

Article

Bayesian Evidence for Angular Symmetry and Spectral Curvature in the Nanohertz Gravitational-Wave Background

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Abstract

Pulsar timing arrays have detected a nanohertz signal exhibiting the Hellings–Downs angular correlation, the expected angular symmetry of an isotropic stochastic gravitational-wave background. This angular symmetry fixes the tensor correlation class of the signal, so its physical origin must be inferred from the frequency spectrum. Using the public NANOGrav 15-year free-spectrum products, we compare five spectral hypotheses through Bayesian evidence: the canonical scale-free power law from purely gravitational-wave-driven supermassive black hole binaries (SMBHBs), a free-slope power law, an environmental SMBHB turnover model, a first-order phase transition template, and an effective cosmic string spectrum. The evidence favors spectra with physical curvature or a characteristic scale, while the strict scale-free SMBHB law is strongly disfavored. Within the tested physical templates, the phase transition model gives the largest compressed spectral evidence; within astrophysical source models, environmental SMBHB hardening is the leading interpretation and links the spectral bend to parsec-scale nuclear stellar densities. The effective cosmic string spectrum shows little evidence gain under the baseline prior. An orbit- and foreground-aware LISA continuation forecast gives this ranking a multi-band check: the PTA-selected QCD-scale phase transition posterior has no appreciable millihertz continuation, whereas an unchanged broad cosmic string extrapolation is LISA-bright and needs additional spectral structure.

Keywords: nanohertz gravitational-wave background; pulsar timing arrays; Hellings–Downs correlations; angular symmetry; spectral curvature; Bayesian model comparison; supermassive black hole binaries; environmental hardening; phase transitions; cosmic strings



Academic Editor: Adailton A. Araújo Filho

Received: 7 June 2026

Revised: 5 July 2026

Accepted: 8 July 2026

Published: 10 July 2026

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1. Introduction

The detection of low-frequency gravitational waves via pulsar timing arrays marks a major breakthrough in astrophysics and cosmology. PTAs monitor the arrival times of pulses from an array of millisecond pulsars spread across the sky, searching for the telltale correlation patterns induced by a nanohertz-frequency gravitational-wave background (GWB) [1,2]. After decades of observations, several PTA collaborations have reported evidence for a stochastic common-spectrum signal in pulsar timing data [3–7]. In particular, the NANOGrav Collaboration has recently announced strong evidence that this common process exhibits the Hellings–Downs spatial correlation pattern expected for an isotropic GWB [8]. This discovery opens up a new window into gravitational-wave sources emitting at nanohertz frequencies.

A key question now is the physical origin of the detected GWB. The astrophysical benchmark is a background from numerous inspiraling supermassive black hole binaries (SMBHBs) [9,10]. In the simplest GW-driven limit, the characteristic strain follows $h_c(f) \propto f^{-2/3}$ (equivalently, timing residual power $P(f) \propto f^{-13/3}$), reflecting the long inspiral phase [9]. Such populations were long predicted to produce nanohertz amplitudes near $A_{\text{GWB}} \sim 10^{-15}$, consistent with the NANOGrav 15-year amplitude [8]. Recent NANOGrav work further shows that environmental coupling can imprint a low-frequency turnover and translate the turnover scale into parsec-scale nuclear densities [11]. This motivates treating SMBHB environmental turnover as a primary astrophysical model.

The nanohertz GWB could also arise from early-Universe physics. A first-order cosmological phase transition near the quantum chromodynamics (QCD) scale, $T \sim 100$ MeV, can peak at $f \sim 10^{-8}$ – 10^{-7} Hz if the transition is strong and slow [12]. Because the Standard Model predicts a smooth QCD crossover, such a signal would point to beyond-Standard-Model dynamics. Another possibility is a stochastic background from cosmic strings or cosmic superstrings, topological defects that radiate through oscillating loops [13]. Broad-band string backgrounds can contribute in the nanohertz regime, with amplitudes set by the dimensionless string tension $G\mu$ [13,14]. Other cosmological sources, including induced gravitational waves and domain walls, have also been proposed [15]; here, we focus on SMBHBs, phase transitions, and cosmic strings as representative models.

Discriminating between these scenarios using PTA data is challenging because several source classes can share the same Hellings–Downs angular correlations. This is also where the connection to gravitational physics and symmetry becomes concrete. The HD curve follows from statistical isotropy, homogeneity, and the tensor polarization response of general relativity: after averaging over an isotropic background, the two-point timing correlation is invariant under rotations of the sky and depends only on the pulsar-pair separation angle. Once this isotropic tensor correlation class is fixed, source discrimination moves to the spectral sector. The strict GW-driven SMBHB inspiral law is approximately scale-free in the PTA band; environmental hardening introduces a bend frequency f_b , first-order phase transitions introduce a peak frequency f_{peak} , and structured defect networks introduce loop, reconnection, or decay scales. The question addressed here is therefore whether the public HD free spectrum identifies such a physical scale or deformation, and which source mechanism explains it most efficiently. In short, the angular sector fixes the symmetry class; the spectral sector identifies the physical scale.

In this paper, the main statistical object is the compressed Bayesian likelihood constructed from the public NANOGrav 15-year HD free-spectrum KDE products. The official timing residual analyses fix the detection and angular correlation layers; the public HD free spectrum then becomes the natural likelihood object for the source-spectrum question. The analysis is organized around the following:

- The HD angular sector, which fixes the isotropic tensor correlation class of the signal under the standard PTA stochastic-background framework;
- Source spectra that test whether the HD free spectrum is strict and scale-free or instead carries a source-dependent spectral deformation;
- Compressed spectral posteriors and marginal likelihoods for SMBHB-PL, SMBHB-env, phase transition, and effective cosmic string models;
- Robustness checks across HD-component KDE products and low-frequency-bin ablations.

Building on the official timing residual analyses, this work uses the released HD-correlated free-spectrum compression as a reproducible public likelihood for source-spectrum ranking. We add an environmental SMBHB turnover model with density propagation, compare it against cosmological peak and effective string templates under matched

evidence calculations, and test the ranking across KDE products, priors, and low-frequency-bin ablations. Figure 1 summarizes this inference structure.

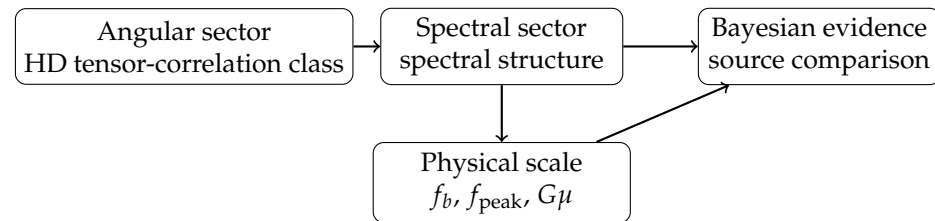


Figure 1. Symmetry structure of the inference. The angular sector tests the HD isotropic tensor correlation signature of a stochastic background and fixes the symmetry class of the signal. The spectral sector then measures departures from a strict scale-free SMBHB power law, and Bayesian evidence ranks the source mechanisms that introduce the relevant physical scale.

The public HD free spectrum favors physical curvature or a characteristic scale: a free-slope fit shows the non-canonical shape, while environmental and phase transition templates provide the leading physical explanations. Environmental SMBHB turnover is the leading astrophysical interpretation because it explains much of the structure with known binary-hardening physics and maps the bend to plausible parsec-scale nuclear densities. A first-order phase transition template is the leading cosmological challenger, whereas the simple cosmic string effective-slope model is weak.

The remainder of this paper is organized as follows. Section 2 defines the HD angular sector, the spectral compression, and the source models. Section 3 gives the Bayesian evidence calculation, including the free-spectrum KDE likelihood and its ρ convention. Section 4 summarizes the public NANOGrav inputs used in the compressed analysis. Section 5 reports the spectral evidence, model-family evidence, and robustness tests. Section 6 discusses the astrophysical and cosmological implications, and Section 8 states the resulting inference hierarchy.

2. Methods: Angular Sector, Spectral Compression, and Source Models

2.1. Timing-Likelihood Conventions and Spectral Compression

In a PTA experiment, the primary observable is the timing residual: the difference between an observed pulse time of arrival and the prediction of a deterministic pulsar-timing model. A GWB perturbs these residuals coherently across pulsars, so the signal is inferred through correlated timing deviations in excess of pulsar-specific noise.

We denote by $\mathbf{r}$ the vector of residuals for all pulsars. Under the Gaussian likelihood commonly adopted in PTA analyses, the covariance matrix C contains the pulsar noise and GWB contributions [16], and

$$\mathcal{L}(\mathbf{r}|\theta, M) = \frac{1}{\sqrt{(2\pi)^N \det C(\theta)}} \exp\left(-\frac{1}{2}\mathbf{r}^T C(\theta)^{-1}\mathbf{r}\right), \quad (1)$$

where N is the total number of TOA measurements and $C(\theta)$ is the parameter-dependent covariance matrix. We write

$$C = C_{\text{WN}} + C_{\text{RN}} + C_{\text{GWB}}. \quad (2)$$

Here, C_{WN} is uncorrelated white noise, C_{RN} is pulsar-specific red noise, and C_{GWB} is the common GWB covariance with the Hellings–Downs spatial structure. Full timing analyses also model chromatic propagation, clock, and ephemeris channels [16–18]. In this paper, those nuisance marginalizations enter through the published free-spectrum products.

The public HD free-spectrum products are therefore the appropriate input for the present source-spectrum comparison: they carry the timing analysis marginalization into a frequency-domain object on which competing physical spectra can be evaluated directly.

In the one-sided PSD convention used here, the residual covariance induced by an isotropic GWB can be written as

$$(C_{\text{GWB}})_{ij}(t, t') = \int_0^\infty df S_r(f) \Gamma_{ij} \cos [2\pi f(t - t')] , \quad (3)$$

where $S_r(f)$ is the one-sided timing residual power spectral density and Γ_{ij} is the Hellings–Downs overlap reduction function (normalized so that $\Gamma_{ii} = 1$). These quantities obey

$$h_c^2(f) = f S_h(f) , \quad S_r(f) = \frac{h_c^2(f)}{12\pi^2 f^3} = \frac{S_h(f)}{12\pi^2 f^2} . \quad (4)$$

In practice, we implement the likelihood in the Fourier domain using a finite set of low-frequency Fourier modes per pulsar. Writing $\Phi(f_k) \equiv S_r(f_k) \Delta f$ for the power at discrete frequencies $\{f_k\}$ and $\mathbf{F}_k$ for the corresponding Fourier design vector, the GWB covariance takes the standard Kronecker form

$$(C_{\text{GWB}})_{(i,a),(j,b)} = \Gamma_{ij} \sum_{k=1}^K \Phi(f_k) F_{i,a,k} F_{j,b,k} , \quad (5)$$

which preserves the full temporal structure and separates the angular correlation (Γ) from the spectral power in each Fourier bin [16].

The log-likelihood corresponding to Equation (1) is

$$\ln \mathcal{L}(\mathbf{r}|\theta, M) = -\frac{1}{2} \left[\mathbf{r}^T C^{-1} \mathbf{r} + \ln \det C + N \ln(2\pi) \right] , \quad (6)$$

up to an additive constant. This full likelihood is the object used by PTA collaborations for detection and angular-correlation evidence. The source Bayes factors reported below use the public free-spectrum compression derived from that likelihood.

