

Article

The Pmmm QCD Condensate Lattice: Nominal Wyckoff Occupation as the Ground State and Topological Defects as the Geometric Origin of Particle Excitations

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Abstract

We propose a lattice structure and space group symmetry, Pmmm (No. 47), for the QCD condensate ground state, whose Wyckoff positions are occupied by the four light quarks and antiquarks u , d , $\bar{u}$, $\bar{d}$. These serve as the fundamental building blocks of both the condensate lattice ground state and the baryonic and leptonic particle excitations embedded within it as topological defects of the nominal Wyckoff occupation, offering a more structured alternative to the QCD instanton liquid picture. Building on Bloch quark wave solutions of a tight-binding Hamiltonian defined on this lattice, we propose a generalization of Einstein's Equivalence Principle: composite particles embedded in the lattice and propagating by tunnelling cannot distinguish acceleration by gravity, the strong, weak, or electromagnetic forces, or curvature of the lattice itself, arising from local variation in unit cell shape. We derive an eight-by-eight tight-binding Hamiltonian that decouples into two four-by-four blocks separating the quark and antiquark sectors. Electrons, positrons, protons, neutrons, deuterons, and α -particles are embedded in the lattice as defect-induced deviations from the nominal Wyckoff occupation, with their spin and helicity emerging structurally from this picture. We further propose that the lattice's unit cells carry a small nonzero rest mass, whose collective gravitational effect across a galactic halo may account for the discrepancy between visible mass and rotation curves, identifying the Pmmm condensate as a dark matter candidate. Finally, we outline a mechanism near black hole horizons by which local melting of the condensate lattice followed by quark reactions that conserve the number and flavor of the quarks could yield a new route to baryon asymmetry. We propose a framework that goes several steps beyond the Standard Model by introducing a Pmmm space group unit cell for the QCD condensate ground state, built from the four light quarks and antiquarks u , d , $\bar{u}$, $\bar{d}$. We further propose that topological defects of the Pmmm condensate lattice are the geometric origin of particle excitations.

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1. Introduction

According to instanton liquid theory [1], the QCD condensate is described as an interacting liquid of instantons and anti-instantons with an estimated density of one instan-

ton per cubic femtometer. The ground state of QCD is a dense state composed of gauge fields and quarks, in which instantons act as potential wells where light quarks can form bound states. Quarks can travel over large distances by hopping from one instanton to another, in close analogy with electron motion in a solid-state conductor. The chiral condensate is connected to the density of states of the Dirac operator near zero energy through the Banks–Casher relation [2], which can in turn be related to the zero-energy bound states of quarks in the instanton liquid potential wells. Is the instanton liquid massless?

Erwin Schrödinger introduced the concept of a wave-packet particle and showed that a Gaussian wave packet remains coherent within a harmonic potential. However, Schrödinger and Darwin [3] demonstrated that in free space the wave packet disperses: the width of a Gaussian wave packet grows rapidly with time. If an electron wave packet is initially localized on the scale of atomic dimensions ($\sim 10^{-10}$ m), its width doubles in approximately 10^{-16} s, and after roughly one millisecond it expands to the order of a kilometer, a physically unreasonable result for a point-like particle [4]. This raises a fundamental question: is the Schrödinger equation invalid for free electrons, or is our understanding of empty space itself incorrect, as Dirac suggested in his lecture on the electron and the vacuum [5]? If the latter is true, can the QCD condensate provide the physical substrate that prevents dispersion and sustains coherent electron wave-packet evolution in free space?

Quark masses are fundamental parameters of the QCD Lagrangian that cannot be measured directly, since quarks are permanently confined inside hadrons. Lattice QCD provides the framework for their determination by discretizing spacetime onto a four-dimensional Euclidean grid with spacing $a \sim 0.05\text{--}0.1$ fm, replacing the QCD path integral with a finite-dimensional sum amenable to Monte Carlo evaluation. The light quark masses m_u and m_d are extracted by tuning the bare lattice parameters until simulated pseudoscalar meson masses reproduce their experimentally measured values, exploiting the leading-order chiral perturbation theory relation $M_\pi^2 \propto (m_u + m_d)\Lambda_{\text{QCD}}$. Separating m_u and m_d further requires incorporating isospin-breaking effects from both QCD and QED, typically through joint QCD+QED simulations or dispersive input from $\eta \rightarrow 3\pi$ decays. According to the Flavor Lattice Averaging Group (FLAG) 2021 review [6], $m_u^{\overline{\text{MS}}}(2\text{ GeV}) = 2.14 \pm 0.08$ MeV and $m_d^{\overline{\text{MS}}}(2\text{ GeV}) = 4.70 \pm 0.12$ MeV, with the mass splitting $m_d - m_u \simeq 2.56\text{ MeV}$ established at the few-percent level. The d and u quarks are light but not massless and their mass can be determined by lattice QCD computations. Dark matter is the unseen mass, inferred from gravitational effects such as galactic rotation curves and gravitational lensing, that is believed to make up roughly five-sixths of the matter in the universe yet has never been directly detected as a particle. Is the QCD condensate massless and can it play the role of dark matter?

If the QCD condensate lattice carries even an extremely small nonzero rest mass per unit cell, set by the energy gap at the Γ point of the tight-binding dispersion derived in Section 2, the astronomically large number of unit cells filling a galactic halo could produce a collective gravitational effect sufficient to account for the observed discrepancy between visible mass and galactic rotation curves, without requiring any new particle species beyond the light quarks already known to exist. We return to this estimate, of order eV/c^2 , in Section 9.

The same lattice structure also offers a structural origin for the internal quantum numbers of embedded leptons. Electrons and positrons embedded in the QCD condensate propagate by tunnelling between Wyckoff positions, and through rapid underlying quark permutations they form self-confined clouds rather than bare free particles. We propose that their spin and helicity states are encoded by this tunnelling structure itself: spin is determined by whether the tunnelling carrier is a quark or an antiquark defect, while helicity depends on whether the motion of the larger quark charge is parallel or antiparallel to the motion of the embedded electron charge. This picture is developed in Section 4.

Finally, we propose that near black hole horizons, where the QCD condensate lattice melts locally, Pionic tetraquarks may react to produce deuteron–antideuteron pairs, which in turn react with further Pionic tetraquarks to yield protons, electrons, and anti-neutrons. If the antineutrons preferentially fall beneath the event horizon while protons and electrons escape as Hawking radiation, this offers a new mechanism for Baryon asymmetry, discussed in Section 8.

Bloch quark waves and the QCD condensate $Pmmm$ space group tight-binding k -space hopping Hamiltonian and its energy bands are presented in Section 2. The QCD condensate lattice equivalence principle is described in Section 3. Topological defects as the origin of particle excitations, embedded electrons and positrons in the QCD condensate lattice, are discussed in Section 4. Section 5 addresses the embedded Cooper pair, while Section 6 focuses on electron–positron creation and annihilation within the condensate. Topological defects as the origin of particle excitations, embedded protons, neutrons, deuterons, and α -particles, are described in Section 7. An alternative model for Baryon asymmetry, the matter–antimatter separation, is proposed in Section 8. Section 9 presents a discussion.

2. The QCD Condensate Lattice $Pmmm$ Space Group Ground State

We assume that the four light quarks and antiquarks $u, d, \bar{u}, \bar{d}$ are the basic constituents of the QCD vacuum [7], condensing into a periodic lattice with $Pmmm$ space group symmetry and simultaneously serving as the fundamental components of both baryonic and leptonic matter, an assignment reminiscent of Harari’s four-sub-particle $T, V, \bar{T}, \bar{V}$ model [8]. The resulting unit cell is illustrated in Figure 1.

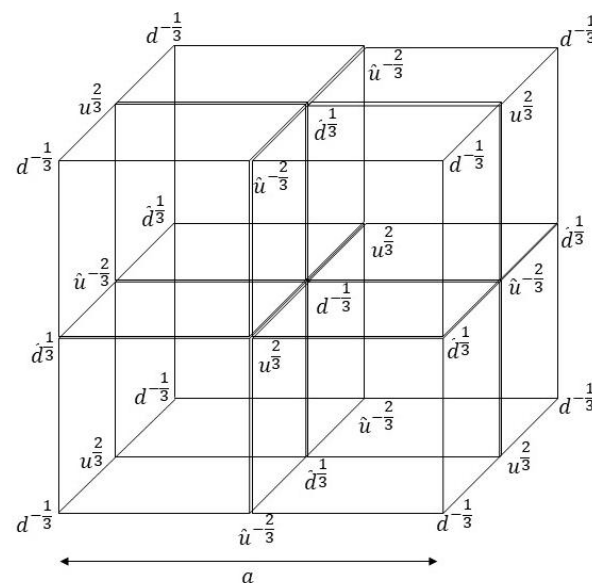


Figure 1. Illustration of the QCD condensate lattice $Pmmm$ space group unit cell.

The unit cell has $Pmmm$ space group symmetry (No. 47, point group D_{2h} , order 8), which is a subgroup of the $Pm\bar{3}m$ space group (No. 221, point group O_h , order 48). We assume that the $Pmmm$ space group of the QCD condensate arises from a spontaneous symmetry breaking of the parent $Pm\bar{3}m$ space group: the four distinct light quarks and antiquarks occupy different Wyckoff positions, thereby breaking the full cubic O_h symmetry down to D_{2h} . This is consistent with the prediction of QCD theory that the QCD vacuum breaks chiral symmetry [9]. The $Pmmm$ space group is adopted as an ansatz, motivated by two physical assumptions about the QCD ground state. First, only the two lightest, stable quark flavors, u and d , are taken to populate the ground state condensate, consistent with

the standard picture of the QCD vacuum as a chiral condensate, $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle + \langle \bar{d}d \rangle$, whose nonvanishing value drives spontaneous chiral symmetry breaking and gives rise to the pion as its (pseudo-)Goldstone boson [10]. Since the ground state may accordingly be understood as a condensate of pions [11], built exclusively from u , d , $\bar{u}$, $\bar{d}$, its elementary constituents are restricted to these four species. Second, the number of quarks and antiquarks in the ground state must be equal, since the ground state unit cell should be electrically neutral and QCD colorless. A cubic lattice is the natural starting point for a free, flat-space ground state with no preferred direction; however, requiring that the four species occupy structurally equivalent sublattices, with none privileged over the others, forces a specific ordered arrangement that reduces the cubic symmetry to the orthorhombic $Pmmm$ space group (point group D_{2h} , No. 47). The physical distinctions between u and d quarks, their different masses and electric charges, break the cubic space group symmetry, resulting in the proposed $Pmmm$ space group ansatz illustrated in Figure 1.

Since the QCD condensate is modelled as a lattice in which the light quarks and antiquarks occupy the $Pmmm$ -symmetric Wyckoff positions described above, the QCD vacuum wavefunction can be expressed as a Bloch quark spinor, and a tight-binding Hamiltonian model can be applied. The $Pmmm$ space group symmetry of the condensate lattice gives rise to a highly symmetric eight-by-eight Hamiltonian in k -space, which decouples into two four-by-four block Hamiltonians separating the quark and antiquark sectors upon enforcing symmetry-allowed tunnelling selection rules within the nearest-neighbor approximation. Each block can be solved analytically, yielding eight energy bands in closed form.

We assume that the light quarks and antiquarks, as the fundamental elementary constituents of the QCD condensate lattice ground state, are absolutely stable and do not decay, and that their total number and flavor content are conserved within the condensate, including during the formation, propagation, and annihilation of the embedded particle excitations discussed below. This is a cornerstone hypothesis of the present model, departing from the standard treatment of current quarks in the QCD vacuum, where quark–antiquark pairs are continuously created and annihilated as quantum fluctuations; we do not claim the present hypothesis to be established by, or generally consistent with, that picture. Rather, it is restricted to the low-energy regime addressed by this manuscript, in which the $Pmmm$ condensate lattice is assumed to be stable. We propose that at sufficiently high energies, for example, in supernova explosions, near black hole horizons, or in high-energy collider environments, the assumed condensate lattice melts, the strict low-energy conservation of quark number and flavor may no longer apply, and heavier quark flavors absent from the low-energy condensate may appear. A consequence of the low-energy stability hypothesis is the need to account for the cosmological scarcity of antimatter; Section 8 proposes that antiquarks are not absent but sequestered below the event horizons of black holes, through a Hawking radiation process in the vicinity of the event horizon that separates matter from antimatter particles, with antimatter preferentially falling inward while matter escapes as radiation.

