

Article

A Symmetric SFS-DEMATEL-TODIM Model for Online Movie Review Usefulness Ranking: Integrating Adaptive Weights and Hesitation Penalties

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Abstract

This study examines the characteristics of Group Multi-Attribute Decision Making (GMADM), including highly ambiguous information, divergent expert opinions, and bounded rationality among decision-makers. From the perspective of symmetry modeling and bias control, we propose an adaptive decision-making framework based on Spherical Fuzzy Sets (SFS). First, a spherical fuzzy quantification system for online reviews is constructed to map multi-source asymmetric information within reviews to Spherical Fuzzy Numbers. Second, an adaptive expert weighting mechanism is developed that integrates individual expert performance with the level of group consensus, dynamically adjusting weights to suppress the asymmetric interference of outlier opinions. Subsequently, we design the Credibility-based Spherical Weighted Arithmetic Mean (CSWAM) to preserve the dominance of expert judgments in a nonlinear manner and construct the Spherical Fuzzy Score function with Adaptive Hesitation Penalty (HP-SC) to ensure robustness and non-negativity in the defuzzification process. Furthermore, we extend DEMATEL and TODIM to the SFS environment, constructing a comprehensive evaluation model that captures causal relationships among attributes and asymmetric information, such as decision-makers' loss aversion. Finally, empirical results from online movie review usefulness rankings demonstrate that this model can accurately identify and mitigate asymmetric information biases while maintaining decision symmetry equilibrium and exhibiting higher ranking stability.

Keywords: spherical fuzzy set (SFS); group multi-attribute decision making (GMADM); symmetric perceptual decision making; information asymmetry regulation; adaptive weights; DEMATEL-TODIM

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1. Introduction

In the era of digital consumption and intelligent decision-making, online reviews have become a core basis for consumer decisions and a vital data source for personalized recommendations. Although previous research has shown that online product reviews significantly influence consumer decisions [1,2], the exponential growth in review volume leads to severe information overload [3], making it difficult for consumers to quickly iden-

tify high-value information amid vast amounts of heterogeneous text. This problem is particularly pronounced for experiential products such as movies. Since the value of a movie is highly dependent on the audience's subjective experience, movie reviews exhibit high linguistic ambiguity [4], leading to strong asymmetry and uncertainty in the review data. Accurately assessing and ranking the usefulness of such reviews is key to alleviating information overload; however, traditional mathematical models based on exact values struggle to capture these asymmetric and uncertain characteristics effectively. This often disrupts the symmetric equilibrium required for decision-making, thereby undermining the credibility of the ranking results. In contrast, Spherical Fuzzy Sets (SFS) [5], which leverage their mathematical advantage of allowing decision-makers to independently assign membership, non-membership, and hesitancy in a three-dimensional space, unify symmetric expression spaces and asymmetric parameter values, providing an ideal quantitative tool for handling the fuzziness of online reviews.

Evaluating the usefulness of online reviews is essentially a highly complex Group Multi-Attribute Decision Making (GMADM) problem. The core of constructing a scientific and robust GMADM evaluation system lies in a reasonable attribute weighting and alternative ranking model. In the field of attribute weighting, relevant research has proposed numerous classical weighting methods, such as ANP [6], SWARA [7], and MEREC [8], but these methods often overlook the dynamic asymmetric causal relationships among evaluation indicators and the inherent fuzziness in subjective judgments when dealing with complex review systems; In the realm of alternative ranking, methods such as WASPAS [9], COPRAS [10], EDAS [11] and TOPSIS [12] have been widely applied in various fuzzy decision-making scenarios due to their excellent stability and computational flexibility. However, most of these ranking models assume fully rational decision-makers and fail to capture asymmetric psychological responses to gains and losses.

To address these shortcomings, this study employs the DEMATEL method [13] during the attribute weighting stage to identify asymmetric causal relationships among attributes in complex systems. In recent years, the DEMATEL method has been widely extended to fuzzy environments due to its ability to identify and quantify asymmetric causal relationships among attributes. It has been applied to analyze complex systems under conditions of uncertainty, such as product iteration [14], movie ranking [15], and production risk assessment [16]. Meanwhile, in the solution ranking stage, this paper employs the prospect theory-based TODIM method [17] to quantify decision-makers' limited rational behaviors, such as reference dependence and loss aversion. The TODIM method is one of the few existing GMADM methods that incorporate decision-makers' psychological states; it has now been extended to various fuzzy decision-making environments, including Intuitionistic Fuzzy Sets [18], Pythagorean Fuzzy Sets [19], Probabilistic Linguistic Term Sets [20], and Credibility-Based Interval Hesitant Fuzzy Sets [21] have been applied in various GMADM domains, including commercial insurance selection [22], sustainable supplier selection [23], and product ranking and selection [24].

Although SFS, DEMATEL, and TODIM provide robust decision-making frameworks for addressing GMADM problems in scenarios with uncertain information, there remain many theoretical gaps in their application to the ranking of reviews. First, movie review data is highly susceptible to interference from extreme sentiments, paid trolls, or fan bias. If the expert opinion aggregation phase relies solely on static weights or subjective weighting, without a dynamic mechanism to suppress outlier opinions, it can easily distort genuine group consensus when extreme data are present. Second, traditional spherical fuzzy aggregation operators focus solely on experts' global weights, ignoring differences in credibility across dimensions for the same expert, thereby losing an expert's judgmental advantage in specific domains. Finally, the ambiguity inherent in movie review texts can easily trigger significant hesitation within the decision-making group. Existing

spherical fuzzy scoring functions, however, rely excessively on accuracy functions, which not only increase computational complexity but also fail to account for the group's overall level of hesitation. Consequently, the aggregated results struggle to accurately reflect the true reliability of the group's decision-making environment.

Based on the above analysis, this study builds on the SFS, DEMATEL, and TODIM methods and focuses on the specific challenges in ranking the usefulness of online movie reviews, thereby proposing a novel adaptive multi-attribute group decision-making framework. The innovations of this paper primarily lie in the following three aspects:

1. **Proposal of an Adaptive Expert Weight Mechanism:** A dynamic weight adjustment mechanism was developed that integrates individual evaluation performance with the level of group consensus, enabling the automatic identification and suppression of poor individual performance and outlier opinions, and dynamically adjusting expert weights.
2. **Design of a Credibility-based Spherical Weighted Arithmetic Mean (CSWAM):** This operator performs nonlinear aggregation of spherical fuzzy information based on experts' credibility coefficients across different evaluation dimensions, effectively preserving experts' local judgment advantages in their areas of expertise.
3. **Construction of the Spherical Fuzzy Score Function with Adaptive Hesitation Penalty (HP-SC):** By introducing the group average hesitation degree as a penalty factor, this approach eliminates dependence on exact functions while ensuring the robustness and non-negativity of the defuzzification process, thereby aligning the aggregation results with the group's level of uncertainty.

Finally, this paper integrates the aforementioned innovations into the SFS-extended DEMATEL-TODIM framework to construct a complete symmetric-aware decision-making model. To verify the system's reliability in handling local weight biases and environmental disturbances, this paper conducts multiple sensitivity analyses, including single-attribute weight perturbations, further demonstrating the robustness of this decision-making framework and the stability of its rankings.

The remainder of this paper is organized as follows: Section 2 introduces relevant background knowledge; Section 3 elaborates on the mathematical logic of the adaptive expert weighting mechanism, CSWAM, and HP-SC; Section 4 defines the research problem; Section 5 presents the computational results of the case study; Section 6 discusses the results; and Section 7 summarizes the paper and outlines future research directions.

2. Preliminaries

This section provides an overview of some basic definitions required for this study.

2.1. Spherical Fuzzy Set (SFS)

Spherical Fuzzy Set (SFS) is a further extension of the Fuzzy set [25], Intuitionistic fuzzy set (IFS) [26], and Pythagorean Fuzzy Set (PFS) [27]. By relaxing the implicit symmetry constraints imposed by previous-generation fuzzy sets on membership and non-membership, it can more flexibly characterize the high levels of fuzziness and uncertainty present in real-world decision-making. Figure 1 illustrates the differences between IFS, PFS, and SFS. From a geometric perspective, IFS restricts the expression space of decision information to a triangular region, while PFS extends it to a spherical surface within the first quadrant. SFS, on the other hand, is represented in three dimensions as a portion of a solid unit sphere within the first quadrant. Within this solid region, decision-makers can independently assign their membership, non-membership, and hesitancy, and all three can be defined separately on the interval $[0, 1]$, subject only to the constraint that their sum of squares does not exceed 1. This not only allows decision-makers to independently allo-

cate membership, non-membership, and hesitancy in their views across a broader domain but also unifies “symmetry in the representation space” and “asymmetry in parameter values” within fuzzy set theory. Leveraging this advantage, SFS provides a more expressive mathematical framework for characterizing indecision and ambiguity in online reviews.

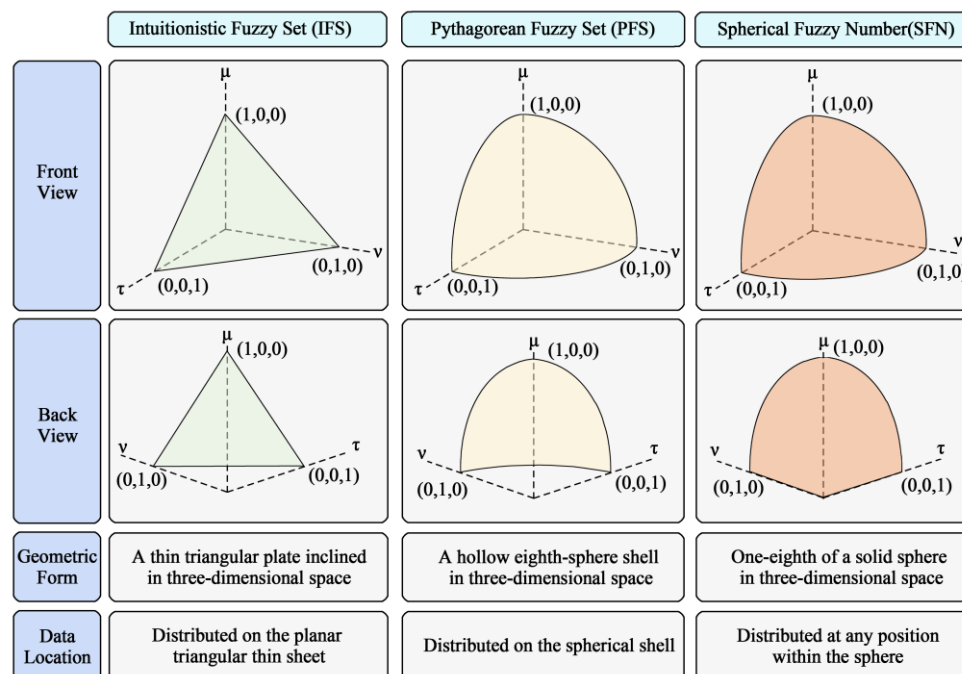


Figure 1. Geometric representations and decision spaces of IFS, PFS, and SFS.

In this section, we present the definition of SFS, its arithmetic operation rules, and its aggregation and defuzzification operations [28,29].

Definition 1. Let domain $X = \{x_1, x_2, \dots, x_n\}$ be a non-empty finite set, and let $A = \{(x, \mu_A(x), \nu_A(x), \tau_A(x)) \mid x \in X\}$ be a Spherical Fuzzy Set (SFS) on domain X . Here, $\mu_A(x): X \rightarrow [0,1]$ denotes the membership degree of element x in set A , representing the degree of support for $x \in A$. $\nu_A(x): X \rightarrow [0,1]$ denotes the non-membership degree of element x in set A , representing the degree of opposition to $x \in A$. $\tau_A(x): X \rightarrow [0,1]$ is the hesitancy of element x regarding set A , representing the degree of uncertainty and indecision regarding $x \in A$. For any $x \in X$, $0 \leq \mu_A(x)^2 + \nu_A(x)^2 + \tau_A(x)^2 \leq 1$. In particular, the ordered pair $A = (\mu_A, \nu_A, \tau_A)$ consisting of membership, non-membership and hesitancy is called a Spherical Fuzzy Number (SFN).

Basic Properties 1. Let $\tilde{\alpha}_1 = (\mu_1, \nu_1, \tau_1)$ and $\tilde{\alpha}_2 = (\mu_2, \nu_2, \tau_2)$ be any two SFNs in the domain X . The arithmetic operations for Spherical Fuzzy Numbers are defined as follows:

1 Union

$$\tilde{\alpha}_1 \cup \tilde{\alpha}_2 = \left(\begin{array}{cc} \max\{\mu_{\tilde{\alpha}_1}, \mu_{\tilde{\alpha}_2}\}, & \min\{v_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_2}\}, \\ \min\left\{1 - \left(\left(\max\{\mu_{\tilde{\alpha}_1}, \mu_{\tilde{\alpha}_2}\}\right)^2 + \left(\min\{v_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_2}\}\right)^2\right)^{\frac{1}{2}}, \max\{\tau_{\tilde{\alpha}_1}, \tau_{\tilde{\alpha}_2}\}\right\} \end{array} \right)$$

2 Intersection

$$\tilde{\alpha}_1 \cap \tilde{\alpha}_2 = \left(\begin{array}{cc} \min\{\mu_{\tilde{\alpha}_1}, \mu_{\tilde{\alpha}_2}\}, & \max\{v_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_2}\}, \\ \max\left\{1 - \left(\left(\min\{\mu_{\tilde{\alpha}_1}, \mu_{\tilde{\alpha}_2}\}\right)^2 + \left(\max\{v_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_2}\}\right)^2\right)^{\frac{1}{2}}, \min\{\tau_{\tilde{\alpha}_1}, \tau_{\tilde{\alpha}_2}\}\right\} \end{array} \right)$$

3 Addition

$$\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \left(\sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2 \mu_2^2}, v_1 v_2, \sqrt{(1 - \mu_1^2) \tau_1^2 + (1 - \mu_2^2) \tau_2^2 - \tau_1^2 \tau_2^2} \right)$$

4 Multiplication

$$\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \left(\mu_1 \mu_2, \sqrt{v_1^2 + v_2^2 - v_1^2 v_2^2}, \sqrt{(1 - v_1^2) \tau_1^2 + (1 - v_2^2) \tau_2^2 - \tau_1^2 \tau_2^2} \right)$$

5 Multiplication by the same scalar $(\lambda > 0)$

$$\lambda \tilde{\alpha}_1 = \left(\sqrt{1 - (1 - \mu_1^2)^\lambda}, v_1^\lambda, \sqrt{(1 - \mu_1^2)^\lambda - (1 - \mu_1^2 - \tau_1^2)^\lambda} \right), \lambda > 0$$

6 Power of $\tilde{\alpha}_1$ $(\lambda > 0)$

$$\tilde{\alpha}_1^\lambda = \left(\mu_1^\lambda, \sqrt{1 - (1 - v_1^2)^\lambda}, \sqrt{(1 - v_1^2)^\lambda - (1 - v_1^2 - \tau_1^2)^\lambda} \right), \lambda > 0$$

Basic Properties 2. Let SFS $\tilde{\alpha}_1 = (\mu_1, v_1, \tau_1)$ and $\tilde{\alpha}_2 = (\mu_2, v_2, \tau_2)$ be given. Under condition $\lambda, \lambda_1, \lambda_2 > 0$, the following operational rules hold.

$$\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \tilde{\alpha}_2 \oplus \tilde{\alpha}_1$$

$$\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \tilde{\alpha}_2 \otimes \tilde{\alpha}_1$$

$$\lambda(\tilde{\alpha}_1 \oplus \tilde{\alpha}_2) = \lambda \tilde{\alpha}_1 \oplus \lambda \tilde{\alpha}_2$$

$$\lambda_1 \tilde{\alpha}_1 \oplus \lambda_2 \tilde{\alpha}_1 = (\lambda_1 + \lambda_2) \tilde{\alpha}_1$$

$$(\tilde{\alpha}_1 \otimes \tilde{\alpha}_2)^\lambda = \tilde{\alpha}_1^\lambda \otimes \tilde{\alpha}_2^\lambda$$

$$\tilde{\alpha}_1^{\lambda_1} \otimes \tilde{\alpha}_1^{\lambda_2} = \tilde{\alpha}_1^{\lambda_1 + \lambda_2}$$