2.2. Angular Correlation Structure

A central signature of an isotropic stochastic GWB is the quadrupolar angular correlation between pulsar timing residuals. Hellings and Downs [1] derived the expected correlation coefficient as a function of the angle γ between the directions of two pulsars in the sky. For two distinct pulsars a and b , define

$$x_{ab} = \frac{1 - \cos \gamma_{ab}}{2} .$$

The distinct-pulsar overlap reduction function used throughout this paper is

$$\Gamma_{ab}(\gamma_{ab}) = \frac{1}{2} + \frac{3}{2} x_{ab} \ln x_{ab} - \frac{x_{ab}}{4} , \quad a \neq b , \quad (7)$$

with the limiting value $x \ln x \rightarrow 0$ as $x \rightarrow 0$. The auto-correlation term is conventionally set separately, $\Gamma_{aa} = 1$. With the distinct-pair normalization in Equation (7), $\Gamma_{ab}(\gamma \rightarrow 0) = 1/2$ and $\Gamma_{ab}(180^\circ) = 1/4$.

In the context of our covariance matrix, the Hellings–Downs overlap function Γ_{ij} enters multiplicatively with the spectral power in each Fourier bin, as in Equation (3). This preserves the colored frequency dependence while imposing the tensor polarization angular symmetry predicted by general relativity. Clock errors and Solar System ephemeris errors instead induce monopolar or dipolar correlation patterns; they are therefore in-

cluded as nuisance channels or checked against the HD-correlated interpretation in the model comparison.

Under the isotropic-background approximation, the correlation function carries no memory of an absolute sky direction and depends only on angular separation. Finite pulsar sampling produces fluctuations around the mean HD curve, but the ensemble template is fixed by the angular symmetry of the tensor background [19].

2.3. Source Models and Expected Spectra

We now describe the specific models for the GWB spectrum in terms of the residual PSD $S_r(f)$ or, equivalently, the characteristic strain $h_c(f)$, related to $S_r(f)$ by the equations above:

1. **SMBHB Astrophysical Background (Model $\mathcal{M}_{\text{SMBHB}}$):** We assume the background is generated by an ensemble of inspiraling supermassive black hole binaries in the 10^8 – $10^{10} M_\odot$ mass range distributed throughout the Universe. The characteristic strain spectrum for such a population is expected to be a power law to the first approximation:

$$h_c(f) = A_{\text{GWB}} \left(\frac{f}{f_{\text{yr}}} \right)^\alpha, \quad (8)$$

where $f_{\text{yr}} = 1 \text{ yr}^{-1} \approx 3.17 \times 10^{-8} \text{ Hz}$, and α is the spectral index. For circular, GW-driven binaries in the inspiral regime, general relativity predicts $\alpha = -2/3$ [9], equivalently $S_r(f) \propto f^{-13/3}$. We use this fixed slope as the SMBHB reference model. Environmental coupling can bend the low-frequency spectrum, and recent NANOGrav work shows that such a turnover can encode parsec-scale nuclear densities [11].

2. **SMBHB Environmental-Turnover Background (Model $\mathcal{M}_{\text{SMBHB-env}}$):** We include an astrophysical turnover model in which binary hardening by stars, gas, or multi-body interactions suppresses the lowest PTA frequencies. Taking A to be the high-frequency GW-driven strain normalization referenced to f_{yr} , we parameterize the one-sided residual power spectrum with a smooth broken power law (SBPL),

$$S_r(f) = \frac{A^2}{12\pi^2 f_{\text{yr}}^3} \left(\frac{f}{f_{\text{yr}}} \right)^{-\gamma_{\text{hi}}} \left[1 + \left(\frac{f_b}{f} \right)^{\Delta\gamma/\zeta} \right]^{-\zeta}, \quad (9)$$

where $\gamma_{\text{hi}} = 13/3$ is the high-frequency GW-driven index, f_b is the bend frequency, $\Delta\gamma$ controls the low-frequency flattening, and ζ controls the smoothness. The default analysis fixes $\zeta = 1$ and samples $(A, f_b, \Delta\gamma)$.

3. **Cosmic String Background (Model $\mathcal{M}_{\text{CS}}$):** Cosmic strings are topological remnants of symmetry breaking and generate a stochastic background from oscillating loops [13,14]. The detailed spectrum depends on the string tension $G\mu$ and loop distribution; in the PTA band, broad-string-network spectra can be represented by an effective amplitude–slope spectrum,

$$h_c(f) \approx A_{\text{CS}} \left(\frac{f}{f_{\text{yr}}} \right)^\beta, \quad (10)$$

where β captures the effective PTA-band slope, expected to lie between roughly -1 and $-1/2$ for standard loop scenarios [13]. We use (A_{CS}, β) as an effective PTA-band string spectrum and interpret A_{CS} as an order-of-magnitude guide to $G\mu$. As with the other stochastic models, the spatial correlation is assumed to be Hellings–Downs.

4. **Phase Transition Background (Model $\mathcal{M}_{\text{PT}}$):** A first-order phase transition in the early Universe can generate gravitational waves through bubble nucleation, growth, and collisions, as well as from plasma sound waves and turbulence. Physically, this is

the channel in which PTA data would be probing a first-order symmetry-breaking event in the visible or hidden sector. We define the phase transition template in terms of the present-day gravitational-wave energy density. Let $y = f/f_{\text{peak}}$. We use the peak-normalized broken power law

$$\Omega_{\text{PT}}(f) = \Omega_{\text{peak}} \frac{(a+b)y^a}{b + ay^{a+b}}, \quad (11)$$

with $a = 3$ for the causal low-frequency rise and $b > 0$ for the high-frequency fall. The characteristic strain is then obtained from

$$h_c(f) = \left(\frac{3H_0^2}{2\pi^2} \right)^{1/2} f^{-1} \Omega_{\text{PT}}^{1/2}(f). \quad (12)$$

This implies $h_c \propto f^{a/2-1} = f^{1/2}$ at low frequency for $a = 3$, and $h_c \propto f^{-1-b/2}$ at high frequency. The conversion is therefore internally consistent and keeps the physical distinction between Ω_{gw} and h_c . We explore f_{peak} around 10^{-8} – 10^{-7} Hz, corresponding to QCD-scale or hidden-sector transitions for plausible transition durations, and sample the peak energy density and high-frequency index within physically motivated ranges. As with the other stochastic models, the spatial correlation is assumed to be Hellings–Downs.

The model set combines source templates at different physical resolution. The strict SMBHB and SMBHB-env cases are astrophysical templates tied to binary evolution. The phase transition model is a peak-normalized cosmological template, and the cosmic string case is an effective PTA-band representation of broad-string-network spectra. The evidence values rank these specified templates and priors.

Phase Transition Parameter Mapping

The phenomenological template in Equation (11) can be related to transition strength α_* , inverse duration $(\beta/H)_*$, wall speed v_w , relativistic degrees of freedom g_* , and source efficiency κ [20]. We use this mapping to choose a PTA-relevant prior domain for $(\Omega_{\text{peak}}, f_{\text{peak}}, b)$ and to interpret the resulting frequency scale. Frequencies near 10^{-8} Hz point to QCD-scale or nearby hidden-sector transitions for plausible durations and wall speeds. Because the NANOGrav new-physics analysis has an erratum concerning spectral-shape normalization in its phase transition implementation [15], our comparison is formulated directly in the corrected, peak-normalized Ω_{GW} convention of Equation (11) before conversion to h_c .

Each model $\mathcal{M}$ therefore predicts a different spectral scale structure. A strict SMBHB power law is scale-free in the PTA band, whereas the curved alternatives introduce f_b , f_{peak} , or network-dependent string scales. Bayesian evidence is the appropriate comparison metric because it rewards improved spectral fit while penalizing unused prior volume.

3. Bayesian Analysis Framework

We use Bayesian evidence and Bayes factors to compare source spectra. The inference has two linked likelihood layers. The full timing residual likelihood acts on the residual vector $\mathbf{r}$ in Equation (1) and establishes the common-process and HD angular correlation sector in the official PTA analyses. The compressed HD free-spectrum likelihood acts on the public KDE product d_{KDE} and is the statistical object used here for source-spectrum evidence. After the angular tensor correlation class is fixed, the calculation below quantifies the spectral structure in the published HD free-spectrum compression.

The public compressed likelihood is frequency resolved and quick to evaluate, so we can compare source spectra and prior choices without rerunning the full timing model. It retains the NANOGrav pulsar-noise, clock, ephemeris, monopole, dipole, and HD component modeling used to produce the released free spectra. The likelihood chain is traceable: the official timing likelihood supplies the detection and angular correlation anchor, the public KDE product supplies the spectral likelihood used for the main evidence tables, and the released ENTERPRISE-style scripts document the corresponding timing-likelihood validation route.

This defines a two-layer Bayesian construction. The released free-spectrum product is the non-parametric layer: each Fourier bin has an HD-correlated posterior independent of a source law. The present calculation adds a parametric source layer by projecting SMBHB-env, phase transition, and string spectra into those bins and integrating over their priors. This preserves the frequency-resolved reconstruction while making direct evidence comparisons among physical-scale hypotheses.

Mathematically, the first layer marginalizes the timing likelihood over nuisance timing and noise parameters to obtain a spectral compression,

$$\mathcal{L}_{\text{fs}}(\rho_1, \dots, \rho_N \mid \Gamma^{\text{HD}}) \equiv \int d\eta \mathcal{L}_{\text{TOA}}(\mathbf{r} \mid \eta, \rho_1, \dots, \rho_N, \Gamma^{\text{HD}}) \pi(\eta).$$

The public release represents this object with one-dimensional KDE factors after division by the free-spectrum prior. The second layer composes a physical source map $\theta_{\text{spec}} \mapsto \{\rho_k(\theta_{\text{spec}})\}$ with this nuisance-marginal likelihood and integrates over the source prior. This formulation uses the public frequency-resolved likelihood directly in the source comparison. It carries the released timing-model marginalization into the source test, keeps the free-spectrum reconstruction independent of any source law, and makes the source-prior volume explicit in the evidence.

The framework produces three testable outputs. Bayes factors test whether the HD-correlated spectrum is scale-free or carries a characteristic nanohertz scale. Posterior samples of f_b or f_{peak} propagate that scale to astrophysical or cosmological parameters, such as ρ_* or a QCD-scale transition frequency. The same factorization separates the spectral likelihood from the angular kernel, so future PTA products with angular coefficients can be inserted into a joint spectral-angular evidence calculation.

This factorization creates a direct handoff to LISA-type anisotropy work. A LISA analysis can use a time-dependent detector response to infer angular modes at millihertz frequencies. The present calculation supplies the nanohertz spectral-evidence factor for the already observed HD-correlated PTA signal: it identifies whether the source spectrum is scale-free or carries a characteristic scale, and which physical source family supplies that scale. These are the quantities a multi-band analysis needs before asking whether the same source population has millihertz continuation or angular structure. Together, the PTA and LISA calculations define a joint program: PTA fixes the observed nanohertz source scale in the HD sector, while LISA-type analyses test the associated angular response and high-frequency continuation.

