

Review

Machine Learning for Multi-Messenger Probes of New Physics and Cosmology: Review and Perspective

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Abstract

The multi-messenger exploration of dark matter and physics beyond the Standard Model has emerged as a central direction in modern astro-particle physics, particularly following the discovery of gravitational waves. In this work, we present a comprehensive review and forward-looking perspective on machine-learning-enhanced multi-messenger approaches, combining information from gravitational waves, cosmic rays, gamma rays, neutrinos, and collider experiments. We summarize the current state of the field, discuss recent methodological developments, and outline a coherent research program aimed at integrating heterogeneous datasets within a unified inference framework. We collaboratively propose a plan for forthcoming analyses aiming at extracting information on the properties and interactions of dark matter, and finally on its genesis, combining multi-messenger astronomy techniques and inputs from laboratory physics. The main objectives planned in this line of research comprise: (i) the multi-messenger analysis of new physics in cosmology, including mainly, but not only, several different models of dark matter; (ii) the phenomenology of new physics signatures in ground-based cosmic rays experiments, with cross-correlation to the corresponding physical, astrophysical and cosmological observations; (iii) the development of machine learning methods for data analysis in ground-based cosmic rays experiments,



Academic Editors: Vasilis K. Oikonomou and Alberto Ruiz-Jimeno

Received: 18 January 2026

Revised: 15 April 2026

Accepted: 22 April 2026

Published: 30 June 2026

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in light of the new physics signatures. We note that several groups have explored the use of multi-messenger observations, including gravitational waves, to probe alternative dark matter candidates. The present work builds on these developments by focusing on the role of machine learning in integrating heterogeneous datasets. We foresee that a cross-fertilizing approach combining the information that arises from very different experimental methodologies will represent the right and successful path to extract information about the very elusive dark matter particles and provide answers to the main questions that are left in fundamental physics.

Keywords: machine learning; multi-messenger astronomy; beyond the standard model; dark matter; high energy cosmic rays; gravitational waves

1. Introduction and General State-of-the-Art

One of the most pressing questions in modern physics concerns the nature of dark matter (DM) and its interplay with gravitational and cosmological observations. Addressing this problem requires combining information from multiple observational channels, including electromagnetic radiation (e.g., CMB measurements by Planck, ACT, Ali-CPT) [1–3], cosmic rays and neutrinos (LHAASO, HAWC, HESS, IceCube) [4–7], collider experiments (LHC, HL-LHC, CEPC) [8–10], and gravitational-wave observations (LIGO–Virgo, LISA, DECIGO, BBO) [11–14], as well as pulsar timing arrays (FAST, SKA, IPTA) [15–17].

The emergence of multi-messenger astrophysics has opened a qualitatively new avenue for probing dark matter and physics beyond the Standard Model. By combining heterogeneous datasets across vastly different energy scales and detection techniques, multi-messenger analyses can significantly enhance sensitivity to non-standard scenarios, particularly those that are difficult to access within single-channel approaches.

Despite rapid progress in recent years, the field remains fragmented: existing studies typically focus on individual observables or limited combinations of datasets, and a systematic framework for consistently integrating multi-messenger information is still lacking.

1.1. Experiments for the Detection of Cosmic Rays and Multi-Messenger Probes for New Physics and Cosmology

We discuss here a novel approach to the multi-messenger analysis of DM models beyond the WIMP miracle paradigm, investigating several possible models provided by the theoretical literature, while resorting to a sharply developed methodology that accounts for machine learning techniques hitherto applied in the literature. The observables for our multi-messenger analysis will be primarily provided by data on cosmic ray spectra, electromagnetic radiation (gamma rays) and on GW that are about to be released either by direct measurements through gravitational interferometers or indirect measurements by radio-telescopes. Indeed, within this vast and puzzling scenario, the direct GW detection, which was finally attained only a few years ago, represents an unprecedented possibility to unveil the micro-physics of DM, providing through current and forthcoming experiments a sizeable amount of information that can be deployed to solve at least some of the current shortcomings. At the same time, data on cosmic ray spectra that will arise from very sensitive experiments like LHAASO [18,19] will enable us to explore energy ranges from 100 GeV to 100 PeV, thus providing further unprecedented possibilities to test these models.

We will then discuss here the possibility to carry out multi-messenger numerical analyses of DM candidates that are either minimal extensions of the Standard Model (SM) of particle physics or arise as more convoluted theoretical frameworks. Among the first examples, we will focus on axion-like particles, the dark photon, the dark atoms, and

several possible instantiations of majoron models. Among the latter models, we will consider supergravity candidates and models that emerge from unification attempts of the fundamental forces. Our purpose will then be to derive tighter constraints than the ones hitherto presented in the literature on the aforementioned models, through their comparison with the sensitivities of GW interferometers and radio telescopes, with the spectrum of primary and secondary gamma rays from the DM candidates decays (satellite experiments), with the possible signatures of Higgs or Higgs-like portals within the same decays (CEPC and collider experiments like Fermi-LAT), and with the refined measurements on the spectra of cosmic rays (low energies in PAMELA and AMS-02, and high energies in LHAASO), on which we will specifically focus.

1.2. State of the Art of Very-High-Energy-Cosmic-Rays (VHECR) Experiments

The discovery, by the IceCube collaboration, of very high-energy (VHE) astrophysical neutrinos [7] and then the very first direct detection of GW by the LIGO-Virgo collaboration [11], marked the dawn of the new field of multi-messenger astronomy, opening a new window to explore the Universe. The experimental synergy recently disclosed by successful ongoing experiments, which is provided by the existence of multiple channels of detection, including the electromagnetic radiation, gravitational radiation, cosmic rays and neutrinos, and which is complemented by the data provided by ground based colliders, is supposed to boost progresses in the understanding of some of the most urgent questions that pertain fundamental physics, and to allow the investigation of several aspects of astro-particle physics and cosmology. Within this framework the Large High Altitude Air Shower Observatory (LHAASO) experiment, built at an altitude of 4410 m in the Sichuan province of China, stands as a high-sensitivity TeV gamma-ray telescope characterized by a wide field of view and high duty cycle. Other notable experiments that will shed light on the physics of cosmic rays include HAWK and HESS.

For instance, the LHAASO experiment is currently measuring, with unprecedented sensitivity, the spectrum, the composition and the anisotropy of cosmic rays, spanning within a range of energies between 10^{12} eV and 10^{18} eV. Furthermore, it is providing a wide aperture (one stereo-radiant), continuously-operated gamma ray telescope that is able to probe the energy range between 10^{11} eV and 10^{15} eV. LHAASO will be continuously surveying the TeV sky for steady and transient sources within the energy range between 100 GeV and 1 PeV. This allows the direct observation of high-energy cosmic ray sources in the range of 100–1000 TeV. While addressing observables of different types, including electronic, muonic and Cherenkov/fluorescence components, LHAASO and the novel generation of cosmic rays experiments will enable the investigation, with very accurate resolution, of the origin, acceleration and propagation of radiation. This will be allowed by detailed measurements of the energy spectrum, composition and anisotropy of the cosmic rays.

The data that have been currently taken and will be further collected are already providing a boost for the studies on active galactic nuclei (AGN), especially on blazars, the majority of AGN detected by gamma-ray telescopes [20,21]. Through its enhanced flux sensitivity and the wide field of view accessible by the experiment, which are unprecedented, especially when compared to other Cherenkov Telescope facilities that are currently acquiring data, a very promising possibility for the search for clear evidence of very high-energy cosmic rays emitted by blazars is expected to be provided. In particular, the search for hard spectra, with energies larger than 10 TeV and emitted either from the extreme blazars, such as 1ES 0229+200, or from nearby blazars, such as Mrk 421, and the search for TeV photons from distant blazars with redshift $z \sim 1$ will provide information to reconstruct the cosmic TeV background and the luminosity function of TeV blazars.

This furnishes an outstanding possibility to understand the origin of ultra-high-energy cosmic rays (UHECR) and PeV neutrinos, and, at the same time, to probe not only the fate of Lorentz symmetries, assessing either their violation or their deformation, but also to unveil the nature of DM by testing possible axion-like particles (ALPs). Furthermore, a plethora of equally unprecedented and rich amounts of information will be provided for the understanding of relativistic jet physics, such as high-energy radiation mechanisms and acceleration of particles, and the determination of extragalactic background light (EBL).

For completeness, we note that each of the experimental facilities mentioned above has an extensive dedicated literature; representative references are provided in the bibliography.

1.3. Perspective and Scope

While multi-messenger analyses and machine learning techniques have both been widely explored, a key limitation of the current state of the field is the absence of a unified inference framework capable of consistently integrating heterogeneous datasets across different observational channels. Existing approaches typically rely on channel-specific analyses or loosely coupled combinations of observables, which limits their ability to extract robust and globally consistent constraints on dark matter models.

The central perspective of this work is that machine learning can provide the missing layer of integration, enabling the construction of joint inference pipelines that operate directly on multi-modal data. In this sense, the goal is not merely to apply machine learning to individual observables, but to develop architectures and methodologies that allow a coherent exploration of theory space across multiple messengers within a single framework.

In particular, we identify a key limitation of current approaches, namely the lack of unified frameworks capable of consistently integrating heterogeneous multi-messenger datasets. We argue that machine learning provides a natural pathway to address this challenge, enabling joint inference across observational channels and opening the possibility of coherent, high-dimensional exploration of dark matter parameter spaces.

Therefore, this manuscript is intended as a comprehensive review and forward-looking perspective. Rather than presenting new quantitative constraints or experimental results, our goal is to systematize existing approaches and outline a coherent research program for multi-messenger analyses of dark matter and beyond-Standard-Model physics using machine learning techniques.

2. Models of Dark Matter

2.1. Dark Matter from Supergravity

The lack of positive evidence for supersymmetry (SUSY) at the LHC energies may reflect the extreme case that the SUSY scale is very high. In such a case, collider searches for SUSY particles would become impossible, but the advantage of such models is the possibility of unifying them with gravity in the framework of supergravity models. Such models can predict a stable superheavy gravitino as a DM candidate as well as provide a physical basis for Starobinsky supergravity, reproducing the Starobinsky inflation scenario on a supergravity basis. Probes for such models are related to their cosmological consequences, which can lead to observable effects, like Primordial Black holes (PBH) and effects of their evaporation or superheavy DM effects in high-energy cosmic rays.

In the minimal Starobinsky–Polonyi $\mathcal{N} = 1$ Supergravity, superheavy gravitinos are produced in decays of the inflaton and Polonyi fields. This framework links the physics of inflation to the physics of superheavy gravitino DM in these models [22]. The price for such unification of all four fundamental forces and a unified description of inflation and DM is the loss of supersymmetric solutions for the SM problems of divergence of the Higgs boson mass and origin of the electroweak energy scale, as well as the lack of direct

experimental probes for DM particles. This makes indirect effects of supermassive DM crucial for Supergravity probes, thus making the search for such effects in the LHAASO, HAWK, and HESS experiments challenging.

2.2. Axion-like Particles

Axion-like particles (ALPs) comprise a wide class of light pseudo-Goldstone bosons (PGB) that arise from several different theoretical scenarios. These particles, primarily originated by the spontaneous symmetry breaking of global symmetries beyond the Standard Model (BSM), include QCD axions [23–28], familons and arions [29,30], majorons [31,32], and baryo-majorons [33]. As PGBs, ALPs retain interactions with the SM particles that are suppressed by energy scales regulating their spontaneous symmetry breaking, and individuating their decay constants, usually denoted with f . String phenomenology may also provide a rich variety of ALP candidates beyond QCD axion mass and couplings [34–36] that arise, for instance, as moduli scalar fields either from the compactifications of Calabi–Yau manifolds, or even from anomalous Peccei–Quinn symmetries resulting from the intersection of D-branes—the violation of the Peccei–Quinn symmetry is usually induced by world-sheet, brane, new gauge and gravitational instantons.

Within this wide scenario, it is worth mentioning that QCD instantons were originally proposed as a way to solve the strong CP problem, instantiating a shift of the θ -parameter in the topological part of the QCD Lagrangian. Thus, the QCD axion can be understood in terms of the breakdown of the $U(1)_{PQ}$ symmetry, induced by the triangle-anomaly involving gluons. A periodic potential for the axion particles is hence generated by the non-perturbative topological fluctuation of the gluon fields, with a minimum being reached by the axion field at the decay constant value f [23–25,37]. This effect realises a screening of the CP-violating θ -term. Furthermore, anomalous triangular diagrams, as the ones induced by the interaction of the electromagnetic Pontryagin density with the axion field, may generate ALPs-photons, thus opening a pathway to ALPs detection. Photo-Pontryagin-axion interaction terms are hence responsible for a photon-ALPs mixing as a source by a magnetic field background. This is the Primakoff effect [38,39], which also motivated searches for ALPs in laboratory experiments with high magnetic fields—see e.g., the CAST experiment at CERN [40].

From a phenomenological perspective, cosmic magnetic fields along the sight line might trigger ALPs oscillations, which can then be observed thanks to VHE gamma ray sources [41–45]. The distance of sources may then act as an amplifier parameter. Among such sources stand blazars and specific AGN characterized by gamma beams that are pointing toward our line of sight. Blazars are among the most energetic (and distant) gamma-ray sources hitherto observed. Pair production and the interaction with the radiation backgrounds affect the mean free path for VHE photons above 100 GeV. In the presence of extragalactic background light (EBL), these effects indeed induce a non-negligible probability for VHE gammas to annihilate into electron–positron pairs [46–48]. The average path length hence decreases with the energy while effects due to EBL become dominant. On the other hand, for energies below 100 GeV, the pair production effect is negligible, being that the average path length is comparable with the Hubble radius. We may further consider photon-ALP-photon oscillations occurring in the intergalactic space before reaching the terrestrial detectors, which affects the estimated flux in a competitive way to the electron–positron pair production [41–43]. Finally, photon-ALP conversion happening inside blazars shall also be taken into account. The conversion into photons may hence occur even inside our galaxy [44,45]. This effect would then provide an even larger average path length of VHE gammas than expected. This would provide the picture of a more transparent Universe at high energies. Remarkably, if the hardening of the blazar

spectra, in turn controlled by the coupling with ALPs, would be observed, this would provide a hint for photon-ALPs oscillation in the $1 \div 100$ TeV range of energies.

It is also worth mentioning that an axion-like particle could be considered as a source of baryon asymmetry within the spontaneous baryogenesis model. Arising as an angular degree of freedom of a complex scalar field after symmetry breaking, the axion-like particle is derivatively coupled to the fermion current, which causes baryon and lepton numbers non-conservation, while it oscillates. Such models are considered in refs. [49–51]. It is demonstrated in the aforementioned papers that in flat space-time, baryon asymmetry in this model, in the case of small oscillations, is proportional to the cube of the initial phase of the axion-like particle: $(n_b - n_{\bar{b}}) \propto \theta_i^3$. Due to mathematical complexity, there is no analytical result for baryon asymmetry in the case of expanding space-time, and it could be studied elsewhere.

2.3. PBH

The idea of black holes of primordial origin in the early universe was proposed in 1967 by Zeldovich and Novikov [52] and also, separately, by Hawking and Carr [53–55]. PBH have been a subject of great interest for dozens of years. Indeed, they can also be considered as DM candidates [56]. At the moment, constraints are imposed on a wide range of masses of a PBH as a DM candidate [57]. It has also been shown in [58–60] that PBH could be formed in clusters, and that within this scenario, constraints that have been imposed on black holes as DM candidates should be reconsidered [59,61,62].

Currently, several different mechanisms for the formation of PBHs have been suggested: conventional density fluctuations [53,54], phase transitions of the first-order [55], cosmic string collapse [63], the appearance of black holes in hybrid inflation models [64], phase transitions of the second-order [65] - one can find a review of PBHs formation mechanisms in [66]. Recently, it was also shown that there is a new mechanism of formation of black holes in the multidimensional modified $f(R)$ -gravity model [67], containing tensor and Ricci scalar squared corrections. In addition to this fact, a modified $f(R)$ -gravity offers solutions to many cosmological problems [68], thus the possibilities of modified $f(R)$ -gravity have been widely studied [69,70].

It is of great interest that the axion-like field and modified gravity could produce clusters of PBHs. They are both cases of a second-order phase transition. There are papers considering clusters of PBHs as DM candidates [71,72]. The idea is that mass distribution within the cluster and its finite size would lead to the weakening of the dynamical constraints on black holes as DM candidates. Unfortunately, the analysis performed in [72] indicates that clusters of black holes should obey basically the same dynamical constraints as single black holes.

2.4. Hadronic, Hadron-like and Composite Dark Matter

2.4.1. Hadronic and QCD-like Composite Dark Matter

Extensions of the Standard Model are strongly motivated by its well-known theoretical and phenomenological limitations. A promising option is the addition to the set of SM fields of extra heavy fermions in confinement that interact in a vector manner with standard gauge bosons [73]. These scenarios of the SM extension are referred to as hypercolor (H-color) models. The symmetry group of the extended model in our case is $SU(2n_F) \times G_{SM}$, where $SU(2n_F)$ stands for the H-color group, and n_F is the number of H-quark generations. The analysis of the group structure of H-color models, as well as the emergence of pseudo-Nambu-Goldstone bosons resulting from symmetry breaking and other peculiarities of this scenario (including the introduction of massive bound states of H-quarks in the framework

of linear σ -model), were studied in detail in the minimal version for $n_F = 2, 3$ —see e.g., [74].

An important feature of H-color models is the presence of stable neutral objects, which can be associated with the hidden mass candidates. In the $SU(4) \rightarrow Sp(4)$ scenario, these are the lightest neutral objects in the triplet of H-pion states and H-baryons (di-hyperquarks). The latter ones are stable due to conservation of the H-baryon number, but the neutral H-pion stability is provided by the specific symmetry leading to the conservation of a new quantum number, G-parity. Thus, in the $SU(4) \rightarrow Sp(4)$ model, two particles are neutral and stable; in the $SU(6) \rightarrow Sp(6)$ H-color extension with an extra singlet H-quark, two more charged objects turn out to be stable, because of the symmetry with respect to two $U(1)$ groups. Such stable neutral particles with the masses ~ 1 TeV are interpreted as the DM carriers, resulting in the necessary value of the DM abundance in the freeze-out scheme. Obviously, they are WIMPs, but due to the specific structure of the set of states in the hidden mass sector [75], some non-standard manifestations of these objects are possible, as we review below.

Also, we can consider some alternative type of the hidden mass originated from new massive hadrons strongly interacting with each other and with ordinary matter by gluon and/or electroweak exchanges [76]. Measurements at underground detectors (LUX, XENON1) [77,78] imposed some strict bounds on the cross sections for the interaction of these particles with nucleons. Such new hadrons arise from a chiral symmetry model or from a model with an additional $SU(2)$ singlet heavy quark—similar scenarios with heavy (meta-) stable hadrons are considered in the fourth-generation extensions or in “mirror” models. In these scenarios, the genesis and prerequisites for the interpretation of new heavy hadrons—i.e., strongly interacting massive particles (SIMPs)—as the DM particles have been studied carefully.

More exactly, in the SIMP model, the hadronic hidden mass candidate is a heavy neutral hadron, qQ , composed of a standard light quark, q , and a new heavy quark, Q , which has vector interaction with photons and a Z-boson but does not interact directly with a W-boson. Contributions of new quarks to boson polarization functions (Peskin–Tackeuchi parameters) are small, and the model is in agreement with the EW data. Moreover, in the simplest version, we can suppose that the new heavy quark does not mix with the standard ones. Importantly, it was found that any additional neutral currents with a change of flavor are absent in this scenario.

This additional heavy quark participates in strong interactions with standard quarks as in QCD. At the stage of nucleosynthesis, strongly connected bound two-quark and three-quark states of the form (qQ) , (qqQ) , (qQQ) and (QQQ) are formed. Main properties of new heavy mesons, namely, of the simplest two-quark states (qQ) , were intensively analyzed using, in particular, modified methods of heavy quarks effective theory (HQET) [79]. It allowed us to consider the excited states’ structure and low-energy interaction of new heavy stable hadrons with photons, leptons, and nucleons, and to use, for the lowest excited levels, an analogy with the standard heavy-light mesons. Introducing the effective vertex for the interaction of the new heavy hadrons with the W-boson, we can quantitatively analyze the cross-section of the lepton scattering on the hadron DM. Also, strong low-energy interaction of new hadrons and their scattering on nucleons were carried out within the effective meson-exchange model based on dynamical realization of $SU(3)$ -symmetry. Namely, the cross section of new heavy hadrons interaction with nucleons at low energies has been calculated in an analytic form and can be used to study interaction of the cold hidden mass with the matter—clouds of hydrogen or dust, scattering of cosmic rays on the DM halo, etc.—in the Galaxy. Using the known data on the DM relic abundance, the new hadron’s mass was estimated as ≈ 10 TeV.

The symmetry properties of the model entail the evaluation of fine splitting magnitude between masses of charged and neutral components in the new meson doublet. The value of this splitting is small, it is $\sim 1 \div 10$ MeV. Additionally, exploring the structure of excited states can prove the existence of a hyperfine splitting between them with a small magnitude (≈ 2 keV or slightly higher) [80,81].

The fine and hyperfine splitting can lead to the observation, in principle, of a glow (low-energy radiation) of hadronic DM, either in space or in ground-based observatories. In particular, this radiation would be more intensive and visible when generated by clumps of the hidden mass. The glow can indeed be seen in processes when the heavy DM particles scatter on the target, resulting in the appearance of excited states that radiate low-energy photons. This radiation can also explain events with $E \sim \text{KeV}$ at the XENON1T [78], but still with a low confidence level.

The model of SIMPs also predicts the emergence of long-lived charged states. In principle, manifestations of such heavy charged and (almost) stable states can be observed in anomalous events in the atmosphere—heavy charged projectiles should create traces of ionized (and/or excited) atoms and molecules together with fast muons, pions, electrons and photons. More exactly, an extensive atmospheric shower (EAS) would occur that would be characterized by a specific profile. These showers can be seen presumably at low altitudes, and the question is how to separate such events produced by exotic heavy mesons from analogous showers generated by the “standard” cosmic rays. These scenarios, accounting for an additional heavy quark, can manifest in astrophysical and ground-based observations, as well as in collider experiments. Coming back to hyper-color scenarios inspired by technicolor and sigma model (for construction of the set of new bound states of hyper-quarks), we may note that the small mixing between the Higgs boson and an additional scalar field (an analog of the known low-energy sigma meson) ensures the necessary smallness of the Peskin-Takeuchi parameters [82]. This provides robustness to the aforementioned scenarios in terms of interpretation of observed phenomena and prediction of the new ones.

As it was mentioned, in the hyper-color models, the hidden mass sector is multi-component, and the mass splitting between the stable neutral DM candidates is ≈ 160 MeV, being nearly constant in the whole region of model parameters. The DM candidates' masses have been evaluated from the numerical solution of the kinetic equation system for the hidden mass burnout during evolution to the freeze-out point. In this scenario, the diffuse photon spectrum generated by DM annihilation and the cross sections for the DM pairs' transformation into the SM particles were also calculated. Due to the lack of LHC data for the heavy hidden mass, interactions of galactic and intergalactic very high-energy cosmic rays (VHECR) with the DM are an important source of specific DM manifestations. Specifically, from the calculations of these scattering processes, it has been shown that the contributions of intermediate scalar exchanges can be especially relevant [83].

