

Effects of Dark Matter on the Properties of Strange Quark Stars

Jing Huang, Gan Wu, Xiao-Yang Zhang, Jin-Biao Wei  and Huan Chen ^{*} 

School of Mathematics and Physics, China University of Geosciences, Lumo Road 388, Wuhan 430074, China; 1202310917@cug.edu.cn (J.H.); 1202510948@cug.edu.cn (X.-Y.Z.)

^{*} Correspondence: huanchen@cug.edu.cn

Abstract

We investigate the effects of dark matter on the properties of strange quark stars within the framework of general relativity with two fluids coupled only by gravity. Adopting the color–flavor-locked model for strange quark matter and considering both fermionic (free fermion gas) and bosonic (polytropic) equations of state for dark matter, we systematically study the structure and tidal deformability of dark matter-admixed strange stars. Our results show that the presence of dark matter significantly modifies the mass–radius relations, with the maximum mass of dark matter-admixed strange stars exhibiting a non-monotonic dependence on the dark matter mass fraction χ , which reaches a minimum at an intermediate value of χ . The tidal deformability Λ of dark matter-admixed strange stars shows complex behavior depending on both the stellar mass and dark matter fraction, with $\Lambda - \beta$ (the compactness parameter) relations deviating from the universal relations observed for pure strange stars or dark stars. Our findings demonstrate that dark matter-admixed strange stars with different configurations but identical masses and radii can be distinguished by their tidal deformabilities, providing potential observational signatures for detecting dark matter in compact astrophysical objects. The results are compared with current astrophysical constraints from gravitational wave observations and pulsar measurements.

Keywords: strange quark stars; dark matter; tidal deformability

1. Introduction

As the most compact observable celestial objects, neutron stars (NSs), with their extreme interior environments, serve as natural laboratories for studying fundamental interactions and physics beyond the Standard Model [1]. Theoretical investigations of NSs can date back nearly a century [2–6]. Since the first observation of a pulsar [7], substantial progress has been made in relation to observations. In August 2017, the binary neutron star merger event GW170817 [8] confirmed gravitational waves as a second and equally powerful observational window alongside electromagnetic signals, inaugurating the era of multi-messenger astronomy. Currently operating detectors include LIGO, Virgo and KAGRA, and a new generation of ground-based and space-based instruments will soon expand the network [9–11]. Observations from these detectors have already provided tight constraints on the theoretical models of NSs [12,13], such as their tidal deformability. In addition, traditional electromagnetic observation methods also provide a large number of high-precision observations to constrain the properties of NSs, e.g., PSR J0740+6620 [14,15], PSR J0030+0451 [16,17], HESS J1731-347 [18], and XTE J1814-338 [19].

However, up to now, there are still many uncertainties in the internal composition of NSs, which have motivated a wealth of sophisticated models [20]. The concept of strange



Academic Editors: Parada Tobel
Paraduan Hutaauruk, Heribertus Bayu
Hartanto and Jesus Javier
Cobos-Martinez

Received: 17 February 2026

Revised: 8 April 2026

Accepted: 13 April 2026

Published: 16 April 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and

conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

stars (SS) composed entirely of self-bound strange quark matter (SQM) [21–24] has been studied for nearly half a century. It provides a unique observational window into the properties of matter within the low-temperature, high-density regime of the quantum chromodynamics (QCD) phase diagram. Unlike NSs, which are gravitationally bound and possess a low-density crust, SSs are self-bound by the strong interaction. This self-binding nature gives rise to a sharp, well-defined surface with a density discontinuity. Consequently, SSs are typically more compact than NSs of the same mass, with radii in the range of 10–12 km for masses around $1.4 M_{\odot}$. Another distinctive feature is that SSs can have arbitrarily small masses. Recent astronomical observations [25–28] provide an increasing number of candidates for SSs, such as the well-studied HESS J1731-347 [18].

The equations of state (EOSs) of SQM reflect its macroscopic properties and provide a key input for studying the structure of SSs. Yet many uncertainties persist due to the challenges in solving nonperturbative QCD. Therefore, various effective models have been proposed, such as the MIT bag model [29,30], the color-flavor-locked (CFL) model [31], the Nambu–Jonna–Lasino model [32], the quasi-particle model [33], the confining density-dependent mass model [34], and models based on the Dyson–Schwinger equations [35–39]. These models aim to reproduce some of the most important properties of QCD, such as asymptotic freedom, confinement, dynamical chiral symmetry breaking and its restoration, as well as color superconductivity, and can provide reasonable descriptions of hybrid NSs or SSs.

On the other hand, dark matter (DM), first hypothesized by Zwicky [40] in the 1930s and steadily corroborated by subsequent observations [41–43], has become one of the most pressing open questions in contemporary physics. Its extremely weak interaction with ordinary matter leaves its intrinsic properties almost entirely unknown. Various conjectures on the particle nature of DM have been proposed [44], but none have been experimentally confirmed to date [45]. During the past three decades, several groups have advanced the idea that NSs, whose global properties are relatively well-characterized, can be used as astrophysical laboratories to probe DM [46–49]. As a result, a growing body of work has systematically incorporated the effects of DM on NSs [50–57]. In this work, we focus on dark-matter-admixed SSs (DSSs), which have also attracted much attention recently [58–67]. Previous studies have investigated the properties of DSSs, such as mass–radius relations, tidal deformability, and radial oscillations with various EOSs of SQM and DM, and have compared these results with astrophysical observations.

In this paper, we investigate the effects of DM on the properties of SSs, adopting various models of SQM and DM. In particular, we adopt the CFL model for SQM. The CFL phase of SQM has been shown to be the ground state of QCD at extremely high densities [31] and might occur in compact stars. It is characterized by a specific pattern of chiral and color symmetry breaking: $SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R$ is broken down to the diagonal $SU(3)_{\text{color}+L+R}$. For DM, we will investigate two fundamental particle types: fermionic and bosonic DM. The two-fluid system considered here also possesses a consistent symmetry structure: SQM and DM are coupled only through gravity, where each fluid independently conserves its energy–momentum tensor. This gravitational coupling also preserves the individual $U(1)$ symmetries associated with particle number conservation for each component. These symmetry structures are manifested in the two-fluid Tolman–Oppenheimer–Volkoff (TOV) equations and radial oscillation equations [64], as well as the observable properties of DSSs.

We explore in detail how the presence of DM influences the mass–radius relations and tidal deformability of SSs. Concerning the conventional capture scenarios [68], previous studies in the literature [58–61] usually investigate cases with small DM fractions in DSSs. However, to facilitate a systematic theoretical exploration of DM’s influence, we treat the

DM mass fraction in DSSs χ as a phenomenological parameter ranging from zero to one in this work. We emphasize that DM fractions of order unity are challenging to realize in conventional capture scenarios without invoking additional interactions or exotic formation mechanisms. The formation mechanisms of such DSSs are not discussed in this work.

The paper is organized as follows. In Section 2, we introduce the EOSs for SQM and DM. In Section 3, we introduce the equations governing the static structure and tidal deformability of two-fluid stars. In Section 4, we present our numerical results on the structure and tidal deformability of DSSs. In the last section, we present a summary and the conclusions. Throughout this paper, we adopt natural units with $c = \hbar = 1$ while retaining the gravitational constant G explicitly.

2. EOSs of SQM and DM

The EOS describes the relationship between pressure p and the energy density ϵ of matter, usually written as $p = p(\epsilon)$. EOSs play a fundamental role in studying the structure and evolution of stars, as different EOSs yield distinct predictions for their structural properties. In the study of NSs and SSs, EOSs directly determine the mass, radius, and internal structure of these stars. In the following subsections, we present the EOSs adopted in this work for SQM and DM.

2.1. Strange Quark Matter

In the density region of the interior of compact stars, perturbative QCD and first-principle non-perturbative QCD calculations face significant challenges. Therefore, phenomenological models are widely used to describe the EOS of SQM. In this work, we adopt the CFL model to describe SQM.

The CFL model is an improved bag model that incorporates color superconductivity at high densities. The MIT bag model, introduced in Refs. [29,30], describes quark matter as free particles confined within a bag, with a bag constant B to simulate quark confinement. Specifically, a bag constant B contributes negatively to the pressure and positively to the energy density. This implies that the EOS of SQM is shifted relative to that of a free Fermi gas, leading to a lower pressure for SQM at a given energy density. Its EOS takes an extremely simple form in the zero-temperature and massless quark limit, given explicitly as

$$p = \frac{1}{3}(\epsilon - 4B). \quad (1)$$

Due to the simplicity, it is often used as a benchmark for studying SQM and SSs.

The CFL model, introduced in Ref. [31], improves the bag model by accounting for the effect of the energy gap Δ of the color superconductivity. The corresponding EOS given by Refs. [69,70] can be expressed as follows:

$$\epsilon(p) = 4B + 3p - \frac{9\alpha\mu^2}{\pi^2}, \quad (2)$$

$$\alpha = \frac{2\Delta^2}{3} - \frac{m_s^2}{6}, \quad (3)$$

$$\mu^2 = -3\alpha + \sqrt{9\alpha^2 + \frac{4}{3}\pi^2(B + p)}, \quad (4)$$

where m_s is the strange quark mass and Δ is the color superconductivity energy gap. The CFL model has been extensively studied in Ref. [70] with respect to observational constraints on NSs. The influences of the parameters on the SS structure and nonradial oscillations have been studied. It was found that increasing the parameter Δ can stiffen the EOS and increase the maximum mass of SSs. In contrast, increasing the bag constant or the strange quark mass will soften the EOS and decrease the maximum mass of SSs. In

comparison with EOSs from the MIT bag model, which are generally too soft to support the observed maximum mass of NSs, such as PSR J0740+6620 [16,17], the CFL model with proper parameters can satisfy the observational constraints on NSs much more effectively.

2.2. Dark Matter

Given the lack of experimental knowledge regarding the fundamental nature of DM, its EOS is highly uncertain, and various EOS models have been proposed to describe DM. DM particles are generally hypothesized to be either fermionic or bosonic. In this work, we adopt two representative EOS models for DM. The first is a free fermion gas EOS, representing fermionic DM, and the other is a polytropic EOS [71], representing bosonic DM.

The free fermion gas model has been widely used in studying DM-admixed NSs or DSSs [50,60,62,64]. For the free fermion gas with a finite DM particle mass m_D , hereafter referred to as DPM, the pressure and energy density are given by

$$\epsilon_{free} = \frac{m_D^4}{8\pi^2} [(X_F + 2X_F^3)\sqrt{1 + X_F^2} - \sinh^{-1}(X_F)], \quad (5)$$

$$p_{free} = \frac{m_D^4}{24\pi^2} [(-3X_F + 2X_F^3)\sqrt{1 + X_F^2} + 3\sinh^{-1}(X_F)], \quad (6)$$

where $X_F = \frac{k_F}{m_D}$ and k_F is the Fermi momentum. We note that in this model, the only free parameter is m_D , and we investigate the effects of varying m_D in this work.

For bosonic DM, we adopt a widely used EOS based on the Gross–Pitaevskii equation, a polytropic EOS that describes matter in the Bose–Einstein condensate [71,72]. The corresponding EOS is expressed as follows:

$$p(\epsilon) = K\epsilon^2 = \frac{2\pi a}{m_D^3}\epsilon^2, \quad (7)$$

where a is the scattering length and m_D is DPM. Similarly to the fermionic model, we also focus on the effects of DPM on stellar properties. The stiffness of the bosonic DM EOS and consequently the maximum mass of DM stars (DSs) are also influenced by the scattering length a , with larger (smaller) values of a leading to stiffer (softer) EOS and higher (lower) maximum masses of DSs. Following common practice in the literature [71,73–76], we fix the scattering length $a = 1$ fm as a representative value. A systematic investigation of the effects of a on DSS properties is left for future work.