The Wyckoff positions of the quark and antiquark in the $Pmmm$ space group with a unit cell length a are: d quark: $(0,0,0)$ $(a,0,0)$ $(0,a,0)$ $(0,0,a)$ $(0,a,a)$ $(a,a,0)$ $(a,0,a)$ (a,a,a) $(a/2,a/2,a/2)$; $\bar{d}$ antiquark: $(0,0,a/2)$ $(a,0,a/2)$ $(0,a,a/2)$ $(a,a,a/2)$ $(a/2,a/2,0)$ $(a/2,a/2,a)$; u quark: $(0,a/2,0)$ $(a,a/2,0)$ $(0,a/2,a)$ $(a,a/2,a)$ $(a/2,0,a/2)$ $(a/2,a,a/2)$; and $\bar{u}$ antiquark: $(a/2,0,0)$ $(a/2,a,0)$ $(a/2,0,a)$ $(a/2,a,a)$ $(0,a/2,a/2)$ $(a,a/2,a/2)$ summarized in Table 1 below. Accordingly, the $Pmmm$ space group unit cell Bloch quark spinor, where each spinor component is a Bloch wave with a periodic part located at the Wyckoff positions and a long-wavelength plane wave factor, is as follows:

$$\hat{\psi} = \begin{bmatrix} \psi_{d_{1a}}^{(0,0,0)} \\ \psi_{d_{1b}}^{(\frac{a}{2}, \frac{a}{2}, \frac{a}{2})} \\ \psi_{u_1}^{(\frac{a}{2}, 0, \frac{a}{2})} \\ \psi_{u_2}^{(0, \frac{a}{2}, 0)} \\ \psi_{d_{1a}}^{(0,0,\frac{a}{2})} \\ \psi_{d_{1b}}^{(\frac{a}{2}, \frac{a}{2}, 0)} \\ \psi_{\tilde{u}_1}^{(\frac{a}{2}, 0, 0)} \\ \psi_{\tilde{u}_1}^{(0, \frac{a}{2}, \frac{a}{2})} \end{bmatrix} \quad (1)$$

The *Pmmm* unit cell is assumed to be colorless, like a hadron. The description of quarks by Bloch waves is not obvious and might appear to contradict the short-range nature of the strong force and quark confinement. However, within the present model, quarks and antiquarks can only exchange positions within the QCD condensate lattice and can therefore never separate and exit the condensate lattice structure, consistent with the established confinement of quarks in QCD [12]. Confinement is thus enforced by the *Pmmm* condensate lattice structure. The melting temperature of the QCD condensate lattice is assumed to be extremely high, with melting occurring only under the extreme conditions of supernova explosions or ultra-high-energy heavy-ion collisions. A key advantage of the proposed tight-binding Hamiltonian model is that the interaction potential between quarks, the strong-force potential, does not need to be specified explicitly. The only input parameters of the model are the quark self-energies ε_d and ε_u , the hopping amplitude t_{du} (for both the quark and antiquark sectors), the unit cell length a , and the Wyckoff positions of the *Pmmm* space group.

Table 1. Wyckoff positions of the four quarks in the *Pmmm* unit cell.

Quark Flavor	Wyckoff Position	Representative Position	Site Symmetry	Per Cell
d	1a + 1b	(0,0,0) and (a/2, a/2, a/2)	Oh (order 48)	2
$\tilde{d}$	3d (z-direction)	(a/2, a/2, 0)	D4h (order 16)	2
u	3c (xz-direction)	(a/2, 0, a/2)	D4h (order 16)	2
$\tilde{u}$	3d (x-direction)	(a/2, 0, 0)	D4h (order 16)	2
Total				8

The *Pmmm* unit cell contains eight quark/antiquark positions per primitive cell: $2d + 2\tilde{u} + 2u + 2\tilde{d}$.

Color confinement is not derived within the present model but is assumed as a precondition, consistent with the standard understanding that only color-singlet combinations of quarks and gluons can exist as physical, observable states [13]. The tight-binding Hamiltonian formalism described below operates at a coarser level of description than the underlying SU(3) color dynamics: as is common practice in condensed-matter tight-binding models, where microscopic interaction physics is absorbed into effective on-site energies and hopping amplitudes rather than written out explicitly, the strong-force dynamics responsible for color confinement is absorbed here into the hopping amplitudes and on-site energies of the model, rather than introduced through explicit color degrees of free-

dom or SU(3) gauge potentials. Color is accordingly not represented explicitly, but is assumed to be saturated: each unit cell of the Pmmm condensate lattice, and the condensate lattice as a whole, is taken to be a colorless object, in direct analogy with a hadron having vanishing net color charge despite being composed of colored quarks.

2.1. The QCD Vacuum Tight-Binding Hamiltonian Model

The tight-binding Hamiltonian [14] in a second-quantized form is written as

$$H = H_{on-site} + H_{hop} \quad (2)$$

The on-site part assigns self-energy ϵ_α to each quark flavor $\alpha \in \{d, u, \bar{d}, \bar{u}\}$, representing the effective single-particle energy at its Wyckoff position:

$$H_{on-site} = \epsilon_d \sum C_{d,R,i}^\dagger C_{d,R,i} + \epsilon_u \sum C_{u,R,j}^\dagger C_{u,R,j} + \epsilon_{\bar{d}} \sum C_{\bar{d},R,k}^\dagger C_{\bar{d},R,k} + \epsilon_{\bar{u}} \sum C_{\bar{u},R,l}^\dagger C_{\bar{u},R,l} \quad (3)$$

where R labels the primitive cell index and i, j label the two inequivalent sites of each species within the cell. The index $i = 1$ refers to the Wyckoff 1a-type site (corner-derived) and $i = 2$ to the Wyckoff 1b-type site (body-center-derived) for the d quark; similarly, $j = 1$ refers to the xz-face-center site and $j = 2$ to the y-edge-midpoint site for the u quark and similarly the k and l indexes represent the two positions of the $\bar{d}$ and $\bar{u}$ antiquarks.

The hopping part of the Hamiltonian describes the amplitude for a quark to tunnel from one Wyckoff site to a neighboring Wyckoff site in the lattice:

$$H_{hop} = -\sum_{R,R'} \sum_{\alpha\beta} t_{\alpha\beta}(R-R') C_{\alpha,R}^\dagger C_{\beta,R'} + \text{h.c.} \quad (4)$$

where $t_{\alpha\beta}(R-R')$ is the hopping integral from species β at cell R' to species α at cell R . The hopping integrals are determined by the quark flavors and the overlap of the quark wavefunctions at the two sites, which depend on both the distance and the allowed hopping selection rules described below.

2.2. The Hopping Selection Rules Separating the Quark and Antiquark Sectors

We assume that the only allowed hoppings are $d \leftrightarrow u$ (quark sector hopping) and $\bar{d} \leftrightarrow \bar{u}$ (antiquark sector hopping), where both d and u quarks and $\bar{d}$ and $\bar{u}$ antiquark numbers are conserved in the quark exchange tunnelling/hopping. The enforced selection rules decouple the tight-binding hopping Hamiltonian model and couple the d and u quarks as required by the nonzero vacuum expectation value (VEV) in the QCD ground state.

t_{du} : $d \leftrightarrow u$ hopping amplitude (quark sector)

$t_{\bar{d}\bar{u}}$: $\bar{d} \leftrightarrow \bar{u}$ hopping amplitude (antiquark sector)

All other hopping amplitudes, $t_{d\bar{u}}, t_{d\bar{d}}, t_{u\bar{u}}, t_{u\bar{d}}, t_{dd}, t_{uu}, t_{\bar{d}\bar{d}}, t_{\bar{u}\bar{u}}$, are assumed to vanish, producing an eight-by-eight block-diagonal Hamiltonian model:

$$H = H_q \oplus H_{\bar{q}} \quad (5)$$

where H_q operates in the $\{d, u\}$ quark subspace and $H_{\bar{q}}$ operates in the $\{\bar{d}, \bar{u}\}$ antiquark subspace.

2.3. Structure Factor Calculation with the Nearest-Neighbors Approximation

According to the Bloch theorem [15], the quark eigenstates of a periodic Hamiltonian can be written as Bloch waves:

$$|k, \alpha\rangle = \frac{1}{\sqrt{N}} \sum_R e^{ik \cdot (R + \tau\alpha)} C^\dagger(\alpha_R) |0\rangle \quad (6)$$

where R is the Bravais lattice vector of primitive cell R , $\tau\alpha$ is the position of site α within the lattice cell, and N is the total number of cells. In the Bloch basis the hopping Hamiltonian matrix element between species α and β is

$$H_{\alpha\beta} = \langle k, \alpha | H | k, \beta \rangle = \sum_{\Delta R} t_{\alpha\beta}(\Delta R) e^{ik(\Delta R + \tau\beta - \tau\alpha)} \quad (7)$$

where $\Delta R = R' - R$ is the lattice vector difference. In the nearest-neighbor approximation, only the shortest displacement vectors $\delta = \Delta R + \tau\beta - \tau\alpha$ contribute, and the hopping amplitude $t_{\alpha\beta}$ is assumed equal for equivalent neighbors. The Hamiltonian matrix element then simplifies to the structure factor:

$$H_{\alpha\beta}(k) = f_{\alpha\beta}(k) = t_{\alpha\beta} \sum_{\delta} e^{ik\delta} \quad (8)$$

where the sum runs over all nearest-neighbor displacement vectors δ connecting a Wyckoff site β to Wyckoff site α . For the $Pmmm$ centrosymmetric lattice, the structure factors are real, because every hop vector δ is accompanied by its inversion-related symmetric partner $-\delta$, and $e^{ik\delta} + e^{-ik\delta} = 2 \cos(k\delta)$ is real. For the $Pmmm$ lattice the cosine factors appear with arguments $\frac{ka}{2}$ because the nearest-neighbor vectors between Wyckoff sites are of magnitude $\frac{a}{2}$, half the lattice constant. cx , cy and cz are defined as

$$cx(k_x) = \cos(k_x a/2), \quad cy(k_y) = \cos(k_y a/2) \quad \text{and} \quad cz(k_z) = \cos(k_z a/2) \quad (9)$$

2.3.1. Structure Factor B(k): d ↔ u Hopping via the xz Face-Diagonal

Site d_{1a} at Wyckoff 1a: $\tau_{d_{1a}} = (0,0,0)$.

Site u_1 at Wyckoff 3c: $\tau_{u_1} = (\frac{a}{2}, 0, \frac{a}{2})$ (xz face-center representative).

Both sites belong to the same primitive cell $R = 0$. The intra-cell displacement vector from d_{1a} to u_1 is

$$\tau_{u_1} - \tau_{d_{1a}} = \left(\frac{a}{2}, 0, \frac{a}{2}\right) - (0,0,0) = \left(\frac{a}{2}, 0, \frac{a}{2}\right)$$

There are four equivalent displacement vectors:

$$\begin{aligned} n_1 = 0, \quad n_3 = 0: \quad \delta &= \left(+\frac{a}{2}, 0, +\frac{a}{2}\right), & |\delta| &= a/\sqrt{2} \\ n_1 = -1, \quad n_3 = 0: \quad \delta &= \left(-\frac{a}{2}, 0, +\frac{a}{2}\right), & |\delta| &= a/\sqrt{2} \\ n_1 = 0, \quad n_3 = -1: \quad \delta &= \left(+\frac{a}{2}, 0, -\frac{a}{2}\right), & |\delta| &= a/\sqrt{2} \\ n_1 = -1, \quad n_3 = -1: \quad \delta &= \left(-\frac{a}{2}, 0, -\frac{a}{2}\right), & |\delta| &= a/\sqrt{2} \end{aligned}$$

The four displacement vectors lie in the xz plane ($y = 0$ component), reflecting the xz-type character of the 3c Wyckoff position occupied by u_1 .

The structure factor $B(k)$ is the sum $e^{ik\delta}$ over all four nearest-neighbor vectors, multiplied by the hopping amplitude t_{du} :

$$\begin{aligned} B(k_x, k_z) &= t_{du} \left[e^{\left(\frac{ik_x a}{2} + \frac{ik_z a}{2}\right)} + e^{\left(-\frac{ik_x a}{2} + \frac{ik_z a}{2}\right)} + e^{\left(\frac{ik_x a}{2} - \frac{ik_z a}{2}\right)} + e^{\left(-\frac{ik_x a}{2} - \frac{ik_z a}{2}\right)} \right] \\ B(k_x, k_z) &= t_{du} \left[e^{\left(\frac{ik_x a}{2}\right)} + e^{\left(-\frac{ik_x a}{2}\right)} \right] \left[e^{\left(\frac{ik_z a}{2}\right)} + e^{\left(-\frac{ik_z a}{2}\right)} \right] \\ B(k_x, k_z) &= 4t_{du} \cos(k_x a/2) \cos(k_z a/2) = 4t_{du} cx \, cz \end{aligned} \quad (10)$$

2.3.2. Structure Factor C(k): d ↔ u Hopping via the y-Direction

Site d_{1a} at Wyckoff 1a: $\tau_{d_{1a}} = (0,0,0)$.

