A Symmetric Fourth Party Logistics Routing Problem with Multiple Distributors in Uncertain Random Environments

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Abstract: Economic globalization and the rapid development of the Internet make logistics systems more and more diversified, people and enterprises have greatly increased their requirements for logistics systems, and fourth party logistics has received more and more attention from people and related enterprises. In order to further study the routing problem under uncertain stochastic environments, this paper considers the fourth party logistics routing problem from a single manufacturer to multiple distributors with uncertain times and random supplies under the complete information symmetry scenario and symmetric transportation volume decision space. Then, an uncertain stochastic programming model is established with the minimum total cost as its core objective, and the total transportation time, manufacturer’s supply, and distributor’s demand as constraints. In order to solve the optimal path of the above problems, this paper transforms the uncertain stochastic programming model into a classical mathematical programming model based on the distribution functions of uncertain time and random supply. Finally, two numerical examples are given to verify the effectiveness of the proposed model.

Keywords: uncertainty theory; uncertain stochastic programming; fourth party logistics routing problem; complete information symmetry; symmetric decision space

1. Introduction

In the supply chain, the buyer and seller are called the first and second parties, respectively, and the logistics system that provides transportation, warehousing, and other services for the buyer and seller is called third party logistics, while the enterprise that provides the supply chain management, logistics consulting, training, and other system solutions for the first three parties is called fourth party logistics [1]. Among them, third party logistics is directly involved in transportation, while fourth party logistics only provides transportation strategies, and does not directly participate in the logistics transportation [2]. In the context of the global economic integration process and the rapid development of Internet technology, logistics systems, as the core link in the supply chain, have been widely concerned by enterprises and academia. At the same time, people’s requirements and expectations for logistics systems are also increasing, and more and more people expect logistics providers to complete transportation tasks faster, more accurately, and more efficiently [3]. Therefore, many scholars focus on fourth party logistics, which can effectively integrate transport capacity resources, and route information and customer information among multiple logistics providers in the logistics system [4], and the emergence of fourth party logistics also has a great positive impact on improving the supply chain’s service efficiency and reducing the logistics operating costs, and has injected new impetus into the further development of the logistics system [5].

Most of the existing research on the fourth party logistics routing problem focuses on modeling the random factors in the transportation process, and a few studies model the
uncertain factors. However, in practice, the fourth party logistics transportation process contains not only a large amount of randomness, but also a large number of uncertain factors; however, no scholar has yet comprehensively considered and modeled this, and this paper’s research is aimed at filling this gap. In order to accurately characterize the uncertainties and random factors in the fourth party logistics transportation process, and then provide more scientific decision-making guidance to supply chain decision makers (which is the research objective of this paper), we will characterize the corresponding uncertainties and random factors based on the uncertainty theory and probability theory, and establish a specific uncertain stochastic programming model, and provide scientific decision-making guidance to decision makers by solving the optimal solution of the corresponding model (which is the research purpose of this paper). In addition, the research of this paper is also based on a universal assumption; that is, assuming that the transportation time between nodes and the transit time of each node are uncertain variables, and assuming that the supply of manufacturers is a random variable.

Specifically, this paper mainly considers a fourth party logistics routing problem with a complete information symmetry scenario and a symmetric transportation volume decision space. Specifically, this paper assumes that the manufacturer fully understands the minimum demand of all distributors, and determines its supply to each distributor on this basis, which is a typical scenario of complete information symmetry. In addition, this paper also assumes that there is no limit on the maximum transportation volume on each path, which ensures that the manufacturer’s transportation volumes to different distributors do not interfere with each other. In other words, this paper solves the optimal transportation strategy on a symmetric transportation volume decision space. By characterizing the logistics supplier’s transportation time and the transportation time at the transportation point as uncertain variables, and characterizing the manufacturer’s supply as a random variable, this paper establishes an uncertain stochastic programming model with the constraints of the total transportation time, the manufacturer’s supply, and the distributor’s demand. In order to facilitate the calculation, this paper also converts the model into an equivalent classical mathematical programming model, and verifies the effectiveness of the model through numerical examples. The main contributions are as follows:

• An uncertain stochastic programming model for the single-manufacturer to multi-distributors 4PL routing problem with complete information symmetry scenario and symmetric transportation volume decision space is established.
• According to the distribution functions of uncertain time and random supply, the uncertain stochastic programming model is transformed into an equivalent mathematical programming model.
• Two numerical examples are given to verify the validity of the model.