Basic Properties 3. In the group decision-making process, aggregation operators must be introduced to combine the opinions of different experts into a comprehensive evaluation; the most commonly used traditional aggregation operator is the Spherical Weighted Arithmetic Mean (SWAM). Let $w = (w_1, w_2, \dots, w_n)$ be the set of weights cor-

responding to a set of SFNs $A = (\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$, and let them satisfy $w_i \in [0, 1]; \sum_{i=1}^n w_i = 1$. Then SWAM is defined as:

$$SWAM_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = w_1 \tilde{\alpha}_1 + w_2 \tilde{\alpha}_2 + \dots + w_n \tilde{\alpha}_n \\ = \left\{ \left[1 - \prod_{i=1}^n (1 - \mu_{\tilde{\alpha}_i}^2)^{w_i} \right]^{1/2}, \prod_{i=1}^n v_{\tilde{\alpha}_i}^{w_i}, \left[\prod_{i=1}^n (1 - \mu_{\tilde{\alpha}_i}^2)^{w_i} - \prod_{i=1}^n (1 - \mu_{\tilde{\alpha}_i}^2 - \tau_{\tilde{\alpha}_i}^2)^{w_i} \right]^{1/2} \right\}$$

Basic Properties 4. Since the SFN consists of three components—membership, non-membership, and hesitancy—it cannot be directly compared in terms of magnitude or fed into the subsequent GMADM framework. The most common approach to addressing this issue is to define a scoring function to convert the SFN into a real-valued number. However, in the traditional defuzzification process, when the results of the scoring function are identical, an accuracy function is required to determine the relative ranking. Let SFS $\tilde{\alpha}_1 = (\mu_1, \nu_1, \tau_1)$ be given. Define its scoring function $SC(\tilde{\alpha}_1)$ and accuracy function $AC(\tilde{\alpha}_1)$ as follows:

$$SC(\tilde{\alpha}_1) = (\mu_1 - \tau_1)^2 - (\nu_1 - \tau_1)^2 \\ AC(\tilde{\alpha}_1) = \mu_1^2 + \nu_1^2 + \tau_1^2$$

$$\tilde{\alpha}_1 \prec \tilde{\alpha}_2 \quad \text{if and only if} \quad SC(\tilde{\alpha}_1) < SC(\tilde{\alpha}_2), \quad \text{or} \quad SC(\tilde{\alpha}_1) = SC(\tilde{\alpha}_2) \quad \text{and} \\ AC(\tilde{\alpha}_1) < AC(\tilde{\alpha}_2).$$

2.2. DEMATEL

DEMATEL (Decision-Making Trial and Evaluation Laboratory) is a systematic analysis method based on graph theory and matrix analysis that effectively identifies and quantifies asymmetric causal relationships and the degree of mutual influence among attributes within the GMADM framework. By quantifying the direct and indirect influences among criteria, this method calculates the centrality and causality of each criterion, thereby enabling the ranking of criterion importance and the classification of causal relationships. This provides a scientific and quantitative basis for determining criterion weights in subsequent multi-criteria decision-making. Based on this, this study attempts to use SFS as the information input format for the DEMATEL method. Within a framework that unifies the symmetric expression space provided by SFS with its asymmetric information distribution, the DEMATEL method is further extended to a spherical fuzzy set environment to better characterize subjective fuzziness and cognitive hesitation. The following outlines the DEMATEL operational process:

Step 1: After inviting experts to score the items, an initial direct influence matrix A is formed, where x_{ij} represents the strength of the direct effect of factor i on factor j .

$$A = \begin{bmatrix} 0 & x_{12} & \dots & x_{1n} \\ x_{21} & 0 & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & 0 \end{bmatrix}$$

Step 2: Normalize matrix A to obtain the normalized matrix $\hat{B}$.

$$B = \frac{x_{ij}}{\max \left(\sum_{j=1}^n x_{ij} \right)}$$

Step 3: Calculate the comprehensive influence matrix T . The comprehensive system matrix T reflects the combined effects resulting from the interactions among the various attributes within the system, where I denotes the identity matrix.

$$T = B(I - B)^{-1} = (B + B^2 + \dots + B^k) = \sum_{K=1}^{\infty} B^k$$

Step 4: After calculating the degree of influence D_i and the degree of being influenced F_i , further calculate the degree of centrality M_i and the degree of cause R_i

$$D_i = \sum_{j=1}^n x_{ij}, (i = 1, 2, \dots, n)$$

$$F_i = \sum_{j=1}^n x_{ji}, (i = 1, 2, \dots, n)$$

$$M_i = D_i + F_i$$

$$R_i = D_i - F_i$$

Step 5: Normalize the M_i and R_i for each attribute and calculate the weight values for each attribute, where z_j represents the initial weight corresponding to attribute f_j , and satisfies $z_j \in [0, 1]$ and $\sum_{j=1}^n z_j = 1$.

$$z_j = \frac{D_i + F_i}{\sum_{i=1}^n (D_i + F_i)}$$

2.3. TODIM

The TODIM method, proposed by Gomes and Lima in 1991, effectively captures decision-makers' reference dependence and loss-averse behavior in alternative ranking [17], thereby challenging the rational assumption in traditional expected utility theory that decision-makers exhibit symmetric risk preferences in both gain and loss scenarios. Specifically, when evaluating the relative advantages of alternatives, TODIM employs a piecewise value function based on prospect theory. This function is concave for gains and convex for losses, thereby reflecting decision-makers' risk-seeking tendencies and their heightened sensitivity to losses. This construction enables TODIM to capture the two asymmetric psychological behaviors—risk aversion when facing gains and risk-seeking when facing losses—more closely approximating real-world decision-making behavior. The following outlines the computational process of TODIM:

Step 1: Generate the initial decision matrix Y as input data for the TODIM, where $i = 1, 2, \dots, m; j = 1, 2, \dots, n$.

$$Y = \begin{bmatrix} y_{11} & \cdots & y_{1j} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{i1} & \cdots & y_{ij} & \cdots & y_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{m1} & \cdots & y_{mj} & \cdots & y_{mn} \end{bmatrix}_{m \times n}$$

Step 2: Standardize the metrics in the original decision matrix.

$$d_{ij} = \frac{y_{ij} - \min(y_{ij})}{\max(y_{ij}) - \min(y_{ij})}$$

Step 3: Select the indicator with the highest weight as the reference indicator ω_r , and calculate the relative weight ω_{jr} of each indicator relative to the reference indicator ω_r .

$$\omega_r = \max\{\omega_j | j = 1, 2, \dots, n\}$$

$$\omega_{jr} = \frac{\omega_j}{\omega_r}, j = 1, 2, \dots, n$$

Step 4: For each attribute, calculate the relative advantage $\Phi_j(R_i, R_v)$ of Alternative R_i over Alternative R_v , where θ represents the loss attenuation coefficient, which is a constant assigned by the decision-maker based on the specific circumstances. Generally, the smaller the value of θ , the higher the decision-maker's aversion to loss.

$$\Phi_j(R_i, R_v) = \begin{cases} \sqrt{\frac{\omega_{jr}(d_{ij} - d_{vj})}{\sum_{j=1}^m \omega_{jr}}}, & d_{ij} > d_{vj} \\ 0, & d_{ij} = d_{vj} \\ -\frac{1}{\theta} \sqrt{\frac{\sum_{j=1}^m \omega_{jr}(d_{ij} - d_{vj})}{\omega_{jr}}}, & d_{ij} < d_{vj} \end{cases}$$

Step 5: Calculate the combined dominance degree $\varphi(R_i, R_v)$, where $i, v = 1, 2, \dots, m$.

$$\varphi(R_i, R_v) = \sum_{j=1}^m \Phi_j(R_i, R_v)$$

Step 6: Calculate and rank the global dominance degree $\Psi(R_i)$. The global dominance degree indicates the relative quality of the alternative solutions, and they are ranked based on the magnitude of this value; the higher the value, the higher the rank.

$$\Psi(R_i) = \frac{\sum_{j=1}^n \varphi(R_i, R_v) - \min \sum_{j=1}^n \varphi(R_i, R_v)}{\max \left\{ \sum_{j=1}^n \varphi(R_i, R_v) \right\} - \min \left\{ \sum_{j=1}^n \varphi(R_i, R_v) \right\}}$$

2.4. Related Research and Methods

In recent years, the Group Multi-Attribute Decision Making (GMADM) method has garnered widespread attention and has been applied to address complex evaluation problems. In terms of attribute weighting, methods such as the Analytic Hierarchy Process

(AHP) and its fuzzy extensions are widely used to assign subjective and objective weights to multi-level criteria; the SWARA method has demonstrated excellent performance in reducing expert cognitive load and extracting subjective preferences; while the MEREC method, based on the removal effect, offers a novel and reliable perspective for extracting objective weights. In the alternative ranking and evaluation stage, classical methods such as WASPAS, COPRAS, and EDAS are widely used across various fuzzy decision-making scenarios due to their robustness and computational flexibility.

However, existing research still has certain limitations when applied to evaluating the usefulness of online reviews. First, most of the aforementioned classical ranking methods assume fully rational decision-makers and thus fail to effectively capture asymmetric psychological perceptions of decision-makers when faced with gains and losses. Second, although some studies have incorporated SFS into decision-making frameworks to address high uncertainty, existing methods often rely on static expert weights or traditional scoring functions during the expert opinion aggregation and fuzzy information refinement stages. This not only lacks a dynamic suppression mechanism for outlier expert opinions but also fails to establish an adaptive control mechanism for such outliers.

Table 1 presents the methodological characteristics of recent relevant studies.

Table 1. Methodological characteristics of recent related studies.

Reference Number	Fuzzy Environment	Expert Weights	Information Aggregation	Defuzzification	Attribute Weighting	Ranking Model
[30]	The Triangular Fuzzy Number (TFN)	/	/	Possibility degree	FAHP	COPRAS
[31]	Circular Intuitionistic Fuzzy Set (CIFS)	Use the CIFS linguistic terms to determine	CIFWG	Traditional CIFS scoring function and accuracy function	CIFS-SWARA	CIFS-WASPAS
[32]	Probabilistic Reliable Linguistic Term Set (PRLTS)	Equal Weighting	PRLWA	Expectation function	PRL-MEREC	PRL-EDAS
[33]	Spherical Fuzzy Sets (SFS)	Deriving static weights via linguistic variables	SWGM	A New Spherical Fuzzy Scoring Function	SF-MPSI	SF-RAPS
[34]	Interval Type-2 Fuzzy Sets (IT2FSs)	Equal Weighting	Arithmetic Mean Aggregation	An Ordering Value Function Based on Interval Type-II Fuzzy Numbers	Best-Worst Method (BWM)	IT2F TOPSIS
[35]	The Triangular Fuzzy Number (TFN)	Equal Weighting	Arithmetic Mean Aggregation	Basic Defuzzification Functions	An Expert Rating Method Based on TFN	Fuzzy TOPSIS Fuzzy VIKOR Fuzzy GRA
[36]	Spherical Fuzzy Sets (SFS)	Calculated objectively based on SF entropy (SFE) measure	SFGDWA/SFGDWG	SF score function involving the natural logarithm of the refusal degree	BWM-EWM	SF-COPRAS

3. Logic of Mathematical Models

3.1. Adaptive Expert Weighting Mechanism

In Group Multi-Attribute Decision Making (GMADM), subjective biases stemming from differences in experts' professional backgrounds, cognitive preferences, and decision-making hesitation [37] can affect the quality of expert evaluations. Traditional equal-weighting or subjective weighting methods not only struggle to quantify such influences but also fail to mitigate the distorting effect of outlier opinions on group consensus. To address this, this paper proposes an adaptive expert weighting mechanism that integrates individual performance with group consensus. This mechanism primarily consists of two processes: First, it quantifies the quality of individual expert evaluations based on three criteria—information certainty, distinctiveness of viewpoints, and group consensus—and calculates the base weights for each expert accordingly; second, it introduces a group consensus benchmark to dynamically penalize outlier experts whose evaluations deviate from the group consensus, thereby achieving adaptive regulation of expert weights.

3.1.1. Determination of Base Weights Based on Individual Performance

To assess the quality of individual expert evaluations, this study employs three dimensions—certainty, discriminative power, and similarity—to determine the baseline weights for each expert. Among these, the certainty metric based on fuzzy entropy reflects the clarity of expert evaluations; the discriminative power metric captures the intensity of experts' preferences between supporting and opposing views, manifested as the degree of difference between membership and non-membership in the SFN; the similarity measure assesses the level of consistency between an expert and other group members, reflecting the degree to which an expert's individual cognition aligns with group consensus. The specific calculation process is shown in Figure 2.

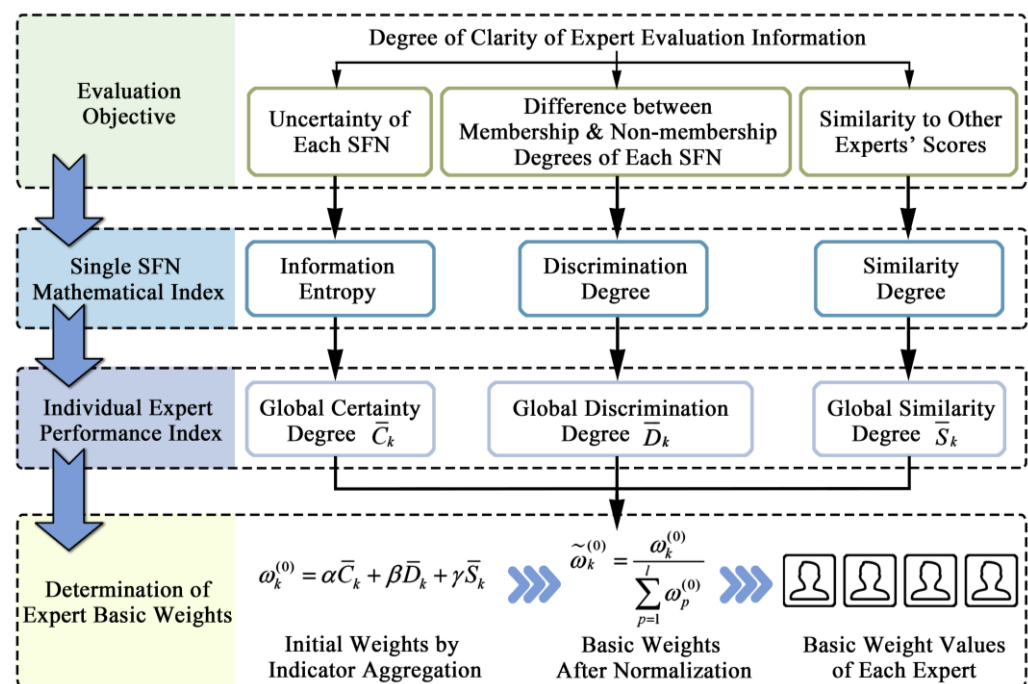


Figure 2. Determination of expert basic weights based on individual performance.

In a group decision-making scenario, let there be l experts $E_k (k=1,2,\dots,l)$ who evaluate n factors $F_i (i=1,2,\dots,n)$ under m alternatives $S_j (j=1,2,\dots,m)$. The spherical fuzzy evaluation $A_{kji} = (\mu_{kji}, \nu_{kji}, \tau_{kji})$ provided by expert E_k for factor F_i under alternative S_j satisfies $\mu_{kji}, \nu_{kji}, \tau_{kji} \in [0,1]$ and $0 \leq \mu_{kji}^2 + \nu_{kji}^2 + \tau_{kji}^2 \leq 1$.

