

Article

A One-Dimensional Dynamic Model for a Thin-Walled U-Shaped Boom Segment Considering Cross-Section Deformation

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Abstract: This article presents a one-dimensional dynamic model for a thin-walled U-shaped telescopic crane boom segment, considering cross-section deformation, to address complex and inefficient dynamic modeling issues. The symmetric U-shaped cross-section provides a uniform distribution of mass and stress, enhancing the beam's stability and bending stiffness. This symmetry allows for a simplified analysis in dynamic modeling, reducing the number of variables that need to be considered. The cross-section deformation is captured by basis functions satisfying displacement continuity conditions, which lays the foundation for constructing the initial model formulation based on the Hamilton principle. The variation forms of the cross-section are obtained by the decoupling eigenvalue problem, and then the principal component analysis is carried out to identify major cross-section deformation. The identified cross-section deformation features are hierarchically structured and have real physical significance. Finally, the initial one-dimensional higher-order dynamics model is improved by using the identified deformation mode. Numerical examples are presented in order to validate the three-dimensional dynamic properties and transient dynamic behavior of the U-shaped boom segment. The proposed model demonstrated high accuracy compared to ANSYS models, with relative errors below 2%. In addition, the method can be widely applied to a thin-walled U-shaped boom segment with a slenderness ratio of more than four.

Keywords: thin-walled U-shaped boom segment; one-dimensional higher-order model; principal component analysis; cross-section deformation



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1. Introduction

Thin-walled structures are crucial structural components with a significant impact on the fields of mechanical, automotive, and aeronautical engineering because of their excellent collision absorption capacity and lightweight attributes [1–3]. In order to minimize crane telescopic boom weight as much as possible, the configuration of the arm section has transitioned from traditional polygons to irregular shapes such as U-shapes, ellipses, and even the latest duck egg shapes [4]. The symmetrical U-shaped section helps to make more efficient use of the material, avoiding unnecessary weight gain while maintaining high strength and stiffness. The U-shaped section is a relatively reasonable section form obtained through optimization calculation. The symmetry of the thin-walled structure of the U-section helps to evenly distribute the load and reduce stress concentration. This in turn reduces the deformation and vibration of the beam during operation and improves the smoothness and safety of the crane. A telescopic boom is an important part of the all-terrain crane, and it consists of several separated TBSs, which hoist the load and expand the operational scope. The TBS consists of dissimilar thicknesses and high-strength steel, and the cross-section is a U-shaped section [4,5]. U-shaped booms are widely used in heavy loading and unloading equipment such as cranes because of their high carrying capacity [6]. Due to the benefit of its lifting capacity and convenient mobility, it plays an important role in many construction fields.

In terms of engineering, the crane telescopic boom with a high height-to-span ratio makes it simple to achieve stiff cross-section deformation when subjected to outside forces [7]. It has a direct impact on the performance of engineering equipment. The main beam unit theories commonly used when modeling and analyzing the dynamics of beams are Timoshenko and Euler–Bernoulli beams [8–12]. Alok proposed a new algorithm for calculating the natural frequencies and modes of non-uniform Timoshenko beams [8]. Xu derived the three-dimensional Euler–Bernoulli eccentric beam element of the U-shaped telescopic arm. On the premise of considering the discretization of gravity and wind load, internal degrees of freedom of the substructure are condensed to the boundary nodes, forming a geometrical nonlinear super-element [5]. In order to describe the dynamic performance of the arm joints as accurately as possible, solid or plate-shell units are often used for modeling in engineering. Compared with the traditional beam model, these methods are computationally inefficient and difficult to quickly analyze the structural dynamic response. However, the U-shaped telescopic arm structure exhibits complex higher-order cross-sectional deformation, which cannot be considered by the traditional Timoshenko beam theory. Higher-order beam theory can better consider the U-shaped section configuration, which becomes an important solution idea and research direction. Therefore, it is of great significance to study the higher-order deformation pattern recognition of a thin-walled U-shaped boom segment of the crane.

Currently, researchers mainly focus on two aspects of the deformation mode of thin-walled structure cross-sections. One aspect is improving beam models based on traditional beam theory by considering deformation modes of certain special behaviors. Yoon et al. [13,14] established the warping function for various thin-walled structures based on the classical beam theory and developed the warping deformation mode of the thin-walled beam model. Capdevielle et al. [15] provided a method for modeling by composite finite element structure and derived the torsional warping function of arbitrary section shape. To further emphasize the importance of warping modes in modeling, Li et al. [16] established a new finite element model for thin-walled beams in nonlinear analysis while introducing a higher-order interpolation method for the warping displacement field.

Another aspect involves identifying deformation modes through eigenvector decoupling differential equations based on the generalized beam theory. With the rapid development of generalized beams, Vieira et al. [17,18] presented a decoupling dynamic equation criterion for beam deformation. This derivation produces a set of higher-order deformation modes that can reflect the model. Bebiano et al. [19] investigated the buckling behavior of thin-walled members with different cross-sectional shapes under varied loading and support situations. Silvestre et al. [20] conducted an elastic–plastic analysis of thin-walled members based on GBT to examine the impact of deformation modes on the load-carrying capacity of beams beyond the yield limit. Additionally, variations in the load, deflection, and deformation mode configurations were applied to study the consequences. The research results indicate that the higher the beam load-bearing capacity, the larger the contribution of the local deformation mode to the beam damage. Ádány et al. [21,22] used a constrained finite element method to combine generalized beam theory and the constrained finite bar method. This method can also be used to identify thin-walled members with holes or reinforced plates in order to generate various deformation modes.

In terms of higher-order model operations, Choi et al. [23,24] proposed a higher-order beam theory with additional degrees of freedom for the cross-sectional deformation modes on the basis of Vlasov’s assumptions. This theory addresses the bending response issues that the traditional Timoshenko beam theory is unable to analyze. Jin et al. [25] proposed a new recognition criterion based primarily on mechanical characteristics as opposed to conventional deformation shapes based on three kinds of fundamental deformation: global deformation, local deformation, and distortion. The results suggested that the three deformation modes based on the new definitions were orthogonal to each other with respect to the stiffness of the member, which accelerated the computational efficiency of the dynamics model. Pagani et al. [26,27] studied the efficiency of the radial basis function

method applied to higher-order theories. This theory applies the notion of imaginary displacement to derive the equations of motion for the beam on free vibration and employs the Carrera Unified Formulation (CUF) to describe the displacement field of a general order beam model. Le et al. [28] proposed a thin-walled beam model for variable-section structures that does not require a pre-definition of warping of the cross-section but instead defines the deformation mode from the solution of the problem. Argyridi et al. [29] proposed a two-stage approach to higher-order beam theory. The principle considers the common deformation and warping modes using the boundary cell method in the first stage and the coupled solution using the finite element method in the second stage. For various directional displacement loads, the method demonstrates the involvement of higher-order deformation modes. Wu et al. [30] established an experimental platform system for obtaining modal properties of members by the resonance method. It was shown that preload can not only improve the overall dynamic model performance of the member but also help to enhance the local stiffness. The previous study demonstrates that existing research methods for thin-walled structures can handle the mechanical behavior of almost arbitrary cross-sectional deformation. But, it is difficult to solve the differential equations of the established higher-order model. Motivated by this, a simplified method that avoids invalid cross-sectional features and retains the essential features of the cross-sectional deformation is efficient for the calculation of the dynamical model of thin-walled structures.