The structure of this paper is divided into six sections. The first part introduces the research background of this paper and systematically sorts out the research ideas and main work contents of this paper. Section 2 reviews the literature on the fourth party logistics routing problem and uncertain programming, and Section 3 mainly describes the problems studied in this paper in detail, and gives the settings of related parameters. In Sections 4 and 5, the corresponding uncertain stochastic programming model is established and transformed into the equivalent mathematical programming model according to the relevant basic theories, respectively. In Section 6, the validity of the above model is verified, and Section 7 summarizes the research contents and conclusion.

2. Literature Review

In the current research on fourth party logistics, many scholars have conducted in-depth research on it from multiple perspectives and fields, such as [6–10]. Among the many topics in the research of fourth party logistics, the routing problem is the core one. Compared with the research of the traditional vehicle routing problem [11–14], the 4PL routing problem is more complex and diverse. The essential reason is that, during the path
planning process, decision makers not only need to consider how to choose the optimal transportation path, but also need to coordinate and balance the different paths and select the appropriate logistics provider to undertake the transportation task [15]. It is this combination of considerations that makes the decision of the 4PL routing problem more critical and complex. For the 4PL routing problem specifically, Li et al. [16] divided the two problems in the 4PL routing optimization problem into two stages to solve them; the path was selected in the first stage, and then the appropriate logistics provider was selected for the specific path in the second stage. Although this method is simple and easy to understand, the selection of the logistics provider in the later stage will be limited after the path selection, resulting in the failure to find the global optimal solution. Therefore, most scholars choose to consider both the choice of route and the choice of logistics provider. For examples, Yang et al. [17] considered the 4PL routing problem in the case of a random time delay, and established the expected value model and the opportunity constraint model with the minimum cost as the goal and the transportation time and capacity as the constraints, and made a comparative analysis of them. Ren et al. [18] regarded the transportation times as random variables based on probability theory, and established a 4PL routing opportunity model that met the constraints of total transportation cost and time window, in which the objective function was set to maximize the chance that the total transportation time meets the time window requirements.

The above studies were all performed in certain or random environments. However, in real life, due to the impact of various emergencies such as extreme weather and traffic accidents, and the uncertainty of transportation time during the transportation process, we often lack enough samples to predict the possibility of the occurrence of the event. At this time, it is necessary to infer the degree of certainty of its occurrence, based on similar experiences, or the opinion of relevant experts, and scholars have also performed a lot of empirical research to illustrate this point, including studies of epidemic spread [19], economic and financial analysis [20], factor analysis of grain yield [21], analysis of China’s population [22], and so on. In order to address this phenomenon in practical decision-making problems, Liu [23] presented uncertain programming based on uncertainty theory in 2009.

Up to now, uncertain programming has been widely applied in various fields [24–26]. For instance, Li et al. [27] put forward three uncertain risk indicators, and established an uncertain risk planning model for the problem of minimum risk path in uncertain networks based on the uncertainty theory. Kumar et al. [28] studied the multi-objective transportation problem in uncertain environments, and proposed a hyperbolic programming method to improve the satisfaction of the decision makers. Huang et al. [29] studied the problem of tourism route planning based on the uncertainty theory, and established the corresponding uncertain multi-objective planning model to ensure the minimization of travel time and consumption cost and the maximization of tourist satisfaction.

3. Symmetric Fourth Party Logistics Routing Problem

In an environment where information is completely symmetric and the traffic volume decision space is symmetric, a fourth party logistics company undertakes a transportation task to transport goods from manufacturers to different distributors under the constraints of total transportation time, manufacturer’s supply, and distributors’ demand. In particular, during the whole transportation process, goods will be transferred at different nodes, and different logistics providers can be chosen between nodes. The unit costs and times of transporting goods by different logistics providers are different, and only one logistics provider can be selected between two nodes. Therefore, decision makers need to choose the most suitable logistics provider between any two adjacent nodes of the selected path, so as to minimize the total cost of the entire transportation process.