3. Structure and Tidal Deformability of a Dark Matter-Admixed Strange Star

By combining the above EOSs, one can obtain macroscopic properties of NSs by solving the equations from general relativity. In the following, we will show the static structure equations that govern the self-gravitating equilibrium of DSSs. We will also show the equation for the tidal deformability of a DSS, presenting its linear response to an external tidal field during a binary inspiral.

3.1. Structural Equations

The so-called TOV equations proposed in the 1930s [5,6] provided the first structural equations for a spherically symmetric static NS.

To obtain the properties of compact stars under a strong gravitational field, one starts from the Einstein field equation, i.e.,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi GT_{\mu\nu}, \quad (8)$$

where $g_{\mu\nu}$ and $T_{\mu\nu}$ represent the metric tensor and the energy–momentum tensor, respectively, $R_{\mu\nu}$ and R represent the Ricci tensor and the Ricci scalar, respectively, which are determined by the metric tensor and its partial derivatives. For the static and spherically symmetric case, the metric takes the form

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2, \quad (9)$$

where $\nu(r)$ and $\lambda(r)$ represent the metric functions, and we adopt the signature $(+, -, -, -)$. The matter in an NS is treated as a perfect fluid, so its energy–momentum tensor can be written as

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu}, \quad (10)$$

where $u^\mu = \frac{dx^\mu}{d\tau}$ represents the contravariant form of a four-velocity. For a static star, the energy–momentum tensor could be written as $T^\mu_\nu = \text{diag}(\epsilon, -p, -p, -p)$.

Putting the metric tensor and the energy–momentum tensor into Equation (8), one can obtain the famous TOV equations:

$$\frac{dp}{dr} = -\frac{G(m + 4\pi r^3 p)}{r^2(1 - 2Gm/r)}(p + \epsilon), \quad (11)$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon. \quad (12)$$

By specifying a central pressure p_c and integrating from $r = 0$ to $p(r = R) = 0$, one can obtain the mass $M = m(R)$ and radius R of a NS. The metric functions can be obtained as

$$e^{-\lambda} = 1 - \frac{2Gm}{r}, \quad (13)$$

$$\frac{dv}{dr} = \frac{2G(m + 4\pi r^3 p)}{r^2(1 - 2Gm/r)}, \quad (14)$$

with the boundary condition that $\nu(r = \infty) = 0$.

For a two-fluid system coupled only through gravity, its energy–momentum tensor can be written as a summation of the two individual fluids $T^{\mu\nu} = \sum_i T_i^{\mu\nu}$ with the subscript i representing SQM or DM. Each fluid satisfies energy–momentum conservation, i.e., $\nabla_\mu T_i^{\mu\nu} = 0$. The corresponding static structural equations for a DSS can be obtained in the following form [77]:

$$\frac{dp_i}{dr} = -\frac{G(m_{tot} + 4\pi r^3 p_{tot})}{r^2(1 - 2Gm_{tot}/r)}(p_i + \epsilon_i), \quad (15)$$

$$\frac{dm_i}{dr} = 4\pi r^2 \epsilon_i, \quad (16)$$

where $m_{tot} = \sum_i m_i$ and $p_{tot} = \sum_i p_i$. Equations (15) and (16) can be solved numerically. At the center, we set $m_q = m_d = 0$ and choose specific central pressures $p_{c,q}$ and $p_{c,d}$ for SQM and DM, respectively. Then we integrate Equations (15) and (16) using a fourth-order Runge–Kutta scheme with adaptive step size control. When both the pressures of the two fluids vanish, i.e., $p_q(r = R_q) = 0$ and $p_d(r = R_d) = 0$, we obtain the structure of a DSS. NB, there are two different radii R_q and R_d for SQM and DM, respectively. We usually take the larger one as the radius R of a DSS, and the mass of a DSS is the total mass $M = m_{tot}(R)$. However, when observing pulsars via electromagnetic methods—which are insensitive to DM—the measured visible radius corresponds to the SQM radius R_q . When $R_q < R_d$, the outer stellar layer composed of DM is called the dark halo. Of course, in such cases, we must account for the gravitational effects of the dark halo when analyzing the

electromagnetic signals from the visible radius R_q , e.g., path changes or light redshift. In this work, we also aim to construct sequences of DSSs with a fixed DM fraction $\chi \equiv M_d/M$. We fix the SQM central pressure $p_{c,q}$ and employ the bisection method to determine the corresponding DM central pressure $p_{c,d}$ that yields the target χ , by exploiting the monotonic relationship between χ and $p_{c,d}$. The convergence tolerance for χ is set to 10^{-5} .

3.2. Tidal Deformability

Tidal deformability is also an important property of NSs, which can be measured via gravitational wave emissions from the binary NS inspiral. Tidal deformability Λ can be obtained by

$$\Lambda = \frac{2}{3}k_2\beta^{-5}, \quad (17)$$

where $\beta = \frac{GM}{R}$ represents the compactness of a star and k_2 is the $l = 2$ tidal Love number, which can be expressed as

$$\begin{aligned} k_2 = & \frac{5}{8}(1-\beta)^2\beta^5[2\beta(y_R-1-y_R+2)] \\ & \times \{2\beta[(4y_R+1)\beta^4 + (6y_R-4)\beta^3 + (26-22y_R)\beta^2 + 3(5y_R-8)\beta - 3y_R+6] \\ & + 3(1-2\beta)^2[2\beta(y_R-1)-y_R+2]\ln(1-2\beta)\}^{-1}, \end{aligned} \quad (18)$$

where the variable $y_R \equiv y(R) = [rH'(r)/H(r)]_{r=R}$ is the logarithmic derivative of the metric perturbation $H(r)$ [78,79]. The function $y(r)$ satisfies the differential equation [80,81]:

$$r\frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2Q(r) = 0, \quad (19)$$

with

$$F(r) = \frac{r - 4\pi Gr^3 \sum_i (\epsilon_i - p_i)}{r - 2GM}, \quad (20)$$

$$\begin{aligned} Q(r) = & \frac{4\pi Gr}{r - 2GM} \times \left\{ \sum_i [5\epsilon_i + 9p_i + \frac{\partial \epsilon_i}{\partial p_i} (\epsilon_i + p_i)] - \frac{6}{4\pi Gr^2} \right\} \\ & - \left[\frac{8\pi Gr^3 \sum_i p_i + 2GM}{r(r - 2GM)} \right]^2. \end{aligned} \quad (21)$$

Equation (19) depends on the internal distributions of pressure and energy density in a DSS and can be solved simultaneously with the structural equations Equations (15) and (16). In addition, the boundary condition $y(r=0) = 2$ is essential, where a detailed derivation is given in Ref. [78]. Furthermore, the energy density of SQM does not decrease to zero at its surface, i.e., it exhibits a step function behavior. Consequently, the term $\frac{\partial \epsilon_i}{\partial p_i}$ in $Q(r)$ behaves as a δ function, which results in an additional contribution to the integral of $y(r)$. Therefore, we need to introduce a correction to $y(r)$ on the SQM surface R_q [79,82]:

$$y(R_q^+) - y(R_q^-) = -\frac{4\pi R_q^3 \Delta \epsilon}{m_{tot}(R_q) + 4\pi R_q^3 p_{tot}(R_q)}, \quad (22)$$

where $\Delta \epsilon = \epsilon(R_q^+) - \epsilon(R_q^-)$ represents the energy density jump at the SQM surface.

4. Results and Discussion

In this section, we show our results for the EOSs of SQM and DM, the properties of pure SSs and DSs, and the properties of DSSs.

4.1. Properties of Pure Strange Stars and Dark Matter Stars

Employing the analytical expressions of Equations (1), (2) and (7), we can obtain the various EOSs of SQM and DM. Our numerical results for these EOSs are presented in Figure 1. For the CFL model of SQM, there are three key parameters, the bag constant B , the color superconductivity energy gap Δ and the strange quark mass m_s . These parameters have been thoroughly investigated in Ref. [70] to fit current astrophysical observations. Here, we adopt the set of parameters, $B = 70 \text{ MeVfm}^{-3}$, $\Delta = 160 \text{ MeV}$, $m_s = 150 \text{ MeV}$, which best fits the NS observation data presented in Figure 2. For comparison, we also present the results for the MIT bag model with bag constant $B = 60 \text{ MeVfm}^{-3}$ and $B = 90 \text{ MeVfm}^{-3}$ —these models are hereafter abbreviated as MIT60 and MIT90, respectively (along with the CFL model for SQM). One can see that the EOS from the CFL model are stiffer than those from the MIT bag model. Note that there is always a lower bound on the energy density for SQM, induced by quark confinement via the bag constant. Concerning the typical mass scale $\sim 1 \text{ GeV}$ in the interior of SSs, and to exemplify the influences of DM on the structure of the DSS, we adopt $m_D = 0.5 \text{ GeV}$, 1 GeV , and 2 GeV to represent three typical cases. Hereafter, the free fermionic EOSs and polytropic bosonic EOSs are abbreviated as FF and PB, respectively, with the subsequent number denoting the corresponding DPM. We find that for each DM model, the EOSs become stiffer as the DPM decreases. In comparison to the EOSs of free fermionic DM, the polytropic EOSs of bosonic DM with the same DPM are softer at low densities but stiffer at high densities; see for example FF0.5 and PB0.5.

By combining with the aforementioned EOSs, we numerically solve the TOV equations and derive the mass–radius relations of pure SSs and DSs. The results for SSs are shown in Figure 2. Meanwhile, we also include a set of recent observational constraints from pulsars in Figure 2, including GW170817 [12], HESS J1731-347 [18], PSR J0030+0451 [16,17], XTE J1814-338 [19], GW190814 [83], PSR J0952+0607 [84], and PSR J0740+6620 [14,15]. We note that the analysis of the NS radius from GW170817 [12] adopted EOS-insensitive relations and parameterized EOSs of hadronic NSs, which needs to be corrected if these NSs are reinterpreted as SSs. The other observational constraints employ gravitational or electromagnetic observational methods that are independent of the assumptions about the internal composition of the compact objects. In comparison to most astrophysical constraints—particularly those related to the maximum mass—the EOSs from the MIT bag model are too soft. By contrast, SQM with the CFL model can support SSs with $M_{\text{max}} > 2.5M_{\odot}$, which is consistent with the current observational constraints on the maximum mass of pulsars. With the parameters we adopt, the CFL model best fits nearly all the pulsar observational data in Figure 2, except that for XTE J1814-338. Note that the data for XTE J1814-338 cannot be explained by SSs simply by adjusting the parameters of the CFL or MIT bag model.

