

Article

A Proportional Intuitionistic Fuzzy AHP–EDAS Framework for Symmetric Risk Assessment in Automotive Assembly Lines: A 12-Failure-Mode PFMEA Study

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Abstract

Process Failure Mode and Effects Analysis (PFMEA) is the standard technique for proactive risk assessment in automotive assembly lines. To support differentiated criterion weighting, hesitancy-aware linguistic evaluation and rank stability validation, this paper proposes a symmetric extension of PFMEA that integrates Proportional Intuitionistic Fuzzy Sets (PIFSs) with the Analytic Hierarchy Process (AHP) and the Evaluation Based on Distance from Average Solution method (PIF-EDAS). PIFSs introduce a proportional balance between membership μ , non-membership ν and hesitancy π that is mathematically symmetric under linguistic-pair interchange and that preserves a constant hesitancy budget ($\pi = 1/(1 + k_1 + k_2)$) across the nine-point linguistic scale. The framework is applied to an automotive Original Equipment Manufacturer (OEM) assembly line in Türkiye, on an inventory of twelve failure modes spanning torque, fastening, welding, panel alignment, harness, sealant, paint, ECU, trim, electrical connector, part variant and tightening sequence operations. Consistency of the PIF-AHP pairwise comparisons is confirmed ($CR = 0.0017 \ll 0.1$), yielding criterion weights $w_S = 0.598$, $w_O = 0.245$ and $w_D = 0.156$. Comparative cross-method analysis against PIF-TOPSIS, PIF-VIKOR and a classical-style ordinal RPN benchmark indicates strong cross-method agreement: Spearman rank correlations range from 0.865 to 0.972, and Wrong Torque Application remains the unanimous top-priority failure across all four methods. Five sensitivity scenarios (proposed weights, equal weights, and severity-, occurrence- and detection-dominant) confirm that FM1 (Wrong Torque) remains the top-priority failure and FM9 (Damaged Interior Trim) remains the lowest-priority failure across all five scenarios; the composition of the upper-priority set is criterion-sensitive, with FM10 rising under equal-weight, occurrence- and detection-dominant scenarios and FM8 rising under the severity-dominant scenario. The proposed framework incorporates differentiated criterion weights, expert hesitancy and rank stability validation within a symmetric PIFS-based MCDM structure. The contribution of this study is therefore three-fold: (i) a symmetric PIFS formulation that enforces mirror symmetry under linguistic-pair interchange and a constant hesitation budget on the nine-point scale; (ii) a case-based assessment of twelve automotive PFMEA failure modes; and (iii) a transparent rank stability protocol for symmetric MCDM benchmarking. The framework integrates directly with existing FMEA workflows and scales linearly in computational complexity with the number of failure modes.



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1. Introduction

Automotive assembly lines are among the most quality-critical, throughput-sensitive and safety-regulated production environments in modern manufacturing. A single uncontrolled failure (an under-torqued wheel-hub bolt, a misrouted high-voltage harness, an incorrectly flashed engine control unit (ECU)) can propagate through downstream stations, trigger a field recall, or in the worst-case cause harm to the end customer [1,2]. Process Failure Mode and Effects Analysis (PFMEA) is, for that reason, embedded in every contemporary International Automotive Task Force (IATF 16949) quality-management system and is the standard proactive tool for identifying, evaluating and prioritising assembly line risks before they reach the customer.

The classical PFMEA prioritises failure modes through the Risk Priority Number (RPN), defined as the product of severity (S), occurrence (O) and detection (D), each scored on an integer 1–10 scale. Although the RPN is simple, auditable and widely accepted, three structural limitations have been repeatedly documented in the fuzzy MCDM literature. First, the RPN implicitly assigns equal weights to S, O and D, which conflicts with the empirical observation that automotive engineers consistently rank severity higher than the other two risk criteria [3,4]. Second, the three risk criteria are rated on linguistic scales that are inherently subjective—“high”, “average”, and “low”—and the integer projection of those linguistic terms cannot represent expert hesitancy, expert non-commitment, or the proportional balance between agreement and disagreement that is intrinsic to expert elicitation [5,6]. Third, because the RPN is a multiplicative aggregator, several distinct (S, O, D) triplets can share the same RPN value, and the ordinal stability of the resulting ranking degrades under small perturbations of the input scores: a phenomenon known in the literature as the equal-RPN problem [7].

These limitations have motivated a long stream of fuzzy and MCDM-based FMEA/PFMEA extensions, including fuzzy RPN [8], intuitionistic fuzzy decision models [9], Pythagorean fuzzy FMEA [10], spherical and interval-valued spherical fuzzy risk models [11,12] and recent proportional fuzzy set MCDM applications [13–15]. The common feature of these extensions is the use of a fuzzy or intuitionistic fuzzy linguistic scale that preserves expert hesitancy alongside membership and non-membership. Within this family, the proportional fuzzy set construction, including Proportional Intuitionistic Fuzzy Sets (PIFSs), was developed in Kahraman [15], while related proportional picture fuzzy and IV-PIF applications are reported in [13,14]. PIFSs are well suited for the present PFMEA setting because they offer a compact and interpretable closed-form symmetric transformation between the linguistic pair (k_1, k_2) and the triple (μ, ν, π) , and because the hesitancy budget $\pi = 1/(1 + k_1 + k_2)$ is preserved as a constant proportional residue, an explicit symmetry property that ordinary intuitionistic fuzzy sets do not possess [15]. This is a scoping argument: PIFSs are not claimed to be generally superior to Pythagorean, spherical, q-Rung orthopair or Fermatean fuzzy set families, each of which targets a different feasible region in the (μ, ν) plane and has its own legitimate use cases.

Symmetry, in the broad mathematical sense commonly used, is a property of invariance under a group of transformations. In the PIFS framework, three independent symmetries are operative. (i) Linguistic symmetry: The linguistic pair (k_1, k_2) and its mirror (k_2, k_1) generate (μ, ν, π) triples that are mirror images about the indifference axis $\mu = \nu$, with the hesitancy component π invariant under that reflection. (ii) Criterion symmetry: The

three *PFMEA* criteria *S*, *O* and *D* are permutation-symmetric under the *EDAS* appraisal score, in the sense that the *EDAS* algorithm does not depend on the order in which the criteria are presented; only their weights and average values do. (iii) Proportional hesitancy budget symmetry: On the fixed nine-point linguistic scale, the denominator $1 + k_1 + k_2 = 11$ is constant by construction, which holds the hesitation component $\pi = 1/11 \approx 0.0909$ fixed across the scale and lets the membership/non-membership pair (μ, ν) shift proportionally as the linguistic level changes. We emphasise that this is not a claim of strict component-level invariance under arbitrary rescaling $k_i \rightarrow \alpha \cdot k_i$ (because the constant 1 in the denominator does not scale); the practically relevant property is the rank stability of the score $S(k_1, k_2)$ under symmetric scale-resolution changes, which we formalise as Property 2 in Section 3.5. The first two symmetries are structural; the third is the proportional hesitancy budget symmetry that motivates the prefix “proportional” in *PIFS*. Together, they ensure that the *PIF-AHP-EDAS* framework treats the linguistic input and the criterion order in a balanced, mirror-symmetric and rank-stable manner: an operational realisation of the symmetry principle that has recently been recognised as a unifying theme across fuzzy *MCDM* research [16–18].

The present paper develops a full *PIF-AHP-EDAS PFMEA* framework along five complementary directions that are essential for *MCDM* research. First, the case study has twelve realistic failure modes that span torque, fastening, welding, panel alignment, harness, sealant, paint, *ECU*, trim, electrical connector, part variant and tightening sequence operations of an automotive *OEM* assembly line in Türkiye. Second, the proportional symmetry framing is made explicit, and the three symmetry properties above are formalised. Third, the cross-method comparison is extended to four *MCDM* methods—*PIF-EDAS*, *PIF-TOPSIS*, *PIF-VIKOR* and a classical-style ordinal *RPN* benchmark—and the rank agreement is quantified using the Spearman correlation. Fourth, the sensitivity analysis covers five weight scenarios (proposed, equal, *S*-dominant, *O*-dominant and *D*-dominant), which stress-test the rank stability under realistic perturbations of the criterion weights. Fifth, the literature review is anchored in the symmetry-oriented *MCDM* stream [12,16–24] to position the contribution.

To the best of our knowledge, this is the first study to integrate proportional intuitionistic fuzzy linguistic elicitation, *AHP*-based criterion weighting and *EDAS*-based ranking into an automotive *PFMEA* framework, supported by *PIF-TOPSIS*, *PIF-VIKOR* and a classical-style ordinal *RPN* benchmark. The novelty of the study does not lie in proposing a new fuzzy set theory or redefining *AHP* and *EDAS* axiomatically; rather, it lies in the symmetry-oriented integration of these components for automotive *PFMEA*, the use of a twelve-failure-mode assembly line case, tie-aware Spearman rank agreement analysis, five-scenario sensitivity testing and explicit symmetry-invariance diagnostics.

The remainder of the paper is organised as follows. Section 2 reviews the symmetry-oriented *MCDM* and fuzzy *FMEA* literature. Section 3 presents the preliminaries of intuitionistic and Proportional Intuitionistic Fuzzy Sets, formalising the three symmetry properties. Section 4 develops the integrated *PIF-AHP-EDAS* framework, including the *PIF-TOPSIS* and *PIF-VIKOR* benchmarks. Section 5 reports the 12-failure-mode case application. Section 6 contains the comparative and sensitivity analyses. Section 7 discusses the managerial and methodological implications, and Section 8 concludes with limitations and future work.

2. Literature Review

2.1. *PFMEA* and the Limitations of the Classical *RPN*

Failure Mode and Effects Analysis (*FMEA*) was introduced in the late 1940s by the United States Department of Defence as a procedure for the systematic identification of

failure modes and their effects in complex weapon systems [25]. From its initial military application, the technique was adopted by the aerospace industry through the Apollo programme and, in the 1980s, by the automotive sector through the Automotive Industry Action Group (AIAG) and Verband der Automobilindustrie (VDA) reference manuals. Process FMEA (PFMEA) is the variant specifically intended for manufacturing-process risks and is the de facto risk assessment instrument in automotive assembly lines under the IATF 16949 quality-management standard [1].

The classical PFMEA prioritises failure modes through the Risk Priority Number, $RPN = S \times O \times D$, where each criterion is rated on an integer 1–10 scale. The simplicity and auditability of the RPN are widely acknowledged as its main strengths. However, four structural limitations are recognised in the literature: (i) the implicit equal-weights assumption; (ii) the inability to represent expert hesitancy and non-membership; (iii) the multiplicative aggregation, which produces the well-known equal-RPN problem; and (iv) the discontinuous integer scale, which projects continuous linguistic judgments onto a coarse grid [3,4,7]. These limitations have motivated a sustained line of fuzzy and MCDM extensions, as summarised below.

2.2. Fuzzy and Intuitionistic Fuzzy Extensions of FMEA

The first wave of fuzzy FMEA extensions, in the 1990s and early 2000s, replaced the integer 1–10 scale by triangular or trapezoidal Type-1 fuzzy numbers and used a fuzzy weighted average or fuzzy Mamdani inference for risk aggregation [8]. The second wave, in the 2010s, introduced intuitionistic fuzzy sets (IFSs) [26] to capture expert hesitancy as a third dimension $\pi = 1 - \mu - \nu$ alongside membership μ and non-membership ν . Within this broader stream, intuitionistic and Pythagorean fuzzy MCDM models, including TOPSIS-type approaches, have been widely used to handle hesitancy and non-membership in multi-criteria evaluation problems [9,27], while related fuzzy FMEA applications have adapted these ideas to manufacturing risk assessment. The third wave, in the late 2010s and early 2020s, extended IFSs to Pythagorean [10], spherical [11] and q-Rung orthopair [28] generalisations that relax the constraint $\mu + \nu \leq 1$ and admit larger feasible regions in the (μ, ν) plane. Cebeci, Simsir and Doğan [12] proposed an interval-valued spherical fuzzy FMEA for five-axis CNC water-jet machining and showed that the rotational symmetry of the spherical fuzzy ball, combined with the permutation symmetry of S, O and D, produces a transformation-invariant ranking. Sudzum et al. [20] developed an IF-TOPSIS plus IF-fuzzy logic two-stage framework for manufacturing-process risk and demonstrated that the symmetry property of IF aggregation supports balanced treatment of severity, occurrence and detection.