In this paper, a one-dimensional higher-order dynamic model for the thin-walled U-shaped boom segment was established considering cross-sectional deformation. The outline of the paper is as follows. First, the basic deformations of the section based on appropriate discrete points are defined to establish the section displacement field in Section 2. The initial one-dimensional higher-order model of a thin-walled structure with U-shaped cross-section has been established in order to address complex computational issues. In Section 3, pattern recognition is used to decompose the eigenvector of the initial one-dimensional higher-order model. The identified characteristic deformation is used to improve the higher-order model. In Section 4, experimental results demonstrate that the method can preserve both computational accuracy and efficiency while simplifying the higher-order model. The paper concludes with a summary in Section 5.

2. One-Dimensional Higher-Order Beam Model

2.1. Displacement Field

The research object of this paper is a thin-walled U-shaped boom segment shown in Figure 1. The parameters L , h , b , and t , respectively, represent the overall length, half of the web height, half of the top plate width, and wall thickness of the U-shaped thin wall. To describe the displacement field, the space coordinate system (x, y, z) is set with the center of the section as the origin considering that a U-shaped cross-section is a closed section, as shown in Figure 1a. The z -axis is parallel to the beam axis, and the x -axis and y -axis represent the tangential and normal directions of the cross-section, respectively. The local coordinate system (n, v, p) is adopted on the center line of the thin-walled beam section. The displacement at the central line of the section is shown in Figure 1b, where v_i and n_i at the edge j are tangential and outward normal coordinates, and p_j is axial coordinates ($j = 1, 2, 3, 4, 5$).

For the U-shaped thin-walled structure, N nodes are introduced on the section to capture the deformation of the section. Tangential, axial, normal, and torsional unit displacements are applied at each node. A node only affects the displacement of two adjacent segments, and the displacement at other nodes is not affected. Linear functions are used to interpolate axial and tangential displacements, and Hermite polynomials are used to interpolate normal displacements. The basis functions are defined by form functions, each of which represents a deformation mode. The overall procedure is sketched in Figure 2, and details will be presented below.

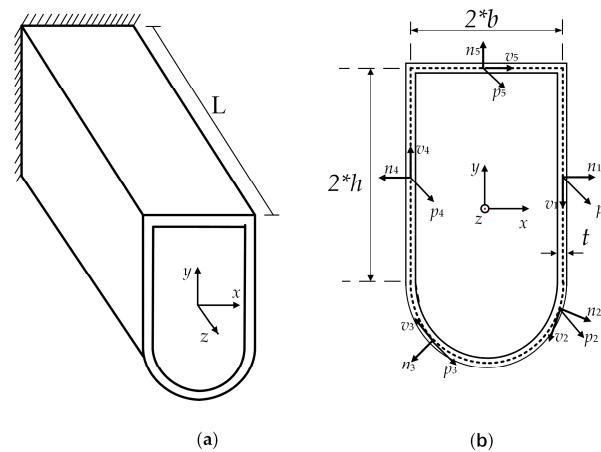


Figure 1. A thin-walled U-shaped boom segment. (a) The global coordinate system (x, y, z) ; (b) the local coordinate system (n, v, p) .

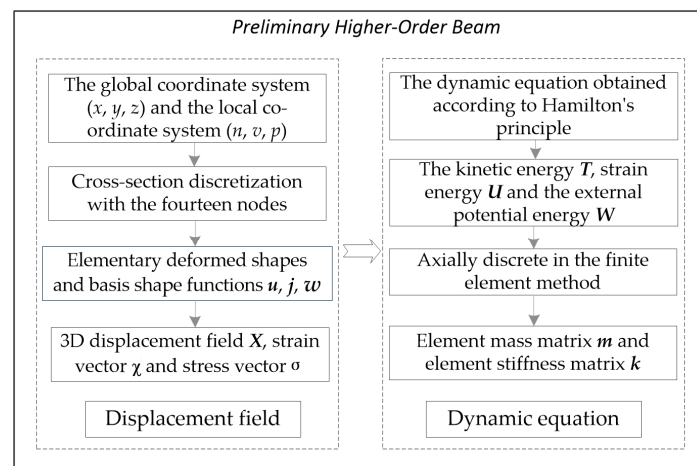


Figure 2. Flowchart of one-dimensional higher-order beam model.

Then, what needs to be considered is how to discretize the cross-section. In practical engineering applications, the higher the degree of model discretization, the more impact it has on the computational efficiency. It is related to the degree of dispersion of the model cross-sections. This is due to the possibility of unequal distribution caused by changes in model structure. It subsequently affects the generation of displacement fields and the number of cross-section deformations in pattern recognition. In general, the more discrete nodes there are, the more distinctive the deformation mode and the more displacement fields there are. Therefore, the cross-sectional discretization needs to be pre-defined as a criterion. The U-shaped cross-section can be divided into a set of walls, connected by some natural nodes, which connects adjacent walls or locates at the free ends [31]. The semicircular arc is divided into several segments according to the size of the radius, and some arc segments in the higher-order model are represented in Figure 3. The parameters a and θ represent the discrete side length and angle of the U-shaped thin-wall bottom plate, respectively. Additionally, three artificial nodes of three webs are employed to contribute to the capability of capturing cross-section deformation from the viewpoint of interpolation.

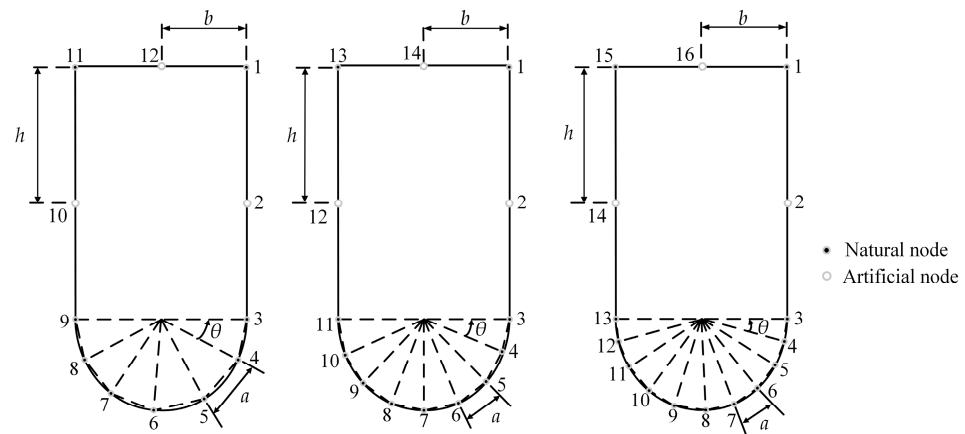


Figure 3. Discrete node processing method of the U-shaped cross-section.

