symmetry

MDPI

Article

Confidence Intervals for the Common Coefficient of Variation of Delta-Inverse Gaussian Distributions

Wasurat Khumpasee, Sa-Aat Niwitpong * and Suparat Niwitpong

Department of Applied Statistics, Faculty of Applied Sciences, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand; k.wasurat@email.kmutnb.ac.th (W.K.); suparat.n@sci.kmutnb.ac.th (S.N.)

* Correspondence: sa-aat.n@sci.kmutnb.ac.th

Abstract

This study investigates the construction of confidence intervals for the common coefficient of variation (CV) of multiple delta-inverse Gaussian (delta-IG) distributions, which are suitable for modeling skewed and zero-inflated data. Existing methods for CV inference are generally limited to single populations or standard distributions and do not adequately address the combined challenges of skewness, excess zeros, and multiple populations. To overcome these limitations, five approaches are considered: generalized confidence intervals (GCI), adjusted GCI (AGCI), parametric bootstrap percentile confidence intervals (PBPCI), Bayesian credible intervals (BCI), and highest posterior density intervals (HPDCI). A Monte Carlo simulation study is conducted to evaluate performance in terms of coverage probability (CP) and average width (AW) across various parameter settings and population sizes. The results reveal a clear trade-off between coverage accuracy and interval efficiency. Among the methods, HPDCI provides coverage probabilities closest to the nominal level, though it still exhibits some undercoverage, whereas BCI offers reasonably stable performance. In contrast, PBPCI produces the narrowest intervals but suffers from severe undercoverage, and GCI and AGCI fail to achieve a satisfactory balance. An application to traffic accident data further supports the simulation findings. Overall, the HPDCI methods are recommended for reliable inference in delta-IG settings.

Keywords: generalized confidence interval; adjusted generalized confidence interval; parametric bootstrap percentile confidence interval; Bayesian credible interval; highest posterior density approach

Academic Editors: Pedro José Fernández de Córdoba Castellá and Shufei Wu

Received: 25 April 2026
Revised: 10 July 2026
Accepted: 13 July 2026
Published: 15 July 2026

1. Introduction

The coefficient of variation (CV) is a widely used measure of relative dispersion, defined as the ratio of the standard deviation to the mean. Unlike variance or standard deviation, the CV is scale-independent, making it particularly useful for comparing variability across populations with different units or magnitudes [1,2]. Consequently, statistical inference for the CV has received considerable attention across fields such as environmental science, reliability engineering, finance, and biomedical research.

In many practical situations, data are collected from multiple populations that are assumed to share a common level of variability. This leads to the concept of the common coefficient of variation, which plays an important role in comparative studies and meta-analysis [3,4]. Estimating a common CV allows combining information across groups, resulting in more efficient and reliable statistical inference than analyzing each population separately [5]. Several studies have addressed inference for the common CV under different distributions, such as normal, lognormal, gamma, and Birnbaum-Saunders distributions [6–10]. However, these approaches are often based on assumptions that may not hold in practice, particularly when data exhibit skewness or contain excess zeros.

The inverse Gaussian (IG) distribution has been widely used for modeling positively skewed data due to its flexibility and strong theoretical foundation [11–13]. It has applications across diverse areas, including survival analysis, reliability studies, environmental modeling, and insurance data analysis [14,15]. Nevertheless, real-world data frequently exhibit not only skewness but also a substantial proportion of zero observations. To accommodate this feature, extensions such as zero-adjusted and delta-inverse Gaussian (delta-IG) distributions have been proposed by incorporating a point mass at zero [16,17]. These models provide a more realistic framework for analyzing mixed discrete–continuous data encountered in practice.

Despite growing interest in delta-IG models, statistical inference for their variability measures remains significantly limited. Recent advances have primarily focused on constructing confidence intervals for individual CVs or conducting pairwise comparisons [18], which are not directly applicable to simultaneous inference across multiple groups [19–23]. Consequently, existing methodologies largely fail to adequately address the combined challenges of zero inflation, right-skewness, and multiple nuisance parameters when estimating a common parameter [24–26]. This gap is important because the common CV provides a unified, scale-free measure of relative variability across multiple populations, which is highly relevant in applications involving heterogeneous yet structurally similar data sources. Therefore, the objective of this study is to develop and evaluate robust methods for constructing confidence intervals for the common CV under multiple delta-inverse Gaussian distributions. Five distinct approaches, including the GCI, AGCI, PBPCI, BCI, and HPDCI, are considered and compared through extensive simulation studies. Each method is specifically motivated by its unique capability to tackle the inherent complexities of multiple delta-IG populations. While GCI and AGCI are designed to eliminate multiple nuisance parameters without relying on large-sample asymptotic properties, PBPCI provides a computationally intensive frequentist alternative that bypasses strict distributional constraints via simulation. Furthermore, the Bayesian framework (BCI and HPDCI) is motivated by its ability to robustly capture joint parameter uncertainty. Given that the delta-IG structure simultaneously involves three distinct parameters (μ, λ, δ), the utilization of prior distributions and posterior computations enables the Bayesian methods to better accommodate high zero-inflation and data scarcity than classical frequentist estimation.