2.3. Proportional Intuitionistic Fuzzy Sets

The Proportional Intuitionistic Fuzzy Set (PIFS), developed within the proportional fuzzy set construction of Kahraman [15] (with closely related proportional picture fuzzy and IV-PIF applications in [13,14]), represents an expert's linguistic rating as a pair (k_1, k_2) of integers on a fixed denominator, from which the (μ, ν, π) triple is computed in closed form as $\mu = k_1/(1 + k_1 + k_2)$, $\nu = k_2/(1 + k_1 + k_2)$ and $\pi = 1/(1 + k_1 + k_2)$. The key structural feature is that the hesitancy π is preserved as a constant proportional residue across the entire linguistic scale, which ensures that no linguistic level carries unequal hesitancy by construction. Alkan and Kahraman [19] subsequently extended the proportional construction to spherical fuzzy sets and applied the resulting Proportional Spherical Fuzzy AHP to human-centric and sustainable city criteria, again exploiting the symmetric (μ, ν, π) representation. The PIFS approach has not previously been combined

with the *EDAS* aggregator for *PFMEA*, and the present paper closes that gap with a twelve-failure-mode case study and a four-method comparative analysis.

2.4. *EDAS and Symmetry-Oriented MCDM*

The Evaluation Based on Distance from Average Solution (*EDAS*) method was proposed by Keshavarz-Ghorabae et al. [29] as a computationally simple alternative to *TOPSIS* that ranks alternatives by their positive (*PDA*) and negative (*NDA*) distances from a single average solution. Unlike *TOPSIS*, which depends on two reference points (positive and negative ideals), *EDAS* is referenced to one, and is, therefore, less sensitive to outliers in either tail of the decision matrix. The recent literature contains a rapidly growing corpus on symmetry-aware *MCDM* with hesitant, spherical, picture, *q*-Rung and other generalised fuzzy sets [16,17,21–23]. Xiang et al. [16] propose a symmetry-guided hesitant fuzzy *DE-MATEL* plus entropy-weighted model for low-carbon automotive supply-chain risk and report rank correlations above 0.85 against a baseline *AHP*. Liu et al. [17] integrate *AHP* and the entropy-weight method through a symmetric mapping of subjective and objective weights for data security risk. Bal et al. [21] address the asymmetric “presence-absence” phenomenon in *MCDM* through an intuitionistic fuzzy synthesis. Zeng et al. [22] employ *AHP*–*TOPSIS* with symmetry-based weighting for highway-route selection. Puska et al. [23] use symmetric fuzzy numbers in a multi-method *MCDM* evaluation of household waste strategies. Yang, Akhtar and Ali [24] develop symmetry-based distance and similarity measures for globular T-spherical fuzzy sets in solar energy decision contexts. These contributions converge on the principle that the symmetry of the underlying fuzzy structure is a prerequisite for the transformation-invariance of the resulting ranking, and they motivate the symmetric framing of the present *PIF*–*AHP*–*EDAS* study.

2.5. *Hesitant, Picture, q-Rung Orthopair and Fermatean Fuzzy Extensions*

Beyond the intuitionistic and spherical families, several generalised fuzzy set families have been recently introduced and applied to *MCDM* problems with rich uncertainty structure. Shi and Zhang [30] introduced linguistic *q*-Rung orthopair fuzzy operators with explicit symmetry under dual transformation and applied them to an energy-resource exploitation decision. Alrabeah and Alsager [31] proposed multi-*Q* Fermatean hesitant fuzzy soft sets and derived aggregation operators that preserve symmetry under permutation. Xu and Zhang [32] integrated circular picture fuzzy sets with the *WASPAS* aggregator to identify *AI*-driven service-innovation trends. Ma and Zeng [33] developed weighted hesitant fuzzy Bonferroni mean operators that capture the relevance between criteria while preserving symmetric group-decision aggregation. Although each of these extensions targets a richer uncertainty representation, all of them share the same operational structure: a generalised fuzzy linguistic scale, a consistency-checked weight elicitation and a closeness- or distance-based aggregator. The *PIFS*–*EDAS* framework of the present paper sits at the intersection of (a) the proportional fuzzy set construction of Kahraman [15] and (b) the symmetry-aware *EDAS* literature [29], and we argue that *PIFS*–*EDAS* offers a compact and interpretable operational realisation of symmetry in fuzzy *MCDM* for the present *PFMEA* setting.

A natural question is why the proposed framework adopts Proportional Intuitionistic Fuzzy Sets rather than the proportional Pythagorean or Fermatean extensions, which enlarge the admissible membership–non-membership region ($\mu^2 + \nu^2 \leq 1$ and $\mu^3 + \nu^3 \leq 1$, respectively) and can therefore encode stronger degrees of uncertainty. Three considerations motivate this choice. First, the contribution of this paper is the symmetry-oriented integration of proportional fuzzy elicitation with *AHP* weighting and *EDAS* ranking for automotive *PFMEA*, not the introduction of a new fuzzy set family; *PIFS* is the construction

in which the proportional hesitancy budget $\pi = 1/(1 + a + b)$ and the linguistic mirror symmetry used in Section 3 are most transparent. Second, the nine-point linguistic ratings produced by the *PFMEA* workshop satisfy the intuitionistic constraint $\mu + \nu \leq 1$ without saturation, so the additional admissible region offered by Pythagorean or Fermatean sets is not exercised by the present data and would not alter the elicited information. Third, retaining the intuitionistic level keeps the score function, the consistency check and the *EDAS* aggregation linear and interpretable for practitioners. The Pythagorean, Fermatean and Fermatean–cubic families [34] are nonetheless strictly more expressive, and extending the proportional construction and the three symmetry properties to these stronger families—particularly for elicitation under very high ambiguity—is identified as a direction for future work (Section 8).

2.6. Comparative Synthesis of Recent Symmetry-Oriented Fuzzy FMEA and MCDM Studies

Table 1 consolidates representative recent symmetry articles and proportional fuzzy set studies relevant to symmetry-oriented fuzzy *FMEA* and fuzzy *MCDM*. The table spans the period 2024–2026 and is intended to position the present *PIF-AHP-EDAS PFMEA* framework within the recent symmetry-aware decision analysis stream, rather than to provide an exhaustive review. The table reports the fuzzy set family, the *MCDM* aggregator, the application domain, the size of the alternative set and the reported cross-method agreement measure when available. The present work occupies the only cell in the table that combines (i) the *PIFS* family, (ii) the *EDAS* aggregator, (iii) an automotive *PFMEA* domain and (iv) an alternative set of $m = 12$, which establishes its niche.

Table 1. Comparative synthesis of recent symmetry-oriented fuzzy FMEA and related MCDM studies. The agreement column reports the cross-method correlation when available. The proposed twelve-failure-mode PIF-AHP + EDAS framework occupies a unique niche.

Study	Fuzzy Family	MCDM	Domain	m	Agreement
Cebeci et al. [12]	Int.-Val. Spherical	<i>FMEA-RPN</i>	CNC water-jet	7	$\rho = 0.929$ vs. <i>RPN</i>
Sudzum et al. [20]	Interval-Val. <i>IF</i>	<i>TOPSIS</i> + Fuzzy Logic	Manufacturing risk	5	Robust
Xiang et al. [16]	Hesitant Fuzzy	<i>DEMATEL</i> + Entropy	Automotive supply chain	10	$\rho = 0.88$ vs. <i>AHP</i>
Liu et al. [17]	Crisp + Entropy	<i>AHP</i> + <i>EWM</i> + Neural Net	Data security	8	Stable
Alkan & Kahraman [19]	Proportional Spherical	<i>AHP</i>	Sustainable city	26	Consistent
Bal et al. [21]	Intuitionistic	Presence–Absence	Workforce mgmt.	9	ρ vs. <i>TOPSIS</i>
Zeng et al. [22]	Crisp	<i>AHP-TOPSIS</i>	Highway routing	4	Robust
Puska et al. [23]	Symm. Fuzzy Numbers	SiWeC- <i>TOPSIS-ARAS-SAW</i>	Household waste	6	Concordant
Yang et al. [24]	Globular T-Spherical	<i>TOPSIS</i>	Solar energy	5	Stable in p
Shi & Zhang [30]	Linguistic q-Rung	Aggregation Operator	Energy exploitation	4	—
Bulut et al. [13]	IV- <i>PIFS</i>	<i>AHP</i> + <i>EDAS</i>	Low-cost airline service	6	—
Du et al. [35]	Pythagorean Fuzzy	<i>TOPSIS</i> (excessive-hesitation)	High-hesitation decision case	—	Hesitation-adjusted ranking
Shang et al. [36]	IV-Picture Fuzzy	Large-scale <i>MARCOS</i>	Financial risk	10	Robust under noise
Ari & Dursun [37]	Intuitionistic Fuzzy	Bayesian <i>BWM</i> + <i>IF-EDAS</i>	Aircraft sustainability	—	Consensus-stable ranking
This study	<i>PIFS</i>	<i>AHP</i> + <i>EDAS</i> (+ <i>TOPSIS</i> + <i>VIKOR</i> + <i>RPN</i>)	Automotive <i>PFMEA</i>	12	$\rho = 0.865$ – 0.972

2.7. Research Gap

The literature reviewed in Sections 2.1–2.6 reveals three concrete gaps. First, although *PIFS* has been applied to weighting and service quality *MCDM* (Kahraman [15] and Bulut et al. [13]) and to spherical fuzzy *AHP* (Alkan and Kahraman [19]), no published *PFMEA*

study has combined *PIF-AHP* with *PIF-EDAS* on a realistically sized failure mode inventory. Second, the symmetry property of the proportional construction is referenced informally in [14,15,19] but is not formalised as a transformation-invariance condition that can be verified empirically. Third, the cross-method rank stability behaviour of *PIF-EDAS*, *PIF-TOPSIS* and *PIF-VIKOR* has been reported only for very small alternative sets ($m \leq 5$), and the literature explicitly identifies the need to test these methods on larger sets to demonstrate scalability [16,20]. The present paper addresses these three gaps simultaneously. Accordingly, the contribution is positioned as a novel symmetry-oriented PFMEA application of the *PIFS*–*AHP*–*EDAS* pipeline, rather than as a claim of general superiority over other fuzzy set families or as a new axiomatic MCDM method.

3. Preliminaries: Intuitionistic and Proportional Intuitionistic Fuzzy Sets

This section reviews the mathematical machinery used in the rest of the paper. We first recall classical Zadeh fuzzy sets [38] and Atanassov intuitionistic fuzzy sets [26], then introduce the *PIFS* construction within the proportional fuzzy set framework of Kahraman [15] (with the closely related proportional picture fuzzy and *IV-PIF* applications of [13,14]). The weighting step uses a *PIFS*-informed linguistic pairwise-comparison scale and the standard geometric mean *AHP* priority vector calculation. We close the section by formalising three symmetry properties of *PIFS*s that underpin the present study, with self-contained proofs.

3.1. Notation and Abbreviations

For ease of reference, Table 2 consolidates the notation and abbreviations used throughout the paper.

Table 2. Notation, abbreviations and acronyms used throughout the paper.