There will be some impact on the accuracy depending on the different treatments of the curve sections. For example, the circular part of the model was discretized into eight segments to create a displacement field. Then, the values of a and θ can be determined. Tangential, axial, normal, and torsional unit displacements are applied at each node. A total of fifty-six deformation modes are generated as the model considers fourteen degrees of freedom. The four basic deformation modes of a node after displacement are shown in Figure 4. The parameters z , x , y , σ represent axial, tangential, normal, and torsion angles, respectively.

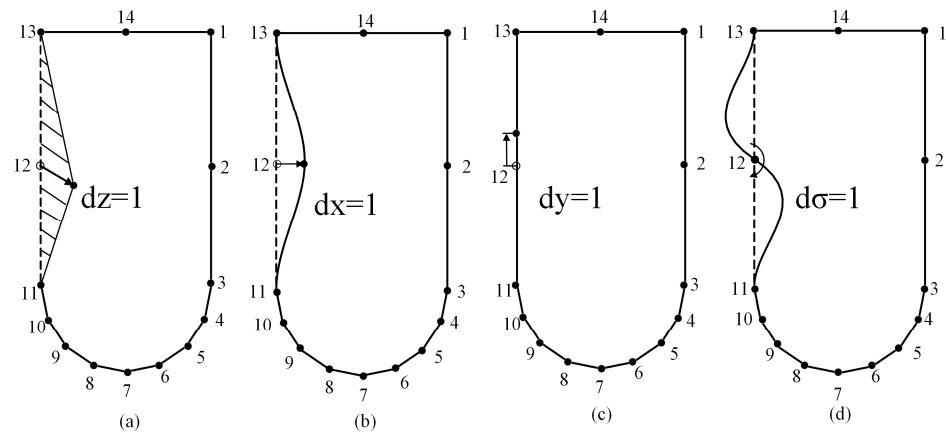


Figure 4. Shape functions resulted from unit deformations of node 12. (a) axial unit displacement; (b) tangential unit displacement; (c) normal unit displacement; (d) torsional unit displacement.

The three-dimensional displacement is approximated by the product form of the one-dimensional deformation and the corresponding section shape function, namely $u(v,p)$, $j(v,p)$, and $w(v,p)$ for the axial, tangential, and normal displacement components of the beam, respectively.

$$u(v,p) = \sum_{i=1}^{56} \beta_i^T(v) \alpha_i(p), j(v,p) = \sum_{i=1}^{56} \varphi_i^T(v) \alpha_i(p), w(v,p) = \sum_{i=1}^{56} \mu_i^T(v) \alpha_i(p) \quad (1)$$

where $\beta_i(v)$ is the out-of-plane warping basis function, and $\varphi_i(v)$, $\mu_i(v)$ denote the in-plane distorted basis function. A weight vector $\alpha(z)$ is introduced to describe the axial displacement changes of the thin-walled beam, while $\alpha(z) = [\alpha_1(z) \ \alpha_2(z) \ \dots \ \alpha_{56}(z)]^T$.

The three-dimensional displacement field $X(u, j, w)$ is as follows:

$$X = R\phi\alpha \quad (2)$$

where $\mathbf{R}$, $\boldsymbol{\phi}$ are the differential vector and coordinate transformation matrix. They are respectively expressed as:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -n \frac{\partial}{\partial p} \\ 0 & 1 & -n \frac{\partial}{\partial v} \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$\boldsymbol{\phi} = \begin{Bmatrix} \beta_1(v) & \cdots & \beta_{14}(v) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \varphi_1(v) & \cdots & \varphi_{42}(v) \\ 0 & \cdots & 0 & \mu_1(v) & \cdots & \mu_{42}(v) \end{Bmatrix} \quad (4)$$

2.2. Dynamic Equation and Finite Element

The strain component of a thin-walled beam was given based on the Kirchhoff hypothesis for small displacements. It consists of assuming a straight line perpendicular to the middle of the wall before and after the deformation, ignoring thickness variations, and not squeezing and stretching the intermediate surfaces against each other.

$$\boldsymbol{\chi} = \mathbf{C}\mathbf{X} \quad (5)$$

where $\mathbf{C}$ is the differential operator.

$$\mathbf{C} = \begin{bmatrix} 0 & \frac{\partial}{\partial v} & 0 \\ \frac{\partial}{\partial v} & \frac{\partial}{\partial p} & 0 \\ \frac{\partial}{\partial p} & 0 & 0 \end{bmatrix} \quad (6)$$

According to the linear elastic instantiation relationship, the stress component of a thin-walled beam is:

$$\boldsymbol{\sigma} = \mathbf{E}_q \boldsymbol{\chi} \quad (7)$$

where $\mathbf{E}_q$ is the constitutive matrix. It can be expressed as:

$$\mathbf{E}_q = \begin{bmatrix} \frac{Ev}{1-v_1^2} & 0 & \frac{E}{1-v_1^2} \\ 0 & G & 0 \\ \frac{E}{1-v_1^2} & 0 & \frac{Ev}{1-v_1^2} \end{bmatrix} \quad (8)$$

where E describes the elastic modulus, G denotes the shear elastic modulus, and v_1 represents the Poisson's ratio of the material.

The dynamic equation of the U-shaped thin-walled structure is obtained according to Hamilton's principle. In Hamilton's principle, the true trajectory of motion is determined by making the amount of action take a very small value.

$$\int_{t_1}^{t_2} (\delta \mathbf{T} + \delta \mathbf{U} + \delta \mathbf{W}) dt = 0 \quad (9)$$

where t_1 and t_2 are boundary time, and $\mathbf{T}$, $\mathbf{U}$, and $\mathbf{W}$ are the kinetic energy, strain energy, and the external potential energy. Each component is represented as:

$$\mathbf{T} = \frac{1}{2} \iiint_V \rho \frac{\partial \mathbf{X}^T}{\partial t} \frac{\partial \mathbf{X}}{\partial t} dV \quad (10)$$

$$\mathbf{U} = \frac{1}{2} \iiint_V \boldsymbol{\chi}^T \boldsymbol{\sigma} dV \quad (11)$$

$$\mathbf{W} = - \iiint_V \mathbf{X}^T \mathbf{f} dV \quad (12)$$

where ρ is the thin-walled beam material density, f is the column vectors of the distributed forces acting on the cross-section, and V is the volume of the U-shaped thin-walled structure. The parameter f can be expressed as:

$$f = [f_x \quad f_y \quad f_z]^T \quad (13)$$

By substituting Equations (10)–(12) into Equation (9), the dynamic equation of the U-shaped thin-walled structure can be obtained:

$$\iiint_V \rho \delta X^T \frac{\partial^2 X}{\partial t^2} dV - \iiint_V \delta X^T f dV + \iiint_V \delta (CX)^T E_q CX dV = 0 \quad (14)$$

Solving the analytical solutions of motion control differential equations requires very complex mathematical operations and is only applicable to the case of simple boundary conditions. The thin-walled U-shaped boom segment dynamics model in this paper is suitable to be solved numerically due to its complexity under different boundary treatments. With the rapid development and wide application of computer technology, numerical methods have become an important solution tool. The field of numerical analysis methods has seen the emergence of a range of methods such as the finite difference method, the finite element method, and the boundary element method [32,33].