2. The CI for the Common CV of Multiple Delta-IG Distributions

Let $X_{ij} = (X_{i1}, X_{i2}, \dots, X_{in_i})$, for $i = 1,2,\dots,k, j = 1,2,\dots,n_i$ denote a non-negative random sample drawn from a delta-IG distribution, written as $X_{ij} \sim$ delta-IG$(\delta_i, \mu_i, \lambda_i)$ Here, μ_i is the mean, λ_i is the shape parameter, and δ_i represents the probability of zero outcomes. The probability density function is given by

$$f_{\text{delta-IG}}(X_{ij}; \delta_i, \mu_i, \lambda_i) = \delta_i I_{\{X_{ij}=0\}} + (1-\delta_i) I_{\{X_{ij}>0\}} \sqrt{\frac{\lambda_i}{2\pi X_{ij}^3}}\, exp\left[\frac{-\lambda_i (X_{ij}-\mu_i)^2}{2\mu_i^2 X_{ij}}\right], \tag{1}$$

where $I(\cdot)$ is an indicator function, $P(X_{ij}=0) = \delta_i$, and $\mu_i, \lambda_i > 0$. Each sample consists of n_i observations, which can be partitioned into zero observations $n_{i(0)}$ and nonzero observations $n_{i(1)}$, such that $n_i = n_{i(0)} + n_{i(1)}$. The number of zeros follows a binomial distribution, $n_{i(0)} \sim \text{Bin}(n_i, \delta_i)$.

https://doi.org/10.3390/sym18071194

Following Folks and Chhikara [27], the maximum likelihood estimators (MLEs) of the parameters are $\hat{\delta}_i = \frac{n_{i(0)}}{n_i}, \hat{\mu}_i = \bar{X}_i = \frac{1}{n_{i(1)}}\sum_{j=1}^{n_{i(1)}} X_{ij}$, and $n_{i(1)}\hat{\lambda}_i^{-1} = V_i = \sum_{j=1}^{n_{i(1)}}(X_{ij}^{-1} - \bar{X}_i^{-1})$. The mean and variance of X_{ij} are $E(X_{ij}) = (1-\delta_i)\mu_i$ and $Var(X_{ij}) = \mu_i^2(1-\delta_i)\left(\frac{\mu_i}{\lambda_i}+\delta_i\right)$. The CV for the delta-IG distribution is $CV(X_{ij}) = \theta_i = \sqrt{(\mu_i + \lambda_i\delta_i)/(1-\delta_i)\lambda_i}$, with the estimator of θ_i given by

$$\hat{\theta}_i = \sqrt{(\hat{\mu}_i + \hat{\lambda}_i\hat{\delta}_i)/(1-\hat{\delta}_i)\hat{\lambda}_i}. \tag{2}$$

Based on Khumpasee, Niwitpong, and Niwitpong [28], the asymptotic variance of $\hat{\theta}_i$, obtained via the Delta method using a Taylor series expansion as follows:

Denote

$$A_i = \frac{(1-\hat{\delta}_i)\hat{\lambda}_i}{\hat{\mu}_i+\hat{\lambda}_i\hat{\delta}_i}.$$

Computing the partial derivatives with respect to $\hat{\mu}_i$, $\hat{\lambda}_i$, and $\hat{\delta}_i$ as

$$\frac{\partial\hat{\theta}_i}{\partial\hat{\mu}_i} = \frac{1}{2}A_i^{1/2}\cdot\frac{1}{(1-\hat{\delta}_i)\hat{\lambda}_i},$$

$$\frac{\partial\hat{\theta}_i}{\partial\hat{\lambda}_i} = \frac{1}{2}A_i^{1/2}\cdot\frac{(1-\hat{\delta}_i)\hat{\lambda}_i^2-(\hat{\mu}_i+\hat{\lambda}_i\hat{\delta}_i)(-\hat{\lambda}_i)}{\left[(1-\hat{\delta}_i)\hat{\lambda}_i\right]^2},$$

$$\frac{\partial\hat{\theta}_i}{\partial\hat{\delta}_i} = \frac{1}{2}A_i^{1/2}\cdot\frac{(1-\hat{\delta}_i)\hat{\lambda}_i\hat{\delta}_i-(\hat{\mu}_i+\hat{\lambda}_i\hat{\delta}_i)(1-\hat{\delta}_i)}{\left[(1-\hat{\delta}_i)\hat{\lambda}_i\right]^2}.$$

Therefore,

$$Var(\hat{\theta}_i) = \left(\frac{\partial\hat{\theta}_i}{\partial\hat{\mu}_i}\right)^2\left(\frac{\hat{\mu}_i{}^3}{\hat{\lambda}_i n_{i(1)}}\right) + \left(\frac{\partial\hat{\theta}_i}{\partial\hat{\lambda}_i}\right)^2\left(\frac{2\hat{\lambda}_i{}^2}{n_{i(1)}}\right) + \left(\frac{\partial\hat{\theta}_i}{\partial\hat{\delta}_i}\right)^2\left(\frac{\hat{\delta}_i(1-\hat{\delta}_i)}{n_i}\right). \tag{3}$$

Consequently, the common coefficient of variation across k delta-IG populations is estimated using a weighted average and is given by

$$\hat{\theta} = \frac{\sum_{i=1}^{k}[\hat{\theta}_i/Var(\hat{\theta}_i)]}{\sum_{i=1}^{k}[1/Var(\hat{\theta}_i)]}, \tag{4}$$

where $i = 1,2,\ldots,k$.

The next subsection describes the methods used to construct the corresponding confidence intervals.

2.1. Generalized Confidence Interval (GCI)

A GCI offers a flexible approach for interval estimation, particularly in settings where conventional methods are unreliable or difficult to implement. Classical confidence intervals often rely on restrictive assumptions, such as normality or large sample sizes, whereas the GCI framework relaxes these conditions and remains effective in more complex scenarios [29].

The GCI is constructed based on a generalized pivotal quantity (GPQ), a function whose distribution is free from unknown parameters [19]. In practice, GPQs are repeatedly generated via Monte Carlo simulation, and their empirical percentiles are used to obtain the confidence limits. This simulation-based strategy enables the GCI method to achieve accurate coverage probabilities even with small sample sizes or non-standard distributions. As a result, GCI has become widely used in areas such as biostatistics and engineering [30].

Ye, Ma, and Wang [24] proposed a GPQ-based method for constructing confidence intervals for the mean of IG distributions across multiple populations. Their approach derives GPQs for the key parameters of the IG distribution, namely the shape parameter λ and the mean parameter μ.