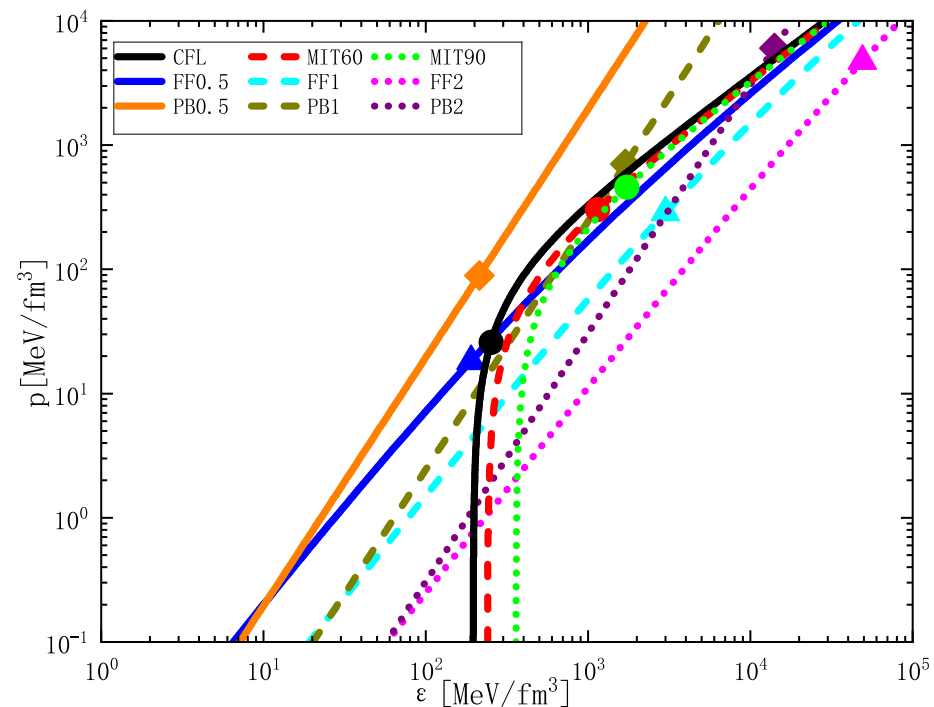


Figure 1. The pressure as a function of energy density for SQM and DM. The different line types/colors indicate different models/parameters: CFL quark model(CFL; solid black), MIT bag model with $B = 60 \text{ MeVfm}^{-3}$ (MIT60; dashed red), or $B = 90 \text{ MeVfm}^{-3}$ (MIT90; dotted green); free fermionic DM with $m_D = 0.5 \text{ GeV}$ (FF0.5; solid blue), $m_D = 1 \text{ GeV}$ (FF1; dashed cyan), and $m_D = 2 \text{ GeV}$ (FF2; dotted magenta); polytropic bosonic DM with $m_D = 0.5 \text{ GeV}$ (PB0.5; solid orange), $m_D = 1 \text{ GeV}$ (PB1; dashed olive), and $m_D = 2 \text{ GeV}$ (PB2; dotted purple). The p_c and ϵ_c of pure SSs and DSs with their maximum masses are also marked.

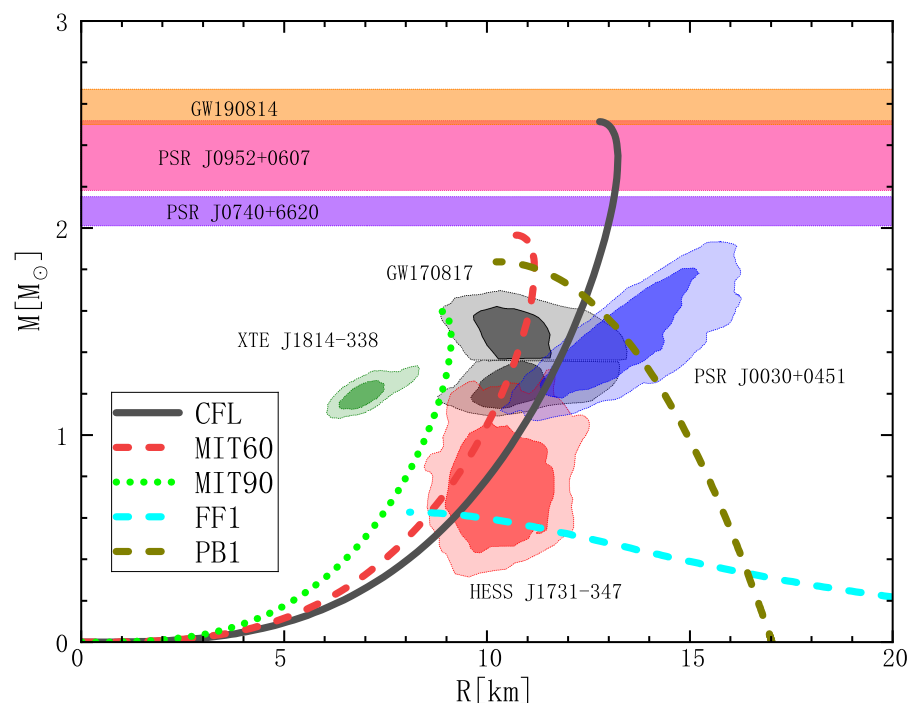


Figure 2. The mass as a function of radius for pure SSs and DSs with various EOSs. The correspondence between line types/colors and EOSs are the same as in Figure 1. The colored regions represent a selection of recent pulsar observational constraints, with the lighter and darker shades indicating approximately 1σ and 2σ confidence intervals, see the text for more details.

We also present results for pure DSs adopting the FF1 and PB1 models in Figure 2. Results of DSs with other parameters are not shown in Figure 2 due to their too large/small mass/radius, which can be found in following sections. With such models and parameters, the DSs have radii with a similar magnitude, whose radii decrease with increasing stellar mass and are slightly smaller than those of SSs. The maximum mass and typical radius of pure DSs depend strongly on the DPM values. Both quantities decrease rapidly as the DPM increases. For example, the maximum mass and corresponding radius of pure DSs with fermionic EOS decrease by more than one order of magnitude with DPM increasing from 0.5 GeV to 2 GeV. The maximum masses, corresponding radii, and central densities of pure SSs and DSs for various EOSs and parameter sets are summarized in Table 1. It is worth noting that the maximum mass of DSs with large DPMes cannot reach $1.4 M_{\odot}$ —e.g., for the FF1, FF2, and PB2 models—and consequently, the corresponding $\Lambda_{1.4M_{\odot}}$ are not reported in Table 1. In comparison of the two DM models, the maximum mass of pure DSs with the polytropic bosonic EOS is larger than that of pure DSs with the free fermionic EOS for the same DPM, which can be inferred from the stiffness of the EOSs. We note the unphysical behavior of zero-mass DSs with a non-zero radius: a finite radius for the bosonic polytropic EOS and an infinite radius for the free fermionic EOS. This unphysical behavior arises from the idealized yet unphysical behavior of the EOSs at low densities, for which a quantitative analysis is provided in Ref. [85].

Table 1. Some typical properties of pure SSs and DSs with various EOSs and parameters. Some $\Lambda_{1.4}$ are absent because the maximum stellar masses are lower than $1.4M_{\odot}$ with corresponding models and parameters.

Model	$M_{max}(M_{\odot})$	$R_{M_{max}}(\text{km})$	$\epsilon_{c,M_{max}}(\text{MeVfm}^{-3})$	$\Lambda_{1.4}$
CFL	2.51	12.77	808	862
MIT60	1.97	10.72	1146	417
MIT90	1.60	8.75	1731	118
FF0.5	2.50	32.28	190	1,948,900
FF1	0.63	8.11	2994	-
FF2	0.16	2.01	49,160	-
PB0.5	5.20	28.98	214	658,065
PB1	1.84	10.27	1698	660
PB2	0.65	3.60	14,060	-

In Figure 3, we present the tidal deformability of pure SSs and DSs as a function of the stellar mass (left panel) and the compactness parameter (right panel) respectively. The observational constraint from GW170817 is also included in Figure 3. In all cases, tidal deformability decreases as the stellar mass increases. The minimum tidal deformability values are similar for SSs across different models, and the dependence of tidal deformability on the compactness parameter is nearly universal. For SSs with the CFL model, the corresponding tidal deformability at $M = 1.4M_{\odot}$ is somewhat higher than the observational constraint and that from the MIT bag model. For pure DSs, the minimum tidal deformability values are strongly dependent on the adopted DM models but relatively insensitive to the DPM values. For pure DSs with free fermionic EOSs, the tidal deformability values are all above 1000, while the minimum values for polytropic bosonic EOSs can be as low as 100, similarly to those of SSs. For DSs, the dependence of tidal deformability on the compactness parameter deviates from the universal relation for SSs and depends on the specific DM model, but not on the DPM values. One can see that the maximum of the compactness of DSs with free fermionic EOSs is small, which can partly explain why the corresponding minimum tidal deformability is so high.

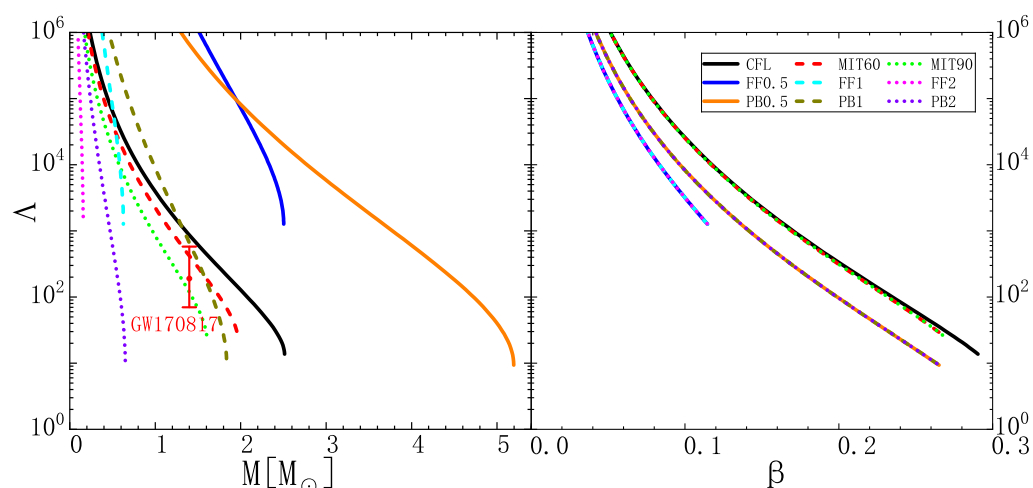


Figure 3. The tidal deformability Λ as a function of mass M (left panel) and compactness β (right panel) for pure SSs and DSs with various EOSs. The correspondence between line types/colors and EOSs are the same as in Figure 1. The observational constraint from GW170817 is indicated by a red error bar.

4.2. Properties of Dark Matter-Admixed Strange Stars

By combining the aforementioned EOSs of SQM and DM, we solve the equations for two-fluid stars and derive the structure and tidal deformability of DSSs.