Definition 2. The certainty of expert evaluations is an important criterion for measuring their quality; the clearer the evaluation information and the less hesitation there is, the higher its reference value. Based on this, this paper quantifies the clarity of expert evaluations from the perspectives of information entropy and certainty. For a spherical fuzzy number $A_{kji} = (\mu_{kji}, \nu_{kji}, \tau_{kji})$, the information entropy $\varepsilon(A_{kji})$, certainty degree $C(A_{kji})$, and the global certainty degree $\bar{C}_k$ of expert E_k are defined as:

$$\varepsilon(A_{kji}) = \frac{1}{\ln 3} \left[\mu_{kji}^2 \ln \left(\frac{1}{\mu_{kji}^2} \right) + \nu_{kji}^2 \ln \left(\frac{1}{\nu_{kji}^2} \right) + \tau_{kji}^2 \ln \left(\frac{1}{\tau_{kji}^2} \right) \right] \quad (1)$$

$$C(A_{kji}) = 1 - \varepsilon(A_{kji}) \quad (2)$$

$$\bar{C}_k = \frac{1}{m \times n} \sum_{j=1}^m \sum_{i=1}^n C(A_{kji}) \quad (3)$$

where the information entropy $\varepsilon(A_{kji})$ satisfies $0 \leq \varepsilon(A_{kji}) \leq 1$, and a larger value indicates that the SFN information is more ambiguous. Specifically, when a component of the SFN (membership, non-membership and hesitancy) is 0, this paper follows the standard limit convention of $0 \cdot \log \frac{1}{0} \triangleq \lim_{x \rightarrow 0^+} x \log \frac{1}{x} = 0$ in information theory and specifies that $0 \cdot \log \frac{1}{0} = 0$ in this case. Additionally, when $A_{kji} = (1, 0, 0)$ or $(0, 1, 0)$ (complete certainty), $\varepsilon(A_{kji})$ takes the minimum value of 0; when $A_{kji} = (0, 0, 0)$ (invalid data), $\varepsilon(A_{kji})$ takes the maximum value of 1; the larger the global certainty degree $\bar{C}_k$ of expert E_k , the more explicit the expert's viewpoint.

Definition 3. The clarity of an expert's position also affects the quality of the evaluation: the greater the difference between membership and non-membership, the clearer the expert's stance, and the higher the evaluation weight should be. This paper defines "discrimination degree" to characterize the strength of an expert's position. The discrimination degree $D(A_{kji})$ of the spherical fuzzy number $A_{kji} = (\mu_{kji}, \nu_{kji}, \tau_{kji})$ and the global discrimination degree $\bar{D}_k$ of expert E_k are defined as:

$$D(A_{kji}) = (\mu_{kji}^2 - \nu_{kji}^2)^2 \quad (4)$$

$$\bar{D}_k = \frac{1}{m \cdot n} \sum_{j=1}^m \sum_{i=1}^n D(A_{kji}) \quad (5)$$

where a larger discrimination degree $D(A_{kji})$ means a greater difference between the membership degree and non-membership degree of A_{kji} and a clearer opinion. When $A_{kji} = (0, 0, 0)$ (the expert provides no clear judgment), $D(A_{kji}) = 0$, which conforms to practical intuition. The global discrimination degree $\bar{D}_k$ reflects the average clarity of expert E_k 's judgments across all alternatives and factors.

Definition 4. In group decision-making, the consistency between expert evaluations and group consensus is also an important criterion for assessing the reliability of opinions: the smaller the deviation from the majority of experts' views, the more likely the opinion is to be accepted by the group. This paper quantifies the level of cognitive consistency between individual experts and other experts by defining "similarity degree". For the same factor F_i under the same alternative S_j , the similarity degree $S_{kq}^{(ji)}$ between the evaluation $A_{kji} = (\mu_{kji}, \nu_{kji}, \tau_{kji})$ by expert E_k and the evaluation $A_{qji} = (\mu_{qji}, \nu_{qji}, \tau_{qji})$ by expert E_q , and the global similarity degree $\bar{S}_k$ of expert E_k are defined as:

$$S_{kq}^{(ji)} = 1 - \frac{1}{3} \left[\left| \mu_{kji}^2 - \mu_{qji}^2 \right| + \left| \nu_{kji}^2 - \nu_{qji}^2 \right| + \left| \tau_{kji}^2 - \tau_{qji}^2 \right| \right] \quad (6)$$

$$\bar{S}_k = \frac{1}{m \cdot n \cdot (l-1)} \sum_{j=1}^m \sum_{i=1}^n \sum_{\substack{q=1 \\ q \neq k}}^l S_{kq}^{(ji)} \quad (7)$$

where the global similarity degree $\bar{S}_k$ reflects the average similarity between expert E_k and all other experts across all factors.

Definition 5. After synthesizing the global certainty degree $\bar{C}_k$, global discrimination degree $\bar{D}_k$, and global similarity degree $\bar{S}_k$ that measure the individual performance of each expert, we obtain the experts' basic weights through normalization to quantify the quality of their individual evaluations. The initial weight $\omega_k^{(0)}$ and basic weight $\omega_k^{(0)}$ of expert E_k are defined as:

$$\omega_k^{(0)} = \alpha \bar{C}_k + \beta \bar{D}_k + \gamma \bar{S}_k \quad (8)$$

$$\omega_k^{(0)} = \begin{cases} \frac{\omega_k^{(0)}}{\sum_{p=1}^l \omega_p^{(0)}}, & \sum_{p=1}^l \omega_p^{(0)} \neq 0 \\ \frac{1}{l}, & \text{otherwise} \end{cases} \quad (9)$$

where $\alpha, \beta, \gamma > 0$ are weight coefficients satisfying $\alpha + \beta + \gamma = 1$, and the default values in this study are $\alpha = 0.4, \beta = 0.3, \gamma = 0.3$. The basic weight $\omega_k^{(0)}$ is obtained by normalizing the initial weight $\omega_k^{(0)}$. In Equation (9), if all experts provide completely invalid information such that $\sum_{p=1}^l \omega_p^{(0)} = 0$, all expert opinions are regarded as uninformative, and the basic weights degenerate to equal weights, i.e., $\omega_k^{(0)} = \frac{1}{l}$.

3.1.2. Dynamic Weight Adjustment Based on Group Consensus

Although the base weights account for the quality of individual experts' evaluations, they fail to constrain outlier opinions that deviate from the group consensus, which can easily lead to extreme viewpoints unduly influencing decision outcomes. Therefore, to avoid the excessive influence of outlier experts, this study introduces a penalty adjustment mechanism driven by group consensus. First, using the experts' base weights, the SWAM operator is applied to calculate the weighted aggregation center for each factor at the group level—that is, the group center—which serves as the group consensus benchmark for that factor. Subsequently, the degree of deviation of each expert relative to the group center is measured, and these values are used to apply appropriate penalty adjustments to each expert's final weight. This ensures that outlier experts with greater deviation have their final weights suppressed more strongly, while experts with lower deviation retain relatively stable weights. Ultimately, this yields final expert weights that balance both individual quality and group consensus. The calculation process is shown in Figure 3.

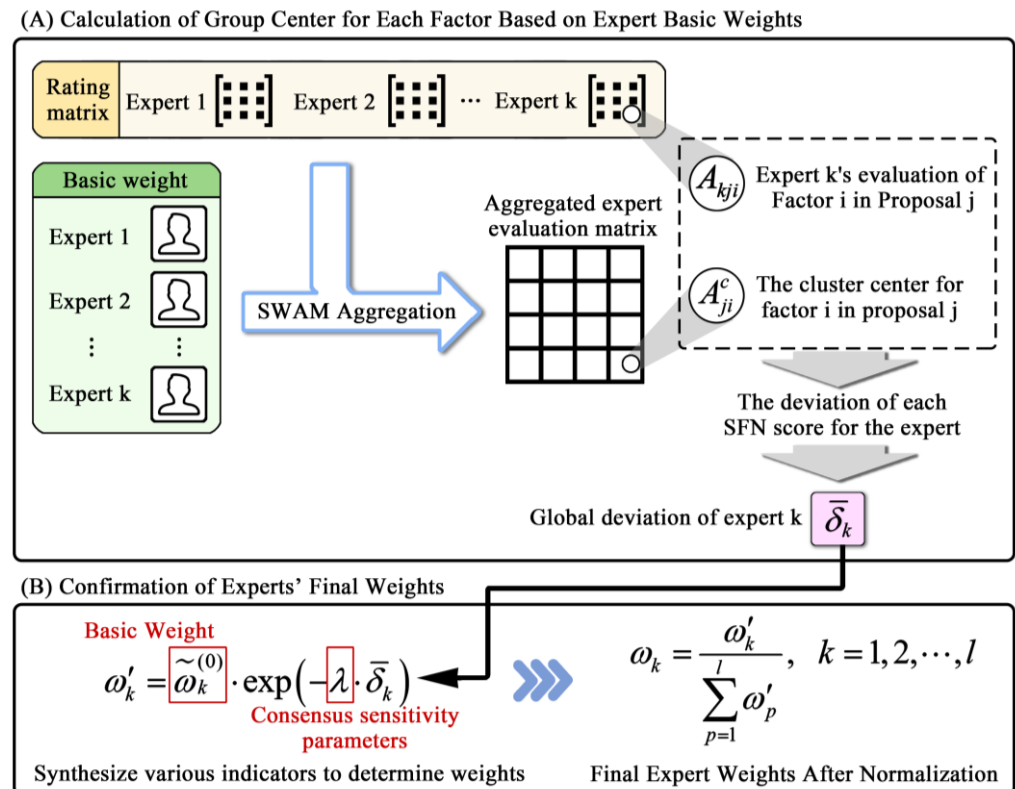


Figure 3. Determination of experts' final weights based on group consensus.

Definition 6. Based on the base weights assigned to each expert as determined earlier, the SWAM algorithm is used to aggregate the experts' evaluations to represent the group consensus. Define the group center of factor F_i under alternative S_j as $A_{ji}^c = (\mu_{ji}^c, \nu_{ji}^c, \tau_{ji}^c)$, which is given by the Spherical Weighted Arithmetic Mean (SWAM) based on the basic expert weights $\omega_k^{(0)}$:

$$A_{ji}^c = (\mu_{ji}^c, \nu_{ji}^c, \tau_{ji}^c) = \text{SWAM}_{\omega^{(0)}}(A_{1ji}, A_{2ji}, \dots, A_{lji})$$

$$= \left(\left[\frac{1 - \prod_{k=1}^l (1 - \mu_{kji}^2)^{\omega_k^{(0)}}}{\prod_{k=1}^l (1 - \mu_{kji}^2)^{\omega_k^{(0)}} - \prod_{k=1}^l (1 - \nu_{kji}^2 - \tau_{kji}^2)^{\omega_k^{(0)}}} \right]^{1/2}, \frac{\prod_{k=1}^l \nu_{kji}^{\omega_k^{(0)}}}{\prod_{k=1}^l (1 - \mu_{kji}^2)^{\omega_k^{(0)}} - \prod_{k=1}^l (1 - \nu_{kji}^2 - \tau_{kji}^2)^{\omega_k^{(0)}}} \right)^{1/2} \quad (10)$$

Definition 7. The evaluations of outlier experts can distort group consensus. This paper quantifies the degree to which each expert's evaluation deviates from the group center using a measure of deviation, and uses this as the basis for weight penalties. Define the deviation degree δ_{kji} of the evaluation A_{kji} by expert E_k on factor F_i under alternative S_j relative to the group center A_{ji}^c , and the global deviation degree $\bar{\delta}_k$ of expert E_k , as follows:

$$\delta_{kji} = \sqrt{\frac{1}{3} \left[\left| \mu_{kji}^2 - (\mu_{ji}^c)^2 \right| + \left| \nu_{kji}^2 - (\nu_{ji}^c)^2 \right| + \left| \tau_{kji}^2 - (\tau_{ji}^c)^2 \right| \right]} \quad (11)$$

$$\bar{\delta}_k = \frac{1}{m \cdot n} \sum_{j=1}^m \sum_{i=1}^n \delta_{kji} \quad (12)$$

where the global deviation degree $\bar{\delta}_k$ reflects the average deviation of expert E_k from the corresponding group center across all factors.

Definition 8. Based on the global deviation degree of each expert, the base weights are dynamically adjusted using an adaptive penalty mechanism, and the final weights for the experts are obtained after normalization. Define the final weight ω_k of expert E_k :

$$\omega'_k = \omega_k^{(0)} \cdot \exp(-\lambda \cdot \bar{\delta}_k) \quad (13)$$

$$\omega_k = \frac{\omega'_k}{\sum_{p=1}^l \omega'_p}, \quad k = 1, 2, \dots, l \quad (14)$$

where $\lambda > 0$ is the consensus sensitivity parameter, set as $\lambda = 1.5$ in this study. A larger λ indicates a heavier penalty on outlier experts.

3.2. A Novel Spherical Fuzzy Aggregation Operator Based on Credibility Weighting

Traditional spherical fuzzy aggregation operators typically consider only the global weights assigned by experts, ignoring the fact that the same expert may exhibit varying levels of evaluation credibility across different factors—that is, experts tend to be more certain in their evaluations within familiar domains and more hesitant in unfamiliar ones. To address this phenomenon and shortcoming, this study proposes a Credibility-based Spherical Weighted Arithmetic Mean (CSWAM). Building upon expert weights, CSWAM introduces credibility coefficients for each factor to achieve more refined information aggregation. The following sections present, in order, the definition of the credibility coefficient, the form of the aggregation operator, and the proof of its mathematical properties.

In a group decision-making context, for factor F_i under alternative S_j , the evaluation given by expert E_k is $A_{kji} = (\mu_{kji}, \nu_{kji}, \tau_{kji})$.

Definition 9. Define the credibility coefficient ρ_{kji} of expert E_k with respect to factor F_i under alternative S_j as:

$$\rho_{kji} = \begin{cases} \frac{\omega_k \cdot C(A_{kji})}{\sum_{p=1}^l \omega_p \cdot C(A_{pji})}, & \sum_{p=1}^l \omega_p \cdot C(A_{pji}) \neq 0 \\ \omega_k, & \text{otherwise} \end{cases} \quad (15)$$

where ω_k is the final weight of expert E_k , and $C(A_{kji})$ is the certainty degree of the evaluation A_{kji} by expert E_k , obtained from Equations (1) and (2). In Equation (15), when all experts express complete hesitation in evaluating factor F_i under alternative S_j , i.e., $\sum_{p=1}^l \omega_p \cdot C(A_{pji}) = 0$, it indicates that no credible judgment advantage exists in this dimension. In this case, the credibility coefficient ρ_{kji} degenerates to the global final weight ω_k of the expert.

Definition 10. For factor F_i under alternative S_j , the Credibility-based Spherical Weighted Arithmetic Mean (CSWAM) is adopted to compute the group aggregation result.

$$A_{ji}^{agg} = \text{CSWAM}_{\rho} (A_{1ji}, A_{2ji}, \dots, A_{lji}) = (\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg})$$

$$= \left(\left[1 - \prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} \right]^{\frac{1}{2}}, \prod_{k=1}^l \nu_{kji}^{\rho_{kji}}, \left[\prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \right]^{\frac{1}{2}} \right) \quad (16)$$

Theorem 1. *Component Boundedness of the Aggregated SFN*

To ensure the mathematical validity of CSWAM, it must be proven that the aggregated SFN still satisfies the component boundedness condition: $A_{ji}^{agg} = (\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg})$ satisfies $0 \leq \mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg} \leq 1$.

Proof of Theorem 1. It is known that each input $A_{kji} = (\mu_{kji}, \nu_{kji}, \tau_{kji})$ satisfies $0 \leq \mu_{kji}, \nu_{kji}, \tau_{kji} \leq 1$ and $(\mu_{kji})^2 + (\nu_{kji})^2 + (\tau_{kji})^2 \leq 1$, the credibility coefficient $\rho_{kji} \geq 0$ and $\sum_{k=1}^l \rho_{kji} = 1$. $\square$

Lemma 1. For any $x \in [0, 1], \rho \geq 0$, we have $x^{\rho} \in [0, 1]$. In particular, if $x = \rho = 0$, we define $x^{\rho} = 1$.

Lemma 2. For any $x_k \in [0, 1]$, we have $\prod_{k=1}^l x_k \in [0, 1]$.

Lemma 3. For any real numbers $x \geq y \geq 0$ and $\rho \geq 0$, the inequality $x^{\rho} \geq y^{\rho} \geq 0$ always holds. The equality holds if and only if $\rho = 0$.

For $0 \leq \mu_{ji}^{agg} \leq 1$:

Since $0 \leq (\mu_{kji})^2 \leq 1$, we derive $1 - \mu_{kji}^2 \in [0, 1]$. For $\rho_{kji} > 0$, it follows from Lemma

1 that $(1 - \mu_{kji}^2)^{\rho_{kji}} \in [0, 1]$. According to Lemma 2: $\prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} \in [0, 1]$, Thus

$1 - \prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} \in [0, 1]$, and we further obtain: $\left[1 - \prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} \right]^{\frac{1}{2}} \in [0, 1]$.