The GPQs for λ_i and μ_i, as developed by Ye, Ma, and Wang, are expressed as follows:

$$G_{\lambda_i} = \frac{n_{i(1)}\lambda_i V_i}{n_{i(1)}v_i} \sim \frac{\chi^2_{n_{i(1)}-1}}{n_{i(1)}v_i}, i = 1,2,\ldots,k, \quad (5)$$

where $\chi^2_{n_{i(1)}-1}$ is denoted as a Chi-square distribution with $n_{i(1)} - 1$ degrees of freedom, $i = 1,2,\ldots,k$ and

$$G_{\mu_i} = \frac{\bar{X}_i}{\left|1+\frac{\sqrt{n_{i(1)}\lambda_i}(\bar{X}_i-\mu_i)}{\mu_i\sqrt{\bar{X}_i}}\sqrt{\frac{\bar{X}_i}{n_{i(1)}G_{\lambda_i}}}\right|} \sim \frac{\bar{X}_i}{\left|1+Z_i\sqrt{\frac{\bar{X}_i}{n_{i(1)}G_{\lambda_i}}}\right|}, \quad (6)$$

where Z_i is approximately distributed as a standard normal distribution, denoted by $N(0,1)$.

In this study, the parameter δ_i is estimated using a variance-stabilizing transformation (VST). Khumpasee, Niwitpong, and Niwitpong [31] showed that the VST approach provides more reliable performance than several alternative methods. The concept of variance stabilization, originally introduced by Dasgupta [32], was later extended by Wu and Hsieh [33], who applied the arcsine square-root transformation to stabilize the variance of binomial data. Accordingly, the GPQ for δ_i is defined as in (7), where the transformed variable is approximately standard normal for large samples.

Accordingly, the GPQ for δ_i, following Wu and Hsieh, can be expressed as follows:

$$G_{\delta_i} = sin^2\left[arcsin\sqrt{\hat{\delta}_i} - \frac{T}{2\sqrt{n_i}}\right], \quad (7)$$

where $T = 2\sqrt{n_i}(arcsin\sqrt{\hat{\delta}_i} - arcsin\sqrt{\delta_i}) \sim N(0,1),\ n_i \to \infty$.

Using the GPQs for λ_i, μ_i, and δ_i from (5)–(7), the GPQ for the coefficient of variation θ_i is constructed as

$$G_{\theta_i} = \sqrt{(G_{\mu_i} + G_{\lambda_i}G_{\delta_i})/(1 - G_{\delta_i})G_{\lambda_i}}, \quad (8)$$

where $i = 1,2,\ldots,k$.

The GPQ for the variance of $\hat{\theta}_i$, denoted $G_{Var(\hat{\theta}_i)}$, is obtained by replacing $(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i)$ in Equation (3) with their corresponding GPQs $(G_{\mu_i}, G_{\lambda_i}, G_{\delta_i})$, is given by

$$G_{Var(\hat{\theta}_i)} = Var(\hat{\theta}_i)|_{\hat{\mu}_i\to G_{\mu_i},\hat{\lambda}_i\to G_{\lambda_i},\hat{\delta}_i\to G_{\delta_i}}. \quad (9)$$

The GPQ for the common coefficient of variation across k populations, obtained as a weighted combination of individual GPQs, as defined from (8) and (9), is

$$G_\theta = \frac{\sum_{i=1}^{k}\left[G_{\theta_i}/G_{Var(\hat{\theta}_i)}\right]}{\sum_{i=1}^{k}\left[1/G_{Var(\hat{\theta}_i)}\right]}. \quad (10)$$

Consequently, the $100(1-\alpha)\%$ two-sided confidence interval for θ is determined from the empirical percentiles of the GPQs from the GCI method as

$$CI_\theta^{\mathrm{GCI}} = \left[L_\theta^{GCI}, U_\theta^{GCI}\right] = [G_\theta(\alpha/2), G_\theta(1-\alpha/2)], \quad (11)$$

where $G_\theta(\alpha/2)$ and $G_\theta(1-\alpha/2)$ are the $100\frac{\alpha}{2}$-th and $100\left(1-\frac{\alpha}{2}\right)$-th percentiles of the distribution of G_θ, respectively.

In summary, the GCI approach constructs a GPQ for the coefficient of variation whose distribution is independent of unknown parameters, thereby avoiding reliance on asymptotic approximations. Confidence limits are obtained by simulating the GPQ distribution and extracting the relevant percentiles. The detailed computational procedure is outlined in Algorithm 1.

Algorithm 1: GCI

1.	Create $X_{ij} = (X_{i1}, X_{i2}, \ldots, X_{in_i})$, for $i = 1,2,\ldots,k$, $j = 1,2,\ldots,n_i$ from $X_{ij} \sim$ delta-IG$(\delta_i, \mu_i, \lambda_i)$, then estimate for $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$.
2.	Generate $\chi^2_{n_{i(1)}-1}$ and $t_{n_{i(1)}-1}$ from Chi-square and Student's t-distributions, respectively.
3.	Calculate G_{λ_i}, G_{μ_i}, and G_{δ_i} using (5), (6), and (7), respectively.
4.	Iterate steps 1–3 for k = 1, 2, …, 5000 times to acquire CI_θ^{GCI} as (11).

2.2. Adjusted Generalized Confidence Interval (AGCI)

Building on the GCI framework for inverse Gaussian distributions, Krishnamoorthy and Tian [34] later introduced an alternative GPQ for the parameter of interest. The AGCI method employed in this study adopts this modified GPQ for μ, replacing the conventional form of Equation (6) used in the standard GCI approach. The GPQ for μ_i, as proposed by Krishnamoorthy and Tian, is defined as

$$G_{\tilde{\mu}_i} = \frac{\bar{X}_i}{max\left\{0, 1 + t_{n_{i(1)}-1}\sqrt{\frac{\bar{X}_i v_i}{n_{i(1)}-1}}\right\}}, \tag{12}$$

where $t_{n_{i(1)}-1}$ is Student's t-distribution with $n_{i(1)} - 1$ degrees of freedom. The estimation of δ_i continues to rely on the VST method, consistent with the GCI method.