For simple geometries and boundary conditions, the finite difference method is usually computationally efficient. But, the accuracy of the solution depends on the density and shape of the mesh. And local errors may lead to inaccuracies in the numerical solution in the presence of higher order derivatives. The boundary element method transforms a partial differential equation into an equation that integrates only on the boundary. It is suitable for problems where boundary conditions dominate. The finite element method works by splitting the region into discrete finite elements. The partial differential equations are transformed into a system of algebraic equations by approximating the spatial distribution of the solution using basis functions. It is particularly suitable for complex geometries and unstructured meshes and is able to deal more flexibly with a variety of boundary conditions and changes in material properties.

Based on the above considerations, this paper adopts the finite element method to solve the dynamics of the U-shaped thin-walled beam segments. The finite element method is used to solve the one-dimensional higher-order model, and the thin-walled beam is discretized into n elements by the Lagrange linear interpolation function:

$$\alpha = N d_i, i = 1, 2, \dots, n \quad (15)$$

where i denotes the node number, N is the linear interpolation function, and d is the nodal displacement vector.

$$N = \begin{bmatrix} \zeta_1 & & & \zeta_2 & & \\ & \zeta_1 & & & \zeta_2 & \\ & & \ddots & & & \ddots \\ & & & \zeta_1 & & \zeta_2 \end{bmatrix} \quad (16)$$

$$d_i = [\psi_1 \quad \dots \quad \psi_{56}(i) \quad \psi_1(i+1) \quad \dots \quad \psi_{56}(i+1)]^T \quad (17)$$

where ζ_1, ζ_2 are the interpolation function, and $(i), (i+1)$ are both ends of the unit.

The total displacement vector of the nodes is expressed as:

$$D = [\alpha_1(1) \quad \alpha_2(2) \quad \alpha_3(3) \quad \alpha_4(4) \quad \dots \quad \alpha_{56}(n)]^T \quad (18)$$

Substituting Equations (15)–(17) into Equation (9), it can be sorted out as follows:

$$\rho \iint_{bB} (\mathbf{R}\boldsymbol{\phi}\boldsymbol{\alpha})^T \mathbf{R}\boldsymbol{\phi}\boldsymbol{\alpha} \frac{\partial^2 \mathbf{d}}{\partial t^2} d\mathbf{B}d\boldsymbol{\zeta} - \iint_{bB} (\mathbf{R}\boldsymbol{\phi}\boldsymbol{\alpha})^T \mathbf{f} d\mathbf{B}d\boldsymbol{\zeta} + \iint_{bB} (\mathbf{C}\mathbf{R}\boldsymbol{\phi}\boldsymbol{\alpha})^T E_q \mathbf{C}\mathbf{R}\boldsymbol{\phi}\boldsymbol{\alpha} N d\mathbf{B}d\boldsymbol{\zeta} = 0 \quad (19)$$

where b is the axial integral region, and B is the section integral region.

The form of the dynamics equation can be arranged as:

$$\mathbf{m} \frac{\partial^2 \mathbf{d}}{\partial t^2} + \mathbf{k} \mathbf{d} = \mathbf{c} \quad (20)$$

where $\mathbf{m}$ is the element mass matrix, $\mathbf{k}$ is the element stiffness matrix, and $\mathbf{c}$ is the element load matrix. They are denoted as:

$$\mathbf{m} = \frac{1}{2} l \rho \int_V (\mathbf{R}\boldsymbol{\phi}N)^T \mathbf{R}\boldsymbol{\phi}N d\mathbf{B}d\boldsymbol{\zeta} \quad (21)$$

$$\mathbf{k} = \frac{1}{2} l \int_V (\mathbf{C}\mathbf{R}\boldsymbol{\phi}N)^T E_q \mathbf{C}\mathbf{R}\boldsymbol{\phi}N d\mathbf{B}d\boldsymbol{\zeta} \quad (22)$$

$$\mathbf{c} = \frac{1}{2} l \iint_{bB} (\mathbf{R}\boldsymbol{\phi}N)^T \mathbf{f} d\mathbf{B}d\boldsymbol{\zeta} \quad (23)$$

where l is the length of the unit.

The thin-walled beam in this paper is in a free vibration state without damping, so the load vector $\mathbf{f}$ is treated as zero. The overall mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ of the U-shaped thin-walled structure are expressed as:

$$\mathbf{M} = \sum_{i=1}^n \mathbf{T}_i^T \mathbf{m} \mathbf{T}_i, \mathbf{K} = \sum_{i=1}^n \mathbf{T}_i^T \mathbf{k} \mathbf{T}_i \quad (24)$$

where the transformation matrix of the node displacement vector $\mathbf{d}_i$ to the total node displacement vector $\mathbf{D}$ is represented by $\mathbf{T}_i$.

3. Pattern Recognition

Aiming at the problem of large degrees of freedom of the initial higher-order model, a data-driven approach is proposed to identify the cross-section deformation modes by principal component analysis of the free vibration deformation data. In this aspect, principal component analysis takes particular advantage since it can directly extract deformation patterns hiding in free vibration behaviors, achieving dimension reduction with minimum information loss. A set of principal deformation modes is obtained and selected to update the initial thin-walled U-shaped boom segment model, resulting in an improved higher-order beam model with high accuracy and efficiency.

3.1. Data Processing

The dynamic model of the U-shaped thin-walled structure is established according to the fifty-six deformation modes of the section. Of course, the model can be solved by theoretical calculation, but it is complicated due to the consideration of too many deformation modes. Therefore, it is necessary to identify reasonable and effective deformation modes from the above deformation to improve the model. The overall procedure is sketched in Figure 5, and details will be presented below.

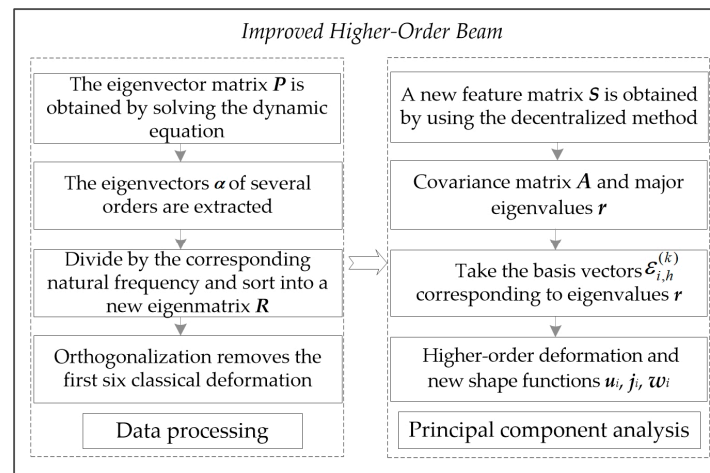


Figure 5. Flowchart of improved higher-order beam model.