As an interesting test bench for neural network methods in data analysis, one may suggest considering multicomponent DM luminescence due to transitions between the mass-split components. Indeed, radiation of diffuse photons with energies up to 10 GeV, especially from regions of high hidden mass density, is possible [84,85]. The effect is found in the H-color extension of the SM, where heavy neutral H-pions and H-baryons are stable and originated from different hypercolor structures [86]. However, in any multi-component hidden mass sector, analogous effects are possible, being detectable by cosmic telescopes depending on the mass splitting values. The problem is how to reliably select such photons from cosmic photon noise of various frequencies, determine the bounds of the energy range and detect the source of radiation and its location, if possible. Furthermore, the H-color scheme can be extended up to $SU(6) \rightarrow Sp(6)$, where an analogous set of heavy stable

fields arises. However, two stable states with fractional charge also appear. This urges us to ponder carefully their dynamics, manifestations and related cosmological consequences. These are open questions, available to future investigations.

Clearly, the hyper-color vector model does not solve all the SM problems. Nonetheless, except for some interesting and characteristic effects for the multicomponent heavy DM, the model can be complemented, for instance, by the clockwork scheme [87,88]. This provides the necessary smallness of the neutrino masses and couplings, due to the specific hierarchy of extra fields. Immediate questions emerge about how observable and measurable the manifestations of such additional fields can be. It seems reasonable that methods of data analysis that are proposed by neural networks for such multi-messenger investigations can represent effective tools for the classification of prominent data structures, and the research of the sets of successive signals from interacting additional fields. In other words, the analysis of sequences of similar signals—final photons with eventually diminishing energies, stable or decaying leptons that are radiated by intermediate fields, having defined energies, etc.—can be done more accurately by “eyes and brains”, which are learned by modern neural networks.

2.4.2. Stable Multiple Charged Constituents of Dark Atoms

Over the last decades the mainstream studies of cosmological DM were related to the predictions of supersymmetric (SUSY) models, which had the advantage of solving the internal SM problems. The solution for the problem of divergence of the mass of the Higgs boson was one of the most attractive features of SUSY. Simultaneously, the lightest supersymmetric particle could be stable and have a mass in the range tens-hundreds GeV, thus playing the role of Weakly Interacting Massive Particle (WIMP) candidate for DM—see [89–92] for a review and related references.

SUSY particles have not been hitherto found at the LHC, and WIMPs seem not to have been detected in the underground direct searches for DM. It appealing to explore a more extensive field of BSM physics and, in particular, to address possible nonsupersymmetric solutions, cutting the divergence of the Higgs boson mass. Models of composite Higgs boson can offer such an alternative solution and, as it was shown in the framework of Walking Technicolor models [93,94], can also lead to a new approach to the nature of DM, revealing its composite dark atom character [95–97].

Atoms of DM, in which new stable charged particles are bound by the Coulomb force, were first proposed in [95]. However, it was shown in [96] that the prediction of single-charged stable particles inevitably leads to the overproduction of anomalous hydrogen, severely constrained in the terrestrial matter. Stable particles with charge $+1$ bind with electrons, forming atoms of anomalous hydrogen. On the other hand, stable particles with charge -1 bind with primordial helium in $+1$ charged ion immediately after Big Bang Nucleosynthesis, and this $+1$ charged ion also forms with electron anomalous hydrogen. Only stable particles with even negative charge $-2n$ avoid this immediate trouble and bind with primordial helium in neutral atoms [97].

However, the existence of particles with charge $-2, -4 \dots$ should be inevitably accompanied by the existence of the corresponding positively charged particles that can bind with electrons and form anomalous helium, beryllium, etc., respectively. The abundance of such anomalous isotopes can be suppressed in the terrestrial matter if the new particles have additional long-range interactions, causing positively charged constituents of anomalous isotopes to recombine with the corresponding negatively charged particles in the terrestrial matter, thus reducing their abundance below the experimental upper limits [98].

In Walking Technicolor Models, $-2n$ charged stable techniparticles can be generated in excess over their $+2n$ charged partners, which is equilibrated by sphaleron transitions

with the baryon excess. The relationship between the excess of $-2n$ and baryon asymmetry can explain the observed ratio of baryonic and DM densities. In this case DM is in the form of dark atoms, in which the excessive $-2n$ charged particles are bound by the Coulomb force with n helium nuclei. We call such dark atoms OHe for $n = 1$, being a Bohr-like atom with lepton core of O^{--} with helium shell with Bohr radius equal to the size of He nucleus or XHe for $n > 1$, being the Thomson-like atom with $-2n$ leptons situated inside a nuclear droplet of n He nuclei, and plan to study their structure, interaction with matter and possible cosmological impact.

The Dark atom model has the advantage of explaining the puzzles of direct DM searches by annual modulations of their low-energy binding with Na nuclei. Strongly interacting shell of dark atoms provides their slowing down in terrestrial matter, making this form of DM elusive for the strategy of WIMP searches because it involves significant nuclear recoil. However, dark atom interaction with nuclei can provide a low-energy binding, and the corresponding effect experiences annual modulations.

This explanation [99] is based on the following picture of OHe interaction with nuclei. OHe is a neutral atom in the ground state, perturbed by the Coulomb and nuclear forces of the approaching nucleus. The sign of OHe polarization changes with the distance. At larger distances, a Stark-like effect takes place—the nuclear Coulomb force polarizes OHe so that the nucleus is attracted by the induced dipole moment of OHe, while as soon as the perturbation by the nuclear force starts to dominate, the nucleus polarizes OHe in the opposite way, so that He is situated more closely to the nucleus, resulting in the repulsive effect of the helium shell of OHe. Qualitatively, this leads to a shallow potential well with a low-energy bound state in the OHe-Na system, while such a state does not exist for heavy nuclei like Xenon. A quantitative description of OHe-nucleus interaction with a self-consistent account for the effects of nuclear and Coulomb forces is crucial for the description of the OHe (or XHe) evolution in stars. One can expect that dark atoms can be ionized and Supernova explosions can lead to the formation of an anomalous cosmic ray flux of stable multiple-charged leptons. The search for such a component and the possibility of discriminating the corresponding air showers are challenges for the new generation of cosmic ray experiments.

2.5. Mirror Dark Matter

DM may come from a parallel mirror sector of particles, with properties identical (due to parity acting between two sectors) or similar (if mirror parity is broken) to properties of ordinary matter. Mirror matter cosmology has been extensively studied in the literature [100–103] and extensively developed [104,105] by some of us. Mirror baryons represent a case of asymmetric DM (originated due to baryon asymmetry in the mirror sector) of atomic/dissipative type, with several implications for the early Universe's evolution and potentially testable in high precision data on primordial element abundances, cosmic microwave background, and distribution of matter at large and small scales. Mirror stars can exist, and first high-mass mirror stars could form PBHs of astrophysical origin. Mirror neutrinos are natural candidates for sterile neutrinos. Within the case of broken mirror parity, mirror neutrinos with keV-range masses can also constitute DM. The baryon and lepton-violating interactions between ordinary and mirror particles may provide a co-genesis mechanism of baryon asymmetries in both ordinary and mirror sectors, and thus naturally explain the near coincidence of the baryonic matter and DM mass fractions in the Universe. Mixing phenomena between the neutral ordinary particles and their mirror twins are of special interest, e.g., ordinary and mirror photon kinetic mixing that can be a portal for DM direct detection, or ordinary-mirror (active-sterile) neutrino mixings. Possible mixing between ordinary and mirror neutrons can be an explanation of the neutron

lifetime problem—10 s discrepancy between the neutron lifetimes measured via the bottle (neutron disappearance) and beam (proton appearance) experiments—and can have interesting implications for the evolution and properties of neutron stars, for ultra-high-energy cosmic rays, presence of antimatter in galactic cosmic rays, etc. In the near future several experiments are planned for direct search of the neutron-mirror neutron conversions at the Paul Scherrer Institute (PSI, Villingen, Switzerland), Institut Laue-Langevin (ILL, Grenoble, France), Oak Ridge National Laboratory (ORNL, Oak Ridge, TN, USA), and in the perspective of large-scale experimental projects, neutron disappearance and regeneration experiments at the European Spallation Source (ESS, Lund, Sweden). All these aspects make mirror matter an appealing candidate, which can be cross-tested via the multi-messenger approach.

The principal possibility for multi-messenger probes for mirror DM comes from the correlation of signals in GW detectors from violent processes in mirror objects with observations in other tools of multi-messenger astronomy.

3. Phenomenological Probes and Multi-Messenger Constraints

3.1. Constraining DM from VHECR

3.1.1. Interaction of VHECRs with the DM Particles in the Milky Way Galaxy

Because high-energy cosmic rays (CR) consisting of protons, electrons, neutrinos and possibly hidden mass carriers (they can be accelerated in the process of CR scattering on the DM) have energies which are inaccessible for ground-based colliders, the SM extensions structure can be observed and quantified in a variety of astrophysical events at energy scales of about $10 \div 10^5$ TeV and higher. Thus, careful and detailed analysis of final states for high-energy electrons, photons and neutrinos scattered on (or producing, in the inelastic scattering reaction) the DM particles can yield important conclusions about parameters and properties of new (stable) states from the SM extensions.

There are a number of processes of VHECR interactions with the DM demonstrating previously unknown effects and signals, which acquire particular importance due to new possibilities of cosmic telescopes and ground-based observatories like JAMES WEBB, LHAASO, HAWK, HESS and others. These new signals can be registered both separately and in correlation with neutrino events observed at the IceCube, Baikal-GVD, KM3NeT, Hyper-K, etc. The analysis of these multi-particle events in such a wide energy interval is a very complex and cumbersome task. Thus, the application of methods suggested by neural networks in order to detect regularities and anomalies in the data set, patterns of specific configurations of final particles, selections of specific energy intervals (where some set of events can be unambiguously explained by specific physical processes), is not only possible but also necessary. In more detail, some events can be detected at either IceCube or LHAASO or other ground-based observatories due to specific forms of rare EAS generated by fast heavy neutral objects. Such EAS should be separated from other events in the atmosphere due to the study of special classes of variables describing EAS. Note that Artificial Intelligence is already in use for the extensive air showers analysis and simulation [106].

Using the data on EAS generated by the VHECR, it is possible to determine the nature and type of interaction of the DM components with standard particles from cosmic rays, and, therefore, to understand whether the DM objects belong to WIMPs, SIMPs or are something else [73–75]. It also allows us to study the spatial distribution of DM near the Galaxy center, in the Galaxy halo, to search for DM inhomogeneities (clumps) and to trace VHECR interaction with DM or with gas and dust clouds in the galaxy. We also suppose that the elastic and inelastic reactions of cosmic rays with the particles of hidden mass

bring information about the DM dynamics, and that its spatial distribution may influence VHECR's energy spectra and propagation in space.

As the most informative reactions, inelastic or quasi-elastic scattering of CR particles (protons, electrons, neutrinos, photons) on DM components [80,83] should be investigated. The products of these reactions will also be neutrinos, leptons, photons, standard (unstable) mesons, and hadronic jets. Some of these reactions have already been studied both in the framework of supersymmetry, where cosmic rays are scattered by neutralinos (wino, higgsino...), and in the hypercolor SM extension. It was shown that: the scattering cross sections for electrons and neutrinos by hyperparticles strongly depend on the DM mass; secondary particles are emitted mostly forward; channels are found that dominantly contribute into the reaction cross section; the probability of secondary neutrinos registration at the IceCube facility is small when the lepton component of CR is scattered by DM [83]; high-energy photons from quasar jets can effectively produce neutrinos of high energies interacting with components of hidden mass [107].

It is a hard but important task to study the inelastic scattering of cosmic rays by DM particles in a halo surrounding AGNs. As a result of such VHECR scattering, not only fast secondary neutrinos arise, but also stable DM particles from halos can be accelerated to high ($\simeq 10 \div 100$ TeV) energies—these are the so-called secondary VHECRs. Such neutral particles with high energies carry information not only about the source of the primary VHECRs, but also about the nature of the interaction, properties and spatial distribution of DM objects in the halo. Signals of such reactions, which are generated by the interaction of secondary VHECRs with the Earth's atmosphere, can be specific in composition (corresponding EAS has a small number of muons), in the number of particles (broad showers arising from either the interaction of several neutrinos in the atmosphere at different depths or at close depths). These EAS can contain fast (heavy) neutral particles—these are DM particles accelerated in previous scattering processes. These specific signals can be searched both among the EAS at the facilities like LHAASO [4] and in correlation with the neutrino signals at the IceCube. Ultrahigh-energy photons can also lead to the appearance of fast intergalactic neutrinos or accelerated particles DM when photons interact with the halo DM surrounding the AGN. It is also necessary to study correlations between the neutrino fluxes detected at ground-based neutrino facilities, with photons measured by space telescopes (such as Fermi-LAT) and the composition and energy distribution of EAS recorded at the same time by cosmic ray experiments. Such correlations may indicate the reaction of deep inelastic scattering of intergalactic VHECRs on the DM particles, which simultaneously generate several secondary neutrinos and hadronic jets (in the scattering of protons from VHECR) and/or accelerated DM objects (or their excited states decaying with the formation of fast standard mesons).

Physical effects that occur as a consequence of the VHECR interaction with DM are diverse and informative, and their signals can be measured by different ground-based detectors. These data can help to study both the DM structures and SM extensions. These processes are studied both theoretically—calculating analytically the cross sections of VHECR scattering on DM particles in several different scenarios—and phenomenologically by deploying the new generation of cosmic rays experiments. In any case, such a huge bulk of data can be effectively studied only with powerful instruments that enable the selection of “suspicious” data sets and their relative comparison, while searching for correlations based on dynamics and symmetries. Machine learning methods provide suitable tools for this analysis—see e.g., Ref. [108].

3.1.2. Theoretical Predictions for Cosmic Rays Experiments

Cosmic rays may hide invaluable indirect information on DM physics. Observations of any odd effects in CR are interpreted as possible DM signals. Positron anomaly in CR, observed in [109–111], at energies $E \leq 1$ TeV, possible cosmic electron–positron excess at $E \simeq 1 \div 1.5$ TeV [112] are vivid examples. Decay or annihilation of DM particles with production of high-energy $e^\pm$ could account for these effects. However, these top-down models of CR origin inevitably lead to gamma-production, which is hardly consistent with existing data on cosmic gammas, first of all, at the highest energy part of the spectrum. Gamma yield can be changed by modifying the DM decay or annihilation model, though it is not possible to fully eliminate it. So, data on cosmic gamma-radiation can provide a test ground for similar DM models, and ongoing cosmic rays experiments can significantly extend the energy range to be probed. The essential point in probing these models is that the contribution to gamma-radiation from DM with respect to astrophysical background attains maximal values at their highest energies—close to DM particle masses, as a rule—which are eventually inaccessible to the existing experiments. The appearance of such a growing contribution to gamma-radiation at the high-energy edge, in relation to the origin of DM, provides a distinctive feature that encourages experimental investigations.

3.2. *The Multi-Messenger Cosmological Probes of New Physics in Cross Correlation with Its Physical and Astrophysical Effects*

Effects of new physics in the very early Universe in addition to the necessary elements of the now standard cosmology (inflation, baryosynthesis and DM) inevitably lead to additional model dependent consequences, like primordial GW and PBHs, primordial non-linear structures, decaying particles or early matter/dark energy dominance. The analysis of these multi-messenger cosmological probes for new physics is of special interest for super-high-energy scales [22,89,92]. The systematic studies of the cosmological impact of new physics were extensively developed by some of us. The approach of cosmoparticle physics then provides the chance for a cross-disciplinary study of new physics in a proper combination of its physical, astrophysical and cosmological signatures [113].

Cosmological, astrophysical and experimental physical probes for new physics are complementary. This complementarity of studies of physics corresponding to a new physical scale V follows from the fact that laboratory effects of new physics are reduced at the energies $E \ll V$ to search for rare processes suppressed by some power of (E/V) , while astrophysical and cosmological effects may correspond to the high energy $E \gg V$, at which new physics can manifest in its full strength. This observation is unique in the theoretical analysis of the cosmological consequences of new physics, in cross correlation with astrophysical and experimental physical constraints, an important part of the development of the program of studies of new physics in the new generation of cosmic ray experiments.

3.3. *Detection of Decaying DM with Cosmic Rays Experiments*

Cosmic ray interactions in the interstellar medium provide a guaranteed source of neutrinos and gamma rays with an energy range above 10 TeV. Apart from this guaranteed source, other “unexpected” sources might appear in the sky, like the DM decay signal, which has unique spectral and imaging properties and could be readily distinguished from the diffuse flux from cosmic ray interactions.

Decaying DM produces both neutrinos and gamma-rays in hadronic decay channels. The cosmological contribution to the gamma-ray signal is suppressed in the gamma-ray band because of the pair production of the photons of the Extragalactic Background

Light (EBL). This is not the case for the neutrino signal, which still has a cosmological (isotropic) component.

Neutrinos are detected by the IceCube experiment at the South Pole. Hard neutrino fluxes at high energies $E > 200$ TeV, measured in the muon neutrino channel, can be explained by conventional astrophysical sources. However, the highly diffused neutrino flux measured by the IceCube experiment at $E < 100$ TeV cannot be explained by extragalactic sources, since the corresponding diffused gamma-ray background would overproduce diffused gamma-ray measurements by the Fermi-LAT experiment. This excess neutrino flux should have a Galactic origin. Also, because of the IceCube neutrino limit on signal from the Galactic plane, this excess cannot be due to usual cosmic ray interactions with the gas in the Galaxy. DM decays can explain the excess in neutrino flux measured by IceCube, but at the same time provide a large flux of galactic gamma-rays which can be detected by LHAASO [114].

In the mass range $10 \text{ TeV} < m_{DM} < 0.3 \text{ EeV}$ LHAASO will provide a factor of 3-to-100 improvement of sensitivity compared to present limits from HAWC, IceCube and KASCADE. The best improvement of sensitivity is expected in the 10 PeV mass range, corresponding to 100 TeV gamma-ray sensitivity by LHAASO [114].

For decaying DM, with particle masses at the PeV scale, photons are produced inevitably even in leptophilic decaying DM models. They appear as final state radiation (FSR) and as a result of inverse Compton scattering $e^\pm$ from DM decay off background photons. However, the absorption probability of these energetic photons varies with the energy. Photons scatter off different electromagnetic backgrounds (CMB, optical, radio), and that prevents them from coming from the whole Universe. This provides a non-trivial gamma-spectrum from DM decays that carries, together with signals in the neutrinos channel, rare information on the DM physics and its distribution in space. The expected gamma-spectrum can be estimated in the near future in its dependence on m_{DM} .

4. Gravitational-Wave Probes

The unprecedented possibility disclosed within the last decade to probe high-energy physics resorting to gravitational-wave (GW) probes can be accounted for along the following proposed lines:

- **GW probes for Higgs sector couplings in the SM and beyond.** GWs can be used as a source of information on triple/quartic Higgs coupling complementary to ongoing and planned measurements at colliders—see e.g., Refs. [115–117]. One can then investigate to what extent the GW can probe possible compositeness of the Higgs sector helping to distinguish between the composite versus elementary Higgs scenarios. Since composite QCD-like scenarios typically predict rather large self-interaction couplings, these may strongly enhance the strength of the phase transitions associated with the dynamical EW symmetry breaking. Similar studies have already been developed by some of us to probe the Higgs sector properties via GWs [118–120]. A cross-analysis of both GW observability limits and collider constraints then becomes mandatory. Composite models often predict scalar composite heavy DM candidates that might strongly affect the strength and other characteristics of high-scale phase transitions.
- **GW probes for Higgs and gauge sector couplings in the SM and beyond.** GWs from cosmological first-order phase transitions provide information on Higgs-sector interactions (including the triple/quartic Higgs couplings) that is complementary to collider measurements, see e.g., Refs. [115–117]. More generally, the GW signal is controlled by the finite-temperature scalar potential, and is therefore sensitive to Higgs-portal, gauge and Yukawa couplings in BSM scenarios; this enables quantitative GW-collider complementarity, particularly in models with dynamical electroweak

symmetry breaking (such as composite Higgs setups) where enhanced self-interactions can strengthen the transition and may correlate with additional scalar/DM states. Building on earlier studies [118–120], recently supercooled phase transitions in conformal $U(1)'$ extensions (including Majoron/seesaw realizations) and in conformal dark sectors were investigated, confronting them with current LVK/PTA data and forecasting the reach of LISA/ET [121–123]. Correlated cosmological relics were further explored, such as primordial black holes and primordial magnetic fields [124]. A combined GW–collider–cosmology analysis is thus essential.

- **GW probes for light and ultra-light DM models** (ALPs, specifically Majoron/Axion-like). The GW spectrum measurements provide a source of information about new physics models featuring very light pseudo-Goldstone (axion-like) scalar states that, under certain conditions, may play the role of DM. The presence of such states may indicate an approximate global continuous symmetry. This is, for instance, the case of the family symmetry at high scales that is both spontaneously and explicitly broken at low scales. Despite a light pseudo-Goldstone mass, the presence of such states affects the vacuum structure and can lead to additional phase-transition patterns that otherwise would not exist [119]. We may pose a question about a possible complementarity of the GW data with the direct DM detection experiment data, such as the recent hint from the XENON1T measurement.
- **GW probes for neutrino mass generation mechanisms.** Future GW measurement can probe the scale of Majorana neutrinos and lepton-number breaking patterns, hence zooming into the properties of the neutrino spectrum—see e.g., Ref. [119].
- **GW probes for the new physics energy scale in BSM scenarios.** Probing the high-scale phase transitions in LR-symmetry/family symmetry/GUT theory models—see e.g., Refs. [125–127]—can be envisaged. This would offer a way to disentangle different new physics scenarios beyond the reach of particle collider measurements.
- **GW probes for PBHs.** Future GW astronomy will open a new window into the early Universe by probing two key signatures: high-redshift black hole merger events and the stochastic GW background over a wide range of frequencies. The statistical properties and frequency content of these signals will enable us to conduct a powerful test of the existence of PBHs [128–131], potentially distinguishing them from their astrophysical counterparts.

All these items require building a sophisticated Machine-Learning-based tool that would comprise both constraints coming from collider measurements and analyses of the phase transitions for phenomenologically allowed points.

4.1. GW Probes for Higgs Sector Couplings in SM and Beyond

A historically unique era of particle physics and cosmology has begun with the launch of operations of the Large Hadron Collider (LHC) at CERN, the largest experiment ever, as well as many large- and small-scale neutrino physics, DM, cosmic-ray and GW astrophysics experiments. SM has appeared to be the most phenomenologically successful theory of the subatomic world, proven to work impressively well in interpreting all the experimental data collected at particle colliders so far.

It is unquestionable, however, that there is overwhelming phenomenological evidence that strongly suggests the need for a more complete theory. Indeed, the SM does not naturally incorporate a neutrino oscillations mechanism, has issues with vacuum stability and does not contain suitable DM candidates. Additionally, it provides neither a reasonable explanation of the observed strong fermion mass and mixing hierarchies nor the baryon asymmetry in the Universe. In this regard, the question about the accessibility of new BSM phenomena through direct measurements becomes more and more precious. Cur-

rently, the absence of new physics indications either suggests that new particles and/or interactions can only show up at a larger energy scale beyond the current reach of collider measurements or is due to a lack of sensitivity of the current measurements to very rare phenomena. Clearly, the greater challenge in probing such new phenomena means a weaker interplay and interaction between the SM and new physics sectors, indicating a growing demand for new methods and tools.

A poorly known structure and dynamics of the ground state of the Universe are ultimately responsible for the majority of the unsolved problems in particle physics. It determines the particle spectra and interactions that we observe through the mechanism of spontaneous symmetry breaking. In particular, a specific shape of the potential of the Higgs field leads to electroweak symmetry breaking, where the ground state features a constant Higgs field value. The properties of the Higgs field potential and interactions determine the values of all particle masses and also many of the coupling strengths. This occurs through the electroweak phase transition, a few picoseconds after the Big Bang, when the current laws of physics are determined. In order to understand exactly how and why this happens we need to probe the Higgs field potential phenomenologically.