Accordingly, the GPQs for the parameters are G_{λ_i}, $G_{\tilde{\mu}_i}$, and G_{δ_i}, given in (5), (12), and (7), respectively. Using these, the GPQ for the coefficient of variation θ_i is constructed as

$$G_{\tilde{\theta}_i} = \sqrt{(G_{\tilde{\mu}_i} + G_{\lambda_i}G_{\delta_i})/(1 - G_{\delta_i})G_{\lambda_i}}, \tag{13}$$

where $i = 1,2,\ldots,k$.

The corresponding GPQ for the variance of $\hat{\theta}_i$ under the AGCI method is obtained by replacing $(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i)$ in Equation (3) with their corresponding GPQs $(G_{\tilde{\mu}_i}, G_{\lambda_i}, G_{\delta_i})$. This is given by

$$G_{\tilde{V}ar(\hat{\theta}_i)} = Var(\hat{\theta}_i)|_{\hat{\mu}_i \to G_{\tilde{\mu}_i}, \hat{\lambda}_i \to G_{\lambda_i}, \hat{\delta}_i \to G_{\delta_i}}. \tag{14}$$

The GPQ for the common coefficient of variation across k populations is obtained as a weighted combination of the individual GPQs and is defined as

$$G_{\tilde{\theta}} = \frac{\sum_{i=1}^{k}\left[G_{\tilde{\theta}_i}/G_{Var(\hat{\theta}_i)}\right]}{\sum_{i=1}^{k}\left[1/G_{Var(\hat{\theta}_i)}\right]}. \tag{15}$$

Consequently, the $100(1-\alpha)\%$ two-sided confidence interval for θ is determined from the empirical percentiles of this GPQ from the AGCI method, which is given by

$$CI_\theta^{\mathrm{AGCI}} = \left[L_\theta^{AGCI}, U_\theta^{AGCI}\right] = [G_{\tilde{\theta}}(\alpha/2), G_{\tilde{\theta}}(1-\alpha/2)], \tag{16}$$

where $G_{\tilde{\theta}}(\alpha/2)$ and $G_{\tilde{\theta}}(1-\alpha/2)$ are the $100_{\frac{\alpha}{2}}$-th and $100_{\left(1-\frac{\alpha}{2}\right)}$-th percentiles of the distribution of $G_{\tilde{\theta}}$, respectively.

In essence, the AGCI method extends the GCI framework by incorporating an adjustment to the GPQ that helps reduce bias. This modification is particularly beneficial in small-sample settings or when skewness is strong, where the standard GCI approach may yield inaccurate coverage probabilities. As a result, the AGCI method provides more reliable confidence intervals with improved finite-sample performance. The detailed computational procedure is outlined in Algorithm 2.

Algorithm 2: AGCI	
1.	Produce $X_{ij} = \left(X_{i1}, X_{i2}, \ldots, X_{in_i}\right)$, for $i = 1, 2, \ldots, k$, $j = 1,2, \ldots, n_i$ from $X_{ij} \sim$ delta-IG$(\delta_i, \mu_i, \lambda_i)$, then compute for $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$.
2.	Generate $\chi^2_{n_{i(1)}-1}$ and $t_{n_{i(1)}-1}$ from Chi-square and Student's t-distributions, respectively.
3.	Estimate G_{λ_i}, $G_{\widetilde{\mu}_i}$, and G_{δ_i} using (5), (12), and (7), respectively.
4.	Iterate steps 1–3 for $t = 1, 2, \ldots, 5000$ times to get CI_θ^{AGCI} as (16).

2.3. Parametric Bootstrap Percentile Confidence Interval (PBPCI)

The PBPCI, introduced by Efron and Tibshirani [21], is a simulation-based method for constructing confidence intervals for a parameter of interest. Unlike traditional approaches that rely on asymptotic theory or strict distributional assumptions, this method assumes that the data follow a specified parametric model, such as a normal or Weibull distribution, and estimates its parameters from the observed sample.

The core idea is to generate a large number of bootstrap samples from the fitted parametric distribution rather than resampling directly from the data. For each bootstrap sample, the statistic of interest is computed, producing an empirical distribution. The confidence interval is then obtained from the $\alpha/2$ and $1-\alpha/2$ percentiles of this distribution. For example, a 95% confidence interval corresponds to the 2.5th and 97.5th percentiles.

When the assumed model is appropriate, PBPCI often provides greater efficiency and accuracy than nonparametric bootstrap methods, particularly for small sample sizes. In this study, the PBPCI method is implemented using 5000 bootstrap replications, and the parameter δ_i is estimated via the VST approach. The full procedure is summarized in Algorithm 3.

Let X^*_{ij} denotes the bootstrap samples. Define $n^*_{i(0)}$ and $n^*_{i(1)}$ as the numbers of zero and nonzero observations, respectively, such that $n^*_i = n^*_{i(0)} + n^*_{i(1)}$. The estimator of CV for the delta-IG distribution is given by

$$\hat{\theta}^*_i = \sqrt{(\hat{\mu}^*_i + \hat{\lambda}^*_i\hat{\delta}^*_i)/(1-\hat{\delta}^*_i)\hat{\lambda}^*_i}, \tag{17}$$

where $\hat{\delta}^*_i = n^*_{i(0)}/n^*_i$, $\hat{\mu}^*_i = \bar{X}^*_i = \frac{1}{n^*_{i(1)}}\sum_{j=1}^{n^*_{i(1)}} X^*_{ij}$, $n^*_{i(1)}\hat{\lambda}^{*-1}_i = V^*_i = \sum_{j=1}^{n^*_{i(1)}}(X^{*\,-1}_{ij} - \bar{X}^{*\,-1}_i)$, and $i = 1,2,\ldots,k$.