Site u_2 at Wyckoff 3c: $\tau_{u_2} = (0, \frac{a}{2}, 0)$ (y-edge-midpoint representative).

$$\tau_{u_2} - \tau_{d_{1a}} = \left(0, \frac{a}{2}, 0\right) - (0,0,0) = \left(0, \frac{a}{2}, 0\right)$$

There are two equivalent displacement vectors:

$$\begin{aligned}
n_1=0, n_2=0, n_3=0: \quad \delta &= \left(0, +\frac{a}{2}, 0\right), \quad |\delta| = a/2 \\
n_1=0, n_2=-1, n_3=0: \quad \delta &= \left(0, -\frac{a}{2}, 0\right), \quad |\delta| = a/2 \\
C(k_y) &= t_{du} \left[e^{\left(\frac{ik_y a}{2}\right)} + e^{\left(-\frac{ik_y a}{2}\right)} \right] = 2 t_{du} \cos(k_y a/2) = 2 t_{du} c_y \quad (11)
\end{aligned}$$

2.3.3. Structure Factor F: $\tilde{d}_1 \leftrightarrow \tilde{u}_1$ Hopping via the xy Face-Diagonal

Site $\tilde{d}_1$ is at $(a/2, a/2, 0)$ (xy-face center). Site $\tilde{u}_1$ is at $(a/2, 0, 0)$ (x-edge midpoint). The nearest-neighbor displacement vectors connecting these two site types lie in the xy plane:

$$\delta = (\pm a/2, \pm a/2, 0) \quad 4 \text{ xy face-diagonal vectors}$$

There are four equivalent displacement vectors:

The Fourier sum similar to the $B(k_x, k_z)$ calculation above gives

$$F(k_x, k_y) = 4 t_{\tilde{d}\tilde{u}} c_x c_y$$

Note that the structure factor F involves c_x and c_y the xy face-diagonal in contrast to the quark sector B factor which involves c_x and c_z . This xy vs. xz distinction is the direct geometric consequence of the antiquark species occupying a different set of Wyckoff positions.

2.3.4. Structure Factor G: $\tilde{d}_1 \leftrightarrow \tilde{u}_2$ Hopping via the z-Direction

Site $\tilde{u}_2$ is at $(0, a/2, a/2)$ (yz-face center). The displacement vectors from $\tilde{d}_1$ at $(a/2, a/2, 0)$ to $\tilde{u}_2$ involve z-direction hops:

There are two equivalent displacement vectors:

$$\delta = (0, 0, \pm a/2) \quad 2 \text{ z-direction vectors}$$

$$G(k_z) = 2 t_{\tilde{d}\tilde{u}} c_z \quad (12)$$

The antiquark G factor involves c_z alone corresponding to z-direction hopping while the quark sector C factor involves c_y alone. The substitution $y \leftrightarrow z$ relative to the quark sector reflects the axis permutation in the Pmmm unit cell between the quark and antiquark Wyckoff positions.

2.3.5. The Inversion-Symmetry Cross-Coupling Pattern

The structure factors for d_{1b} are obtained by the same Fourier sum, but starting from $\tau_{d_{1b}} = (a/2, a/2, a/2)$ instead of $\tau_{d_{1a}} = (0, 0, 0)$. The key result derivable by an identical calculation is that the roles of B and C are crossed for d_{1b} compared to the d_{1a} position:

$$d_{1a} \leftrightarrow u_1: \quad \text{structure factor B} = 4 t_{du} c_x c_z$$

$$d_{1a} \leftrightarrow u_2: \quad \text{structure factor C} = 2 t_{du} c_y$$

$$d_{1b} \leftrightarrow u_1: \quad \text{structure factor C} = 2 t_{du} c_y$$

$$d_{1b} \leftrightarrow u_2: \quad \text{structure factor B} = 4 t_{du} c_x c_z$$

This inversion symmetry is due to the point group symmetry [16]: the inversion operation maps $(x, y, z) \rightarrow (-x, -y, -z)$, which maps d_{1a} at $(0, 0, 0) \leftrightarrow d_{1b}$ at $(a/2, a/2, a/2)$ (related by inversion through the sub-cell center $(a/4, a/4, a/4)$), and simultaneously maps u_1 at $(a/2, 0, a/2) \leftrightarrow u_2$ at $(0, a/2, 0)$. Inversion therefore swaps both the d-site labels and the u-site labels, crossing the coupling assignments. This crossed pattern symmetry produces the distinctive off-diagonal block structure of $\mathbf{H}_q(\vec{k})$ and the inversion symmetry generates further the symmetric/antisymmetric band splitting $B \pm C$ that appears in the energy bands.

2.4. The Decoupled Quark Block $\mathbf{H}_q$ and Antiquark Block $\mathbf{H}_{\bar{q}}$

Assembling the structure factors into the 4×4 quark block, with basis ordering $|d_{1a}\rangle$, $|d_{1b}\rangle$, $|u_1\rangle$ and $|u_2\rangle$:

$$\mathbf{H}_q(\vec{k}) = \begin{vmatrix} \varepsilon_d & 0 & 4t_{du}cx\,cz & 2t_{du}cy \\ 0 & \varepsilon_d & 2t_{du}cy & 4t_{du}cx\,cz \\ 4t_{du}cx\,cz & 2t_{du}cy & \varepsilon_u & 0 \\ 2t_{du}cy & 4t_{du}cx\,cz & 0 & \varepsilon_u \end{vmatrix} \quad (13)$$

The antiquark sector is derived analogously, but the geometric roles of the coordinate axes are permuted. The $\bar{d}$ antiquark occupies the z-direction 3d position while $\bar{u}$ occupies the x-direction 3d position, as shown in Figure 1. This axis permutation produces a different pair of structure factors. The structure factors with basis ordering $|\bar{d}_1\rangle$, $|\bar{d}_2\rangle$, $|\bar{u}_1\rangle$ and $|\bar{u}_2\rangle$:

$$\mathbf{H}_{\bar{q}}(\vec{k}) = \begin{vmatrix} \varepsilon_{\bar{d}} & 0 & 4t_{\bar{d}\bar{u}}cx\,cy & 2t_{\bar{d}\bar{u}}cz \\ 0 & \varepsilon_{\bar{d}} & 2t_{\bar{d}\bar{u}}cz & 4t_{\bar{d}\bar{u}}cx\,cy \\ 4t_{\bar{d}\bar{u}}cx\,cy & 2t_{\bar{d}\bar{u}}cz & \varepsilon_{\bar{u}} & 0 \\ 2t_{\bar{d}\bar{u}}cz & 4t_{\bar{d}\bar{u}}cx\,cy & 0 & \varepsilon_{\bar{u}} \end{vmatrix} \quad (14)$$

2.5. The Eight Energy Bands—Diagonalizing the Hamiltonian Blocks

Each 4×4 Hamiltonian (Equations (13) and (14)) is symmetric and can be diagonalized analytically by a unitary transformation, exploiting the $Z_2 \times Z_2$ symmetry of the block: both $\mathbf{H}_q$ and $\mathbf{H}_{\bar{q}}$ are invariant under the simultaneous exchange $d_{1a} \leftrightarrow d_{1b}$ combined with $u_1 \leftrightarrow u_2$ (or $\bar{d}_1 \leftrightarrow \bar{d}_2$ combined with $\bar{u}_1 \leftrightarrow \bar{u}_2$). This symmetry is generated by the Pmmm inversion operation, which maps each Wyckoff site to its inversion-related partner.

The unitary symmetric (+) and antisymmetric (−) transformations are

$$|d^+\rangle = (|d_{1a}\rangle + |d_{1b}\rangle) / \sqrt{2}$$

$$|d^-\rangle = (|d_{1a}\rangle - |d_{1b}\rangle) / \sqrt{2}$$

$$|u^+\rangle = (|u_1\rangle + |u_2\rangle) / \sqrt{2}$$

$$|u^-\rangle = (|u_1\rangle - |u_2\rangle) / \sqrt{2}$$

In the transformation's symmetric and antisymmetric bases, the 4×4 quark block becomes block-diagonal after the unitary transformation $\mathbf{H}_q' = \mathbf{U} \mathbf{H}_q \mathbf{U}^\dagger$, separating into two independent 2×2 matrices, H_+ and H_- :

$$\mathbf{H}_q'(\vec{k}) = \begin{vmatrix} \varepsilon_d & 0 & 4t_{du}cx\,cz + 2t_{du}cy & 0 \\ 0 & \varepsilon_d & 0 & 4t_{du}cx\,cz - 2t_{du}cy \\ 4t_{du}cx\,cz + 2t_{du}cy & 0 & \varepsilon_u & 0 \\ 0 & 4t_{du}cx\,cz - 2t_{du}cy & 0 & \varepsilon_u \end{vmatrix} \quad (15)$$

2.5.1. Symmetric Quark Block ($|d^+\rangle$, $|u^+\rangle$)

$$H_+ = \begin{vmatrix} \varepsilon_d & 4t_{du}cx\,cz + 2t_{du}cy \\ 4t_{du}cx\,cz + 2t_{du}cy & \varepsilon_u \end{vmatrix} \quad (16)$$

The off-diagonal element $B + C = 4t_{du}cx\,cz + 2t_{du}cy = 2t_{du}(2cx\,cz + cy)$.

The eigenvalues of the 2 × 2 H_+ symmetric matrix are

$$E_{1,2} = \frac{\varepsilon_d + \varepsilon_u}{2} \pm \sqrt{\left(\frac{\varepsilon_d - \varepsilon_u}{2}\right)^2 + (2t_{du})^2(2cx\,cz + cy)^2} \quad (17)$$

2.5.2. Antisymmetric Quark Block ($|d^-\rangle$, $|u^-\rangle$)

$$H_- = \begin{vmatrix} \varepsilon_d & 4t_{du}cx\,cz - 2t_{du}cy \\ 4t_{du}cx\,cz - 2t_{du}cy & \varepsilon_u \end{vmatrix} \quad (18)$$

The off-diagonal element is $B - C = 4t_{du}cx\,cz - 2t_{du}cy = 2t_{du}(2cx\,cz - cy)$.

The eigenvalues of the 2×2 H_- antisymmetric matrix are

$$E_{3,4} = \frac{\varepsilon_d + \varepsilon_u}{2} - \sqrt{\left(\frac{\varepsilon_d - \varepsilon_u}{2}\right)^2 + (2t_{du})^2(2cx\,cz - cy)^2} \quad (19)$$

2.5.3. Antiquark Bands

The same procedure applied to $\mathbf{H}_{\bar{q}}$ with symmetric/antisymmetric combinations of $|\bar{d}^+\rangle$, $|\bar{d}^-\rangle$, $|\bar{u}^+\rangle$ and $|\bar{u}^-\rangle$ gives

Off-diagonal element $F + G = 4t_{\bar{d}\bar{u}}cx\,cy + 2t_{\bar{d}\bar{u}}cz = 2t_{\bar{d}\bar{u}}(2cx\,cy + cz)$:

$$E_{5,6} = \frac{\varepsilon_{\bar{d}} + \varepsilon_{\bar{u}}}{2} - \sqrt{\left(\frac{\varepsilon_{\bar{d}} - \varepsilon_{\bar{u}}}{2}\right)^2 + (2t_{\bar{d}\bar{u}})^2(2cx\,cy + cz)^2} \quad (20)$$

Off-diagonal element $F - G = 4t_{\bar{d}\bar{u}}cx\,cy - 2t_{\bar{d}\bar{u}}cz = 2t_{\bar{d}\bar{u}}(2cx\,cy - cz)$:

$$E_{7,8} = \frac{\varepsilon_{\bar{d}} + \varepsilon_{\bar{u}}}{2} - \sqrt{\left(\frac{\varepsilon_{\bar{d}} - \varepsilon_{\bar{u}}}{2}\right)^2 + (2t_{\bar{d}\bar{u}})^2(2cx\,cy - cz)^2} \quad (21)$$

In summary, the k-space tight-binding Hamiltonian model is

$$H(\vec{k}) = \begin{vmatrix} \varepsilon_d & 0 & 4t_{du}cx\,cz & 2t_{du}cy & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_d & 2t_{du}cy & 4t_{du}cx\,cz & 0 & 0 & 0 & 0 \\ 4t_{du}cx\,cz & 2t_{du}cy & \varepsilon_u & 0 & 0 & 0 & 0 & 0 \\ 2t_{du}cy & 4t_{du}cx\,cz & 0 & \varepsilon_u & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{\bar{d}} & 0 & 4t_{\bar{d}\bar{u}}cx\,cy & 2t_{\bar{d}\bar{u}}cz \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_{\bar{d}} & 2t_{\bar{d}\bar{u}}cz & 4t_{\bar{d}\bar{u}}cx\,cy \\ 0 & 0 & 0 & 0 & 4t_{\bar{d}\bar{u}}cx\,cy & 2t_{\bar{d}\bar{u}}cz & \varepsilon_{\bar{u}} & 0 \\ 0 & 0 & 0 & 0 & 2t_{\bar{d}\bar{u}}cz & 4t_{\bar{d}\bar{u}}cx\,cy & 0 & \varepsilon_{\bar{u}} \end{vmatrix} \quad (22a)$$