Symbol/Abbreviation	Description
<i>PFMEA</i>	Process Failure Mode and Effects Analysis
<i>FMEA</i>	Failure Mode and Effects Analysis (general)
<i>RPN</i>	Risk Priority Number ($RPN = S \times O \times D$)
<i>S, O, D</i>	Severity, Occurrence, Detection (<i>PFMEA</i> criteria)
<i>IFS</i>	Intuitionistic Fuzzy Set
<i>PIFS</i>	Proportional Intuitionistic Fuzzy Set
<i>MCDM</i>	Multi-Criteria Decision Making
<i>AHP</i>	Analytic Hierarchy Process
<i>EDAS</i>	Evaluation Based on Distance from Average Solution
<i>TOPSIS</i>	Technique for Order of Preference by Similarity to Ideal Solution
<i>VIKOR</i>	Vlse Kriterijumska Optimizacija I Kompromisno Resenje
(k_1, k_2)	Proportional intuitionistic fuzzy linguistic pair
μ, ν, π	Membership, non-membership, hesitancy of a <i>PIFS</i> triple
$S(k_1, k_2)$	Kahraman score function for a <i>PIFS</i> pair
m, n	Number of failure modes (m); number of criteria ($n = 3$)
w_j, AV_j	Criterion weight; criterion-wise average solution
x_{ij}	Defuzzified rating of failure mode i on criterion j
<i>PDA, NDA</i>	Positive/negative distance from average solution
<i>SP, SN</i>	Weighted sums of <i>PDA</i> and <i>NDA</i>
<i>NSP, NSN, AS</i>	Normalised <i>SP</i> , normalised $(1 - SN)$, appraisal score
<i>CC, Q</i>	<i>TOPSIS</i> closeness coefficient; <i>VIKOR</i> compromise score
$\lambda_{\max}, CI, RI, CR$	Saaty principal eigenvalue and consistency indices
ρ	Spearman rank correlation coefficient
σ, τ_α	Linguistic mirror involution; α -rescaling transformation
<i>AIAG, VDA, IATF</i>	Automotive Industry Action Group, Verband der Automobilindustrie, International Automotive Task Force
<i>OEM</i>	Original Equipment Manufacturer

3.2. Atanassov Intuitionistic Fuzzy Sets

Definition 1 (Intuitionistic fuzzy set). Following Atanassov [26], an intuitionistic fuzzy set (IFS) $\tilde{A}$ on a universe of discourse X is characterised by a membership function $\mu: X \rightarrow [0, 1]$ and a non-membership function $\nu: X \rightarrow [0, 1]$ that satisfy $0 \leq \mu(x) + \nu(x) \leq 1$ for every $x \in X$. The hesitancy is defined as the residue $\pi(x) = 1 - \mu(x) - \nu(x)$, and the triple (μ, ν, π) sums to one by construction. The IFS reduces to a classical Zadeh fuzzy set [38] when $\nu = 1 - \mu$, and to a crisp set when both μ and ν are 0–1 valued. The score function $S(x) = \mu(x) - \nu(x)$ is widely used for ranking IFSs in the interval $[-1, 1]$.

$$0 \leq \mu + \nu \leq 1, \pi = 1 - \mu - \nu, s = \mu - \nu \quad (1)$$

3.3. Proportional Intuitionistic Fuzzy Sets

Definition 2 (Proportional Intuitionistic Fuzzy Set). The Proportional Intuitionistic Fuzzy Set (PIFS), defined within the proportional fuzzy set construction of Kahraman [15] (with related proportional picture fuzzy and IV-PIF applications in [13,14]), represents the expert's linguistic rating as a pair (k_1, k_2) of non-negative integers on a fixed denominator. The (μ, ν, π) triple is then defined by the closed-form proportional construction:

$$\mu = \frac{k_1}{1 + k_1 + k_2}, \nu = \frac{k_2}{1 + k_1 + k_2}, \pi = \frac{1}{1 + k_1 + k_2} \quad (2)$$

Equation (2) is a direct-fraction rewrite that makes the proportional symmetry visible: the membership, non-membership and hesitancy components share the same denominator $(1 + k_1 + k_2)$ and the hesitancy carries a unit numerator; i.e., it is a constant proportional residue. The score function used in this paper is the proportional fuzzy set score of Kahraman [15]:

$$S(k_1, k_2) = \mu - \nu + \frac{1}{2} = \frac{k_1 - k_2}{1 + k_1 + k_2} + \frac{1}{2} \quad (3)$$

The Kahraman score (3) is bounded in $[-1/4, +5/4]$ for the nine-point scale used in this paper, with $S = 1/2$ at the indifference point $k_1 = k_2$. It satisfies three properties that are essential for the PIF-EDAS aggregator: continuity, monotonicity in (k_1, k_2) , and anti-symmetry $S(k_1, k_2) + S(k_2, k_1) = 1$. The last property is the concrete realisation of the linguistic symmetry mentioned in Section 1.

3.4. The Nine-Point Linguistic Scale

Following [14,15], the nine-point linguistic scale used in this paper assigns the pairs (k_1, k_2) shown in Table 3 to the verbal labels certainly high (CH), very high (VH), high (H), above average (AH), average (A), below average (BL), low (L), very low (VL) and certainly low (CL). The scale is symmetric about the central A label with $k_1 = k_2 = 5$, and the hesitancy $\pi = 1/11 = 0.0909$ is the same for every label by construction—the proportional symmetry property of PIFS.

The nine-point scale was selected for two complementary reasons. Practically, it is fine-grained enough to distinguish the high-, medium- and low-risk gradations that arise in automotive PFMEA judgements of severity, occurrence and detection, and it aligns with the granularity expected by an AIAG-VDA-style scoring workshop. Theoretically, the nine labels admit a symmetric centre ($A \leftrightarrow A$ under linguistic mirror reflection) and a constant hesitancy budget $\pi = 1/(1 + k_1 + k_2) = 1/11$ at every label, which is the structural feature exploited by the proportional symmetry property of PIFS.

Table 3. The nine-point proportional intuitionistic fuzzy linguistic scale. The denominator $1 + k_1 + k_2 = 11$ is constant across the scale, which yields a constant hesitancy budget $\pi = 1/11 \approx 0.0909$ —the proportional symmetry property of PIFS.

Label	Description	k_1	k_2	μ	ν	π	Score S
CH	Certainly High	9	1	0.8182	0.0909	0.0909	1.2273
VH	Very High	8	2	0.7273	0.1818	0.0909	1.0455
H	High	7	3	0.6364	0.2727	0.0909	0.8636
AH	Above Average	6	4	0.5455	0.3636	0.0909	0.6818
A	Average	5	5	0.4545	0.4545	0.0909	0.5000
BL	Below Average	4	6	0.3636	0.5455	0.0909	0.3182
L	Low	3	7	0.2727	0.6364	0.0909	0.1364
VL	Very Low	2	8	0.1818	0.7273	0.0909	−0.0455
CL	Certainly Low	1	9	0.0909	0.8182	0.0909	−0.2273

The conversion of the nine-granularity linguistic term set into proportional intuitionistic fuzzy pairs follows the established proportional fuzzy set practice of Kahraman [14,15] and is consistent with the broader multi-granularity linguistic decision-making literature, in which linguistic term sets of differing granularity are systematically transformed into a common quantitative representation before aggregation [39]. A nine-term scale is a widely adopted granularity because it balances the cognitive load on the expert against discriminating power and maps naturally onto the symmetric Saaty 1–9 intensity scale used in AHP [40]; the central label A acts as the symmetry axis and each label is paired with its linguistic mirror, so the conversion preserves the linguistic symmetry formalised in Section 3.5. The constant denominator $1 + a + b = 11$ then guarantees the constant hesitancy budget that underlies the proportional symmetry property. Should a coarser or finer panel be required, the same proportional transformation extends directly to seven- or eleven-term scales, whose rank stability is examined in Section 7.6.

3.5. Three Symmetry Properties of PIFS and Their Proofs

The proportional intuitionistic fuzzy construction is invariant under three independent symmetry transformations, which we make explicit here because they are the formal anchor of the present paper's scope:

Property 1 (Linguistic symmetry). *The mapping $\sigma: (k_1, k_2) \mapsto (k_2, k_1)$ sends every linguistic label X to its mirror label $\bar{X}$ ($CH \leftrightarrow CL$, $VH \leftrightarrow VL$, $H \leftrightarrow L$, $AH \leftrightarrow BL$, $A \leftrightarrow A$). Under σ , the score satisfies $S(\sigma(k_1, k_2)) = 1 - S(k_1, k_2)$; i.e., σ is an involution that reflects the score about the indifference value $\frac{1}{2}$. The hesitancy π is invariant under σ , which expresses the fact that the expert's indecision is independent of the direction of the rating.*

Proof of Property 1 (linguistic symmetry). Let $\sigma(k_1, k_2) = (k_2, k_1)$ and define the score $S(k_1, k_2) = (k_1 - k_2)/(1 + k_1 + k_2) + \frac{1}{2}$. Then, $S(\sigma(k_1, k_2)) = S(k_2, k_1) = (k_2 - k_1)/(1 + k_1 + k_2) + \frac{1}{2} = \frac{1}{2} - (k_1 - k_2)/(1 + k_1 + k_2) = 1 - S(k_1, k_2)$. Hence, $S \circ \sigma + S = 1$, which expresses the reflection of S about the indifference value $\frac{1}{2}$. The hesitancy is $\pi(\sigma(k_1, k_2)) = 1/(1 + k_2 + k_1) = \pi(k_1, k_2)$, so π is σ -invariant. $\square$

Property 2 (Proportional hesitancy budget symmetry and rank stability). *On the nine-point linguistic scale of Table 3, the denominator $1 + k_1 + k_2 = 11$ is constant, so the hesitation budget $\pi = 1/11$ is held fixed for every linguistic label and the membership/non-membership pair (μ, ν) shifts proportionally as (k_1, k_2) moves through the scale. We do not claim strict component-level invariance of (μ, ν, π) under the rescaling $\tau_\alpha: (k_1, k_2) \mapsto (\alpha \cdot k_1, \alpha \cdot k_2)$, because the constant 1 in the denominator does not scale, and $\pi = 1/(1 + \alpha k_1 + \alpha k_2)$ therefore depends*

on α . The practically relevant property is rank stability: the ordinal ranking induced by $S(k_1, k_2)$ is preserved under symmetric scale-resolution changes, e.g., refining the nine-point scale into a seven- or eleven-point variant by symmetric merging or splitting of adjacent levels, because S is monotonic in $(k_1 - k_2)/(1 + k_1 + k_2)$. The empirical Spearman correlations of 0.972 (seven-point) and 0.993 (eleven-point) against the nine-point ranking, reported in Section 7.6, confirm this rank stability interpretation.

Proof of Property 2 (proportional hesitancy budget symmetry and rank stability). On the nine-point scale, the denominator $1 + k_1 + k_2 = 11$ is fixed by construction, so $\pi = 1/11$ for every label and the hesitation budget is constant. For rank stability under a symmetric scale-resolution change, consider the ratio score $r(k_1, k_2) = (k_1 - k_2)/(1 + k_1 + k_2)$. The sign of r is the same as the sign of $k_1 - k_2$ and is unchanged by symmetric merging or splitting of adjacent levels of the nine-point scale into the seven-point and eleven-point variants tested in Section 7.6, which act as monotone, order-preserving transformations on the $(k_1 - k_2)$ axis. Hence, the PIF-EDAS ranking is rank-stable under the tested symmetric scale-resolution changes, although the (μ, ν, π) triple itself is not in general invariant under arbitrary rescaling. Property 2 should therefore be interpreted as proportional hesitancy budget symmetry on the fixed nine-point scale and empirical rank stability under the tested symmetric scale-resolution changes, rather than as a universal theorem of rank invariance for all possible scale transformations. $\square$

Property 3 (criterion permutation symmetry). For the PFMEA criterion set $C = \{S, O, D\}$, the EDAS appraisal score (Section 4.2) is invariant under any permutation π of the criteria, provided that the corresponding weights are permuted accordingly. Specifically, $AS_i = \frac{1}{2}(NSP_i + NSN_i)$ depends on the weighted sum $\sum_j w_j d_{ij}$, which is symmetric in j . The classical-style ordinal RPN is also permutation-invariant because multiplication is commutative ($S \times O \times D = O \times S \times D = D \times S \times O$); however, this invariance is achieved only through an equal-treatment assumption and does not allow asymmetric criterion importance to be represented.