This scope also fixes where the Bayesian framework choice enters the calculation. The public KDE product remains the non-parametric free-spectrum posterior; the added layer is the conditional source-spectrum evidence

$$\mathcal{Z}_{\text{spec}}(\mathcal{M}) = \int d\theta_{\text{spec}} \pi(\theta_{\text{spec}} \mid \mathcal{M}) \mathcal{L}_{\text{KDE}}(d_{\text{KDE}} \mid \rho_k(\theta_{\text{spec}}), \Gamma^{\text{HD}}),$$

where the HD angular kernel is inherited from the timing analysis and the integration is over source-spectrum parameters. Changing the source prior or template family can

therefore change the Bayes factors, which is why the results below include prior sensitivity, KDE product, and low-frequency-bin checks. An anisotropy or LISA-type calculation extends this structure by promoting the angular sector itself to model parameters and integrating over detector-response or sky-map coefficients. Section 7 presents these two evidence targets side by side as a bridge: the present $\mathcal{Z}_{\text{spec}}$ is the PTA spectral factor, and the LISA/anisotropy likelihood supplies the angular response factor for future joint analyses.

3.1. Prior and Posterior

For a given model M with parameter vector θ , Bayes' theorem gives the posterior distribution

$$p(\theta|d, M) = \frac{\mathcal{L}(d|\theta, M) \pi(\theta|M)}{\mathcal{Z}(d|M)}, \quad (13)$$

where $\mathcal{L}(d|\theta, M)$ is either the full timing likelihood or the compressed KDE likelihood defined below, $\pi(\theta|M)$ is the prior probability density for the parameters under model M , and

$$\mathcal{Z}(d|M) = \int d\theta \mathcal{L}(d|\theta, M) \pi(\theta|M) \quad (14)$$

is the Bayesian evidence (also called marginal likelihood) for model M . The evidence $\mathcal{Z}$ is the probability of the data given the model, integrated over all possible parameter values weighted by the prior. It encapsulates both the quality of fit (based on the likelihood) and the complexity or predictiveness of the model (based on the volume of parameter space allowed by the prior).

This evidence calculation is central to the structure of this paper. The first Bayesian layer asks whether there is a common process; the second asks whether its cross-correlations follow the HD angular template rather than an uncorrelated, monopolar, or dipolar alternative; the third asks which physical source spectrum best explains the frequency dependence once the angular tensor correlation class is fixed. This layered structure turns the symmetry argument into an inference strategy: the angular sector tests the isotropic tensor response correlation class, and the spectral sector tests the emergence of a physical nanohertz scale.

We adopt priors for each model's parameters that reflect our state of knowledge:

- For the strict common power law SMBHB model, we take a log-uniform prior on the amplitude A_{GWB} over 10^{-17} – 10^{-14} and fix the residual-PSD slope to $\gamma = 13/3$, equivalently $\alpha = -2/3$ in characteristic strain. We also run a free-slope power law with $\gamma \sim \mathcal{U}[0, 7]$ to measure spectral tilt directly.
- For the SMBHB environmental turnover model, we use the same prior range for the high-frequency normalization A as for the power law amplitude, fix $\gamma_{\text{hi}} = 13/3$ in the baseline run, take $\Delta\gamma \sim \mathcal{U}[0, 4]$, and use $\log_{10}(f_b/\text{Hz}) \sim \mathcal{U}[-9.4, -8.0]$ so that the bend is allowed to occupy the lowest PTA frequency bins where environmental hardening should be most visible. We keep $\zeta = 1$ unless otherwise stated.
- For the effective cosmic string spectrum, we use a log-uniform amplitude prior over the same strain amplitude range as the common spectrum and a uniform PTA-band slope prior $\beta \sim \mathcal{U}[-1, -1/2]$. The amplitude gives an order-of-magnitude mapping to $G\mu$.
- For the phase transition model, f_{peak} and Ω_{peak} are given log-uniform priors informed by cosmology. In the baseline compressed comparison, we use $\log_{10}(f_{\text{peak}}/\text{Hz}) \sim \mathcal{U}[-9, -7]$, $\log_{10} \Omega_{\text{peak}} \sim \mathcal{U}[-12, -5]$, and a continuous high-frequency slope prior $b \sim \mathcal{U}[2, 4]$.
- The released direct timing-likelihood workflow uses standard pulsar-noise, clock, ephemeris, and chromatic process priors as in NANOGrav's analysis [16]; their marginalization enters here through the public free-spectrum product.

Full timing residual posteriors can be sampled with Markov Chain Monte Carlo, thermodynamic integration, nested sampling, and related methods implemented in PTA tools such as PTMCMCSAMPLER, ENTERPRISE, and DYNESTY [16,21,22]. The evidence values reported in Table 1 were obtained using the compressed free-spectrum KDE likelihood defined below, which isolates the source-spectrum question addressed by the public data.

Table 1. Compressed spectral evidence values from the public HD free-spectrum KDE comparison. The strict SMBHB power law with fixed $\gamma = 13/3$ is the reference. The free-slope PL row is an empirical spectral-shape benchmark; family rows are equal-prior averages over source classes.

Model	$\Delta \ln Z$ vs. PL	BF vs. PL
Strict SMBHB-PL	0.00 ± 0.01	1.00
Free-slope PL	5.73 ± 0.04	306.7
SMBHB-env/SBPL	3.87 ± 0.02	48.0
Effective CS spectrum	0.39 ± 0.02	1.48
PT, Ω_{gw} template	4.94 ± 0.02	139.5
Curved family	4.14	63.0
Env+PT family	4.54	93.7

3.2. Bayes Factors for Model Comparison

To compare two competing models M_1 and M_2 in the Bayesian framework, one uses the Bayes factor

$$\text{BF}_{12} = \frac{\mathcal{Z}(d|M_1)}{\mathcal{Z}(d|M_2)}, \quad (15)$$

which is the ratio of evidence values. If $\text{BF}_{12} > 1$, the data favor model M_1 over model M_2 (with BF_{12} quantifying the strength of evidence), and if $\text{BF}_{12} < 1$, model M_2 is favored. By convention, one often takes logarithms and quotes $\ln \text{BF}$. For example, $\ln \text{BF} > 5$ (roughly $\text{BF} > 150$) might be considered “strong evidence” on the Jeffreys scale, while $\ln \text{BF} < 1$ is negligible evidence.

The Bayes factors of interest separate three questions. NANOGrav’s official analyses establish the common process and the HD angular sector, with decisive evidence for a common red process and Bayes factors of order 200–1000 for HD correlations over an uncorrelated common process [8]. This paper focuses on the third comparison, BF_{M_i, M_j} , which ranks source spectra such as SMBHB-env, phase transition, cosmic strings, and strict SMBHB-PL under the same HD-correlated stochastic background assumption. The evidence automatically applies an Occam penalty to prior volume that does not explain the measured free-spectrum shape.

Because Bayesian evidence is integrated over prior predictive volume, the prior ranges are part of each model definition. We therefore report both baseline evidence values and prior-sensitivity checks, making the comparison a reproducible measurement of which physical scales the public spectrum favors under the stated prior domains.

3.3. Compressed HD Free-Spectrum Likelihood and ρ Convention

For the source-model comparison reported below, the data object is the public NANOGrav HD-correlated 30-frequency free-spectrum KDE product, using the corrected v1.1.0 Zenodo release [23]. It provides the Fourier bin frequencies `freqs.npy`, the `log10rho` grid, and the per-frequency posterior KDE density values for the HD free-spectrum amplitudes; the rapid-refitting and KDE construction methodology is described by Lamb, Taylor, and van Haasteren [24]. We use the same Fourier power convention as Equation (3):

$$\rho_k^2 \equiv \Phi_k = S_r(f_k) \Delta f, \quad (16)$$

where $\Delta f = 1/T_{\text{obs}}$ for the discrete Fourier basis. Given any source model M , we convert its predicted $h_c(f_k)$ or $\Omega_{\text{GW}}(f_k)$ to $S_r(f_k)$ using Equations (4) and (12). Since the public KDEs represent posterior densities for the free-spectrum amplitudes, the corresponding compressed likelihood is obtained by removing the free-spectrum prior density:

$$\ln \mathcal{L}_{\text{KDE}}(\theta_M) = \sum_{k=1}^{30} \ln \left\{ \frac{\hat{p}_k[\log_{10} \rho_k^{(M)}(\theta_M) | d_{\text{KDE}}]}{\pi_{\text{fs},k}[\log_{10} \rho_k^{(M)}(\theta_M)]} \right\}. \quad (17)$$

Here, $\hat{p}_k$ is the public posterior KDE density for the k th Fourier bin and $\pi_{\text{fs},k}$ is the free-spectrum prior density. For the public products used here, the prior is flat in $\log_{10} \rho_k$ across the evaluated support, so the denominator contributes only a model-independent constant for predictions inside the support. Equation (17) evaluates the product of one-dimensional marginal KDEs, treating the Fourier-bin-likelihood factors as independent after marginalization. This independence is an approximation imposed by the released one-dimensional KDE products rather than a statement about the full timing likelihood. Neglected inter-bin covariance can change the absolute evidence scale by changing the effective number of independent spectral constraints. Its sign is not fixed without the full covariance: positive correlations can reduce the effective information content, while anticorrelations can sharpen relative spectral-shape tests. The product likelihood should therefore be read as a reproducible compressed-evidence calculation tied to the public one-dimensional products, with the prior and leave-one-frequency-out checks below used to test whether the ranking is driven by a single bin or by the distributed spectrum. It defines the compressed likelihood used for the source-spectrum evidence tables, whose numerical values are read from the generated JSON and sample files.

4. Data Description and Analysis Setup

We apply the above framework to the NANOGrav 15-year data set [8,15,25,26], which is a publicly released collection of pulse timing data for 68 millisecond pulsars observed over approximately 15 years (2004 to mid-2020) using the Arecibo Observatory and Green Bank Telescope, among others. We briefly summarize the salient features of this dataset and the preprocessing steps involved in our analysis.

Each pulsar in the 15-year dataset comes with time-of-arrival measurements typically at a bi-weekly to monthly cadence, often at multiple radio observation frequencies to allow the correction of dispersion delays caused by the interstellar medium. The data release provides the timing residuals after subtracting a best-fit timing model for each pulsar (including spin frequency, spin-down, astrometric parameters, and binary orbital parameters if applicable) [25]. Additionally, the NANOGrav analysis implements the following measures to mitigate noise:

- **White noise calibration:** For each pulsar and each observing backend/receiver system, nuisance parameters such as EFAC (error factor) and EQUAD (added noise term) are introduced to ensure the TOA uncertainties are appropriately scaled. These were either fixed from per-pulsar noise analysis or included as parameters in the full Bayesian fit with priors.
- **Dispersion Measure (DM) variations:** Irregular variations in the electron column density along the line of sight to a pulsar can cause frequency-dependent time delays. NANOGrav models these with chromatic stochastic processes and related deterministic terms informed by multi-frequency data. In the present compressed analysis, this marginalization is inherited from the public free-spectrum product rather than re-estimated from TOAs.