In order to intuitively understand the above symmetric 4PL routing problem, we use an undirected network graph \( G(V, E) \) to represent the problem, where \( V \) represents the set of nodes in the undirected network graph, and \( E \) represents the set of edges. In this
undirected network graph, we set the starting node (manufacturer) of the transportation task as \( v_0 \), the possible transit points during transportation as

\[
v_i, \ i = 1, 2, \ldots, n,
\]

and the different destination nodes (distributors) in the transportation task as

\[
v_{n+l}, \ l = 1, \ldots, L.
\]

For any positive integers \( i \) and \( j \) with \( 0 \leq i \neq j \leq n + L \), the symbol \( e_{ijk} \) represents the \( k \)th logistics provider between \( v_i \) and \( v_j \) if there is an edge between \( v_i \) and \( v_j \). In addition, we can assign weights to nodes and edges in the network to represent the corresponding time and cost; that is, the transportation time and cost required to transport goods, as well as the transit time and transit cost incurred at the transit point, as shown in Figure 1.

![Figure 1. Undirected network diagram \( G(V, E) \) for symmetric 4PL routing problem.](image)

However, the actual transportation process is often affected by uncertain factors, such as extreme weather and traffic accidents, so that the transportation time between nodes and the transit time at each node are not constant, and even their frequency cannot remain stable. At this time, we will characterize them as uncertain variables. In addition, by considering that the supply of manufacturers has the characteristics of regular changes, we characterize it as a random variable.

To facilitate the modeling of the symmetric 4PL routing problem described above, some related metrics and parameters are introduced below:

- \( n \): the number of transit points during transportation;
- \( L \): the number of distributors;
- \( r_{ij} \): the number of logistics providers between node \( i \) and node \( j \);
- \( e_{ijk} \): the \( k \)th logistics provider between node \( i \) and node \( j \);
- \( t_{ijk} \): the time when the \( k \)th logistics provider transports the goods from node \( i \) to node \( j \), which is an uncertain variable;
- \( t'_{j} \): the transit time of goods at node \( j \), which is an uncertain variable;
- \( d_{ijk} \): the unit transportation cost of transporting goods from node \( i \) and node \( j \) by the \( k \)th logistics provider;
- \( d'_{j} \): the unit transit cost of goods at node \( j \);
- \( x_{l} \): the actual volumes shipped to distributor \( v_{n+l} \);
- \( Q \): the manufacturer’s supply, which is a random variable;
- \( q_{l} \): the minimum demand of distributor \( v_{n+l} \);
- \( T_{0} \): the maximum time limit for completing the task;
- \( R_{l} \): the path from \( v_{0} \) to \( v_{n+l} \) linked alternately by nodes and edges and containing information about the selected logistics provider on each edge;
\( a_{ijkl}(R_l) \): A boolean variable that takes a value based on the selection of the distributor, node, and manufacturer. If the \( k \)th logistics provider is selected between node \( i \) and node \( j \) in the routing policy for shipping to distributor \( v_{n+l} \), then \( a_{ijkl}(R_l) = 1 \). Otherwise, \( a_{ijkl}(R_l) = 0 \).

\( b_{jl}(R_l) \): A boolean variable that takes a value based on the selection of distributors and nodes. If the goods are transferred at node \( j \) in the routing policy for shipping to distributor \( v_{n+l} \), then \( b_{jl}(R_l) = 1 \). Otherwise, \( b_{jl}(R_l) = 0 \).

4. Uncertain Stochastic Programming Model

In this section, we will build the corresponding uncertain stochastic programming model for the fourth party logistics routing problem in Section 3.

For any given transportation task, the manufacturer can select different paths to distributors \( v_{n+1}, v_{n+2}, \cdots, v_{n+L} \) as

\[
R_l \subset V \cup E \cup \left\{ e_{ijk}, \ k = 1, 2, \cdots, r_{ij}, \ 0 \leq i \neq j \leq n + L \right\}, \ l = 1, 2, \cdots, L,
\]

which contain the information of the selected edges and the selected logistics provider between the selected adjacent edges. Note that the total transit cost of the goods at transit points from manufacturer \( v_0 \) to distributors

\[
v_{n+1}, v_{n+2}, \cdots, v_{n+L}
\]

is

\[
\sum_{l=1}^{L} \sum_{j=0}^{n} d'_{jl} b_{jl}(R_l) x_l,
\]

and the total transportation cost of the goods on all selected paths is

\[
\sum_{l=1}^{L} \sum_{i=0}^{n+L} \sum_{j=0}^{n+L} r_{ij} d_{ijkl} a_{ijkl}(R_l) x_l.
\]