As noted earlier, there are two independent central pressures for SQM and DM, respectively, that determine the configuration of a DSS. Therefore, the mass–radius relations can not be represented by a single curve, as is the case for single-fluid stars. In Figure 4, we present the masses of DSSs as a function of the total radius (left panels) and the SQM radius (right panels) for a fixed DM mass fraction $\chi = \frac{M_d}{M}$. We choose $\chi = 0, 0.1, 0.3, 0.5, 0.7, 0.9$ and 1 as the representative values, and from the top panels to the bottom panels, we adopt DPM values $m_D = 0.5, 1$ and 2 GeV, respectively. We find that for a small DM mass fraction (e.g., $\chi = 0.1$), the maximum mass of DSSs is slightly lower than that of pure SSs. Meanwhile, the radius of DSSs—defined by the SQM radius—is also slightly smaller than that of SSs with the same mass. As the stellar mass decreases, the radius of DSSs evolves in the same manner as that of SSs until the SQM radius equals the DM radius. Then, the radius of DSSs—defined by the DM radius—increases as the stellar mass decreases further, and may even intersect with curves for smaller χ . In contrast, the SQM radius always follows the same trend as that of SSs and decreases as χ increases, as illustrated in the right panels. As the DM fraction increases, the maximum mass of DSS decreases and then increases until $\chi = 1$. With a large χ , the DM radius can be larger than the SQM radius even at the maximum mass of the DSSs, and the DM radius always increases as the stellar mass decreases. In general, for a small DSS mass or a large DM mass fraction, R_d becomes larger than R_q , which results in the formation of a DM halo. In the comparison of the results for different DPM values, we find qualitatively similar trends in the effect of DM on the maximum stellar masses and the M–R curves. Quantitatively, for larger DPM values, the DM radius dominates at lower stellar masses or higher DM mass fractions. For the XTE J1814–338 data, we note that the visible SQM radius always decreases with χ and can match the observed radius for XTE J1814–338. Therefore, when the DPM is sufficiently small such that the maximum mass of the corresponding DSSs does not drop too rapidly with increasing χ (e.g., FF0.5, PB0.5 and PB1), DSSs can be consistent with the XTE J1814–338 data within the current observational uncertainties.

In Figure 5, we present the masses of DSSs as a function of the total radius (left panels) and the SQM radius (right panels) for the polytropic bosonic DM EOS. Again, we find qualitative trends similar to those observed in Figure 4. Quantitatively, the M–R relations of

DSSs depend strongly on the structural properties of pure DSs. Since the maximum mass and radius of pure DSs with the polytropic bosonic EOS are larger than those with the free fermionic EOS for the same DPM, the M-R relations of DSSs with the polytropic bosonic EOS are similar to those with the free fermionic EOS but for a smaller DPM value. Note that the behavior of the zero-mass DSSs with a non-zero radii can be traced to a similar property exhibited by pure DSs.

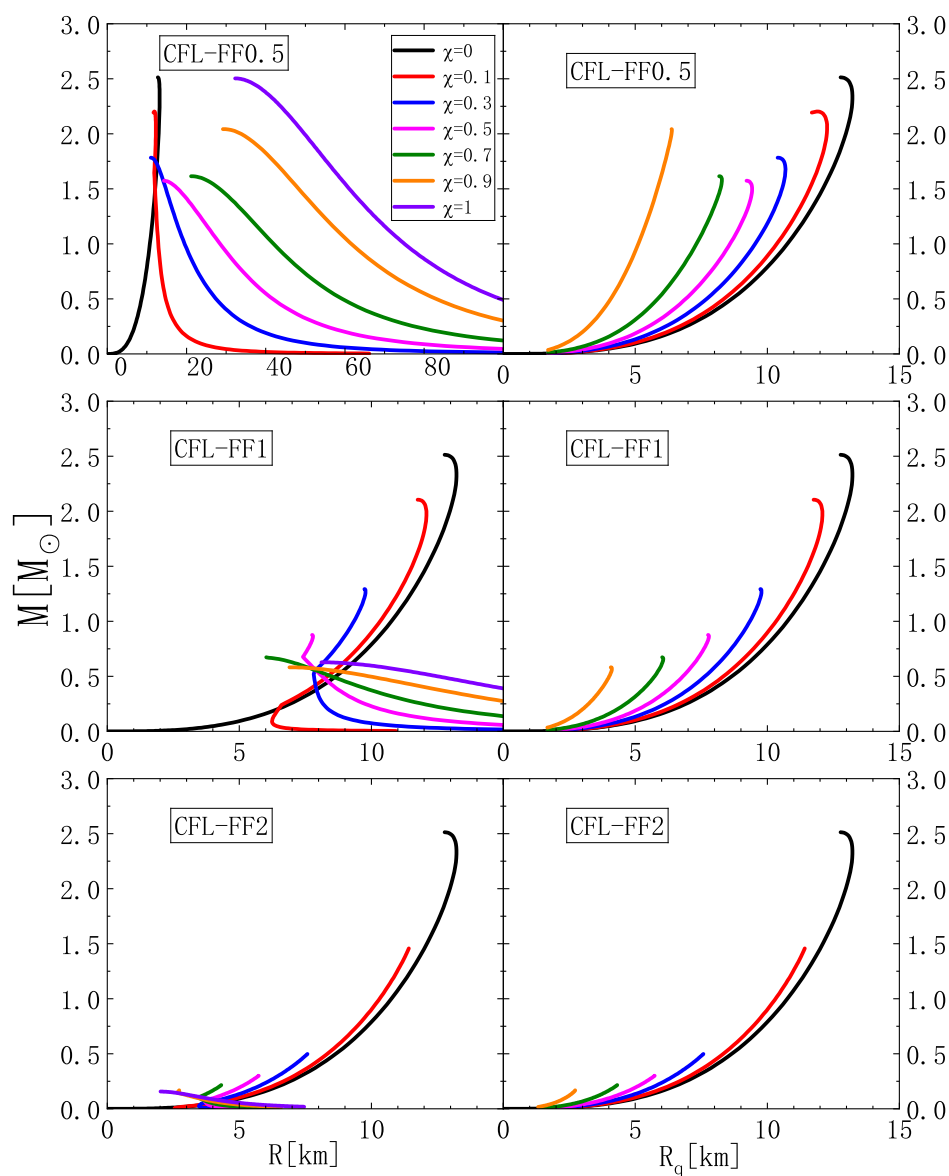


Figure 4. The mass M as a function of the radius R (left panels) or SQM radius R_q (right panels) for DSSs with free fermionic EOS of DM. From the top to the bottom panels, $m_D = 0.5$ GeV, 1 GeV, and 2 GeV, respectively. Different line colors represent different DM fractions.

To illustrate the variation more clearly, we present the variation of the maximum DSS mass as a function of χ in Figure 6. Qualitatively, a common feature across all cases is that the DSS maximum mass decreases from $\chi = 0$ to a minimum and then increases until $\chi = 1$. Quantitatively, the position of the minimum on each curve strongly depends on the maximum mass of pure DSs. When the maximum masses of pure DSs and pure SSs are similar (e.g., the case CFL-FF0.5), the minima appear close to $\chi = 0.5$. Similar results are reported in the literature [65,66], where the minimum occurs exactly at $\chi = 0.5$ due to the DM-SQM symmetry in the mirror DM model. However, when the maximum masses of DSs

and SSs differ significantly, the minimum position lies close to the lower end of the χ range. For example, for the cases of CFL-FF2 and CFL-PB0.5, the minimum positions are $\chi = 0.99$ and $\chi = 0.17$, respectively. We also note that when the DPM is small and pure DSs have a sufficiently large maximum mass, the maximum mass of DSSs increases rapidly with increasing χ . These masses can reach the so-called lower mass gap between NSs and stellar-mass black holes, a region that has attracted considerable attention following gravitational wave detections of events such as GW190814 [83]. For example, in the CFL-PB0.5 case, the maximum mass of DSSs reach $2.5\text{--}5 M_{\odot}$ for $\chi \gtrsim 0.5$. Our results suggest that DSSs with sufficiently low DPM values and high DM mass fractions can explain such mass gap objects, consistent with recent studies that also found that DM-admixed NSs can populate this mass range [86].

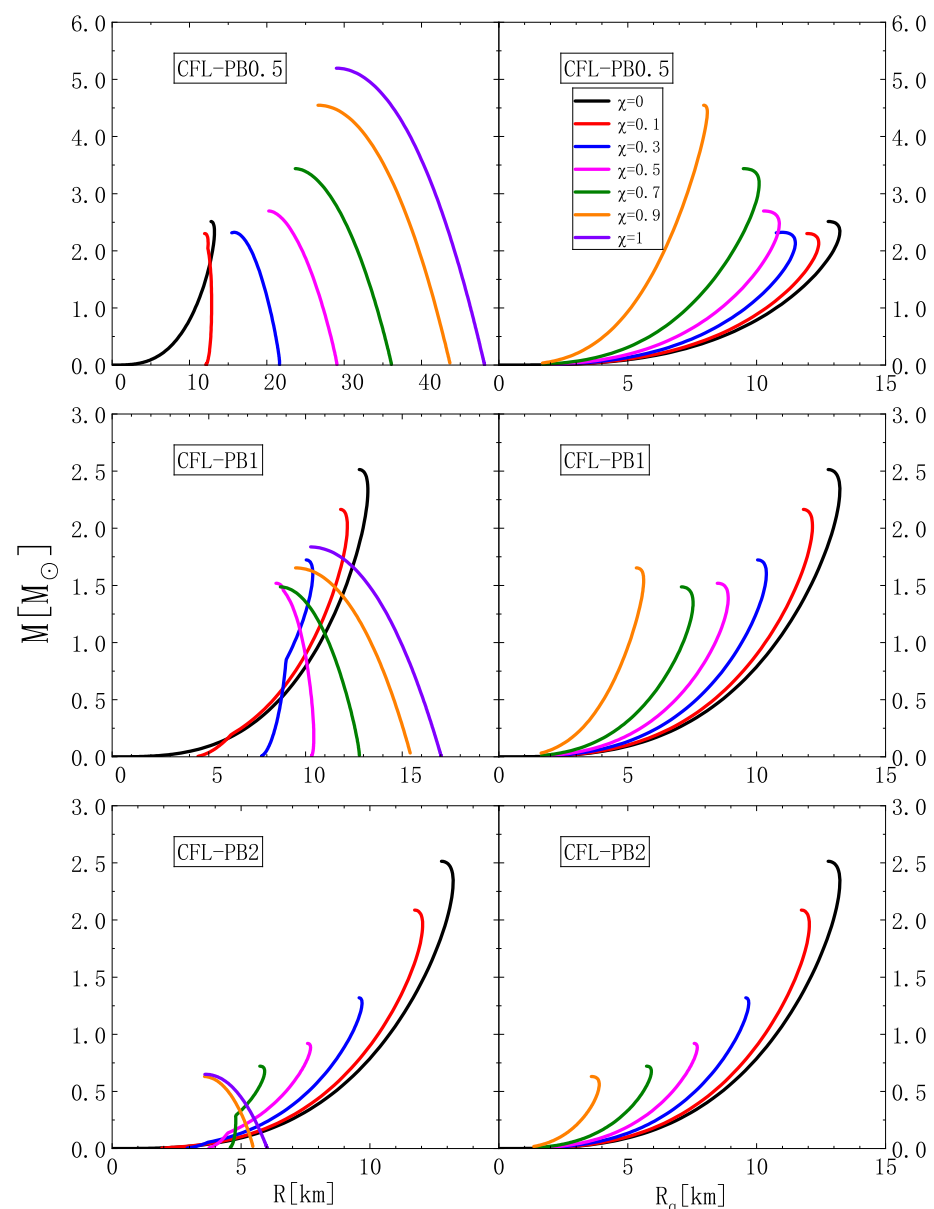


Figure 5. The mass M as a function of radius R (left panels) and SQM radius R_q (right panels) for DSSs with polytropic bosonic EOS of DM. From the top to the bottom panels, $m_D = 0.5$ GeV, 1 GeV, and 2 GeV, respectively. Different line colors represent different DM fractions.