The SM predicts a minimal, well-defined structure for the Higgs-field potential. At present, only the potential near its minimum has been experimentally probed; its behavior at both large and small field values remains essentially unconstrained. This can be addressed by experimentally probing the strengths of the triple and quartic Higgs self-interaction couplings, λ_{hhh} and λ_{hhhh} , respectively. The SM predicts their specific values, which are, however, highly sensitive to possible extensions of the Higgs sector. Popular BSM theories require additional scalar fields in the scalar potential that significantly affect their properties, in particular, by changing the value of λ_{hhh} and λ_{hhhh} . Therefore, it is mandatory to measure these couplings with sufficient precision as one of the most critical indirect tests of the SM.

The concepts of compositeness and supersymmetry represent the two distinct paradigms for new physics model building. The compositeness concept relies on the existence of a new strongly-coupled dynamics at large energy scales responsible for the formation of a whole family of heavy bound states composed of elementary constituents, in a close resemblance to the hadron spectra in Quantum Chromodynamics (QCD). In this approach, the Higgs boson is considered to be among the lightest composite states while the electroweak symmetry is broken dynamically via a condensation of new fermions at a new large confinement scale—the phenomenon is in some sense similar to spontaneous chiral symmetry breaking in QCD. A typical realization of such a dynamical electroweak symmetry breaking provides a natural solution to the hierarchy problem and the vacuum stability issue in the SM. An alternative concept of supersymmetry, which is a new extended spacetime symmetry that relates bosons and fermions, offers another pathway towards the solution of the hierarchy problem, guaranteeing the cancellation of quadratic divergences to the Higgs boson mass to all orders in perturbation theory.

Many particular realizations of both such concepts have been proposed in the literature so far, while the measurements cut out deeper regions of the parameter space, leaving less freedom in such models and thus rendering further progress in new physics searches more difficult. Development of new methods and tools is mandatory at this stage to increase the sensitivity to possibly weak and yet elusive signatures of new physics.

The measurement of the Higgs boson pair production at the LHC will provide direct access to the triple-Higgs coupling value. Its possible deviation from the SM prediction represents an efficient tool for probing the possible existence of additional scalar fields as well as new fermion states entering the triple Higgs coupling through radiative corrections. Particular effort should be devoted to the study of the triple Higgs coupling, both

experimentally and theoretically. Indeed, one can extract crucial information about the structure of the key benchmark new physics scenarios through a comparison of the model predictions for λ_{hhh} to the experimental data. This information would enable us to set further constraints on model parameter space and interactions in both the minimal composite Higgs model (MCHM) and the Minimal Supersymmetric Standard Model (MSSM) and to potentially distinguish between these models if a statistically significant deviation of λ_{hhh} from the SM value is experimentally observed.

Cosmological phase transitions (PTs) offer an inspiring possibility to probe physics beyond the SM. If first-order PTs took place at early cosmological times, a GW spectrum can be induced, with crucial observational consequences for current and future GW experiments. The recent observation of cosmic GW spectrum opened up a new channel for probing new physics in complementarity with ongoing new physics searches at particle colliders [115–117]. Models with extended scalar sectors, typical for both compositeness and supersymmetry scenarios, feature a possibility of a strong first-order electroweak phase transition (EW-PT) in the early Universe. In fact, such a possibility is not realized in the SM, making it impossible to explain the origin of baryon asymmetry in the modern Universe, thus further motivating non-minimal Higgs sectors and a more complicated structure of the scalar potential. The first-order EW-PT is capable of generating a stochastic background of primordial GWs, which, if detected, can open up a gravitational portal and an efficient probe for BSM physics. The shape and evolution of the Higgs-field effective potential at finite temperatures strongly depend on the microscopic structure of the underlined theory. Future measurements of primordial GW spectra at planned GW spectrometers, such as LISA, will provide an access to the triple-Higgs coupling complementary to that from the LHC measurements.

One of our key goals would be to study the potential for precision measurement of λ_{hhh} , which allows us to distinguish between the supersymmetry and composite dynamics through a combination of future experimental data from both LISA and LHC. For the purpose of studies of interplay and efficient combination of collider and GW observables we plan to develop an inclusive computational framework based on state-of-the-art deep learning techniques. We will start with an analysis of low-energy effective supersymmetric and composite theories from the perspective of triple-Higgs coupling as well as the GW observables. The focus will be on the relative comparison of MCHM and MSSM implications on λ_{hhh} that also contain new vector-like fermions (VLFs). Besides influencing the triple-Higgs coupling through radiative corrections, the VLFs might be able to generate observable signatures of primordial GWs, address the $g-2$ anomalies as well as provide new Higgs and exotic fermion decay signatures at colliders. Thus, we plan to perform an inclusive study of all these observables. The inevitable consequence of first-order PT is the formation of PBHs in bubble collisions. It provides the link between the GW spectrum and the spectrum of PBHs, created in such a transition, as well as implies studies of the observable effects of such PBHs.

In order to be able to develop these studies in an efficient way, it is necessary to provide a novel computational framework based on the state of the art of deep learning techniques. For this purpose, it is possible to interface model building and Monte Carlo software tools while applying Deep Learning techniques in order to combine all available theoretical and phenomenological information. This will include an option to evaluate the impact of collider constraints or infer predictions for the primordial GW stochastic background. The final aim is to combine this knowledge with the experimental measurements of the triple-Higgs coupling to help point the way toward a consistent theory of the fundamental interactions.

4.2. GW Probes for PBHs

The direct detection of GW in 2015 [11] provided a significant boost to the study of PBHs. The idea that some of the detected black hole merger events have a primordial rather than stellar origin is extremely intriguing and widely discussed [132–137]. PBH binaries can form both in the very early Universe deep in the radiation-dominated stage due to pairwise decoupling from the Hubble flow [133,138,139], and in the relatively contemporary Universe in galactic and sub-galactic structures, due to dynamical channels: two-body [132, 134] and three-body interactions [140]. Corresponding estimates of present-day PBH merger rate vary by several orders of magnitude depending on the binary formation channel. The point is that the prediction of the number of PBH mergers depends in a very complex way on the fraction of PBHs in the DM $f_{PBH} = \Omega_{PBH}/\Omega_{DM}$, moreover, there is a “mutual influence” of the binary formation channels on each other due to nonlinear gravitational dynamics during PBH clustering.

As was shown in Refs. [141–144], when the contribution of PBHs to the DM density is sufficiently large $f_{PBH} \gtrsim 0.01$, natural Poisson clustering of PBHs arises, and gravitationally bound dark structures form long before the first proto-galaxies. Unlike Section 2.3, where the possibility of weakening constraints on PBHs through clustering was raised, this Section focuses exclusively on the observable GW manifestations of this effect. In a cluster environment, complex dynamical effects of interactions between black holes occur, which affect both the population of binaries and the internal evolution of the cluster itself [137,145–147]. In addition, Refs. [148–150] showed that such effects, on the one hand, lead to a decrease in the merger rate of PBH binaries formed in the early Universe, but on the other hand, cluster dynamics lead to an increase in the population of binaries formed through dynamical channels—see also Ref. [140]. Moreover, both these PBH merger channels are comparable in the case of $f_{PBH} \gtrsim 0.1$ at relatively small cosmological distances with redshifts $z \lesssim 1$, which is within the sensitivity threshold of current ground-based GW detectors. Several theoretical models predict the initial clustering of PBHs [65,151–154]. In this case, the physical distances between PBHs are much smaller on small spatial scales than on average at large scales. Furthermore, the dynamical interactions between PBHs are much stronger than in the case of Poisson clustering, significantly affecting the PBH merger rate [155–161]. The temporal evolution of the merger rate in both Poisson and initial PBH clustering cases differs significantly from the predictions of astrophysical black hole mergers. Both models also produce distinctive spectral features in the stochastic GW background [161–165]. These features, which arise in the PBHs scenario, can be tested through future GW observations.

Another factor influencing the dynamics of PBH binaries, and consequently the theoretical prediction of their merger rate, is the accretion of DM halos onto PBH binaries. The binaries themselves interact with the DM particles of this halo, and this interaction leads to the evolution of their orbital parameters [166–169], in a manner similar to the GW emission. Note that this consideration does not at all imply that PBHs do not make up all DM. PBHs with masses significantly smaller than the components of the binary system can play the role of “light” DM particles. In particular, the possibility that PBHs with masses $M_{PBH} \sim 10^{18}–10^{22}$ g can make up all the DM is still open. In the latter case, the PBHs forming a halo around the binary system would be the cold component, since the interaction between such light black holes is negligible. It should also be noted that if DM poses self-interactions, the evolution of the binaries orbital parameters will generally differ due to this interaction [170–174]. In any case, such interaction between the binary system and DM particles will be reflected in the GW signal from the black hole merger, which can then be used to study the nature of DM.

Current observations make it virtually impossible to determine whether merging black holes are stellar or primordial. However, future GW detectors will possess unprecedented

sensitivity, enabling us to detect black hole mergers at high redshifts $z \gtrsim 10$, i.e., at an epoch when significant astrophysical black hole populations had not yet formed, and study GW across a wide range of frequencies [12,175–178]. This would also likely allow the orbital parameters of merging binaries to be probed, as the detectors will be sensitive to the long inspiral phase that precedes coalescence. An important point to note here is that, unlike astrophysical black holes, PBH binaries are expected to form with high eccentricities. This will allow us to answer the question about the existence of PBHs more reliably and to identify the most promising models for their formation. Achieving this ambitious goal depends on advances in engineering, improvements to physics facilities and the development of data analysis methods, including machine learning approaches.

5. Machine Learning Applied to Particle Physics

The possible adaptation of machine learning techniques to quantum field theory, and specifically particle physics, in order to attain improved performances of the codes that are used to calculate scattering processes, their associated cross-sections and the related spectra for cosmic rays, is a very fascinating and promising point of the research directions we are exploring here. In the next sections we will introduce a number of basic concepts coming from computational complexity theory, artificial intelligence, and machine learning essential for drawing future directions of development for DM multi-messenger astronomy. Machine learning is not introduced here as a replacement for physical modeling, but as an inference and representation tool enabling the exploration, correlation, and compression of high-dimensional theory–data spaces inherent to multi-messenger physics.

5.1. Computational Complexity Theory

Computation is an essential part of the technical and scientific methodology. Given a specific input to a specific set of operation models, the desired output will be produced according to the parameters of the model. The key issue consists of finding the suitable parameters that are supposed to feed the model. This captures the essence of machine learning. The complexity of the problem defines the complexity of the computation to the extent that it defines the complexity of the search space. A basic task of theoretical computer science is to sort problems into complexity classes. A complexity class contains all problems that can be solved within a given resource. Theoretical computer science identified two complexity classes of problems, respectively P (polynomial time) problems and NP (nondeterministic polynomial time) problems. The conjecture that these two classes are distinct in the standard Turing model of computation is the most important open question in the field. In particular, the question can be formulated by asking whether every problem whose solution can be verified in polynomial time can also be solved in polynomial time, that is, in such a way that the time to complete the task varies as a polynomial function on the size of the input to the algorithm (as opposed to, say, exponential time).

As a generality, each physical theory supports computational models whose power is limited by the physical theory. It is well known that classical physics supports a multitude of implementations of the Turing machine. Non-Abelian topological quantum field theories exhibit the mathematical features necessary to support a model capable of solving all #P problems (sharp P-complete problems), a computationally intractable class, in polynomial time. Specifically, Witten [179] has identified expectation values in a certain SU(2)-field theory with values of the Jones polynomial [180] that are #P-hard [181]. This suggests that some physical system whose effective Lagrangian contains a non-Abelian topological term might be manipulated to serve as an analog computer capable of solving NP or even #P-hard problems in polynomial time. Defining such a system and addressing the accuracy issues inherent in preparation and measurement is a major unsolved problem.

A polynomial-time algorithm for solving a #P-complete problem, if it existed, would solve the P versus NP problem by implying that P and NP are equal. No such algorithm is known, nor is a proof known that such an algorithm does not exist.

In 1993 computer scientists Ethan Bernstein and Umesh Vazirani defined a new complexity class called BQP, which is an acronym that stands for “bounded-error quantum polynomial time” [182]. More specifically, BQP is the class of problems solvable with high probability in polynomial time by a quantum computer. Around the same time, Bernstein and Vazirani also proved that quantum computers can solve all the problems that classical computers can solve. That is, BQP contains all the problems that are in P. Moreover, BQP contains problems not found in another important class of problems known as PH, a different acronym that stands for “polynomial hierarchy” [183]. PH is a generalization of NP. In particular, PH contains almost all well-known complexity classes such as P, NP, and co-NP. This means PH contains all the problems if one starts with a problem in NP and makes it more complex by layering qualifying statements like “there exists” and “for all”. Classical computers today cannot solve most of the problems in PH, but theoretical computer science assumes that PH is the class of all problems classical computers could solve if P turned out to equal NP. In other words, to compare BQP and PH is to determine whether quantum computers have an advantage over classical computers that would survive even if classical computers could (unexpectedly) solve many more problems than they can today.

Neto et al. have proved that Quantum Neural Network models coupled with a non-unitary operator can solve the 3-SAT problem in polynomial time, a result that no classical neural network was able to obtain previously [184]. As we will discuss in Section 5.7, one of the approaches one can pursue consists of making use of (Topological) Quantum Neural Networks for building models of particle physics. This approach allows to deal with the complexity of the computational space associated with this problem by exploiting the computational resources coming from the quantum computing platform and, therefore, possibly solving problems unsolvable by using a classical platform. In this sense, it should be obvious why the computational capabilities of the underlying platform could play an important role in shifting not only the computational efficiency of the learning algorithm but also the class of problems that could eventually be faced from an algorithmic point of view.

5.2. Machine Learning: Perspectives and Approaches

A machine learning model is a mathematical function $Y = f(X)$ between an input X and an output Y . The goal of different machine learning approaches is to learn this function given some observed input and output data. It is useful to consider machine learning problems from several perspectives:

- **Machine learning as probabilistic inference.** A first perspective is that machine learning tasks are often tasks involving probabilistic inference of the learned model from the training data and prior probabilities. In fact, the two primary principles for deriving learning algorithms are the probabilistic principles of Maximum Likelihood Estimation—in which the learner seeks the hypothesis that makes the observed training data most probable—and Maximum a Posteriori Probability (MAP) estimation—in which the learner seeks the most probable hypothesis, given the training data plus a prior probability distribution over possible hypotheses. The perspective that machine learning algorithms are performing probabilistic inference is very compatible with the perspective we are going to mention, according to which machine learning algorithms are solving an optimization problem. In most cases, deriving a learning algorithm based on the MLE or MAP principle involves first defining an objective function in

terms of the parameters of the hypotheses and the training data, then applying an optimization algorithm to solve for the hypothesis parameter values that maximize or minimize this objective.

- **Machine learning as optimization.** Machine learning tasks are often formulated as optimization problems. For example, in training a neural network containing millions of parameters, we typically frame the learning task as one of discovering the parameter values that optimize a particular objective function such as minimizing the sum of squared errors in the network outputs compared to the desired outputs given by training examples. When machine learning tasks are framed as optimization problems, the learning algorithm is often itself an optimization algorithm. Sometimes we use general-purpose optimization methods such as gradient descent (e.g., to train neural networks) or quadratic programming (e.g., to train Support Vector Machines). In other cases, we can derive and use more efficient methods for the specific learning task at hand (e.g., methods to calculate the maximum likelihood estimates of parameters for a naïve Bayes classifier).
- **Machine learning as parametric programming.** Another perspective we can take on the same learning programs is that they are choosing parameter values that define a function or a computer program written in a programming language, which is defined by their hypothesis space. For example, we can view deep neural networks as implementing parameterized programs, where the learned network parameters instantiate a specific program out of a set of potential programs predefined by the given network structure. As we move from simple feedforward networks to networks with recurrent (feedback) structure and with trainable memory units, the set of representable (and potentially learnable) programs grows in complexity.

Regarding the main approaches, the literature distinguishes between parametric, non-parametric, and semi-parametric models.

- A parametric approach assumes the functional form (i.e., the shape) of the mathematical function f by construction. That is, it assumes that the function belongs to a particular family of mathematical functions such as, for instance, linear, quadratic, etc. The goal is now to determine the coefficients (parameters) of the different components of the function on the basis of the training data. More specifically, a parametric model will assume some finite set of parameters θ . Given the parameters, θ , future predictions X will be independent of the observed data D : $P(X|\theta, D) = P(X|\theta)$. Therefore, θ captures everything there is to know about the observed data D . As a consequence, the complexity of the model is bounded even if the amount of data is unbounded. Yet, parametric models suffer from being poorly flexible to the extent that the shape of the function is defined a priori. Linear regression is an example of such an approach. It is assumed that the input and output follow the relation $Y = \beta_0 + \beta_1 X$ and the goal is to determine the values of the parameters, i.e., the coefficients β_0 and β_1 . Note that, in this case, the function belongs to the family of linear functions and different values of the parameters will generate different linear functions. The main task of a parametric approach is equivalent to estimating the vector of parameters. Logistic regression, k-means and hidden Markov models are all examples of parametric models.
- Non-parametric models assume that the data distribution cannot be defined in terms of such a finite set of parameters. But they can often be defined by assuming an infinite-dimensional θ . Usually we think of θ as a function. The amount of information that θ can capture about the observed data D can grow as the amount of data grows. This makes them more flexible. In this sense, a non-parametric approach does not make any assumption about the functional form; it is very flexible and can take any shape. It could be a very complex function, a combination of an extremely large number of

non-linear functions, a rule like a large margin boundary, or a simple estimation of density or discriminant outcome in the desired input space. Gaussian approaches, k-nearest neighbor and decision trees are examples of non-parametric approaches. The term non-parametric does not mean that such models are completely lacking parameters, but that the number of parameters is flexible and not fixed a priori.

- Finally, semi-parametric modeling is a hybrid of the parametric and non-parametric approaches of statistical models. It may appear at first that semi-parametric models include non-parametric models; however, semi-parametric models are considered to be “smaller” than a completely non-parametric model because we are often interested only in the finite-dimensional component of β . By contrast, in non-parametric models, the primary interest is in estimating the infinite-dimensional parameter. In result, the estimation is statistically harder in non-parametric models compared to semi-parametric models. While parametric models are easy to understand and easy to work with, they fail to give a fair representation of what is happening in the real world. Semi-parametric models allow us to have the best of both worlds: a model that is understandable and offers a fair representation of the messiness that is involved in real life. Semi-parametric regression models take many different structures. One is a form of regression analysis in which a part of the predictors does not take pre-determined forms, and the other part takes known forms with the response. For example, f_1 may be known (assume linear) and f_2 is unknown. In this case, the semi-parametric form will be written as $Y = \beta_0 + \beta_1 x_1 + f(x_2)$. In this setting, the relationship between x_1 and the response is linear, but the relationship between the response and x_2 is unknown. The most illustrative example is the scatter plot smoother. The approach is model-based and utilizes the basic principles like Maximum Likelihood Estimation (MLE). Mixed model-based smoother can be extended to a full hierarchical Bayesian model when analyzed via Markov chain Monte Carlo [185].

Direct implementation of neural network models is based on the use of common architectures and their modifications, such as the multilayer perceptron (MLP), single-layer and multilayer n -dimensional convolutional neural networks (CNNs), and recurrent neural networks (RNNs), and is usually included through the modern LSTM or GRU cell models. MLPs are commonly implemented via dense or fully-connected pre-defined layers in popular APIs such as Keras, PyTorch or MATLAB Deep Learning Toolbox, and can be used on their own to solve a wide range of problems, or as an integral part of hybrid models. Convolutional neural networks excel in image processing tasks such as spectrogram and telescope data processing, strong lensing detection, and feature extraction, and have recently been widely used in GW data processing. Recurrent neural networks demonstrate high efficiency in time series processing tasks, for example, in processing gamma-ray burst data and identifying patterns in changes of observed stars' characteristics. Transformer neural networks, as well as graph neural networks (GNNs)[186], are rapidly gaining popularity in solving problems in various fields, including computer vision and time-series analysis. Each of the mentioned architectures can be part of a more complex organizational structure, such as a Variational Auto-Encoder (VAE) [187–189] or a Generative Adversarial Network (GAN) [190]. More general approaches to machine learning include supervised and unsupervised methods such as Support Vector Machines (SVMs), Random forests and gradient boosting algorithms, for example, XGBoost [191], clustering methods (KMeans and others), K-nearest neighbors algorithm. With the introduction of physics-informed machine learning (PIML) [192–196] and neural networks (PINNs) [197–199], the possibility was opened to combine the flexibility and versatility of neural networks with the accuracy and ability of physical interpretation of partial differential equations. This approach allows us to find numerical data-driven solutions to problems formulated in terms of nonlinear

PDEs. This result is achieved by taking into account physical constraints, patterns and domain knowledge through the use of a neural network as a solver, and by encoding information about the structure and solution of the differential equation in the memory of the neural network during training. The same approach also enables us to solve the problem of the discovery of differential equations based on neural network calculations, both for discrete-time models and continuous-time models, as well as for problems with dynamic and periodic boundary conditions.

5.3. Explainable AI and Knowledge Discovery

In science, the formal definition of a theory is difficult, but commonly it refers to a comprehensive explanation of a subfield of nature that is supported by a body of evidence [200]. In physics, the term theory is generally associated with a mathematical framework derived from a set of basic axioms, which allows us to generate experimentally testable predictions for such a subfield of physics. Typically, these systems are highly idealized, in that the theories describe only certain aspects of reality. Verifiability of a theory has been progressively substituted by falsifiability [201]. Generally speaking, the use of machine learning algorithms increases the level of abstraction realized by physical theories as well as the explainability power of physical theories. Machine learning methods have been remarkably successful for a wide range of application areas in the extraction of essential information from data. An exciting and relatively recent development is the uptake of machine learning in the natural sciences, where the major goal is to obtain novel scientific insights and discoveries from observational data. A prerequisite for obtaining a scientific outcome is domain knowledge, which is needed to gain explainability, but also to enhance scientific consistency. Explainable machine learning in view of applications in the natural sciences appears to play a major role. Three core elements are relevant in this context: transparency, interpretability, and explainability.

Transparency connotes some sense of understanding the mechanism by which the model works. A machine learning approach is transparent if the processes that extract model parameters from training data and generate labels from testing data can be described and motivated by the design approach. For instance, we can obviously describe how stochastic gradient descent works. Generally, to expect a machine learning method to be completely transparent in all aspects is rather unrealistic; usually, there will be different degrees of transparency. For instance, in the case of neural networks, the choice of hyperparameters such as learning rate, batch size, etc., has a more heuristic, non-transparent algorithmic nature. The same remark can be applied to the number of units or layers of the topology of the network. Due to the presence of several local minima, the solution is usually not easily reproducible as well; therefore, the specific solution obtained is not fully algorithmically transparent.

Interpretability means to present some of the properties of machine learning models in terms understandable to a human. Ideally, one should be able to answer a question such as “can we understand on what the machine learning algorithm bases its decision?” Note that, in contrast to transparency, to achieve interpretability, the data is always involved.

Explainability deals with three classes of questions: (1) what-questions, such as “What event happened?”; (2) how-questions, such as “How did that event happen?”; and (3) why-questions, such as “Why did that event happen?”. The goal of the machine learning “user” is very relevant regarding explainability. There are essentially four reasons to seek explanations: to justify decisions, to enhance control, to improve models, and to discover new knowledge. It is important to differentiate between scientific explanations and algorithmic explanations.