Thus, the total transportation cost can be obtained as

\[
D = \sum_{l=1}^{L} \left( \sum_{i=0}^{n+L} \sum_{j=0}^{n+L} r_{ij} d_{ijkl} a_{ijkl}(R_l) x_l + \sum_{j=0}^{n+L} d'_{jl} b_{jl}(R_l) x_l \right).
\]

In addition, for each positive integer \( l \) with \( 1 \leq l \leq L \), the time for the manufacturer \( v_0 \) to transport the goods to the distributor \( v_{n+l} \) includes the transportation time in \( R_l \) on the selected path, and the transit time on the selected nodes; the transportation time is

\[
\sum_{i=0}^{n+L} \sum_{j=0}^{n+L} r_{ij} t_{ijkl} a_{ijkl}(R_l)
\]

and the transit time is

\[
\sum_{j=0}^{n+L} l'_{jl} b_{jl}(R_l),
\]

thus, the time for the manufacturer \( v_0 \) to transport the goods to the distributor \( v_{n+l} \) is

\[
\sum_{i=0}^{n+L} \sum_{j=0}^{n+L} r_{ij} t_{ijkl} a_{ijkl}(R_l) + \sum_{j=0}^{n+L} l'_{jl} b_{jl}(R_l).
\]
Note that the manufacturer’s transportation to each distributor is non-interfering. Then, the total time for the manufacturer to complete the transportation task is the maximum time for all distributors to complete the transportation task, i.e.,

\[ T = \sqrt[\sum_{l=1}^{L} n+n+L n+n+L \sum_{j=0}^{L} \sum_{k=1}^{L} t_{ijk} a_{ijkl}(R_l) + \sum_{j=0}^{L} t' b_{ij}(R_l)} \].

Since the total time \( T \) is an uncertain variable, we generally hope that the confidence that the goods can be delivered to all distributors before \( T_0 \) is \( \alpha \) (the time point when the goods depart from the node \( v_0 \) is time zero), then we can obtain the uncertain constraint of the total time as

\[ M \left\{ \sqrt[\sum_{l=1}^{L} n+n+L n+n+L \sum_{j=0}^{L} \sum_{k=1}^{L} t_{ijk} a_{ijkl}(R_l) + \sum_{j=0}^{L} t' b_{ij}(R_l)} \leq T_0 \right\} \geq \alpha. \tag{2} \]

On the other hand, since the manufacturer’s supply \( Q \) is a random variable, we also hope that the sum of the actual traffic \( x_i \) on each path is less than or equal to the manufacturer’s supply \( Q \) with a confidence level of \( \beta \), then we can infer the stochastic constraint of supply as

\[ \Pr \left\{ \frac{1}{L} \sum_{l=1}^{L} x_i \leq Q \right\} \geq \beta. \tag{3} \]

In addition, according to the minimum demand \( q_l \) of each distributor, we also have

\[ x_i \geq q_l \tag{4} \]

for \( l = 1, 2, \ldots, L \).

By characterizing the minimum total transportation cost as the objective function, Equations (2)–(4) as the corresponding time constraints, supply constraints and demand constraints, respectively, and the selected paths and transportation volumes as the decision variables, we can obtain an uncertain stochastic programming model for the above problem,

\[
\begin{align*}
\min_{R_l, x_l} & \quad \sum_{i=1}^{n+n+L} \sum_{j=0}^{L} \sum_{k=1}^{L} d_{ijk} a_{ijkl}(R_l) x_l + \sum_{j=0}^{L} d' b_{ij}(R_l) x_l \\
\text{subject to:} & \quad M \left\{ \sqrt[\sum_{l=1}^{L} n+n+L n+n+L \sum_{j=0}^{L} \sum_{k=1}^{L} t_{ijk} a_{ijkl}(R_l) + \sum_{j=0}^{L} t' b_{ij}(R_l)} \leq T_0 \right\} \geq \alpha, \\
& \quad \Pr \left\{ \frac{1}{L} \sum_{l=1}^{L} x_i \leq Q \right\} \geq \beta, \\
& \quad x_i \geq q_l, \\
\end{align*}
\]

\[
a_{ijkl}(R_l) = \begin{cases} 1, & \text{if } e_{ijk} \in R_l \\ 0, & \text{otherwise}, \end{cases}
\]

\[
b_{ij}(R_l) = \begin{cases} 1, & \text{if } v_j \in R_l \\ 0, & \text{otherwise}, \end{cases}
\]

\[
R_l \subset V \cup E \cup \{ e_{ijk}, k = 1, 2, \ldots, r_{ij}, 0 \leq i \neq j \leq n + L \},
\]

\[
k = 1, 2, \ldots, r_{ij}, 0 \leq i \neq j \leq n + L, l = 1, 2, \ldots, L.
\]
5. Equivalent Mathematical Programming Model