And the eight energy bands are

$$E_{1,2} = \frac{\varepsilon_d + \varepsilon_u}{2} + \sqrt{\left(\frac{\varepsilon_d - \varepsilon_u}{2}\right)^2 + (2t_{du})^2(2cx\,cz + cy)^2} \quad (22b)$$

$$E_{3,4} = \frac{\varepsilon_d + \varepsilon_u}{2} - \sqrt{\left(\frac{\varepsilon_d - \varepsilon_u}{2}\right)^2 + (2t_{du})^2(2cx\,cz - cy)^2} \quad (22c)$$

$$E_{5,6} = \frac{\varepsilon_{\bar{d}} + \varepsilon_{\bar{u}}}{2} + \sqrt{\left(\frac{\varepsilon_{\bar{d}} - \varepsilon_{\bar{u}}}{2}\right)^2 + (2t_{\bar{d}\bar{u}})^2(2cx\,cy + cz)^2} \quad (22d)$$

$$E_{7,8} = \frac{\varepsilon_{\bar{d}} + \varepsilon_{\bar{u}}}{2} - \sqrt{\left(\frac{\varepsilon_{\bar{d}} - \varepsilon_{\bar{u}}}{2}\right)^2 + (2t_{\bar{d}\bar{u}})^2(2cx\,cy - cz)^2} \quad (22e)$$

2.5.4. K-Space Γ , X, R High-Symmetry Point Behavior—The QCD Lattice Unit Cell Mass

At k-space Γ point ($k_x = k_y = k_z = 0$): $cx = cy = cz = 1$, giving $B + C = 6t_{du}$ and $B - C = 2t_{du}$. The symmetric and antisymmetric bands are split by $4t_{du}$, which may represent the QCD condensate lattice unit cell mass $4t_{du} = m_{cell}c^2$. The QCD lattice is not massless, and by the universality of free fall established for all massive bodies [17], the QCD condensate lattice should fall in a gravitational field like any massive body.

At the k-space X point ($k_x = \pi/a$, $k_y = k_z = 0$): $cx = 0$, $cy = cz = 1$. Then $B = 0$ and $C = 2t_{du}$, giving $B + C = B - C = C$. The symmetric and antisymmetric bands are degenerate at the X point.

At the k-space R point ($k = (\pi/a, \pi/a, \pi/a)$): $cx = cy = cz = 0$. All structure factors vanish: $B = C = F = G = 0$. All eight bands collapse to the four on-site energies, with complete decoupling of all sites at the Brillouin zone corner. There is an asymptotic freedom limit with no coupling between quarks.

The inversion symmetry of the Pmmm space group permits simultaneous symmetric position exchanges of the d and u quarks, correlated with corresponding exchanges of

the $\tilde{d}$ and $\tilde{u}$ antiquarks, which can be viewed as a symmetric wave propagating through the QCD condensate lattice. The QCD condensate lattice can carry momentum and transmit forces over distances far exceeding the nuclear scale of 1 fm via Bloch waves.

3. The QCD Condensate Lattice Equivalence Principle

The cosine terms of the QCD tight-binding Hamiltonian, which multiply the hopping amplitude t_{du} , contain the product of the crystal momentum and the unit cell length inside the cosine argument. A representative example is

$$E_{3,4} = \frac{\varepsilon_d + \varepsilon_u}{2} \pm \sqrt{\left(\frac{\varepsilon_d - \varepsilon_u}{2}\right)^2 + 2t_{du}^2 \left(\cos \frac{k_x a}{2} \cos \frac{k_z a}{2} - \cos \frac{k_y a}{2}\right)^2}$$

Assuming the unit cell length $a(\mathbf{r})$ varies on a scale much larger than the femtometer lattice spacing, consistent with lattice QCD formulations in which the lattice geometry is made an explicit function of a curved background spacetime metric [18], the energy bands acquire a corresponding spatial variation. For instance, a gradual increase in the cell length along the x -axis may give rise to a force F_x proportional to the energy gradient:

$$F_x = -\frac{\partial E}{\partial x} = -\frac{\partial E}{\partial a} \frac{\partial a}{\partial x} \quad (23)$$

3.1. Approximate Dispersion and the Force on an Embedded Composite Particle

Assuming that the hopping amplitude t_{du} is small compared to half the on-site energy difference, $t_{du} \ll (\varepsilon_d - \varepsilon_u)/2$, the square root in the energy band expressions (Equation (22A–C)) can be expanded using $(1+x)^{1/2} \approx 1+x/2$ for $x \ll 1$. Applying this expansion to the antisymmetric quark block (Equation (22C)), the $E_{3,4}$ bands simplify to

$$E_3(k_x, k_y, k_z) \approx \varepsilon_d + \frac{\left[4t_{du} \left(\cos \frac{k_x a}{2} \cos \frac{k_z a}{2} - \cos \frac{k_y a}{2}\right)\right]^2}{\varepsilon_d - \varepsilon_u} \quad (24a)$$

$$E_4(k_x, k_y, k_z) \approx \varepsilon_u - \frac{\left[4t_{du} \left(\cos \frac{k_x a}{2} \cos \frac{k_z a}{2} - \cos \frac{k_y a}{2}\right)\right]^2}{\varepsilon_d - \varepsilon_u} \quad (24b)$$

Since $\varepsilon_d > \varepsilon_u$, the hopping pushes E_3 above ε_d and E_4 below ε_u for all k , with a bandwidth proportional to $t_{du}^2/(\varepsilon_d - \varepsilon_u)$.

3.2. The Energy Gradient and the Generalized Equivalence Principle

Following the standard semiclassical treatment of Bloch wave dynamics in a slowly varying environment [19], we derive an effective force from the gradient of the local tight-binding energy. Differentiating the expanded E_3 band with respect to the unit cell length a , and introducing the shorthand $c_i = \cos(k_i a/2)$ and $s_i = \sin(k_i a/2)$ for $i = x, y, z$, yields

$$\frac{\partial E_3}{\partial a} = \frac{-16 t_{du}^2 (c_x c_z - c_y)}{\varepsilon_d - \varepsilon_u} \cdot [k_x s_x c_z + k_z c_x s_z - k_y s_y] \left[\frac{\text{Joul}}{m} \right] \quad (25)$$

The force on the d quark Bloch wave along the x -direction is therefore

$$F_x = \frac{16 t_{du}^2 (c_x c_z - c_y)}{\varepsilon_d - \varepsilon_u} \cdot [k_x s_x c_z + k_z c_x s_z - k_y s_y] \cdot \frac{\partial a}{\partial x} \left[\frac{\text{Joul}}{m} \right] \quad (26)$$

In one-dimensional motion along the x -axis with $k_y = k_z = 0$, F_x is reduced to

$$F_x = -\frac{16 t_{du}^2 k_x \sin\left(\frac{k_x a}{2}\right) \left(1 - \cos \frac{k_x a}{2}\right)}{\varepsilon_d - \varepsilon_u} \cdot \frac{\partial a}{\partial x} \left[\frac{\text{Joul}}{m} \right] \quad (27)$$

Similar to the atmospheric barometric formula, the QCD condensate density in the gravitational field of the Milky Way galaxy is assumed to decrease with galactocentric radius r , with a Fermi-like function shown in Figure 2 below:

$$a(x) = \frac{a_s e^{\frac{G m_{\text{cell}} M_{\text{MW}} (x-x_0)}{k_B T}}}{1 + e^{\frac{G m_{\text{cell}} M_{\text{MW}} (x-x_0)}{k_B T}}} \quad [\text{m}] \quad (28)$$

$$\frac{da}{dx} = \frac{a_s m_{\text{cell}} G M_{\text{MW}} e^{\frac{m_{\text{cell}} G M_{\text{MW}} (x-x_0)}{k_B T}}}{k_B T \left(1 + e^{\frac{G m_{\text{cell}} M_{\text{MW}} (x-x_0)}{k_B T}} \right)^2} \quad (29)$$

The QCD condensate density and the unit cell length are related by

$$\rho(x) = m_{\text{cell}}/a(x)^3 \quad \left[\frac{\text{Kg}}{\text{m}^3} \right] \quad (30)$$

where m_{cell} is the mass of the QCD lattice unit cell. Assuming $x_0 = 100 \text{ kpc}$, $a_s = 0.25 \times 10^{-13} \text{ m}$, $T = 10^8 \text{ Kelvin}$ at the galaxy center, the values of the QCD lattice unit cell, m_{cell} , and the QCD condensate unit cell $a(x=0)$ are determined self-consistently by solving the coupled Equations (31) and (32). The density integral in Equation (32) uses the $a(x)$ expression from Equation (28). $M_{\text{DM}} = 1.2 \times 10^{12} M_{\odot}$ is the assumed missing Milky Way galaxy dark matter mass and $M_{\text{MW}} = 6.4 \times 10^{10} M_{\odot}$ is the baryonic mass [20].

$$m_{\text{cell}} = -\frac{k_B T \ln\left(\frac{a(x=0)}{a_s/2}\right)}{G M_{\text{MW}} x_0} \quad (31)$$

$$M_{\text{DM}} = 4\pi \int_0^{200 \text{ kpc}} x^2 m_{\text{cell}}/a(x)^3 dx = 1.2 \times 10^{12} M_{\odot} \quad [\text{Kg}] \quad (32)$$

The Newton gravitational force along the x-axis is

$$F_x = -\frac{G m_{\text{cell}} M_{\text{MW}}}{x^2} \quad \left[\frac{\text{Joul}}{\text{m}} \right] \quad (33)$$

Combining Equations (27) and (29) above gives

$$F_x = -\frac{16 t_{du}^2 k_x \sin\left(\frac{k_x a}{2}\right) \left(1 - \cos\frac{k_x a}{2}\right)}{\varepsilon_d - \varepsilon_u} \cdot \frac{a_s m_{\text{cell}} G M_{\text{MW}} e^{\frac{m_{\text{cell}} G M_{\text{MW}} (x-x_0)}{k_B T}}}{k_B T \left(1 + e^{\frac{G m_{\text{cell}} M_{\text{MW}} (x-x_0)}{k_B T}} \right)^2} \left[\frac{\text{Joul}}{\text{m}} \right] \quad (34)$$

According to the equivalence principle, Equations (33) and (34) should be equal, and this gives the following expression for $k_x^2(x)$, assuming $\frac{k_x a}{2} \ll 1$:

$$k_x^2(x) = \frac{(\varepsilon_d - \varepsilon_u) k_B T \left(1 + e^{\frac{G m_{\text{cell}} M_{\text{MW}} (x-x_0)}{k_B T}} \right)^2}{x^2 16 t_{du}^2 a(x) a_s e^{\frac{m_{\text{cell}} G M_{\text{MW}} (x-x_0)}{k_B T}}} \quad (35)$$

Figure 2 illustrates the unit cell length $a(x)$ for the full 200 kpc Milky Way galaxy range according to Equation (28) above.

The solution of the coupled Equations (31) and (32) above gives an extremely small mass to the QCD condensate $m_{\text{cell}} = 1.016 \times 10^{-36} m_e$, where m_e is the electron mass. Due to the huge volume of the Milky Way galaxy, the extremely small mass of the condensate generates the estimated dark matter huge mass given by Equation (32).

A generalization of Einstein's Equivalence Principle (EEP [17,21,22]) is: a composite particle made of quarks embedded in the QCD condensate lattice cannot distinguish whether it is being accelerated by gravity, by the strong force, by the weak force, by the electromagnetic force, or by a curvature of the QCD condensate lattice.

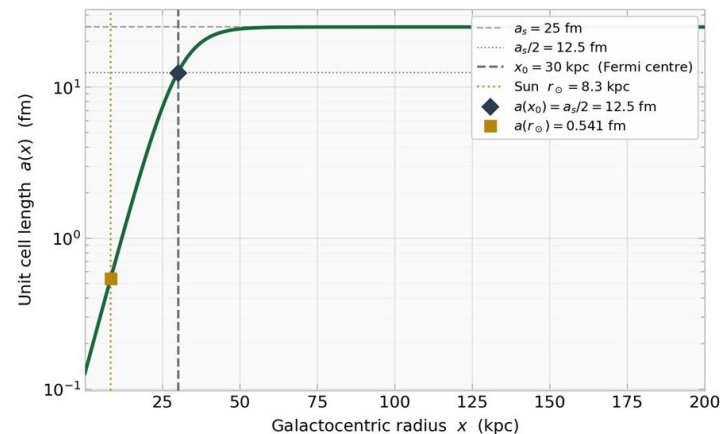


Figure 2. Illustration of the QCD condensate lattice unit cell length $a(x)$.

Figure 3 illustrates the QCD condensate density $\rho(x)$ of Equation (30).