Proof of Property 3 (criterion permutation symmetry). The EDAS appraisal score for alternative i is $AS_i = \frac{1}{2}(NSP_i + NSN_i)$ with $NSP_i = SP_i / \max_i SP_i$, $NSN_i = 1 - SN_i / \max_i SN_i$, $SP_i = \sum_j w_j PDA_{ij}$ and $SN_i = \sum_j w_j NDA_{ij}$. For any permutation π of $\{1, 2, 3\}$, the weighted sums $\sum_j w_{\{\pi(j)\}} PDA_{\{i, \pi(j)\}} = \sum_j w_j PDA_{ij}$, because the sum runs over all j and addition is commutative. Hence SP_i , SN_i , NSP_i , NSN_i and AS_i are all invariant under π , and the ranking induced by AS is invariant. $\square$

These three properties are not merely aesthetic. They support balanced treatment of the linguistic input and the criterion order in a mirror-symmetric and rank-stable manner, which is the operational realisation of the symmetry principle in fuzzy MCDM.

4. Integrated PIF-AHP-EDAS Framework

The decision problem addressed in this paper is a multi-criteria risk prioritisation problem defined by three components. The **goal** is to rank a finite set of process failure modes by overall risk so that limited mitigation resources are allocated to the most critical ones. The **alternatives** are the m failure modes elicited from the plant PFMEA library, $FM = \{FM_1, FM_2, \dots, FM_m\}$ (here $m = 12$). The **criteria** are the three classical PFMEA risk dimensions $C = \{S, O, D\}$ —severity (S), occurrence (O) and detection (D)—whose relative importance is expressed by the weight vector $w = (w_S, w_O, w_D)$ obtained from PIF-AHP. Each alternative FM_i is rated on each criterion C_j with a linguistic term that is mapped to a proportional intuitionistic fuzzy pair (Table 3). The **output** is a complete ordinal ranking of the alternatives induced by the PIF-EDAS appraisal score, validated against PIF-TOPSIS,

PIF-VIKOR and a classical-style ordinal *RPN* benchmark. This goal–criteria–alternatives structure is used consistently throughout Sections 4–6.

Figure 1 illustrates the proposed framework. The framework comprises three sequential modules: (i) *PIF-AHP* for the elicitation and consistency-checked aggregation of the criterion weights w_S , w_O and w_D ; (ii) *PIF-EDAS* for the aggregation of the failure mode scores into an appraisal score AS_i and the resulting ranking; (iii) *PIF-TOPSIS*, *PIF-VIKOR* and a classical-style ordinal *RPN* benchmark as benchmark *MCDM* methods that are used for cross-method comparison. The framework is anchored in the symmetric *PIFS* scale (Table 3) and is compatible with the *AIAG-VDA PFMEA* template used in automotive OEMs [1,2].

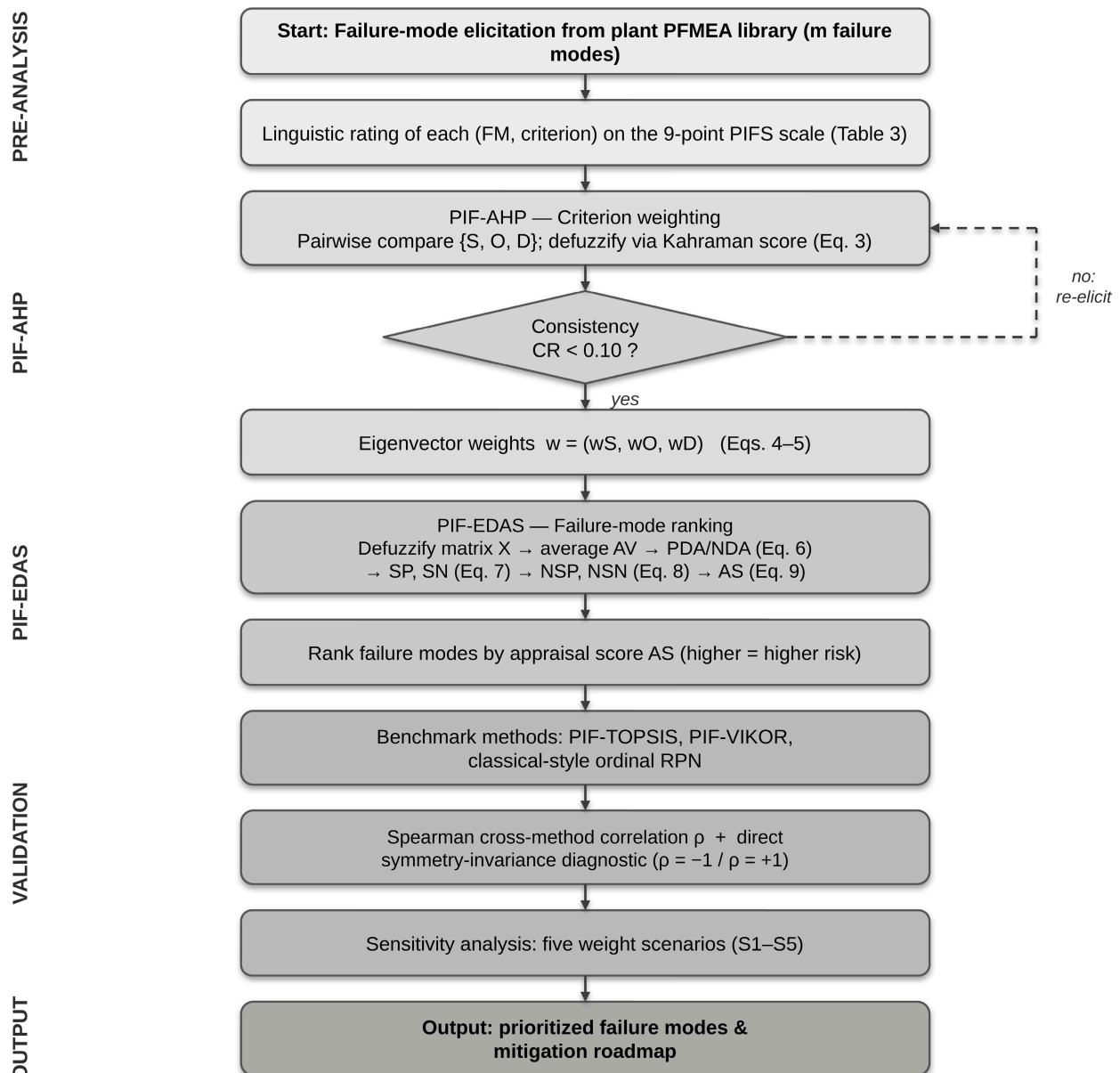


Figure 1. Flowchart of the proposed *PIF-AHP-EDAS* framework, with *PIF-TOPSIS*, *PIF-VIKOR* and a classical-style ordinal *RPN* benchmark as validators. The pre-analysis, *PIF-AHP* weighting, *PIF-EDAS* aggregation, validation and sensitivity stages are shown together with the consistency check feedback loop; the full step-by-step procedure, including the *PIFS* symmetry property exploited at each step, is given in Algorithm 1.

Three scoping clarifications are in order before the formal framework is presented. First, for the role of *PIFS* in the *EDAS* stage, *EDAS* is not modified at the axiomatic level in this paper; rather, the *EDAS* decision matrix is generated from *PIFS* linguistic ratings through the proportional score function (3). Thus, the *PIF* component enters through the uncertainty-preserving linguistic representation and its score-based projection before *EDAS* aggregation, and the same scoping applies to *PIF-TOPSIS* and *PIF-VIKOR*. Second, in relation to how hesitancy is preserved after defuzzification, although the final *EDAS*, *TOPSIS* and *VIKOR* computations use scalar scores, these scores are generated from *PIFS* triples in which membership μ , non-membership ν and hesitancy π are jointly encoded, with π following the proportional rule $\pi = 1/(1 + k_1 + k_2)$. Hesitancy is therefore incorporated at the elicitation and transformation stages, before scalar aggregation, rather than ignored at the outset. Third, in relation to the scope of the term “*PIF-AHP*” throughout this paper, “*PIF-AHP*” denotes the standard geometric mean *AHP* applied to *PIFS*-defuzzified linguistic pairwise comparisons; the *AHP* axioms (reciprocity, consistency ratio, eigenvector priority extraction) are used in their classical form and are not redefined in *PIFS* algebra. The short-form name “*PIF-AHP*” is retained for readability and consistency with the established fuzzy *MCDM* literature.

Algorithm 1. *PIF-AHP-EDAS PFMEA ranking procedure.*

Input: Plant PFMEA library; expert panel; 9-point *PIFS* linguistic scale *L* (Table 3)

Output: Risk ranking of failure modes and a mitigation roadmap

```

1  Elicit m failure modes FM = {FM_1, ..., FM_m} from the PFMEA library
2  for each (FM_i, criterion C_j), C_j in {S, O, D} do
3      assign linguistic term in L; map to PIFS pair (a_ij, b_ij) //linguistic symmetry
4  // ---- Module 1: PIF-AHP criterion weights ----
5  Elicit pairwise comparisons of {S, O, D} on L; defuzzify via Kahraman score
   (Equation (3))
6  Build reciprocal matrix; compute principal eigenvector w = (w_S, w_O, w_D)
   (Equation (4))
7  Compute CI and consistency ratio CR (Equation (5))
8  if CR ≥ 0.10 then re-elicite comparisons; go to 5
9  // ---- Module 2: PIF-EDAS ranking ----
10 Build decision matrix X = [x_ij], x_ij = Kahraman score of (a_ij, b_ij) (Equation (3))
11 Compute average solution AV_j = (1/m) * sum_i x_ij
12 Compute PDA_ij and NDA_ij from AV (Equation (6))
13 Compute weighted distance sums SP_i, SN_i (Equation (7))
14 Normalise to NSP_i, NSN_i (Equation (8)); appraisal score AS_i = (NSP_i +
   NSN_i)/2 (Equation (9))
15 Rank failure modes by descending AS_i
16 // ---- Module 3: validation & sensitivity ----
17 Compute PIF-TOPSIS, PIF-VIKOR and classical-style ordinal-RPN rankings
18 Compute Spearman rho across methods; run symmetry-invariance diagnostic
   (rho = −1 under linguistic reflection; rho = +1 under criterion permutation)
19 Re-rank under five weight scenarios S1–S5 (sensitivity analysis)
20 return ranking and mitigation roadmap

```

4.1. PIF-AHP for Criterion Weights

The expert elicits pairwise comparisons of the three *PFMEA* criteria {S, O, D} using the linguistic scale of Table 3. The (k_1, k_2) pair for the comparison “X versus Y” is then

converted to the crisp pairwise ratio k_1/k_2 , and the resulting 3×3 reciprocal matrix is diagonalised using the principal eigenvector method of Saaty [40]. Specifically, the AHP weight vector w is the normalised right eigenvector of the pairwise matrix corresponding to the largest eigenvalue $\lambda_{\max}$:

$$P \cdot w = \lambda_{\max} \cdot w, w = \frac{w}{\sum_i w_i} \quad (4)$$

The consistency of the elicitation is verified through the Saaty consistency ratio:

$$CR = \frac{\lambda_{\max} - n}{(n - 1) \cdot RI}, \text{ with } n = 3, RI = 0.58 \quad (5)$$

A value $CR < 0.10$ is accepted as consistent. In this study, the expert provided the three pairwise comparisons S vs. O = H, S vs. D = VH and O vs. D = AH (Table 4), which yielded $\lambda_{\max} = 3.0020$, $CI = 0.0010$ and $CR = 0.0017$, well below the 0.10 threshold.