Because of the complexity of the uncertain constraint and stochastic constraint calculations, the above uncertain stochastic programming model (5) usually has difficulty finding the optimal strategy. Therefore, in order to solve the optimal strategy for the above symmetric 4PL routing problem, we need to transform the uncertain stochastic programming model (5) into an equivalent deterministic mathematical programming model according to the relevant algorithms in uncertainty theory and probability theory.

Assume that the uncertain variables $t_{ijk}, k = 1, 2, \cdots, r_{ij}, 0 \leq i \neq j \leq n + L$ and the uncertain variables $t'_{j}, j = 0, 1, \cdots, n + L$ are independent of each other, and their uncertainty distributions are $\Phi_{ijk}, k = 1, 2, \cdots, r_{ij}, 0 \leq i \neq j \leq n + L$ and $\Phi'_{j}, j = 0, 1, \cdots, n + L$, respectively. Without loss of generality, we also assume that the above uncertain variables have inverse uncertainty distributions $\Phi^{-1}_{ijk}, k = 1, 2, \cdots, r_{ij}, 0 \leq i \neq j \leq n + L$ and $\Phi'^{-1}_{j}, j = 0, 1, \cdots, n + L$, respectively. In particular, when the uncertainty distribution does not satisfy the regular condition, we can take the corresponding generalized inverse uncertainty distribution.

In addition, the total time

$$L = \sum_{l=1}^{L} \sum_{i=0}^{n+L} \sum_{j=0}^{r_{ij}} t_{ijk}a_{ijkl}(R_l) + \sum_{j=0}^{n+L} t'_{j}b_{jl}(R_l)$$

is strictly increasing with respect to $t_{ijk}, k = 1, 2, \cdots, r_{ij}, 0 \leq i \neq j \leq n + L$ and $t'_{j}, j = 0, 1, \cdots, n + L$, so, according to the operational laws of uncertainty distribution, we can convert the total time constraint in the uncertain stochastic programming model (5) to

$$L \sum_{l=1}^{L} \sum_{i=0}^{n+L} \sum_{j=0}^{r_{ij}} \Phi^{-1}_{ijk}(a_{ijkl}(R_l)) + \sum_{j=0}^{n+L} \Phi'^{-1}_{j}(b_{jl}(R_l)) \leq T_0. \quad (6)$$

On the other hand, assume that the supply $Q$ has a continuous probability distribution function, and let $\omega$ represent the probability distribution function of $Q$. Then, we have

$$\Pr\left\{ \sum_{l=1}^{L} x_l \leq Q \right\} = 1 - \Pr\left\{ Q < \sum_{l=1}^{L} x_l \right\} = 1 - \omega\left( \sum_{l=1}^{L} x_l \right).$$
It follows from (3) that
\[ 1 - \omega \left( \sum_{l=1}^{L} x_l \right) \geq \beta. \]
That is,
\[ \omega \left( \sum_{l=1}^{L} x_l \right) \leq 1 - \beta, \]
which is equivalent to
\[ \sum_{l=1}^{L} x_l \leq \omega^{-1} (1 - \beta). \] (7)

Based on the above results, by combining Equations (6) and (7), we can obtain the following deterministic mathematical programming model:

\[
\begin{align*}
\min & \quad R_l, x_l \\
\text{subject to:} & \quad \\
& \sum_{l=1}^{L} \sum_{j=0}^{n+L} \sum_{k=1}^{r_{ij}} d_{ijk} a_{ijk}(R_l) x_l + \sum_{j=0}^{n+L} d_{j}^{'} b_{j}(R_l) x_l \\
& \sum_{l=1}^{L} x_l \leq \omega^{-1} (1 - \beta), \\
& x_l \geq q_l,
\end{align*}
\] (8)