- **Solar System ephemeris and clock errors:** A mismodeling of planetary ephemerides can introduce a dipolar correlated signal across pulsars, while terrestrial time-standard errors produce a monopolar common signal. NANOGrav performed tests for these channels and published HD-component products that separate or marginalize common angular patterns [8]. Our robustness checks below therefore compare the HD-only product with HD components extracted in the presence of monopole, dipole, and common-process channels.

After constructing the residuals and noise design matrices for all pulsars, a full analysis forms the likelihood as in Section 2. The dimensions of C (number of residual points) are large, but Fourier-domain likelihoods and time-domain block-matrix methods make the calculation tractable. The official NANOGrav timing analysis reports decisive evidence for the common process and an inferred amplitude $A_{\text{GWB}} \sim 2 \times 10^{-15}$ at f_{yr} for simple common-spectrum models [8]. We use those timing-likelihood results as the detection anchor and carry out the source-spectrum evidence calculation with the public free-spectrum compression.

At the timing-likelihood level, the angular separation can be viewed schematically as a decomposition of the correlated cross-pulsar covariance:

$$C_{ab}^{\text{corr}}(f_k) = \Gamma_{ab}^{\text{HD}} S_k^{\text{HD}} + \Gamma_{ab}^{\text{MP}} S_k^{\text{MP}} + \Gamma_{ab}^{\text{DP}} S_k^{\text{DP}} + \delta_{ab} S_k^{\text{CP}}.$$

Here, Γ_{ab}^{HD} is the quadrupolar Hellings–Downs tensor response pattern, Γ_{ab}^{MP} represents a monopolar clock-like channel, Γ_{ab}^{DP} represents a dipolar ephemeris-like channel, and the last term represents an uncorrelated common red process. The present paper does not refit these angular components from the timing residuals. Instead, it uses the released HD-component free-spectrum products and checks whether the source-spectrum ranking is stable when the non-HD channels are included in the NANOGrav extraction.

The KDE products used here are frequency-resolved posteriors for the HD-correlated component of the common process. They provide a spectral compression of the angular decomposition already carried out in the NANOGrav timing analysis. Operationally, the HD-only product fits the quadrupolar HD template; the HD+MP+DP product fits monopolar clock-like and dipolar ephemeris-like sectors simultaneously and reports the extracted HD amplitudes; the HD+MP+DP+CP product adds a common uncorrelated red process. Comparing the evidence ranking across these HD-component products is therefore an empirical check that the spectral preference is tied to the HD-correlated component rather than to monopole or dipole leakage. These products provide the spectral inputs for the HD-component comparisons below; explicit anisotropy studies can be built by adding an angular basis to the same evidence logic.

This gives a concrete traceability test for the angular sector. The baseline HD-only KDE supplies the 30 frequency-bin posteriors after fitting the quadrupolar HD template. The HD+MP+DP product asks whether the same spectral conclusion survives when monopolar clock-like and dipolar ephemeris-like channels are fitted at the same time; the reported object is still the extracted HD component. The HD+MP+DP+CP product further allows an uncorrelated common red process and is therefore a more conservative stress test of the HD spectral component. In all three cases, the source evidences below are evaluated only on the released HD-component spectrum. The HD-component comparison below is therefore the empirical counterpart to the angular statement: monopole, dipole, and common-process channels can change the extracted HD spectrum, but the source-ranking question is always asked of the HD-correlated component.

For the source-model comparison reported below, we use the public NANOGrav HD-correlated 30-frequency free-spectrum KDE product [23,24] and evaluate each source

spectrum with the ρ convention and compressed likelihood in Section 3.3. This is the likelihood object for the central question: whether the public free-spectrum shape favors a strict SMBHB power law, an SMBHB environmental turnover, a cosmic-string-like slope, or a phase transition peak. Specifically, we evaluate the following:

- The standard power law model ($\mathcal{M}_{\text{SMBHB}}$) with free α (uninformed) and with α fixed to $-2/3$.
- The environmental SMBHB model ($\mathcal{M}_{\text{SMBHB-env}}$) with the SBPL bend frequency and turnover strength sampled under the priors above.
- An effective cosmic string spectrum ($\mathcal{M}_{\text{CS}}$) parameterized by an amplitude and a PTA-band strain slope β , with the amplitude mapped approximately to $G\mu$.
- A phase transition model ($\mathcal{M}_{\text{PT}}$) with Ω_{peak} , f_{peak} , and b sampled under the priors defined in Section 3, using Equations (11) and (12) to predict each Fourier bin.

All reported spectral Bayes factors, posterior medians, and prior-sensitivity checks are read from the generated JSON and sample files under `analysis_outputs/kde_model_comparison`. The reproducibility package follows this layout: `data_sources/` contains the public NANOGrav timing and KDE inputs, `analysis/` contains the scripts used for the KDE evidence calculation and validation checks, `analysis_outputs/` contains the JSON tables and posterior samples used in the manuscript, and `repro/` records the environment and run commands. The archived numerical evidence products are the compressed-KDE JSON tables and posterior samples; the released repository also includes ENTERPRISE-style timing-likelihood scripts as validation workflows.

5. Results

5.1. Detection of a Common Spectrum and Hellings–Downs Correlation

The detection layer of this analysis is anchored to the official NANOGrav timing-residual study. NANOGrav reports extremely strong evidence for a common red process and Bayes factors of the order of a few hundred for Hellings–Downs spatial correlations over an uncorrelated common process, depending on spectral and noise assumptions [8]. We therefore treat an HD-correlated nanohertz common process as the established observational input. The new calculation in this paper addresses the next layer of inference: which source spectra best describe the measured free-spectrum shape.

Figure 2 shows the distinct-pair HD curve used in the covariance model and in the public data cross-check below. All source hypotheses considered here share this tensor polarization angular symmetry, so the model discrimination problem is primarily spectral.

As a public data cross-check, the pipeline ingests the wideband `.par/.tim` pairs for the first 24 pulsars, bins residuals into 30-day windows, removes a low-order trend per pulsar, and computes weighted pairwise cross-correlations. Figure 3 shows the resulting 273 usable pairs and an HD-template coefficient 0.455 ± 0.102 , consistent with the official NANOGrav timing-likelihood result. This check is not used as an independent evidence calculation; it fixes the angular bin and normalization conventions used in the source-spectrum analysis.

Taking the HD-correlated common process as the observational input, we proceed to characterize its spectrum under different model assumptions.

Figure 4 visualizes the evidence calculation. The upper panel shows which source spectra pass through the public free-spectrum posterior intervals; the residual panel shows where each template misses the free-spectrum median; and the evidence panel summarizes the Occam-weighted fit relative to the strict SMBHB power law. The environmental SMBHB curve improves the low-frequency shape by introducing a bend, so it is most sensitive to the first PTA Fourier bins. The phase transition curve introduces a peak and is less tied to one endpoint, which explains why its evidence remains more distributed in the leave-one-frequency-out test. The effective cosmic string spectrum tests an amplitude–slope

approximation to a broad string background. Its weak baseline evidence means that this proxy lacks the PTA-band curvature favored by the current free spectrum; it does not exclude more structured string network spectra.

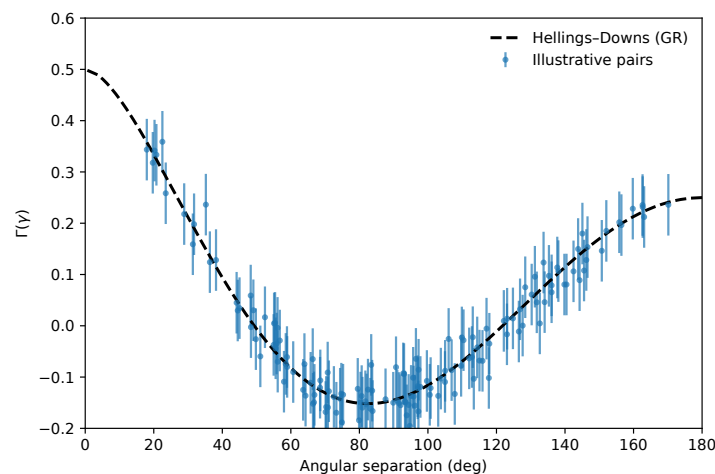


Figure 2. Hellings–Downs overlap reduction function. The dashed curve shows the theoretical distinct-pair function $\Gamma(\gamma)$ from Equation (7), normalized to $\Gamma(0^\circ) = 0.5$ for distinct pulsars and $\Gamma(180^\circ) = 0.25$. The scatter shows the scale of pairwise estimators.

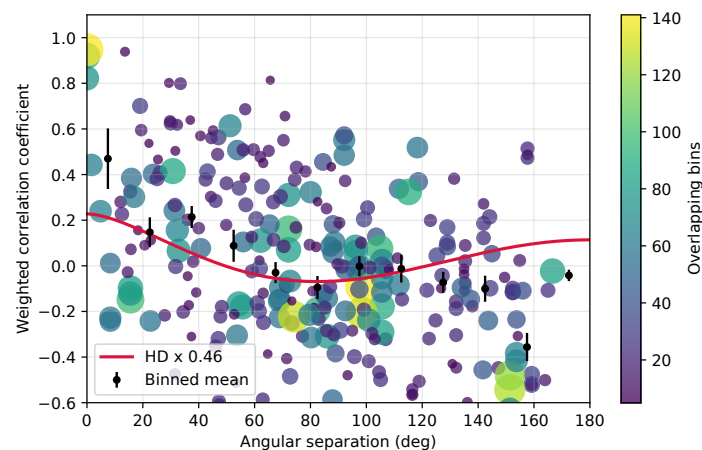


Figure 3. Hellings–Downs public data cross-check. Each point corresponds to a pulsar pair; the color indicates the number of overlapping 30-day bins contributing to the correlation estimate. Black circles show angle-binned averages with standard errors. The plot fixes the angular bin and normalization conventions used below.

5.2. Common-Process Spectrum: Amplitude and Slope

Assuming a simple power law form for the common process, the official NANOGrav timing analysis and the public free-spectrum product imply an amplitude consistent with the SMBHB scale. Writing $h_c(f) \propto f^{(3-\gamma_{\text{GWB}})/2}$, or equivalently $S_r(f) \propto f^{-\gamma_{\text{GWB}}}$, the reported strain amplitude at $f_{\text{yr}} = 1/\text{yr}$ is $A_{\text{GWB}} \simeq 2.4 \times 10^{-15}$ [8]. This amplitude agreement makes the strict SMBHB law the natural astrophysical reference, while the free-spectrum evidence below measures the spectral departure from its canonical index.