Regarding natural sciences, a broad framework leverages unsupervised learning approaches to learn low-complexity representations of physical process observations [202]. A syntax-direct variational autoencoder (SD-VAE) has been recently introduced, where syntax and semantic constraints are used in a generative model for structured data. As an application, the drug properties of molecules are predicted [187]. The learned latent space is visually interpreted, while the diversity of the generated molecules is interpreted using domain expertise. In a similar way, the classical unsupervised algorithms like principal component analysis and the derived interpretable principal components have been used for the exploration of different phases, phase-transitions, and crossovers in classical spin models [203]. Implementations of symbolic systems based on semantic technologies suitable for improving explanations for non-insiders have been adopted. The paper [204] proposes a symbolic regression approach to the study of Gray-Body Factors (GBFs), which play a crucial role in the derivation of Hawking radiation and are recognized for their computational complexity. The authors explore simple analytical forms for the GBFs of the Schwarzschild black hole. To achieve this, authors employed the open-source PySR Python library as the basis for their pipeline, ReGrayssion. While the traditional approach involves determining unknown coefficients or distributions based on observed or synthetic data, symbolic regression results in analytical expressions obtained by machine learning, artificial intelligence, and heuristic algorithms. This approach allows us to identify hidden patterns in observational data that would be lost through purely numerical calculations and that are not detected by traditional fitting methods. Symbolic regression offers a unique opportunity to obtain physically justified and observationally supported expressions that are not directly derived analytically, but also retain the possibility of interpretation. The machine learning-related components, such as input features, hidden layers, and computational units, and predicted output of deep learning models, can be mapped into entities of Knowledge Graphs or concepts and relationships of ontologies (knowledge matching). Traditionally, these ontology artifacts are the results of conceptualizations and practices adopted by experts from various disciplines [205].

5.4. Collider Physics Application

Along the multi-messenger perspective that motivates this review, at least two main lines of investigation can be identified for the consistent transposition of machine learning toolkits, either borrowed from artificial intelligence or developed ad hoc for fundamental physics applications. These two directions concern, data related to the observation of very high-energy cosmic and gamma rays and data associated with gravitational wave observations, respectively. In the former case, particular attention can be devoted to the analysis of very high-energy gamma-ray and cosmic-ray spectra measured by the new generation of cosmic-ray experiments—for a general overview across the energy and intensity frontiers see e.g., [194]. In this context, data-analysis strategies and computational techniques originally developed for collider physics can be naturally adapted and transferred. This represents one of the main objectives of the present program. Many of these techniques are already available within the machine learning framework and can be applied to air-shower reconstruction with minimal modifications, while others may require further methodological development.

Concerning applications inspired by collider physics, we identify the following points of immediate relevance for data analysis and dark-matter phenomenology:

1. Reliable models have been developed to simulate nuclear interactions in contexts such as ion therapy, including the Boltzmann–Langevin One Body (BLOB) model or efforts in QCD matter at extreme conditions [195], which describes heavy-ion interactions up to a few hundreds of MeV [206]. In this approach, the final state is represented as a

probability density function (PDF) describing the likelihood of finding a nucleon at a given point in phase space. Due to the large computational cost of BLOB simulations, deep-learning-based emulation strategies have been developed [188]. These methods rely on the discretization of the PDF and the training of a Variational Auto-Encoder to reproduce it. In particular, Ref. [188] demonstrates the successful emulation of BLOB PDFs for ^{12}C – ^{12}C interactions at 62 MeV, achieving distributions consistent with the original simulations at negligible computational cost. Furthermore, enhanced control over the generation process has been obtained by reorganizing the VAE latent space to explicitly encode the dependence on the impact parameter, together with the training of a dedicated classifier. As discussed in the work program section, this strategy is not restricted to the energy range for which it was originally developed and can be generalized to other physical scenarios.

2. A second major development is the deployment of machine learning algorithms at the trigger level, including implementations on FPGAs and heterogeneous hardware. At the LHC and future colliders, ML-enhanced triggers enable real-time event selection under severe latency and bandwidth constraints, allowing rare or unconventional signatures to be retained already at the earliest stages of data acquisition. Techniques such as quantized neural networks, graph-based inference, and low-latency autoencoders have been successfully ported to FPGA architectures [207,208], demonstrating that sophisticated ML models can operate reliably in real-time experimental environments. This development is of direct relevance for multi-messenger experiments, where similar constraints arise in radio telescopes, gravitational-wave interferometers, and extensive air-shower arrays, and where rapid identification of candidate events is essential for coordinated follow-up observations.
3. Another important class of applications concerns model-agnostic searches for new physics, where the goal is to detect deviations from the Standard Model without committing to specific signal hypotheses. Recently proposed approaches [209] based on anomaly-detection frameworks exploit unsupervised or weakly supervised learning to identify statistically significant excesses in suitably learned feature spaces, rather than in predefined kinematic variables. These methods are particularly powerful in scenarios involving composite dark matter, hidden sectors, or long-lived particles, where the signal morphology may not be well captured by traditional analyses. Crucially, the same philosophy can be applied to multi-messenger data sets, for instance, to search for unexpected spectral or temporal features in cosmic-ray fluxes, neutrino events, or stochastic gravitational-wave backgrounds, without relying on detailed signal templates.
4. A further application concerns the assessment of hadronic interaction models used in air-shower reconstruction from very-high-energy cosmic-ray observations, namely SIBYLL 2.3c [210], QGSJet II-04 [211], and EPOS-LHC [212]. A systematic scan of the corresponding output spaces can be performed using graph neural networks, which have already proven effective in the classification and regression of hadronic jets at the LHC. Compared to CNN-based approaches, GNNs naturally overcome the requirement of data structured on regular 2D or 3D grids and can be applied directly to generic three-dimensional point clouds without imposing geometric symmetries. Similar techniques have been used at ATLAS to analyze atypical hadronic jets, such as those arising from displaced decays of long-lived neutral particles in hidden-sector or hidden-valley models. This approach appears particularly well suited for experiments such as LHAASO and can be further optimized for real-time and trigger-level applications on high-speed processors. Methods based on explainable AI can

- also be integrated to optimize model performance and parameter tuning, enabling a comparative assessment of different hadronic models within a unified framework.
5. Assessment of the three hadronic models hitherto applied to air-shower reconstructions from VHECR observations, i.e., Sybil 2.3c [210], QGSJet II-04 [211] and EPOS-LHC [212], and consequently a scan of the output space, can be achieved within this perspective applying peep neural networks based on graph-neural network already used for classification problems and regression of hadronic showers (jet) of several different types at LHC. This procedure would allow us to solve the problem related to convolutional neural networks (CNN) implemented in deep learning visual applications, of obtaining data structured in symmetric meshes (2D and 3D pixels). Furthermore, the same procedure could be applied to any 3D point clouds, without any specified request for geometric symmetry associated to the detectors. Atypical hadronic jets at ATLAS have already been analyzed by adopting these techniques; this encodes, for instance, hadronic jets produced by the displaced decays of long lifetime neutral particles in hidden models, including either the hidden sector or the “hidden valley”. This situation seems particularly suitable to be adapted to the case of LHAASO, and one can naturally adapt these algorithms to high-speed processors for trigger/real-time analysis applications. Methods connected to explainable AI can also be applied to this purpose, in a different way than standard AI models. Therefore, an optimization strategy can be achieved, while making use of parameter tuning, through the synthesis of the best performing tCNN adapted to the trigger FPGA, as applied to air-shower reconstruction, making use of the three different hadronic models previously highlighted.
 6. A general and essential feature of machine learning applications to collider physics is the ability to handle high-dimensional ensembles of data and to perform fast, real-time processing. A broad review of deep learning applications to LHC physics can be found in [193]. Techniques developed for collider experiments can therefore be directly transferred to the reconstruction of air showers, combining information from Cherenkov detectors, as well as electronic and muonic channels. Deep-learning methods are particularly effective in extracting physical quantities from large data sets when detailed analytical models are unavailable, due to complex detector geometries or large experimental uncertainties. In this respect, deep neural networks act as universal approximators, capable of learning hierarchical representations of the data. Each layer of the network performs a transformation that progressively builds more abstract and informative representations, enabling the extraction of relevant physical features directly from experimental observations.

In summary, collider-driven ML techniques foster a shift from traditional, channel-by-channel analyses toward a representation-based view of experimental data, where events are embedded into latent spaces that capture their essential physical content. Physics-informed ML has also become quite an attractive proposition for collider physics—see e.g., [192–196]. Physics-informed machine learning (PINNs and related approaches) has become an established direction [192]. Altogether, these collider-inspired machine learning strategies provide the computational and conceptual tools needed to efficiently explore high-dimensional parameter spaces and to coherently integrate collider, cosmic-ray, and gravitational-wave data within a unified multi-messenger inference framework for dark matter and physics beyond the Standard Model.

5.5. Illustrative Multi-Messenger Machine Learning Pipeline

The purpose of this section is to provide an illustrative example that should be interpreted as a conceptual demonstration of the proposed framework rather than a full quantitative implementation.

We consider a simplified proof-of-concept scenario in which three representative classes of observables are combined:

1. gamma-ray spectra associated with dark matter annihilation or decay;
2. high-energy neutrino fluxes compatible with IceCube-like sensitivities;
3. stochastic gravitational-wave spectra generated by first-order phase transitions.

Each observational channel is encoded through a dedicated feature extractor (e.g., convolutional or fully connected neural networks), producing latent representations that are subsequently combined within a joint inference architecture. The resulting model maps multi-messenger inputs to underlying physical parameters, such as the dark matter mass, interaction cross section, and phase-transition strength.

We stress that this construction is intended as a conceptual template illustrating how heterogeneous datasets can be consistently integrated. In particular, no detector-level simulations, background modeling, or systematic uncertainties are included at this stage.

A complete quantitative implementation would require:

- realistic mock data generation for each experimental channel,
- detector response modeling,
- training and validation on statistically meaningful datasets,
- evaluation of parameter reconstruction accuracy and uncertainty calibration.

Such an analysis constitutes a substantial, dedicated study and is beyond the scope of the present review. Instead, the goal here is to highlight the architectural principles and to clarify how multi-messenger information can be combined within a machine learning framework.

This schematic example serves to bridge the conceptual discussion of previous sections with the research directions outlined in the concluding roadmap.

5.6. Spanning the Parameter Space of Dark Matter Models with ML

The collider-based applications discussed above illustrate how machine learning enables the extraction of physically meaningful information from high-dimensional data and the identification of non-standard signatures in complex experimental environments. However, the ultimate challenge in the multi-messenger program is not limited to analyzing individual experiments in isolation, but rather to consistently explore and constrain the vast parameter spaces of dark-matter and BSM scenarios across multiple observational channels. In this broader context, machine learning becomes a key instrument for spanning, organizing, and correlating theory spaces with heterogeneous data sets. Here, we focus on ML-based strategies for efficient parameter-space exploration and global inference, emphasizing their role in unifying collider, gravitational-wave, cosmic-ray, and cosmological probes within a single coherent framework.

The Standard Model provides an extremely successful description of particle interactions, yet it does not account for dark matter, neutrino masses, or the baryon asymmetry. We refer to the Introduction for a general discussion of these motivations. The search for new physics is a great challenge of contemporary science, which nonetheless has not been hitherto successful in recovering detectable signatures that could be distinguished from noise backgrounds. Within this framework, machine learning techniques have been recently deployed to overcome related phenomenological problems. Probably the discovery of the Higgs has provided the most striking chance for the exploitation of techniques borrowed from artificial intelligence. Nonetheless, there still exist several impeding issues related

to the statistical limitations of the samples employed for the training, the way of dealing with uncertainties, the reliability of the results and the occurrence of model-dependent interpretations. Within this broad scenario one can identify a plethora of possible searches that can be beneficial to artificial intelligence methods and that can be used for concrete demonstrations of toolkits. These include mainly DM candidate searches, which originated from the decay of new particles, as predicted by supersymmetry, supergravity and other models that we have been reviewing in the preceding sections. Among these possible searches, we can also consider specific parameter space regions of either ALP particles or “dark photons”, as light particles that belong to novel hidden sectors not yet observed because they are too feebly interacting with ordinary matter.

Within this wide perspective, one may deploy machine learning techniques that can be adapted to span the large parameter spaces of the different models hitherto proposed. Parameter spaces can be so extended that concretely assessing specific regions might require a sizable computational time, hence affecting the concrete capability to accomplish phenomenological analyses. Machine learning techniques allow us to avoid these restrictions. Developing both theoretically and numerically the phenomenology of the DM models to be assessed in light of the multi-messenger perspective is an urgent point to unveil the nature of the dark universe.

5.6.1. The Multi-Messenger Approach and Cross-Correlation with Other Channels

The multi-messenger strategy requires us to process joint information arising from the gravitational radiation channel, the electromagnetic radiation channel, the cosmic rays and neutrino channel and the terrestrial collider channel. This strategy involves methods of cosmoparticle physics, which includes system of analysis of multi-messenger cosmological probes for new physics, as well as indirect searches for DM, and new frameworks developed to make contact between deep neural network and quantum field theories. This theoretical treatment aims to specify, in the analysis of the data of multi-messenger astronomy, possible parameters of new physics and make predictions for observable features in cosmic ray experiments. The methodology we are going to specify for this purpose will make heavily use of machine learning techniques, which has become a distinctive and essential feature of phenomenological analyses.

Training samples needed for the implementation of the toolkits that are proper of deep learning are constituted by theoretical predictions. This choice enables a simulation that may allow us to scan the parameter spaces of several different DM models. The machine learning techniques deployed and further developed to this purpose can help restoring the spaces to be scanned, increasing the numerical performances of the phenomenological analysis, while decreasing the required computational time. The different methodologies, deployed along the different observation channels that can be selected, can then be finally recomposed in a unified analytical framework so as to limit the parameter spaces of the theories to be assessed. This cross-fertilizing strategy may then enhance the sensitivity of the scrutinized models to the observations provided by the relevant experiments.

5.6.2. Interaction Networks, First Order Phase Transitions and BSM Models

We are currently living in the data/information era, with new trends emerging from the creation to storing to finally reading and interpreting the information conveyed by the data itself. From the scientific community’s perspective, this trend started to emerge in the last decade because of the rapid investment in many types of experiments/sections. The fast-paced evolution of hardware and the data availability within international co-operations are the driving forces of such a shifting trend. Encoding information and reasoning about objects, relations and physics is one of the main domains humans take for

granted, and it is among the most basic and important aspects of intelligence. Nearly all of the physics we experience on a daily basis can be described in terms of the interaction rules (symmetries, forces, space-time transformations, . . .) between their components (particles, fields, objects, . . .).

A recent work [213] demonstrated a way to leverage the inductive biases from the Interaction Networks (INs) [186] in a Graph Network, to learn models of particle systems at different domains. In this respect, a similar approach can be deployed to tackle the rather challenging problem of using the combined information from particle collisions, cosmic rays and future space-based GWs detectors as a way to probe and constrain BSM theories.

To have a better understanding of the challenges we will face it is first necessary to lay down the basic concepts of GWs generated by the early universe. The GWs produced during the earlier stages of the universe are a consequence of the cosmological first-order phase transition (FOPT), in which the universe changes from a meta-stable high-energy (symmetric) phase to a stable lower energy (broken) phase [214]. This can occur via quantum tunneling or thermal energy transfer, and a consequence of this change is the appearance of bubbles of the broken phase, separated from the surrounding symmetrical phase by a thin wall. These bubbles expand at relativistic velocities and push the surrounding plasma; eventually, some bubbles can also collide or coalesce. This process generates a large amount of energy, which can be converted into GWs [215,216], and the characteristic power-spectrum of these GWs are directly tied with the theory model.

We still do not fully comprehend all the steps our Universe took to transit from the earlier symmetric to the current broken phase, and knowing these steps is crucial for a better understanding of nature's behavior from low to higher energy scales. There is a plethora of BSM models, each one capable of explaining and predicting phenomena associated with particular energy scales—see e.g., Refs. [118–120]. These models are often parametrized in terms of effective field operators, which provides a scheme that offers an easy and powerful method to understand the interactions between the fields involved in a given theory. The effective operators are composed of the interaction fields (scalars, fermions, gauge, etc) and a constant that encodes the information and the energy scale at which the interaction occurs.

One important aspect of such an approach is that we can structure and build any BSM model as an expansion of effective operators, where these new operators can modify the behavior of the phase transition and consequently the GW power spectrum. This aspect can potentially be used as a way to track the effects of these operators on the GW. To perform such a task it is crucial to leverage the inductive biases that emerged from the Interaction Networks in their generalized form, the Graph Network. A main objective is to use Graph Networks as a way to learn from simulated data—these can be produced by means of packages such as CosmoTransition [217] and/or PhaseTracer [218]—the subtle effects in the GW spectrum that can arise from the interaction terms predicted by the BSM. This may indeed allow us to derive a symbolic expression bridging the model parameters to the GW data, with a consequent better understanding of the individual roles of the BSM models at the FOPT during the earlier stages of the Universe. One can then build a robust analysis for the GW from FOPT and a new framework based on machine learning capable of predicting a GW spectrum for a given theory, as well as constrain BSM theories and get new insights from simulations in order to prepare the theoretical background for analyzing the data of future space-based experiments.

5.7. Adapting Models of Particle Physics to Quantum Neural Networks

BSM models of particle physics can be developed by casting them directly on the graph configurations of quantum neural networks. This strategy should be implemented

along the directions outlined by Refs. [219–221], which specify how a topological quantum field theory (TQFT) can be associated to a class of neural networks that represent a quantum version of deep neural networks in machine learning. In TQFT, indeed, one considers principal bundles of the relevant gauge Lie group, and hence takes into account: (i) the holonomies of the Lie algebra connection that is associated to the principle bundle: these are the new configuration variables of the theory; (ii) the fluxes of the frame field, which are the conjugated variables to the holonomies. This theoretical construction is also known as a *BF* theory, serving as a prototype of TQFT. There exists a precise mathematical path one can resort to in order to recast Yang–Mills theory directly in terms of TQFT, as their deformations. This traces back to the formulation of those “extended” (namely, not anymore topological) *BF* theories that are constructed on the principle bundle of the Lie group, the Yang–Mills theory of which we aim at representing—see e.g., [222].

This construction is not the only one provided hitherto in the literature. Pretty recently, it has been suggested in Ref. [213] that a discretization strategy may be applied to systems with finite matter degrees of freedom. The smearing process is supported by graphs that are at the base of the neural network’s construction. This strategy is then extended to encode scalar matter degrees of freedom, not only vector gauge fields. In other words, a discretization procedure can be accounted for, either resorting to the association path summarized in [213] or to the discretization method put forward in [219]. Within the former case, an analogy among graph network theory and Newtonian mechanics allows us to associate nodes to particles, a pair of nodes to a pair of interacting particles, an edge among nodes to forces acting on the latter, and so on. Within this latter case, a detailed discretization procedure can be derived from lattice formulations of field theories [223], including vector fields and scalar fields. Furthermore, fermionic fields can be encoded in the picture, discretizing degrees of freedom at the nodes of the graphs.

Within this scenario, the very novelty of the approach proposed in Refs. [219–221] is that dynamics is implemented on the 2-complexes associated to the neural network structures. Hidden layers are supported on graphs, rather than on nodes, as in previous attempts in the literature [224]. This allows us to provide a functorial dynamical evolution of the states of the theory, which are supported on 1-complexes, i.e., graphs, and is provided by the discretized path-integral on the lattice (scalar, gauge and fermionic) theories considered. Developing such a framework immediately provides some advantages. First, a non-perturbative treatment is naturally implemented within this context, which allows us to accommodate wider inspection into the parameter spaces of the theory without the necessity to account for perturbativity constraints anymore. Furthermore, issues related to gauge invariance, which are often encountered while resorting to cut-off regulators to perform loop-integral evaluations, are easily avoided. These issues can actually affect in a relevant manner the estimated observables of the theory, and thus in general provide a source of ambiguity and a limit of the predictive power of the model. Finally, the discretization on the lattice of quantum fields is well tailored to achieve a natural implementation of the algorithms developed from the side of machine learning, at least the ones for which states of the Hilbert space are supported on the graphs of the quantum neural networks. Hence, a proper span of the parameter space that characterizes models that arise as SM extensions can, in principle, be better accommodated by the development of non-perturbative methods. Nonetheless, the latter can be linked through quantum neural networks to machine learning techniques, making feasible the task of optimizing the parameters of the model [221].

6. Cosmological Inferences from Machine Learning

The complexity and scale of modern cosmological data, particularly from surveys like Euclid [225], LSST [226], and DESI [227], require inference frameworks that are not only

computationally efficient but also capable of managing non-Gaussian, high-dimensional, and noisy data distributions. Machine learning and more specifically deep learning thus emerges as a robust framework capable of enhancing or replacing traditional inference methods, especially when likelihood functions are analytically intractable or prohibitively expensive to evaluate [228–236].

In Bayesian inference, one seeks the posterior distribution

$$p(\theta|\mathbf{d}) = \frac{L(\mathbf{d}|\theta)\pi(\theta)}{Z(\mathbf{d})}, \quad (1)$$

where $L(\mathbf{d}|\theta)$ is the likelihood, $\pi(\theta)$ the prior, and $Z(\mathbf{d})$ the evidence. However, when the likelihood is not analytically tractable, methods such as Approximate Bayesian Computation (ABC) or simulation-based inference become necessary.

Neural networks, particularly artificial neural networks (ANNs), can be trained to emulate forward models such that the mapping $\theta \mapsto \mathbf{d}$ can be learned. Let $f_{\text{NN}} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ represent the trained neural mapping from a parameter space to a simulated data space. Then inference proceeds via inverse learning

$$\theta_{\text{MAP}} = \arg \max_{\theta} p(\theta|f_{\text{NN}}^{-1}(\mathbf{d}_{\text{obs}})). \quad (2)$$

This approach, known as likelihood-free inference (LFI) [237–240], enables posterior sampling via emulators such as the AGB (Artificial Gaussian Bayesian) [241–244] networks used in recent cosmological analyses. We illustrate likelihood-free inference in Figure 1 by reconstructing a posterior distribution using only simulations and a distance-based kernel, without requiring a closed-form likelihood.

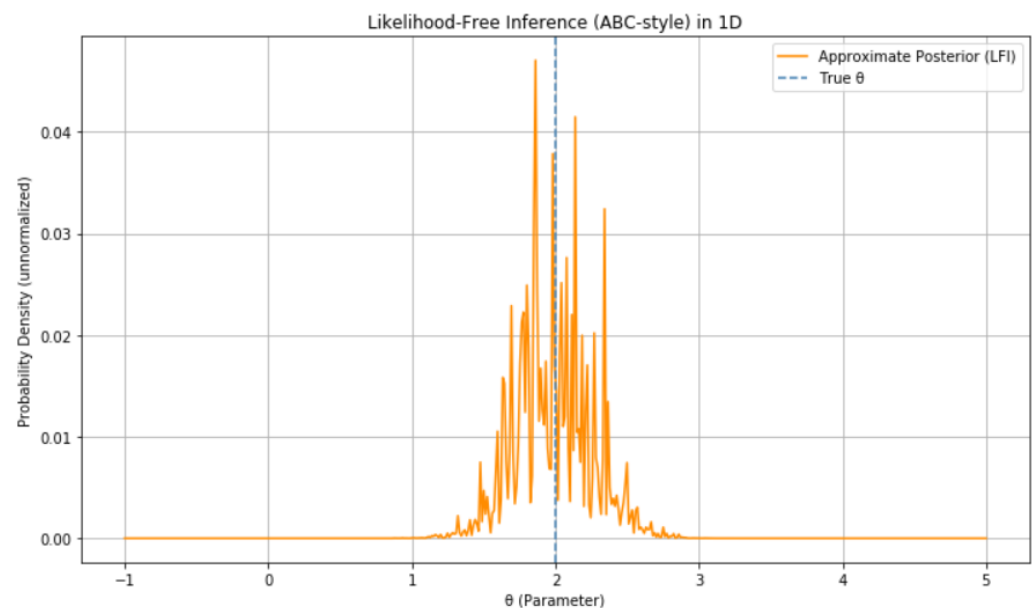


Figure 1. Demonstration of likelihood-free inference (LFI) using a simulation-based approximate posterior in a 1D toy cosmological parameter estimation task. Unlike traditional Bayesian inference, which requires an analytical likelihood, this approach uses distance-weighted simulation outputs to recover the posterior around the true parameter value. Such methods are foundational in machine learning-based cosmological inference pipelines where forward simulations are available but explicit likelihoods are intractable.