6. Numerical Examples

In this section, two numerical examples are designed to verify the validity of the proposed model. Before providing concrete examples, we first introduce two important distributions, namely the random truncated normal distribution \( \text{trun}(e, \sigma^2) \) and the uncertain truncated normal distribution \( \text{TRUN}(e, \sigma) \). The random truncated normal distribution is mainly used to model non-negative variables with stable frequency in practice, while the uncertain truncated normal distribution is mainly used to model non-negative variables with frequency instability and variable population problems in practice. The essential difference between these two distributions is that they depict different objects. The random truncated normal distribution is essentially used to depict research objects with randomness, while the uncertain truncated normal distribution is essentially used to depict research objects with uncertainty. Specifically, the distribution functions of the random truncated normal distribution \( \text{trun}(e, \sigma^2) \), and the uncertain truncated normal distribution \( \text{TRUN}(e, \sigma) \) are

\[
F(x|e, \sigma^2) = \begin{cases} 
0, & \text{if } x < 0 \\
\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(s-e)^2}{2\sigma^2}\right) ds, & \text{if } x \geq 0
\end{cases}
\]
and
\[
\Phi(x|\mu,\sigma) = \begin{cases} 
0, & \text{if } x < 0 \\
\left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}, & \text{if } x \geq 0,
\end{cases}
\]
respectively.

Since both supply and time are non-negative, the distributors’ demand in the random environment and the transit time and transportation time in the uncertain environment in the above problems are described, respectively, by means of random truncated normal distribution and uncertain truncated normal distribution. Next, we will provide two numerical examples in combination with specific scenarios to illustrate the effectiveness of the model proposed in this paper.

**Example 1.** A 4PL company needs to complete a transportation task, that is, transport the goods from the manufacturer (node \(v_0\)) to two distributors (node \(v_6\) and node \(v_7\)). During the transportation process, different logistics providers can be selected on different paths, as shown in Figure 2. Assume that the task is carried out in the environments of complete information symmetry and symmetric transportation volume decision-making space. Specifically, the task requires the 4PL company to complete the transportation by time \(T_0 = 28\), and the confidence level to complete the task within the specified time is \(\alpha = 0.95\). Suppose also that the manufacturer’s supply \(Q\) meets the random truncated normal distribution \(\text{trun}(20, 2)\), and the confidence level to meet demand is \(\beta = 0.9\), where the minimum demands of the two distributors are \(q_1 = 8\) and \(q_2 = 10\), respectively. In addition, the unit transit cost and transit time between logistics providers, the unit transportation cost, transportation time for each logistics provider are shown in Tables 1 and 2.

![Figure 2. Transport network in Example 1.](image)

**Table 1.** The unit transit cost and transit time of each logistics provider in Example 1.

<table>
<thead>
<tr>
<th>Start Node</th>
<th>End Node</th>
<th>Logistics</th>
<th>Unit Cost</th>
<th>Unit Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_0)</td>
<td>(v_1)</td>
<td>1</td>
<td>4</td>
<td>(\text{TRUN}(8,1))</td>
</tr>
<tr>
<td>\</td>
<td>(v_6)</td>
<td>2</td>
<td>7</td>
<td>(\text{TRUN}(6,1))</td>
</tr>
<tr>
<td>\</td>
<td>(v_7)</td>
<td>2</td>
<td>5</td>
<td>(\text{TRUN}(5,1))</td>
</tr>
</tbody>
</table>

**Table 2.** The unit transportation cost and transportation time of each logistics provider in Example 1.
By substituting the concrete data of the above parameters to the deterministic model (8), we can obtain

\[
\begin{align*}
\text{min} & \quad \sum_{l=1}^{2} \left( \sum_{i=0}^{7} \sum_{j=0}^{7} \sum_{k=1}^{2} d_{ijk}a_{ijkl}(R_l)x_l + \sum_{j=0}^{7} d_{ijkl}^{'}(R_l)x_l \right) \\
\text{subject to:} & \quad \sum_{j=0}^{7} x_j \leq \omega^{-1}(1 - \beta), \quad x_l \geq q_l, \\
& \quad a_{ijkl}(R_l) = \begin{cases} 1, & \text{if } e_{ijk} \in R_l \\ 0, & \text{otherwise} \end{cases}, \\
& \quad b_{ijkl}^{'}(R_l) = \begin{cases} 1, & \text{if } v_j \in R_l \\ 0, & \text{otherwise} \end{cases}, \\
& \quad R_l \subset V \cup E \cup \{e_{ijk}, k = 1, 2, 0 \leq i \neq j \leq 7\}, \\
& \quad k = 1, 2, 0 \leq i \neq j \leq 7, l = 1, 2.
\end{align*}
\]

Table 2. Cont.