The eigenvectors of the higher-order model are usually used as the basis for identifying deformation modes. The generalized eigenvalues of the higher-order model of the thin-walled beam are transformed to obtain the intrinsic frequencies of the system. Data preprocessing is necessary in order to obtain the cross-sectional feature deformation of thin-walled structures. The eigenvectors of the higher-order model are derived and combined in the matrix P , which can be expressed as follows:

$$P = [\alpha^{(1)} \ \alpha^{(2)} \ \dots \ \alpha^{(k)}] \quad (25)$$

where $\alpha^{(k)}$ is an eigenvector of order k .

Each order eigenvector is transformed into an eigenmatrix, $\alpha^{(k)}$, which is expressed as follows:

$$\alpha^{(k)} = [\alpha_1^{(k)} \ \alpha_2^{(k)} \ \dots \ \alpha_{N_1}^{(k)}] \quad (26)$$

where N_1 is the number of interpolation nodes along the thin-walled beam axis.

Each column of the feature matrix reflects the variation in the one-dimensional deformation, and the pattern recognition of the cross-sectional features is accomplished by decomposing the feature matrix. Since each node considers one out-of-plane and three in-plane degrees of freedom, it can be divided into out-of-plane deformation pattern recognition and in-plane deformation pattern recognition. The vector $\alpha^{(k)}$ is divided by the corresponding natural frequency of each mode. It is then sorted into a new eigenmatrix, which is further normalized to create a new modal vector matrix $R^{(k)}$:

$$R^{(k)} = \left[\frac{\alpha^{(k)}}{f^k} \right] = \begin{bmatrix} \alpha_{14 \times N_1}^o \\ \alpha_{42 \times N_1}^i \end{bmatrix} \quad (27)$$

where f^k is the k th mode natural frequency of a thin-walled U-shaped boom segment, and α^o and α^i are the feature matrix corresponding to the out-of-plane and in-plane deformation modes.

The next goal is to eliminate the interference of the classical deformation of the first six modes to subsequent pattern recognition. The first six classical characteristic deformation vectors and the deformation vectors of each section under each mode are orthogonalized, respectively. The out-of-plane deformation vector matrix α^{o1} and in-plane deformation vector matrix α^{i1} are extracted from the updated matrix W :

$$W = \left[T_1 - \sum_{i=1}^6 (c_i \times \text{dot} \frac{(g_i, c_i)}{(c_i, c_i)}), \dots, p_{(n \times m)} - \sum_{i=1}^6 (c_i \times \text{dot} \frac{(g_i, c_i)}{(c_i, c_i)}) \right] = \begin{bmatrix} \alpha_{14 \times N_1}^{o1} \\ \alpha_{42 \times N_1}^{i1} \end{bmatrix} \quad (28)$$

where, T_1 is the characteristic deformation vector of the section in the modal vector matrix $R^{(k)}$, c_i is the classical deformation eigenvector in the first six modes, and the g_i meets the following:

$$g_{i+1} = T - \sum_1^i (c_i \times \text{dot} \frac{(g_i, c_i)}{(c_i, c_i)}), (g_1 = T_1) \quad (29)$$

3.2. Principal Component Analysis

The plane deformation mode identified in this paper is based on the principal component analysis of the generalized eigenvalue vector of the eigenmatrix, which is a process of converting the original multiple variables into several linearly unrelated variables by linear transformation. At the same time, the magnitude of the eigenvalue is used to measure the contribution of the corresponding eigenvectors.

The feature matrix needs to be pre-processed before pattern recognition. A new feature matrix, S , is obtained by using the decentralized method.

$$S^{(k)} = \alpha_j^{(k)} - \frac{1}{m} \sum_{j=1}^m \alpha_j^{(k)} \quad (30)$$

where m is number of column vector elements in the feature matrix.

The covariance matrix A is constructed for the new identity matrix $S^{(k)}$.

$$A = \frac{S^T S}{n - 1} \quad (31)$$

where n is the total freedom of the sample.

The covariance matrix A is decomposed into eigenvalues and corresponding eigenvectors, where the deformation feature corresponding to the largest eigenvalue is called the major deformation mode and the deformation feature corresponding to the remaining eigenvalues is called the secondary deformation mode.

The eigenvalues, λ , are sorted from the largest to the smallest, that is:

$$\lambda_1^{(k)} > \lambda_2^{(k)} > \lambda_3^{(k)} \dots > \lambda_R^{(k)} \quad (32)$$

where R is the rank of the covariance matrix A .

The number of effective deformation modes generated by each order of eigenmatrix decomposition depends on Q . Q is the smallest positive integer satisfying the following equation:

$$\sum_{i=1}^Q \lambda_i^{(k)} / \sum_{i=1}^R \lambda_i^{(k)} \geq T \quad (33)$$

where T is the retention rate ($0 < T < 1$). The parameter T indicates the degree of retention of the primary element.

The eigenvector corresponding to the top r eigenvalues is taken as the basis vector ξ . Since a weight vector is the weight of the basis function, the weight vector $\alpha(z)$ in each order of the eigenvector of the original model is projected into a component that is co-linear with the basis vector. It is expressed as follows:

$$\varepsilon_{i,h}^{(k)} = (\xi_h^{(k)})^T \alpha_i^{(k)} \quad (34)$$

where $\varepsilon_{i,h}^{(k)}$ is the weight coefficient ($1 \leq i \leq 52, 1 \leq h \leq r$).

In the process of pattern recognition of a thin-walled U-shaped boom segment, some deformation modes may recur in different modal order categories. The column vectors in the weight matrix need to satisfy linear irrelevance to ensure their orthogonality.

$$\text{rank}(\varepsilon^{(k)}) = Q \quad (35)$$

The feature vector generated by the decomposition of the feature matrix is the weight vector of the extracted deformation basis functions. Final weight matrix is the weight set of the fifty-six deformation modes of the cross-section. Multiple out-of-plane and in-plane cross-sectional feature deformation vectors are integrated into new vector matrices. They are denoted as follows:

$$u_i(v) = \sum_{i=1}^{14} (\varepsilon_{i,h}^{(k)} \beta_i(v)) \quad (36)$$

$$j_i(v) = \sum_{i=14}^{56} (\varepsilon_{i,h}^{(k)} \varphi_i(v)) \quad (37)$$

$$w_i(v) = \sum_{i=14}^{56} (\varepsilon_{i,h}^{(k)} \mu_i(v)) \quad (38)$$

where $u_i(v)$, $j_i(v)$, and $w_i(v)$ are the axial displacement component, tangential displacement component, and normal displacement component.