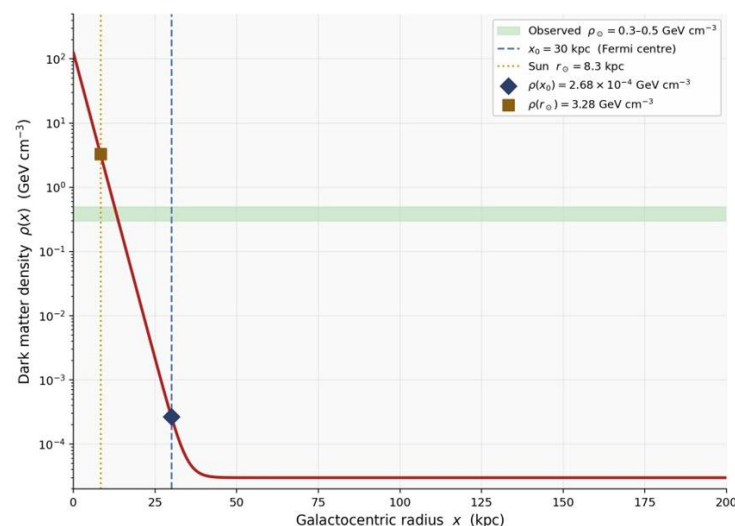


Figure 3. Illustration of the QCD condensate lattice density $\rho(x)$.

The following sections show how the QCD condensate lattice can accommodate embedded electrons, positrons, electron–positron pairs, Cooper pairs, protons, neutrons, deuterons, and α -particles, all of which, like quarks, are confined by the lattice structure. The physical motivation for treating these particles as quark-based composites embedded in the QCD condensate lattice comes from the kinematics of neutron β -decay, as described below.

4. Topological Defects as the Origin of Particle Excitations—Embedded Electrons and Positrons in the Pmmm QCD Condensate Lattice

Conversion between a QCD condensate unit cell, shown in Figure 1, and an embedded electron cell, shown below in Figure 4, requires the exchange of a u quark for a d quark. It is worth noting that, beyond Harari’s schematic model of quarks and leptons as composite particles [8], Davidson has recently proposed that leptons are composed of three pre-leptons carrying fractional charges at a more fundamental level [23].

The symmetric quark exchange reaction between the QCD lattice Pmmm cell and embedded electron cell at the QCD condensate lattice neighboring sites is described in Equation (36), in which a d quark and a u quark are exchanged. Since the reactants and products are identical, this reaction is analogous to a symmetric double-well potential

problem and can proceed by quantum tunnelling through the intervening potential barrier, in direct analogy with the inversion of the ammonia molecule [24].

$$Pmmm_cell_e(d) + Pmmm_cell(u) \rightarrow Pmmm_cell(u) + Pmmm_cell_e(d) \quad (36)$$

An analogous exchange of $\bar{d}$ and $\bar{u}$ antiquarks may take place in the QCD condensate lattice according to Equation (37), illustrated in the embedded electron Type II in Figure 5.

$$Pmmm_cell_e(\bar{u}) + Pmmm_cell(\bar{d}) \rightarrow Pmmm_cell(\bar{d}) + Pmmm_cell_e(\bar{u}) \quad (37)$$

The symmetric $d \leftrightarrow u$ quark exchange and $\bar{d} \leftrightarrow \bar{u}$ antiquark exchange reactions between the embedded electron cell and the Pmmm cell within the QCD condensate lattice can be interpreted as a hidden internal symmetry of the QCD condensate lattice. By Noether's theorem [25], this symmetry generates the conservation of the embedded electric charge and current. Specifically, electric charge and current are carried by the $d \leftrightarrow u$ quark exchange described in Equation (36) and the $\bar{d} \leftrightarrow \bar{u}$ antiquark exchange described in Equation (37), which can occur simultaneously within the proposed QCD condensate lattice after the topological defects, particle excitations, are created as illustrated in Figures 4 and 5 below and Figures 8–15 further below in next sections.

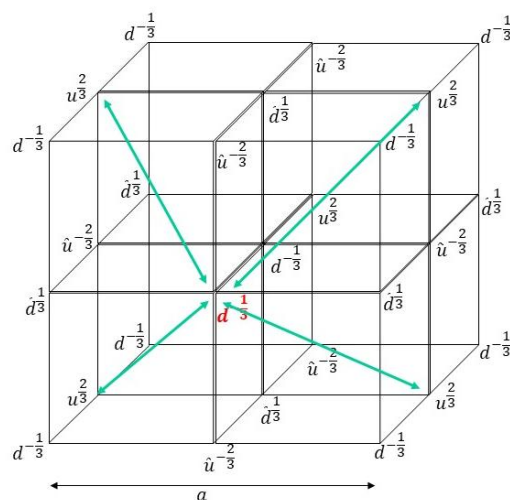


Figure 4. Illustration of embedded electron Type I in the QCD condensate unit cell.

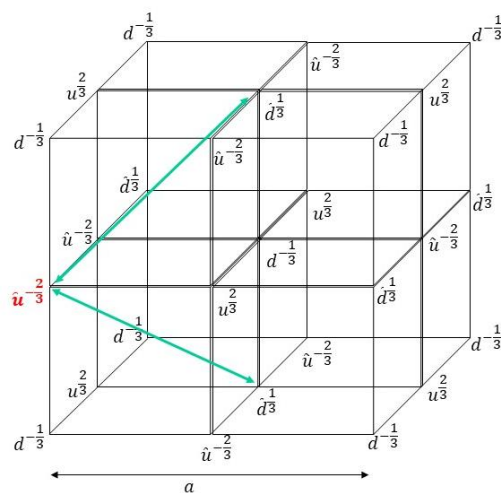
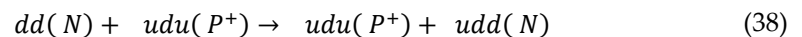


Figure 5. Illustration of embedded electron Type II in the QCD condensate unit cell.

It should also be noted that an analogous quark exchange reaction may occur within a deuterium nucleus, where d and u quarks are exchanged between the proton and the neutron, as shown in Equation (38).



Since the proposed QCD condensate lattice unit cell is built from Pionic tetraquarks, rapid quark permutations between embedded electron tetraquarks and neighboring Pionic tetraquarks in adjacent unit cells can occur. As a result, the embedded electron does not reside at a fixed location but instead forms a delocalized confined cloud within the QCD condensate lattice. The rapidity of these quark permutation reactions makes it impossible to determine the precise position of the composite embedded electron at any given instant, in full consistency with the Heisenberg uncertainty principle of quantum mechanics [26].

Embedded electrons and positrons can be generated within the Pmmm QCD condensate lattice by topological defects of the nominal Wyckoff positions substituting a quark or an antiquark at a specific high-symmetry Wyckoff position, as illustrated in the figures below. Figure 4 depicts the Type I embedded electron cell, formed by replacing a u quark with a d quark at Wyckoff site $(\frac{a}{2}, 0, \frac{a}{2})$ of the unit cell (the d quark is shown in Figure 4 with red color), which thereby acquires a net electric charge of -1 . The four green arrows in Figure 4 indicate the symmetry-allowed hoppings of the substituted d quark within the unit cell, connecting Wyckoff u quark site $(\frac{a}{2}, 0, \frac{a}{2})$, to four other u quark positions, $(0, \frac{a}{2}, 0)$, $(a, \frac{a}{2}, 0)$, $(0, \frac{a}{2}, a)$ and $(a, \frac{a}{2}, a)$.

The hopping of an embedded electron within the QCD condensate lattice, mediated by quark and antiquark exchanges, gives rise to two distinct electron types. In the Type I configuration, the negatively charged d quark (charge $-1/3$) hops in one direction while the positively charged u quark (charge $+2/3$) hops in the opposite direction. The relative directions of these hops determine the helicity of the embedded electron: in this case the larger positive charge hops antiparallel to the direction of the electron charge motion. The symmetry-allowed quark and antiquark hoppings between Wyckoff positions proceed by quantum tunnelling and preserve the quantum invariants of the embedded electron or positron, namely spin and helicity.

These quark permutations extend across adjacent unit cells of the Pmmm chiral condensate lattice, giving rise to a delocalized embedded electron confinement cloud. This is analogous to a self-trapped polaron state in a tight-binding lattice [27]: the electron's wavefunction is delocalized over several unit cells rather than a single site, while remaining genuinely bound, since its presence induces the self-curvature of the surrounding lattice that in turn confines it. The permutations are assumed to occur at the extremely high frequency of Dirac's Zitterbewegung [28]; as a consequence, the physical embedded electron is a dressed particle [5] and exists as a confined cloud sustained by the continuous underlying quark and antiquark motion within the QCD condensate lattice. Furthermore, the unit cell hosting the embedded electron, shown in Figure 4, may be locally curved so that its symmetry may differ from that of the neutral vacuum unit cell shown in Figure 1, which represents flat, charge-free empty space.

Figure 5 illustrates the Type II embedded electron cell, generated by substituting a $\bar{d}$ antiquark for a $\bar{u}$ antiquark at Wyckoff site $(0, 0, \frac{a}{2})$ (the $\bar{u}$ antiquark is shown in Figure 5 with red color). The associated permutation exchanges the $\bar{u}$ antiquark at Wyckoff site $(0, 0, \frac{a}{2})$ with the $\bar{d}$ antiquark at Wyckoff site $(\frac{a}{2}, \frac{a}{2}, 0)$: the negatively charged $\bar{u}$ antiquark (charge $-2/3$) hops from site $(0, 0, \frac{a}{2})$ to site $(\frac{a}{2}, \frac{a}{2}, 0)$, while simultaneously the positively charged $\bar{d}$ antiquark (charge $+1/3$) hops in the opposite direction, from site $(\frac{a}{2}, \frac{a}{2}, 0)$ to site $(0, 0, \frac{a}{2})$. The two green arrows in Figure 5 indicate the symmetry-allowed

hoppings of the substituted $\bar{u}$ antiquark within the unit cell. The combined motion of these two antiquarks generates the second electron helicity configuration, in which the direction of the net negative charge motion aligns with the direction of the embedded electron charge movement, in contrast to the Type I case, where the two directions are antiparallel.

Metropolis-Weighted Classical Random Walk in a Curved Condensate

The self-confinement of the embedded electrons is modelled by a Metropolis-weighted classical random walk [29,30] in a curved Pmmm QCD condensate lattice using a curved cell length model for $a(r)$ shown in Figure 6 below.

$$a(r) = \frac{a_s e^{\frac{\alpha(r-r_e)}{r_e}}}{1 + e^{\frac{\alpha(r-r_e)}{r_e}}} \quad [\text{m}] \quad (39)$$

The curved cell length $a(r)$ model encodes the self-induced curvature of the QCD condensate around the topological defect, the substituted quark at the Wyckoff position: it rises from $a(0) = 0.012\text{fm}$ at the origin, passes through its inflection point $a(r_e) = \frac{a_s}{2} = 0.125\text{ fm}$ at the classical electron radius $r_e = 2.82\text{fm}$ (Equation (40)), and saturates at the assumed unperturbed condensate spacing $a_s = 0.25\text{fm}$ far from the topological defect. The model introduces two parameters, $\alpha_e = 3$, the exponential slope that determines the $a(0)$ value, and r_e , the classical electron radius.

$$r_e = \frac{e^2}{4\pi \epsilon_0 m_e c^2} = 2.82 \text{ fm} \quad (40)$$

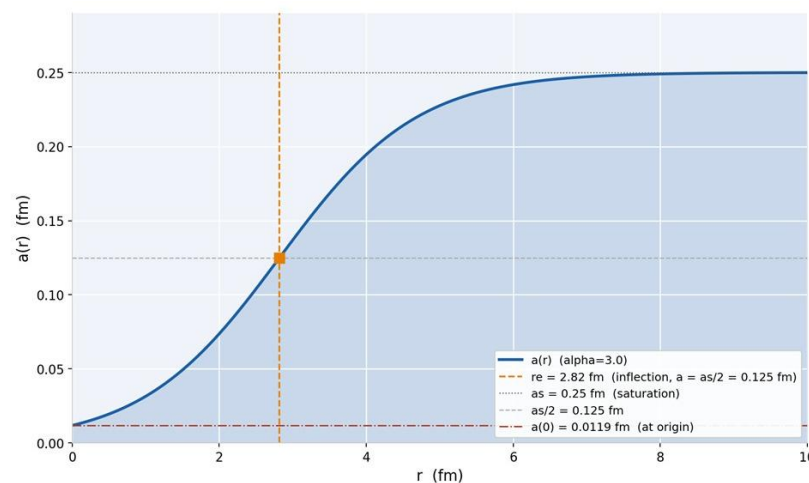


Figure 6. Illustration of the curved embedded electron cell length model $a(r)$.