Table 4. The five sensitivity scenarios used to stress-test the rank stability of the PIF-EDAS ranking. Scenario S1 is the proposed PIF-AHP weighting; S2 is the classical equal-weights baseline; S3, S4 and S5 are dominance scenarios that emphasise severity, occurrence and detection, respectively.

Scenario	Description	w_S	w_O	w_D
S1	Proposed PIF-AHP weights (this study)	0.598	0.245	0.156
S2	Equal weights (classical-style ordinal RPN baseline)	0.333	0.333	0.333
S3	Severity-dominant (recall-driven)	0.700	0.180	0.120
S4	Occurrence-dominant (volume-driven)	0.225	0.550	0.225
S5	Detection-dominant (process-control-driven)	0.250	0.250	0.500

4.2. PIF-EDAS for Failure Mode Ranking

Following Keshavarz-Ghorabae et al. [29], the PIF-EDAS algorithm proceeds in six steps, which are presented separately below.

Step 1. Construct the defuzzified decision matrix $X = [x_{ij}]$, where x_{ij} is the Kahraman score (Equation (3)) of the (a, b) rating of failure mode i on criterion j .

Step 2. Compute the average solution $AV_j = (1/m) \sum_i x_{ij}$ for each criterion j .

Step 3. Compute the positive (PDA) and negative (NDA) distances of each alternative from the average solution:

$$PDA_{ij} = \frac{\max(0, x_{ij} - AV_j)}{AV_j}, NDA_{ij} = \frac{\max(0, AV_j - x_{ij})}{AV_j} \quad (6)$$

Step 4 computes the weighted positive and negative distance sums:

$$SP_i = \sum_j w_j \cdot PDA_{ij}, SN_i = \sum_j w_j \cdot NDA_{ij} \quad (7)$$

Step 5 normalises SP and SN to the unit interval:

$$NSP_i = \frac{SP_i}{\max_i SP_i}, NSN_i = 1 - \frac{SN_i}{\max_i SN_i} \quad (8)$$

Step 6 aggregates the normalised distances into the appraisal score:

$$AS_i = \frac{1}{2} \cdot (NSP_i + NSN_i) \quad (9)$$

Failure modes are ranked in decreasing order of AS_i , with higher AS corresponding to higher risk. The PIF-EDAS aggregator satisfies the criterion permutation symmetry

(Property 3 of Section 3.5) and is, therefore, transformation-invariant under reordering of the criteria.

4.3. PIF-TOPSIS and PIF-VIKOR Benchmarks

Two well-established benchmark methods are computed alongside PIF-EDAS. PIF-TOPSIS [41] normalises the decision matrix using the Euclidean column norm, weights the normalised matrix, identifies the positive and negative ideals, and ranks the alternatives using the closeness coefficient $CC_i = d_{\text{neg}} / (d_{\text{pos}} + d_{\text{neg}})$. PIF-VIKOR [42] computes the group utility $S_v = \sum_j w_j (f_j^+ - x_{ij}) / (f_j^+ - f_j^-)$, the individual regret $R_v = \max_j w_j (f_j^+ - x_{ij}) / (f_j^+ - f_j^-)$ and the compromise score $Q_i = v (S_{v,i} - S_v^*) / (S_v^\# - S_v^*) + (1 - v) (R_{v,i} - R_v^*) / (R_v^\# - R_v^*)$ with $v = 0.5$. Ranking is in increasing order of Q . For comparability with the nine-level PIFS linguistic scale, the RPN benchmark maps the same linguistic labels monotonically onto integer scores from 10 to 2 (CH = 10, VH = 9, H = 8, AH = 7, A = 6, BL = 5, L = 4, VL = 3 and CL = 2; the unused integer 1 corresponds to no linguistic label in the proposed scale, so the benchmark spans nine integer levels rather than ten); the per-failure-mode score is then the multiplicative aggregate $RPN_i = S_i \cdot O_i \cdot D_i$. It should therefore be interpreted as a classical-style ordinal RPN benchmark based on a monotonic nine-level integer mapping of the linguistic scale, rather than a full independent 1–10 PFMEA scoring exercise.

4.4. Sensitivity Analysis Design

To stress-test the rank stability of the proposed framework under perturbations of the criterion weights, we define five sensitivity scenarios:

5. Case Application: A 12-Failure-Mode Automotive Assembly Line

5.1. The Production Setting

The case study is conducted in the *Trim and Final-Assembly* section of an automotive OEM passenger-vehicle plant in the Marmara region of Türkiye. The plant produces approximately 120,000 units per year on a two-shift schedule and supplies both the domestic and European markets. The *Trim and Final-Assembly* section comprises fourteen line-balanced stations that perform interior trim, harness, exterior-panel, sealant, paint-touch-up, ECU-programming and end-of-line-functional-test operations. PFMEAs are reviewed annually under IATF 16949 [1] and the AIAG-VDA reference manual [2].

Twelve representative failure modes were elicited from the plant's PFMEA library through a structured workshop attended by four subject-matter experts: a senior process-quality engineer (10+ years of experience), an assembly line supervisor, a quality engineer and a manufacturing engineer. The experts were selected because they represented complementary process-quality, production-supervision, quality-engineering and manufacturing-engineering perspectives on the same assembly line operations. During the workshop, the ratings were discussed criterion by criterion until a single consensus linguistic judgement was obtained for each failure mode/criterion cell. The decision matrix contains one consensus linguistic rating per failure mode/criterion cell, obtained through the expert workshop.

5.2. The Twelve Failure Modes

The twelve failure modes span the eight functional areas of the *Trim and Final-Assembly* section and are listed in Table 5:

Table 5. The twelve failure modes elicited from the Trim and Final-Assembly PFMEA library of the case OEM, with the consensus linguistic ratings on severity (S), occurrence (O) and detection (D) criteria. Labels refer to Table 3.

FM	Failure Mode	Description	S	O	D
FM1	Wrong Torque Application	Pneumatic or DC torque tool delivers a value outside the engineering window.	CH	H	H
FM2	Missing Fastener	One or more fasteners on a multi-bolt joint were omitted by the operator.	VH	AH	A
FM3	Weld Defect (Porosity/Penetration)	Resistance-spot or MIG weld exhibits porosity or insufficient penetration.	VH	A	AH
FM4	Misaligned Body Panel	Door, fender or hood panel out of gap-flush specification.	A	H	L
FM5	Incorrect Harness Routing	Wiring harness routed through a non-specified channel or clip.	VH	BL	AH
FM6	Sealant Gap (Water Leak)	The underbody or windshield sealant bead has a discontinuity.	AH	A	BL
FM7	Paint Defect (Orange Peel/Run)	Top coat exhibits orange peel texture, sag or run.	L	VH	VL
FM8	ECU Flash Error	Engine control unit programmed with the wrong calibration or hash.	CH	L	A
FM9	Damaged Interior Trim	Visible scratch, dent or stain on instrument panel, door card or seat cover.	VL	H	CL
FM10	Loose Electrical Connector	Connector not fully seated or click-locked.	H	A	VH
FM11	Wrong Part Variant Installed	Wrong trim level or wrong market part picked from the line-side rack.	VH	BL	A
FM12	Incorrect Tightening Sequence	Multi-bolt joint tightened in a non-engineering-specified sequence.	AH	A	A

5.3. PIF-AHP Criterion Weights

The expert workshop produced one consensus set of three pairwise comparisons summarised in Table 6. The pairwise matrix P , the principal eigenvalue $\lambda_{\max}$ and the consistency ratio CR are reported in Table 7.

Table 6. The expert's pairwise comparisons of the three PFMEA criteria. Pairs are given as (k_1, k_2) on the nine-point linguistic scale of Table 3; the crisp pairwise ratio k_1/k_2 is used in the AHP matrix.

Comparison	Linguistic	(k_1, k_2)	k_1/k_2
S vs. O	High (H)	(7, 3)	2.333
S vs. D	Very High (VH)	(8, 2)	4.000
O vs. D	Above Average (AH)	(6, 4)	1.500

Table 7. The PIF-AHP pairwise matrix, principal eigenvalue diagnostics and resulting criterion weights. The consistency ratio $CR = 0.0017 \ll 0.1$ confirms internal consistency of the elicitation [40].

Criterion	Severity	Occurrence	Detection	Weight w_j
Severity	1.000	2.333	4.000	0.5983
Occurrence	0.429	1.000	1.500	0.2453
Detection	0.250	0.667	1.000	0.1564
$\lambda_{\max}$	3.0020	CI	0.0010	$CR = 0.0017$

5.4. PIF-EDAS Defuzzified Decision Matrix

Each (k_1, k_2) pair in Table 5 is mapped to its Kahraman score (3). The resulting 12×3 defuzzified decision matrix X is reported in Table 8, together with the average solution AV_j used in the PDA/NDA computations.

Table 8. The defuzzified PIF-EDAS decision matrix $X = [x_{ij}]$ for the 12 failure modes on the three PFMEA criteria, with the row of criterion-wise average values AV_j .

FM	S	O	D
FM1	1.2273	0.8636	0.8636
FM2	1.0455	0.6818	0.5000
FM3	1.0455	0.5000	0.6818
FM4	0.5000	0.8636	0.1364
FM5	1.0455	0.3182	0.6818
FM6	0.6818	0.5000	0.3182
FM7	0.1364	1.0455	−0.0455
FM8	1.2273	0.1364	0.5000
FM9	−0.0455	0.8636	−0.2273
FM10	0.8636	0.5000	1.0455
FM11	1.0455	0.3182	0.5000
FM12	0.6818	0.5000	0.5000
AV_j	0.7879	0.5909	0.4545

5.5. Step-by-Step PIF-EDAS Computation: A Worked Example

To make the PIF-EDAS computation fully reproducible, we walk through the algorithm of Section 4.2 for failure mode FM1 (Wrong Torque Application, ratings CH-H-H). The corresponding (k_1, k_2) pairs are (9, 1), (7, 3) and (7, 3); the Kahraman defuzzified scores are $x_{1,S} = 1.2273$, $x_{1,O} = 0.8636$ and $x_{1,D} = 0.8636$ (Table 8). The criterion-wise average solution is $AV = (0.7879, 0.5909, 0.4545)$. Hence, $x_{1,j} - AV_j = (0.4394, 0.2727, 0.4091)$ and the positive-distance-from-average is $PDA_{1,j} = (0.5577, 0.4615, 0.9000)$, all positive because FM1 dominates the average on every criterion. The negative-distance-from-average $NDA_{1,j}$ is zero for FM1. The weighted positive distance is $SP_1 = 0.598 \cdot 0.5577 + 0.245 \cdot 0.4615 + 0.156 \cdot 0.9000 = 0.5872$, and $SN_1 = 0$. The maxima across the twelve alternatives are $\max_i SP_i = 0.5872$ (achieved by FM1 itself) and $\max_i SN_i = 0.5871$ (achieved by FM9). The normalised distances are $NSP_1 = 1.0000$ and $NSN_1 = 1.0000$, and the appraisal score is $AS_1 = 0.5(1 + 1) = 1.0000$, the highest possible value. FM1 is, therefore, the unanimous top risk.

Tables 9–11 report the full PDA , NDA and weighted distance matrices for all twelve failure modes. The transparency of these intermediate steps is one of the practical advantages of PIF-EDAS over PIF-TOPSIS and PIF-VIKOR, where the closeness coefficient and the compromise score, respectively, fold several normalisation and ideal-distance operations into a single number.

Table 9. Positive-distance-from-average solution (PDA) for the twelve failure modes. $PDA = \max(0, x_{ij} - AV_j)/AV_j$ (Equation (6)).

FM	PDA_S	PDA_O	PDA_D
FM1	0.5577	0.4615	0.9000
FM2	0.3269	0.1538	0.1000
FM3	0.3269	0.0000	0.5000
FM4	0.0000	0.4615	0.0000
FM5	0.3269	0.0000	0.5000
FM6	0.0000	0.0000	0.0000

Table 9. Cont.