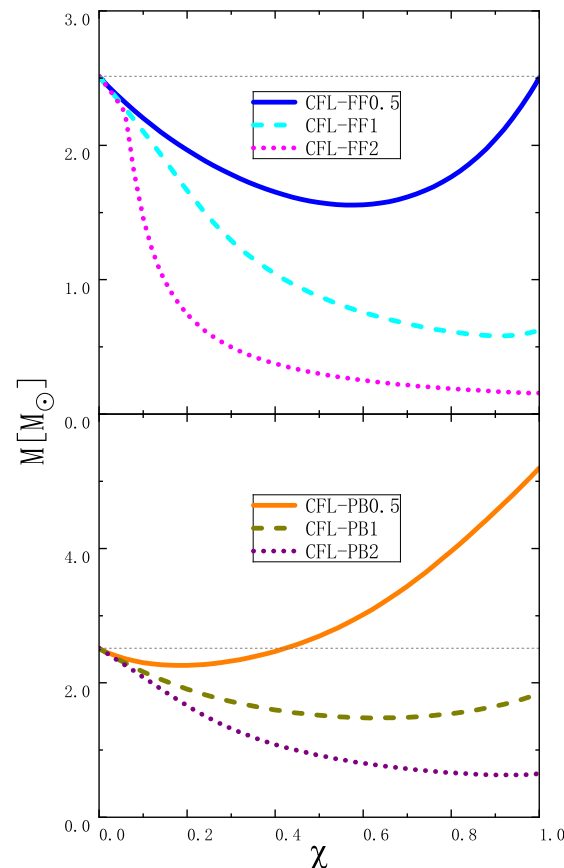


Figure 6. The maximum mass as a function of DM fraction for DSSs with free fermionic EOS of DM (**upper panel**) or polytropic bosonic EOS of DM (**lower panel**). The correspondence between line types/colors and EOS of DM are the same as in Figure 1. The thin dotted line indicates the maximum mass of a pure SS.

In Figure 7, we further present the dependence of DSS masses on the total central energy density $\epsilon_c = \epsilon_{c,q} + \epsilon_{c,d}$, along a fixed χ . We find that with a fixed χ , $\frac{dM}{d\epsilon_c} \geq 0$ until the maximum masses is reached, i.e., $dM/dR|_\chi = 0$ and $\frac{dM}{d\epsilon_c}|_\chi = 0$ coincide at the maximum masses.

For DSS stability, strictly speaking, stability is determined by the positivity of the square of the lowest radial oscillation frequency. For single-fluid stars, it is well established that the condition $dM/d\epsilon_c = 0$ also marks the transition from stable to unstable configurations against radial perturbations. Except in rare exceptional cases, $dR/d\epsilon_c = 0$ would not emerge simultaneously, so $dM/d\epsilon_c = 0$ is also consistent with $dM/dR = 0$. For two-fluid stars, the situation is more subtle. Several studies on the radial oscillations of two-fluid stars [50,64,87] have numerically demonstrated that the maximum mass configurations along sequences of fixed DM fraction χ coincide with the onset of instability. However, to the best of our knowledge, a rigorous mathematical proof analogous to that for the single-fluid case has not yet been developed for the two-fluid case. Therefore, in the present work we treat the maximum mass for a fixed χ as an empirical indicator of the stability boundary, and in Figures 5 and 6 we truncate the curves at the maximum mass for each fixed χ .

In Figure 8, we show the contour plots of the mass of DSS as a function of the central pressures of SQM and DM, for the fermionic EOS (left panels) and the polytropic bosonic EOS (right panels), respectively, with $m_D = 0.5$ GeV, 1 GeV, and 2 GeV from the top to the bottom panels. The color gradient indicates regions corresponding to different DSS masses. The solid curves show configurations with fixed DM mass fractions $\chi = 0.1, 0.2, \dots, 0.9$ from the bottom to the top, and they terminate at the corresponding maximum masses. The

dashed curves are determined by the maximum mass points with various DM fractions, i.e., $dM/dR|_{\chi} = 0 = \frac{dM}{d\epsilon_c}|_{\chi}$. We present these as empirical stability boundaries for DSS configurations. Obviously, when one central pressure is fixed and the other is varied, the point of maximum mass does not coincide with our empirically defined critical stable point. Furthermore, at the center of the DSSs, the pressures of SQM and DM can be much larger than the maximum central pressures of pure SSs or DSs, as seen, for example, in the top-left and bottom-left panels.

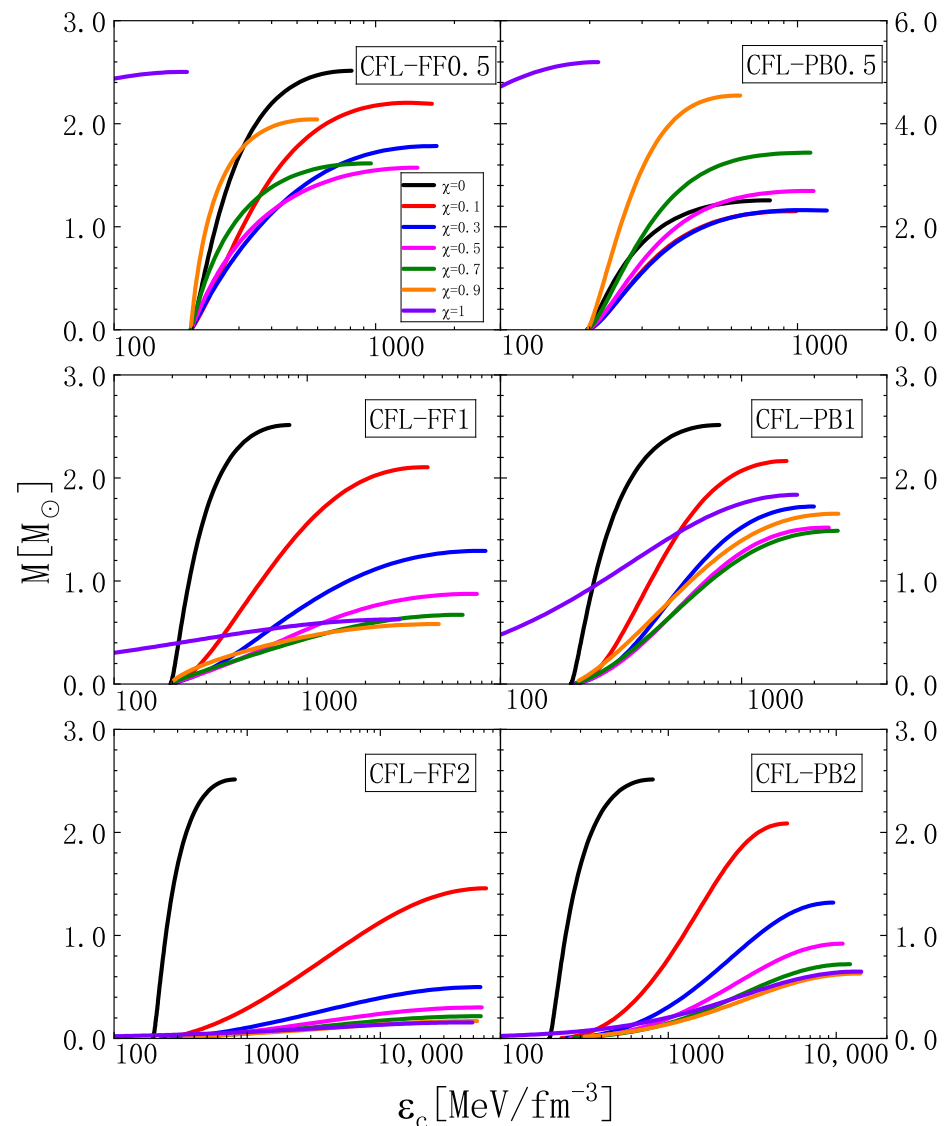


Figure 7. The mass as a function of the central energy density ϵ_c for DSSs. From top to bottom panels, $m_D = 0.5$ GeV, 1 GeV, and 2 GeV, respectively. Different line colors represent different DM fractions.

In Figure 9, we present the tidal deformability Λ of DSSs as a function of the stellar mass (left panels), the compactness parameter $\beta = \frac{M}{R}$ (middle panels), and $\beta_q = \frac{M}{R_q}$ (right panels). We adopt the free fermionic EOS of DM with $m_D = 0.5, 1, 2$ GeV from the top to the bottom panels. Similarly to one-fluid stars, Λ of DSSs with a fixed χ decreases monotonically with mass, reaching a minimum at the maximum mass. For DSSs with small DM fractions, the minimum value of Λ increases with χ , but remains lower than that of a DSS with the same mass but a smaller χ . As the stellar mass decreases, Λ of a DSS increases rapidly, and at low masses it turns to increase with χ . In contrast, for DSSs with large χ , Λ of DSS with the same mass increases with χ in all mass regions. This trend is especially obvious when the DPM is small, e.g., in the top-left panel for the FF2 EOS of DM.

These variations of Λ are qualitatively similar to the variations in the radius of DSS with χ . However, there are still quantitative differences. The intersection points of the Λ - M curves differ from the intersection points of the M - R curves. Thus different configurations with the same mass M and radius R have different Λ . Comparing the three panels with different DPM values, one can see that, when DPM is small, large values of Λ can occur in DSSs with large χ . As the DPM increases, DSSs with large χ can only achieve a small maximum mass and large values of Λ occur only in the low-mass region. For example, when $m_D = 1$ or 2 GeV only DSSs with low χ can reach $1.4 M_\odot$ and the corresponding Λ is smaller than that of a pure SS.

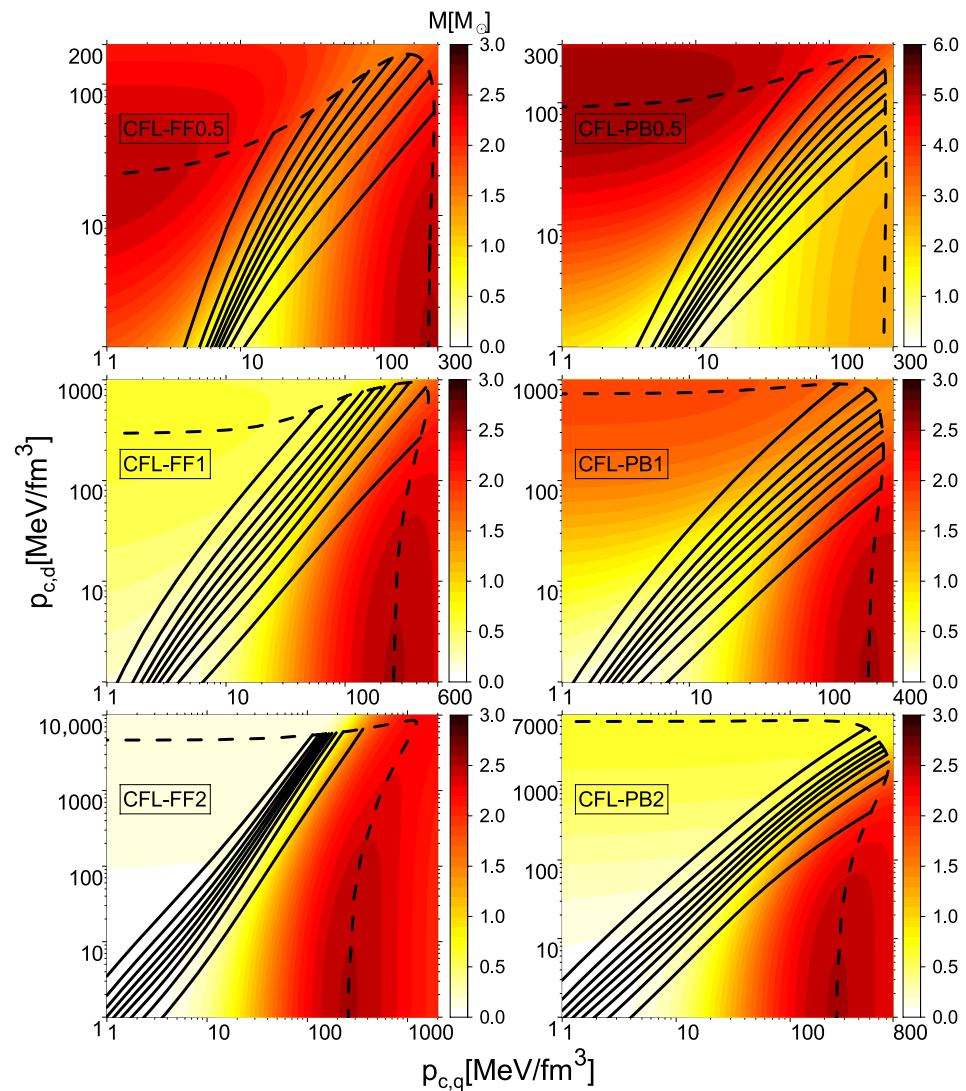


Figure 8. Contour plot of DSS mass as a function of SQM and DM central pressures. The solid curves in each panel, from bottom to top, correspond to constant χ consequences with $\chi = 0.1, 0.2, \dots, 0.9$. The dashed black curves represent empirical stability boundary lines, determined by the maximum masses along fixed χ sequences, i.e., $\frac{dM}{dR}|_\chi = 0 = \frac{dM}{d\epsilon_c}|_\chi$.