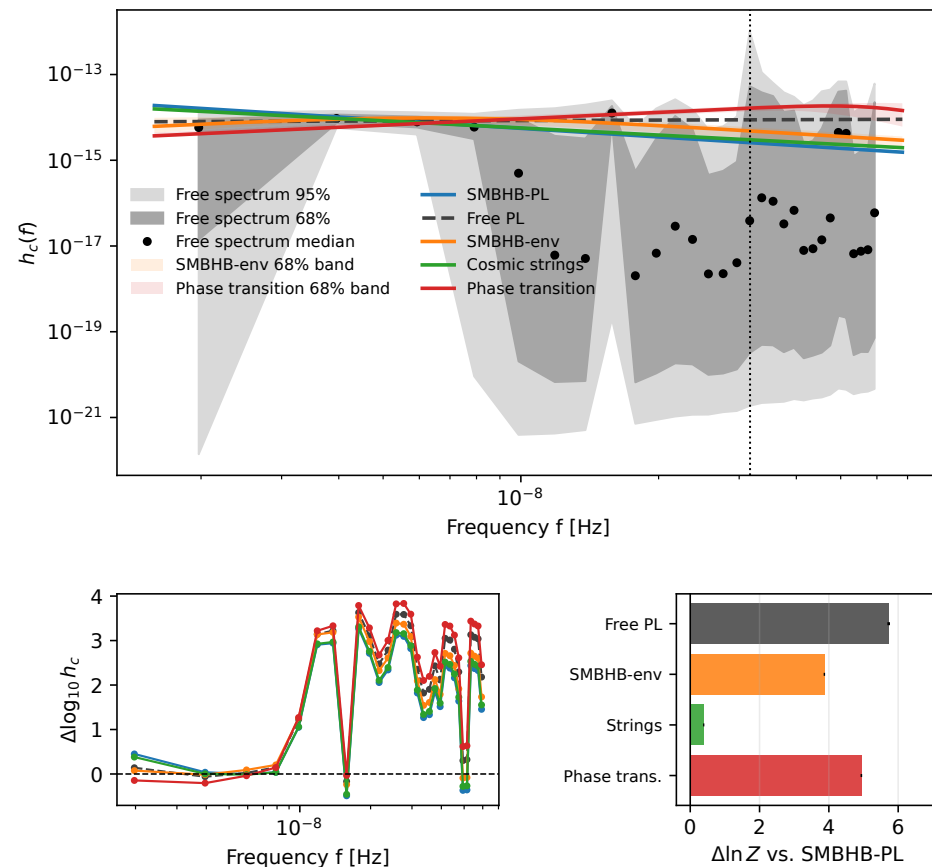


Figure 4. Public free-spectrum data and model comparison. (**Top**): Median and 68/95% credible intervals from the public HD free-spectrum KDE product, overlaid with posterior predictive median spectra for the source models and a dashed free-slope power law. Shaded bands show the central 68% posterior predictive intervals for the two leading curved templates. (**Bottom left**): Model residuals relative to the free-spectrum median. (**Bottom right**): Compressed KDE evidence differences relative to the strict SMBHB power law. The panels follow the two-layer evidence construction: the top panel gives the non-parametric HD free-spectrum layer, the residual panel shows how each physical source map deforms that spectrum, and the evidence panel gives the source-prior-marginalized ranking. Together, these panels connect Section 3 to the observed preference for scale-carrying spectra. The SMBHB-env curve follows Equation (9); the phase transition curve is generated from Equations (11) and (12).

5.3. Bayesian Model Comparison: Astrophysical vs. Cosmological Sources

We now turn to the core question of this study: given the public NANOGrav 15-year HD free-spectrum shape, which source spectra are preferred? Table 1 gives the reproducible compressed KDE evidence comparison. The reference model is the strict SMBHB power law with fixed $\gamma = 13/3$, the GR-driven scale-free prediction for circular inspiraling binaries. The free-slope power law gives the largest empirical improvement, directly showing that the public HD free spectrum supports a non-canonical spectral tilt. Among physical source templates, the phase transition model gives the largest compressed evidence value, while the environmental SMBHB model gives the leading astrophysical interpretation because it explains much of the curvature with known binary-hardening physics and maps the bend to nuclear stellar density. The simple cosmic string effective-slope model remains near neutral under the baseline prior. Figure 5 presents the timing-likelihood validation diagnostics used alongside this comparison.

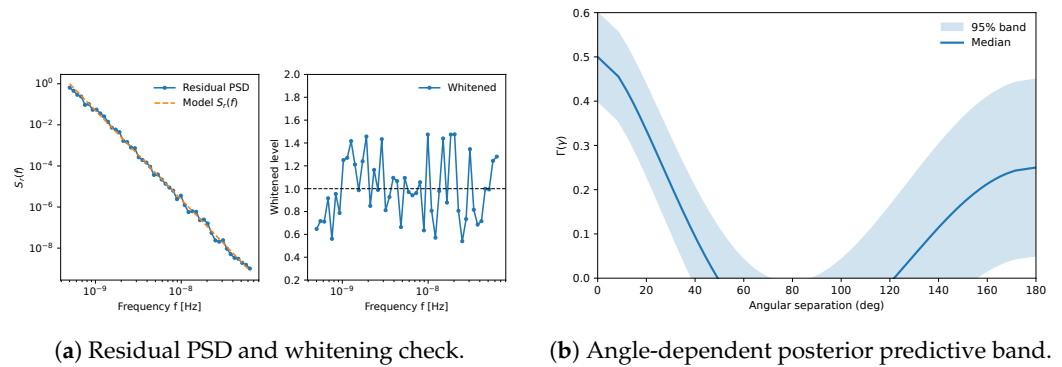


Figure 5. Validation diagnostics for the timing-likelihood workflow: residual whitening and angle-dependent posterior predictive checks. In the whitening panel, the dashed horizontal line marks the unit level expected after successful whitening.

The main result is therefore two-fold. First, the public free-spectrum shape favors non-canonical spectral structure. Second, the physical reading depends on the model class: the PT template is the strongest cosmological competitor in the tested set, while SMBHB environmental hardening is the strongest astrophysical explanation. This distinction avoids treating the highest numerical evidence value and the leading astrophysical interpretation as the same statement. Figure 6 presents the posterior distribution for the SMBHB-env amplitude, bend frequency, turnover strength, and density inferred from the bend.

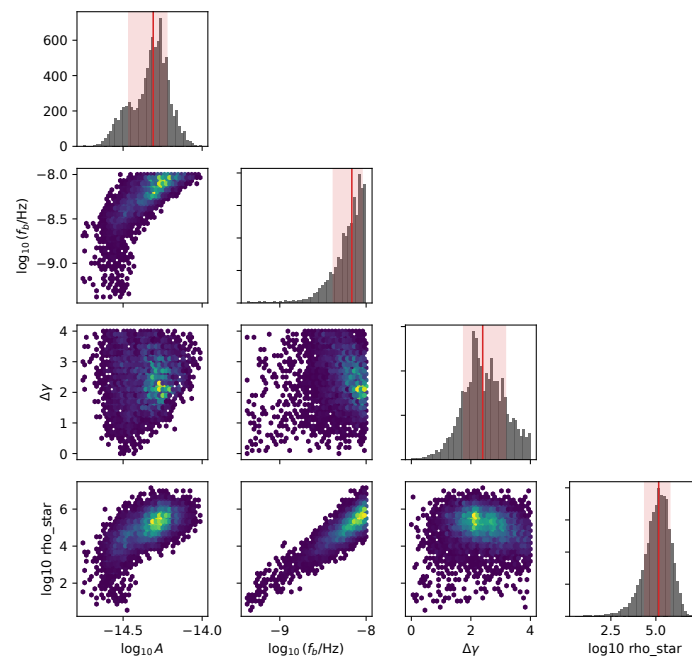


Figure 6. SMBHB-env posterior and density propagation. Marginal and pairwise posterior projections for the high-frequency normalization $\log_{10} A$, $\log_{10}(f_b/\text{Hz})$, $\Delta\gamma$, and the derived $\log_{10}(\rho_\star/M_\odot\text{pc}^{-3})$. Gray histograms show the one-dimensional marginalized posteriors; red vertical lines mark the medians and red bands the central 68% intervals. In the pairwise panels, the purple-to-yellow scale indicates increasing posterior sample density. The density coordinate uses the hardening-balance mapping described in Section 6.2.

The posterior summaries from the same run sharpen this picture. For strict SMBHB-PL, the median amplitude is $\log_{10} A = -14.59$. For the free-slope power law, the posterior is $\gamma = 2.93^{+0.24}_{-0.19}$, far from the canonical $\gamma = 13/3$ and consistent with the non-canonical tilt seen in the free spectrum. For SMBHB-env, the median high-frequency normalization is $\log_{10} A = -14.31$, with $\log_{10}(f_b/\text{Hz}) = -8.17$ and $\Delta\gamma = 2.41$. For the phase transition

template, the median peak is $\log_{10}(f_{\text{peak}}/\text{Hz}) = -7.21$ and the high-frequency falloff is $b = 2.98^{+0.71}_{-0.68}$, consistent with the continuous prior $b \sim \text{U}[2, 4]$. The cosmic string spectrum has median slope $\beta = -0.55$, close to a shallow power law, and is therefore unable to exploit the observed curvature strongly.

The prior-sensitivity results in Table 2 show how the ranking changes under wider or narrower prior domains. Broadening the SMBHB-env bend-frequency prior from $\log_{10} f_b \in [-9.4, -8.0]$ to $[-10.0, -8.0]$ changes $\Delta \ln Z$ from 3.87 to 3.57; narrowing it to $[-9.0, -8.3]$ gives $\Delta \ln Z = 3.20$. Broadening the phase transition peak prior to $\log_{10} f_{\text{peak}} \in [-10, -6]$ gives $\Delta \ln Z = 4.45$. The robustness checks in Tables 3 and 4, together with the leave-one-frequency-out diagnostic below, further support the spectral curvature result across KDE products and frequency choices.

Table 3 compares different public HD-component KDE products. The HD-only row gives the main evidence values. The HD+MP+DP and HD+MP+DP+CP rows repeat the calculation after the HD component has been extracted from broader fits including monopole, dipole, and common-process channels. The large HD+MP+DP values have a direct within-product interpretation: the zero point is the strict $\gamma = 13/3$ SMBHB-PL fit to the same HD+MP+DP KDE product, and that fixed law is especially disfavored for this extracted HD spectrum. The absolute values are therefore not meant to be compared across rows as one common evidence scale. They are larger in the HD+MP+DP row because the extracted HD spectral shape in that product is especially poorly described by the fixed canonical slope used as the row reference. As a robustness stress test, this row shows that when monopole and dipole sectors are fitted simultaneously, the extracted HD component still favors non-canonical spectral structure. The absolute zero point is product-specific because each row uses its own strict-PL reference.

Table 2. Prior-sensitivity checks from the generated KDE comparison output. Values are $\Delta \ln Z$ relative to strict SMBHB-PL.

Model	Prior Variation	$\Delta \ln Z$
SMBHB-env	$\log_{10} f_b \in [-9.4, -8.0]$ baseline	3.87
SMBHB-env	$\log_{10} f_b \in [-10.0, -8.0]$	3.57
SMBHB-env	$\log_{10} f_b \in [-9.0, -8.3]$	3.20
Cosmic strings	$\beta \in [-1.2, -0.3]$	2.60
Phase transition	$\log_{10} f_{\text{peak}} \in [-10, -6]$	4.45

Table 3. Robustness of compressed spectral evidence across public HD-component KDE products. Values are $\Delta \ln Z$ relative to strict SMBHB-PL fitted to the same KDE product, using the same priors as Table 1. The HD+MP+DP and HD+MP+DP+CP products extract the HD component while also fitting monopole, dipole, and common-process channels, making the table a within-product check such that the HD spectral component continues to favor non-canonical structure.