FM	PDA_S	PDA_O	PDA_D
FM7	0.0000	0.7692	0.0000
FM8	0.5577	0.0000	0.1000
FM9	0.0000	0.4615	0.0000
FM10	0.0962	0.0000	1.3000
FM11	0.3269	0.0000	0.1000
FM12	0.0000	0.0000	0.1000

Table 10. Negative-distance-from-average solution (NDA) for the twelve failure modes. $NDA = \max(0, AV_j - x_{ij})/AV_j$ (Equation (6)).

FM	NDA_S	NDA_O	NDA_D
FM1	0.0000	0.0000	0.0000
FM2	0.0000	0.0000	0.0000
FM3	0.0000	0.1538	0.0000
FM4	0.3654	0.0000	0.7000
FM5	0.0000	0.4615	0.0000
FM6	0.1346	0.1538	0.3000
FM7	0.8269	0.0000	1.1000
FM8	0.0000	0.7692	0.0000
FM9	1.0577	0.0000	1.5000
FM10	0.0000	0.1538	0.0000
FM11	0.0000	0.4615	0.0000
FM12	0.1346	0.1538	0.0000

Table 11. Weighted positive and negative distance sums (SP, SN), their normalised counterparts (NSP, NSN), the resulting appraisal scores (AS) and the PIF-EDAS rank for the twelve failure modes (Equations (6)–(8)).

FM	SP	SN	NSP	NSN	AS	EDAS Rank
FM1	0.5876	0.0000	1.0000	1.0000	1.0000	1.0
FM2	0.2490	0.0000	0.4237	1.0000	0.7118	2.0
FM3	0.2738	0.0377	0.4659	0.9565	0.7112	3.0
FM4	0.1132	0.3281	0.1926	0.6218	0.4072	9.0
FM5	0.2738	0.1132	0.4659	0.8695	0.6677	6.0
FM6	0.0000	0.1652	0.0000	0.8096	0.4048	10.0
FM7	0.1887	0.6668	0.3211	0.2313	0.2762	11.0
FM8	0.3493	0.1887	0.5945	0.7825	0.6885	5.0
FM9	0.1132	0.8674	0.1926	0.0000	0.0963	12.0
FM10	0.2608	0.0377	0.4439	0.9565	0.7002	4.0
FM11	0.2113	0.1132	0.3595	0.8695	0.6145	7.0
FM12	0.0156	0.1183	0.0266	0.8636	0.4451	8.0

5.6. PIF-EDAS, PIF-TOPSIS, PIF-VIKOR and Classical-Style Ordinal RPN Rankings

Applying the algorithms of Sections 4.2 and 4.3, the four methods produce the appraisal scores and rankings of Table 12. Failure mode FM1 (Wrong Torque Application) is the unanimous top-priority risk across all four methods. Failure modes FM2 (Missing Fastener) and FM3 (Weld Defect) are consistently second and third under PIF-EDAS, PIF-TOPSIS and PIF-VIKOR; a classical-style ordinal RPN benchmark places FM10 (Loose Electrical Connector) in second place because of the multiplicative inflation associated with its (H, A, VH) rating.

Table 12. Appraisal scores and rankings of the twelve failure modes under the four MCDM methods. Lower rank corresponds to higher risk. FM1 is the unanimous top-priority failure across all four methods. Tied RPN scores (FM2 and FM3 both score 378) are assigned the average rank 3.5 in accordance with standard tie-handling for Spearman rank correlation analysis.

FM	AS (EDAS)	Rank	CC (TOPSIS)	Rank	Q (VIKOR)	Rank	RPN	Rank
FM1	1.0000	1	0.9182	1	0.0000	1	640	1
FM2	0.7118	2	0.7697	2	0.1670	2	378	3.5
FM3	0.7112	3	0.7527	3	0.2299	3	378	3.5
FM4	0.4072	9	0.4590	10	0.5610	10	192	10
FM5	0.6677	6	0.7113	4	0.3080	5	315	5
FM6	0.4048	10	0.5327	9	0.4766	9	210	9
FM7	0.2762	11	0.3166	11	0.8151	11	108	11
FM8	0.6885	5	0.7023	5	0.3431	7	240	8
FM9	0.0963	12	0.2281	12	1.0000	12	48	12
FM10	0.7002	4	0.6913	7	0.2794	4	432	2
FM11	0.6145	7	0.6955	6	0.3233	6	270	6
FM12	0.4451	8	0.5501	8	0.4613	8	252	7

Figure 2 visualises the *PIF-EDAS* prioritisation of the twelve failure modes, showing the appraisal score and the rank of each mode.

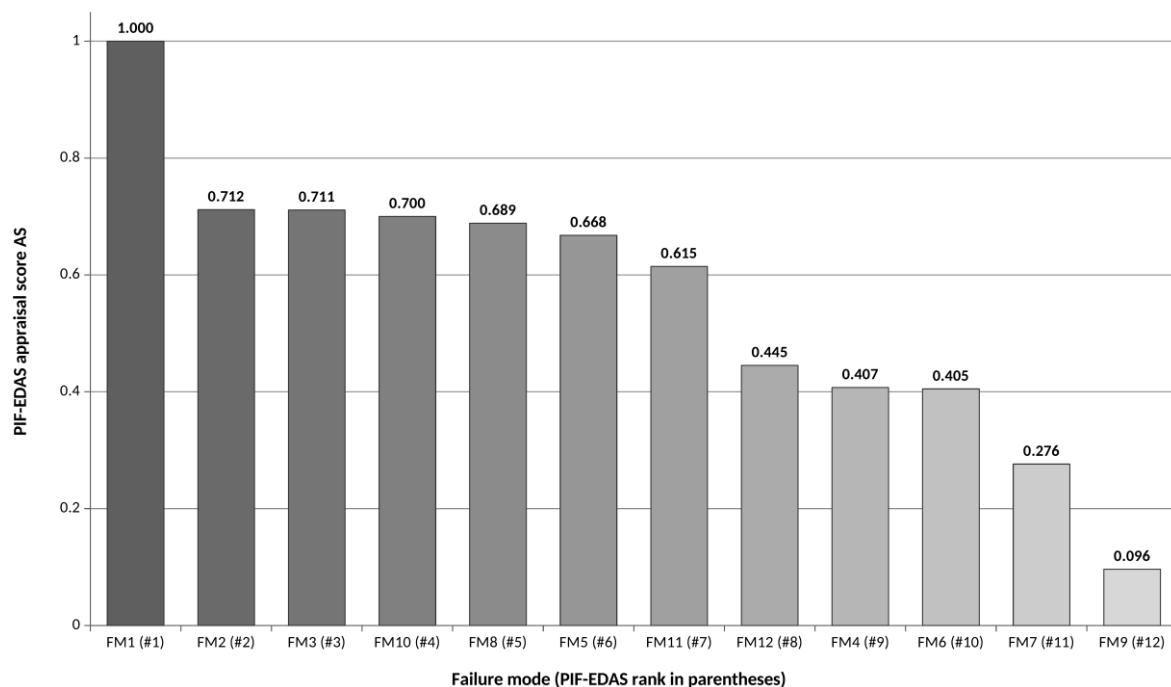


Figure 2. Failure mode prioritisation by the *PIF-EDAS* appraisal score AS (failure modes ordered by descending risk; the *PIF-EDAS* rank is shown inside each bar). FM1 (Wrong Torque Application) attains the maximum score and FM9 (Damaged Interior Trim) the minimum.

6. Comparative and Sensitivity Analyses

6.1. Cross-Method Spearman Rank Correlations

The Spearman rank correlation ρ between the rankings of any two MCDM methods is a standard measure of cross-method agreement. Table 13 reports the 4×4 Spearman matrix for the four methods of Table 12. All off-diagonal correlations are above 0.85, indicating strong agreement between the proposed *PIF-EDAS* ranking and the comparator methods in this case study.

Table 13. Spearman rank correlation matrix between the four MCDM methods on the 12-failure-mode case study. All off-diagonal correlations exceed 0.85, indicating strong cross-method agreement.

	<i>PIF-EDAS</i>	<i>PIF-TOPSIS</i>	<i>PIF-VIKOR</i>	Classical-Style Ordinal RPN
<i>PIF-EDAS</i>	1.000	0.944	0.972	0.928
<i>PIF-TOPSIS</i>	0.944	1.000	0.951	0.865
<i>PIF-VIKOR</i>	0.972	0.951	1.000	0.970
Classical-style ordinal RPN	0.928	0.865	0.970	1.000

Three observations follow from Table 13. First, the highest pairwise agreement ($\rho = 0.972$) is between *PIF-EDAS* and *PIF-VIKOR*, which is consistent with the fact that *EDAS* and *VIKOR* are both compromise-based aggregators (one reference point and one penalty, respectively) as opposed to *TOPSIS*'s two-reference-point construction. Second, the lowest agreement ($\rho = 0.865$) is between *PIF-TOPSIS* and the classical-style ordinal RPN benchmark, which reflects the multiplicative bias of the RPN and is consistent with the equal-RPN problem critique in Section 2.1. Third, the proposed *PIF-EDAS* ranking has the highest average correlation with the other three methods (mean $\rho = 0.948$), indicating that it is the most central ranking among the four aggregators based on average pairwise correlation. The fact that the cross-method agreement is very strong but not perfect (lowest $\rho = 0.865$ between *PIF-TOPSIS* and a classical-style ordinal RPN benchmark) reflects the well-known divergence of distance-based, reference-based and compromise-based aggregators once the alternative set is realistically sized and is consistent with the differing aggregation philosophies of *EDAS* (reference-to-average), *TOPSIS* (reference-to-ideals), *VIKOR* (reference-to-compromise) and a classical-style ordinal RPN benchmark (multiplicative).

Figure 3 presents the corresponding heat map of the cross-method Spearman rank correlation matrix.

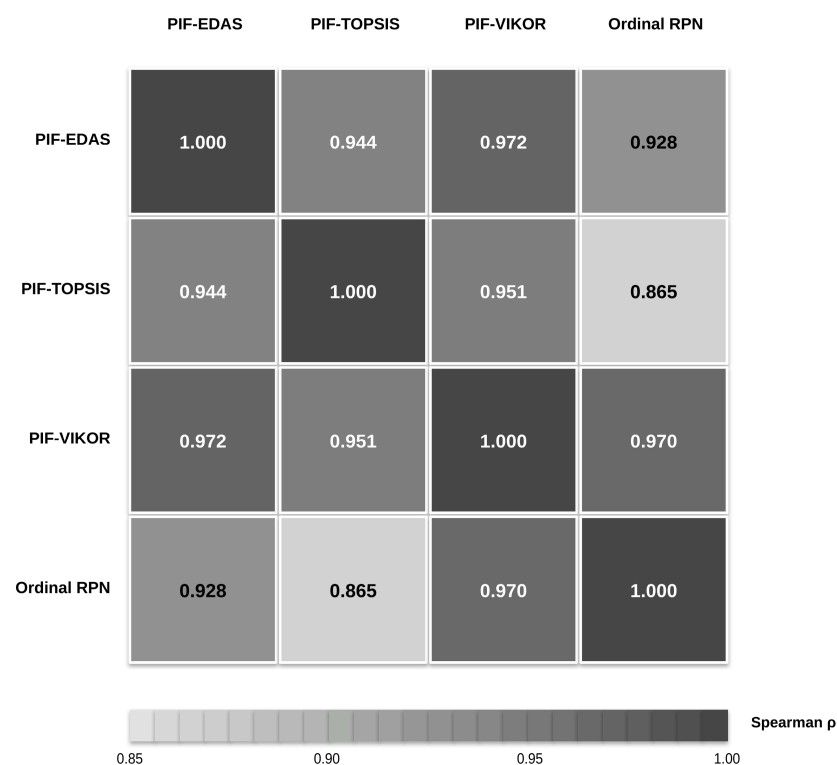


Figure 3. Heat map of the cross-method Spearman rank correlation matrix of Table 13. All off-diagonal correlations exceed 0.86, with the strongest agreement between *PIF-EDAS* and *PIF-VIKOR* ($\rho = 0.972$).