For the $\Lambda - \beta$ relations of DSSs shown in the middle panels and the $\Lambda - \beta_q$ relations shown in the right panels, it can be seen that the curves of DSSs deviate from the universal relations of both SSs and DSs. Quantitatively, all the $\Lambda - \beta$ curves of DSSs are lower than the corresponding pure SS curve, and some are even lower than the pure DS curve, especially when DPM is small, e.g., in the top-middle panel for CFL-FF0.5 model. The $\Lambda - \beta_q$ curves of DSSs also deviate from those of pure SSs, particularly for cases with low

DPM values and large χ , as illustrated in the top-right panel of Figure 9. Such a deviation from the universal relations of SSs can also be an indicator of the DM content in SSs.

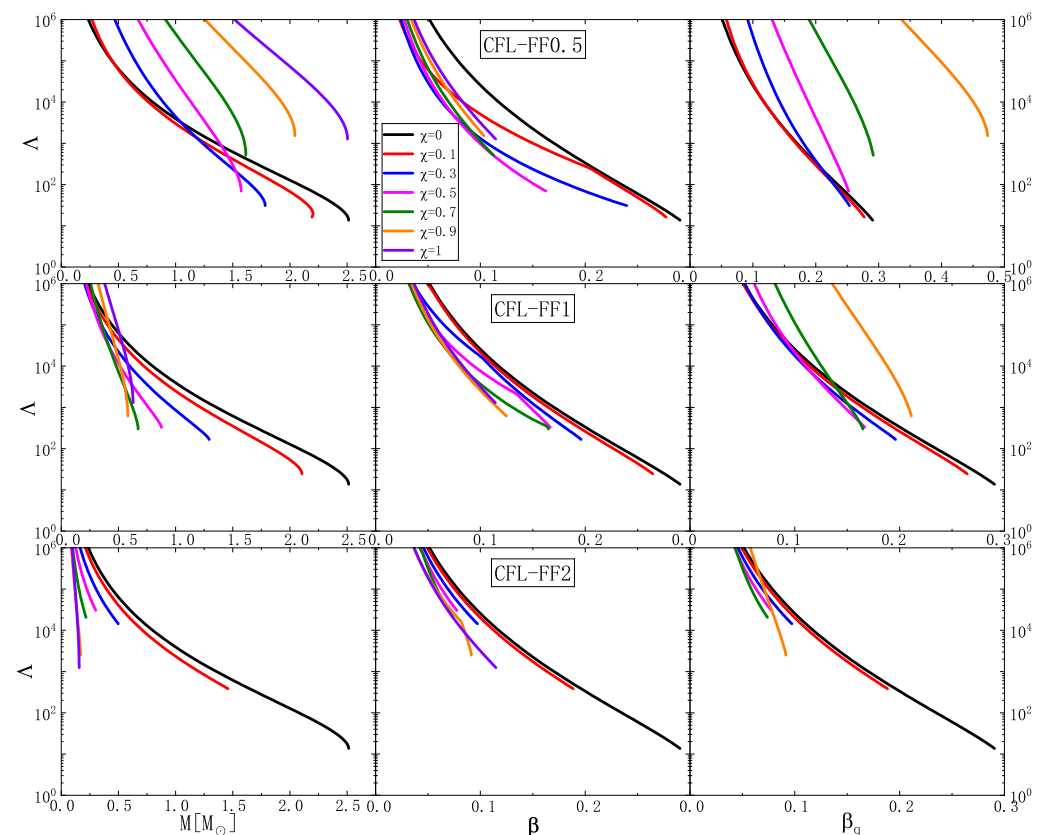


Figure 9. The tidal deformability Λ as a function of mass M (left panel) and the compactness parameter $\beta = \frac{M}{R}$ (middle panels) and $\beta_q = \frac{M}{R_q}$ (right panels) for DSSs with free fermionic EOS of DM. From the top to the bottom panels, $m_D = 0.5$ GeV, 1 GeV, and 2 GeV, respectively.

In Figure 10, we present the tidal deformability of DSSs with the polytropic bosonic EOS for DM, as a function of the stellar mass (left panels) and the compactness parameter (right panels) respectively, with $m_D = 0.5, 1, 2$ GeV from the top to the bottom panels. Qualitatively, the results of $\Lambda - M$, $\Lambda - \beta$ and $\Lambda - \beta_q$ relations are all similar to those with the free fermionic EOS but smaller DPM values. Quantitatively, the variation in the minimum Λ of DSSs with χ is smaller, since the minimum Λ of a pure DS is similar to the SS, while the minimum Λ of a pure DS with free fermionic EOS is much larger. From an observational perspective, we focus on different configurations with the same mass M and visible radius R_q . For example, in the top-right panel of Figure 5, there are two configurations with $M \approx 2.33 M_\odot$ and $R_q \approx 10.83$ km, corresponding to $\chi = 0.3$ or $\chi = 0.5$, respectively. However, their tidal deformabilities differ substantially: $\Lambda \approx 13$ for $\chi = 0.3$ and $\Lambda \approx 200$ for $\chi = 0.5$, representing a difference of more than an order of magnitude. This demonstrates that tidal deformability measurements from gravitational wave observations can serve as a powerful tool to probe the DM content in compact stars, even when traditional electromagnetic observations fail to distinguish them.

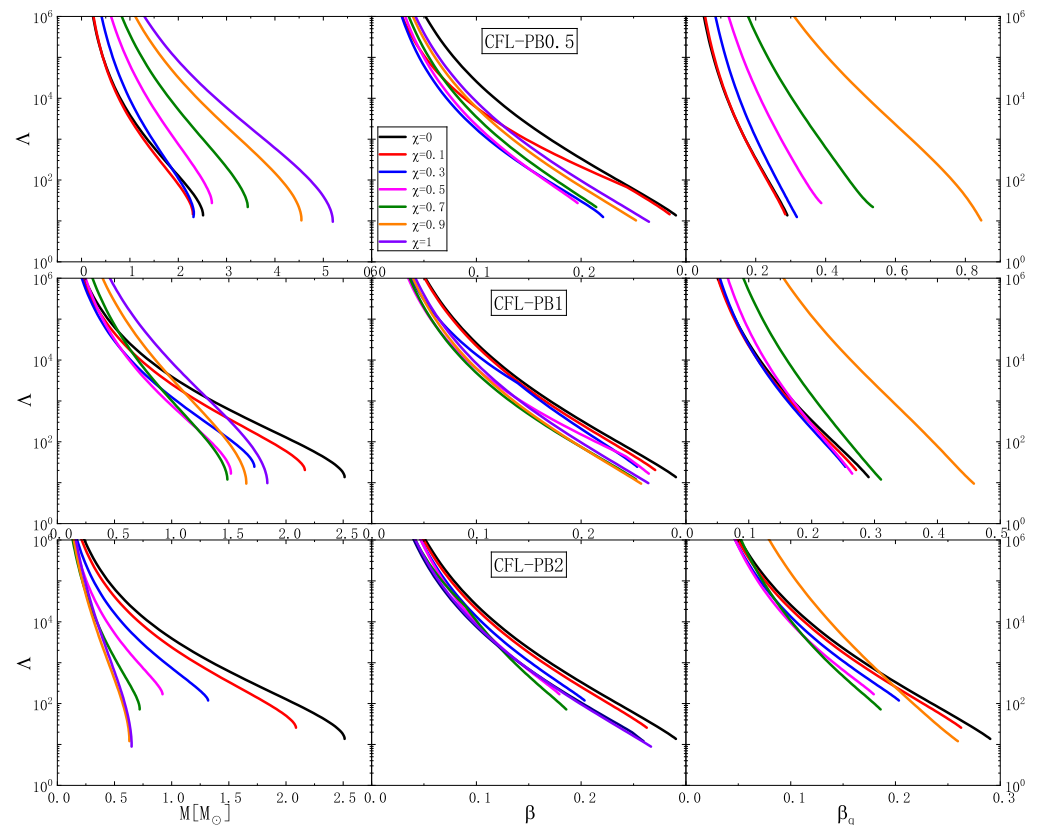


Figure 10. The tidal deformability Λ as a function of mass M (left panel) and the compactness parameter $\beta = \frac{M}{R}$ (middle panels), $\beta_q = \frac{M}{R_q}$ (right panels) for DSSs with polytropic bosonic EOS of DM. From the top to the bottom panels, $m_D = 0.5$ GeV, 1 GeV, and 2 GeV, respectively.

5. Summary

In this work, we systematically investigate DSSs, exploring how DM influences the structural properties and tidal deformability of these compact astrophysical objects. We employ a two-fluid formalism to describe the coupled system of SQM and DM, solving the two-fluid TOV equations for static, spherically symmetric configurations. For the EOSs, we adopt the CFL model of SQM and two types of models for DM, the free fermion gas model and the polytropic boson model. We quantitatively compare the different influences of these two models, as well as the impacts of the DPM, which is the key parameter in DM models. To more clearly illustrate the qualitative effects of DM, we consider three representative DPM values, $m_D = 0.5$ GeV, 1 GeV, and 2 GeV, which are similar to the typical mass scale of SSs. The qualitative trends for higher or lower DPM values can be inferred from our results.

In comparison with recent studies on DSSs, which focus on specific DM models/parameters [65,66] or DSSs with small DM fractions [58–62], our work systematically investigates the whole DM fraction range with various representative DM models/parameters and reveals several key findings. (1) The mass–radius relations of DSSs exhibit complex behavior depending on the DM fraction χ , with the maximum mass showing a characteristic minimum at an intermediate value of χ . With low values of DPM, the maximum masses of DSSs quickly turn to increase with χ and can reach the so-called low mass gap region between NSs and low-mass black holes. (2) The presence of DM generally reduces the (visible) SQM radius compared to a pure SS of the same mass, which can give a possible explanation for the small radius of XTE J1814–338, as pointed out in Ref. [66]. On the contrary, the DM radius can be larger for large DM fractions or for small DM fractions combined with small stellar masses, forming a DM halo. (3) The tidal deformability Λ of

DSSs with significant DM fractions displays distinctive patterns that deviate from those of pure SSs and pure DSs, with the $\Lambda - M$, $\Lambda - \beta$, and $\Lambda - \beta_q$ relations providing a potential observational discriminant for DM content beyond the mass–radius degeneracy, which was also discussed in earlier literature on DM-admixed NSs [54,55]. (4) The specific behavior depends sensitively on the DM EOS and particle mass, with fermionic and bosonic DM models yielding quantitatively different predictions.