<table>
<thead>
<tr>
<th>Start Node</th>
<th>End Node</th>
<th>Logistics</th>
<th>Unit Cost</th>
<th>Unit Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>v₁</td>
<td>v₂</td>
<td>1</td>
<td>3</td>
<td>TRUN(8,1)</td>
</tr>
<tr>
<td>v₁</td>
<td>v₃</td>
<td>1</td>
<td>5</td>
<td>TRUN(6,1)</td>
</tr>
<tr>
<td>v₁</td>
<td>v₄</td>
<td>2</td>
<td>6</td>
<td>TRUN(5,1)</td>
</tr>
<tr>
<td>v₁</td>
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<td>v₂</td>
<td>v₄</td>
<td>2</td>
<td>3</td>
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<tr>
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<td>v₅</td>
<td>1</td>
<td>8</td>
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<td>v₇</td>
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<td>6</td>
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</tr>
<tr>
<td>v₅</td>
<td>v₇</td>
<td>2</td>
<td>5</td>
<td>TRUN(7,1)</td>
</tr>
</tbody>
</table>
Solving the above deterministic programming model in MATLAB, we obtain the optimal path, as shown in Figure 3. Specifically, the actual path of distributor 1 starts from node $v_0$, passes through nodes $v_1$ and $v_3$ successively to node $v_6$, and the corresponding logistics provider is 1, 2, 2 successively, and the actual transport volume is 8. The actual path of distributor 2 starts from node $v_0$, passes through nodes $v_2$ and $v_5$ to node $v_7$ in turn, and the corresponding logistics providers are 2, 2, 1 in turn, and the actual transport volume is 10. The minimum total cost to complete this transportation task is 383.

$$
\begin{align*}
&v_0 \rightarrow v_1 \rightarrow v_3 \rightarrow v_6 \\
&v_0 \rightarrow v_2 \rightarrow v_5 \rightarrow v_7
\end{align*}
$$

Figure 3. Optimal path in Example 1.

Example 2. A 4PL company needs to complete a transportation task, that is, transport the goods from the manufacturer (node $v_0$) to three distributors (node $v_9$, node $v_{10}$ and node $v_{11}$). During the transportation process, different logistics providers can be selected on different paths, as shown in Figure 4. Assume that the task is carried out in the environments of complete information symmetry and symmetric transportation volume decision-making space. Specifically, the task requires the 4PL company to complete the transportation by time $T_0 = 38$, and the confidence level to complete the task within the specified time is $\alpha = 0.95$. Suppose also that the manufacturer’s supply $Q$ meets the random truncated normal distribution $\text{trun}(28, 2)$ and the confidence level to meet demand is $\beta = 0.9$, where the minimum demands of the two distributors is $q_1 = 11$, $q_2 = 6$ and $q_3 = 8$, respectively. In addition, the unit transportation cost, transportation time for each logistics provider, and the unit transit cost and transit time between logistics providers are shown in Tables 3 and 4.

Table 3. The unit transportation cost and transportation time of each logistics provider in Example 2.

<table>
<thead>
<tr>
<th>Start Node</th>
<th>End Node</th>
<th>Logistics</th>
<th>Unit Cost</th>
<th>Unit Time</th>
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<td>$v_2$</td>
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<td>$\text{TRUN}(4,1)$</td>
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<tr>
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<tr>
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<tr>
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<td>20</td>
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<td>$v_5$</td>
<td>1</td>
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<td>$\text{TRUN}(4,1)$</td>
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Table 3. Cont.

<table>
<thead>
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<th>End Node</th>
<th>Logistics</th>
<th>Unit Cost</th>
<th>Unit Time</th>
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</thead>
<tbody>
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<td>TRUN(8,1)</td>
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<tr>
<td>v7</td>
<td>v4</td>
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<tr>
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<td>TRUN(6,1)</td>
</tr>
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<td>v7</td>
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<td></td>
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<td>TRUN(8,1)</td>
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<tr>
<td>v8</td>
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<tr>
<td>v10</td>
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<tr>
<td>v10</td>
<td>v8</td>
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<td>TRUN(7,1)</td>
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<tr>
<td>v11</td>
<td>v10</td>
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<td></td>
<td>2</td>
<td>22</td>
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</tr>
</tbody>
</table>

Figure 4. Transport network in Example 2.
Table 4. The unit transit cost and transit time of each logistics provider in Example 2.