The corresponding variance of $\hat{\theta}^*_i$ under the PBPCI method is obtained by replacing $\left(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i\right)$ in Equation (3) with their corresponding estimators $(\hat{\mu}^*_i, \hat{\lambda}^*_i, \hat{\delta}^*_i)$ is given by

$$Var\left(\hat{\theta}^*_i\right) = Var\left(\hat{\theta}_i\right)\big|_{\hat{\mu}_i\to\hat{\mu}^*_i, \hat{\lambda}_i\to\hat{\lambda}^*_i, \hat{\delta}_i\to\hat{\delta}^*_i}. \tag{18}$$

The common coefficient of variation across multiple delta-IG samples is then estimated as a weighted average, given by

$$\hat{\theta}^* = \frac{\sum_{i=1}^k[\hat{\theta}^*_i/Var(\hat{\theta}^*_i)]}{\sum_{i=1}^k[1/Var(\hat{\theta}^*_i)]}. \tag{19}$$

Finally, the $100(1-\alpha)\%$ two-sided CI of θ for the delta-IG distribution based on the PBPCI method is given by

$$CI_\theta^{\text{PBPCI}} = \left[\hat{\theta}^*(\alpha/2), \hat{\theta}^*(1-\alpha/2)\right], \tag{20}$$

where $\hat{\theta}^*(\alpha/2)$ and $\hat{\theta}^*(1-\alpha/2)$ are the $100\frac{\alpha}{2}$-th and $100_{\left(1-\frac{\alpha}{2}\right)}$-th percentiles of the distribution of $\hat{\theta}^*$.

Algorithm 3: PBPCI

1. Produce $X_{ij} = \left(X_{i1}, X_{i2}, \dots, X_{in_i}\right)$, for $i = 1,2,\dots,k$, $j = 1,2,\dots,n_i$ from $X_{ij} \sim$ delta-IG$(\delta_i, \mu_i, \lambda_i)$.
2. Collect the bootstrap sample $X_{i1}^*, X_{i2}^*, \dots, X_{in_i}^*$ from $X_{i1}, X_{i2}, \dots, X_{in_i}$ attained in step 1.
3. Estimate the interest parameters $\hat{\theta}^*$ from the bootstrap sample.
4. Iterate steps 1–3 5000 times.
5. Estimate CI_θ^{PBPCI} using (20).

2.4. Bayesian Credible Interval (BCI)

A BCI provides a range of plausible values for an unknown parameter based on Bayesian inference. It is derived from the posterior distribution, which combines prior information with the likelihood obtained from observed data via Bayes' theorem. For the delta-IG distribution, the posterior is obtained by integrating the likelihood function with the chosen prior distributions. The choice of prior is critical, as it can substantially affect the resulting interval, particularly in small samples. The likelihood function of the delta-IG distribution is given by

$$L(\theta|X_i) \propto {\delta_i}^{n_{i(0)}}(1-\delta_i)^{n_{i(1)}}\lambda_i^{n_{i(1)}/2}\exp\left(-\frac{\lambda_i}{2\mu_i^2}\left(t_1 - 2n_{i(1)}\mu_i + t_2\mu_i^2\right)\right), \quad (21)$$

where $t_1 = \sum_{j=1}^{n_i} X_{ij}$, and $t_2 = \sum_{j=1}^{n_i} X_{ij}^{-1}$, $i = 1,2,\dots,k$.

The posterior density is then proportional to the product of the prior and the likelihood, given by

$$f(\theta|X_i) \propto f(\theta)L(\theta|X_i). \quad (22)$$

Since the resulting posterior distributions are analytically intractable, Markov Chain Monte Carlo (MCMC) methods, specifically Gibbs sampling, are used to generate the posterior samples. In this study, non-informative priors are assigned as follows: a Uniform(0, M) prior for μ_i, where M is set sufficiently large relative to the observed data range, a Gamma(0.001, 0.001) prior for λ_i, and a Beta(1,1) prior for δ_i, the latter being equivalent to a Uniform(0,1) distribution. The computation is implemented using the R2OpenBUGS package (version 3.2-3.2.1) in the R programming language (version 4.2.1).

Based on the posterior samples, the estimator of the CV for the delta-IG distribution is defined as

$$\hat{\theta}_i^{BS} = \sqrt{(\hat{\mu}_i^{BS} + \hat{\lambda}_i^{BS}\hat{\delta}_i^{BS})/(1-\hat{\delta}_i^{BS})\hat{\lambda}_i^{BS}}, \quad (23)$$

The corresponding variance of $\hat{\theta}_i^{BS}$ under the BCI method is obtained by replacing $\left(\hat{\mu}_i, \hat{\lambda}_i, \hat{\delta}_i\right)$ in Equation (3) with their corresponding estimators $(\hat{\mu}_i^{BS}, \hat{\lambda}_i^{BS}, \hat{\delta}_i^{BS})$ and is given by

$$Var\left(\hat{\theta}_i^{BS}\right) = Var\left(\hat{\theta}_i\right)\big|_{\hat{\mu}_i \to \hat{\mu}_i^{BS}, \hat{\lambda}_i \to \hat{\lambda}_i^{BS}, \hat{\delta}_i \to \hat{\delta}_i^{BS}}. \quad (24)$$

The common CV across multiple populations is then estimated as a weighted average, defined as

$$\hat{\theta}^{BS} = \frac{\sum_{i=1}^{k}\left[\hat{\theta}_i^{BS}/Var\left(\hat{\theta}_i^{BS}\right)\right]}{\sum_{i=1}^{k}\left[1/Var\left(\hat{\theta}_i^{BS}\right)\right]}. \quad (25)$$

Finally, the $100(1-\alpha)\%$ two-sided CI of θ for the delta-IG distribution based on the BCI method is given by

$$CI_\theta^{\text{BCI}} = \left[\hat{\theta}^{BS}(\alpha/2), \hat{\theta}^{BS}(1-\alpha/2)\right], \quad (26)$$

where $\hat{\theta}^{BS}(\alpha/2)$ and $\hat{\theta}^{BS}(1-\alpha/2)$ are the $100_{\frac{\alpha}{2}}$-th and $100_{\left(1-\frac{\alpha}{2}\right)}$-th percentiles of the distribution of $\hat{\theta}^{BS}$.