For $0 \leq \nu_{ji}^{agg} \leq 1$:

Since $0 \leq \nu_{kji} \leq 1$ and $\rho_{kji} > 0$ with $\sum_{k=1}^l \rho_{kji} = 1$, we have $(\nu_{kji})^{\rho_{kji}} \in [0, 1]$ by Lemma

1. Specifically, the term degenerates to 1 when $\rho_{kji} = 0$. Combining with Lemma 2:

$\prod_{k=1}^l (\nu_{kji})^{\rho_{kji}} \in [0, 1]$

For $0 \leq \tau_{ji}^{agg} \leq 1$:

We know $(\mu_{kji})^2 + (\nu_{kji})^2 + (\tau_{kji})^2 \leq 1$. Since $(\nu_{kji})^2 \geq 0$, we can deduce $(\mu_{kji})^2 + (\tau_{kji})^2 \leq 1$. Let: $X = 1 - (\mu_{kji})^2, Y = 1 - (\mu_{kji})^2 - (\tau_{kji})^2$. Apparently, $X \geq Y \geq 0$. For $\rho_{kji} > 0$, Lemma 3 gives $X^{\rho_{kji}} \geq Y^{\rho_{kji}} \geq 0$. Since the product of non-negative numbers preserves the inequality sign, we have $\prod_{k=1}^l X^{\rho_{kji}} \geq \prod_{k=1}^l Y^{\rho_{kji}} \geq 0$. Substitute X and Y: $\prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \geq 0$. As proven above, $\prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} \leq 1$ and $\prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \geq 0$. Therefore: $\prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \leq 1$. To sum up: $\left[\prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \right]^{\frac{1}{2}} \in [0, 1]$.

Theorem 2. No Amplification of the Spherical Constraint

Proof that the SFN spherical constraint is not amplified after aggregation: After aggregation by CSWAM, the spherical constraint still holds: $0 \leq (\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2 \leq 1$.

Proof of Theorem 2. Expand the sum of the squares of the three components:

$$\begin{aligned} & (\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2 \\ &= 1 - \prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} + \left(\prod_{k=1}^l \nu_{kji}^{\rho_{kji}} \right)^2 + \prod_{k=1}^l (1 - \mu_{kji}^2)^{\rho_{kji}} - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \\ &= 1 + \left(\prod_{k=1}^l \nu_{kji}^{\rho_{kji}} \right)^2 - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \end{aligned}$$

From the original spherical fuzzy constraint $\nu_{kji}^2 \leq 1 - \mu_{kji}^2 - \tau_{kji}^2$ and $\rho_{kji} > 0$, the monotonicity of the power function gives $(\nu_{kji}^2)^{\rho_{kji}} \leq (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}}$.

Thus $\prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \geq \prod_{k=1}^l (\nu_{kji}^2)^{\rho_{kji}} = \left(\prod_{k=1}^l \nu_{kji}^{\rho_{kji}} \right)^2$, which can be rearranged as $-\prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} + \left(\prod_{k=1}^l \nu_{kji}^{\rho_{kji}} \right)^2 \leq 0$.

Substituting into the squared sum yields:

$$(\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2 = 1 + \left(\prod_{k=1}^l \nu_{kji}^{\rho_{kji}} \right)^2 - \prod_{k=1}^l (1 - \mu_{kji}^2 - \tau_{kji}^2)^{\rho_{kji}} \leq 1$$

Meanwhile, the squares of all three components are nonnegative, so $0 \leq (\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2 \leq 1$. In summary, $0 \leq (\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2 \leq 1$. $\square$

The above theorems prove that the output of CSWAM is bounded within the spherical fuzzy space, ensuring that the results are always valid SFNs and providing a unified data guarantee for subsequent decision-making calculations.

3.3. Spherical Fuzzy Score Function Based on Adaptive Hesitancy Penalty (HP-SC)

The scoring function is a crucial tool for converting Spherical Fuzzy Numbers into real-valued numbers that can be directly compared or used, and it is an essential step in implementing solution ranking. However, traditional scoring functions require optimization in terms of computational complexity: if the results of multiple SFNs' scoring functions are identical, the exact function must be used for auxiliary comparison; more critically, these functions ignore the impact of the overall hesitation level in group decision-making on the reliability of the results, and the computed results fail to reflect the overall uncertainty of the decision-making environment. To address this, this paper proposes a Spherical Fuzzy Score function with Adaptive Hesitation Penalty (HP-SC). HP-SC not only eliminates the reliance on the accuracy function and improves computational efficiency but also introduces the group's average hesitation level as a penalty factor. This imposes a moderate penalty on the overall uncertainty of group decision-making, enabling the final results to more comprehensively reflect the quality characteristics of the group's decision-making information.

Definition 11. *The overall environment of hesitation in group decision-making affects the reliability of all outcomes: when the overall evaluation is generally hesitant, the credibility of all outcomes should decrease. HP-SC achieves adaptive adjustment of the penalty intensity by defining a hesitancy penalty factor. For the aggregated result $A_{ji}^{agg} = (\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg})$ of factor F_i under alternative S_j , the hesitancy penalty factor η and the score function $\text{Score}(A_{ji}^{agg})$ are defined as follows.*

$$\eta = \frac{1}{m \cdot n} \sum_{j=1}^m \sum_{i=1}^n (\tau_{ji}^{agg})^2 \quad (17)$$

$$\text{Score}(A_{ji}^{agg}) = \begin{cases} \text{clip} \left(\frac{(1-\eta) \cdot (\mu_{ji}^{agg})^2 - (1+\eta) \cdot [(\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2] + 1}{2}, 0, 1 \right) & \text{if } \mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg} \neq 0 \\ 0 & \mu_{ji}^{agg} = \nu_{ji}^{agg} = \tau_{ji}^{agg} = 0 \end{cases} \quad (18)$$

where $\eta \in [0, 1]$ is the hesitancy penalty factor, which can be adaptively adjusted according to the group average hesitancy level of the aggregated expert evaluations. This allows the penalty intensity to be dynamically tuned based on the overall decision-making environment: when the group exhibits high overall hesitancy, the scores of all aggregated results are more strongly suppressed; conversely, the penalty effect is weakened.

In particular, when there is no expert group decision and only a single spherical fuzzy number $A = (\mu, \nu, \tau)$ needs to be converted into a crisp value, no penalty is imposed on the score, i.e., $\eta = 0$. In this case, the score function is
$$\text{Score}(A) = \frac{\mu^2 - (\nu^2 + \tau^2) + 1}{2}.$$

For any spherical fuzzy numbers A and B:

If $\text{Score}(A) > \text{Score}(B)$, then $A \succ B$;

If $\text{Score}(A) = \text{Score}(B)$, then $A \sim B$;

If $\text{Score}(A) < \text{Score}(B)$, then $A \prec B$.

Theorem 3. Weak Monotonicity of HP-SC

The monotonicity of HP-SC is an important property that ensures the validity of the ordering. The following proves the weak monotonicity of HP-SC: $\text{Score}(A_{ji}^{agg})$ is weakly monotonically increasing with respect to the membership degree μ_{ji}^{agg} , and weakly monotonically decreasing with respect to the non-membership degree ν_{ji}^{agg} , hesitancy degree τ_{ji}^{agg} and penalty factor η .

Proof of Theorem 3. Let the unclipped internal function of $\text{Score}(A_{ji}^{agg})$ be:

$$f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta) = \frac{(1-\eta) \cdot (\mu_{ji}^{agg})^2 - (1+\eta) \cdot [(\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2] + 1}{2}.$$

Compute the partial derivative of f with respect to the membership degree μ_{ji}^{agg} :

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \mu_{ji}^{agg}} = (1-\eta)\mu_{ji}^{agg}.$$
 Given $\eta \in [0,1]$ and $\mu_{ji}^{agg} \in [0,1]$, we obtain:

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \mu_{ji}^{agg}} \geq 0.$$

Compute the partial derivative of f with respect to the non-membership degree ν_{ji}^{agg} :

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \nu_{ji}^{agg}} = -(1+\eta)\nu_{ji}^{agg}.$$
 Given $\eta \in [0,1]$ and $\nu_{ji}^{agg} \in [0,1]$, we obtain:

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \nu_{ji}^{agg}} \leq 0.$$

Compute the partial derivative of f with respect to the hesitancy degree τ_{ji}^{agg} :

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \tau_{ji}^{agg}} = -(1+\eta)\tau_{ji}^{agg}.$$
 Given $\eta \in [0,1]$ and $\tau_{ji}^{agg} \in [0,1]$, we obtain:

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \tau_{ji}^{agg}} \leq 0.$$

Compute the partial derivative of f with respect to the penalty factor η :

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \eta} = -\frac{(\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2}{2} \leq 0.$$
 Since $(\mu_{ji}^{agg})^2 + (\nu_{ji}^{agg})^2 + (\tau_{ji}^{agg})^2 \in [0,1]$,

$$\frac{\partial f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)}{\partial \eta} \leq 0.$$

we obtain:

In summary, the internal function $f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta)$ is strictly monotonically increasing in μ_{ji}^{agg} , and strictly monotonically decreasing in $\nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta$.

The score function $\text{Score}(A_{ji}^{agg})$ involves the clipping operation, which makes it non-differentiable at the clipping boundaries. When $f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta) \in (0,1)$,

$\text{Score}(A_{ji}^{agg})$ maintains strict monotonicity. When $f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta) \leq 0$ or $f(\mu_{ji}^{agg}, \nu_{ji}^{agg}, \tau_{ji}^{agg}, \eta) \geq 1$, $\text{Score}(A_{ji}^{agg})$ degenerates to 0 and 1 respectively. Therefore, $\text{Score}(A_{ji}^{agg})$ presents weak monotonicity as a whole. This completes the proof. $\square$

Theorem 4. Boundary Cases

Under the condition $\eta = 0$, the score function satisfies:

If $A = (1, 0, 0)$ (full affirmation), then $\text{Score}(A) = 1$.
 If $A = (0, 1, 0)$ (full negation), then $\text{Score}(A) = 0$.
 If $A = (0, 0, 1)$ (full hesitancy), then $\text{Score}(A) = 0$.

To further verify the effectiveness, applicability, and advantages of the HP-SC method proposed in this study compared to traditional methods, this paper presents multiple sets of test cases for comparative validation. We assume aggregated results corresponding to 3 alternatives and 3 factors ($m = n = 3$). The aggregation result matrix is:

$$A = \begin{bmatrix} (0.70, 0.50, 0.10) & (0.55, 0.15, 0.10) & (0.20, 0.70, 0.10) \\ (0.35, 0.65, 0.15) & (0.80, 0.20, 0.50) & (0.40, 0.40, 0.10) \\ (0.50, 0.30, 0.20) & (0.60, 0.40, 0.30) & (0.20, 0.80, 0.50) \end{bmatrix}$$

Table 2 lists a comparison of the calculation results between the traditional method and HP-SC.

Table 2. Comparison of HP-SC and traditional method calculation results.

Element	SFN	Traditional Method		HP-SC
		SC	AC	
A_{11}	(0.70, 0.50, 0.10)	0.2	0.75	0.58615
A_{12}	(0.55, 0.15, 0.10)	0.2	0.335	0.62211
A_{13}	(0.20, 0.70, 0.10)	−0.35	0.54	0.24923
A_{21}	(0.35, 0.65, 0.15)	−0.21	0.5675	0.31692
A_{22}	(0.80, 0.20, 0.50)	0	0.93	0.63922
A_{23}	(0.40, 0.40, 0.10)	0	0.33	0.48230
A_{31}	(0.50, 0.30, 0.20)	0.08	0.38	0.54538
A_{32}	(0.60, 0.40, 0.30)	0.08	0.61	0.53153
A_{33}	(0.20, 0.80, 0.50)	0	0.93	0.03922

It can be seen from the comparison in Table 2 that the HP-SC proposed in this paper has the following main advantages over the traditional method:

First, HP-SC eliminates dependence on the accuracy function and avoids the extra computation required when the score function values are identical (e.g., A_{11} and A_{12}), thereby reducing overall time complexity. Second, the traditional method fails in some

special cases (e.g., A_{22} and A_{33}) and cannot directly compare SFNs, whereas HP-SC remains valid and produces distinct results. Third, HP-SC introduces a hesitancy penalty mechanism that applies an appropriate penalty based on the overall hesitancy level, making it suitable for group decision-making across diverse backgrounds. Finally, the traditional method may yield negative score values when processing certain SFNs (e.g., A_{13} and A_{21}), which makes it difficult to directly use in GMADM methods such as DEMATEL and the entropy weight method that require non-negative input data. In contrast, HP-SC ensures that all score results are non-negative, expanding the application scope of spherical fuzzy score functions and improving compatibility with other GMADM methods in terms of data input formats.

4. Problem Statement

4.1. Online Movie Review Problem

With improvements in living standards, people's demand for entertainment-oriented products in the spiritual dimension has become increasingly urgent. As a typical experience product, the value of movies depends heavily on the audience's subjective experience, leading their reviews to be characterized by more sensitive emotional expression and greater divergence of opinions. Therefore, movies have gradually become an ideal carrier for testing the rationality of fuzzy review tools and the robustness of multi-attribute decision-making methods. In addition, movie audiences have diverse backgrounds and professional perspectives, and the preferences and criteria for judging the dimensions of a review may vary significantly among decision-makers. Such trade-offs require a decision-making strategy that considers multiple factors, such as expert opinions, subjective impacts, outlier degrees, and credibility.

This section aims to demonstrate the practicality of the proposed method through a case study that ranks the usefulness of online movie reviews.

4.2. Online Review Usefulness Evaluation System

Establishing an evaluation system for the usefulness of film reviews is fundamental to assessing their value. However, the raw data from reviews typically exhibit significant asymmetry in data format and information distribution. For example, "Usefulness Vote" [38,39] consists of discrete count data, "Star Rating" [40] is ordinal data, "Reviewer Professionalism" [41] involves the integration of continuous variables across multiple dimensions, and "Text Features" [39] are derived from natural language processing techniques. This type of information is difficult to compare and integrate directly. Traditional methods also struggle to effectively capture inherent implicit characteristics such as fuzziness and indecision.

Spherical fuzzy sets, however, leverage a three-dimensional symmetric structure comprising membership, non-membership, and hesitancy to uniformly characterize the degrees of support, opposition, and uncertainty across various types of information. To this end, this study constructs a spherical fuzzy quantification system tailored for online reviews; this evaluation system covers both non-textual and textual information. To achieve a unified representation of heterogeneous data, this paper uniformly maps various types of asymmetric heterogeneous data in each dimension to the spherical fuzzy space, providing a unified data foundation for subsequent ranking of review usefulness. Figure 4 illustrates the construction process of this system and the detailed metrics involved.