At each time step, the quark walker, representing the tunnelling d quark (Type I) or $\bar{u}$ antiquark (Type II), evaluates the eight body-diagonal candidate positions $(\pm 1, \pm 1, \pm 1)$ permitted by the Pmmm hopping selection rules, computes the local cell length $a(\mathbf{r}_i)$ at each candidate from Equation (39), and selects one candidate with probability proportional to the Boltzmann weight $w_i \propto \exp(-\beta E_{\text{band}}(a(\mathbf{r}_i)))$, where $\beta = \frac{1}{k_B T}$ is the inverse condensate temperature, identical for both types. Because $a(r)$ is smaller near the origin where the condensate is most compressed, and because the energy band increases with $a(r)$ through the cosine argument, $\cos(K_0 a(r_i)/2)$, at each step of the random walk the Boltzmann factor $\exp(-\beta E(a(r)))$ is evaluated independently for each of the eight candidate hops, and is largest for whichever candidates land closest to the origin, where $a(r)$ is smallest. The Metropolis scheme therefore favors inward steps at every individual selection, and this local bias, repeated over many steps, generates a net inward restoring force that confines the quark cloud within the classical electron radius.

The energy band entering the Boltzmann weight is evaluated using the single-band tight-binding dispersion derived from the 8×8 excitation Hamiltonian with only the same-flavor coupling $t_{\bar{u}\bar{u}}$ (Type I) or $t_{\bar{d}\bar{d}}$ (Type II) nonzero. The Bloch sum over the eight body-diagonal nearest-neighbor vectors connecting same-flavor Wyckoff sites gives the structure factor $h(k_x, k_y, k_z) = 8 \cos(k_x a(r)/2) \cos(k_y a(r)/2) \cos(k_z a(r)/2)$; since the embedded electron's confinement is radially symmetric, all three wavevector components are equal, $k_x = k_y = k_z = K$, reducing the structure factor to $8 \cos^3(Ka(r)/2)$. The eight-by-eight tight-binding Hamiltonian splits into four two-by-two block Hamiltonians for each quark and antiquark. The resulting lower-band energy is $E_{e,type I} = \varepsilon_u - 8t_{uu} \cos^3(Ka(r)/2)$, with ε_u and t_{uu} being the hopping amplitude of the embedded electron Type I. $E_{e,type II} = \varepsilon_{\bar{d}} - 8t_{\bar{d}\bar{d}} \cos^3(Ka(r)/2)$, with $\varepsilon_{\bar{d}}$ and $t_{\bar{d}\bar{d}}$ being the hopping amplitude of the embedded electron and $t_{\bar{u}\bar{u}}$ the hopping amplitude of the embedded electron Type II. These energy bands are evaluated at the local cell size $a(r)$ of each candidate position and apply identically to both embedded electron types.

The condensate cell mass m_{cell} can be calculated by Equation (41):

$$m_e(r_e, \alpha_e) = 4\pi \int_0^{r_e} \left(\frac{m_{cell}}{a(r)^3} - \frac{m_{cell}}{a_s^3} \right) r^2 dr = 0.511 \text{ MeV} \quad (41)$$

The density-excess mass integral reflects the energy stored in the curved QCD condensate by the topological defect at the Wyckoff position for the embedded electron. Topological defects that curve the QCD condensate lattice may be geometric sources of the rest mass of embedded particle excitations in the QCD condensate lattice. The density-excess integral yields $m_{cell} = 1.33 \times 10^{-36} \text{ kg}$, establishing the absolute mass scale of the condensate unit cell from the electron rest mass. The condensate unit cell is extremely light, $1.46 \times 10^{-6} m_e$, however, not massless, and as described above, may be a dark matter candidate.

A similar density-excess equation can be used for a proton; for example:

$$m_p(r_p, \alpha_p) = 4\pi \int_0^{r_p} \left(\frac{m_{cell}}{a(r)^3} - \frac{m_{cell}}{a_s^3} \right) r^2 dr = 983.5 \text{ MeV} \quad (42)$$

Figure 7 shows the probability density of the two embedded electrons' random walks in the QCD condensate lattice. The two embedded electrons' d quark and $\bar{u}$ antiquark are confined by the QCD condensate curved model below with $r_e = 2.82 \text{ fm}$.

Figure 8 presents the initial configuration of the Type I embedded positron unit cell. This configuration is obtained by substituting a d quark at Wyckoff site $(a/2, a/2, a/2)$, the body-center position of the Pmmm unit cell, with a u quark, which thereby acquires a net electric charge of +1. The four green arrows indicate the symmetry-allowed hopping directions available to the u quark, corresponding to four of the eight symmetry-allowed hops within the QCD Pmmm lattice.

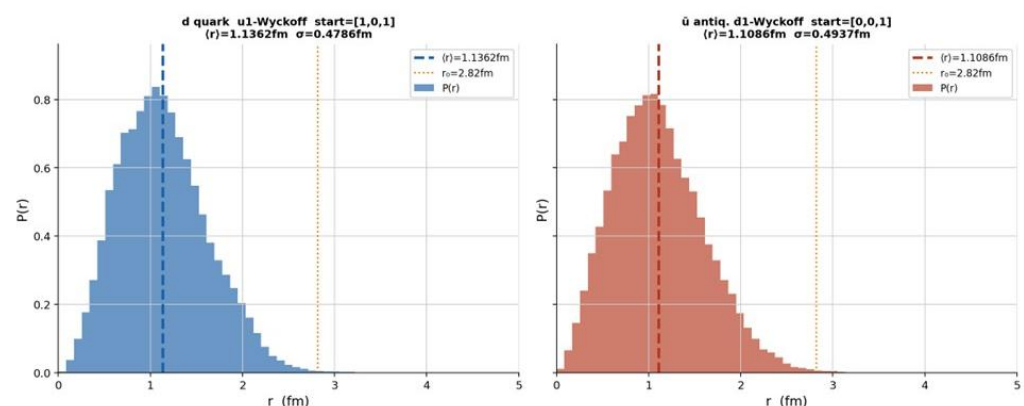


Figure 7. Illustration of self-confined embedded electrons in curved QCD condensate lattice.

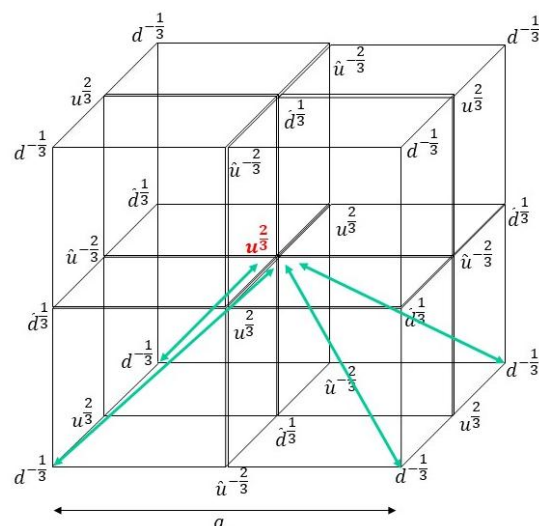


Figure 8. Illustration of embedded positron Type I in the QCD condensate unit cell.

Figure 9 presents the initial configuration of the Type II embedded positron unit cell. This configuration is obtained by substituting a $\bar{u}$ antiquark at Wyckoff site $(a/2, 0, 0)$ with a $\bar{d}$ antiquark, which thereby acquires a net electric charge of +1. The two green arrows indicate the symmetry-allowed hopping directions available to the $\bar{d}$ antiquark, corresponding to four of the eight symmetry-allowed hops within the QCD Pmmm lattice.

The examples above demonstrate how the spin and helicity states of the embedded electron are structurally and dynamically encoded by the Pmmm QCD condensate lattice. The spin state is determined by the nature of the tunnelling carrier, whether it is a quark (matter) or an antiquark (antimatter), while the helicity is determined by whether the larger fractional quark charge hops parallel or antiparallel to the direction of the embedded electron or positron charge motion.

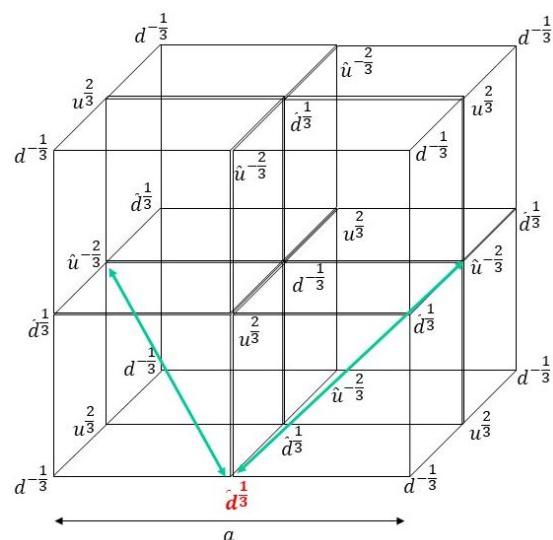


Figure 9. Illustration of embedded positron Type II in the QCD condensate unit cell.

5. Embedded Cooper Pair in the QCD Condensate Lattice

Within the chiral condensate lattice, a Cooper pair can be formed by simultaneously substituting a u quark with a d quark at Wyckoff position $(a/2, 0, a/2)$ and a $\bar{d}$ antiquark with a $\bar{u}$ antiquark at Wyckoff position $(0, 0, a/2)$. The two resulting negative charges are localized on the rectangular face of the unit cell defined by Wyckoff sites

$(a/2, 0, a/2)$, $(0, 0, a/2)$, $(a/2, 0, 0)$, and $(0, 0, 0)$, which is spanned by two d quarks and two $\bar{u}$ antiquarks. The green arrows in Figure 10 depict the correlated parallel motion of the two embedded electrons arising from their simultaneous symmetry-allowed hoppings—the d quark hopping in the quark sector and the $\bar{u}$ antiquark hopping in the antiquark sector—which defines the paired quantum-walk state of the Cooper pair within the condensate lattice.

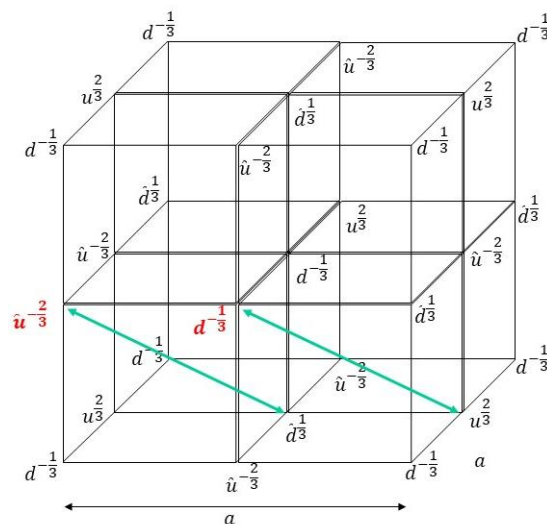


Figure 10. Illustration of the embedded Cooper pair in the QCD condensate unit cell, with d quark and $\bar{u}$ antiquark simultaneous parallel motion shown with green arrows.

According to Bardeen–Cooper–Schrieffer (BCS) theory, Cooper pairs in the superconducting state consist of electrons with opposite spin and momentum near the Fermi surface, bound by an attractive interaction mediated by the motion of the lattice ions [31]. BCS theory accounts for the isotope effect in superconductors through the dependence of this phonon-mediated coupling on the ionic mass. The present framework suggests an alternative interpretation of this effect. We propose that the density and structure of the QCD condensate lattice depend on the mass of the ion, such that heavier isotopes increase the local density of the condensate and curve its unit cells more strongly. Within the present framework, a Cooper pair is formed from two embedded electrons of different types—one quark-based and one antiquark-based—occupying different Wyckoff sites of the condensate lattice. Increased local curvature effectively shortens the separation between these two embedded electrons, strengthening the pairing interaction between them. The effective attraction between Cooper pairs is therefore attributed, within the present framework, to the ion-mass-dependent curvature of the QCD condensate lattice acting on the two embedded electron types, consistent with the proposed generalized equivalence principle.

Superconducting gravimeters are used to monitor with high precision the temporal variations in Earth’s gravitational field arising from the orbital motion of the Moon and Earth’s revolution around the Sun [32,33]. By analogy with the isotope effect in superconductors, a quantum gravity theory based on the chiral condensate might account for these periodic gravitational variations through periodic modulations of the chiral condensate density induced by the motions of the Moon and Earth. This hypothesis implies that spatial gradients in condensate density mediate the transmission of long-range gravitational forces, with the QCD condensate density decreasing with distance from massive bodies, in direct analogy with the decrease in atmospheric density with altitude above the Earth’s surface [34].

6. Embedded Electron–Positron Creation and Annihilation

Embedded electron–positron pairs can be generated within the QCD condensate by exchanging, for example, a u quark at Wyckoff position $(a/2, 0, a/2)$ with a d quark at Wyckoff position $(0, 0, 0)$ as illustrated in Figure 11.