Bayesian Neural Networks (BNN) offer probabilistic predictions by placing a distribution over the weights w of the network. Given data D , the predictive distribution for a new input x^* is

$$p(y^*|x^*, D) = \int p(y^*|x^*, w)p(w|D) dw. \quad (3)$$

This integral is typically approximated using variational inference or Monte Carlo dropout. BNNs have been employed for redshift estimation, modified gravity model selection, and classification of theoretical cosmological models from power spectra [245]. They provide not only point estimates but also credible intervals, crucial for cosmological parameter constraints.

One can also consider Convolutional Neural Networks, which are employed to directly map image-based cosmological observables (e.g., weak lensing maps, 21 cm intensity cubes) to parameters. For input tensors $X \in \mathbb{R}^{H \times W \times C}$, CNNs use convolutional layers defined as:

$$(X * K)_{i,j} = \sum_{m,n} X_{i+m,j+n} \cdot K_{m,n}, \quad (4)$$

where K is the convolution kernel. This allows feature extraction without the need for explicit summary statistics. Applications include axion mass constraints, GW parameter estimation, and weak lensing analyses [246–248]. In Figure 2 we simulate synthetic 2D sky-like maps with statistical features tied to a cosmological parameter, Ω_m , and train a CNN to regress that parameter.

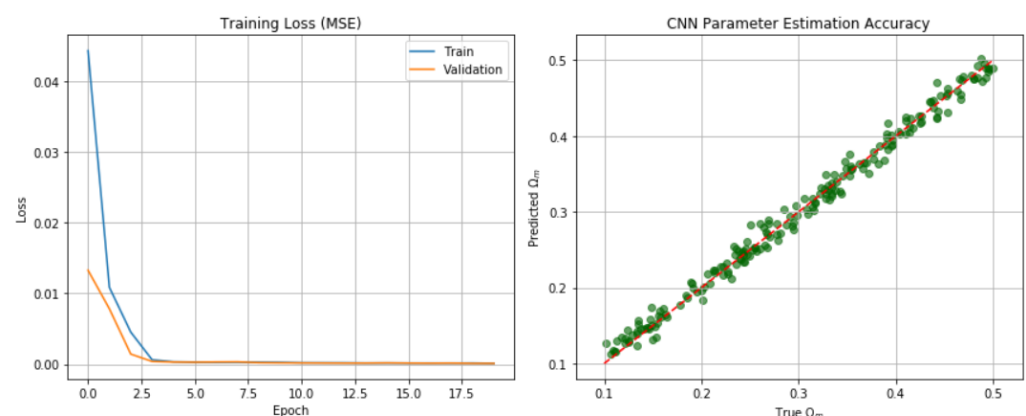


Figure 2. Convolutional Neural Network (CNN) trained to estimate the cosmological parameter Ω_m directly from synthetic 2D “sky map” images. The (**left panel**) shows the training and validation mean squared error (MSE) loss over epochs, indicating successful learning. The (**right panel**) compares predicted and true Ω_m values on held-out data, demonstrating the model’s strong regression performance. This illustrates the potential of CNN-based inference in cosmology.

The left panel shows rapid convergence of training and validation loss, indicating stable learning. The right panel plots true vs predicted Ω_m , demonstrating the CNN’s high accuracy and generalization. This exemplifies how deep learning enables direct cosmological inference from image-like observables, bypassing handcrafted summary statistics.

One also employs Normalizing Flows (NFs) [249] models, which take in complex, high-dimensional distributions by transforming a simple base distribution $p_z(z)$ (usually Gaussian) via a series of invertible and differentiable mappings f_i

$$\theta = f_n \circ \dots \circ f_1(z), \quad z \sim \mathcal{N}(0, I). \quad (5)$$

The transformed distribution $p(\theta)$ is then obtained via the change-of-variables formula:

$$p(\theta) = p_z(z) \left| \det \left(\frac{\partial z}{\partial \theta} \right) \right|. \quad (6)$$

This formulation allows flexible modeling of cosmological posteriors. Frameworks like EMUFLOW use NFs to combine constraints from multiple datasets and mitigate dimensionality issues in joint posterior modeling [250–253].

Another possible avenue of significant interest in the cosmological regard is that of diffusion models [253,254]. These models construct posteriors by reversing a stochastic noise process. Starting from white noise, they iteratively “denoise” to generate samples from a target distribution [254]. Let x_T be a noise-corrupted sample and x_0 the original data, then the forward process is

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1 - \beta_t}x_{t-1}, \beta_t \mathbf{I}), \quad (7)$$

and the generative model learns the reverse transitions $p(x_{t-1}|x_t)$. These models are well-suited for large-scale structure inference and CMB simulations, with high fidelity and tractable uncertainty quantification [255,256]. What we have hitherto discussed does not encompass any novel method with respect to previously existing ones, but is rather supposed to be an overview of the practices covered in the literature so far for ML approaches to Cosmology.

Reference [257] is devoted to the development of a hierarchical pipeline for classification using deep learning. The authors discuss the new enhancements of ResNet-based deep learning code—AresGW, and employ the modified model to detect new GW candidate events in data from a network of interferometric detectors. The original model consists of 27 residual blocks, comprising two convolutional layers. Five blocks are used to decrease the dimensionality, and two blocks are used to reduce the number of channels and reduce the problem to a binary classification problem. The improved model includes a set of hierarchical conditional filters for different frequencies. The proposed development made it possible to reduce the number of false alarms by at least 70% in the Default Low-Pass triggers class, and by 90% in the Selective Noise Rejection triggers class, compared to the number of false alarms in the Default Low-Pass triggers class alone. The proposed algorithm made it possible to identify 10 events with a probability $p_{astro} \geq 0.99$ on a test data and prior events GW190916_200658, GW200305_084739, GW190906_054335, GW200106_134123, with probabilities 1.00, 1.00, 0.99, 0.95, respectively.

Detailed accuracy estimates are provided in Ref. [258]. The authors propose a high-precision supervised interpolation model using AutoGluon to infer ALP DM parameters from reconstructed NS mass–radius curves. The developed model provided $R^2 > 0.998$ for axion-like particles mass estimations, with MAE being equal to 3.1661, RMSE = 10.1552, MAPE = 2.0339 %. The estimates for DM Fermi momentum were 0.9992, 3.73×10^{-5} , 3.99×10^{-4} , 0.3399% for R^2 , MAE, RMSE, MAPE, respectively. A hybrid approach was proposed in [189], where the authors use VAE to learn low-dimensional latent embeddings of three synthetic datasets for black hole masses and spins. Extracted data are processed using a set of machine learning algorithms, specifically, KMeans and Random forest. The supervised random forest algorithm provided accuracy estimates within the 0.8–0.95 range, while the unsupervised KMeans approach achieved only 0.51–0.56.

Reference [197] then proposed a physics-informed neural network based on CNN architecture that can infer the mass parameters of binary black hole merger systems to within 7% from only the time-frequency representation of the GW signal. The problem of modeling gravitational collapse, a critical phenomenon in astrophysics and cosmology, is discussed in [198], where an MLP-based PINN is proposed. The authors introduce

two Schrödinger–Poisson informed neural networks that solve the nonlinear equations to simulate the gravitational collapse of fuzzy dark matter (FDM) in both 1D and 3D settings.

Reference [259] finally provided a comparison between an autoencoder-based Bayesian inference algorithm for the GWB measurement and the cross-correlation method. The proposed MSMH autoencoder achieved good sensitivity much faster than the cross-correlation method, being able to estimate a weaker GWB component simultaneously. The approach allowed authors to measure a CBC GWB of amplitude 10^{-9} and a cosmological GWB of amplitude 1.3×10^{-10} . Multiple models for forward and inverse modeling of the Boltzmann equation for freeze-in DM particle yield in alternative cosmology are provided in [199]. The authors employed multiple dense layers in PINN architectures, which are restricted by Boltzmann equation and initial conditions, particle interactions and cosmological constraints, to derive the estimates for the pair-wise relationship between the power-law exponent and particle interactions. Additionally, Bayesian methods are used to quantify the epistemic uncertainty of theoretical parameters found in inverse problems. The most promising results are obtained for inverse problems. Figure 3 from [199] (originally, figure number 6 there) illustrates the approach.

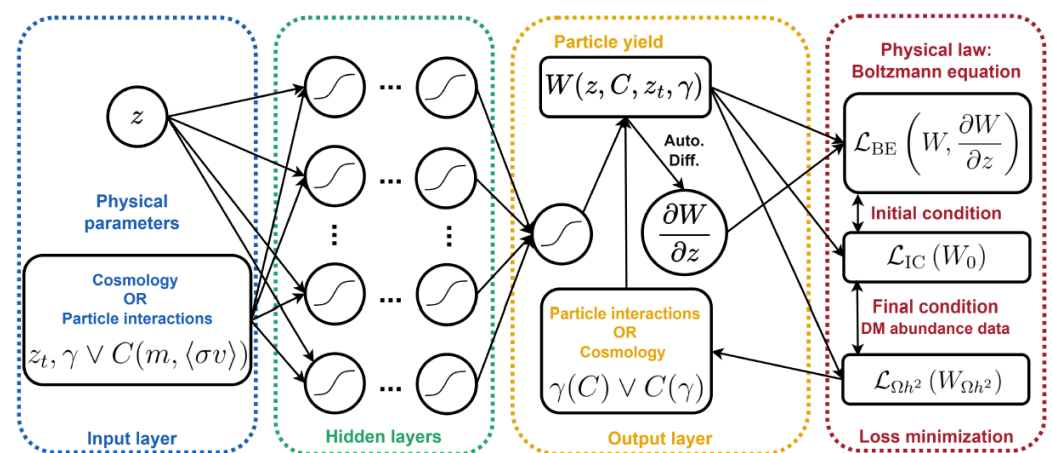


Figure 3. Figure 6 from [199], shows a schematic representation of the PINN structure. This is relevant to tackle inverse problems while modeling the Boltzmann equation for freeze-in DM particle yield, in alternative cosmology.

In general, the successful deployment of ML in cosmology can introduce a new paradigm in which

- Posterior estimation is data-driven, relying on accurate training simulations $\{(\theta_i, d_i)\}$;
- Forward models (Boltzmann solvers, N-body simulations) are replaced or accelerated by differentiable emulators;
- Likelihood-free inference replaces Gaussian assumptions with flexible neural mappings.

However, the mathematical rigor of these methods still faces challenges:

1. **Calibration of Uncertainties:** ML posteriors must be statistically calibrated to avoid overconfident predictions.
2. **Explainability:** Black-box networks can obscure physical interpretability. Work on physics-informed architectures and interpretable flows is ongoing.
3. **Generalization:** Trained networks must generalize across cosmologies; adversarial training or Bayesian ensembles may help.
4. **Posterior consistency:** For ML-based posteriors $p_{\text{ML}}(\theta)$, we require convergence in probability to the true posterior as $N \rightarrow \infty$ (in sample size), which remains an open theoretical problem.

The authors of [257] note the difficulty of solving the binary classification problem (distinguishing a GW signal from a noise signal) using classical methods based on minimizing the mean squared error or maximizing the logarithmic likelihood function (as in minimizing cross-entropy). Events of interest in GW analysis are rare, necessitating working with unbalanced datasets. Classical loss functions in these situations can exhibit insufficient classification accuracy and suppress the detection of events in the tails of the distribution. However, machine learning approaches can mitigate these issues by using alternative loss functions, such as the Kullback-Leibler divergence, as well as hierarchical and multi-stage methods and models. Another important aspect is that many machine learning models, in particular those based on convolutional neural networks and transformer neural networks, allow for the efficient use of parallel computing to obtain results significantly faster than traditional approaches based on pure Bayesian inference. For example, the model, proposed in [248], allowed the generation of 10,000 posterior samples in just 0.05 s using CNN, running on an NVIDIA GeForce RTX 2080 Ti GPU. The proposed approach allows us to reduce time costs by several orders of magnitude compared to traditional methods, while maintaining comparable accuracy, with only a single outlier, and reducing the likelihood of effects caused by the instrument bottleneck during the inference. Another example is [260]. Here, the authors introduce a technique to enhance the reliability of GW parameter estimation using attention maps, obtained from spectrograms, in order to provide the model with the ability to focus on the most important data sections. The total estimation was performed in six minutes on an NVIDIA GeForce RTX 3090 using simulated data for GW150914, proving that machine learning can significantly reduce the computation time. Reference [261] proposes a hybrid transformer deep learning network for PBH populations probing using GW data, which could be run on a single GPU. The accuracy is comparable to that obtained via hierarchical Bayesian inference. The proposed model's inference takes $\mathcal{O}(1)$ second on a GPU and 10 s on a CPU. The authors provide an estimate for an MCMC sampler, which takes about one week to converge.

It is also worth noting that the development of likelihood-free inference (LFI) and simulation-based inference (SBI) has progressed rapidly in recent years, becoming a central methodological framework in modern cosmological data analysis. Systematic reviews of SBI techniques have highlighted how neural density estimators, neural ratio estimation and neural posterior estimation have significantly improved the efficiency and scalability of inference pipelines for high-dimensional datasets [262,263], like the ones relevant for cosmology [264]. Similarly, Bayesian neural networks and probabilistic deep learning approaches have matured to the point where they can provide calibrated uncertainties and principled posterior approximations, allowing their application to problems such as cosmological parameter estimation, model selection and large-scale structure reconstruction. There are comprehensive discussions of these developments, including methodological comparisons in recent reviews and methodological papers [265–269].

7. Future Perspectives

The directions outlined in this work can be organized into a concrete research program:

1. development of joint multi-messenger pipelines combining data from LHAASO, IceCube, Fermi-LAT and gravitational-wave observatories;
2. implementation of machine learning frameworks for global parameter inference across heterogeneous datasets;
3. systematic exploration of benchmark dark matter models within this unified framework;
4. application of anomaly-detection techniques for model-independent searches;

5. investigation of next-generation methods, including physics-informed and quantum-enhanced machine learning.

These directions outline a possible path toward transforming multi-messenger astrophysics.

In concluding this review of theoretical and numerical methods, we mention a few urgent points to be developed in the multi-messenger investigation on DM.

First of all, from the side of the DM models to be constrained, specific features of several types of SM extensions must be carefully considered, focusing on the additional fermionic degrees of freedom and effects that are associated with the presence and possible manifestation of DM particles in various reactions. Several points shall be taken into account.

- Qualitative and quantitative analyses of the processes of high-energy neutrinos (with energies in the range from 10 to 1000 TeV) scattering on the DM particles—with the production in the final state of both electronic and muon neutrinos and/or the acceleration of the target (so-called scattered-up reactions)—can be provided. In the scatterings, charged components of the hyperpion triplet can be produced with subsequent decay. The possibility of registering the products of such reactions at the LHAASO and IceCube is then a direction to be investigated.
- An in-depth study of the processes of high-energy proton and photon interactions with the DM in the Galaxy halo can be carried out. These reactions can result in the production of charged partners of the DM particles, and their decays can generate neutral stable particles together with high-energy fluxes of electrons, positrons and neutrinos. These fast secondary particles can produce specific EAS with a low muon content accompanied by neutrinos of various flavors and/or heavy neutral stable particles.
- The dependence of the secondary neutrino and lepton fluxes on the spatial distribution of the DM in the Galaxy can be considered in detail. In particular, one can study the scattering of cosmic rays by inhomogeneities in the Galactic halo. This investigation can be considered in connection with the problem of studying the internal dynamics of DM in the Galaxy.
- Since the hypercolor model has a whole set of H-hadrons that are located higher in mass than the stable DM particles, there should also be excited unstable states of di-hyperquarks. They can also manifest themselves in scattering reactions. To clarify the SM extension type, the study of the unstable H-hadrons and the analysis of their excitation channels and decay modes are very important. Signals of such decays will also be fluxes of ordinary (decaying) mesons, accompanied possibly by stable neutral heavy DM objects. Consequently, one can expect the appearance of EAS products, rare and specific in composition and angular distribution, which can be detected at the LHAASO, HAWK and HESS facilities.
- For a complete mass spectrum study in the H-color model, it is beneficial to consider vacuum condensate structures in the SM extensions with additional fermions. In a sense, the model contains a kind of QCD duplication but with a smaller number of quark flavors. The symmetry violation also requires the introduction of an H-quark vacuum condensate along with a nonzero vacuum condensate of H-gluons. Thus, it becomes possible to study the H-hadron characteristics using a previously unknown analog of the QCD sum rules. At the same time, the presence of a certain hierarchy in the structure of vacuum condensates of H-color models, as well as the stability of their vacuum state, can be also investigated. For issues related to the conditions for the applicability of these methods, the possibility to extract information from new types of sum rules and the data on the masses of new heavy states shall be investigated within this framework.

- Interaction processes (inelastic and quasi-elastic scattering) of high-energy particles of dark and ordinary matter can be investigated, and detailed analyses of the possibilities to register the heavy metastable hadrons signals can be carried out: the conditions and channels for the production of new heavy hadrons, the types and intensity of their annihilation signals shall be considered in detail.
- It is also essential to carry out a study of the DM halo interaction with the gas-dusty and solid components of the Galaxy. Such an analysis may allow us to obtain (after a quantitative consideration and classification of types of signals) additional information on the possible detection of DM particles.
- The basis for a detailed study of the conditions and specificity of the luminosity of hadronic DM is an analysis of the hyperfine splitting between excited states. This is crucial to characterize the possible observed manifestations of DM both in the hadronic and in the hypercolor scenario. For this purpose, the character of the splitting and the conditions for the metastability of excited states of new hadrons are equally important for various scenarios of the SM extensions considered.

Concerning the numerical toolkits exploited to carry out phenomenological analyses, specific techniques derived from ML can be applied, in a multi-messenger perspective, to achieve air-shower reconstruction of cosmic ray detection and to scan the parameter spaces of DM models from the future measurements of the GW stochastic background, as produced by FOPTs. We list below a few of the most relevant methods that can be implemented, consistently with community-level directions summarized in [196].

- Enhanced VAE techniques currently under development allow us to structure the latent space and control it much better than in previous studies [270–272]. Elements of the latent space that are unique will be orthogonal to the different latent variables, hence allowing for more efficient simulations. These techniques can be applied to multiple contexts, without being limited to the low-energy regime investigated so far. Instead, they belong to a framework of generative deep learning that is useful to simulate any kind of process. While it is possible to use a deep learning approach to emulate a model developed to simulate nuclear reactions in any energy regime, toolkits implementing the generation part in C++ must still be developed, and then interfaced with a common Monte Carlo toolkit such as Geant4.
- Tests can be carried out employing gradient boosted decision trees (XGBoost), Gaussian processes based on Bayesian regression, deep neural networks (DNN) and convolutional neural networks (CNN). Improved sensitivities by a factor of 2 to 5 in XGBoost can be achieved within several models—see e.g., Ref. [191]. These preliminary results can be consolidated with a systematic comparison of the methods implemented, and with related tests of their robustness against input variations. In particular, in order to overcome possible biases, generative adversarial networks [190] will be used to study the dependencies on simulated Monte Carlo (MC) samples of the training processes. Concerning the issues related to explainable AI in air-shower reconstructions, studies may be developed that are based on convolutional neural network (CNN) for image classification that are trained to distinguish background processes from signals that can be used to map clusters of hadrons (jets) in 3D. This methodology has already been checked against overtraining, ensuring that an accuracy above 90% can be achieved. This is actually susceptible to factor 2–3 improvements with respect to previous methods [273]. Along this direction one can also adapt Graph Neural Network (GNN) techniques [274], hence enhancing sensitivities to new physics through machine learning techniques, while maintaining the transparency of the involved section processes. Thus, the implementation of explainability becomes crucial to

ensure the correct physical interpretation and the scientific consistency of results that are derived.

- A novel computational framework based on the state of the art of deep learning techniques can be applied to the study of cosmological phase transitions induced by thermal corrections that may leave gravitational footprints in the form of a stochastic GW background. Such a GW background may carry imprints of phase transitions above the electroweak scale, offering access to physics beyond the reach of collider experiments. It is possible to interface model building and Monte Carlo software tools while applying Deep Learning techniques in order to combine all available theoretical and phenomenological information. This will include an option to evaluate the impact of collider constraints or infer predictions for the primordial GW stochastic background. This knowledge can be combined, in this way, with the experimental measurements of the triple-Higgs coupling, to help point the path toward a consistent theory of the fundamental interactions.

Author Contributions: Conceptualization, M.K. (Maxim Khlopov); Methodology, A.A., K.B., V.B., T.B., D.C., F.F., K.J., A.K., V.K., M.K. (Maxim Krasnov), A.M. (Atharv Mahajan), A.M. (Andrey May-ov), D.S. and V.S.; Writing—original draft, J.L.S.; Writing—review & editing, S.G., A.M. (Antonino Marciandò), A.M. (Antonio Morais), R.P. and O.T. All authors have read and agreed to the published version of the manuscript.

Funding: The research of A.K., T.B., M.K. (Maxim Khlopov), D.S. and V.K. was carried out in the Southern Federal University with financial support from the Ministry of Science and Higher Education of the Russian Federation (State contract FENW-2026-0028). The work of V.S. was funded by the Ministry of Science and Higher Education of the Russian Federation, Project “Studying physical phenomena in the micro- and macro-world to develop future technologies” FSWU-2026-0010.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Planck Collaboration. Planck 2018 results. I. Overview and the cosmological legacy of Planck. *Astron. Astrophys.* **2020**, *641*, A1.
2. ACT Collaboration. The Atacama Cosmology Telescope: DR4 Maps and Cosmological Parameters. *J. Cosmol. Astropart. Phys.* **2020**, *12*, 047.
3. Li, H.; Li, S.-Y.; Liu, Y.; Li, Y.-P.; Cai, Y.; Li, M.; Zhao, G.-B.; Liu, C.-Z.; Li, Z.-W.; Xu, H.; et al. AliCPT: A ground-based CMB polarization experiment. *Natl. Sci. Rev.* **2019**, *6*, 145–154. <https://doi.org/10.1093/nsr/nwy019>.
4. Addazi, A.; Belotsky, K.; Beylin, V.; Cirelli, M.; DiSciaccio, G.; Esmaili, A.; Fornengo, N.; Gan, Q.; Kachekriess, M.; Khlopov, M.; et al. Dark Matter Searches for Heavy Dark Matter with LHAASO. In Proceedings of the 37th International Cosmic Ray Conference (ICRC 2021), Virtual, 12–23 July 2021; p. 474. <https://doi.org/10.22323/1.395.0574>.
5. HAWC Collaboration. The HAWC Gamma-Ray Observatory: Design, Calibration, and Operation. *Astrophys. J.* **2017**, *843*, 39.
6. H.E.S.S. Collaboration. Observations of the Crab nebula with H.E.S.S. *Astron. Astrophys.* **2006**, *457*, 899–915.
7. IceCube Collaboration. Observation of High-Energy Astrophysical Neutrinos in Three Years of IceCube Data. *Phys. Rev. Lett.* **2014**, *113*, 101101. <https://doi.org/10.1103/PhysRevLett.113.101101>.
8. Evans, L.; Bryant, P. LHC Machine. *J. Instrum.* **2008**, *3*, S08001.
9. Apollinari, G.; Béjar Alonso, I.; Brüning, O.; Fessia, P.; Lamont, M.; Rossi, L.; Taviani, L. High-Luminosity Large Hadron Collider (HL-LHC): Technical Design Report. *CERN Yellow Rep. Monogr.* **2017**, *4*. <https://doi.org/10.23731/CYRM-2017-004> (accessed on 21 April 2026).
10. CEPC Study Group. CEPC Conceptual Design Report: Volume 2—Physics and Detector. *arXiv* **2018**, arXiv:1811.10545.
11. LIGO Scientific Collaboration and Virgo Collaboration. Observation of Gravitational Waves from a Binary Black Hole Merger. *Phys. Rev. Lett.* **2016**, *116*, 061102. <https://doi.org/10.1103/PhysRevLett.116.061102>.
12. Amaro-Seoane, P.; Audley, H.; Babak, S.; Baker, J.; Barausse, E.; Bender, P.; Berti, E.; Binetruy, P.; Born, M.; Bortoluzzi, D.; et al. Laser Interferometer Space Antenna. *arXiv* **2017**, arXiv:1702.00786.

