi_\theta$ and let $(a, b)_{(x, y)} \in G \times H$. Since $(a, b)_{(x, y)} = a_x \times b_y$, $a_x \in G$ and $b_y \in H$. Thus, there exist $U \in \Psi$ and $V \in \Phi$ such that $a_x \in U \subseteq Cl_{\Psi_\omega}(U) \subseteq G$ and $b_y \in V \subseteq Cl_{\Phi_\omega}(V) \subseteq H$. Also, by Proposition 3 of [51], $Cl_{\Psi_\omega}(U) \times Cl_{\Phi_\omega}(V) \subseteq Cl_{(pr(\Psi \times \Phi))_\omega}(U \times V)$. Therefore, we have $U \times V \in pr(\Psi \times \Phi)$ and by $(a, b)_{(x, y)} = a_x \times b_y \in U \times V \subseteq Cl_{\Psi_\omega}(U) \times Cl_{\Phi_\omega}(V) \subseteq Cl_{(pr(\Psi \times \Phi))_\omega}(U \times V) \subseteq G \times H$, $G \times H \in (pr(\Psi \times \Phi))_{\theta_\omega}$. $\square$

Theorem 25. Let $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ be a soft function. If $f_{p^\#u^\#} : (R, \Psi, M) \rightarrow (R \times L, pr(\Psi \times \Phi), M \times N)$ is soft F - θ_ω -C, then $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft F - θ_ω -C.

Proof. Let $H \in \Phi_{\theta_\omega}$. Since $1_M \in \Psi_{\theta_\omega}$, by Theorem 24, $1_M \times H \in (pr(\Psi \times \Phi))_{\theta_\omega}$. Since $f_{p^\#u^\#} : (R, \Psi, M) \rightarrow (R \times L, pr(\Psi \times \Phi), M \times N)$ is soft F - θ_ω -C, by Theorem 19 (b), $f_{p^\#u^\#}^{-1}(1_M \times H) \in \Psi$. By Lemma 2, $f_{p^\#u^\#}^{-1}(1_M \times H) = 1_M \tilde{\cap} f_{pu}^{-1}(H) = f_{pu}^{-1}(H)$, and hence $f_{pu}^{-1}(H) \in \Psi$. Thus, again by Theorem 19 (b), $f_{pu} : (R, \Psi, M) \rightarrow (L, \Phi, N)$ is soft F - θ_ω -C. $\square$

5. Conclusions

Many aspects we face daily need to deal with uncertainty. Soft set theory and its related concepts are considered one of the most important ideas proposed for dealing with uncertainty. Soft topology is considered one of the most important frameworks emerging from soft set theory. This study focuses on the concept of soft continuity, which is considered one of the most important concepts of soft topology.

This work introduces soft faint continuity, a weaker version of soft weak continuity, and soft faint θ_ω -continuity, which is a weaker version of soft weak θ_ω -continuity. Several characterizations of them are obtained. It is shown that they may be preserved under soft restrictions. Furthermore, it is demonstrated that a soft function is soft faintly continuous (respectively, soft faintly θ_ω -continuous) if its soft graph function is also soft faintly continuous. Furthermore, it is demonstrated that a soft function with a soft almost regular (respectively, soft extremally disconnected) co-domain is soft faintly continuous if it is soft almost continuous (respectively, soft δ -continuous). In addition, it is shown that a soft faintly continuous surjective function is a soft set-connected function, and as a result, it is explained that soft faintly continuous functions maintain soft connectivity. Finally, the symmetry of our new concepts and their topological counterparts is studied.

Future research might look into the following topics: (1) defining soft faintly semi-continuous functions; (2) defining soft faintly semi-continuous functions; (3) defining soft faintly β -continuous functions; (3) defining fuzzy soft faint continuity; (4) finding a use for our new soft continuity notions in a “decision making problem”.

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