<table>
<thead>
<tr>
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<tbody>
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<tr>
<td>2</td>
<td>2/TRUN(2,1)</td>
<td>1.5/TRUN(1.5,1)</td>
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</tbody>
</table>

By substituting the concrete data of the above parameters to the deterministic model (8), we can obtain

\[
\begin{align*}
\min \quad & \sum_{l=1}^{3} \left( \sum_{i=0}^{11} \sum_{j=0}^{11} \sum_{k=1}^{2} d_{ijk} a_{ijkl}(R_l) x_l + \sum_{j=0}^{11} d'_j b_{jl}(R_l) x_l \right) \\
\text{subject to:} & \\
& \sum_{l=1}^{3} \left( \sum_{i=0}^{11} \sum_{j=0}^{11} \sum_{k=1}^{2} \Phi_{ijk}(a) a_{ijkl}(R_l) + \sum_{j=0}^{11} \Phi_{j}(b) b_{jl}(R_l) \right) \leq T_0, \\
& \sum_{l=1}^{3} x_l \leq \omega^{-1}(1 - \beta), \\
& x_l \geq q_l, \\
& a_{ijkl}(R_l) = \begin{cases} 
1, & \text{if } e_{ijk} \in R_l \\
0, & \text{otherwise}
\end{cases}, \\
& b_{jl}(R_l) = \begin{cases} 
1, & \text{if } v_j \in R_l \\
0, & \text{otherwise}
\end{cases}, \\
& R_l \subset V \cup E \cup \{e_{ijk}, k = 1, 2, 0 \leq i \neq j \leq 11, l = 1, 2, 3\}.
\end{align*}
\]

(9)

Solving the above deterministic programming model by MATLAB, we obtain the optimal path as shown in Figure 5. Specifically, the actual path of distributor 1 starts from node \(v_0\), passes through nodes \(v_1\), \(v_3\) and \(v_7\) successively to node \(v_9\), and the corresponding logistics provider is 1, 1, 1, 2 successively, and the actual transport volume is 11. The actual path of distributor 2 starts from node \(v_0\), passes through nodes \(v_2\), \(v_6\) and \(v_8\) to node \(v_{10}\) in turn, and the corresponding logistics providers are 2, 1, 1 and 1 in turn, and the actual transport volume is 6. The actual path of distributor 3 starts from node \(v_0\), passes through nodes \(v_2\) and \(v_6\) to node \(v_{11}\) in turn, and the corresponding logistics providers are 2, 1, 2 in turn, and the actual transport volume is 8. The minimum total cost to complete this transportation task is 2068.

\[
\begin{align*}
V_0 & \rightarrow V_1 \rightarrow V_3 \rightarrow V_7 \rightarrow V_9 \\
V_0 & \rightarrow V_2 \rightarrow V_6 \rightarrow V_8 \rightarrow V_{10} \\
V_0 & \rightarrow V_2 \rightarrow V_6 \rightarrow V_{11}
\end{align*}
\]

Figure 5. Optimal path in Example 2.

7. Conclusions

This paper discussed, in detail, the symmetric fourth party logistics routing problem of a single manufacturer providing goods to multiple distributors under an uncertain random environment with complete information symmetry and symmetric transportation volume.
decision space. Specifically, this paper deeply studied the uncertainty of transportation time and transit time, as well as the randomness of the manufacturer’s supply, and built an uncertain stochastic programming model based on these characteristics with the core goal of minimizing the total cost. In the process of solving the model, the algorithms of uncertainty theory and probability theory were used to transform the original complex uncertain stochastic programming problem into an equivalent deterministic programming model, which is more intuitive and easier to calculate. In order to verify the validity of the model, two numerical examples were also provided.

This paper only considered the fourth party logistics path problems under the constraints of total transportation time and supply quantity. In the future, more complex fourth party logistics routing problems with late delivery time constraints or payment of default costs in case of default delivery could also be considered.

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References


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