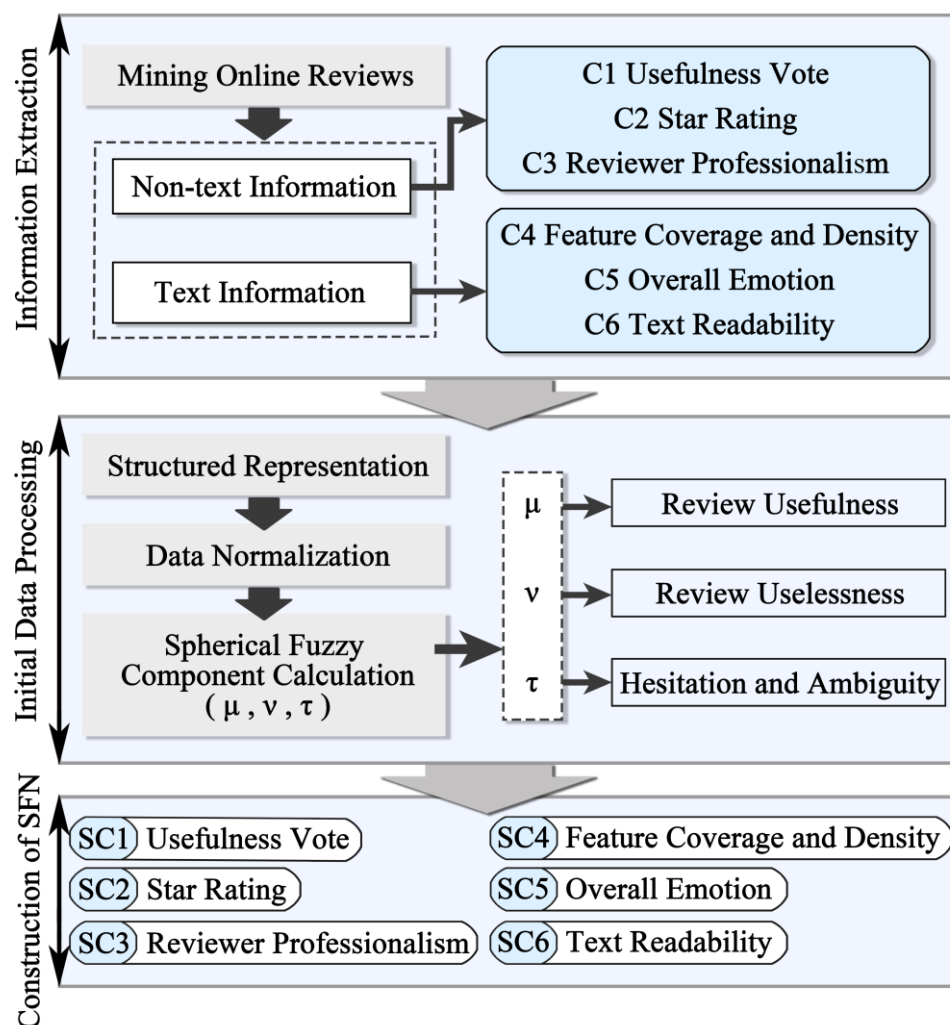


Figure 4. Spherical Fuzzy Set-based evaluation of online review usefulness.

Definition 12. For a given comment ρ , the feature information across all dimensions is mapped to a Spherical Fuzzy Number $\tilde{\alpha} = (\mu, \nu, \tau)$, where μ represents the degree of support for “comment is useful,” ν represents the degree of support for “comment is useless,” τ represents the degree of uncertainty in the judgment, and the following condition $0 \leq \mu^2 + \nu^2 + \tau^2 \leq 1$ is satisfied.

Definition 13. “Usefulness Vote” refers to the degree to which a comment is recognized as valuable. Most review sites use “like” or “dislike” buttons to evaluate the usefulness of online comments [42]. Generally, the more “useful” votes a comment receives—or the fewer “unhelpful” votes it receives—the more authentic and useful it is considered to be. Let $S_{C1}(\rho)$ be the Spherical Fuzzy Number for this dimension of comment ρ .

$$S_{C1}(\rho) = \begin{cases} \left(\frac{H(\rho)}{TV(\rho)}, \frac{U(\rho)}{TV(\rho)} \right) & \text{if } H(\rho) \neq 0 \text{ and } U(\rho) \neq 0 \\ \min \left(1, \frac{1}{\sqrt{TV(\rho)+1}} \right) \cdot \sqrt{1 - \mu_{C1}^2 - \nu_{C1}^2} & \\ (0,1,0) & \text{if } H(\rho) = 0 \text{ and } U(\rho) \neq 0 \\ (1,0,0) & \text{if } H(\rho) \neq 0 \text{ and } U(\rho) = 0 \\ (0,0,1) & \text{if } H(\rho) = 0 \text{ and } U(\rho) = 0 \end{cases} \quad (19)$$

where $H(\rho)$ denotes the number of usefulness votes for review ρ , $U(\rho)$ denotes the number of uselessness votes for review ρ , and the total number of votes is $TV(\rho) = H(\rho) + U(\rho)$. The

term $\min \left(1, \frac{1}{\sqrt{TV(\rho)+1}} \right)$ is an adjustment factor used to quantify the uncertainty caused by small samples, ensuring that the fewer the total votes, the higher the hesitancy degree.

Definition 14. “Star Rating” refers to the rating given by a reviewer for a product or service they have experienced. Generally speaking, extreme ratings tend to convey clearer attitudes, while moderate ratings indicate a more ambiguous attitude on the part of the reviewer [40]. The Spherical Fuzzy Numbers corresponding to different star ratings under Review ρ are shown in Table 3.

Table 3. Spherical Fuzzy Numbers corresponding to different star ratings.

Star Ratings r	Spherical Fuzzy Numbers $S_{C2}(\rho)$
1,2	(0.8,0.1,0.1)
3,4	(0.4,0.3,0.5)
5,6	(0.3,0.3,0.6)
7,8	(0.4,0.3,0.5)
9,10	(0.8,0.1,0.1)
None	(0,0,1)

Definition 15. “Reviewer Professionalism” refers to the level of expertise a reviewer demonstrates on a review website. It typically includes the number of reviews the reviewer has previously posted and the number of times they have used the product or service, serving as an indicator of the user’s level of expertise [43,44]. It is evident that readers of reviews tend to pay more attention to reviews posted by users with a higher level of expertise. The Spherical Fuzzy Number for dimension ρ of the review definition is $S_{C3}(\rho)$. It is composed of three metrics: the number of star ratings the reviewer has given to products or services they have experienced, the number of reviews the reviewer has posted, and the number of rewards granted to the reviewer by the review platform (such as badges and titles). These metrics represent the reviewer’s accumulated experience, engagement in content creation, and community recognition and influence, respectively.

$$Norm(x) = \frac{x - \min(x)}{\max(x) - \min(x)} \quad (20)$$

$$m_p = 0.3Norm(w) + 0.4Norm(r) + 0.3Norm(f) \quad (21)$$

$$\sigma_p = \sqrt{\frac{[Norm(w) - m_p]^2 + [Norm(r) - m_p]^2 + [Norm(f) - m_p]^2}{3}} \quad (22)$$

$$\mu_p = m_p \cdot (1 - 0.5\sigma_p) \quad (23)$$

$$\nu_p = (1 - m_p) \cdot (1 + 0.3\sigma_p) \quad (24)$$

$$\tau_p = 0.5\sigma_p \quad (25)$$

$$S_{c3}(\rho) = \left(\frac{\mu_p}{\sqrt{\mu_p^2 + \nu_p^2 + \tau_p^2}}, \frac{\nu_p}{\sqrt{\mu_p^2 + \nu_p^2 + \tau_p^2}}, \frac{\tau_p}{\sqrt{\mu_p^2 + \nu_p^2 + \tau_p^2}} \right) \quad (26)$$

where w represents the number of star ratings the reviewer has given, r represents the number of reviews the reviewer has posted, f represents the number of rewards the reviewer has received; $\text{Norm}(x)$ is the normalization function, $x = w, r, f$; m_p is the expertise metric, σ_p is the dispersion measure, and μ_p, ν_p, τ_p is the original Spherical Fuzzy component.

Definition 16. “Feature Coverage and Density” refers to the breadth of product features covered in the text of a review and the level of detail in their description; it is an important metric for assessing the information content and reference value of a review. A review that comprehensively covers various aspects of a product and provides a thorough elaboration of the features mentioned typically offers potential readers a more comprehensive basis for decision-making. The Spherical Fuzzy Number for this dimension of review ρ is $S_{c4}(\rho)$.

$$R_{\text{cov}} = \frac{B}{K} \quad (27)$$

$$D = \frac{T}{W} \quad (28)$$

$$\mu_f = R_{\text{cov}} \cdot (1 - |D - 0.5|) \quad (29)$$

$$\nu_f = (1 - R_{\text{cov}}) + 0.5|D - 0.5| \quad (30)$$

$$\tau_f = 0.3|D - 0.5| \quad (31)$$

$$S_{c4}(\rho) = \left(\frac{\mu_f}{\sqrt{\mu_f^2 + \nu_f^2 + \tau_f^2}}, \frac{\nu_f}{\sqrt{\mu_f^2 + \nu_f^2 + \tau_f^2}}, \frac{\tau_f}{\sqrt{\mu_f^2 + \nu_f^2 + \tau_f^2}} \right) \quad (32)$$

where K is the preset total number of product feature categories, B is the number of feature categories actually appearing in the review, satisfying $0 \leq B \leq K$. T is the total number of words related to feature evaluations in the review, and W is the total number of words in the review.

R_{cov} denotes the feature coverage rate, D denotes the feature density, and μ_f, ν_f, τ_f are the original spherical fuzzy components. $|D - 0.5|$ indicates that the ideal feature density is set to 0.5, meaning about half of the review content is used to describe product features.

Definition 17. “Overall Emotion” refers to the intensity of the overall emotional tendency expressed by the review, which directly affects readers’ initial perception of the review content and even the product [43,44]. Generally, extremely positive or negative reviews usually convey clear attitudes, provide stronger support for judging usefulness, and enable readers to quickly identify the reviewer’s position. The spherical fuzzy number of review ρ in this dimension is defined as $S_{c5}(\rho)$.

$$\mu_s = 2|s - 0.5| \quad (33)$$

$$\nu_s = 1 - 2|s - 0.5| \quad (34)$$

$$\tau_s = 4s(1-s) \quad (35)$$

$$S_{C5}(\rho) = \left(\frac{\mu_s}{\sqrt{\mu_s^2 + \nu_s^2 + \tau_s^2}}, \frac{\nu_s}{\sqrt{\mu_s^2 + \nu_s^2 + \tau_s^2}}, \frac{\tau_s}{\sqrt{\mu_s^2 + \nu_s^2 + \tau_s^2}} \right) \quad (36)$$

where $s \in [0, 1]$ is the overall emotional intensity value of the review obtained by sentiment analysis. $s=0$ represents extremely negative, $s=1$ represents extremely positive, and $s=0.5$ represents neutral. μ_s, ν_s, τ_s are the original spherical fuzzy components.

Definition 18. “Text Readability” refers to the readability and clarity of expression of the review text, which is an important factor affecting readers’ understanding and acceptance of the review content [45]. Obviously, reviews with clear expression and reasonable structure are easier for readers to understand quickly, and thus are more likely to be rated as “useful”; whereas reviews with obscure expression and complicated sentence structures may be ignored or abandoned by readers. The spherical fuzzy number of review ρ in this dimension is defined as $S_{C6}(\rho)$.

$$x_{\text{read}} = \frac{\text{Flesch Reading Ease}}{100} \quad (37)$$

$$S_{C6}(\rho) = \left(\frac{x_{\text{read}} \cdot T, (1 - x_{\text{read}}) \cdot T}{\sqrt{1 - (x_{\text{read}} \cdot T)^2 - [(1 - x_{\text{read}}) \cdot T]^2}} \right) \quad (38)$$

where $\text{Flesch Reading Ease} \in [0, 100]$ is a score measuring the readability of the review text [46]. A higher score indicates that the text is easier to understand. x_{read} denotes the normalized readability quantification value. $T = \sqrt{2x_{\text{read}}(1 - x_{\text{read}})} + 1$ represents a nonlinear term, which is designed to achieve the goal that the membership degree (usefulness) grows faster when x_{read} is higher (clearer expression), and the non-membership degree (uselessness) grows faster when x_{read} is lower (more obscure expression).

After constructing the SFNs for each attribute of different reviews, the score function is used to defuzzify them, and the decision matrix $M = (f_{ij})_{m \times n}$ representing the relationship between review attribute performance and reviews is formed.

4.3. Decision Model for Movie Review Usefulness

A comprehensive Group Multi-Attribute Decision Making (GMADM) model is key to solving the problem of review ranking. Therefore, this paper proposes an SFS-based DEMATEL-TODIM decision-making model. By integrating an adaptive expert weighting mechanism, HP-SC and CSWAM, this model attempts to address the issue in TODIM where the calculation of attribute weights overlooks asymmetric causal relationships between attributes, as well as the issue in DEMATEL where it over-relies on experts’ definitive scores while neglecting the inherent ambiguity and cognitive hesitation in expert judgments. The flowchart of this model is shown in Figure 5.

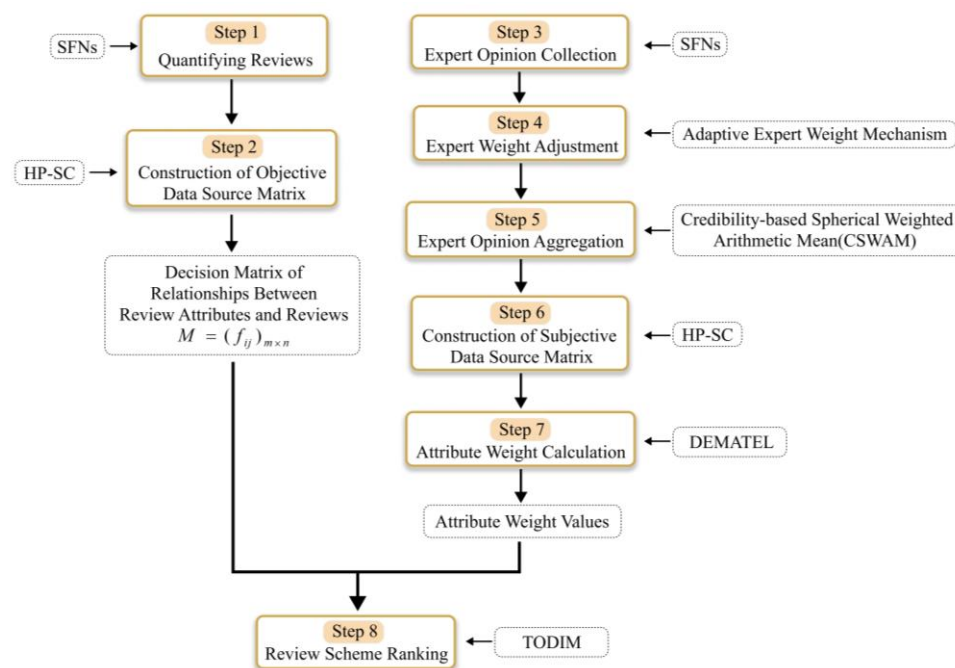


Figure 5. The overall workflow of the model.

This framework covers processes such as representing uncertain information, adaptively calculating expert weights, causal modeling, and risk-based decision-making. The proposed model consists of the following eight steps:

Step 1. Review quantification. Using an online review usefulness evaluation system, the data in raw reviews are quantified as SFNs.

Step 2. Construction of the objective data source matrix. After refining each SFN using HP-SC, a decision matrix $M = (f_{ij})_{m \times n}$ is formed to represent the relationship between review attributes and reviews, serving as the objective data source for the decision model.

Step 3. Collection of expert evaluation data. Experts are invited to evaluate the inter-relationships between attributes using the SFN format. The experts' assessments of the degree of influence and the corresponding SFNs are shown in Table 4.

Table 4. SFS-DEMATEL linguistic terms and the related SFN.

Definition	Abb	μ	ν	τ
No Influence	NI	0	0.9	0.1
Very Weak Influence	VW	0.25	0.75	0.25
Weak Influence	W	0.4	0.6	0.35
Moderate Influence	M	0.5	0.45	0.5
Strong Influence	S	0.6	0.3	0.35
Very Strong Influence	VS	0.75	0.15	0.25
Extreme Influence	EI	0.9	0	0.1
Diagonal elements	0	0	0	0

Step 4. Expert weight adjustment. Under the adaptive expert weighting mechanism, expert weights are dynamically adjusted based on both individual performance and group consensus.

Step 5. Aggregation of Expert Opinions. Based on the adjusted final expert weights, the Credibility-based Spherical Weighted Arithmetic Mean (CSWAM) is used to aggregate each expert's opinions.

Step 6. Construction of the Subjective Data Source Matrix. The aggregated results are processed using HP-SC to obtain precise values, forming an expert direct influence matrix that can be directly input into the DEMATEL algorithm.

Step 7. Calculation of attribute weights. The expert direct influence matrix is input into DEMATEL to calculate attribute weights, and the resulting attribute weights serve as the subjective data source for the decision-making model.

Step 8. Ranking of evaluation schemes. The decision matrix $M = (f_{ij})_{m \times n}$ and the attribute weight values are input into the TODIM method to determine the final ranking of each evaluation scheme.

4.4. Experimental Tools and Data Sources

The online movie review data used in this paper was sourced from the IMDb online review platform. Reviews for the movie F1: The Movie (excluding those containing spoilers) were selected as the experimental dataset; see Table 5 for specific details. The software used for data processing and the dates of data analysis are shown in Table 6.