KDE Product	Free PL	Env	PT	CS	Curved
HD only	5.64	3.88	4.98	0.43	4.18
HD+MP+DP	49.50	46.82	47.84	38.22	47.05
HD+MP+DP+CP	2.12	1.78	0.91	−0.15	1.13

Table 4. Low-frequency-bin ablation tests using the HD-only KDE product. Values are $\Delta \ln Z$ relative to strict SMBHB-PL after removing the indicated Fourier bins and recomputing the compressed evidences.

Ablation	Free PL	Env	PT	CS	Curved	Env+PT
Drop bin 1	3.56	0.96	2.09	−0.20	1.35	1.68
Drop bin 2	6.17	3.91	6.81	0.39	5.76	6.17
Drop bins 1–2	4.07	0.81	3.74	−0.11	2.72	3.10
Drop bins 1–3	3.63	0.41	3.24	−0.10	2.23	2.60

As a frequency influence diagnostic, we also compute a leave-one-frequency-out evidence profile. For each Fourier bin k , the reduced data object $d_{\text{KDE}}^{(-k)}$ is formed by removing that bin from the product in Equation (17), and the evidence contrast

$$I_k(M) = \ln Z_M(d_{\text{KDE}}^{(-k)}) - \ln Z_{\text{SMBHB-PL}}(d_{\text{KDE}}^{(-k)})$$

is recomputed with the same priors. This tests how each frequency bin contributes to the measured curvature. Across the 30 leave-one-out refits, the minimum/median/maximum values of I_k are 1.09/3.81/4.51 for SMBHB-env, 1.74/4.82/6.98 for the phase transition template, 1.41/4.02/5.94 for the curved family, and 1.75/4.41/6.34 for the Env+PT family. The effective cosmic string spectrum remains near neutral, with $-0.19/0.39/0.49$. The preference for scale-carrying spectra is therefore supported across all single-bin refits: the first bin is most influential for the environmental turnover, as expected for a low-frequency bend, while the phase transition evidence is distributed more broadly across the free spectrum. This behavior is also visible in Figure 6: the environmental model uses the posterior bend frequency to flatten the lowest part of the spectrum, so its density interpretation is most directly tied to the first few Fourier bins. The PT template instead uses a peak frequency and high-frequency falloff, so future PTA data at both the low- and high-frequency ends will be important for separating a binary-hardening turnover from a cosmological peak.

Table 5 summarizes the spectral priors used for all source models in the compressed evidence calculation.

Table 5. Spectral priors used in the compressed KDE evidence calculation. Log-uniform priors are used for strictly positive scale parameters and uniform priors for indices.

Parameter	Prior	Notes
A_{GWB} (strict SMBHB)	log-uniform in $[10^{-17}, 10^{-14}]$	at f_{yr}
A (free PL / SMBHB-env)	log-uniform in $[10^{-17}, 10^{-14}]$	strain normalization
γ (strict SMBHB)	fixed 13/3	residual-PSD slope
γ (free PL)	uniform in $[0, 7]$	residual-PSD slope
$\log_{10} f_b$ (SMBHB-env)	uniform in $[-9.4, -8.0]$	Hz
$\Delta\gamma$ (SMBHB-env)	uniform in $[0, 4]$	turnover strength
A_{CS} (string)	log-uniform in $[10^{-17}, 10^{-14}]$	strain amplitude
β (strings slope)	uniform in $[-1, -1/2]$	$h_c \propto f^\beta$ in PTA band
f_{peak} (PT)	log-uniform in $[10^{-9}, 10^{-7}]$ Hz	maps to T_*
b (PT high-f slope)	uniform in $[2, 4]$	source-dependent
Ω_{peak} (PT)	log-uniform in $[10^{-12}, 10^{-5}]$	energy density peak

Figure 7 shows representative sampler traces and the evidence-growth diagnostic.

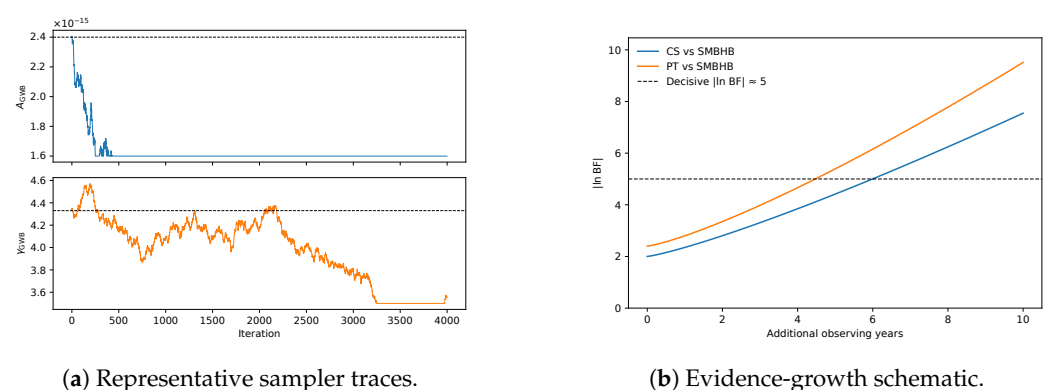


Figure 7. Sampler diagnostics and evidence-growth schematic for the timing-likelihood workflow.

5.4. Implications for Noise and Systematics

The separation between detection, spatial correlation, and source discrimination is essential for noise interpretation. The official NANOGrav timing analyses establish that the common process and HD correlations survive detailed pulsar noise, clock, and ephemeris modeling. The compact KDE comparison then focuses on the following physics question: given the HD-correlated free-spectrum product, which source spectra best fit its shape?

PTA datasets can also exhibit chromatic and non-stationary phenomena such as scattering-related processes and transient chromatic dips. Direct timing-residual extensions benefit from per-pulsar flexibility to capture such effects; Figure 5 shows representative validation products for such a workflow.

Single continuous-wave searches and anisotropy analyses remain complementary tests of the stochastic background interpretation.

6. Discussion

The analysis above turns the NANOGrav 15-year free spectrum into a source discrimination measurement. The public free-spectrum shape moves decisively beyond the canonical fixed-slope SMBHB description as the best compressed spectral model. Among astrophysical interpretations, the leading explanation in our model set is an SMBHB background with environmental low-frequency turnover; among cosmological interpretations, a first-order phase transition is the strongest challenger. In this section, we explain why the environmental reading is physically compelling and how cosmic strings or phase transitions can be sharpened into testable alternatives.

6.1. Consistency with Astrophysical Predictions

The amplitude $A_{\text{GWB}} \sim 2 \times 10^{-15}$ at f_{yr} falls squarely within the expected SMBHB regime. Earlier studies [9] suggested a plausible range from $A_{\text{GWB}} \sim 10^{-16}$ to 10^{-15} , with more recent population models including higher-redshift contributions and updated black hole–galaxy-scaling relations allowing amplitudes up to a few $\times 10^{-15}$ [10]. The observed amplitude points to an active massive-binary population, and the turnover traces how binaries interact with stars, gas, and galactic structure before gravitational radiation takes over.

PTAs are beginning to measure massive-binary astrophysics at parsec scales. In stellar-dynamical language, SMBHBs embedded in triaxial or axisymmetric galactic nuclei can sustain loss-cone refilling and continue to harden down to GW-driven separations [27,28]. The measured background amplitude is naturally aligned with this picture. The PTA signal supports a population in which angular momentum transport operates efficiently in massive galaxy mergers, with the low-frequency bend carrying information about the environment that drives the binaries through the final parsec regime.

6.2. Turnover-to-Density Mapping

The environmental turnover model makes this statement quantitative. Propagating the preferred bend-frequency samples through the stellar-hardening balance gives a representative density estimate $\log_{10}(\rho_{\star}/M_{\odot}\text{pc}^{-3}) = 5.16^{+0.62}_{-0.80}$, consistent with dense galactic nuclei and with the turnover-to-density interpretation emphasized by Chen et al. [11]. This is the main astrophysical contribution of the paper: PTA spectral curvature is translated into a model-calibrated estimate of parsec-scale nuclear environments. The same result connects three observables that are often discussed separately—the common HD-correlated background, the low-frequency spectral bend, and the density scale needed to harden SMBHBs.

The mapping is obtained by equating the stellar-hardening shrinkage rate to the GW-driven shrinkage rate at the separation corresponding to the bend frequency. For a circular binary with total mass $M = m_1 + m_2$,

$$a_b = \left(\frac{GM}{\pi^2 f_b^2} \right)^{1/3}, \quad (18)$$

and the hardening balance gives

$$\rho_\star = \frac{64 G^2 m_1 m_2 M \sigma}{5 c^5 H a_b^5}, \quad (19)$$

where σ is the stellar velocity dispersion, quoted in km s^{-1} in Table 6, and H is the dimensionless stellar-hardening coefficient. In the propagation used for Figure 8, f_b is drawn from the SMBHB-env posterior, while the binary total mass, mass ratio, velocity dispersion, and H are marginalized over astrophysical hyperpriors centered on massive galaxy nuclei. Table 6 lists the hyperpriors used in the released calculation. The quoted density uncertainty combines the bend-frequency posterior width with the population-level spread in binary and nuclear parameters. The resulting $\rho_\star$ distribution is therefore a model-dependent estimate, not a direct measurement of nuclear density; it depends on the assumed binary-evolution channel, circular-binary mapping, hardening coefficient, and population priors.

Table 6. Astrophysical hyperpriors used to propagate the SMBHB-env bend-frequency posterior to the nuclear-density estimate in Figure 8.

Quantity	Prior/Treatment	Role in Equation (19)
$\log_{10}(M/M_\odot)$	Normal(9.3, 0.3 ²)	Total binary mass
$q = m_2/m_1$	Uniform[0.25, 1]	Mass ratio for $m_1 m_2 M$
σ	Normal(200, 30 ²) km s^{-1} , clipped at 50 km s^{-1}	Stellar velocity dispersion
H	Normal(15, 5 ²), clipped at 5	Stellar-hardening coefficient
e	fixed circular	Matches Equation (19)

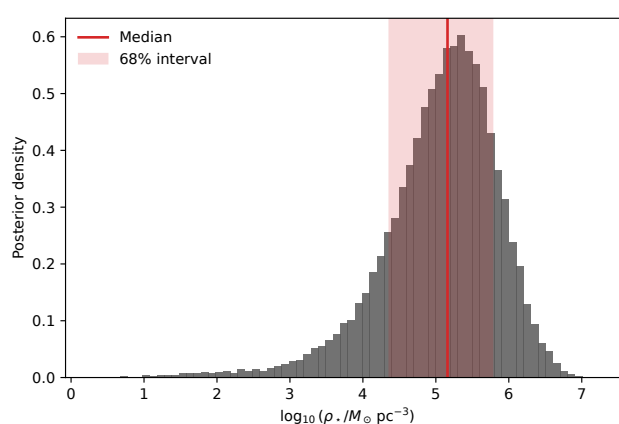


Figure 8. Posterior distribution of the stellar density $\rho_\star$ implied by the SMBHB environmental turnover samples. The median and central credible interval connect the PTA bend frequency to parsec-scale nuclear environments.

The environmental interpretation also gives a direct route to future tests. An astrophysical background should eventually show signatures of a discrete source population: mild anisotropy from local large-scale structure and, at higher frequencies in the PTA band,

possible continuous-wave outliers from the brightest SMBHBs. A primordial background should remain closer to statistically isotropic and Gaussian as sensitivity improves.