After Equations (36)–(38) replace Equation (1), new cross-sectional deformations are added to form new shape function matrixes. The newly formed displacement transformation matrix ϕ is inserted into dynamic equation Equation (19), and the generalized eigenvalues are obtained by solving the mass and stiffness matrices of the U-shaped thin-walled structure in conjunction with Equations (20)–(24). It is important to note that the amplitude of the shape function of the deformation mode is normalized to ensure that its maximum displacement is one. The shape functions of the deformation modes are also solved and used to replace the basis functions integrated in the dynamics equations to simplify the beam model. The number of identified cross-section deformations is adjusted by setting the size of the retention degree T . An appropriate number of eigenvectors can be selected to construct a U-shaped thin-walled structure model that takes into account both computational accuracy and efficiency.

4. Numerical Examples and Model Validation

A thin-walled U-shaped boom segment dynamic model not only helps to optimize the structural design and improve working performance but also enhances the safety, stability, and reliability of the crane. This provides important technical support for engineering implementation and operation to a certain extent. In order to verify the correctness of the model proposed in this paper and the effectiveness of higher-order deformation, this section will compare the natural frequency, slenderness ratio adaptability, model validation, and transient dynamics modules of the ANSYS model. In practice, it fits the geometric dimensions of thin-walled beams and takes into account the ease of design and analysis time cost.

4.1. Numerical Examples

The U-shaped thin-walled structure is shown in Figure 1. Its geometric parameters include length $l = 8.00$ m, width $b = 0.204$ m, height $h = 0.306$ m, thickness $t = 0.0025$ m, density of the material $\rho = 7850$ kg/m³, elastic modulus $E = 200$ Gpa, and Poisson ratio $\nu_1 = 0.3$.

Under the premise of ensuring computational accuracy and reducing model complexity, high-order feature deformations with significant weights are used to reconstruct the initial higher-order model. Since the classical deformation mode accounts for a large

proportion of the first six modes, this paper directly selects modal vectors of the 7th~100th modes for pattern recognition.

A total of fourteen section deformation modes can be identified based on the pattern recognition algorithm from the 7th~100th orders, as shown in Figure 6. With the corresponding principal component vector as the weight, the section deformation modes are combined by linear combination of the fundamental form functions. The six deformations, namely a–c and h–j, correspond to the rigid deformation modes of the plane. The remaining eight deformation modes are the higher-order deformation modes of the plane, which are the higher-order deformation modes of plane distortion and plane warpage, respectively. The identified deformation modes have practical mechanical significance. Their addition can weaken the stiffness of the model and cause the natural frequency to decrease.

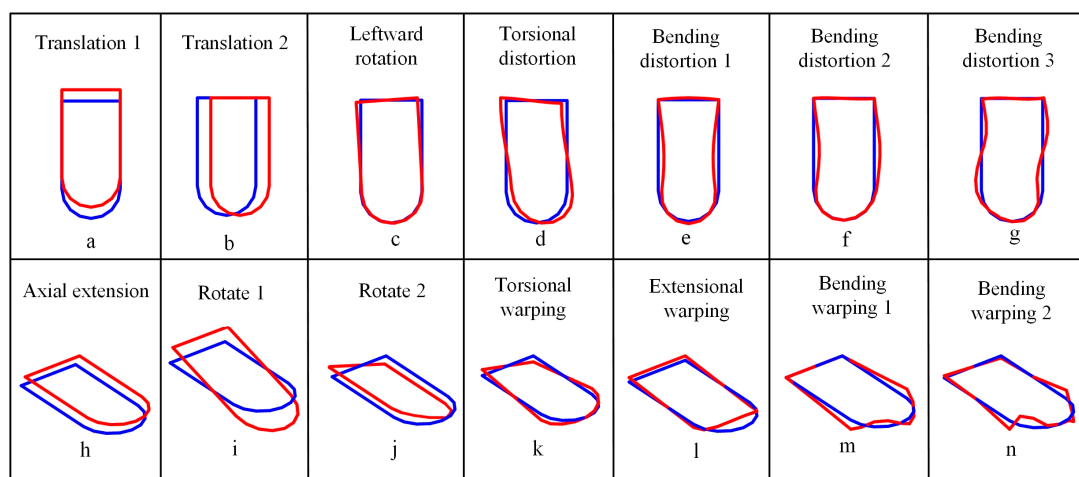


Figure 6. The first fourteen sectional deformation modes identified from the cantilevered thin-walled U-shaped boom segment.

The plane deformation modes of U-shaped thin-walled structures will be different with different recognition modes. When the computational accuracy of the model needs to be improved, the threshold is raised. Then, more deformation modes are introduced to improve accuracy. The initial model has more degrees of freedom and the calculation is more complex due to considering the basic deformation of each node. In order to further reduce the degree of freedom of the model, it is necessary to select the appropriate cross-section characteristic deformation combination to improve the higher-order model. According to the accuracy requirements of the actual model, the appropriate amount of cross-section deformation is selected to improve the accuracy of the model. For example, only six rigid body deformation modes need to be used where the accuracy of engineering calculations is low. In some cases with high precision requirements, the dynamic model of the U-shaped thin-wall structure can be established by gradually increasing the in-plane and out-of-plane higher-order deformation modes.

To verify the accuracy of the model, the one-dimensional higher-order model with fifty-six deformation modes and the improved one-dimensional higher-order model with fourteen deformation modes are used for free vibration analysis. The results are compared with the Shell 181 from ANSYS software. In order to obtain more accurate values, they are discretized into eighty cells in the axial direction by linear interpolation.

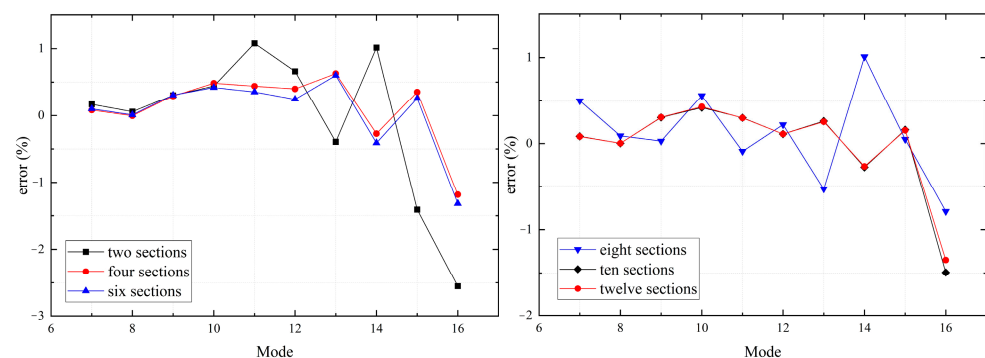
The U-shaped thin-wall structure is discretized into 3900 shell elements, of which 130 elements are along the beam axis and 30 elements are in cross-section. Then, the natural frequency is calculated through modal analysis. The ANSYS natural frequencies are denoted by f , the initial one-dimensional higher-order model and the improved one-dimensional higher-order model are denoted by f_1 and f_2 , and the corresponding errors are denoted by δ_1 and δ_2 , respectively. The results show the comparative results of the free vibration analysis of the U-shaped thin-walled structure in Table 1.