Table 5. Specific details of the online movie reviews crawled.

Website Address	Number of Sentences Crawled	Access Date
https://www.imdb.com/title/tt16311594/reviews/?ref=ttururvgenaism	1618	10 February 2026

Table 6. Data processing software and data analysis date.

Step	Data Processing Software	Processing Date
Reviews Crawling	Python 3.12.x	10 February 2026
Reviews Processing	Python 3.14.3, VADER Sentiment Analysis Library, Flesch Text Readability Calculation Library	11–14 February 2026
Mathematical Computing	Matlab R2022a	15–18 February 2026

After scraping the reviews, this paper uses the Online Review Usefulness Evaluation System to quantify them. In accordance with the data rules of the IMDb platform, the quantification system is configured as follows: “Usefulness Vote” is composed of the number of “upvotes” and “downvotes” a review has received; “Star Rating” represents the reviewer’s star rating for the film; “Reviewer Professionalism” is composed of three metrics: the number of reviews the reviewer has posted, the number of star ratings they have assigned, and the number of badges they have earned; In “Feature Coverage and Density,” the total number of predefined film feature categories is set to $K=7$ (namely, plot narrative, character development, acting, thematic core, audiovisual presentation, overall pacing, and production design, totaling 7 features); “Overall Emotion” consists of the overall emotional intensity value of the review; “Text Readability” consists of the readability of the review text. Figure 6 shows an example of a comment.

Subjective data was derived from expert ratings. To ensure the professionalism and diversity of perspectives in the evaluation, we invited five experts with varying educational backgrounds, work experience, and professional expertise in the film industry to participate. The expert panel consists of: a seasoned general movie viewer (E_1); a content algorithm engineer at a film review platform (E_2); a professional film critic (E_3); a film industry professional with six years of experience (E_4); and a university professor specializing in film and television communication (E_5). The case analysis selected six attributes for evaluating the usefulness of online reviews, as follows: Usefulness Vote (C_1), Star Rating (C_2), Reviewer Professionalism (C_3), Feature Coverage and Density (C_4), Overall

Emotion (C_5), and Text Readability (C_6). All experts possess a deep understanding of these six film attributes.

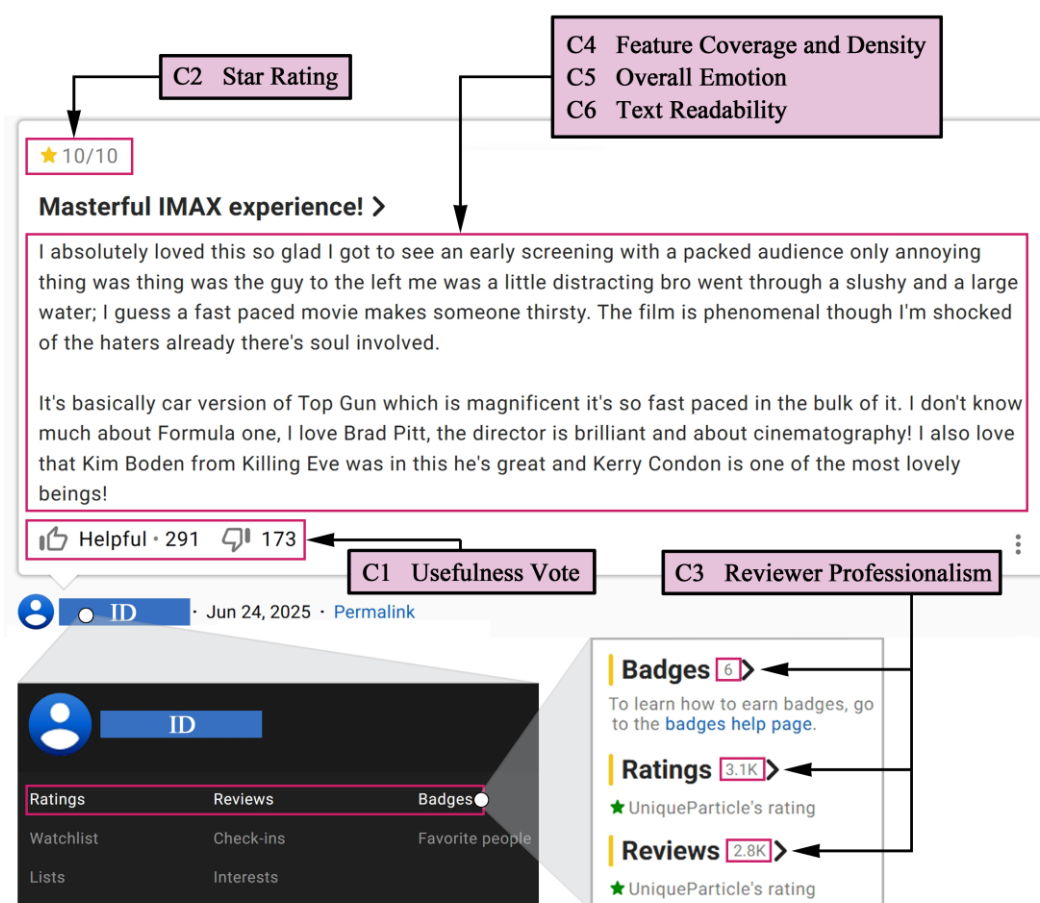


Figure 6. Specific movie review examples and indicator sources.

5. Computational Results

5.1. Case Studies

Step 1. Based on all the reviews we have crawled, we use the online review usefulness evaluation system to quantify them and convert them into SFNs, thereby forming an objective data source decision matrix. Due to the large volume of data and space constraints, only the calculation results for a portion of the comments are listed here, as shown in Table 7.

Table 7. SFN values for each comment.

Reviews	C_1	C_2	C_3	C_4	C_5	C_6
P_1	(0.8261, 0.1739, 0.0188)	(0.8000, 0.1000, 0.1000)	(0.0000, 1.0000, 0.0000)	(0.4429, 0.8925, 0.0845)	(0.5116, 0.4722, 0.7178)	(0.7984, 0.4040, 0.4461)
P_2	(0.8264, 0.1736, 0.0215)	(0.4000, 0.3000, 0.5000)	(0.0000, 1.0000, 0.0000)	(0.1474, 0.9888, 0.0229)	(0.5661, 0.4448, 0.6939)	(0.7949, 0.4086, 0.4485)
P_3	(0.8120, 0.1880, 0.0295)	(0.4000, 0.3000, 0.5000)	(0.0000, 1.0000, 0.0000)	(0.6135, 0.7828, 0.1032)	(0.2528, 0.5889, 0.7669)	(0.7467, 0.4671, 0.4734)
P_4	(0.7709, 0.2291, 0.0358)	(0.4000, 0.3000, 0.5000)	(0.0000, 1.0000, 0.0000)	(0.5194, 0.8533, 0.0450)	(0.6792, 0.3821, 0.6266)	(0.8340, 0.3577, 0.4201)
P_5	(0.7308, 0.2692, 0.0317)	(0.4000, 0.3000, 0.5000)	(0.0025, 0.9999, 0.0017)	(0.3404, 0.9400, 0.0219)	(0.4090, 0.5205, 0.7495)	(0.9397, 0.1927, 0.2824)
...	...	...	...	...	...	...
P_{1617}	(0.0000, 1.0000, 0.0000)	(0.3000, 0.3000, 0.6000)	(0.0000, 1.0000, 0.0000)	(0.2107, 0.9713, 0.1097)	(0.1580, 0.6321, 0.7586)	(0.6936, 0.5273, 0.4907)
P_{1618}	(0.0000, 0.0000, 1.0000)	(0.4000, 0.3000, 0.5000)	(0.0000, 1.0000, 0.0000)	(0.2695, 0.9603, 0.0708)	(0.7637, 0.3273, 0.5564)	(0.8638, 0.3161, 0.3922)

Step 2. Based on the data in Table 7, we use the HP-SC method to convert each SFN into an exact value, ultimately forming the decision matrix $M = (f_{ij})_{m \times n}$, which represents the relationship between review attributes and reviews, as shown in Table 8. Due to the large volume of data and text limitations, only a portion of the calculation results is presented here.

Table 8. Decision matrix for the relationship between comment attribute performance and comments.

Reviews	C_1	C_2	C_3	C_4	C_5	C_6
P_1	0.8260	0.8100	0.0000	0.1962	0.2617	0.6378
P_2	0.8261	0.4100	0.0000	0.0217	0.3205	0.6319
P_3	0.8115	0.4100	0.0000	0.3764	0.0639	0.5577
P_4	0.7703	0.4100	0.0000	0.2698	0.4614	0.6955
P_5	0.7303	0.4100	0.0000	0.1159	0.1673	0.8831
...	...	...	...	...	...	...
P_{1617}	0.0000	0.3200	0.0000	0.0444	0.0250	0.4811
P_{1618}	0.0000	0.4100	0.0000	0.0727	0.5833	0.7462

Step 3. We invited five experts to rate the interrelationships among six attributes used to evaluate the usefulness of online movie reviews—namely, Usefulness Vote (C_1), Star Rating (C_2), Reviewer Professionalism (C_3), Feature Coverage and Density (C_4), Overall Emotion (C_5), and Text Readability (C_6)—using the SFN format. The experts' ratings are shown in Table 9.

Table 9. Experts scoring results.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	VW; VW; VW; VW; W	M; W; M; W; M	W; W; VW; VW; W	W; VW; W; W; M	VW; W; VW; VW; W
C_2	S; W; VS; W; EI	0	VW; VW; VW; VW; VW	W; VW; VW; VW; VW	M; M; S; M; S	VW; VW; VW; VW; VW
C_3	W; M; S; M; S	VW; VW; VW; VW; VW	0	M; EI; S; S; VS	W; M; VW; W; W	W; S; M; M; M
C_4	W; S; M; W; M	VW; NI; NI; NI; NI	W; W; VW; VW; W	0	W; W; VW; W; VW	M; M; VW; VW; W
C_5	VS; W; S; M; VS	W; W; W; W; W	NI; NI; NI; NI; W	W; VW; VW; VW; VW	0	M; VW; VW; VW; VW
C_6	S; M; VS; M; S	NI; VW; NI; NI; NI	W; W; VW; VW; W	W; W; VW; VW; VW	VW; VW; VW; VW; VW	0

Step 4. Based on the adaptive expert weighting mechanism, calculate the initial weights, base weights, and final weights for the five experts, as shown in Table 10.

Table 10. Initial weights, base weights, and final weights for the five experts.

	E_1	E_2	E_3	E_4	E_5
Initial weight $\omega_k^{(0)}$	0.46601	0.48551	0.52991	0.50554	0.49577
Base weight $\omega_k^{(0)}$	0.18770	0.19555	0.21344	0.20362	0.19969
Final weight ω_k	0.18522	0.18979	0.21703	0.20401	0.20395

Step 5. Based on the final weights assigned by the five experts, the aggregated results of the expert opinions were calculated using the Credibility-based Spherical Weighted Arithmetic Mean (CSWAM). The aggregated results are shown in Table 11.

Table 11. Expert decision aggregation results processed by CSWAM.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	(0.26921, 0.73388, 0.26320)	(0.44532, 0.53296, 0.42821)	(0.31544, 0.69107, 0.29481)	(0.36199, 0.63982, 0.34014)	(0.29043, 0.71491, 0.27777)
C_2	(0.80300, 0.00000, 0.19856)	0	(0.25000, 0.75000, 0.25000)	(0.26730, 0.73552, 0.26189)	(0.57293, 0.33953, 0.39957)	(0.25000, 0.75000, 0.25000)
C_3	(0.55271, 0.36957, 0.38338)	(0.25000, 0.75000, 0.25000)	0	(0.80726, 0.00000, 0.21148)	(0.35558, 0.64703, 0.33478)	(0.52140, 0.41702, 0.41702)
C_4	(0.49890, 0.45126, 0.39346)	(0.07998, 0.88383, 0.12524)	(0.31544, 0.69107, 0.29481)	0	(0.31545, 0.69106, 0.29481)	(0.32070, 0.68099, 0.32335)
C_5	(0.69353, 0.21067, 0.29149)	(0.40000, 0.60000, 0.35000)	(0.09298, 0.88201, 0.13077)	(0.26730, 0.73552, 0.26189)	0	(0.26681, 0.73418, 0.27204)
C_6	(0.67160, 0.23034, 0.32332)	(0.08107, 0.88339, 0.12585)	(0.31544, 0.69107, 0.29481)	(0.28816, 0.71699, 0.27622)	(0.25000, 0.75000, 0.25000)	0

Step 6. Based on the data in Table 11, the Spherical Fuzzy Score function with Adaptive Hesitation Penalty (HP-SC) is used to convert each SFN into an exact value and integrate them, thereby forming a direct influence matrix that can be directly input into DEMATEL for computation. The direct influence matrix is shown in Table 12.

Table 12. Direct influence matrix processed by HP-SC.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	0.20748	0.34116	0.24327	0.27904	0.22351
C_2	0.77773	0	0.19366	0.20608	0.50465	0.19366
C_3	0.48947	0.19366	0	0.77806	0.27388	0.43941
C_4	0.42306	0.07547	0.24327	0	0.24328	0.24278
C_5	0.65354	0.31530	0.07747	0.20608	0	0.20411
C_6	0.62452	0.07588	0.24327	0.22176	0.19366	0

Step 7. Enter the data from Table 12 into the DEMATEL method to obtain the weights for each attribute ($C_1 \sim C_6$), as shown in Table 13. The relationships among the attribute weights are illustrated in Figure 7.

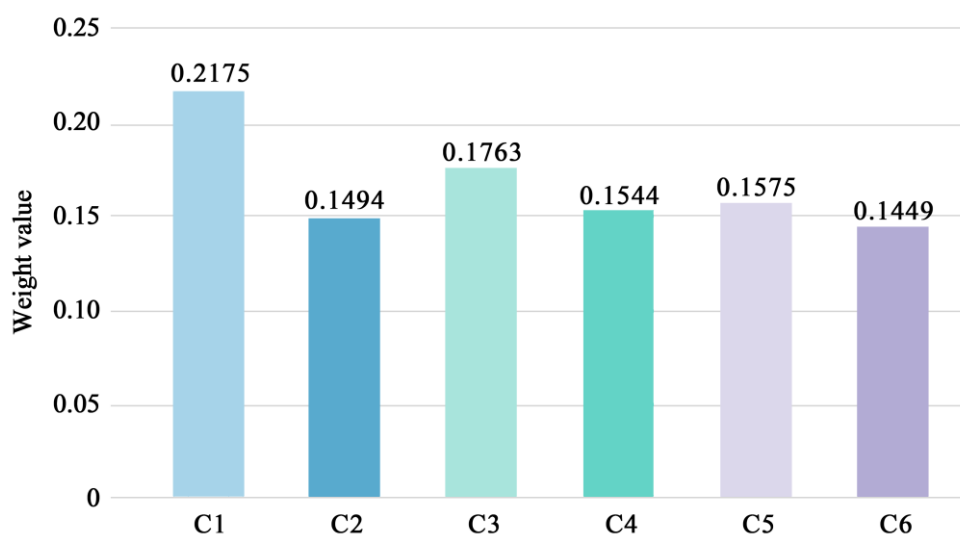


Figure 7. Attribute weights based on the DEMATEL method.

Table 13. Weight values for each attribute.

	C_1	C_2	C_3	C_4	C_5	C_6
Weighting value	0.2175	0.1494	0.1763	0.1544	0.1575	0.1449

Step 8. Using the decision matrix $M = (f_{ij})_{m \times n}$, which depicts the relationship between review attributes and reviews as shown in Table 8, as the objective data source, and the attribute weight values shown in Table 13 as the subjective data source, we input these into the TODIM model, assuming a loss decay factor $\theta = 1$. Due to space constraints, this section only performs ranking validation on the first 20 reviews ($P_1 \sim P_{20}$); the specific results are shown in Table 14.

Table 14. Top 20 Comments Sorting Results.