These results have important implications for multi-messenger astronomy, suggesting that precise measurements of neutron star masses, radii, and tidal deformabilities, from both gravitational wave and electromagnetic observations, could potentially constrain the properties of DM. The non-trivial dependence of stellar properties on DM content provides a new avenue for probing DM through astrophysical observations of compact objects. Nevertheless, much work remains to be undertaken in order to investigate further properties of DM, especially the self-interaction of DM. We leave investigations into the interacting fermion model and studies of the impacts of the scattering length in the bosonic DM model to future work. Future observations with next-generation gravitational wave detectors and X-ray timing missions will further test the theoretical predictions and potentially reveal signatures of DM in compact stars.

Author Contributions: J.H. was the main calculator and drafter of the manuscript. X.-Y.Z. and G.W. participated in the discussion and part of the data verification. J.-B.W. guided the main program calculation, and participated in the discussion and revision of the manuscript. H.C. was the organizer of the project, responsible for organizing the research, the discussion and the revision of the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: We acknowledge financial support from the National Natural Science Foundation of China (Grant No. 12205260).

Data Availability Statement: The data presented in this study are available on request from the corresponding author due to college regulations.

Acknowledgments: We thank Hans-Josef Schulze for helpful discussions.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

References

- Shapiro, S.L.; Teukolsky, S.A. Star Deaths and the Formation of Compact Objects. In *Black Holes, White Dwarfs, and Neutron Stars*; John Wiley and Sons, Ltd.: West Sussex, UK, 1983; Chapter 1; pp. 1–16.
- Chadwick, J. Possible existence of a neutron. *Nature* **1932**, *129*, 312. [[CrossRef](#)]
- Landau, L.D. On the theory of stars. *Phys. Z. Sowjetunion* **1932**, *1*, 285–288.
- Baade, W.; Zwicky, F. Remarks on super-novae and cosmic rays. *Phys. Rev.* **1934**, *46*, 76. [[CrossRef](#)]
- Tolman, R.C. Effect of inhomogeneity on cosmological models. *Proc. Natl. Acad. Sci. USA* **1934**, *20*, 169–176. [[CrossRef](#)] [[PubMed](#)]
- Oppenheimer, J.R.; Volkoff, G.M. On massive neutron cores. *Phys. Rev.* **1939**, *55*, 374. [[CrossRef](#)]
- Hewish, A.; Bell, S.J.; Pilkington, J.D.H.; Scott, P.F.; Collins, R.A. Observation of a Rapidly Pulsating Radio Source. *Nature* **1968**, *217*, 709–713. [[CrossRef](#)]
- Abbott, B.P.; Abbott, R.; Abbott, T.D.; Acernese, F.; Ackley, K.; Adams, C.; Adams, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. GW170817: Observation of gravitational waves from a binary neutron star inspiral. *Phys. Rev. Lett.* **2017**, *119*, 161101. [[CrossRef](#)] [[PubMed](#)]
- Acernese, F.; Agathos, M.; Agatsuma, K.; Aisa, D.; Allemandou, N.; Allocca, A.; Amarni, J.; Astone, P.; Balestri, G.; Ballardin, G.; et al. Advanced Virgo: A second-generation interferometric gravitational wave detector. *Class. Quantum Gravity* **2014**, *32*, 024001. [[CrossRef](#)]
- Abbott, B.P.; Abbott, R.; Abbott, T.; Abraham, S.; Acernese, F.; Ackley, K.; Adams, C.; Adya, V.; Affeldt, C.; Agathos, M.; et al. Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA. *Living Rev. Relativ.* **2020**, *23*, 3. [[CrossRef](#)]

11. Bailes, M.; Berger, B.K.; Brady, P.R.; Branchesi, M.; Danzmann, K.; Evans, M.; Holley-Bockelmann, K.; Iyer, B.; Kajita, T.; Katsanevas, S.; et al. Gravitational-wave physics and astronomy in the 2020s and 2030s. *Nat. Rev. Phys.* **2021**, *3*, 344–366. [\[CrossRef\]](#)
12. Abbott, B.P.; Abbott, R.; Abbott, T.; Acernese, F.; Ackley, K.; Adams, C.; Adams, T.; Addesso, P.; Adhikari, R.X.; Adya, V.B.; et al. GW170817: Measurements of neutron star radii and equation of state. *Phys. Rev. Lett.* **2018**, *121*, 161101. [\[CrossRef\]](#)
13. Abbott, B.P.; Abbott, R.; Abbott, T.; Abraham, S.; Acernese, F.; Ackley, K.; Adams, C.; Adhikari, R.; Adya, V.; Affeldt, C.; et al. GW190425: Observation of a compact binary coalescence with total mass $\sim 3.4 M_{\odot}$. *Astrophys. J.* **2020**, *892*, L3. [\[CrossRef\]](#)
14. Miller, M.; Lamb, F.K.; Dittmann, A.; Bogdanov, S.; Arzoumanian, Z.; Gendreau, K.C.; Guillot, S.; Harding, A.; Ho, W.; Lattimer, J.; et al. PSR J0030+0451 mass and radius from NICER data and implications for the properties of neutron star matter. *Astrophys. J. Lett.* **2019**, *887*, L24. [\[CrossRef\]](#)
15. Riley, T.E.; Watts, A.L.; Bogdanov, S.; Ray, P.S.; Ludlam, R.M.; Guillot, S.; Arzoumanian, Z.; Baker, C.L.; Bilous, A.V.; Chakraborty, D.; et al. A NICER view of PSR J0030+0451: Millisecond pulsar parameter estimation. *Astrophys. J. Lett.* **2019**, *887*, L21. [\[CrossRef\]](#)
16. Miller, M.C.; Lamb, F.; Dittmann, A.; Bogdanov, S.; Arzoumanian, Z.; Gendreau, K.; Guillot, S.; Ho, W.; Lattimer, J.; Loewenstein, M.; et al. The radius of PSR J0740+6620 from NICER and XMM-Newton data. *Astrophys. J. Lett.* **2021**, *918*, L28. [\[CrossRef\]](#)
17. Riley, T.E.; Watts, A.L.; Ray, P.S.; Bogdanov, S.; Guillot, S.; Morsink, S.M.; Bilous, A.V.; Arzoumanian, Z.; Choudhury, D.; Deneva, J.S.; et al. A NICER view of the massive pulsar PSR J0740+6620 informed by radio timing and XMM-Newton spectroscopy. *Astrophys. J. Lett.* **2021**, *918*, L27. [\[CrossRef\]](#)
18. Doroshenko, V.; Suleimanov, V.; Pühlhofer, G.; Santangelo, A. A strangely light neutron star within a supernova remnant. *Nat. Astron.* **2022**, *6*, 1444–1451. [\[CrossRef\]](#)
19. Kini, Y.; Salmi, T.; Vinciguerra, S.; Watts, A.L.; Bilous, A.; Galloway, D.K.; van der Wateren, E.; Khalsa, G.P.; Bogdanov, S.; Buchner, J.; et al. Constraining the properties of the thermonuclear burst oscillation source XTE J1814-338 through pulse profile modelling. *Mon. Not. R. Astron. Soc.* **2024**, *535*, 1507–1525. [\[CrossRef\]](#)
20. Weber, F. Strange quark matter and compact stars. *Prog. Part. Nucl. Phys.* **2005**, *54*, 193–288. [\[CrossRef\]](#)
21. Witten, E. Cosmic separation of phases. *Phys. Rev. D* **1984**, *30*, 272. [\[CrossRef\]](#)
22. Itoh, N. Hydrostatic equilibrium of hypothetical quark stars. *Prog. Theor. Phys.* **1970**, *44*, 291–292. [\[CrossRef\]](#)
23. Farhi, E.; Jaffe, R.L. Strange matter. *Phys. Rev. D* **1984**, *30*, 2379. [\[CrossRef\]](#)
24. Ivanenko, D.; Kurdgelaidze, D. Hypothesis concerning quark stars. *Astrophysics* **1965**, *1*, 251–252. [\[CrossRef\]](#)
25. Drake, J.J.; Marshall, H.L.; Dreizler, S.; Freeman, P.E.; Fruscione, A.; Juda, M.; Kashyap, V.; Nicastro, F.; Pease, D.O.; Wargelin, B.J.; et al. Is RX J1856. 5–3754 a quark star? *Astrophys. J.* **2002**, *572*, 996. [\[CrossRef\]](#)
26. Bombaci, I. Observational evidence for strange matter in compact objects from the x-ray burster 4U 1820-30. *Phys. Rev. C* **1997**, *55*, 1587. [\[CrossRef\]](#)
27. Lattimer, J.M. The nuclear equation of state and neutron star masses. *Annu. Rev. Nucl. Part. Sci.* **2012**, *62*, 485–515. [\[CrossRef\]](#)
28. Dai, Z.G.; Wang, S.Q.; Wang, J.; Wang, L.J.; Yu, Y.W. The most luminous supernova ASASSN-15LH: Signature of a newborn rapidly rotating strange quark star. *Astrophys. J.* **2016**, *817*, 132. [\[CrossRef\]](#)
29. Chodos, A.; Jaffe, R.L.; Johnson, K.; Thorn, C.B.; Weisskopf, V. New extended model of hadrons. *Phys. Rev. D* **1974**, *9*, 3471. [\[CrossRef\]](#)
30. Chodos, A.; Jaffe, R.; Johnson, K.; Thorn, C.B. Baryon structure in the bag theory. *Phys. Rev. D* **1974**, *10*, 2599. [\[CrossRef\]](#)
31. Alford, M.; Rajagopal, K.; Wilczek, F. QCD at finite baryon density: Nucleon droplets and color superconductivity. *Phys. Lett. B* **1998**, *422*, 247–256. [\[CrossRef\]](#)
32. Nambu, Y.; Jona-Lasinio, G. Dynamical model of elementary particles based on an analogy with superconductivity. II. *Phys. Rev.* **1961**, *124*, 246. [\[CrossRef\]](#)
33. Szabó, K.K.; Tóth, A.I. Quasiparticle description of the QCD plasma, comparison with lattice results at finite T and μ . *J. High Energy Phys.* **2003**, *2003*, 008. [\[CrossRef\]](#)
34. Peng, G.; Chiang, H.; Yang, J.; Li, L.; Liu, B. Mass formulas and thermodynamic treatment in the mass-density-dependent model of strange quark matter. *Phys. Rev. C* **1999**, *61*, 015201. [\[CrossRef\]](#)
35. Dyson, F.J. The S matrix in quantum electrodynamics. *Phys. Rev.* **1949**, *75*, 1736. [\[CrossRef\]](#)
36. Schwinger, J. On the Green's functions of quantized fields. II. *Proc. Natl. Acad. Sci. USA* **1951**, *37*, 455–459. [\[CrossRef\]](#)
37. Chen, H.; Yuan, W.; Liu, Y.X. Partial restoration of chiral symmetry versus quark number density at zero temperature. *J. Phys. G Nucl. Part. Phys.* **2009**, *36*, 064073. [\[CrossRef\]](#)
38. Chen, H.; Baldo, M.; Burgio, G.; Schulze, H.J. Hybrid stars with the Dyson-Schwinger quark model. *Phys. Rev. D* **2011**, *84*, 105023. [\[CrossRef\]](#)
39. Klähn, T.; Roberts, C.D.; Chang, L.; Chen, H.; Liu, Y.X. Cold quarks in medium: An equation of state. *Phys. Rev. C—Nuclear Phys.* **2010**, *82*, 035801. [\[CrossRef\]](#)
40. Zwicky, F. Die rotverschiebung von extragalaktischen nebeln. *Helv. Phys. Acta* **1933**, *6*, 110–127.