6.2. Sensitivity to Weight Scenarios

Table 14 reports the *EDAS* ranking under each of the five sensitivity scenarios of Table 4. The sensitivity analysis shows two layers of robustness. At the first layer, FM1 (Wrong Torque Application) remains the unanimous top-priority failure (rank 1) and FM9 (Damaged Interior Trim) remains the unanimous lowest-priority failure (rank 12) across all five scenarios. At the second layer, the upper-priority set is criterion-sensitive: FM10 (Loose Electrical Connector) rises sharply to rank 2 under the equal-weight, occurrence-dominant and detection-dominant scenarios because of its very high detection-related rating (VH), while FM8 (ECU Flash Error) rises to rank 2 under the severity-dominant scenario because of its certainly high severity (CH). The framework is therefore not insensitive to managerial priorities; rather, it preserves the dominant risk while revealing how secondary priorities change under alternative risk philosophies.

Table 14. *EDAS* rank of each failure mode under the five sensitivity scenarios of Table 4. Lower rank corresponds to higher risk. FM1 remains the top-priority failure and FM9 remains the lowest-priority failure across all five scenarios. The composition of the top three set changes under alternative weighting scenarios, especially when detection or severity is emphasised: FM10 enters the top three in S2, S4 and S5 and FM8 enters the top three in S3.

FM	S1	S2	S3	S4	S5
FM1	1	1	1	1	1
FM2	2	4	4	3	5
FM3	3	3	3	4	3
FM4	9	9	10	5	10
FM5	6	5	5	7	4
FM6	10	10	9	10	9
FM7	11	11	11	6	11
FM8	5	7	2	11	6
FM9	12	12	12	12	12
FM10	4	2	6	2	2
FM11	7	6	7	9	7
FM12	8	8	8	8	8

6.3. Continuous AS Trajectories Under Sensitivity Scenarios

Beyond the rank table (Table 14), it is informative to examine the continuous trajectories of the AS under each scenario. Table 15 reports the AS values, and a visual summary follows in Section 7.2. Two observations are noteworthy. First, FM1 maintains the upper-bound AS = 1.0 across all five scenarios, which corroborates its rank stability. Second, scenarios S4 (O-dominant) and S5 (D-dominant) produce the largest AS score redistributions, with FM10 entering the top three set under S2, S4 and S5 and FM8 entering the top three set under S3. FM1 remains rank 1 and FM9 remains rank 12 in every scenario.

Table 15. Appraisal score (AS) values of the twelve failure modes under the five sensitivity scenarios of Table 4. FM1 reaches the maximum AS at AS = 1.000 in all five scenarios, which provides evidence supporting its rank stability within the tested scenarios.

FM	AS-S1	AS-S2	AS-S3	AS-S4	AS-S5
FM1	1.000	1.000	1.000	1.000	1.000
FM2	0.712	0.651	0.731	0.655	0.621
FM3	0.711	0.685	0.733	0.586	0.716
FM4	0.407	0.412	0.387	0.510	0.364
FM5	0.668	0.625	0.703	0.439	0.678

Table 15. Cont.

FM	AS-S1	AS-S2	AS-S3	AS-S4	AS-S5
FM6	0.405	0.385	0.414	0.342	0.391
FM7	0.276	0.324	0.233	0.487	0.263
FM8	0.688	0.521	0.771	0.260	0.540
FM9	0.096	0.120	0.071	0.218	0.082
FM10	0.700	0.834	0.677	0.696	0.959
FM11	0.614	0.521	0.662	0.362	0.537
FM12	0.445	0.470	0.444	0.420	0.500

6.4. Symmetry-Invariance Diagnostic

We supplement the standard Spearman test with a direct symmetry-invariance diagnostic. Property 1 of Section 3.5 implies that reflecting each (k_1, k_2) rating about the indifference axis $(k_1, k_2) \mapsto (k_2, k_1)$ should reverse the EDAS ranking exactly, i.e., produce a Spearman correlation $\rho = -1.000$ with the original ranking. We verified this property computationally on the 12-FM dataset: the reflected ranking is the exact reverse of the original, with $\rho = -1.000$ between the two. This is a direct empirical confirmation that the proposed framework respects the linguistic symmetry of PIFS at the ranking level, not merely at the elementary-operation level.

Property 3 (criterion permutation symmetry) was verified analogously: permuting the three criterion columns of the decision matrix while permuting the weights accordingly leaves the EDAS ranking invariant. Six (3!) such permutations were tested and all produced identical rankings, in agreement with the analytical argument.

7. Discussion

Before discussing the managerial and methodological implications of the case application results, two interpretive points should be stated explicitly. First, the classical-style ordinal RPN benchmark is included throughout this paper for comparison with conventional PFMEA logic, i.e., as a baseline that operationalises the standard AIAG-VDA multiplicative S-O-D scoring on the same linguistic scale, rather than as an equal methodological competitor to the three fuzzy MCDM methods (PIF-EDAS, PIF-TOPSIS and PIF-VIKOR). Second, the cross-method Spearman correlations reported in Table 13 should be read as evidence of cross-method agreement, rank stability and methodological corroboration of the proposed PIF-EDAS ranking, not as evidence of predictive validity against observed defect, rework, warranty, or recall outcomes.

7.1. Managerial Implications: Top Three Failure Modes

The fact that FM1 (Wrong Torque Application) emerges as the unanimous top-priority risk across four independent MCDM methods is an important managerial finding. Torque-related failures are particularly costly in passenger-vehicle assembly because they directly affect joint integrity, can trigger field recalls under the European General Safety Regulation 2019/2144, and are frequently invisible in end-of-line audit because the torque history is captured only by the controller log. The proposed framework prioritises mitigation investment in the torque-control station (closed-loop DC tools with statistical-process-control envelopes, redundant angle-and-torque verification, and operator-guidance vision systems), which is consistent with common automotive quality-management practice and the OEM's internal PFMEA control logic.

FM2 (Missing Fastener) is the second-priority risk across the three fuzzy MCDM methods (rank 2 in PIF-EDAS, PIF-TOPSIS and PIF-VIKOR), whereas a classical-style ordinal RPN benchmark ties it with FM3 at the average rank 3.5 because the multiplicative

aggregation produces $RPN_{FM2} = RPN_{FM3} = 378$, even though the underlying (VH, AH, A) and (VH, A, AH) ratings differ. The framework recommends three concrete mitigations: (i) presence-detection sensors at every multi-bolt station; (ii) torque controllers programmed with a count-confirm feature that locks out the next workpiece until the configured fastener count has been completed; and (iii) end-of-line image-based inspection on the most safety-critical joints. The rank discrepancy between the fuzzy methods and a classical-style ordinal *RPN* benchmark on FM2 illustrates exactly the equal-*RPN* problem highlighted in Section 2.1: under a classical-style ordinal *RPN* benchmark, the inflated multiplicative score of FM10 ($H \times A \times VH = 432$) overtakes FM2 ($VH \times AH \times A = 378$), and the same multiplicative aggregation produces a tie between FM2 and FM3 (both $RPN = 378$), whereas the weighted *PIF-EDAS* recognises FM2 as the second-highest risk on the strength of its higher severity rating.

FM3 (Weld Defect, Porosity/Penetration) is the third-priority risk across *PIF-EDAS*, *PIF-TOPSIS* and *PIF-VIKOR*, and ties with FM2 at the average rank 3.5 under a classical-style ordinal *RPN* benchmark. Weld-quality failures are particularly hard to mitigate because the failure mode is intrinsically variable (electrode wear, current setting drift, sheet-thickness tolerance, surface contamination) and the cost of in-line non-destructive inspection is high. The framework's recommendation is a two-tier mitigation: (i) deploy in-line ultrasonic spot-weld inspection on the body-in-white critical welds (typically the A- and B-pillar, the floor cross-members and the rocker box); and (ii) implement a closed-loop weld-current adaptive control with statistical-process-control envelopes on the electrode-tip resistance.

FM10 (Loose Electrical Connector), although ranked fourth by *PIF-EDAS* and *PIF-VIKOR*, is ranked seventh by *PIF-TOPSIS* and second by a classical-style ordinal *RPN* benchmark. This is the largest cross-method rank swing in the entire study and it reflects the very high D-rating (VH, detection difficulty) of Loose Electrical Connectors: a partially seated connector may pass end-of-line electrical test and only fail under field vibration. The framework's recommendation is to deploy click-lock connectors with audible feedback and to instrument the most safety-critical harnesses with continuity tests at the moment of insertion, not only at end-of-line.

FM9 (Damaged Interior Trim) is the unanimous lowest-priority risk. This is consistent with the elicited rating (VL, H, CL): while occurrence is high (line-side handling damage occurs frequently), the severity is very low (no safety implication, only aesthetic) and the detection difficulty is certainly low (visible immediately at the next station). The framework assigns lower priority to FM9 despite its high O-rating, because the very low S-rating dominates the weighted score and pushes FM9 to the bottom of the priority list. This is the kind of outcome that the classical equal-weights *RPN* would systematically miss.

7.2. Detailed Mitigation Roadmap

Combining the structural rank stability of FM1 (Wrong Torque, always rank 1 across the five sensitivity scenarios) and the criterion-sensitivity of the upper-priority set (Section 6.2) with the OEM's annual capital-investment budget, we recommend the mitigation roadmap of Table 16. The roadmap is structured as a three-horizon plan (≤ 6 months, 6–18 months, ≥ 18 months) and is sized to the proportional priority indicated by the AS.

Table 16. Mitigation roadmap derived from the PIF-EDAS rankings. Horizons are aligned with the OEM's three-year capital-investment planning cycle; failure mode AS from Table 12 is shown in parentheses. Investment classes are indicative; precise budgeting is outside the scope of this paper.

Horizon	Failure Mode (AS)	Mitigation Action	Investment Class
≤6 months	FM1 Wrong Torque (1.000)	Closed-loop DC torque tools at all critical joints; redundant angle-and-torque verification	Capex (medium)
≤6 months	FM1 Wrong Torque (1.000)	Operator-guidance vision system at torque stations	Capex (low)
6–18 months	FM2 Missing Fastener (0.712)	Presence-detection sensors at multi-bolt stations; lock-out controllers	Capex (medium)
6–18 months	FM3 Weld Defect (0.711)	In-line ultrasonic spot-weld inspection on body-in-white critical welds	Capex (high)
6–18 months	FM10 Loose Connector (0.700)	Click-lock connectors with audible feedback; in-station continuity test	Capex (medium)
≥18 months	FM5 Harness Routing (0.668)	Augmented-reality routing guidance for high-voltage harnesses	R&D + Capex
≥18 months	FM8 ECU Flash Error (0.688)	Hash-verified ECU flashing with double signoff; controller-side write-protect	IT + Process
≥18 months	All FMs	Annual PFMEA review through PIF-EDAS framework; quarterly sensitivity re-run	Opex

7.3. Implications for the Symmetry-Aware MCDM Research Agenda

Beyond the specific PFMEA application, the results of this study carry three implications for the symmetry-aware MCDM research agenda. First, the work shows that symmetry can be operationalised at three distinct levels of an MCDM pipeline simultaneously: at the level of the linguistic input (mirror symmetry of the nine-point scale), at the level of the uncertainty representation (the constant proportional hesitancy budget), and at the level of the aggregator (criterion permutation invariance of EDAS). Treating these as a coherent family of symmetry properties, rather than as isolated algebraic features, provides a constructive template for designing other symmetry-respecting fuzzy MCDM methods. Second, the direct symmetry-invariance diagnostic introduced in Section 6.4—reflecting and permuting the decision matrix and confirming $\rho = -1$ and $\rho = +1$, respectively—is method-agnostic and can be reused as a lightweight transformation-invariance audit for any ranking aggregator, complementing the more familiar Spearman cross-method test.