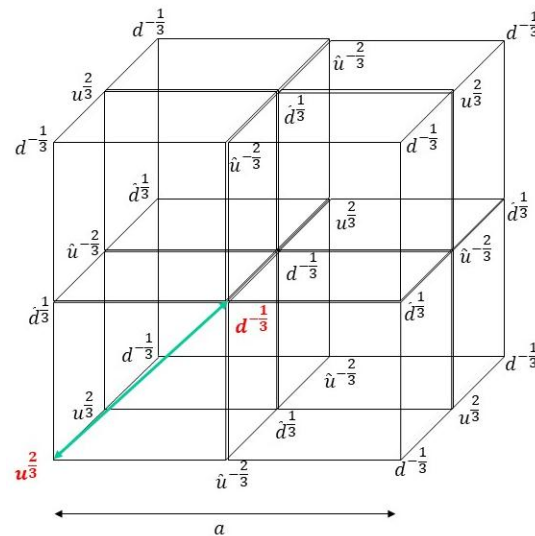
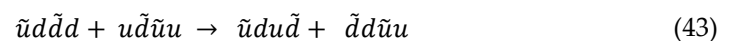


Figure 11. Illustration of an embedded electron–positron pair in the QCD condensate unit cell.

These Wyckoff dislocations require surmounting a potential barrier of 1.022 MeV, equal to twice the rest-mass energy of the electron. Subsequently, the d quark at Wyckoff position $(a/2, 0, a/2)$ can further exchange with a u quark from an adjacent unit cell, and similarly the u quark at Wyckoff position $(0, 0, 0)$ can exchange with a d quark from another neighboring unit cell, thereby separating the electron and positron charges. The symmetry-allowed exchanges convert the two neighboring unit cells into a Type I embedded electron cell and a Type I embedded positron cell, respectively. The overall result is the creation and spatial separation of an electron–positron pair within the chiral condensate lattice. Conversely, the reverse permutation sequence can annihilate the electron–positron pair, and the energy released by this annihilation may propagate as a wave through the QCD condensate lattice, which can be identified with an electromagnetic wave.

The annihilation of an embedded electron–positron pair within the QCD condensate is described by Equation (43), in which the electron and positron tetraquarks recombine to yield two neutral Pionic tetraquarks:



In Equation (43), both the total quark number and quark flavor are strictly conserved: these reactions within the chiral condensate involve no creation or annihilation of quarks, only their rearrangement among the Wyckoff positions of the lattice.

7. Topological Defects as the Origin of Particle Excitations—Embedded Protons, Neutrons, Deuterons and α -Particles

Embedded protons, neutrons, deuterons, and α -particles are generated as topological defects of the QCD condensate unit cell, Wyckoff dislocations, by a procedure analogous to that used for the embedded electron and positron (Figures 4 and 5). In each case, one or more $\tilde{u}$ or $\tilde{d}$ antiquarks are replaced by the appropriate quark species or left as a vacancy at designated high-symmetry Wyckoff positions. The specific Wyckoff substitutions and the resulting quark configurations for each embedded hadron are illustrated in Figures 12–15 below. Topological defects that curve the QCD condensate lattice may be geo-

metric sources of the rest mass of embedded particle excitations in the QCD condensate lattice. We note that according to quantum field theory, particles are excitations of quantum fields and particle mass is received from the interaction with the Higgs field [35–37].

An embedded proton is created by substituting a $\tilde{u}$ antiquark with a u quark at Wyckoff position $(a, a/2, a/2)$, or at an equivalent Wyckoff position, and simultaneously removing a $\tilde{d}$ antiquark from its Wyckoff position $(a, a, a/2)$, as illustrated in Figure 12 with a red asterisk (*). These substitutions impart a net electric charge of $+1$ to the otherwise neutral QCD condensate unit cell and introduce a vacant lattice site, denoted by an asterisk (*), representing a lattice vacancy defect. Such a reaction can occur near a BH event horizon where the antiquarks $\tilde{u}\tilde{d}$ become trapped according to Hawking radiation below the horizon and the matter particles, uud , escape.

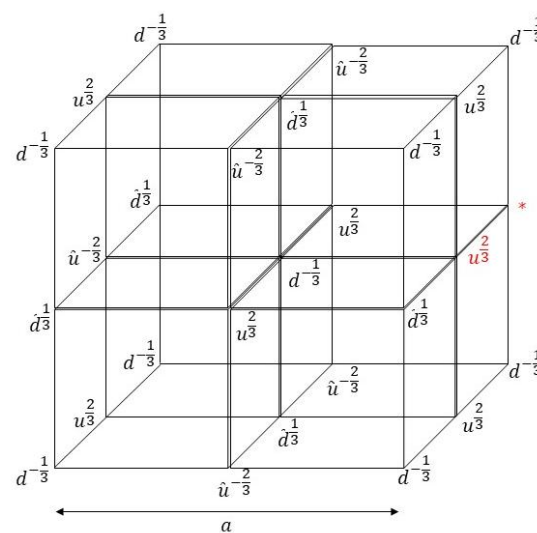


Figure 12. Illustration of the QCD embedded proton in the QCD condensate unit cell.

The motion of the embedded proton through the QCD condensate lattice is more complex than that of the embedded electron and proceeds via a synchronized tunnelling mechanism involving two simultaneous exchanges. In the first exchange, the substituted u quark hops between adjacent $\tilde{u}$ Wyckoff sites along the long body-diagonal direction of the unit cell. Simultaneously, the $\tilde{d}$ -site vacancy exchanges positions with a $\tilde{d}$ antiquark along a principal axis of the unit cell. This synchronized coupling between the u quark hop and the vacancy hop allows the embedded proton to propagate from one QCD condensate lattice cell to an adjacent cell by tunnelling with no energy loss.

By analogy with the embedded electron curvature of Equation (39), the topological defect of the embedded proton is assumed to locally curve the QCD condensate lattice according to the same sigmoid cell-size function, but with distinct curvature parameters entering Equation (42): r_p , the proton confinement radius identified with the measured proton charge radius $r_p = 0.84$ fm, and α_p , the exponential slope governing the steepness of the curvature transition. For the embedded proton and neutron, the tunnelling selection rules governing quark exchange differ from those of the embedded electron, as the proton and neutron involve second-order synchronized hops of both a quark and a vacancy. Accordingly, the tight-binding Hamiltonian and its energy bands take a different form for hadronic excitations. The proton radius is taken directly from experimental data as a phenomenological input parameter, rather than derived from the present model.

An embedded neutron may be created by substituting a $\tilde{u}$ antiquark with a d quark at Wyckoff position $(a, a/2, a/2)$, or at an equivalent Wyckoff position, and simultaneously

removing a $\bar{d}$ antiquark from its Wyckoff position $(a, a, a/2)$, as illustrated in Figure 13 with a red asterisk (*). Unlike the embedded proton, these substitutions carry no net electric charge, so the QCD condensate unit cell remains electrically neutral while acquiring a vacant lattice site, denoted by an asterisk (*), representing a lattice vacancy defect.

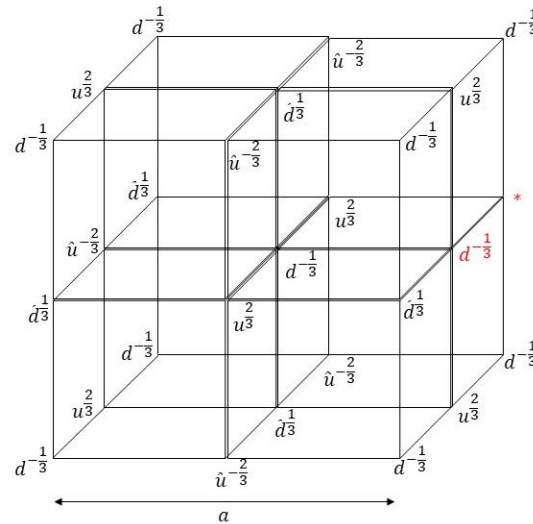
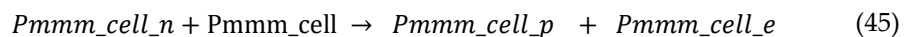


Figure 13. Illustration of the QCD embedded neutron in the QCD condensate unit cell.

The exchange reaction between the embedded neutron cell in Figure 13 QCD_cell_n and an adjacent unperturbed QCD condensate cell, cell QCD_cell , in which the substituted d quark of the neutron at site $(a, \frac{a}{2}, \frac{a}{2})$ is exchanged with the u quark of the adjacent cell at $(a/2, 0, a/2)$, produces two distinct QCD cells: $Pmmm_cell_p$, the embedded proton in Figure 12, and the embedded electron in Figure 4, $Pmmm_cell_e$.



The d quark of the created embedded electron cell can tunnel and exchange positions with u quarks of another QCD cell in the allowed Wyckoff position, which can be written as the following symmetric exchange reaction:



An embedded deuteron may be created by simultaneously substituting two $\bar{u}$ antiquarks with a u quark and a d quark at Wyckoff positions $(0, a/2, a/2)$ and $(a, a/2, a/2)$, respectively, or at equivalent symmetry-related Wyckoff positions, and removing two $\bar{u}$ antiquarks from their Wyckoff positions $(0, 0, a/2)$ and $(a, a, a/2)$, as illustrated in Figure 14 with two red asterisk (*). These substitutions impart a net electric charge of $+1$ to the otherwise neutral QCD condensate unit cell, consistent with the single positive charge of the deuteron, and introduce two vacant lattice sites, each denoted by an asterisk (*), representing two lattice vacancy defects.

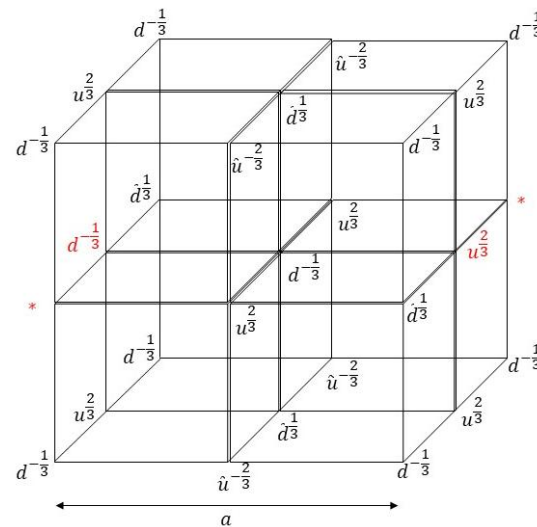


Figure 14. Illustration of the QCD embedded deuteron in the QCD condensate unit cell.

The equivalence principle described above should be applicable also at the nuclei scale. Accordingly, the embedded proton (and neutron) curves the QCD condensate lattice similarly to the embedded electron described in Equations (39) and (41). The embedded proton u quark and $\tilde{d}$ vacancy perform a random walk in the curved QCD condensate lattice and experience the returning force due to the self-curvature that confines the proton to its r_p radius. This mechanism may be equivalent to the Fermi gas mean field approximation applied to the motion of nucleons in the average field of the other nucleons in the nuclei, having a typical momentum K_F . On top of the Fermi gas mean field approximation, deep-inelastic and quasi-elastic electron and proton scattering show that nearly all high-momentum K_F nucleons are paired by short-range correlation (SRC) and that about 90% of those pairs are neutron–proton pairs [38,39].

In the embedded deuteron configuration shown in Figure 14, the d quark of the neutron and the u quark of the proton can tunnel between $\tilde{u}$ Wyckoff sites of the Pmmm lattice at $(a, a, a/2)$ and $(0, 0, a/2)$, while the two $\tilde{d}$ vacancies remain fixed at their $\tilde{d}$ Wyckoff sites throughout the exchange. The simultaneous exchange of the u quark at $(a, a/2, a/2)$ with the $\tilde{u}$ antiquark at $(a/2, 0, a)$ and of the d quark at $(0, a/2, a/2)$ with the $\tilde{u}$ antiquark at $(a/2, a, a)$ exchanges the positions of the embedded proton and neutron within the QCD lattice unit cell without displacing the two $\tilde{d}$ -site vacancies at $(a, a, a/2)$ and $(0, 0, a/2)$ that are shared. This correlated tunnelling, in which the proton and neutron switch the topologically defected vacancies, may further stabilize the embedded deuteron and may be the geometric origin of the short-range nucleon–nucleon correlations (SRCs) observed experimentally. With a bigger nucleus and higher numbers of protons and neutrons, the effect of sharing the $\tilde{d}$ vacancies may be even more significant. Hence, the correlated motion of embedded proton and neutron pairs within the curved QCD condensate lattice may be the geometric mechanism of the short-range correlations (SRCs) observed in the deep-inelastic electron scattering (DIS) in nuclei.