6.3. Viability of Cosmic Strings

Cosmic strings remain an important symmetry-breaking interpretation because they connect PTA data to high-energy topological defects. In the baseline comparison, the simple amplitude–slope spectrum gives only a small gain over strict SMBHB-PL. This statement is prior- and template-specific. The baseline prior $\beta \in [-1, -1/2]$ was chosen to represent broad PTA-band strain slopes expected for standard loop-dominated spectra in the PTA range, with the amplitude treated as an order-of-magnitude proxy for $G\mu$. Table 2 shows that widening the slope prior to $\beta \in [-1.2, -0.3]$ increases the string proxy evidence. The result should therefore not be read as a general exclusion of cosmic strings.

The useful conclusion is more specific: an amplitude–slope proxy is not the most efficient way to explain the present HD free-spectrum curvature. Structured string networks naturally contain additional scales, and stable strings, metastable strings, cosmic superstrings, or nonstandard loop populations can differ substantially in their PTA-band curvature and multi-band continuation [13,29]. A strong string case should combine PTA-band curvature, consistency with LISA and ground-based limits, and viable particle-physics parameters. Table 1 shows that the simplest effective-slope spectrum is weak under the baseline prior, while Table 2 shows the size of the prior dependence.

6.4. Potential Early-Universe Phase Transition

A first-order phase transition around the QCD scale could produce gravitational waves in the PTA band. In the compressed free-spectrum comparison, the phase transition template is the strongest cosmological competitor, with a Bayes factor of 139.5 against the strict SMBHB power law under the baseline prior. The reason is physical: a peaked Ω_{gw} spectrum supplies the same nanohertz scale that appears as curvature in the public free spectrum. This makes the phase transition interpretation the sharpest cosmological target for end-to-end timing-likelihood tests and future PTA data. Because the Standard Model QCD transition is a crossover rather than a first-order transition, a confirmed phase transition reading would point directly to beyond-Standard-Model dynamics near the QCD scale or in a nearby hidden sector.

A slow, strong first-order transition can also produce primordial black holes if delayed false-vacuum regions collapse [12]; QCD-era softening can likewise enhance solar-mass PBH formation [30]. Thus, a PTA phase transition interpretation should be checked against microlensing, CMB, and LIGO/Virgo/KAGRA population constraints.

Another check is cosmological chronology. A QCD-scale transition with $T_* \sim 100$ MeV corresponds to a redshift of order $z \sim 10^{11}–10^{12}$, using the present CMB temperature as the scale reference. This is before standard big bang nucleosynthesis and far before photon decoupling at $z \simeq 1100$. Hidden-sector scenarios can change the relation between the visible temperature and the gravitational-wave peak, but they must still be checked against BBN and CMB constraints through their energy density, entropy injection, and any coupling to the visible sector. Thus, gravitational waves may be the cleanest probe of such a transition, but consistency with early-Universe thermal history remains a required part of the interpretation.

The test is conceptually clean. A phase transition spectrum should reveal a peak in Ω_{gw} , with the energy density falling above the peak according to Equation (11); an environmental SMBHB spectrum should instead show a smoother turnover tied to binary hardening. Better sensitivity across the lowest and highest PTA Fourier bins will separate these cases. A QCD-scale transition would peak in the nanohertz band and be strongly

suppressed by the millihertz LISA band; a much higher-scale transition would peak at much higher frequencies and PTA would see only the low-frequency side, for which Equation (12) gives $h_c \propto f^{1/2}$ for the causal $a = 3$ energy density rise. Thus, if the PTA signal is ultimately interpreted as a phase transition, the frequency scale points to QCD-scale or nearby hidden-sector physics rather than generic high-scale new physics.

Other cosmological sources considered by NANOGrav, including inflationary backgrounds, scalar-induced gravitational waves, and domain walls, can also generate structure in Ω_{gw} [15]. They are outside the baseline model set analyzed here, but the logic is the same: each source must explain the nanohertz scale after the HD angular sector has already fixed the tensor correlation class. The conclusion is therefore a productive hierarchy within the tested templates: phase transitions are the leading cosmological challenger to the environmental SMBHB interpretation, and broader new-physics models can be compared by adding their predicted spectral scale to the same evidence framework.

Across these interpretations, symmetry organizes the inference. The current evidence supports an HD-correlated, approximately isotropic tensor background. The Bayesian source comparison then asks whether the spectrum is most naturally explained by astrophysical environmental coupling, by a first-order symmetry-breaking transition, or by a topological defect network.

6.5. Future Prospects for Discrimination

To distinguish environmental SMBHB curvature from a cosmological peak, future data will need to provide the following:

1. **Sharper GWB spectra:** Longer baselines and more sensitive arrays, including MeerKAT and eventually the SKA, should distinguish a smooth environmental turnover from a narrower cosmological peak [31].
2. **Individual sources or anisotropy:** Continuous waves or anisotropy would support an SMBHB population, whereas persistent Gaussianity and isotropy at higher sensitivity would increase pressure on the astrophysical interpretation [15].
3. **Multi-band observations:** LISA and ground-based searches can test whether a PTA-motivated cosmological spectrum extrapolates consistently beyond the nanohertz band [29,32].
4. **Astrophysical cross-correlations:** If SMBHBs are responsible, the amplitude and turnover scale should connect to merger rates, binary demographics, and nuclear-density distributions.
5. **Additional dark-sector transitions:** The same compressed-evidence machinery can be applied to other dark-sector scenarios once a predictive present-day strain spectrum is available. Examples include SU(2)-motivated dark-sector depercolation and ultralight axion-condensate lump scenarios [33,34]; these are outside the present quantitative ranking because a PTA-band stochastic spectrum has not yet been specified.

The current free-spectrum evidence establishes a non-canonical spectral structure and ranks the tested source classes. Environmental SMBHB hardening supports the leading astrophysical explanation because it maps the bend to plausible parsec-scale nuclear densities, while the phase transition template is the strongest cosmological competitor. PTA astronomy has therefore moved from detection to characterization: the angular sector fixes the HD tensor correlation class, and the spectral sector measures the physical scale that source models must explain.

7. Anisotropy and Continuous-Wave Implications

An astrophysical background formed by a discrete population of SMBHBs is expected to exhibit mild statistical anisotropy at a sufficiently high signal-to-noise ratio, whereas

a primordial cosmological background should be closer to isotropic. Future anisotropy searches should test for departures from the statistically isotropic tensor correlation structure that gives the HD template its simple angular form. NANOGrav's 15-year anisotropy search reports no significant excess beyond the isotropic HD expectation and shows how constraints tighten as more well-timed pulsars are added [35]. Cosmic variance remains part of this problem because an individual PTA realization can show apparent anisotropy even if the underlying background is statistically isotropic [36].

Bayesian anisotropy searches in space-based detectors address the same statistical problem at a different frequency: define angular modes for a stochastic background, evaluate the likelihood for those modes, and compare isotropic and anisotropic hypotheses by evidence. Map-making formalisms, Bayesian spherical harmonic searches, and recent LISA anisotropy studies give the millihertz counterpart [37–41]. In the present PTA analysis, the public NANOGrav KDE product supplies the spectral factor: an HD-correlated, frequency-resolved compression that pairs naturally with angular basis parameters in future PTA products.

The division of labor is direct. LISA supplies angular map-making power because its time-dependent response provides directional modulation. The present PTA calculation supplies the nanohertz source identification factor: it starts from the observed HD-correlated signal and a public frequency-by-frequency spectral likelihood. Its evidence ratios test whether the nanohertz spectrum requires a physical scale and which source family supplies that scale. Together, the two analyses form a joint evidence program: a PTA spectral factor for the detected nanohertz HD component and a LISA angular response factor for millihertz anisotropy and continuation tests.

This difference is clearest at the level of the evidence integral. In the present PTA calculation, the HD angular sector has already been selected by the timing analysis, so the evidence marginalizes over source-spectrum parameters:

$$\mathcal{Z}_{\text{PTA}}(\mathcal{M}_{\text{spec}}) = \int d\theta_{\text{spec}} \pi(\theta_{\text{spec}} | \mathcal{M}_{\text{spec}}) \prod_{k=1}^{30} \frac{\hat{p}_k[\log_{10} \rho_k(\theta_{\text{spec}})]}{\pi_{\text{fs},k}[\log_{10} \rho_k(\theta_{\text{spec}})]}. \quad (20)$$

By contrast, a LISA anisotropy calculation marginalizes over angular response modes together with any spectral parameters:

$$\mathcal{Z}_{\text{LISA}}(\mathcal{M}_{\text{ang}}) = \int d\theta_{\text{spec}} da_{\ell m} \pi(\theta_{\text{spec}}, a_{\ell m} | \mathcal{M}_{\text{ang}}) \mathcal{L}_{\text{LISA}}(\tilde{\mathbf{d}} | \Omega_{\text{gw}}(f; \theta_{\text{spec}}), a_{\ell m}; R_{\text{LISA}}). \quad (21)$$

Equations (20) and (21) separate the two likelihood factors. The PTA evidence is conditional on the observed HD-correlated nanohertz component and directly tests source-scale spectral structure. The LISA evidence uses detector response modulation to test the angular structure at millihertz frequencies. Their product connects the nanohertz source scale selected here with the millihertz angular response measured via LISA-type analyses.

The same decomposition gives a direct PTA route from the present spectral calculation to angular symmetry tests. The public KDE product supplies the spectral factor, and Equation (22) specifies the angular basis parameters for a joint spectral-angular PTA evidence calculation. In the isotropic sector, the angular kernel is the HD overlap function, and the source information enters through $S_r(f_k)$. An anisotropic extension promotes the angular kernel to

$$\Gamma_{ab}(f_k) = \Gamma_{ab}^{\text{HD}} + \sum_{\ell \geq 1} \sum_{m=-\ell}^{\ell} a_{\ell m}(f_k) \Gamma_{ab}^{\ell m}, \quad (22)$$

where $\Gamma_{ab}^{\ell m}$ are PTA response basis functions for sky multipoles. Setting $a_{\ell m} = 0$ recovers the isotropic HD sector used here; allowing band-independent $a_{\ell m}$ tests angular anisotropy; al-

lowing $a_{\ell m}(f_k)$ tests the frequency-dependent angular structure. The Bayesian evidence calculation then defines direct comparisons among $\mathcal{M}_{\text{HD}}$, $\mathcal{M}_{\text{aniso}}$, and joint spectral-angular source models with the same logic used above for the spectral curvature.

This yields concrete evidence targets: an anisotropy Bayes factor at a fixed spectral model, a frequency-dependent angular-structure test using $a_{\ell m}(f_k)$, and source-conditioned comparisons in which SMBHB, phase transition, or string spectra are paired with angular priors. Thus, the present framework supplies the spectral factor for a joint spectral-angular analysis.