Table 1. The natural frequencies and relative errors of the U-shaped thin-walled structure.

Mode	Present Model	ANSYS Shell	Relative Error	Improved Model	Relative Error
	f_1 (Hz)	f (Hz)	δ_1 (%)	f_2 (Hz)	δ_2 (%)
7	10.692	10.826	−1.24	10.884	0.54
8	12.217	12.380	−1.32	12.391	0.09
9	13.995	14.184	−1.33	14.188	0.03
10	14.052	14.223	−1.20	14.302	0.56
11	15.009	15.165	−1.03	15.152	−0.09
12	18.098	18.291	−1.03	18.332	0.22
13	20.807	21.169	−1.71	21.056	−0.53
14	20.824	21.301	−2.24	21.529	1.06
15	22.155	22.677	−2.30	22.689	0.05
16	22.304	23.088	−3.40	22.904	−0.79

One can find that the proposed beam element is far more efficient in catching the natural frequencies. This is manifested in the fact that 80 linear elements have achieved similar precision with that of 3900 shell elements. The initial one-dimensional higher-order model with fifty-six basis functions can obtain the accuracy similar to the two-dimensional plate and shell theory. The relative error is kept within 3.4%. This is because the one-dimensional higher-order model adopts more planar degrees of freedom, which leads to a relatively low frequency. The first sixteen modes of free vibration frequencies are guaranteed to be within 1.06% based on eighty unit beam cells discretized along the axial direction with fourteen plane deformation modes. It ensures the computational efficiency and improves the computational accuracy and verifies the reliability of the identification method in this paper. It can be seen that if enough cross-section deformations are identified, the accuracy of the model may be reduced. It is necessary to select the main deformation modes with higher priority to form a more simplified higher-order model, which can effectively improve the computational efficiency.

The improved one-dimensional higher-order model reduces the number of degrees of freedom from fifty-six to fourteen and improves the computational efficiency. The free vibration frequency tends to be accurate and can be used for practical engineering verification. At the same time, the natural frequency of the model is affected to some extent due to the different discretization methods of the improved one-dimensional higher-order model. The error caused by the number of circular arc segments in the model analysis is shown in Figure 7. Similarly, all of them employ 80 refined beam elements equally distributed along the beam axis. All the improved higher-order models and ANSYS models have error comparisons within 3%. It indicates the extensive suitability of the refined higher-order beam model, even applying it to short beams.

**Figure 7.** Error caused by the number of arc segments in modal analysis.

The error fluctuation is larger in the 7th–16th orders when the arc of the U-shaped cross-section is discretized into two segments. The vibration pattern is relatively stable

with small amplitude when the arc is discretized into other segments. It can be found that the error fluctuation is the smallest in the fourteen discrete arcs. It is not difficult to find that the denser the discrete arc portion of the thin-walled structure, the higher the computational accuracy of the model. This increases the complexity of the model, but it reduces the error and improves the ability of the model to recognize changes.

4.2. Model Validation

In order to further investigate the dynamic behavior of the U-shaped thin walled structure, a comparison is made between the 7th and 14th vibration modes of the improved one-dimensional higher-order model and those of the ANSYS shell.

In Figure 8, the right of each mode is the improved one-dimensional higher-order model, and the left is from the results from ANSYS Shell 181. The identified in-plane deformation modes in the initial higher-order model are reflected in the vibration model, which strengthens the correlation. The results show that both vibration modes are very close to each other, which further validates the accuracy of the improved one-dimensional higher-order model in predicting the dynamic performance of the thin-walled U-shaped boom segment.

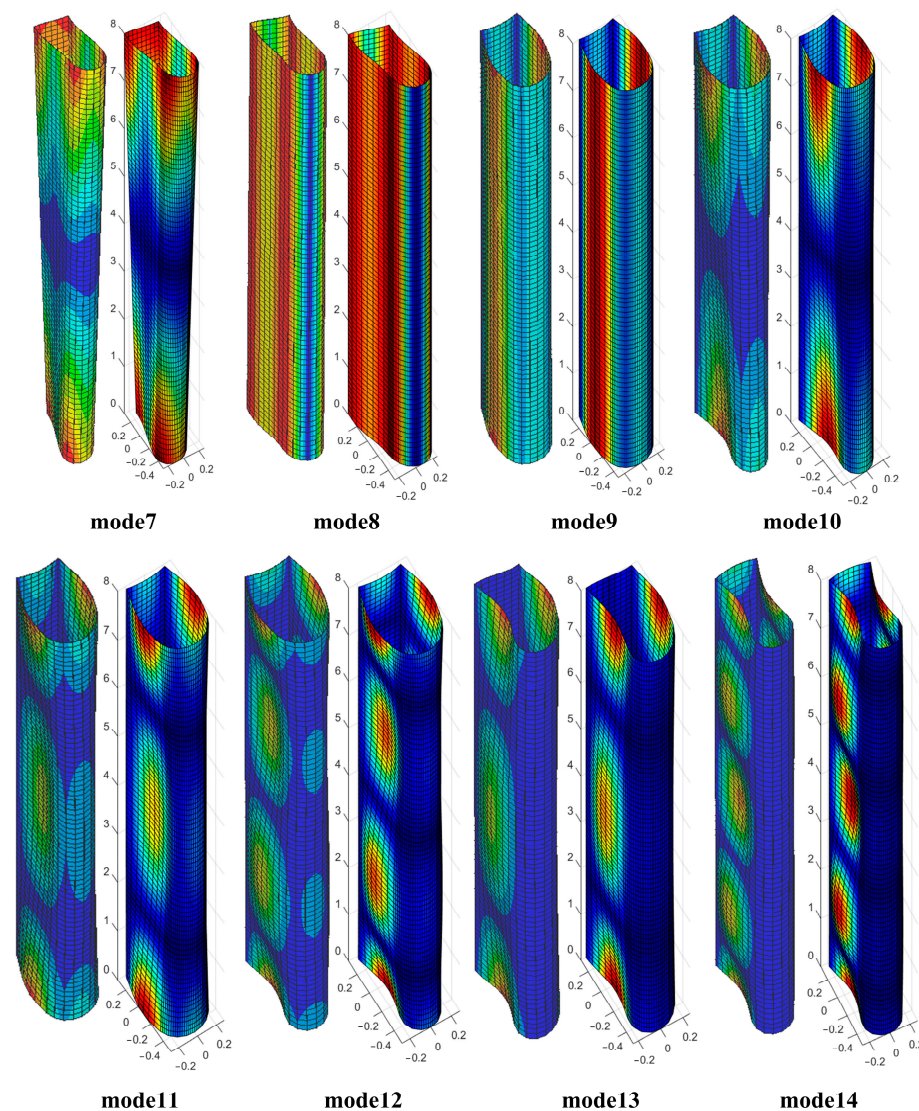


Figure 8. Comparison of modal shapes of the free vibration of the U-shaped thin-walled structure from ANSYS shell model (left) and improved model (right).