2.5. Highest Posterior Density Approach (HPDCI)

The HPD approach is a commonly used Bayesian method for constructing credible intervals for unknown parameters. Unlike frequentist confidence intervals, which rely on repeated sampling properties, HPD intervals are derived directly from the posterior distribution, incorporating both observed data and prior information.

An HPD interval is defined as the shortest interval that contains a specified proportion of the posterior probability mass. It represents the region where the parameter values are most plausible. This feature makes HPD intervals particularly useful when the posterior distribution is skewed, multimodal, or non-normal, situations in which equal-tailed intervals may fail to capture uncertainty accurately. As a result, HPD intervals provide a more informative summary of the posterior distribution. In practice, HPD intervals can be computed in R using the HPDinterval() function from the *coda* package.

Accordingly, the $100(1-\alpha)\%$ credible interval for θ based on the HPD method is defined by

$$CI_{\theta}^{\mathrm{HPD}} = \left[L_{\theta}^{\mathrm{HPD}}, U_{\theta}^{\mathrm{HPD}}\right], \tag{27}$$

where $L_{\theta}^{\mathrm{HPD}}$ and $U_{\theta}^{\mathrm{HPD}}$ are obtained using the HPDinterval() function. The detailed computational steps are presented in Algorithm 4.

Algorithm 4: BCI and HPDCI

1. Generate random variable $X_{ij} = \left(X_{i1}, X_{i2}, \ldots, X_{in_i}\right)$, for $i = 1,2,\ldots,k$, $j = 1,2,\ldots,n_i$ from $X_{ij} \sim$ delta-IG$(\delta_i, \mu_i, \lambda_i)$.
2. Define non-informative prior distributions for each parameter as $\mu_i \sim$ Uniform$(0, M)$, where M is set sufficiently large relative to the observed data range; $\lambda_i \sim$ Gamma$(0.001, 0.001)$; and $\delta_i \sim$ Beta$(1,1)$.
3. Utilize Gibbs sampling in R programming to gain the posterior distribution for three parameters $(\delta_i, \mu_i, \lambda_i)$.
4. Estimate the interest parameter $\hat{\theta}^{BS}$ as (25).
5. Iterate steps 3–4 20,000 times.
6. Burn in 5000 samples.
7. Estimate $CI_{\theta}^{\mathrm{BCI}}$ using (26) and $CI_{\theta}^{\mathrm{HPD}}$ using (27).

3. Simulation Studies and Results

This study evaluates five methods for constructing confidence intervals for the common coefficient of variation of the delta-IG distribution. A Monte Carlo simulation, implemented in the R programming language, is used to assess their performance. The methods are compared based on coverage probabilities (CPs) and average widths (AWs). A desirable method should achieve CPs close to or exceeding the nominal level of 0.95 while producing the narrowest possible interval width. The simulation consists of 5000 replications. For each iteration, data are generated from the delta-IG distribution, $X_{ij} \sim$ delta-IG$(\delta_i, \mu_i, \lambda_i)$, for $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,n_i$. The methods are evaluated under a range of parameter settings across two scenarios with $k = 3$ and $k = 6$. The mean parameters μ_i, shape parameters λ_i, zero-inflation probabilities δ_i, and sample sizes n_i are systematically varied. These configurations are designed to reflect diverse practical conditions, including different levels of skewness, dispersion, and zero inflation. Sample sizes are chosen to represent small-to-moderate cases, while the number of replications is sufficiently large to ensure stable and reliable estimates of CP and AW.

The results presented in Tables 1 and 2 and Figures 1 and 2 summarize the CPs and AWs of the confidence intervals for the common CV under delta-IG distributions across various parameter configurations. The results for $k = 3$ are reported in Table 1 and Figure 1, whereas those for $k = 6$ are presented in Table 2 and Figure 2.

Overall, none of the methods consistently achieved the nominal confidence level of 0.95 across all scenarios. A clear trade-off between coverage accuracy and interval efficiency was observed: methods with higher CPs generally produced wider intervals, while those with narrower widths tended to exhibit substantial undercoverage.

Across all settings, the HPDCI method provided the highest coverage probability among the competing methods. Although the average coverage probability (approximately 0.944) remained slightly below the nominal confidence level of 0.95, it consistently outperformed the alternative methods in terms of coverage accuracy. The BCI method ranked second, whereas the GCI and AGCI methods exhibited more pronounced undercoverage. The PBPCI method had the lowest coverage probability, indicating it did not adequately account for sampling variability of the CV estimator under the delta-IG distribution. Given a Monte Carlo standard error of approximately 0.003 based on 5000 replications, the observed average coverage probability of HPDCI remains statistically below the nominal level. This undercoverage reflects the inherent difficulty of constructing confidence intervals with exact nominal coverage for the delta-IG distribution, where skewness, zero inflation, and multiple nuisance parameters complicate the sampling distribution of the CV estimator. Nevertheless, across most parameter configurations, HPDCI consistently achieved CPs closer to the nominal level than the competing methods, making it the most reliable procedure among those considered in this study.

When interval width was considered, the PBPCI method yielded the narrowest intervals, followed by AGCI, GCI, HPDCI, and BCI. However, the interval width should be interpreted alongside the coverage probability rather than in isolation. Coverage probability is the primary criterion for evaluating confidence interval methods because reliable statistical inference requires the achieved coverage probability to be close to the nominal confidence level. Interval width serves as a secondary criterion to distinguish among methods with comparable coverage performance, with narrower intervals indicating greater precision. Although PBPCI produced the shortest intervals, its substantial undercoverage limits its practical applicability. In contrast, HPDCI maintained CPs closest to the nominal level while producing reasonably narrow intervals, thereby providing the best overall balance between reliability and precision among the competing methods.