13. Kawamura, S.; Ando, M.; Seto, N.; Sato, S.; Nakamura, T.; Tsubono, K.; Kanda, N.; Tanaka, T.; Yokoyama, J.; Funaki, I.; et al. The Japanese space gravitational wave antenna: DECIGO. *Class. Quantum Gravity* **2011**, *28*, 094011. <https://doi.org/10.1088/0264-9381/28/9/094011>.
14. Crowder, J.; Cornish, N.J. Beyond LISA: Exploring future gravitational wave missions. *Phys. Rev. D* **2005**, *72*, 083005. <https://doi.org/10.1103/PhysRevD.72.083005>.
15. Nan, R.; Li, D.; Jin, C.; Wang, Q.; Zhu, L.; Zhu, W.; Zhang, H.; Yue, Y.; Qian, L. The Five-Hundred-Meter Aperture Spherical Radio Telescope (FAST) Project. *Int. J. Mod. Phys. D* **2011**, *20*, 989–1024. <https://doi.org/10.1142/S0218271811019335>.
16. Dewdney, P.E.; Hall, P.J.; Schilizzi, R.T.; Lazio, T.J.L.W. The Square Kilometre Array. *Proc. IEEE* **2009**, *97*, 1482–1496. <https://doi.org/10.1109/JPROC.2009.2021005>.
17. Hobbs, G.; Archibald, A.; Arzoumanian, Z.; Backer, D.; Bailes, M.; Bhat, N.D.R.; Burgay, M.; Burke-Spolaor, S.; Champion, D.; Cognard, I.; et al. The International Pulsar Timing Array project: Using pulsars as a gravitational wave detector. *Class. Quantum Gravity* **2010**, *27*, 084013. <https://doi.org/10.1088/0264-9381/27/8/084013>.
18. Di Sciascio, G. The LHAASO experiment: From Gamma-Ray Astronomy to Cosmic Rays. *Nucl. Part. Phys. Proc.* **2016**, *279–281*, 166–173. <https://doi.org/10.1016/j.nuclphysbps.2016.10.024>.
19. Bai, X.; Bi, B.Y.; Bi, X.J.; Cao, Z.; Chen, S.Z.; Chen, Y.; Chiavassa, A.; Cui, X.H.; Dai, Z.G.; della Volpe, D.; et al. The Large High Altitude Air Shower Observatory (LHAASO) Science White Paper. *arXiv* **2019**, arXiv:1905.02773.
20. Madejski, G.G.; Sikora, M. Gamma-Ray Observations of Active Galactic Nuclei. *Annu. Rev. Astron. Astrophys.* **2016**, *54*, 725. <https://doi.org/10.1146/annurev-astro-081913-040044>.
21. Ajello, M.; Baldini, L.; Ballet, J.; Bastieri, D.; Becerra Gonzalez, J.; Bellazzini, R.; Berretta, A.; Bissaldi, E.; Bonino, R.; Brill, A.; et al. The Fourth Catalog of Active Galactic Nuclei Detected by the Fermi Large Area Telescope—Data Release 3. *Astrophys. J. Suppl. Ser.* **2022**, *263*, 24. <https://doi.org/10.3847/1538-4365/ac9523>.
22. Addazi, A.; Ketov, S.; Khlopov, M.; Marciano, A. Physics of Superheavy Dark Matter in Supergravity. *Int. J. Mod. Phys. D* **2018**, *27*, 1841011.
23. Peccei, R.D.; Quinn, H.R. CP Conservation in the Presence of Instantons. *Phys. Rev. Lett.* **1977**, *38*, 1440.
24. Wilczek, F. Problem of Strong p and t Invariance in the Presence of Instantons. *Phys. Rev. Lett.* **1978**, *40*, 279.
25. Weinberg, S. A New Light Boson? *Phys. Rev. Lett.* **1978**, *40*, 223.
26. Kim, J.E. Weak Interaction Singlet and Strong CP Invariance. *Phys. Rev. Lett.* **1979**, *43*, 103.
27. Shifman, M.A.; Vainshtein, A.I.; Zakharov, V.I. Can Confinement Ensure Natural CP Invariance of Strong Interactions? *Nucl. Phys. B* **1980**, *166*, 493.
28. Zhitnitsky, A.R. On Possible Suppression of the Axion Hadron Interactions. *Sov. J. Nucl. Phys.* **1980**, *31*, 260.
29. Wilczek, F. Axion and Family Symmetry Breaking. *Phys. Rev. Lett.* **1982**, *49*, 1549.
30. Berezhiani, Z.G.; Khlopov, M.Y. The Theory of Broken Gauge Family Symmetry. *Z. Für Phys. C* **1991**, *49*, 73. <https://doi.org/10.1007/BF01570798>.
31. Chikashige, Y.; Mohapatra, R.N.; Peccei, R.D. Are There Real Goldstone Bosons Associated with Broken Lepton Number? *Phys. Lett. B* **1981**, *98*, 265.
32. Gelmini, G.B.; Roncadelli, M. Left-Handed Neutrino Mass Scale and Spontaneously Broken Lepton Number. *Phys. Lett. B* **1981**, *99*, 411.
33. Berezhiani, Z. Through the Looking Glass: Alice’s Adventures in Mirror World. *Eur. Phys. J. C* **2016**, *76*, 705. <https://doi.org/10.1140/epjc/s10052-016-4564-0>.
34. Witten, E. Some Properties of O(32) Superstrings. *Phys. Lett. B* **1984**, *149*, 351.
35. Conlon, J.P. The QCD Axion and Moduli Stabilisation. *J. High Energy Phys.* **2006**, *05*, 078. <https://doi.org/10.1103/PhysRevLett.38.1440>.
36. Svrcek, P.; Witten, E. Axions in String Theory. *J. High Energy Phys.* **2006**, *06*, 051.
37. Peccei, R.D.; Quinn, H.R. Constraints Imposed by CP Conservation in the Presence of Instantons. *Phys. Rev. D* **1977**, *16*, 1791. <https://doi.org/10.1103/PhysRevD.16.1791>.
38. Dicus, D.A.; Kolb, E.W.; Teplitz, V.L.; Wagoner, R.V. Astrophysical Bounds on Axions. *Phys. Rev. D* **1978**, *18*, 1829.
39. Raffelt, G.; Stodolsky, L. Mixing of the Photon with Low-Mass Particles. *Phys. Rev. D* **1988**, *37*, 1237.
40. Andriamonje, S.; Aune, S.; Autiero, D.; Barth, K.; Belov, A.; Beltrán, B.; Bräuninger, H.; Carmona, J.M.; Cebrián, S.; Collar, J.I.; et al. (CAST Collaboration) An Improved Limit on the Axion-Photon Coupling from the CAST Experiment. *J. Cosmol. Astropart. Phys.* **2007**, *0704*, 010.
41. De Angelis, A.; Roncadelli, M.; Mansutti, O. Evidence for a New Light Boson from Cosmological Gamma-Ray Propagation? *Phys. Rev. D* **2007**, *76*, 121301.
42. De Angelis, A.; Mansutti, O.; Persic, M.; Roncadelli, M. Photon Propagation and the VHE Gamma-Ray Spectrum of Blazars. *Mon. Not. R. Astron. Soc.* **2009**, *394*, L21.

43. Mirizzi, A.; Montanino, D. Stochastic Conversion of TeV Photons into Axion-Like Particles in Extragalactic Magnetic Fields. *J. Cosmol. Astropart. Phys.* **2009**, *12*, 004.
44. Simet, M.; Hooper, D.; Serpico, P.D. The Milky Way as a Kiloparsec-Scale Axion Detector. *Phys. Rev. D* **2008**, *77*, 063001.
45. Mirizzi, A.; Raffelt, G.G.; Serpico, P.D. Photon–Axion Conversion as a Mechanism for Supernova Gamma-Ray Emission. *Lect. Notes Phys.* **2008**, *741*, 115.
46. Nikishov, A. Absorption of High-Energy Photons in the Universe. *Sov. Phys. JETP* **1962**, *14*, 393.
47. Fazio, G.G.; Stecker, F.W. Predicted High Energy Break in the Isotropic Gamma-Ray Spectrum. *Nature* **1970**, *226*, 135.
48. Aharonian, F. *Very High Energy Cosmic Gamma Radiation*; World Scientific: Singapore, 2004.
49. Dolgov, A.; Freese, K. Calculation of particle production by Nambu-Goldstone bosons with application to inflation reheating and baryogenesis. *Phys. Rev. D* **1995**, *51*, 2693–2702. <https://doi.org/10.1103/PhysRevD.51.2693>.
50. Dolgov, A.; Freese, K.; Rangarajan, R.; Srednicki, M. Baryogenesis during reheating in natural inflation and comments on spontaneous baryogenesis. *Phys. Rev. D* **1997**, *56*, 6155–6165. <https://doi.org/10.1103/PhysRevD.56.6155>.
51. Cohen, A.G.; Kaplan, D.B. Spontaneous Baryogenesis. *Nucl. Phys. B* **1988**, *308*, 913–928.
52. Zeldovich, Y.B.; Novikov, I.D. The Hypothesis of Cores Retarded during Expansion and the Hot Cosmological Model. *Astron. Zhurnal* **1967**, *10*, 602.
53. Hawking, S.W. Gravitationally Collapsed Objects of Very Low Mass. *Mon. Not. R. Astron. Soc.* **1971**, *152*, 75–78.
54. Carr, B.J.; Hawking, S.W. Black Holes in the Early Universe. *Mon. Not. R. Astron. Soc.* **1974**, *168*, 399–415. <https://doi.org/10.1093/mnras/168.2.399>.
55. Hawking, S.W.; Moss, I.G.; Stewart, J.M. Bubble Collisions in the Very Early Universe. *Phys. Rev. D* **1982**, *26*, 2681–2693. <https://doi.org/10.1103/PhysRevD.26.2681>.
56. Carr, B.; Kühnel, F. Primordial Black Holes as Dark Matter: Recent Developments. *Annu. Rev. Nucl. Part. Sci.* **2020**, *70*, 355–394. <https://doi.org/10.1146/annurev-nucl-050520-125911>.
57. Carr, B.; Kohri, K.; Sendouda, Y.; Yokoyama, J. Constraints on Primordial Black Holes. *Rep. Prog. Phys.* **2021**, *84*, 116902.
58. De Luca, V.; Desjacques, V.; Franciolini, G.; Riotto, A. The Clustering Evolution of Primordial Black Holes. *J. Cosmol. Astropart. Phys.* **2020**, *11*, 028. <https://doi.org/10.1088/1475-7516/2020/11/028>.
59. Belotsky, K.M.; Dokuchaev, V.I.; Eroshenko, Y.N.; Esipova, E.A.; Khlopov, M.Y.; Khromykh, L.A.; Kirillov, A.A.; Nikulin, V.V.; Rubin, S.G.; Svadkovsky, I.V. Clusters of Primordial Black Holes. *Eur. Phys. J. C* **2019**, *79*, 246. <https://doi.org/10.1140/epjc/s10052-019-6741-4>.
60. Berezin, V.; Dokuchaev, V.; Eroshenko, Y.; Smirnov, A. Formation and Clustering of Primordial Black Holes in Brans-Dicke Theory. *Universe* **2020**, *6*, 158.
61. García-Bellido, J.; Clesse, S. Constraints from Microlensing Experiments on Clustered Primordial Black Holes. *Phys. Dark Universe* **2018**, *19*, 144–148.
62. Calcino, J.; García-Bellido, J.; Davis, T.M. Updating the MACHO Fraction of the Milky Way Dark Halo with Improved Mass Models. *Mon. Not. R. Astron. Soc.* **2018**, *479*, 2889–2905.
63. Hawking, S.W. Black Holes from Cosmic Strings. *Phys. Lett. B* **1989**, *231*, 237–239.
64. García-Bellido, J.; Linde, A.; Wands, D. Density perturbations and black hole formation in hybrid inflation. *Phys. Rev. D* **1996**, *54*, 6040–6058. <https://doi.org/10.1103/PhysRevD.54.6040>.
65. Rubin, S.G.; Sakharov, A.S.; Khlopov, M.Y. The Formation of Primary Galactic Nuclei during Phase Transitions in the Early Universe. *J. Exp. Theor. Phys.* **2001**, *92*, 921–929. <https://doi.org/10.1134/1.1385631>.
66. Rubin, S.G.; Sakharov, A.S.; Khlopov, M.Y. Black Holes, Gravitational Waves and Fundamental Physics: A Roadmap. *Class. Quantum Gravity* **2019**, *36*, 143001. <https://doi.org/10.1088/1361-6382/ab0587>.
67. Nikulin, V.V.; Krasnov, M.A.; Rubin, S.G. Compact Extra Dimensions as the Source of Primordial Black Holes. *Front. Astron. Space Sci.* **2022**, *9*, 927144. <https://doi.org/10.3389/fspas.2022.927144>.
68. Capozziello, S.; De Laurentis, M. The Dark Matter Problem from $f(R)$ Gravity Viewpoint. *Ann. Der Phys.* **2012**, *524*, 545–578.
69. De Felice, A.; Tsujikawa, S. $f(R)$ Theories. *Living Rev. Relativ.* **2010**, *13*, 3. <https://doi.org/10.12942/lrr-2010-3>.
70. Capozziello, S.; De Laurentis, M. Extended Theories of Gravity. *Phys. Rep.* **2011**, *509*, 167–321.
71. Krasnov, M.; Belotsky, K. Possibility of Weakening the Constraint on Abundance of Primordial Black Holes from Eridanus II. *Phys. At. Nucl.* **2023**, *86*, 1533–1537. <https://doi.org/10.1134/S1063778824010265>.
72. Belotsky, K.; Krasnov, M.; Pugachev, S. Investigation of the Dynamics of Interaction of a Cluster of Primordial Black Holes with Stellar Cluster. *Int. J. Mod. Phys. D* **2024**, *33*, 2340010. <https://doi.org/10.1142/S0218271823400102>.
73. Pasechnik, R.; Beylin, V.; Kuksa, V.; Vereshkov, G. Chiral-Symmetric Technicolor with Standard Model Higgs Boson. *Phys. Rev. D* **2013**, *88*, 075009.
74. Volchanskiy, N.; Kuksa, V.; Beylin, V. Models of Hypercolor Based on Symplectic Gauge Group with Three Heavy Vectorlike Hyperquarks. *Int. J. Mod. Phys. D* **2018**, *28*, 1941002.

75. Pasechnik, R.; Beylin, V.; Kuksa, V.; Vereshkov, G. Scalar Technibaryon Dark Matter from Vector-Like SU(2) Technicolor. *Int. J. Mod. Phys. A* **2016**, *31*, 1650036.
76. Kuksa, V.; Beylin, V. The Scenario with Hadronic Dark Matter. *Int. J. Mod. Phys. D* **2021**, *30*, 2140006.
77. Aalbers, J.; Akerib, D.S.; Akerlof, C.W.; Al Musalhi, A.K.; Alder, F.; Alqahtani, A.; Alsum, S.K.; Amarasinghe, C.S.; Ames, A.; Ames, A.; et al. (LUX-ZEPLIN Collaboration) First Dark Matter Search Results from the LUX-ZEPLIN (LZ) Experiment. *Phys. Rev. Lett.* **2023**, *131*, 041002. <https://doi.org/10.1103/PhysRevLett.131.041002>.
78. Aprile, E.; Aalbers, J.; Agostini, F.; Alfonsi, M.; Amaro, F.D.; Anthony, M.; Arneodo, F.; Barrow, P.; Baudis, L.; Baudis, L.; et al. (XENON Collaboration) First Dark Matter Search Results from the XENON1T Experiment. *Phys. Rev. Lett.* **2017**, *119*, 181301. <https://doi.org/10.1103/PhysRevLett.119.181301>.
79. Kuksa, V.; Beylin, V. Heavy Quark Symmetry and Fine Structure of Spectrum of Hadronic Dark Matter. *Symmetry* **2020**, *12*, 1906.
80. Beylin, V.; Kuksa, V. Interaction of Hadronic Dark Matter with Nucleons and Leptons. *Symmetry* **2020**, *12*, 567.
81. Kuksa, V.; Beylin, V. Hyperfine Splitting of Excited States of New Heavy Hadrons and Low-Energy Interaction of Hadronic Dark Matter with Photons, Nucleons and Leptons. *Universe* **2020**, *6*, 84.
82. Beylin, V.; Bezuglov, M.; Kuksa, V.; Volchanskiy, N. An Analysis of a Minimal Vectorlike Extension of the Standard Model. *Adv. High Energy Phys.* **2017**, *2017*, 1765340.
83. Beylin, V.; Bezuglov, M.; Kuksa, V.; Tretiakov, E. Quasielastic Lepton Scattering off Two-Component Dark Matter in Hypercolor Model. *Symmetry* **2020**, *12*, 708.
84. Beylin, V.; Bezuglov, M. Possible Manifestations of Transitions within Dark Matter Structure. *Int. J. Mod. Phys. D* **2021**, *30*, 2140009.
85. Beylin, V.; Bezuglov, M. Can the Multicomponent Dark Matter be Visible due to Inner Luminescence? *Phys. Part. Nucl. Lett.* **2023**, *20*, 495.
86. Beylin, V.; Khlopov, M.; Kuksa, V.; Volchanskiy, N. New Physics of Strong Interaction and Dark Universe. *Universe* **2020**, *6*, 196.
87. Giudice, G.F.; McCullough, M. A Clockwork Theory. *J. High Energy Phys.* **2017**, *02*, 036. [https://doi.org/10.1007/JHEP02\(2017\)036](https://doi.org/10.1007/JHEP02(2017)036).
88. Patel, K.M. Clockwork Mechanism for Flavor Hierarchies. *Phys. Rev. D* **2017**, *96*, 115013.
89. Khlopov, M.Y. Fundamental Particle Structure in the Cosmological Dark Matter. *Int. J. Mod. Phys. A* **2013**, *28*, 1330042. <https://doi.org/10.1142/S0217751X13300421>.
90. Aprile, E.; Profumo, S. Focus on Dark Matter and Particle Physics. *New J. Phys.* **2009**, *11*, 105002. <https://doi.org/10.1088/1367-2630/11/10/105002>.
91. Feng, J.L. Dark Matter Candidates from Particle Physics and Methods of Detection. *Annu. Rev. Astron. Astrophys.* **2010**, *48*, 495. <https://doi.org/10.1146/annurev-astro-082708-101659>.
92. Khlopov, M. Cosmological Reflection of Particle Symmetry. *Symmetry* **2016**, *8*, 81.
93. Khlopov, M.Y.; Kouvaris, C. Strong Interactive Massive Particles from a Strong Coupled Theory. *Phys. Rev. D* **2008**, *77*, 065002. <https://doi.org/10.1103/PhysRevD.77.065002>.
94. Khlopov, M.Y.; Kouvaris, C. Composite Dark Matter from a Model with Composite Higgs Boson. *Phys. Rev. D* **2008**, *78*, 065040. <https://doi.org/10.1103/PhysRevD.78.065040>.
95. Glashow, S.L. A Sinister Extension of the Standard Model to $SU(3) \times SU(2) \times SU(2) \times U(1)$. *arXiv* **2005**, <https://doi.org/10.48550/arXiv.hep-ph/0504287>.
96. Fargion, D.; Khlopov, M.Y. Tera-Leptons' Shadows over Sinister Universe. *Gravit. Cosmol.* **2013**, *19*, 219. <https://doi.org/10.1134/S0202289313040063>.
97. Khlopov, M.Y. Composite Dark Matter from 4th Generation. *JETP Lett.* **2006**, *83*, 1–4. <https://doi.org/10.1134/S0021364006010012>.
98. Fargion, D.; Khlopov, M.; Stephan, C.A. Cold dark matter by heavy double charged leptons? *Class. Quantum Gravity* **2006**, *23*, 7305–7354. <https://doi.org/10.1088/0264-9381/23/24/008>.
99. Khlopov, M. Dark Atom Solution for Puzzles of Direct Dark Matter Searches. *J. Phys. Conf. Ser.* **2019**, *1312*, 012011. <https://doi.org/10.1088/1742-6596/1312/1/012011>.
100. Zeldovich, Y.B.; Khlopov, M.Y. Mass of Neutrino in Particle Physics and Cosmology of Early Universe. *Sov. Phys. Uspekhi* **1981**, *24*, 755–774.
101. Blinnikov, S.I.; Khlopov, M.Y. On the Possible Signatures of Mirror Particles. *Sov. J. Nucl. Phys.* **1982**, *36*, 472–474.
102. Blinnikov, S.I.; Khlopov, M.Y. On the Possible Astronomical Effects of Mirror Particles. *Sov. Astron.* **1983**, *27*, 371–375.
103. Khlopov, M.Y.; Beskin, G.M.; Bochkarev, N.G.; Pustilnik, L.A.; Pustilnik, S.A. Observational Physics of Mirror World. *Sov. Astron.* **1991**, *35*, 21–30.
104. Berezhiani, Z.; Comelli, D.; Villante, F. The Early Mirror Universe: Inflation, Baryogenesis, Nucleosynthesis and Dark Matter. *Phys. Lett. B* **2001**, *503*, 362–375. [https://doi.org/10.1016/S0370-2693\(01\)00217-9](https://doi.org/10.1016/S0370-2693(01)00217-9).
105. Berezhiani, Z. Mirror World and Its Cosmological Consequences. *Int. J. Mod. Phys. A* **2004**, *19*, 3775–3806. <https://doi.org/10.1142/S0217751X04020075>.
106. Bozza, C.; Calivà, A.; De Caro, A.; De Gruttola, D.; De Pasquale, S.; Fusco, L.A.; Messuti, G.; Poirè, C.; Scarpetta, S.; Virgili, T. Generative Artificial Intelligence for Air Shower Simulation. *arXiv* **2025**, arXiv:2510.26316.