41. Rubin, V.C.; Ford, W.K., Jr. Rotation of the Andromeda nebula from a spectroscopic survey of emission regions. *Astrophys. J.* **1970**, *159*, 379. [[CrossRef](#)]
42. Clowe, D.; Bradač, M.; Gonzalez, A.H.; Markevitch, M.; Randall, S.W.; Jones, C.; Zaritsky, D. A direct empirical proof of the existence of dark matter. *Astrophys. J.* **2006**, *648*, L109. [[CrossRef](#)]
43. Ade, P.A.; Aghanim, N.; Arnaud, M.; Ashdown, M.; Aumont, J.; Baccigalupi, C.; Banday, A.; Barreiro, R.; Bartlett, J.; Bartolo, N.; et al. Planck 2015 results-xiii. cosmological parameters. *Astron. Astrophys.* **2016**, *594*, A13.
44. Baryakhtar, M.; Caputo, R.; Croon, D.; Perez, K.; Berti, E.; Bramante, J.; Buschmann, M.; Brito, R.; Chen, T.Y.; Cole, P.S.; et al. Dark Matter In Extreme Astrophysical Environments. *arXiv* **2022**, arXiv:2203.07984. [[CrossRef](#)]
45. Billard, J.; Boulay, M.; Cebrián, S.; Covi, L.; Fiorillo, G.; Green, A.; Kopp, J.; Majorovits, B.; Palladino, K.; Petricca, F.; et al. Direct detection of dark matter—APPEC committee report. *Rep. Prog. Phys.* **2022**, *85*, 056201. [[CrossRef](#)] [[PubMed](#)]
46. Goldman, I.; Nussinov, S. Weakly interacting massive particles and neutron stars. *Phys. Rev. D* **1989**, *40*, 3221. [[CrossRef](#)]
47. Kouvaris, C. WIMP annihilation and cooling of neutron stars. *Phys. Rev. D* **2008**, *77*, 023006. [[CrossRef](#)]
48. Kaplan, D.E.; Luty, M.A.; Zurek, K.M. Asymmetric dark matter. *Phys. Rev. D* **2009**, *79*, 115016. [[CrossRef](#)]
49. Kouvaris, C.; Tinyakov, P. Constraining Asymmetric Dark Matter through observations of compact stars. *Phys. Rev. D* **2011**, *83*, 083512. [[CrossRef](#)]
50. Kain, B. Radial oscillations and stability of multiple-fluid compact stars. *Phys. Rev. D* **2020**, *102*, 023001. [[CrossRef](#)]
51. Das, H.; Kumar, A.; Kumar, B.; Patra, S.K. Dark matter effects on the compact star properties. *Galaxies* **2022**, *10*, 14. [[CrossRef](#)]
52. Hippert, M.; Dillingham, E.; Tan, H.; Curtin, D.; Noronha-Hostler, J.; Yunes, N. Dark matter or regular matter in neutron stars? How to tell the difference from the coalescence of compact objects. *Phys. Rev. D* **2023**, *107*, 115028. [[CrossRef](#)]
53. Routaray, P.; Das, H.; Sen, S.; Kumar, B.; Panotopoulos, G.; Zhao, T. Radial oscillations of dark matter admixed neutron stars. *Phys. Rev. D* **2023**, *107*, 103039. [[CrossRef](#)]
54. Liu, H.M.; Wei, J.B.; Li, Z.H.; Burgio, G.; Schulze, H.J. Dark matter effects on the properties of neutron stars: Optical radii. *Phys. Dark Universe* **2023**, *42*, 101338. [[CrossRef](#)]
55. Liu, H.M.; Wei, J.B.; Li, Z.H.; Burgio, G.; Das, H.; Schulze, H.J. Dark matter effects on the properties of neutron stars: Compactness and tidal deformability. *Phys. Rev. D* **2024**, *110*, 023024. [[CrossRef](#)]
56. Bramante, J.; Raj, N. Dark matter in compact stars. *Phys. Rep.* **2024**, *1052*, 1–48. [[CrossRef](#)]
57. Sotani, H.; Kumar, A. Emergence of new oscillation modes in dark matter admixed neutron stars. *Phys. Rev. D* **2025**, *111*, 123013. [[CrossRef](#)]
58. Mukhopadhyay, P.; Schaffner-Bielich, J. Quark stars admixed with dark matter. *Phys. Rev. D* **2016**, *93*, 083009. [[CrossRef](#)]
59. Panotopoulos, G.; Lopes, I. Radial oscillations of strange quark stars admixed with condensed dark matter. *Phys. Rev. D* **2017**, *96*, 083013. [[CrossRef](#)]
60. Panotopoulos, G.; Lopes, I. Radial oscillations of strange quark stars admixed with fermionic dark matter. *Phys. Rev. D* **2018**, *98*, 083001. [[CrossRef](#)]
61. Lopes, L.L.; Das, H. Strange stars within bosonic and fermionic admixed dark matter. *J. Cosmol. Astropart. Phys.* **2023**, *2023*, 034. [[CrossRef](#)]
62. Yang, Y.; Wu, C.; Yang, J.F. Impact of dark matter on strange quark stars described by different quark models. *Eur. Phys. J. C* **2025**, *85*, 426. [[CrossRef](#)]
63. Jiménez, J.C.; Fraga, E.S. Radial oscillations of quark stars admixed with dark matter. *Universe* **2022**, *8*, 34. [[CrossRef](#)]
64. Zhen, Y.; Sun, T.T.; Wei, J.B.; Zheng, Z.Y.; Chen, H. Radial Oscillations of Strange Quark Stars Admixed with Dark Matter. *Symmetry* **2024**, *16*, 807. [[CrossRef](#)]
65. Yang, S.H.; Pi, C.M. Color-flavor locked strange stars admixed with mirror dark matter and the observations of compact stars. *J. Cosmol. Astropart. Phys.* **2024**, *2024*, 052. [[CrossRef](#)]
66. Yang, S.H.; Pi, C.M.; Weber, F. Strange stars admixed with mirror dark matter: Confronting observations of XTE J1814-338. *Phys. Rev. D* **2025**, *111*, 043037. [[CrossRef](#)]
67. Panotopoulos, G.; Rincón, Á.; Lopes, I. Exploring the structural properties of anisotropic dark matter-admixed quark stars. *Phys. Dark Universe* **2025**, *49*, 101972. [[CrossRef](#)]
68. Ellis, J.; Hütsi, G.; Kannike, K.; Marzola, L.; Raidal, M.; Vaskonen, V. Dark matter effects on neutron star properties. *Phys. Rev. D* **2018**, *97*, 123007. [[CrossRef](#)]
69. Lugones, G.; Horvath, J.E. Color-flavor locked strange matter. *Phys. Rev. D* **2002**, *66*, 074017. [[CrossRef](#)]
70. Flores, C.V.; Lugones, G. Constraining color flavor locked strange stars in the gravitational wave era. *Phys. Rev. C* **2017**, *95*, 025808. [[CrossRef](#)]
71. Chavanis, P.H.; Harko, T. Bose–Einstein condensate general relativistic stars. *Phys. Rev. D* **2012**, *86*, 064011. [[CrossRef](#)]
72. Harko, T. Cosmological dynamics of dark matter Bose–Einstein condensation. *PHys. Rev. D Part. Fields Gravit. Cosmol.* **2011**, *83*, 123515. [[CrossRef](#)]
73. Li, X.; Harko, T.; Cheng, K. Condensate dark matter stars. *J. Cosmol. Astropart. Phys.* **2012**, *2012*, 1. [[CrossRef](#)]

74. Gruber, C.; Pelster, A. A theory of finite-temperature Bose–Einstein condensates in neutron stars. *Eur. Phys. J. D* **2014**, *68*, 341. [[CrossRef](#)]
75. Panotopoulos, G.; Lopes, I. Gravitational effects of condensed dark matter on strange stars. *Phys. Rev. D* **2017**, *96*, 023002. [[CrossRef](#)]
76. Jyothilakshmi, O.; Naik, L.J.; Sreekanth, V. Bose–Einstein condensate stars in combined Rastall-Rainbow gravity. *Gen. Relativ. Gravit.* **2024**, *56*, 141. [[CrossRef](#)]
77. Ciarcelluti, P.; Sandin, F. Have neutron stars a dark matter core? *Phys. Lett. B* **2011**, *695*, 19–21. [[CrossRef](#)]
78. Damour, T.; Nagar, A. Relativistic tidal properties of neutron stars. *Phys. Rev. D* **2009**, *80*, 084035. [[CrossRef](#)]
79. Takátsy, J.; Kovács, P. Comment on “Tidal Love numbers of neutron and self-bound quark stars”. *Phys. Rev. D* **2020**, *102*, 028501. [[CrossRef](#)]
80. Hinderer, T. Tidal Love numbers of neutron stars. *Astrophys. J.* **2008**, *677*, 1216. [[CrossRef](#)]
81. Cronin, J.; Zhang, X.; Kain, B. Rotating dark matter admixed neutron stars. *Phys. Rev. D* **2023**, *108*, 103016. [[CrossRef](#)]
82. Postnikov, S.; Prakash, M.; Lattimer, J.M. Tidal Love numbers of neutron and self-bound quark stars. *Phys. Rev. D* **2010**, *82*, 024016. [[CrossRef](#)]
83. Abbott, R.; Abbott, T.; Abraham, S.; Acernese, F.; Ackley, K.; Adams, C.; Adhikari, R.X.; Adya, V.; Affeldt, C.; Agathos, M.; et al. GW190814: Gravitational waves from the coalescence of a 23 solar mass black hole with a 2.6 solar mass compact object. *Astrophys. J. Lett.* **2020**, *896*, L44. [[CrossRef](#)]
84. Romani, R.W.; Kandel, D.; Filippenko, A.V.; Brink, T.G.; Zheng, W. PSR J0952-0607: The fastest and heaviest known galactic neutron star. *Astrophys. J. Lett.* **2022**, *934*, L17. [[CrossRef](#)]
85. Chandrasekhar, S.; Chandrasekhar, S. *An Introduction to the Study of Stellar Structure*; Courier Corporation: Chelmsford, MA, USA, 1957; Volume 2.
86. Maurya, S.; Errehymy, A.; Newton Singh, K.; Aziz, A.; Hansraj, S.; Ray, S. Modeling compact object mergers GW190814 and GW200210 and other self-bound compact stars with dark matter induced by gravitational decoupling and its significance to the mass gap. *Astrophys. J.* **2024**, *972*, 175. [[CrossRef](#)]
87. Leung, S.C.; Chu, M.C.; Lin, L.M. Dark-matter admixed neutron stars. *Phys. Rev. D* **2011**, *84*, 107301. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.