Reviews	Rankings on IMDb	Rankings via Proposed Method	Usefulness Score	Reviews	Rankings on IMDb	Rankings via Proposed Method	Usefulness Score
P_1	1	3	0.77869	P_{11}	11	1	1.00000
P_2	2	14	0.44014	P_{12}	12	17	0.28955
P_3	3	10	0.51445	P_{13}	13	12	0.45963
P_4	4	7	0.64588	P_{14}	14	15	0.39511
P_5	5	11	0.46813	P_{15}	15	5	0.69260
P_6	6	2	0.89798	P_{16}	16	4	0.70291
P_7	7	20	0.00000	P_{17}	17	8	0.57994
P_8	8	16	0.29683	P_{18}	18	19	0.21046
P_9	9	6	0.65649	P_{19}	19	13	0.44669
P_{10}	10	18	0.27261	P_{20}	20	9	0.52916

As shown above, there is a significant discrepancy between our rankings and IMDb's. This suggests that traditional ranking methods may fail to fully reflect the actual value of reviews, which is also a major cause of information overload for viewers when browsing reviews. In contrast, the DEMATEL-TODIM integrated evaluation model based on SFS proposed in this paper integrates objective data from multiple dimensions with expert subjective judgments. This approach enables a more nuanced characterization of a review's performance across attributes and their relative importance, thereby yielding a more reasonable ranking of a review's usefulness.

Furthermore, the distinctiveness of the ranking results reflects the effectiveness of the adaptive expert weighting mechanism, CSWAM, and HP-SC introduced in this study. Collectively, these methods enable the model to effectively suppress the influence of outlier experts, highlight high-quality evaluation information, and reasonably penalize overall uncertainty within a group decision-making context, thereby enhancing the robustness and discriminative power of the ranking results.

5.2. Comparison of Methods

This study proposes a ranking model for evaluating the usefulness of reviews; however, we are still uncertain about the validity and effectiveness of this model. Therefore, in this subsection, we will compare our method (M1) with SFS-CRITIC-TOPSIS (M2), SFS-DEMATEL-TOPSIS (M3) [28], and SFS-DEMATEL-VIKOR (M4) [47] to rank the case studies, respectively, to verify the validity and effectiveness of the ranking model proposed in this paper. Table 15 shows the ranking results calculated by the five GMADM models. To

gain a more intuitive understanding of the relationships among the ranking results, we use the Spearman Rank Correlation Coefficient (SRCC) [48] to measure their correlation; the specific results are shown in Figure 8.

Table 15. Ranking results of comparative experiments.

Methodology	Rankings
This Paper (M1)	$P_{11} \succ P_6 \succ P_1 \succ P_{16} \succ P_{15} \succ P_9 \succ P_4 \succ P_{17} \succ P_{20} \succ P_3 \succ P_5 \succ P_{13} \succ P_{19} \succ P_2 \succ P_{14} \succ P_8 \succ P_{12} \succ P_{10} \succ P_{18} \succ P_7$
SFS-CRITIC-TOPSIS (M2)	$P_{11} \succ P_6 \succ P_{16} \succ P_{15} \succ P_9 \succ P_{20} \succ P_{17} \succ P_{18} \succ P_4 \succ P_1 \succ P_{19} \succ P_3 \succ P_{13} \succ P_5 \succ P_{14} \succ P_2 \succ P_{10} \succ P_{12} \succ P_8 \succ P_7$
SFS-DEMATEL-TOPSIS (M3)	$P_{11} \succ P_6 \succ P_{15} \succ P_{16} \succ P_9 \succ P_{20} \succ P_{17} \succ P_{18} \succ P_4 \succ P_1 \succ P_{19} \succ P_5 \succ P_{13} \succ P_3 \succ P_2 \succ P_{14} \succ P_{10} \succ P_{12} \succ P_8 \succ P_7$
SFS-DEMATEL-VIKOR (M4)	$P_6 \succ P_{11} \succ P_1 \succ P_{15} \succ P_9 \succ P_{17} \succ P_4 \succ P_3 \succ P_{13} \succ P_{19} \succ P_2 \succ P_5 \succ P_{14} \succ P_8 \succ P_{10} \succ P_{12} \succ P_{16} \succ P_7 \succ P_{20} \succ P_{18}$

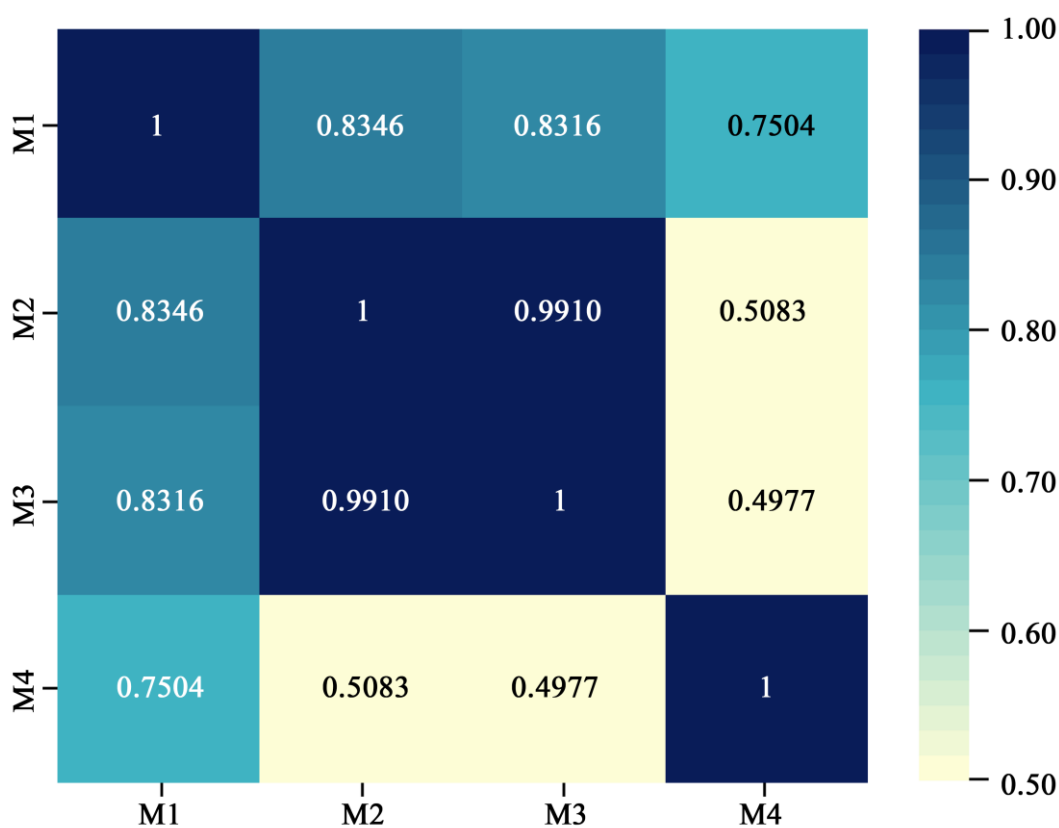


Figure 8. SRCC for alternatives in different methods.

As shown in Table 15, the computational results of the proposed model exhibit a high degree of similarity to those of the other models. Furthermore, Peng et al. [49] note that the closer the SRCC value is to 1, the stronger the correlation between two ranking results is considered to be. As shown in Figure 8, the SRCC values of the proposed model and the other classical GMADM models all exceed 0.7, indicating that their ranking results are highly correlated. In summary, compared with the three classical GMADM methods, the model proposed in this paper exhibits a high degree of similarity in ranking results, further demonstrating the feasibility and effectiveness of the proposed model.

5.3. Sensitivity Analysis

In this paper, the main adjustable parameters include the weight coefficients α, β, γ in Definition 5; the consensus sensitivity parameter λ in Definition 8; and the loss decay factor θ in TODIM. Additionally, variations in the weights of individual attributes may also affect the final ranking results. To investigate how different values of these parameters influence the final ranking results, this section conducts a sensitivity analysis on the aforementioned parameters. Figure 9 shows the final weights of experts under various weight coefficients α, β, γ ; Figure 10 shows the final weights of experts under different consensus sensitivity parameters λ ; Figure 11 presents the results of the sensitivity analysis regarding the loss decay factor θ ; and Table 16 displays the SRCC calculations for each perturbation group compared to the original model in the single-attribute weight perturbation experiment.

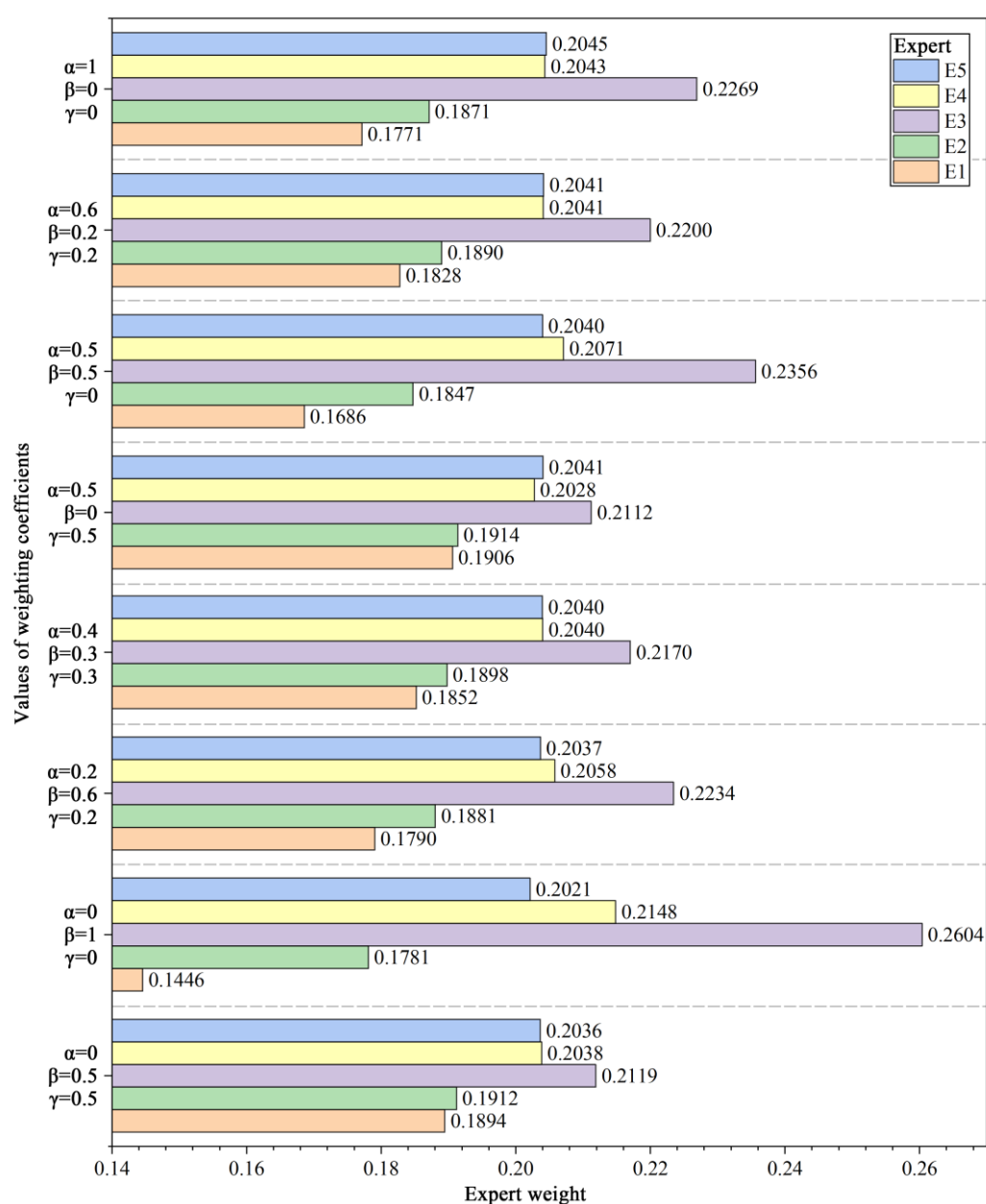


Figure 9. Expert weights for different values of α, β, γ .

Figure 9 shows that, under different values of the weight coefficients α, β, γ , the final weights of each expert exhibit some fluctuation and variation. This is primarily due to the differing proportions of the global certainty degree $\bar{C}_k$, global discrimination degree $\bar{D}_k$, and global similarity degree $\bar{S}_k$ —metrics used to synthesize individual expert performance—which reflect the moderate regulatory effect of parameter adjustments on expert weight results under the adaptive expert weighting mechanism; At the same time, even under different values of the weight coefficient α, β, γ , the ranking of the final weights for each expert remains relatively stable, with no significant fluctuations. For example, the weight of Expert E_3 consistently remains at a high level, while the weight of Expert E_1 is generally lower. This indicates that the adaptive expert weighting mechanism proposed in this paper possesses good robustness, verifying the stability and reliability of this model in group decision-making.

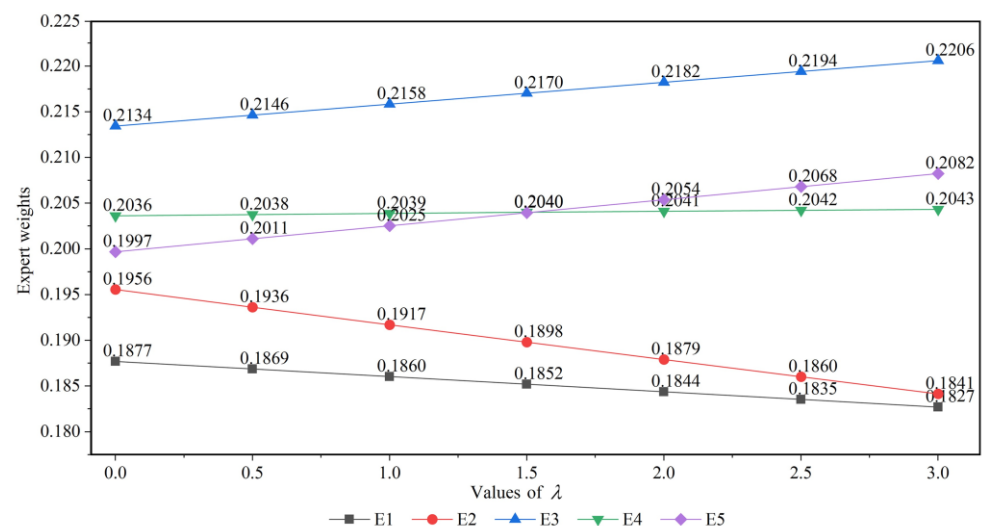


Figure 10. Expert weights for different values of λ .

Figure 10 shows that as the parameter λ gradually increases, the final weights of experts (E_1 and E_2) whose evaluations deviate significantly from the group consensus decrease monotonically; conversely, the final weights of experts (E_3 and E_5) whose evaluations align closely with the group consensus increase monotonically. This demonstrates that the adaptive expert weighting mechanism proposed in this paper can identify outlier opinions and appropriately suppress their interference, while effectively amplifying the decision-making influence of high-quality experts. The effectiveness and discriminatory power of the dynamic penalty mechanism have been validated.

Furthermore, although different penalty intensities λ caused variations in the weights of individual experts, the magnitude of these changes remained within a reasonable range, and the highest and lowest weights were still E_3 and E_1 , respectively. This indicates that even when parameter settings involve subjective uncertainty, the system can still produce decision weights that are logically sound and robust, confirming the scientific validity and reliability of the weight allocation framework proposed in this paper.