107. Beylin, V. Neutrinos Production by Photons Scattered on the Dark Matter. In Proceedings of the 25th Workshop “What Comes Beyond the Standard Model”, Bled, Slovenia, 4–10 July 2022.
108. Abdul Halim, A.; Abreu, P.; Aglietta, M.; Allekotte, I.; Almeida Cheminant, K.; Almela, A.; Aloisio, R.; Alvarez-Muñoz, J.; Ammerman Yebra, J.; Ammerman Yebra, J.; et al. (Pierre Auger Collaboration) Inference of the Mass Composition of Cosmic Rays with Energies from $10^{18.5}$ to 10^{20} eV Using the Pierre Auger Observatory and Deep Learning. *Phys. Rev. Lett.* **2025**, *134*, 021001.
109. Adriani, O.; Barbarino, G.C.; Bazilevskaya, G.A.; Bellotti, R.; Boezio, M.; Bogomolov, E.A.; Bonechi, L.; Bonghi, M.; Bonvicini, V.; Bottai, S.; et al. An anomalous positron abundance in cosmic rays with energies 1.5–100 GeV. *arXiv* **2008**, arXiv:0810.4995.
110. AMS Collaboration. High Statistics Measurement of the Positron Fraction in Primary Cosmic Rays of 0.5–500 GeV with the Alpha Magnetic Spectrometer on the International Space Station. *Phys. Rev. Lett.* **2014**, *113*, 121101. <https://doi.org/10.1103/PhysRevLett.113.121101>.
111. Ackermann, M.; Ajello, M.; Allafort, A.; Atwood, W.B.; Baldini, L.; Barbiellini, G.; Bastieri, D.; Bechtol, K.; Bellazzini, R.; Bellazzini, R.; et al. (Fermi LAT Collaboration) Measurement of Separate Cosmic-Ray Electron and Positron Spectra with the Fermi Large Area Telescope. *Phys. Rev. Lett.* **2012**, *108*, 011103.
112. DAMPE Collaboration. Direct detection of a break in the teraelectronvolt cosmic-ray spectrum of electrons and positrons. *Nature* **2017**, *552*, 63–66.
113. Khlopov, M.Y. *Fundamentals of Cosmic Particle Physics*; CISP-Springer: Cambridge, UK, 2012.
114. Neronov, A.; Semikoz, D. LHAASO Sensitivity for Diffuse Gamma-Ray Signals from the Galaxy. *arXiv* **2020**, arXiv:2001.11881.
115. Huang, P.; Long, A.J.; Wang, L.T. Probing the Electroweak Phase Transition with Higgs Factories and Gravitational Waves. *Phys. Rev. D* **2016**, *94*, 075008. <https://doi.org/10.1103/PhysRevD.94.075008>.
116. Alves, A.; Ghosh, T.; Guo, H.K.; Sinha, K. Resonant Di-Higgs Production at Gravitational Wave Benchmarks: A Collider Study using Machine Learning. *J. High Energy Phys.* **2018**, *12*, 070. [https://doi.org/10.1007/JHEP12\(2018\)070](https://doi.org/10.1007/JHEP12(2018)070).
117. Alves, A.; Gonçalves, D.; Ghosh, T.; Guo, H.K.; Sinha, K. Di-Higgs Production in the $4b$ Channel and Gravitational Wave Complementarity. *J. High Energy Phys.* **2020**, *03*, 053. [https://doi.org/10.1007/JHEP03\(2020\)053](https://doi.org/10.1007/JHEP03(2020)053).
118. Morais, A.P.; Pasechnik, R. Probing multi-step electroweak phase transition with multi-peaked primordial gravitational waves spectra. *J. Cosmol. Astropart. Phys.* **2020**, *2004*, 036. <https://doi.org/10.1088/1475-7516/2020/04/036>.
119. Addazi, A.; Marciandò, A.; Morais, A.P.; Pasechnik, R.; Srivastava, R.; Valle, J.W.F. Gravitational footprints of massive neutrinos and lepton number breaking. *Phys. Lett. B* **2020**, *807*, 135577. <https://doi.org/10.1016/j.physletb.2020.135577>.
120. Vieu, T.; Morais, A.P.; Pasechnik, R. Electroweak phase transitions in multi-Higgs models: The case of Trinification-inspired THDSM. *J. Cosmol. Astropart. Phys.* **2018**, *1807*, 014. <https://doi.org/10.1088/1475-7516/2018/07/014>.
121. Gonçalves, J.; Marfatia, D.; Morais, A.P.; Pasechnik, R. Gravitational waves from supercooled phase transitions in conformal Majoron models of neutrino mass. *J. High Energy Phys.* **2025**, *02*, 110. [https://doi.org/10.1007/JHEP02\(2025\)110](https://doi.org/10.1007/JHEP02(2025)110).
122. Gonçalves, J.; Marfatia, D.; Morais, A.P.; Pasechnik, R. Supercooled phase transitions in conformal dark sectors explain NANOGrav data. *Phys. Lett. B* **2025**, *869*, 139829. <https://doi.org/10.1016/j.physletb.2025.139829>.
123. Bertensam, M.; Finetti, M.; Morais, A.P.; Pasechnik, R.; Rathsmann, J. Gravitational waves from color restoration in a leptoquark model of radiative neutrino masses. *J. Cosmol. Astropart. Phys.* **2025**, *08*, 077. <https://doi.org/10.1088/1475-7516/2025/08/077>.
124. Balaji, S.; Gonçalves, J.; Marfatia, D.; Morais, A.P.; Pasechnik, R. Primordial black holes and magnetic fields in conformal neutrino mass models. *J. Cosmol. Astropart. Phys.* **2025**, *10*, 064. <https://doi.org/10.1088/1475-7516/2025/10/064>.
125. Addazi, A.; Marciandò, A.; Pasechnik, R. Probing Trans-electroweak First Order Phase Transitions from Gravitational Waves. *Physics* **2019**, *1*, 92. <https://doi.org/10.3390/physics1010010>.
126. Camargo-Molina, J.E.; Morais, A.P.; Ordell, A.; Pasechnik, R.; Wessén, J. Scale hierarchies, symmetry breaking and particle spectra in SU(3)-family extended SUSY trinification. *Phys. Rev. D* **2019**, *99*, 035041. <https://doi.org/10.1103/PhysRevD.99.035041>.
127. Camargo-Molina, J.E.; Morais, A.P.; Ordell, A.; Pasechnik, R.; Sampaio, M.O.P.; Wessén, J. Reviving trinification models through an E6-extended supersymmetric GUT. *Phys. Rev. D* **2017**, *95*, 075031. <https://doi.org/10.1103/PhysRevD.95.075031>.
128. Ng, K.K.Y.; Franciolini, G.; Berti, E.; Pani, P.; Riotto, A.; Vitale, S. Constraining High-redshift Stellar-mass Primordial Black Holes with Next-generation Ground-based Gravitational-wave Detectors. *Astrophys. J. Lett.* **2022**, *933*, L41. <https://doi.org/10.3847/2041-8213/ac7aae>.
129. Ng, K.K.Y.; Goncharov, B.; Chen, S.; Borhanian, S.; Dupletsa, U.; Franciolini, G.; Branchesi, M.; Harms, J.; Maggiore, M.; Riotto, A.; et al. Measuring properties of primordial black hole mergers at cosmological distances: Effect of higher order modes in gravitational waves. *Phys. Rev. D* **2023**, *107*, 024041. <https://doi.org/10.1103/PhysRevD.107.024041>.
130. Franciolini, G.; Iacovelli, F.; Mancarella, M.; Maggiore, M.; Pani, P.; Riotto, A. Searching for primordial black holes with the Einstein Telescope: Impact of design and systematics. *Phys. Rev. D* **2023**, *108*, 043506. <https://doi.org/10.1103/PhysRevD.108.043506>.
131. Bagui, E.; Clesse, S.; De Luca, V.; Ezquiaga, J.M.; Franciolini, G.; García-Bellido, J.; Joana, C.; Jain, R.K.; Kuroyanagi, S.; Musco, I.; et al. (LISA Cosmology Working Group) Primordial black holes and their gravitational-wave signatures. *Living Rev. Relativ.* **2025**, *28*, 1. <https://doi.org/10.1007/s41114-024-00053-w>.

132. Bird, S.; Cholis, I.; Muñoz, J.B.; Ali-Haïmoud, Y.; Kamionkowski, M.; Kovetz, E.D.; Raccanelli, A.; Riess, A.G. Did LIGO detect dark matter? *Phys. Rev. Lett.* **2016**, *116*, 201301. <https://doi.org/10.1103/PhysRevLett.116.201301>.
133. Sasaki, M.; Suyama, T.; Tanaka, T.; Yokoyama, S. Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914. *Phys. Rev. Lett.* **2016**, *117*, 061101. <https://doi.org/10.1103/PhysRevLett.117.061101>. Erratum in *Phys. Rev. Lett.* **2018**, *121*, 059901.
134. Clesse, S.; García-Bellido, J. The clustering of massive Primordial Black Holes as Dark Matter: Measuring their mass distribution with Advanced LIGO. *Phys. Dark Universe* **2017**, *15*, 142–147. <https://doi.org/10.1016/j.dark.2016.10.002>.
135. Blinnikov, S.; Dolgov, A.; Porayko, N.K.; Postnov, K. Solving puzzles of GW150914 by primordial black holes. *J. Cosmol. Astropart. Phys.* **2016**, *11*, 036. <https://doi.org/10.1088/1475-7516/2016/11/036>.
136. Ali-Haïmoud, Y.; Kovetz, E.D.; Kamionkowski, M. Merger rate of primordial black-hole binaries. *Phys. Rev. D* **2017**, *96*, 123523. <https://doi.org/10.1103/PhysRevD.96.123523>.
137. Raidal, M.; Spethmann, C.; Vaskonen, V.; Veermäe, H. Formation and Evolution of Primordial Black Hole Binaries in the Early Universe. *J. Cosmol. Astropart. Phys.* **2019**, *02*, 018. <https://doi.org/10.1088/1475-7516/2019/02/018>.
138. Nakamura, T.; Sasaki, M.; Tanaka, T.; Thorne, K.S. Gravitational waves from coalescing black hole MACHO binaries. *Astrophys. J. Lett.* **1997**, *487*, L139–L142. <https://doi.org/10.1086/310886>.
139. Ioka, K.; Chiba, T.; Tanaka, T.; Nakamura, T. Black hole binary formation in the expanding universe: Three body problem approximation. *Phys. Rev. D* **1998**, *58*, 063003. <https://doi.org/10.1103/PhysRevD.58.063003>.
140. Franciolini, G.; Kritos, K.; Berti, E.; Silk, J. Primordial black hole mergers from three-body interactions. *Phys. Rev. D* **2022**, *106*, 083529. <https://doi.org/10.1103/PhysRevD.106.083529>.
141. Afshordi, N.; McDonald, P.; Spergel, D.N. Primordial black holes as dark matter: The Power spectrum and evaporation of early structures. *Astrophys. J. Lett.* **2003**, *594*, L71–L74. <https://doi.org/10.1086/378763>.
142. Inman, D.; Ali-Haïmoud, Y. Early structure formation in primordial black hole cosmologies. *Phys. Rev. D* **2019**, *100*, 083528. <https://doi.org/10.1103/PhysRevD.100.083528>.
143. Tkachev, M.; Pilipenko, S.; Yepes, G. Dark Matter Simulations with Primordial Black Holes in the Early Universe. *Mon. Not. R. Astron. Soc.* **2020**, *499*, 4854–4862. <https://doi.org/10.1093/mnras/staa3103>.
144. Amin, M.A.; Delos, M.S.; Mirbabayi, M. Structure Formation with Warm White Noise: Effects of Finite Number Density and Velocity Dispersion in Particle and Wave Dark Matter. *arXiv* **2025**, arXiv:2503.20881.
145. Vaskonen, V.; Veermäe, H. Lower bound on the primordial black hole merger rate. *Phys. Rev. D* **2020**, *101*, 043015. <https://doi.org/10.1103/PhysRevD.101.043015>.
146. Jedamzik, K. Primordial Black Hole Dark Matter and the LIGO/Virgo observations. *J. Cosmol. Astropart. Phys.* **2020**, *09*, 022. <https://doi.org/10.1088/1475-7516/2020/09/022>.
147. Delos, M.S.; Rantala, A.; Young, S.; Schmidt, F. Structure formation with primordial black holes: Collisional dynamics, binaries, and gravitational waves. *J. Cosmol. Astropart. Phys.* **2024**, *12*, 005. <https://doi.org/10.1088/1475-7516/2024/12/005>.
148. Stasenko, V.; Belotsky, K. Influence of early dark matter haloes on the primordial black holes merger rate. *Mon. Not. R. Astron. Soc.* **2023**, *526*, 4308–4314. <https://doi.org/10.1093/mnras/stad3032>.
149. Stasenko, V. Redshift evolution of primordial black hole merger rate. *Phys. Rev. D* **2024**, *109*, 123546. <https://doi.org/10.1103/PhysRevD.109.123546>.
150. Stasenko, V. Primordial Black Hole Binaries: The Impact of Poisson Clustering. *Phys. Part. Nucl.* **2025**, *56*, 152–156. <https://doi.org/10.1134/S1063779624701338>.
151. Khlopov, M.Y.; Rubin, S.G.; Sakharov, A.S. Primordial structure of massive black hole clusters. *Astropart. Phys.* **2005**, *23*, 265. <https://doi.org/10.1016/j.astropartphys.2004.12.002>.
152. Young, S.; Byrnes, C.T. Signatures of non-gaussianity in the isocurvature modes of primordial black hole dark matter. *J. Cosmol. Astropart. Phys.* **2015**, *04*, 034. <https://doi.org/10.1088/1475-7516/2015/04/034>.
153. Suyama, T.; Yokoyama, S. Clustering of primordial black holes with non-Gaussian initial fluctuations. *Prog. Theor. Exp. Phys.* **2019**, *2019*, 103E02. <https://doi.org/10.1093/ptep/ptz105>.
154. Ding, Q.; Nakama, T.; Silk, J.; Wang, Y. Detectability of Gravitational Waves from the Coalescence of Massive Primordial Black Holes with Initial Clustering. *Phys. Rev. D* **2019**, *100*, 103003. <https://doi.org/10.1103/PhysRevD.100.103003>.
155. Bringmann, T.; Depta, P.F.; Domcke, V.; Schmidt-Hoberg, K. Towards closing the window of primordial black holes as dark matter: The case of large clustering. *Phys. Rev. D* **2019**, *99*, 063532. <https://doi.org/10.1103/PhysRevD.99.063532>.
156. Young, S.; Byrnes, C.T. Initial clustering and the primordial black hole merger rate. *J. Cosmol. Astropart. Phys.* **2020**, *03*, 004. <https://doi.org/10.1088/1475-7516/2020/03/004>.
157. Stasenko, V.D.; Kirillov, A.A. The Merger Rate of Black Holes in a Primordial Black Hole Cluster. *Physics* **2021**, *3*, 372–378. <https://doi.org/10.3390/physics3020026>.
158. Stasenko, V.D.; Kirillov, A.A.; Belotsky, K.M. Dynamics and Merger Rate of Primordial Black Holes in a Cluster. *Universe* **2022**, *8*, 41. <https://doi.org/10.3390/universe8010041>.

159. Eroshenko, Y.; Stasenko, V. Gravitational Waves from the Merger of Two Primordial Black Hole Clusters. *Symmetry* **2023**, *15*, 637. <https://doi.org/10.3390/sym15030637>.
160. Stasenko, V. Coalescence of Primordial Black Holes at High Redshifts with Clustering. *Phys. At. Nucl.* **2025**, *88*, 524–530. <https://doi.org/10.1134/S1063778825600794>.
161. Stasenko, V. Merger history of clustered primordial black holes. *Phys. Dark Universe* **2025**, *50*, 102138. <https://doi.org/10.1016/j.dark.2025.102138>.
162. Braglia, M.; Garcia-Bellido, J.; Kuroyanagi, S. Testing Primordial Black Holes with multi-band observations of the stochastic gravitational wave background. *J. Cosmol. Astropart. Phys.* **2021**, *12*, 012. <https://doi.org/10.1088/1475-7516/2021/12/012>.
163. García-Bellido, J.; Jaraba, S.; Kuroyanagi, S. The stochastic gravitational wave background from close hyperbolic encounters of primordial black holes in dense clusters. *Phys. Dark Universe* **2022**, *36*, 101009. <https://doi.org/10.1016/j.dark.2022.101009>.
164. Mukherjee, S.; Silk, J. Can we distinguish astrophysical from primordial black holes via the stochastic gravitational wave background? *Mon. Not. R. Astron. Soc.* **2021**, *506*, 3977–3985. <https://doi.org/10.1093/mnras/stab1932>.
165. Atal, V.; Blanco-Pillado, J.J.; Sanglas, A.; Triantafyllou, N. Constraining changes in the merger history of BH and PBH binaries with the stochastic gravitational wave background. *Phys. Rev. D* **2022**, *105*, 123522. <https://doi.org/10.1103/PhysRevD.105.123522>.
166. Kavanagh, B.J.; Gaggero, D.; Bertone, G. Merger rate of a subdominant population of primordial black holes. *Phys. Rev. D* **2018**, *98*, 023536. <https://doi.org/10.1103/PhysRevD.98.023536>.
167. Pilipenko, S.; Tkachev, M.; Ivanov, P. Evolution of a primordial binary black hole due to interaction with cold dark matter and the formation rate of gravitational wave events. *Phys. Rev. D* **2022**, *105*, 123504. <https://doi.org/10.1103/PhysRevD.105.123504>.
168. Jangra, P.; Kavanagh, B.J.; Diego, J.M. Impact of dark matter spikes on the merger rates of Primordial Black Holes. *J. Cosmol. Astropart. Phys.* **2023**, *11*, 069. <https://doi.org/10.1088/1475-7516/2023/11/069>.
169. Stasenko, V.D.; Eroshenko, Y.N. Mergers of Binary Primordial Black Holes in Evolving Dark Matter Halos. *Astron. Lett.* **2024**, *50*, 431–439. <https://doi.org/10.1134/S1063773724700348>.
170. Berezhiani, L.; Cintia, G.; De Luca, V.; Khoury, J. Dynamical friction in dark matter superfluids: The evolution of black hole binaries. *J. Cosmol. Astropart. Phys.* **2024**, *06*, 024. <https://doi.org/10.1088/1475-7516/2024/06/024>.
171. Kadota, K.; Kim, J.H.; Ko, P.; Yang, X.Y. Gravitational wave probes on self-interacting dark matter surrounding an intermediate mass black hole. *Phys. Rev. D* **2024**, *109*, 015022. <https://doi.org/10.1103/PhysRevD.109.015022>.
172. Fischer, M.S.; Sagunski, L. Dynamical friction from self-interacting dark matter. *Astron. Astrophys.* **2024**, *690*, A299. <https://doi.org/10.1051/0004-6361/202451304>.
173. Aurrekoetxea, J.C.; Marsden, J.; Clough, K.; Ferreira, P.G. Self-interacting scalar dark matter around binary black holes. *Phys. Rev. D* **2024**, *110*, 083011. <https://doi.org/10.1103/PhysRevD.110.083011>.
174. Alonso-Álvarez, G.; Cline, J.M.; Dewar, C. Self-Interacting Dark Matter Solves the Final Parsec Problem of Supermassive Black Hole Mergers. *Phys. Rev. Lett.* **2024**, *133*, 021401. <https://doi.org/10.1103/PhysRevLett.133.021401>.
175. Luo, J.; Chen, L.-S.; Duan, H.-Z.; Gong, Y.-G.; Hu, S.; Ji, J.; Liu, Q.; Mei, J.; Milyukov, V.; Sazhin, M.; et al. TianQin: A space-borne gravitational wave detector. *Class. Quantum Gravity* **2016**, *33*, 035010. <https://doi.org/10.1088/0264-9381/33/3/035010>.
176. Reitze, D.; Adhikari, R.X.; Ballmer, S.; Barish, B.; Barsotti, L.; Billingsley, G.; Brown, D.A.; Chen, Y.; Coyne, D.; Eisenstein, R.; et al. Cosmic Explorer: The U.S. Contribution to Gravitational-Wave Astronomy beyond LIGO. *Bull. Am. Astron. Soc.* **2019**, *51*, 035.
177. Maggiore, M.; Van Den Broeck, C.; Bartolo, N.; Belgacem, E.; Bertacca, D.; Bizouard, M.A.; Branchesi, M.; Clesse, S.; Foffa, S.; García-Bellido, J.; et al. Science Case for the Einstein Telescope. *J. Cosmol. Astropart. Phys.* **2020**, *03*, 050. <https://doi.org/10.1088/1475-7516/2020/03/050>.
178. Abac, A.; Abramo, R.; Albanesi, S.; Albertini, A.; Agapito, A.; Agathos, M.; Albertus, C.; Andersson, N.; Andrade, T.; Andreoni, I.; et al. The Science of the Einstein Telescope. *arXiv* **2025**, arXiv:2503.12263.
179. Witten, E. Quantum Field Theory and the Jones Polynomial. *Commun. Math. Phys.* **1989**, *121*, 351–399.
180. Jones, V.F.R. A Polynomial Invariant for Knots via von Neumann Algebras. In *Fields Medallists' Lectures*; World Scientific: Singapore, 1997; pp. 448–458.
181. Jaeger, F.; Vertigan, D.L.; Welsh, D.J. On the computational complexity of the Jones and Tutte polynomials. In *Proceedings of the Mathematical Proceedings of the Cambridge Philosophical Society*; Cambridge University Press: Cambridge, UK, 1990; Volume 108, pp. 35–53.
182. Bernstein, E.; Vazirani, U. Quantum Complexity Theory. *SIAM J. Comput.* **1997**, *26*, 1411–1473.
183. Raz, R.; Tal, A. Oracle Separation of BQP and PH. In Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing, Phoenix, AZ, USA, 23–26 June 2019; pp. 13–23.
184. Neto, F.M.; Ludermit, T.B.; Oliveira, W.R.; Silva, A.J. Solving NP-Complete Problems Using Quantum Weightless Neuron Nodes. In *Proceedings of the 2015 Brazilian Conference on Intelligent Systems (BRACIS)*; IEEE: Piscataway, NJ, USA, 2015; pp. 258–263.
185. Ruppert, D.; Wand, M.P.; Carroll, R.J. *Semiparametric Regression*; Cambridge University Press: Cambridge, UK, 2003.