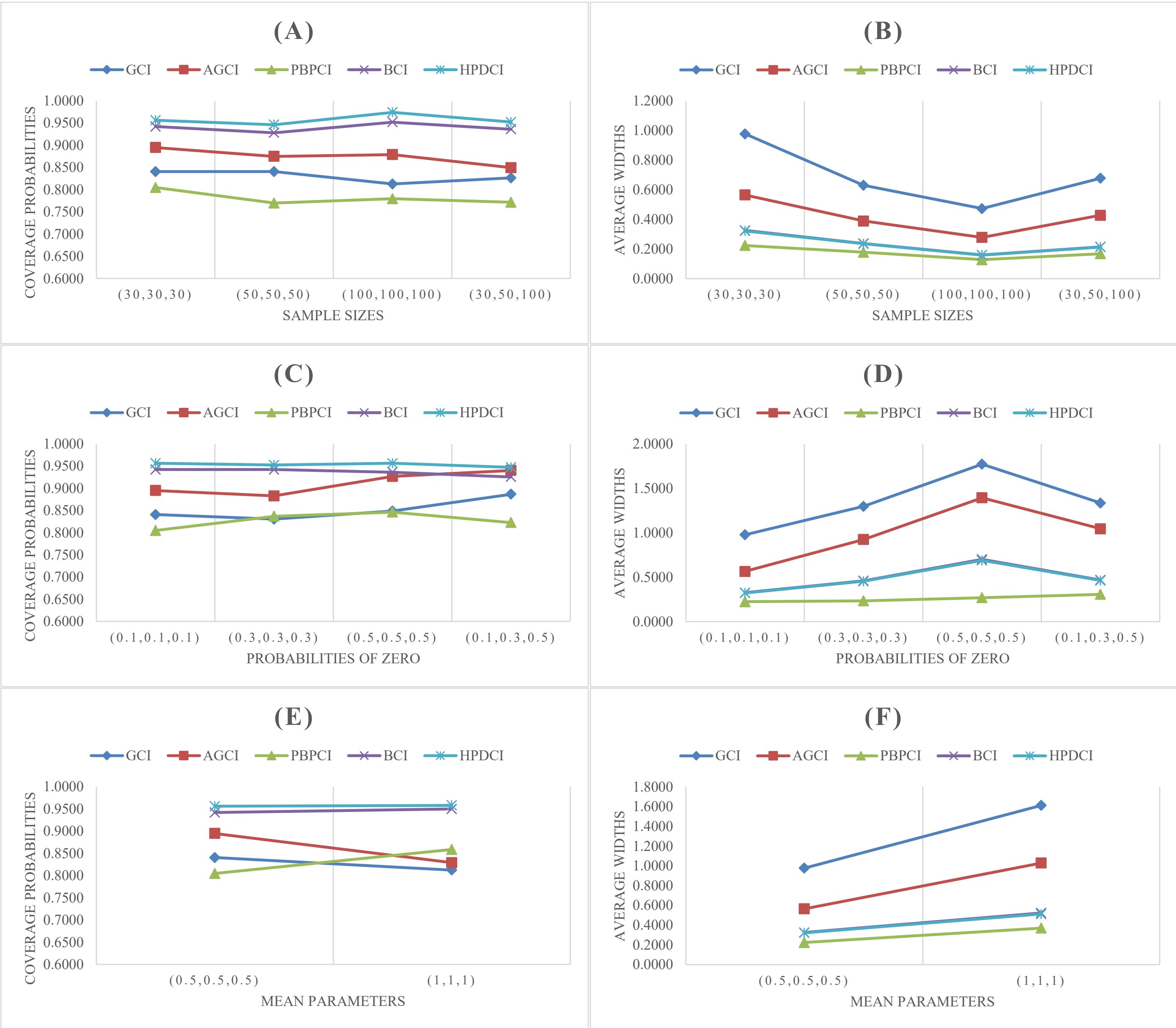
The AGCI method, despite incorporating a bias correction mechanism, did not show substantial improvement over GCI. Its CP remained below the nominal level in most cases, suggesting that the adjustment may be insufficient due to the combined effects of skewness and zero inflation inherent in the delta-IG distribution. Meanwhile, the GCI method exhibited undercoverage but produced wider intervals than AGCI.

The Bayesian-based approaches, namely BCI and HPDCI, clearly outperformed the frequentist methods in terms of coverage stability. This advantage likely stems from their ability to better account for parameter uncertainty, particularly in highly skewed and zero-inflated settings. Among these, HPDCI provided the most desirable balance between coverage accuracy and interval width.

Across different parameter configurations, including varying sample sizes, scale parameters, and zero-inflation levels, the relative performance of the methods remained consistent. Increasing heterogeneity and model complexity tended to exacerbate undercoverage for PBPCI, GCI, and AGCI, while HPDCI and BCI remained comparatively robust. This suggests that methods that rely on asymptotic assumptions or simplified variance structures are not well-suited to the delta-IG framework.

A comparison between $k = 3$ and $k = 6$ further reveals that, although the relative ranking of the methods remains largely unchanged, performance differences become more pronounced as the number of populations increases. In particular, undercoverage issues observed in PBPCI, GCI, and AGCI are more severe when $k = 6$, whereas HPDCI and BCI maintain relatively stable coverage. In addition, interval widths generally increase with k, reflecting the greater uncertainty associated with estimating a common CV across more populations.

Overall, HPDCI consistently achieved the highest coverage probability among all competing methods while maintaining reasonably short interval widths. Although its average coverage probability remained slightly below the nominal level, it provided the best overall compromise between coverage accuracy and interval efficiency.