Frequency and detector geometry set the complementarity: PTAs provide decade-long nanohertz baselines and HD angular correlations, while LISA provides millihertz response maps and access to higher angular modes. In a joint program, PTA evidence identifies the nanohertz scale and LISA evidence tests high-frequency continuation or anisotropy. An environmental SMBHB turnover predicts a nanohertz bend tied to binary hardening and naturally leads to continuous-wave and anisotropy tests from nearby binaries. A QCD-scale phase transition interpretation predicts a nanohertz peak with little millihertz continuation, while higher-scale cosmological models make different LISA-band predictions. This is the operational link between the present source-spectrum evidence calculation and LISA-type anisotropy searches. Table 7 summarizes this division of roles.

We quantify this link with an orbit- and foreground-aware LISA continuation forecast. We propagate the phase transition and effective cosmic-string posterior spectra over 10^{-4} – 10^{-1} Hz and pass them through a four-year equal-arm LISA A/E/T time-delay-interferometry response model [32,42–44]. The calculation includes the Robson–Cornish–Liu instrument noise, the four-year unresolved galactic foreground residual in the A/E noise budget, and a rigid cartwheel sampling of the LISA orbital response. The A and E channels define the network continuation statistic; the T response serves as a null-channel diagnostic. For each posterior draw, we evaluate

$$\mathcal{S}_{\text{TDI}}(\theta) = \left[T_{\text{obs}} \sum_{I=A,E} \int_{10^{-4}\text{Hz}}^{10^{-1}\text{Hz}} \left(\frac{\Omega_{\text{gw}}(f; \theta)}{\Omega_{\text{n},I}^{\text{LISA}}(f)} \right)^2 df \right]^{1/2}, \quad (23)$$

with $T_{\text{obs}} = 4$ yr and with $\Omega_{\text{n},I}^{\text{LISA}}$ including the foreground residual for the A/E channels. This statistic is a posterior-predictive continuation filter: it tests whether a nanohertz-favored spectrum remains visible in the LISA band after the detector response, foreground residuals, and independent A/E channels are included.

The same orbit calculation gives a compact sky-map Fisher forecast. For a real low-multipole sky intensity basis $Y_i(\hat{n})$, we form the fractional response template

$$q_i^I(t) = \frac{\int d\hat{n} R_I(t, \hat{n}) Y_i(\hat{n})}{\int d\hat{n} R_I(t, \hat{n})}, \quad F_{ij}^{\text{sky}}(\theta) = \mathcal{S}_{\text{TDI}}^2(\theta) \left\langle q_i^I(t) q_j^I(t) \right\rangle_{t,I=A,E}, \quad (24)$$

where $R_I(t, \hat{n})$ is the time-dependent equal-arm response. The forecast then becomes a direct map-sensitivity test: if the PTA-favored spectrum continues into the LISA band, the Fisher matrix gives the measurable low-multipole angular structure.

Table 8 reports the resulting posterior probabilities and quadrupole-map sensitivity. The QCD-scale phase transition posterior selected by the PTA data is strongly suppressed in the millihertz band, with a foreground-aware median TDI-network signal-to-noise ratio of 2.6×10^{-5} and $P(\mathcal{S}_{\text{TDI}} > 5) = 0$. It produces no usable LISA sky map. The broad cosmic string proxy gives the opposite result: if extrapolated unchanged as a broad-band power law, every posterior draw is LISA-bright even after the foreground residual is included, and a 10% quadrupolar anisotropy is detected in every draw. This fixes the physical requirement

for string interpretations: a viable string model must combine PTA-band curvature with a structured multi-band spectrum consistent with LISA and higher-frequency constraints. Figure 9 displays the corresponding anisotropy and multi-band forecasts.

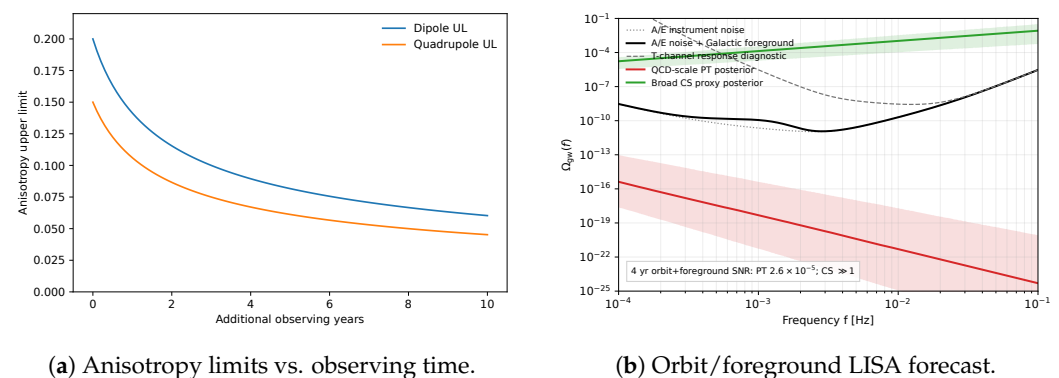


Figure 9. Anisotropy and LISA-band continuation implications. **Left:** Projected improvement of low-multipole anisotropy constraints with additional observation time. **Right:** Posterior propagation of the PTA-favored QCD-scale phase transition template and the broad cosmic string proxy through a four-year orbit- and foreground-aware LISA TDI response model; the black curve includes the unresolved galactic foreground residual, and the dotted curve shows the instrument-only reference.

Table 7. Comparison between the present PTA evidence calculation and Bayesian LISA anisotropy searches.

Aspect	Present PTA Analysis	LISA Anisotropy Framework
Frequency band	Nanohertz PTA band, set by decade-long pulsar timing baselines	Millihertz band, set by space-interferometer arm length and orbital response
Angular information	HD overlap correlations among pulsar pairs fix the isotropic tensor correlation class	Spherical harmonic sky modes describe isotropic and anisotropic stochastic backgrounds
Main data object	Public NANOGrav HD free-spectrum KDE product	LISA stochastic background likelihood for angular response modes
Likelihood factor	Product of one-dimensional HD spectral KDE factors, Equation (20)	Detector response likelihood over angular modes, Equation (21)
Evidence question	Which source spectrum best explains the HD-correlated free-spectrum shape?	Which angular model best explains the detector response to a stochastic background?
Natural extension	Promote the HD kernel to Equation (22) and compare isotropic, anisotropic, and joint spectral-angular models	Infer spherical-harmonic coefficients through the time-dependent LISA response
Spectral role	Spectral curvature is the main discriminator after the HD sector is fixed	Spectrum and angular structure are inferred through the LISA response model
Factor boundary	Supplies the HD spectral compression used for source-scale inference	Supplies the response map factor for angular and millihertz continuation inference
Physical complementarity	Identifies nanohertz source scales such as f_b or f_{peak}	Tests millihertz continuation, angular resolution, and higher-mode structure

Table 8. Orbit- and foreground-aware LISA posterior predictive forecast for the broad-band cosmological templates. The sky-map column reports the median one-sigma uncertainty for a real quadrupole coefficient in the low-multipole Fisher calculation.

Model	Median $\mathcal{S}_{\text{TDI}}$	$P(\mathcal{S}_{\text{TDI}} > 5)$	Median $\sigma_{a_{2m}}$	$P(10\% Q > 3\sigma)$	Implication
QCD-scale phase transition	2.6×10^{-5}	0	1.3×10^5	0	No millihertz continuation for the PTA-selected peak.
Broad cosmic string proxy	2.7×10^{10}	1.00	1.2×10^{-10}	1.00	LISA-bright if extrapolated; viable strings need structure.

Quasi-monochromatic continuous-wave signals from individual massive binaries provide the companion SMBHB probe. Joint analyses of background and continuous-wave channels will sharpen constraints on SMBHB demographics and the background’s origin.

8. Conclusions

We presented a source discrimination analysis for the nanohertz gravitational-wave background on the NANOGrav’s 15-year PTA data. The inference has two separate layers. The official timing analyses establish the common process and the HD angular-correlation sector. The public HD free-spectrum product then allows a source-spectrum evidence calculation. Under this compressed likelihood, the strict fixed $f^{-2/3}$ SMBHB law fails to explain the observed curvature as well as scale-carrying spectra, and the data favor models with a characteristic nanohertz scale.

The evidence ranking and the physical interpretation are distinct. Among the tested physical templates, the first-order phase transition model gives the largest compressed spectral evidence and is the leading cosmological competitor. Among astrophysical explanations, environmental SMBHB hardening is the leading interpretation because it explains much of the same curvature through binary-environment coupling and maps the bend frequency to parsec-scale nuclear stellar densities. The simple cosmic string effective-slope proxy is weak under the baseline prior, but the prior-sensitivity test keeps structured string network spectra in play and calls for direct templates.

This framework gives the nanohertz spectral anchor for a PTA–LISA evidence program. It uses the observed HD-correlated nanohertz spectrum to identify source-scale spectral structure through conditional evidences over source parameters. The LISA anisotropy searches add the matching response map factor over angular coefficients and detector modulation at millihertz frequencies. With orbit modulation and galactic foreground residuals included, the PTA-selected QCD-scale phase transition posterior has no appreciable LISA continuation, whereas a direct broad-band string extrapolation is LISA-bright and strongly constrained by sky-map information. Any durable physical interpretation must pass both filters: PTA data can be used to identify the nanohertz scale and source family, and LISA or other probes to test the angular structure, multi-band continuation, or absence of such continuation. Symmetry fixes the angular tensor correlation class; the spectrum supplies the physical scale that separates astrophysical environmental hardening from early-Universe symmetry-breaking scenarios.

Author Contributions: Conceptualization, H.X. and Y.G.; methodology, H.X. and W.Z.; software, H.X.; validation, H.X. and W.Z.; formal analysis, H.X.; investigation, H.X. and W.Z.; resources, Y.G.; data curation, H.X.; writing—original draft preparation, H.X.; writing—review and editing, W.Z. and Y.G.; visualization, H.X.; supervision, Y.G.; project administration, H.X. and Y.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The NANOGrav 15-year timing data, version 2.1.0, are publicly available from the NANOGrav Collaboration through Zenodo [26]. The free-spectrum likelihood inputs used for the primary analysis and robustness checks are the public NANOGrav KDE representations of the 15-year free spectra, specifically the corrected version 1.1.0 archive [23], whose construction follows the rapid-refitting methodology of Lamb, Taylor, and van Haasteren [24]. The analysis used Python 3.11.14, NumPy 2.3.4, SciPy 1.16.3, dynesty 3.0.0, enterprise-pulsar 3.4.4, enterprise_extensions 3.0.3, PTMCMCSampler 2.1.4, and Matplotlib 3.10.7; the complete environment is recorded in `repro/environment.yml`. The data-only reproducibility archive for this manuscript is available at <https://github.com/HKUST-AARON/nanohertz-gwb-model-comparison-repro> (accessed on 7 July 2026), with the current data-only release archived on GitHub at <https://github.com/HKUST-AARON/nanohertz-gwb-model-comparison-repro/releases/tag/v1.0.5-data-only> (accessed on 7 July 2026). The Zenodo all-versions DOI for the archive series is <https://doi.org/10.5281/zenodo.20581821>. The archive contains `data_sources/` for public inputs, `analysis/` for scripts, `analysis_outputs/` for compressed-KDE JSON tables, posterior samples, and orbit/foreground LISA sky-map forecast outputs, and `repro/` for the environment and run commands supporting the numerical results in this manuscript.

Conflicts of Interest: The authors declare no conflicts of interest.

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