186. Battaglia, P.; Pascanu, R.; Lai, M.; Jimenez Rezende, D.; Kavukcuoglu, K. Interaction Networks for Learning about Objects, Relations and Physics. In Proceedings of the Advances in Neural Information Processing Systems 29 (NIPS 2016), Barcelona, Spain, 5–10 December 2016.
187. Dai, H.; Tian, Y.; Dai, B.; Skiena, S.; Song, L. Syntax-Directed Variational Autoencoder for Structured Data. *arXiv* **2018**, arXiv:1802.08786.
188. Ciardiello, A.; Asai, M.; Caccia, B.; Cirrone, G.A.P.; Colonna, M.; Dotti, A.; Faccini, R.; Giagu, S.; Messina, A.; Napolitani, P.; et al. Preliminary results in using Deep Learning to emulate BLOB, a nuclear interaction model. *Phys. Medica* **2020**, *73*, 65–72. <https://doi.org/10.1016/j.ejmp.2020.04.005>.
189. Steinle, N.; Safi-Harb, S. Machine learning classification of black holes in the mass-spin diagram. *Phys. Rev. D* **2025**, *112*, 103038. <https://doi.org/10.1103/PhysRevD.112.103038>.
190. Lin, J.; Bhimji, W.; Nachman, B. Machine Learning Templates for QCD Factorization in the Search for Physics Beyond the Standard Model. *J. High Energy Phys.* **2019**, *05*, 181. [https://doi.org/10.1007/JHEP05\(2019\)181](https://doi.org/10.1007/JHEP05(2019)181).
191. ATLAS Collaboration. Search for direct production of electroweakinos in final states with one lepton, missing transverse momentum and a Higgs boson decaying into two b -jets in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector. *arXiv* **2019**, arXiv:1909.09226.
192. Karniadakis, G.E.; Kevrekidis, I.G.; Lu, L.; Perdikaris, P.; Wang, S.; Yang, L. Physics-informed machine learning. *Nat. Rev. Phys.* **2021**, *3*, 422–440. <https://doi.org/10.1038/s42254-021-00314-5>.
193. Guest, D.; Cranmer, K.; Whiteson, D. Deep Learning and its Application to LHC Physics. *Annu. Rev. Nucl. Part. Sci.* **2018**, *68*, 161–181. <https://doi.org/10.1146/annurev-nucl-101917-021019>.
194. Radovic, A.; Williams, M.; Rousseau, D.; Kagan, M.; Bonacorsi, D.; Himmel, A.; Aurisano, A.; Terao, K.; Wongjirad, T. Machine learning at the energy and intensity frontiers of particle physics. *Nature* **2018**, *560*, 41–48. <https://doi.org/10.1038/s41586-018-0361-2>.
195. Zhou, K.; Wang, L.; Pang, L.G.; Shi, S. Exploring QCD matter in extreme conditions with Machine Learning. *Prog. Part. Nucl. Phys.* **2024**, *135*, 104084. <https://doi.org/10.1016/j.pnpnp.2023.104084>.
196. Shanahan, P.; Terao, K.; Whiteson, D.; Aarts, G.; Adelmann, A.; Akchurin, N.; Alexandru, A.; Amram, O.; Andreassen, A.; Apresyan, A.; et al. Snowmass 2021 Computational Frontier CompF03 Topical Group Report: Machine Learning. *arXiv* **2022**, arXiv:2209.07559.
197. Smith, L.; Scialpi, M.; di Clemente, F.; Beijer, M. PINNGraPE: Physics Informed Neural Network for Gravitational wave Parameter Estimation. *arXiv* **2025**, arXiv:2510.02858.
198. Mishra, A.K.; Tolley, E. SPINN: Advancing Cosmological Simulations of Fuzzy Dark Matter with Physics Informed Neural Networks. *Astrophys. J.* **2025**, *988*, 114. <https://doi.org/10.3847/1538-4357/ade43e>.
199. Bento, M.; Câmara, H.; Seabra, J. Unraveling particle dark matter with Physics-Informed Neural Networks. *Phys. Lett. B* **2025**, *868*, 139690. <https://doi.org/10.1016/j.physletb.2025.139690>.
200. Humphreys, P. *The Oxford Handbook of Philosophy of Science*; Oxford University Press: Oxford, UK, 2016.
201. Popper, K. *The Logic of Scientific Discovery*; Routledge: Oxfordshire, UK, 2005.
202. Roscher, R.; Bohn, B.; Duarte, M.F.; Garcke, J. Explainable Machine Learning for Scientific Insights and Discoveries. *IEEE Access* **2020**, *8*, 42200–42216.
203. Wang, L. Discovering Phase Transitions with Unsupervised Learning. *Phys. Rev. B* **2016**, *94*, 195105.
204. Yuan, G.W.; Calzà, M.; Pedrotti, D. Machine learning-based analytical expressions for Gray-Body Factors and application to Primordial Black Holes. *Phys. Dark Universe* **2025**, *50*, 102078. <https://doi.org/10.1016/j.dark.2025.102078>.
205. Futia, G.; Vetrò, A. On the Integration of Knowledge Graphs into Deep Learning Models for a More Comprehensible AI—Three Challenges for Future Research. *Information* **2020**, *11*, 122.
206. Napolitani, P.; Colonna, M.; Di Prima, M. Bifurcations in Dissipative Fermionic Dynamics. *J. Phys. Conf. Ser.* **2014**, *515*, 012014. <https://doi.org/10.1088/1742-6596/515/1/012014>.
207. Duarte, J.; Han, S.; Harris, P.; Jindariani, S.; Kreinar, E.; Kreis, B.; Ngadiuba, J.; Pierini, M.; Rivera, R.; Tran, N.; et al. Fast inference of deep neural networks in FPGAs for particle physics. *J. Instrum.* **2018**, *13*, P07027. <https://doi.org/10.1088/1748-0221/13/07/P07027>.
208. Francescato, S.; Giagu, S.; Riti, F.; Russo, G.; Sabetta, L.; Tortonesi, F. Model compression and simplification pipelines for fast deep neural network inference in FPGAs in HEP. *Eur. Phys. J. C* **2021**, *81*, 969. <https://doi.org/10.1140/epjc/s10052-021-09875-2>.
209. Belis, V.; Odagiu, P.; Aarrestad, T.K. Machine learning for anomaly detection in particle physics. *Rev. Phys.* **2024**, *12*, 100091. <https://doi.org/10.1016/j.revip.2024.100091>.
210. Riehn, F.; Engel, R.; Fedynitch, A.; Gaisser, T.K.; Stanev, T. The hadronic interaction model Sibyll 2.3c and extensive air showers. *arXiv* **2019**, arXiv:1912.03300.
211. Ostapchenko, S. Monte Carlo treatment of hadronic interactions in enhanced Pomeron scheme: I. QGSJET-II model. *Phys. Rev. D* **2011**, *83*, 014018. <https://doi.org/10.1103/PhysRevD.83.014018>.

212. Pierog, T.; Karpenko, I.; Katzy, J.M.; Yatsenko, E.; Werner, K. EPOS LHC: Test of collective hadronization with data measured at the CERN Large Hadron Collider. *Phys. Rev. C* **2015**, *92*, 034906. <https://doi.org/10.1103/PhysRevC.92.034906>.
213. Cranmer, M.; Sanchez-Gonzalez, A.; Battaglia, P.; Xu, R.; Cranmer, K.; Spergel, D.; Ho, S. Discovering Symbolic Models from Deep Learning with Inductive Biases. *arXiv* **2020**, arXiv:2006.11287.
214. Coleman, S.R. The Fate of the False Vacuum. 1. Semiclassical Theory. *Phys. Rev. D* **1977**, *15*, 2929–2936. <https://doi.org/10.1103/PhysRevD.16.1248>.
215. Witten, E. Cosmic Separation of Phases. *Phys. Rev. D* **1984**, *30*, 272–285. <https://doi.org/10.1103/PhysRevD.30.272>.
216. Kosowsky, A.; Turner, M.S.; Watkins, R. Gravitational Radiation from Colliding Vacuum Bubbles. *Phys. Rev. D* **1992**, *45*, 4514–4535. <https://doi.org/10.1103/PhysRevD.45.4514>.
217. Wainwright, C.L. CosmoTransitions: Computing Cosmological Phase Transition Temperatures and Bubble Profiles with Multiple Fields. *Comput. Phys. Commun.* **2012**, *183*, 2006–2013. <https://doi.org/10.1016/j.cpc.2012.04.004>.
218. Athron, P.; Balázs, C.; Fowlie, A.; Zhang, Y. PhaseTracer: Tracing Cosmological Phases and Calculating Transition Properties. *Eur. Phys. J. C* **2020**, *80*, 567. <https://doi.org/10.1140/epjc/s10052-020-8035-2>.
219. Marciano, A.; Chen, D.; Fabrocini, F.; Fields, C.; Greco, E.; Gresnigt, N.; Jinkl, K.; Lulli, M.; Terzidis, K.; Zappala, E. Deep Neural Networks as the Semi-Classical Limit of Quantum Neural Networks. *Neural Netw.* **2022**, *153*, 164.
220. Fields, C.; Glazebrook, J.F.; Marciano, A. Sequential Measurements, Topological Quantum Field Theories, and Topological Quantum Neural Networks. *Fortschritte Der Phys.* **2022**, *70*, 2200104. <https://doi.org/10.1002/prop.202200104>
221. Marciano, A.; Chen, D.; Fabrocini, F.; Fields, C.; Lulli, M.; Zappala, E. Deep Neural Networks as the Semi-Classical Limit of Topological Quantum Neural Networks: The Problem of Generalisation. *arXiv* **2022**, arXiv:2210.13741.
222. Cattaneo, A.S.; Cotta-Ramusino, P.; Fucito, F.; Martellini, M.; Rinaldi, M.; Tanzini, A.; Zeni, M. Four-Dimensional Yang-Mills Theory as a Deformation of Topological BF Theory. *Commun. Math. Phys.* **1998**, *197*, 571–621. <https://doi.org/10.1007/s002200050465>.
223. Creutz, M. *Quarks, Gluons and Lattices*; Cambridge University Press: Cambridge, UK, 1984.
224. Beer, K.; Bondarenko, D.; Farrelly, T.; Osborne, T.J.; Salzmann, R.; Scheiermann, D.; Wolf, R. Tensor Network-Based Machine Learning. *Nat. Commun.* **2020**, *11*, 1.
225. Laureijs, R.; Amiaux, J.; Arduini, S.; Auguères, J.-L.; Brinchmann, J.; Cole, R.; Cropper, M.; Dabin, C.; Duvet, L.; Ealet, A.; et al. Euclid definition study report. *arXiv* **2011**, arXiv:1110.3193.
226. Ivezić, Ž.; Kahn, S.M.; Tyson, J.A.; Abel, B.; Acosta, E.; Allsman, R.; Alonso, D.; AlSayyad, Y.; Anderson, S.F.; Andrew, J.; et al. LSST: From Science Drivers to Reference Design and Anticipated Data Products. *Astrophys. J.* **2019**, *873*, 111. <https://doi.org/10.3847/1538-4357/ab042c>.
227. Abdul-Karim, M.; Aguilar, J.; Ahlen, S.; Alam, S.; Allen, L.; Allende Prieto, C.; Alves, O.; Anand, A.; Andrade, U.; Armengaud, E.; et al. (DESI Collaboration) DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints. *arXiv* **2025**, arXiv:2503.14738.
228. Ho, M.; Rau, M.M.; Ntampaka, M.; Farahi, A.; Trac, H.; Poczós, B. A Robust and Efficient Deep Learning Method for Dynamical Mass Measurements of Galaxy Clusters. *Astrophys. J.* **2019**, *887*, 25. <https://doi.org/10.3847/1538-4357/ab4f82>.
229. Peel, A.; Lalande, F.; Starck, J.L.; Pettorino, V.; Merten, J.; Giocoli, C.; Meneghetti, M.; Baldi, M. Distinguishing standard and modified gravity cosmologies with machine learning. *Phys. Rev. D* **2019**, *100*, 023508. <https://doi.org/10.1103/PhysRevD.100.023508>.
230. Caldeira, J.; Wu, W.L.K.; Nord, B.; Avestruz, C.; Trivedi, S.; Story, K.T. DeepCMB: Lensing Reconstruction of the Cosmic Microwave Background with Deep Neural Networks. *Astron. Comput.* **2019**, *28*, 100307. <https://doi.org/10.1016/j.ascom.2019.100307>.
231. He, S.; Li, Y.; Feng, Y.; Ho, S.; Ravanbakhsh, S.; Chen, W.; Poczós, B. Learning to Predict the Cosmological Structure Formation. *Proc. Natl. Acad. Sci. USA* **2019**, *116*, 13825–13832. <https://doi.org/10.1073/pnas.1821458116>.
232. Ravanbakhsh, S.; Lanusse, F.; Mandelbaum, R.; Schneider, J.; Poczós, B. Enabling Dark Energy Science with Deep Generative Models of Galaxy Images. *arXiv* **2016**, arXiv:1609.05796.
233. Escamilla-Rivera, C.; Quintero, M.A.C.; Capozziello, S. A deep learning approach to cosmological dark energy models. *J. Cosmol. Astropart. Phys.* **2020**, *03*, 008. <https://doi.org/10.1088/1475-7516/2020/03/008>.
234. Narayan, G.; Zaidi, T.; Soraisam, M.D.; Wang, Z.; Lochner, M.; Matheson, T.; Saha, A.; Yang, S.; Zhao, Z.; Kececioglu, J.; et al. Machine Learning-based Brokers for Real-time Classification of the LSST Alert Stream. *Astrophys. J. Suppl. Ser.* **2018**, *236*, 9. <https://doi.org/10.3847/1538-4365/aab781>.
235. Lanusse, F.; Ma, Q.; Li, N.; Collett, T.E.; Li, C.; Ravanbakhsh, S.; Mandelbaum, R.; Poczós, B. CMU DeepLens: Deep learning for automatic image-based galaxy–galaxy strong lens finding. *Mon. Not. R. Astron. Soc.* **2018**, *473*, 3895–3906. <https://doi.org/10.1093/mnras/stx1665>.
236. Di Valentino, E.; Said, J.L.; Riess, A.; Pollo, A.; Poulin, V.; Gómez-Valent, A.; Weltman, A.; Palmese, A.; Huang, C.D.; van de Bruck, C.; et al. The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics. *arXiv* **2025**, arXiv:2504.01669.

237. Wang, Y.C.; Xie, Y.B.; Zhang, T.J.; Huang, H.C.; Zhang, T.; Liu, K. Likelihood-free Cosmological Constraints with Artificial Neural Networks: An Application on Hubble Parameters and SNe Ia. *Astrophys. J. Suppl. Ser.* **2021**, *254*, 43. <https://doi.org/10.3847/1538-4365/abf8aa>.
238. Wang, G.J.; Cheng, C.; Ma, Y.Z.; Xia, J.Q.; Abebe, A.; Beesham, A. CoLFI: Cosmological Likelihood-free Inference with Neural Density Estimators. *Astrophys. J. Suppl. Ser.* **2023**, *268*, 7. <https://doi.org/10.3847/1538-4365/ace113>.
239. Gómez-Vargas, I.; Esquivel, R.M.; García-Salcedo, R.; Vázquez, J.A. Neural network reconstructions for the Hubble parameter, growth rate and distance modulus. *Eur. Phys. J. C* **2023**, *83*, 304. <https://doi.org/10.1140/epjc/s10052-023-11435-9>.
240. Manrique-Yus, A.; Sellentin, E. Euclid-era cosmology for everyone: Neural net assisted MCMC sampling for the joint 3×2 likelihood. *Mon. Not. R. Astron. Soc.* **2020**, *491*, 2655–2663. <https://doi.org/10.1093/mnras/stz3059>.
241. Spurio Mancini, A.; Piras, D.; Alsing, J.; Joachimi, B.; Hobson, M.P. CosmoPower: Emulating cosmological power spectra for accelerated Bayesian inference from next-generation surveys. *Mon. Not. R. Astron. Soc.* **2022**, *511*, 1771–1788. <https://doi.org/10.1093/mnras/stac064>.
242. Albers, J.; Fidler, C.; Lesgourgues, J.; Schöneberg, N.; Torrado, J. CosmicNet. Part I. Physics-driven implementation of neural networks within Einstein-Boltzmann Solvers. *J. Cosmol. Astropart. Phys.* **2019**, *09*, 028. <https://doi.org/10.1088/1475-7516/2019/09/028>.
243. Auld, T.; Bridges, M.; Hobson, M.P.; Gull, S.F. Fast cosmological parameter estimation using neural networks. *Mon. Not. R. Astron. Soc.* **2007**, *376*, L11–L15. <https://doi.org/10.1111/j.1745-3933.2006.00276.x>.
244. Wang, G.J.; Li, S.Y.; Xia, J.Q. ECoPANN: A Framework for Estimating Cosmological Parameters using Artificial Neural Networks. *Astrophys. J. Suppl. Ser.* **2020**, *249*, 25. <https://doi.org/10.3847/1538-4365/aba190>.
245. Günther, S.; Lesgourgues, J.; Samaras, G.; Schöneberg, N.; Stadtmann, F.; Fidler, C.; Torrado, J. CosmicNet II: Emulating extended cosmologies with efficient and accurate neural networks. *J. Cosmol. Astropart. Phys.* **2022**, *11*, 035. <https://doi.org/10.1088/1475-7516/2022/11/035>.
246. Sabiu, C.G.; Kadota, K.; Asorey, J.; Park, I. Probing ultra-light axion dark matter from 21 cm tomography using Convolutional Neural Networks. *J. Cosmol. Astropart. Phys.* **2022**, *2022*, 020. <https://doi.org/10.1088/1475-7516/2022/01/020>.
247. Asorey, J.; Crocce, M.; Gaztanaga, E.; Lewis, A. Recovering 3D clustering information with angular correlations. *Mon. Not. R. Astron. Soc.* **2012**, *427*, 1891–1902.
248. Andrés-Carcasona, M.; Martinez, M.; Mir, L.M. Fast Bayesian gravitational wave parameter estimation using convolutional neural networks. *Mon. Not. R. Astron. Soc.* **2023**, *527*, 2887–2894. <https://doi.org/10.1093/mnras/stad3448>.
249. Papamakarios, G.; Nalisnick, E.; Rezende, D.J.; Mohamed, S.; Lakshminarayanan, B. Normalizing flows for probabilistic modeling and inference. *J. Mach. Learn. Res.* **2021**, *22*, 1–64.
250. Bevins, H.T.J.; Handley, W.J.; Lemos, P.; Sims, P.H.; de Lera Acedo, E.; Fialkov, A.; Alsing, J. Marginal post-processing of Bayesian inference products with normalizing flows and kernel density estimators. *Mon. Not. R. Astron. Soc.* **2023**, *526*, 4613–4626. <https://doi.org/10.1093/mnras/stad2997>.
251. Friedman, R.; Hassan, S. HIGlow: Conditional Normalizing Flows for High-Fidelity HI Map Modeling. *arXiv* **2022**, arXiv:2211.12724.
252. Mootoovaloo, A.; García-García, C.; Alonso, D.; Ruiz-Zapatero, J. emuflow: Normalizing flows for joint cosmological analysis. *Mon. Not. R. Astron. Soc.* **2024**, *536*, 190–202. <https://doi.org/10.1093/mnras/stae2604>.
253. Sohl-Dickstein, J.; Weiss, E.; Maheswaranathan, N.; Ganguli, S. Deep Unsupervised Learning Using Nonequilibrium Thermodynamics. In Proceedings of the 32nd International Conference on Machine Learning, Lille, France, 6–11 July 2015; pp. 2256–2265.
254. Song, J.; Meng, C.; Ermon, S. Denoising Diffusion Implicit Models. *arXiv* **2020**, arXiv:2010.02502.
255. Mudur, N.; Cuesta-Lazaro, C.; Finkbeiner, D.P. Cosmological Field Emulation and Parameter Inference with Diffusion Models. In Proceedings of the 37th Conference on Neural Information Processing Systems, New Orleans, LA, USA, 10–16 December 2023.
256. Mudur, N.; Cuesta-Lazaro, C.; Finkbeiner, D.P. Diffusion-HMC: Parameter Inference with Diffusion-model-driven Hamiltonian Monte Carlo. *Astrophys. J.* **2025**, *978*, 64. <https://doi.org/10.3847/1538-4357/ad8bc3>.
257. Koloniari, A.E.; Koursoumpa, E.C.; Nousi, P.; Lampropoulos, P.; Passalis, N.; Tefas, A.; Stergioulas, N. New Gravitational Wave Discoveries Enabled by Machine Learning. *Mach. Learn. Sci. Technol.* **2024**, *6*, 015054. <https://doi.org/10.48550/arXiv.2407.07820>.
258. Thakur, P.; Taridalu, A.; Rather, I.A.; Klangburam, T.; Pongkitivanichkul, C. Constraining axionlike particle mediated dark matter with observational constraints: A statistical and machine learning approach. *Phys. Rev. D* **2026**, *113*, 023001. <https://doi.org/10.1103/j3mm-zjsv>.
259. Einsle, H.; Bizouard, M.A.; Regimbau, T.; Sakellariadou, M. Gravitational-wave background detection using machine learning. *Phys. Rev. D* **2025**, *112*, 063056. <https://doi.org/10.1103/hs9b-drwx>.
260. Iwanaga, H.; Matsuyama, M.; Itoh, Y. Enhancing the reliability of machine learning for gravitational wave parameter estimation with attention-based models. *Phys. Rev. D* **2025**, *112*, 083020. <https://doi.org/10.1103/blfk-7k9f>.
261. Jiang, J.Q.; Huang, H.L.; He, J.; Wang, Y.T.; Piao, Y.S. A fast deep-learning approach to probing primordial black hole populations in gravitational wave events. *arXiv* **2025**, arXiv:2505.15530.

262. Delaunoy, A.; de la Brassinne Bonardeaux, M.; Mishra-Sharma, S.; Louppe, G. Low-Budget Simulation-Based Inference with Bayesian Neural Networks. *arXiv* **2024**, arXiv:2408.15136.
263. Cranmer, K.; Brehmer, J.; Louppe, G. The Frontier of Simulation-Based Inference. *Proc. Natl. Acad. Sci. USA* **2020**, *117*, 30055–30062. <https://doi.org/10.1073/pnas.1912789117>.
264. Alsing, J.; Charnock, T.; Feeney, S.; Wandelt, B. Fast likelihood-free cosmology with neural density estimators and active learning. *Mon. Not. R. Astron. Soc.* **2019**, *488*, 4440–4458. <https://doi.org/10.1093/mnras/stz1960>.
265. Jeffrey, N.; Whiteway, L.; Gatti, M.; Williamson, J.; Alsing, J.; Porredon, A.; Prat, J.; Doux, C.; Jain, B.; Chang, C.; et al. Dark Energy Survey Year 3 results: Likelihood-free, simulation-based w CDM inference with neural compression of weak-lensing map statistics. *Mon. Not. R. Astron. Soc.* **2025**, *536*, 1303–1322. <https://doi.org/10.1093/mnras/stae2508>.
266. Wang, B.; Leja, J.; Villar, V.A.; Speagle, J.S. SBI++: Flexible, ultra-fast likelihood-free inference customized for astronomical applications. *Astrophys. J. Lett.* **2023**, *952*, L10. <https://doi.org/10.3847/2041-8213/acd93d>.
267. Mediato-Diaz, N.; Handley, W. Cosmological Parameter Estimation with Sequential Linear Simulation-Based Inference. *arXiv* **2025**, arXiv:2501.03921.
268. Zhong, F.; Napolitano, N.R.; Comparat, J.; Dolag, K.; Heneka, C.; Huang, Z.; Li, X.; Lin, W.; Longo, G.; Radovich, M.; et al. Machine-learning cosmological parameters by eROSITA data. *arXiv* **2026**, arXiv:2602.20483.
269. Rojas, L.; Espinoza, S.; Gonzalez, E.; Maldonado, C.; Luo, F. A Systematic Literature Review of Machine Learning Techniques for Observational Constraints in Cosmology. *Galaxies* **2025**, *13*, 114. <https://doi.org/10.3390/galaxies13050114>.
270. Kojima, H.; Ikegami, T. Organization of a Latent Space Structure in VAE/GAN Trained by Navigation Data. *Neural Netw.* **2022**, *152*, 234–243. <https://doi.org/10.1016/j.neunet.2022.04.012>.
271. Higgins, I.; Matthey, L.; Pal, A.; Burgess, C.P.; Glorot, X.; Botvinick, M.M.; Mohamed, S.; Lerchner, A. beta-VAE: Learning Basic Visual Concepts with a Constrained Variational Framework. In Proceedings of the International Conference on Learning Representations (ICLR), Toulon, France, 24–26 April 2017.
272. Chen, R.T.Q.; Li, X.; Grosse, R.; Duvenaud, D. Isolating Sources of Disentanglement in VAEs. In Proceedings of the Advances in Neural Information Processing Systems, Montréal, QC, Canada, 3–8 December 2018; Volume 31, pp. 2615–2625.
273. ATLAS Collaboration. Search for light long-lived neutral particles produced in pp collisions at $\sqrt{s} = 13$ TeV and decaying into collimated leptons or light hadrons with the ATLAS detector. *Eur. Phys. J. C* **2020**, *80*, 450.
274. Zhou, J.; Cui, G.; Hu, S.; Zhang, Z.; Yang, C.; Liu, Z.; Wang, L.; Li, C.; Sun, M. Graph Neural Networks: A Review of Methods and Applications. *arXiv* **2018**, arXiv:1812.08434.

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