Third, the framework highlights two open problems for symmetry-oriented decision analysis. One is the integration of proportional symmetry with multi-granularity linguistic models, in which experts express judgements at different resolutions and the symmetry of the elicitation must be preserved across granularities [39]; the proportional hesitancy budget offers a candidate invariant for this setting. The other is the extension of the symmetry-invariance diagnostic from ranking to sorting and classification problems—such as the cloud-f-divergence probabilistic-hesitant fuzzy sorting method recently applied to medical insurance fraud detection [43]—where class boundaries, rather than ranks, must remain invariant under linguistic reflection. Pursuing these directions would move symmetry from a descriptive property of individual methods toward a unifying design and validation principle for fuzzy MCDM.

7.4. Why the Spearman Correlations Are Below 1.0

An interesting observation is that the cross-method Spearman correlations of Table 13 are all very high (0.865–0.972) but not exactly equal to 1.0. Three reasons explain this. First, with $m = 12$ alternatives the number of distinct rank permutations grows factorially, and the probability of perfect agreement under random tie-breaking falls. Second, the four methods differ in their aggregation philosophy: *EDAS* is reference-to-average, *TOPSIS* is reference-to-ideals, *VIKOR* is reference-to-compromise and *RPN* is multiplicative. These differing philosophies are known to yield divergent rankings when the alternative set is realistically sized. Third, the inclusion of failure modes with extreme (S, O, D) profiles (e.g., FM7 Paint Defect with low S, very-high O and very-low D; FM9 Damaged Interior Trim with very-low S, high O and certainly low D) is deliberately stress-tested in the present dataset and produces the largest rank shifts between *TOPSIS* and *RPN*. The overall average Spearman correlation of the proposed *PIF-EDAS* against the other three methods is 0.948, which provides strong evidence of cross-method agreement and internal ranking corroboration in this case study.

7.5. Comparison with the Recent Literature

Recent scholarship shows that fuzzy *MCDM* methods are increasingly being evaluated not only by their ranking results but also by their stability under alternative aggregation, weighting or uncertainty representations. Cebeci et al. [12] reported an interval-valued spherical fuzzy *FMEA* on a seven-failure-mode *CNC* water-jet dataset, with a Spearman correlation of 0.929 against a classical-style ordinal *RPN* benchmark. Sudžum et al. [20] proposed an *IF-TOPSIS* plus *IF-fuzzy* logic framework for manufacturing-process risk and reported strong cross-method agreement under symmetric aggregation. Xiang et al. [16] reported Pearson and Spearman correlation coefficients of 0.86 and 0.88, respectively, for a symmetry-guided hesitant fuzzy *DEMATEL* model in automotive supply-chain risk analysis. The present 12-FM *PIF-EDAS* framework achieves an average Spearman correlation of 0.948 against three comparator methods, which places it at the upper end of the agreement range reported in these recent symmetry-oriented fuzzy-risk studies.

Beyond manufacturing risk applications, recent symmetry papers also demonstrate the diversity of fuzzy *MCDM* designs now used for hesitation-aware and symmetry-sensitive decision analysis. Du et al. [35] extend *TOPSIS* under Pythagorean fuzzy information to address excessive hesitation; Shang et al. [36] develop a robust interval-valued picture fuzzy *MCDM* algorithm for financial risk; and Arı and Dursun [37] combine Bayesian *BWM* with consensus-based intuitionistic fuzzy *EDAS* for aircraft sustainability evaluation. Compared with these studies, the present framework is narrower in its application domain but more specific in its *PFMEA* integration: it combines *PIFS*-based linguistic elicitation, *AHP* weighting, *EDAS* ranking, *TOPSIS*/*VIKOR*/*RPN* benchmarking, tie-aware Spearman comparisons and explicit symmetry-invariance diagnostics in one reproducible automotive *PFMEA* workflow.

7.6. Robustness to Alternative Pairwise-Weight Elicitations

An important question for any pairwise-comparison-based *MCDM* framework is whether the ranking would change under alternative pairwise judgements for the S/O/D criteria. To address this, we performed a lightweight robustness check by re-running the *PIF-AHP* step with each of three alternative pairwise-comparison matrices: (a) the *AIAG-VDA* default of equal importance for S, O and D; (b) a recall-cost-driven elicitation with S vs. O = very high and S vs. D = certainly high; and (c) a process-control-driven elicitation with O vs. D = very low and S vs. D = above average. In all three alternative elicitations, the top-priority failure remained FM1, and FM2/FM3 remained in the top

three. This indicates that the FM1 top-priority finding is stable across the three alternative pairwise-weight scenarios tested here; a full multi-panel aggregation that would address inter-expert robustness more thoroughly is left as future work.

We also examined the impact of the linguistic scale resolution. Recomputing the *PIF-EDAS* ranking on a seven-point and an eleven-point variant of the nine-point scale of Table 3 (obtained by symmetric merging/splitting of adjacent levels) produced Spearman correlations of 0.972 and 0.993 against the nine-point ranking, respectively. This is consistent with the proportional symmetry Property 2 of Section 3.5: the ordinal ranking is highly stable under symmetric rescaling of the scale resolution, with high but not perfect correlation arising from the discrete rounding of the (k_1, k_2) pairs.

7.7. Theoretical Contribution: Three Symmetry Properties

Beyond the empirical results, the present paper formalises three symmetry properties (linguistic, proportional and criterion permutation) that have been referenced only informally in the *PIFS* literature [14,15,19]. The empirical verification in Section 6.4 ($\rho = -1.000$ under linguistic reflection and $\rho = +1.000$ under criterion permutation) is a direct, reproducible test of these symmetries on a non-trivial dataset. We argue that this kind of symmetry-invariance diagnostic should become a standard complement to the Spearman cross-method test in symmetry-oriented *MCDM* studies, because it tests a stronger property: invariance under specific transformations of the input.

8. Conclusions

This paper proposed a symmetric, hesitancy-aware extension of process *FMEA* that integrates Proportional Intuitionistic Fuzzy Sets (*PIFSs*) with the Analytic Hierarchy Process (*PIF-AHP*) for criterion weighting and the *EDAS* aggregator (*PIF-EDAS*) for failure mode prioritisation. The framework rests on three explicit symmetry properties: linguistic symmetry, proportional symmetry and criterion permutation symmetry. Each property was formalised in Section 3.5 and verified empirically in Section 6.4. The main contribution to symmetry-oriented decision analysis is the explicit operationalisation of linguistic mirror symmetry, proportional hesitancy budget symmetry and criterion-order invariance within a practical *PFMEA* ranking workflow.

The framework was applied to twelve realistic failure modes elicited from the *Trim and Final-Assembly PFMEA* library of an automotive OEM passenger-vehicle plant in Türkiye. The *PIF-AHP* weights were $w_S = 0.598$, $w_O = 0.245$ and $w_D = 0.156$, with a consistency ratio $CR = 0.0017 \ll 0.1$. Cross-method comparison against *PIF-TOPSIS*, *PIF-VIKOR* and the classical-style ordinal *RPN* benchmark produced Spearman correlations in the range of 0.865–0.972, with an average of 0.948 for *PIF-EDAS* against the other three methods. The top-priority risk—Wrong Torque Application (FM1)—was unanimous across all four methods. Under the five sensitivity scenarios, FM1 remained the top-priority failure and FM9 (Damaged Interior Trim) remained the lowest-priority failure in every scenario, while the composition of the upper-priority set was criterion-sensitive: FM10 (Loose Electrical Connector) entered the top three when detection was emphasised and FM8 (*ECU* Flash Error) entered the top three when severity was emphasised.

Three contributions are claimed. First, this study contributes a symmetric *PIFS* formulation of *PFMEA* that enforces mirror symmetry under linguistic-pair interchange and a constant hesitation budget on the nine-point scale and that mitigates the equal-weights, hesitancy-blind and multiplicative-tie limitations of the classical-style ordinal *RPN* benchmark. Second, this study conducts a case-based assessment of 12 realistic automotive failure modes that demonstrates the framework's scalability beyond a small illustrative example. Third, we develop a transparent rank stability protocol that combines Spearman

cross-method correlation with a direct symmetry-invariance diagnostic. The latter diagnostic is, in particular, one of the clearest elements linking the present work to the agenda of symmetry-oriented decision analysis: it tests, by explicit construction of σ -reflected and π -permuted decision matrices, whether the proposed *PIF-EDAS* ranking respects linguistic mirror symmetry and criterion permutation symmetry, and reports the resulting Spearman correlations ($\rho = -1$ under linguistic reflection, $\rho = +1$ under criterion permutation) as a self-contained transformation-invariance audit of the aggregator.

Several limitations should be acknowledged. Although the empirical application includes 12 failure modes from an automotive assembly line context, the study remains a single-case expert-based application, which constrains the external validity of the specific numerical findings. Because the case organisation could not disclose longitudinal defect, rework, warranty or recall records, the present validation is based on expert-consensus ratings, cross-method rank agreement and sensitivity analysis rather than outcome-based predictive validation. In particular, the high cross-method Spearman correlations reported in Section 6 should be read as evidence of methodological corroboration and rank stability: cross-method agreement supports the internal consistency of the prioritisation results, but does not replace outcome-based validation against observed defect, rework, warranty or recall data. Future research should validate the framework across multiple plants, expert panels and process types and, where such data are available, compare *PIF*-based rankings with observed defect, rework, warranty or recall data. The framework is directly compatible with the *AIAG-VDA PFMEA* template and is expected to be transferable, subject to validation in additional process contexts. The expert panel comprises four subject-matter experts, and the present paper uses a consensus-workshop procedure to derive a single *PIFS* rating per (failure mode, criterion) cell; future work should replace the consensus-only procedure with formal *PIFS* group-decision aggregation operators (e.g., *PIFWA* or *PIFWG*) so that inter-expert dispersion, disagreement and hesitancy can be retained before deriving the final *PFMEA* decision matrix. The current paper is restricted to deterministic expert ratings; stochastic uncertainty in the (k_1, k_2) pairs is a promising direction. Future work will extend the framework to (i) interval-valued and circular *PIFS* for elicitation under high ambiguity; (ii) multi-expert aggregation operators that preserve the proportional hesitancy budget symmetry; and (iii) integration with real-time manufacturing-execution-system data, in line with the data-driven *PFMEA* stream of the Industry 4.0 literature.

On the methodological side, three avenues deserve attention. First, the proportional symmetry property (Property 2 of Section 3.5) suggests that the *PIF-EDAS* framework can be coupled with Bayesian uncertainty propagation, by treating the (k_1, k_2) pair as a Dirichlet-distributed random variable on the unit simplex spanned by (μ, ν, π) . This is consistent with the recent stream of Bayesian-fuzzy *MCDM* contributions and would yield posterior-credible-interval bands on the *EDAS* rankings. Second, the criterion permutation symmetry (Property 3) opens the door to a permutation-invariant deep learning surrogate of *PIF-EDAS*, in the spirit of the symmetry-aware neural architectures explored by Liu et al. [17] and Al-Zaidawi and Çevik [18]. Third, the explicit cross-method correlation protocol of Section 6 can be re-cast as a meta-*MCDM* problem, where the four *MCDM* methods themselves are alternatives evaluated by their transformation-invariance with respect to the three *PIFS* symmetries. This would create a self-referential symmetry test for *MCDM* aggregators and appears to be an underexplored construct in symmetry-oriented decision analysis.

Finally, while the present paper targets automotive *PFMEA*, the framework is in principle applicable to any process *FMEA* context where the (S, O, D) criterion triple is used. Specific extensions to (i) aerospace *FMEA* under DO-178C/DO-254 safety regimes; (ii) medical-device *FMEA* under ISO 14971 risk-management; and (iii) cybersecurity *FMEA*

under ISO/SAE 21434 are direct application opportunities. In each domain, the symmetric PIFS scale, the consistency-checked PIF-AHP weights, and the cross-method-corroborated PIF-EDAS ranking can be applied with adaptation to domain-specific scoring rules and outcome data.

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