In order to prove the universality of the improved higher-order model, the free vibration analysis of a thin-walled U-shaped boom segment is carried out with a different slenderness ratio, e . It shows the trend of the first seven to sixteen modes of free vibration frequencies as the slenderness ratio e of the U-shaped thin-walled structure increases from four to ten in Figure 9. It can be seen that the vibration frequency error between the improved higher-order model and the ANSYS shell model gradually decreases with the increase in the slenderness ratio e . The improved computational accuracy indicates that the improved one-dimensional higher-order models are applicable to U-shaped thin-walled structures with a slenderness ratio e of more than four.

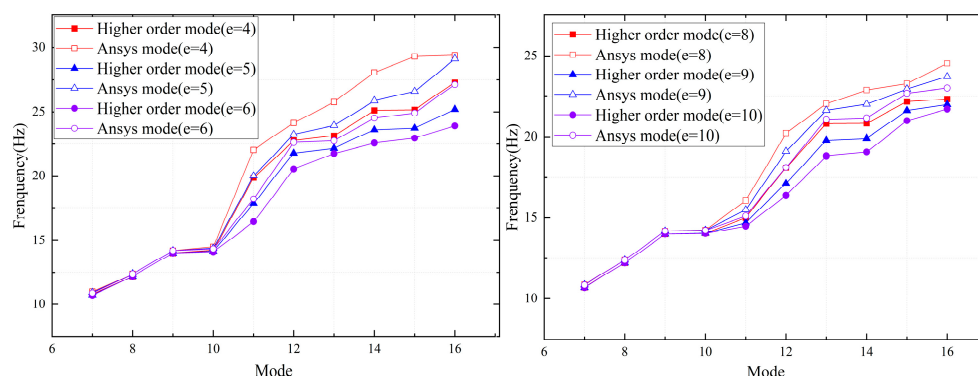


Figure 9. Comparison of the first sixteen free vibration frequencies of the U-shaped thin-walled structures with different slenderness ratios e .

According to the above research, the reliability of the model can be further verified through transient dynamic behavior. One end of the U-shaped thin-walled structure is fixed while the other end is subjected to impact load. In this case, a rectangular impact load $P = 1000\sin(10\pi t)$ Pa is applied in the range of $0.204 \text{ m} \times 0.1 \text{ m}$ on the upper wall of a thin-walled U-shaped boom segment in Figure 10.

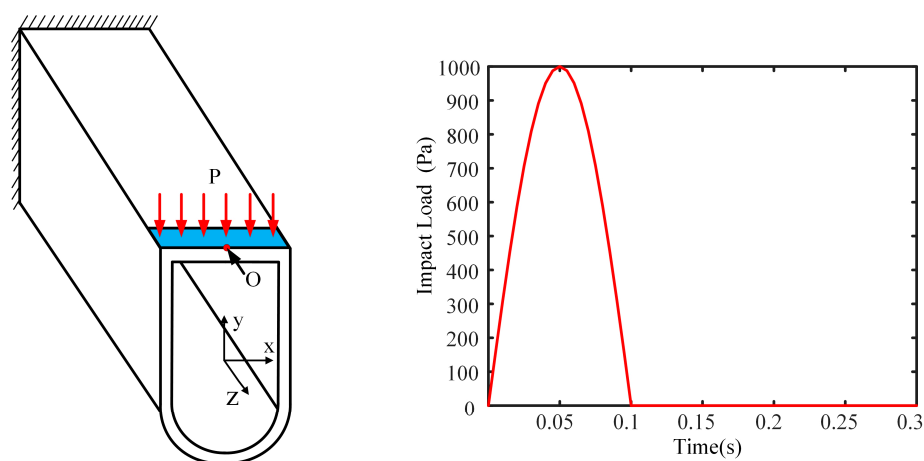


Figure 10. Impact load position (left). Impact load change curve (right).

The one-dimensional dynamic equations of the U-shaped thin-walled structure were developed under undamped conditions and the governing equations were solved using the Newmark method. The time step for both models was 0.005 s during the solution iterations from 0 to 0.3 s. The results are compared with the ANSYS software plate shell element in Figure 11. It shows the displacement curve of the normal direction of point O in the upper wall plate of the U-shaped thin-walled structure. In the initial phase, both fluctuate due to transient responses. Then the transient response rapidly attenuates and disappears due to the magnitude of the force, and gradually only the steady-state vibration is left. Both

consistent variations in normal displacement over time under the same impact load can be seen. This further confirms the reliability and accuracy of the improved one-dimensional higher-order model.

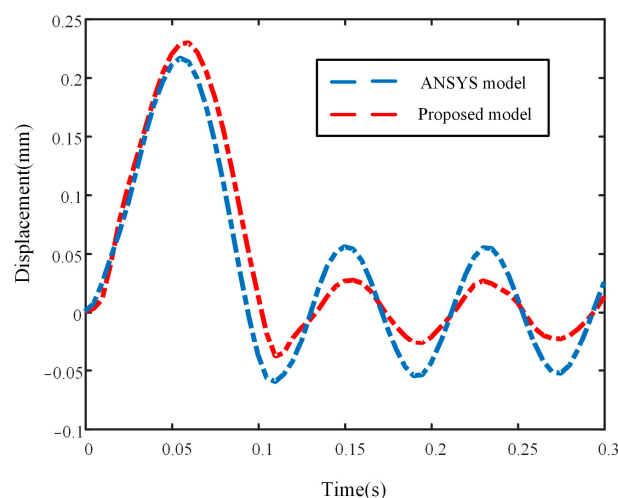


Figure 11. Transient dynamic response of the U-shaped thin-walled structure under impact loading.

5. Conclusions

In this paper, the basis function is constructed to describe the one-dimensional displacement field of the cross-section based on fifty-six plane deformation modes. At the same time, the governing differential equation of a thin-walled U-shaped boom segment is derived according to the Hamiltonian principle. The fourteen deformation modes are determined based on principal component analysis for the 7th to 100th modes of the higher-order model. Participation of higher-order deformation completes the one-dimensional higher-order dynamic model of a thin-walled U-shaped boom segment.

The natural frequencies are compared between the proposed model and the ANSYS model. The comparison shows that the higher-order model can be applied to engineering calculation with very high precision. The error is controlled within 2%, which proves the accuracy of the proposed model. The three-dimensional dynamic properties, slenderness ratio, and transient dynamic behavior of the U-shaped boom segment are verified by numerical examples.

The cross-section selected in this paper is symmetric closed, and the author will analyze the asymmetric or incomplete closed curve cross-section in subsequent studies to verify the reliability of this method more comprehensively. Furthermore, as the process of identifying deformation modes can be used in the non-linear domain, it is expected that this research can be extended to the field of simplified dynamical modeling.

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