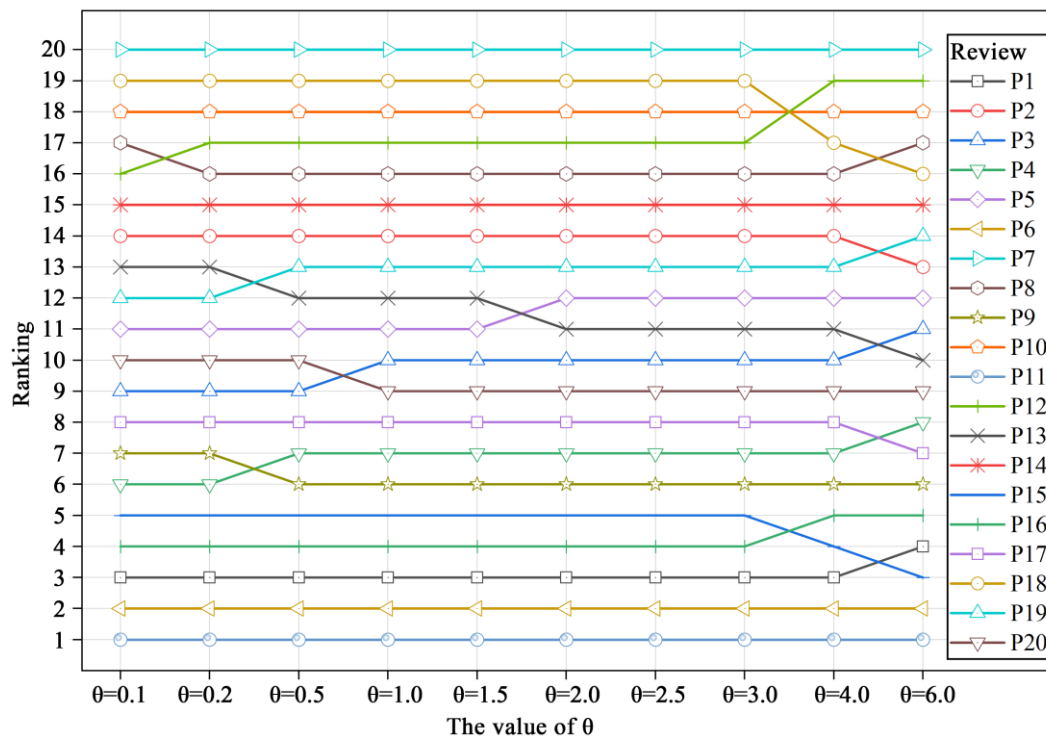


Figure 11. Ranking results for each review under different loss decay factors θ .

Figure 11 shows that, under different settings of the loss decay factor θ , the ranking results do not undergo any drastic changes: the rankings of certain online reviews may shift slightly, indicating that the decision-makers' psychology influences the final ranking results to some extent; at the same time, even when different settings of the loss decay factor θ are applied, the rankings of the best review P_{11} and the worst review P_7 remain stable. This further demonstrates that the overall model in this paper possesses strong robustness and achieves a balance between overall robustness and the ability to measure decision-makers' preferences.

Table 16. SRCC Results for Sensitivity Analysis of Single-Attribute Weight Perturbation.

Perturbed Attribute	+20% Perturbation	−20% Perturbation	+40% Perturbation	−40% Perturbation
C_1	0.9202	0.9263	0.8564	0.8675
C_2	0.9471	0.9495	0.9114	0.9171
C_3	0.9311	0.9354	0.8772	0.8868
C_4	0.9428	0.9461	0.9037	0.9102
C_5	0.9395	0.9432	0.8981	0.9054
C_6	0.9515	0.9542	0.9205	0.9260

As shown in Table 16, the model demonstrates strong robustness under fluctuations in attribute weights. When perturbations were applied to each attribute, the SRCC remained above 0.8 across all experimental groups, indicating that shifts in local weights did not significantly affect the original ranking results. This not only demonstrates that the model retains strong fault tolerance when faced with local measurement errors or changes in weight preferences, but also suggests that it is likely to maintain high system reliability and practical value in complex, dynamic real-world evaluation environments.

Based on the results of the sensitivity analyses from the above groups, we demonstrate the validity of the parameter values used in this paper.

The weighting coefficient used to determine the experts' base weights is set to $\alpha = 0.4, \beta = 0.3, \gamma = 0.3$; the three coefficients correspond to the weighting proportions of global certainty, global discriminative power, and global similarity, respectively, in the calculation of the experts' initial weights. From a theoretical perspective, the certainty calculated from information entropy directly reflects the reliability of expert evaluations and serves as a core indicator for measuring evaluation quality; therefore, it is assigned the highest weight. Discrimination and similarity, as equally important auxiliary evaluation dimensions, are assigned equal weights. This configuration not only prevents ambiguous evaluations resulting from excessively low certainty from interfering with the results but also prevents evaluation biases caused by weight imbalances.

Regarding the consensus sensitivity parameter $\lambda = 1.5$, this parameter is used in the adaptive weighting mechanism to control the severity of penalties applied to outlier expert opinions. Theoretically, if λ is set too low, the penalty strength will be insufficient, failing to effectively suppress the interference of extreme outlier views on group consensus; conversely, an excessively large value of λ would unduly compress the weights of heterogeneous views, erasing the differences in professional perspectives among experts from various fields. Therefore, setting $\lambda = 1.5$ strikes a balance between suppressing outliers and preserving view diversity, while also aligning with the characteristics of film reviews—which are prone to interference from extreme emotions yet require the retention of diverse professional perspectives.

Regarding the loss decay coefficient $\theta = 1$ in the TODIM model, this parameter represents the decision-maker's degree of loss aversion and serves to characterize bounded rationality in decision-making. From a theoretical perspective, setting θ too low would excessively amplify the perception of loss, deviating from the low-risk nature of information-filtering decisions; setting θ too high would weaken the model's ability to capture decision-makers' psychological behavior, causing the model to degenerate into an approximation of perfect rationality and lose the method's inherent advantages. Therefore, $\theta = 1$ aligns with the behavioral characteristics of moderate loss aversion exhibited by users when browsing movie reviews, while balancing rational judgment with real psychological preferences.

5.4. Ablation Experiment

To investigate the effectiveness of each module in the DEMATEL-TODIM integrated evaluation model proposed in this paper, as well as their independent contributions to the final decision results, four sets of ablation experiments were designed in this section. By progressively replacing or removing the adaptive expert weights, CSWAM, HP-SC, DEMATEL and TODIM, the SRCC between the ranking results of each ablation model and those of the proposed method was calculated. The specific results are shown in Table 17.

Table 17. Comparison of SRCC Results Across Different Ablation Experiments.

Exp. No.	Experimental Method					Ablation Methods	SRCC
	Expert Weights	Aggregate Operator	Score Function	Attribute Weights	Proposal Ranking		
This Paper	Expert Adaptive Weight	CSWAM	HP-SC	DEMATEL	TODIM	/	1
1	/	CSWAM	HP-SC	DEMATEL	TODIM	Expert Adaptive Weight	0.8914
2	Expert Adaptive Weight	SWAM	HP-SC	DEMATEL	TODIM	CSWAM	0.8136
3	Expert Adaptive Weight	CSWAM	Traditional SC	DEMATEL	TODIM	HP-SC	0.6952
4	Expert Adaptive Weight	CSWAM	HP-SC	/	TODIM	DEMATEL	0.7427
5	Expert Adaptive Weight	CSWAM	HP-SC	DEMATEL	VIKOR	TODIM	0.7504

The following analysis and conclusions can be drawn from Table 17:

First, when the model is reduced to “equal expert weights” (Exp. 1), the results remain highly correlated with those of the original model. This indicates that the adaptive expert weighting mechanism does not disrupt the overall evaluation trend; rather, by identifying and suppressing extreme outlier opinions, it robustly fine-tunes and calibrates the overall model. Second, when using the traditional SWAM operator to aggregate expert opinions (Exp. 2), the SRCC drops to 0.8136. This fluctuation demonstrates that introducing a local credibility coefficient preserves the evaluative strengths of individual experts on specific evaluation dimensions, thereby avoiding the loss of high-value information that traditional global aggregation can cause.

Furthermore, when attributes were assigned equal weights without using DEMATEL (Exp. 4), the SRCC of the ranking results declined further. This demonstrates the necessity of identifying asymmetric causal relationships among attributes when determining attribute weights; blind equal-weighting masks the dominant role of core attributes and distorts the evaluation scheme’s utility performance in real-world environments. Similarly, when the TODIM method is not used for scheme ranking (Exp. 5), the SRCC also shows a significant decline. This is because the model lacks a characterization of decision-makers’ bounded rationality (such as loss aversion and reference dependence), whereas TODIM effectively captures these asymmetric psychological preferences. This result further confirms that incorporating a prospect theory-driven behavioral decision-making model is indispensable in the problem of ranking review utility.

Finally, the most drastic impact on the decision-making system stems from the ablation of the scoring function. When a traditional exact function was used to replace HP-SC (Exp. 3), the SRCC plummeted to a low of 0.6952. This sharp decline not only reveals the destructive impact of the negative values generated by traditional scoring methods when handling specific spherical fuzziness numbers on the TODIM decision matrix but also demonstrates the crucial role of the “adaptive penalty mechanism for group hesitation” and the non-negativity constraint introduced in this paper in maintaining the data consistency and robustness of the overall decision-making framework.

All of the above ablation experiments demonstrate that removing or degrading any single method in the model proposed in this paper degrades final ranking quality, thereby validating the necessity of the proposed methods and the robustness of the GMADM-integrated framework.

6. Discussion

Based on the previous research and computational results, this section discusses the strengths, weaknesses, and practical implications of the proposed SFS-based DEMATEL-TODIM multi-attribute decision-making model.

The advantages of the model presented in this paper are primarily reflected in the following three aspects:

1. It achieves a good balance between suppressing outlier opinions and preserving the advantages of local judgments. Sensitivity analysis results indicate that as the consensus sensitivity parameter λ increases, the weights of experts who deviate from the group consensus decrease monotonically. This demonstrates that the adaptive expert weighting mechanism can dynamically and effectively suppress the interference of extreme outlier opinions. Meanwhile, ablation experiments show that CSWAM, by introducing a credibility coefficient, effectively reduces the loss of high-value information caused by traditional SWAM aggregation, while achieving asymmetric differentiated weighting within a symmetric aggregation framework.
2. Enhanced the robustness and rationality of the process of converting SFS fuzzy information into precise values. Traditional scoring functions exhibit high computa-

tional complexity when handling specific spherical fuzziness numbers and are prone to producing negative values, which in turn undermines the consistency of subsequent decision-matrix data. By introducing the average group hesitation as an adaptive penalty factor, HP-SC not only ensures the non-negativity of the output but also tightly links the final score to the overall uncertainty of the decision environment, constituting a key step in enhancing the model's overall robustness.

3. Finally, the model demonstrates strong robustness when faced with local parameter perturbations. In sensitivity analysis experiments, such as those involving single-attribute weight perturbations, the SRCC values of the ranking results for each experimental group remained high relative to the original model. This indicates that, although the model in this paper integrates DEMATEL's causal analysis with TODIM's psychological behavioral preferences, incorporating methods such as the adaptive expert weighting mechanism, CSWAM, and HP-SC ensures that it effectively mitigates local measurement errors and maintains high reliability in complex real-world evaluation environments.

Although this study has achieved certain results in terms of theoretical derivation and case validation, the following limitations remain:

1. High computational complexity. To achieve fine-grained information control, the model must sequentially calculate the experts' certainty, discriminative power, similarity, and group deviation. Although it produces more accurate rankings than currently available models, it may encounter computational bottlenecks when handling the massive volume of real-time online comment-ranking requests.
2. Dependence on parameter settings persists. Although sensitivity analysis demonstrates the model's robustness, the weight coefficients α, β, γ in the adaptive expert weighting mechanism, the consensus sensitivity parameter λ , and the loss decay factor θ in TODIM currently rely on prior settings or expert judgment. The uncertainty inherent in these subjective settings may potentially affect the objectivity of ranking results across different domains or data distributions.
3. The model's practical applicability requires further refinement. The overall model's generalization capabilities need further validation across multi-platform and multi-lingual environments; the current data primarily relies on static data and does not incorporate the temporal dimension or user historical behavior. Future research could focus on validation experiments, cross-platform adaptability, data-driven parameter calibration, dynamic penalty mechanisms, and incremental learning to further enhance the model's universality and evolutionary capabilities across different scenarios.

From the perspectives of practical application value and management implications, the model presented in this paper yields ranking results that differ significantly from those of mainstream platforms (such as IMDb), revealing the limitations of traditional methods that rely on a single metric for review ranking in addressing information overload. E-commerce and movie review platforms can adopt the HP-SC hesitation penalty mechanism proposed in this paper. When a review exhibits sparse voting data or the reviewer's expertise is questionable, the system can automatically reduce its ranking weight, thereby lowering the risk of users being misled by low-quality reviews. Additionally, platform reputation systems can adopt the adaptive weight-allocation logic presented in this paper to transition from static "like counts" to dynamic behavioral quality assessments, thereby providing a more precise and robust foundational framework for personalized recommendations.

7. Conclusions

This study focuses on issues such as high information ambiguity, significant expert heterogeneity, and the asymmetric distribution of multi-source data in Group Multi-Attribute Decision Making (GMADM). Building upon existing SFS, DEMATEL, and TODIM theories, this study innovatively proposes an adaptive expert weighting mechanism, the Credibility-based Spherical Weighted Arithmetic Mean (CSWAM), and the Spherical Fuzzy Score function with Adaptive Hesitation Penalty (HP-SC) to address the specific challenges of online movie review ranking. These methods are used to construct a comprehensive decision-making framework for ranking the usefulness of online movie reviews.

To address the limitations of existing expert weighting methods, this paper proposes an adaptive expert weighting mechanism that dynamically adjusts weights by combining individual performance with group consensus, thereby mitigating the influence of outlier opinions. Building on this, we propose the Credibility-based Spherical Weighted Arithmetic Mean (CSWAM), which achieves a more refined preservation of local-judgment advantages when aggregating expert opinions. Furthermore, to overcome the shortcomings of traditional scoring functions—namely, high computational complexity and a tendency to produce negative results—we construct the Spherical Fuzzy Score function with Adaptive Hesitation Penalty (HP-SC). This approach not only eliminates reliance on exact functions but also ensures robust conversion of fuzzy information into precise values. Subsequently, to address the challenge of asymmetric data distribution in online movie reviews, we constructed an SFS quantification index system tailored to review features; finally, by combining the extended DEMATEL and TODIM methods, we ranked review usefulness.

Through an empirical analysis of online movie reviews on the IMDb platform, this study has yielded the following specific findings and research results:

1. **Ranking Validity and Discrimination:** Comparative experiments demonstrate that the model proposed in this paper not only maintains a high correlation with the ranking results of classical methods such as SFS-CRITIC-TOPSIS and SFS-DEMATEL-VIKOR, but also handles extremely ambiguous information—which traditional methods cannot distinguish—with greater precision, thereby achieving higher ranking discrimination.
2. **The Key Role of the HP-SC Scoring Function:** Multiple ablation experiments were conducted to quantify the contributions of each innovative method. The results revealed that HP-SC plays a significant role in maintaining the robustness of the decision-making framework; when HP-SC was omitted and traditional scoring functions were used instead, the system's ranking results deteriorated significantly, with the SRCC value dropping to 0.6952. This further confirms the necessity of applying adaptive penalties to group indecisiveness to eliminate numerical distortions.
3. **Robustness of the Overall Model:** In sensitivity analysis, the SRCC remained at a high level across multiple experimental groups. This not only confirms the model's fault tolerance under weight bias and local measurement errors but also validates the strong interference resistance of the aggregation logic—comprising adaptive expert weights and CSWAM—within the model.
4. **The Indispensability of Asymmetric Phenomena Such as Attribute Causality and Bounded Rationality:** Ablation experiments confirm that omitting either DEMATEL or TODIM leads to a decline in the SRCC of ranking results, further demonstrating the necessity of capturing causal relationships between attributes and asymmetric behaviors such as decision-makers' bounded rationality in the review ranking problem.

In summary, this paper builds upon SFS and, by introducing an adaptive expert weighting mechanism, CSWAM, and HP-SC, extends the application paradigm of DEMATEL-TODIM within the SFS framework, providing an effective, quantitatively sup-

ported framework for review mining and information ranking on online platforms. Future research could focus on deepening the model's cross-scenario adaptability and the adaptive dynamic optimization of parameters, thereby further enhancing the model's universality, adaptability, and evolutionary capabilities, and providing more comprehensive theoretical and methodological support for more complex uncertain group decision-making scenarios.

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Abbreviations

The following abbreviations are used in this manuscript:

SFS	Spherical Fuzzy Set
IFS	Intuitionistic Fuzzy Set
PFS	Pythagorean Fuzzy Set
SFN	Spherical Fuzzy Number
SWAM	Spherical Weighted Arithmetic Mean
DEMATEL	Decision-Making Trial and Evaluation Laboratory
GMADM	Group Multi-Attribute Decision Making
CSWAM	Credibility-based Spherical Weighted Arithmetic Mean
HP-SC	Spherical Fuzzy Score function with Adaptive Hesitation Penalty
SRCC	Spearman Rank Correlation Coefficient

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