An embedded α -particle may be created by simultaneously substituting four $\tilde{u}$ antiquarks with two u quarks and two d quarks at Wyckoff positions $(0, a/2, a/2)$, $(a, a/2, a/2)$, $(a/2, 0, 0)$, and $(a/2, a, a)$, respectively, or at equivalent symmetry-related Wyckoff positions, and removing four $\tilde{u}$ antiquarks from their Wyckoff positions $(0, 0, a/2)$, $(a, a, a/2)$, $(a/2, a/2, 0)$, and $(a/2, a/2, a)$, as illustrated in Figure 15 with four red asterisk (*). These substitutions impart a net electric charge of $+2$ to the otherwise neutral QCD condensate unit cell—consistent with the double positive charge of the α -particle—and introduce four vacant lattice sites, each denoted by an asterisk (*), representing four lattice vacancy defects.

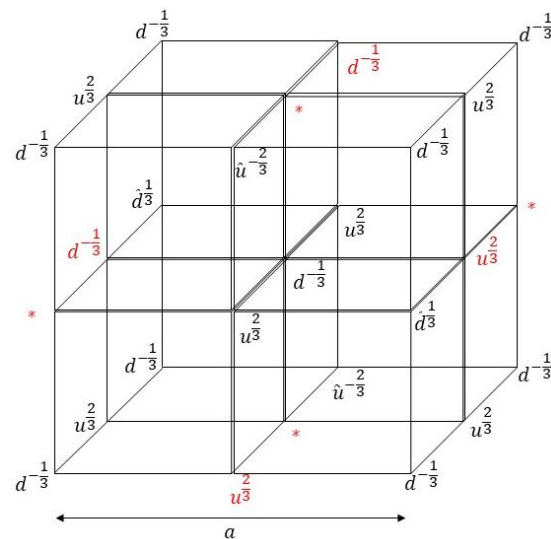


Figure 15. Illustration of the QCD embedded α -particle in the QCD condensate unit cell.

It should be noted that the substitutions proposed above, which generate hadronic particles from the QCD condensate unit cell, can initially occur only under the extreme conditions in the vicinity of black hole horizons, where the QCD condensate lattice partially melts and antiquarks can fall beneath the horizon and become causally disconnected from the exterior region. Once the hadrons are created, they propagate through the QCD condensate lattice by tunnelling between symmetry-allowed Wyckoff positions, in direct analogy with the embedded electrons and positrons, and can be accelerated by spatial curvature of the QCD condensate lattice in accordance with the equivalence principle.

The Pmmm QCD condensate unit cell is highly symmetric despite accommodating four distinct quark species at specific Wyckoff positions that broke the $Pm\bar{3}m$ symmetry. At the same time, its symmetry-allowed tunnelling selection rules permit the quark exchanges and substitutions that generate hadrons and leptons as composite particles embedded in and confined by the QCD condensate lattice. Crucially, neither these composite particles nor their constituent quarks ever exist as free, unconfined entities; confinement is guaranteed by the QCD lattice until it melts.

8. Baryon Asymmetry—The Question of Matter and Antimatter Separation

Which process broke the symmetry between matter and antimatter? In “Baryon Asymmetry, Dark Matter and Quantum Chromodynamics”, Oaknin and Zhitnitsky [40] proposed an innovative explanation rooted in nonperturbative quantum chromodynamics (QCD). Their scenario suggests that at the end of the cosmological QCD phase transition, quarks and antiquarks separated into two distinct phases: ordinary hadrons (and antihadrons), as well as massive lumps (and antilumps) of a novel color superconducting phase that may serve as cosmological cold dark matter.

We propose an alternative matter–antimatter separation mechanism based on balanced quark reaction equations, which could account for the observed matter–antimatter asymmetry of the universe. In this model, antimatter is not annihilated but rather concealed beneath black hole horizons, effectively sequestered from the observable universe. We further assume that the region in the immediate vicinity of a black hole horizon contains an extremely hot and dense Pionic tetraquark liquid, arising from local melting of the QCD condensate lattice. The bosonic tetraquark density can be greatly amplified by the black hole laser effect operating within the ergosphere [41].

Three Pionic tetraquarks from the melted QCD condensate can react to produce a deuteron–antideuteron pair according to Equation (47). The extremely high local density of Pionic tetraquarks then drives further reactions with these products: neutrons are converted to protons and antiprotons are converted to antineutrons according to Equations (48) and (49), respectively. The two antineutrons fall beneath the black hole event horizon and become causally disconnected from the exterior region, while the two protons and two negatively charged tetraquarks, the latter identified as composite electrons, escape into the exterior spacetime via Hawking radiation [42]. The overall balanced reaction in terms of quarks and antiquarks is summarized in Equation (50).

$$3 \tilde{u} u \tilde{d} d \rightarrow u d d u u d + \tilde{u} \tilde{d} \tilde{d} \tilde{u} \tilde{u} \tilde{d} \quad (47)$$

$$u d d + \tilde{u} u \tilde{d} d \rightarrow u u d + d \tilde{d} d \tilde{u} \quad (48)$$

$$\tilde{u} \tilde{u} \tilde{d} + \tilde{u} u \tilde{d} d \rightarrow \tilde{u} \tilde{d} \tilde{d} + \tilde{u} u \tilde{u} d \quad (49)$$

$$5 \tilde{u} u \tilde{d} d \rightarrow 2 \tilde{u} \tilde{d} \tilde{d} + 2 u u d + d \tilde{d} d \tilde{u} + \tilde{u} u d \tilde{u} \quad (50)$$

As a consequence, matter–antimatter symmetry is broken in the vicinity of black hole horizons: Pionic tetraquarks react to form matter–antimatter pairs, antimatter falls beneath the event horizon, and the corresponding matter particles escape into the exterior spacetime. Quark reaction Equations (47)–(50) provide an alternative mechanism for matter–antimatter symmetry breaking in the vicinity of black holes, offering a continuous source of net matter production [43].

It should be noted that the formation of deuteron–antideuteron pairs from three Pionic tetraquarks near black hole horizons, as described by Equation (47), is a non-trivial process. The ALICE Collaboration has recently published a study addressing deuteron and antideuteron formation, proposing a novel approach based on deuteron–pion momentum correlations measured in proton–proton (pp) collisions at the Large Hadron Collider (LHC) [44]. The collision temperatures achieved in pp interactions at the LHC may be comparable to those prevailing in the accretion discs surrounding black holes, conditions under which the chiral condensate lattice may melt locally and thereby enable three Pionic tetraquarks to react and produce deuteron–antideuteron pairs according to Equation (47).

9. Discussion

The proposed Pmmm space group QCD condensate unit cell provides a geometric realization of the QCD condensate lattice ground state that complements and extends the instanton liquid picture. In the present framework, the four light quarks and antiquarks $u, d, \bar{u}, \bar{d}$ condense into an ordered Pmmm lattice, occupying the four highest-symmetry Wyckoff positions of the unit cell. The spontaneous reduction from the parent Pm $\bar{3}$ m cubic symmetry (No. 221, O_h , order 48) to the Pmmm subgroup (No. 47, D_{2h} , order 8) may mirror the chiral symmetry breaking $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ of the QCD vacuum, with the broken symmetry operations generating the three pions π^+, π^-, π^0 as Goldstone bosons. The nonzero Pionic condensate vacuum expectation value (VEV) may be related to the quark hopping amplitude t_{du} . The tight-binding energy bands exhibit a gap at the Γ point that vanishes in the chiral limit, and complete band decoupling at the R point consistent with asymptotic freedom, providing quantitative support for the proposed QCD condensate lattice model.

The question raised in the Introduction of whether the QCD condensate is massless, and whether it can account for the missing dark matter, receives a quantitative answer within the present framework. The QCD condensate ground state density in the Milky Way galaxy can be matched with the estimated missing dark matter of the galaxy [20].

Einstein's Equivalence Principle (EEP) [45–47] is generalized for the QCD condensate lattice; a composite particle made of quarks and embedded in the QCD condensate lattice cannot distinguish whether it is accelerated by gravity, by the strong force, by the weak force, by the electromagnetic force, or by a curvature of the QCD condensate lattice itself. This generalization is made possible by the tight-binding structure of the Hamiltonian, in which the influence of any external force field is encoded entirely in the smooth spatial variation of the unit cell length $a(\mathbf{r})$, entering through the cosine terms $\cos\left(\frac{k_x a}{2}\right)$, $\cos\left(\frac{k_y a}{2}\right)$, $\cos\left(\frac{k_z a}{2}\right)$ of the dispersion relation. The force experienced by the quantum particle is then in the x -direction—for example, $F_x = -\partial E / \partial a \cdot \partial a / \partial x$ —irrespective of its physical origin. The Pmmm unit cell is assumed to deform smoothly and continuously in response to massive particles and charge while remaining structurally intact; only under the extreme conditions of supernova explosions or ultra-high-energy heavy-ion collisions is the QCD condensate lattice expected to melt locally.

The Pmmm QCD condensate lattice, built from the four constituents of its unit cell, the u quark, d quark, $\bar{u}$ antiquark, and $\bar{d}$ antiquark, provides a geometric framework for the origin of particle mass as topological defects in the QCD condensate lattice that does not require the Higgs field [35–37]. As illustrated herein above for the embedded electron (Figures 4 and 5) and the embedded proton (Figure 12), neutron (Figure 13), deuteron (Figure 14), and α -particle (Figure 15), particle excitations may be topological defects, deviations from the nominal Wyckoff occupation of the Pmmm QCD condensate lattice that further locally curve the QCD condensate lattice. The rest mass of embedded particles may be equal to the energy of the self-induced curvature, as derived in Equations (39)–(41) for the electron, for example. In this picture, the Higgs field, which distributes mass to otherwise massless particles through 13 independent Yukawa coupling constants, may be redundant. Mass may be generated by the topological defects at Wyckoff positions. In the proposed geometric picture, leptons, hadrons, and their bound states are embedded defects that locally curve the Pmmm QCD condensate lattice of flat space. The electron arises from a point defect, the substitution of a d quark for a u quark at the $(\frac{a}{2}, 0, \frac{a}{2})$ Wyckoff position of the Pmmm unit cell. This symmetry breaking substitution creates a topological defect in the QCD condensate, a stable, localized curvature of the lattice that carries the electron's rest mass, charge, and spin as geometric properties. These defect-induced deviations from the nominal Wyckoff occupation, or equivalently, topological defects of the Pmmm QCD condensate lattice, may replace the role of the Higgs Yukawa couplings in generating particle mass.

Drawing on Harari's four-sub-particle hypothesis [8], we propose that the electron is a topological defect of the nominal QCD condensate lattice Wyckoff occupation, existing in two distinct structural forms based on quark or antiquark topological defect tunnelling between quark or antiquark Wyckoff positions. This directly addresses the concern raised by Dirac in his lecture [5] that the concept of a bare electron is physically misleading: within the present geometrical framework, the physical embedded electron is never bare; the electron exists as a self-confined topological defect within the QCD condensate lattice performing rapid quark permutations at the Zitterbewegung frequency [28]. Crucially, because the embedded electron is self-confined and propagates as a Bloch wave, these rapid permutations produce no wave-packet dispersion, resolving the Schrödinger–Darwin paradox raised in the Introduction. Particle excitations as topological defects of the QCD condensate lattice ground state, together with the self-induced curvature of the lattice cells they produce, are inherently many-body phenomena involving the collective tunnelling motion of many quarks and antiquarks, consistent with the wave-like description of particles in quantum mechanics, and in contrast to the classical point-like particle picture. Furthermore, the origin of the wave-like nature of quantum mechanics may lie in

the geometry of the QCD condensate lattice ground state and the emergence of particle excitations topological defects as many-body phenomena.

The proposed matter–antimatter separation mechanism based on balanced quark reaction equations (Equations (47)–(50)) offers an alternative to baryogenesis scenarios. In the present model, antimatter is not annihilated but sequestered beneath black hole horizons. The black hole laser effect operating within the ergosphere [41] can exponentially amplify the Pionic tetraquark density. The recent ALICE Collaboration result on deuteron–pion momentum correlations in pp collisions at the LHC [44] provides direct experimental support for the plausibility of Equation (47), suggesting that the three-body Pionic tetraquark reaction producing deuteron–antideuteron pairs is not merely a theoretical construct but is kinematically accessible at temperatures comparable to those of black hole accretion discs.

The proposed structure and correlated dynamics of the embedded proton and neutron pair and the α particle in the QCD condensate lattice, shown in Figures 14 and 15 respectively, may be a geometric model for the short-range nucleon–nucleon correlation (SRC) found in deep-inelastic electron–deuteron scattering [38,39]. The stability of the embedded deuteron and α particle in the QCD condensate lattice represents a promising direction for a follow-up study.

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Abbreviations

The following abbreviations are used in this manuscript:

QCD	Quantum Chromodynamics
QFT	Quantum Field Theory
SM	Standard Model

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