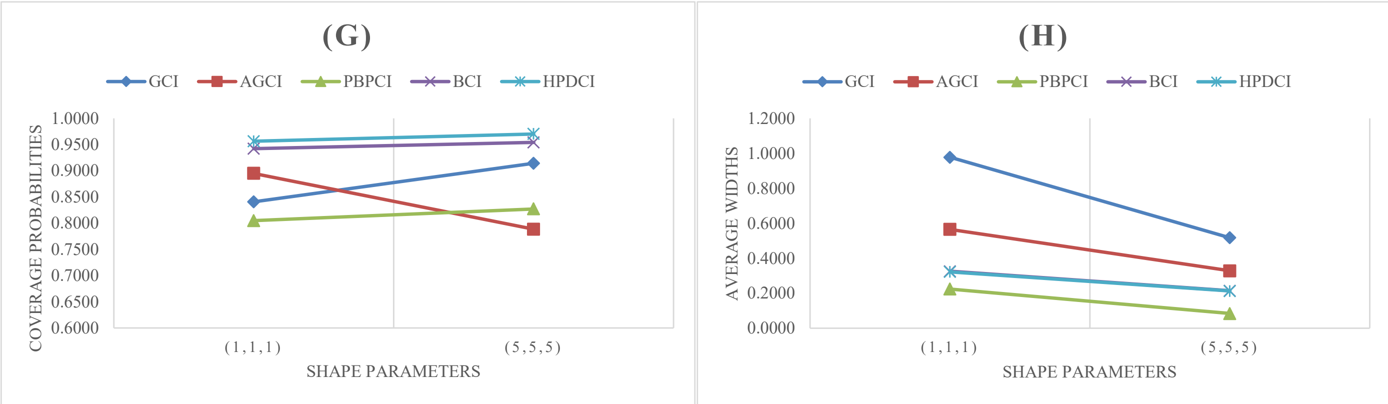
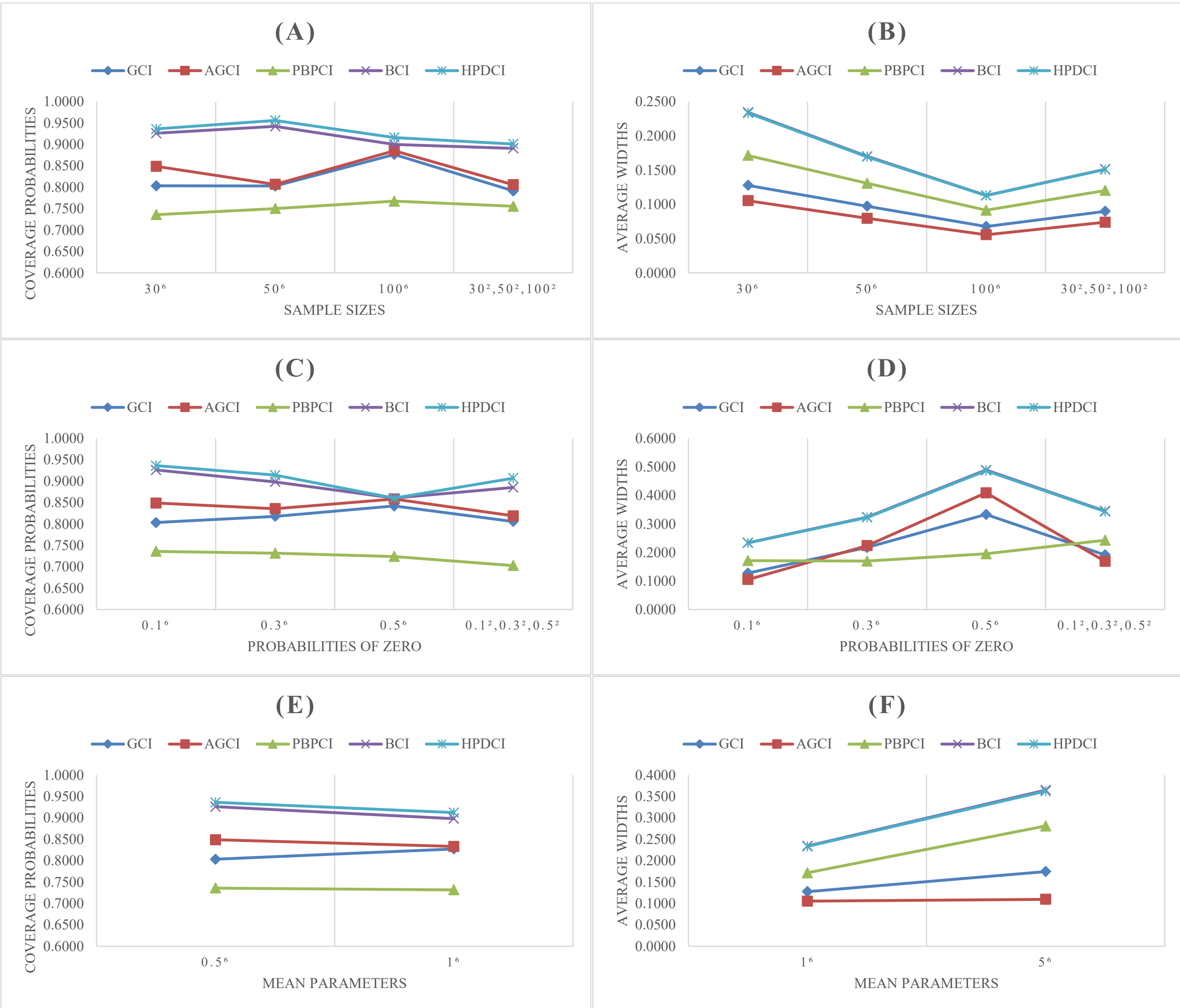


Figure 1. Comparison of the performance of confidence intervals for the common coefficient of variation for several delta-inverse Gaussian distributions for $k = 3$. The performance is evaluated using CP and AW for five methods: GCI, AGCI, PBPCI, BCI, and HPDCI. Panels (**A**,**C**,**E**,**G**) display CP with respect to sample sizes, zero-inflation probabilities δ, mean parameters μ, and shape parameters λ, respectively, while panels (**B**,**D**,**F**,**H**) present the corresponding AW results.



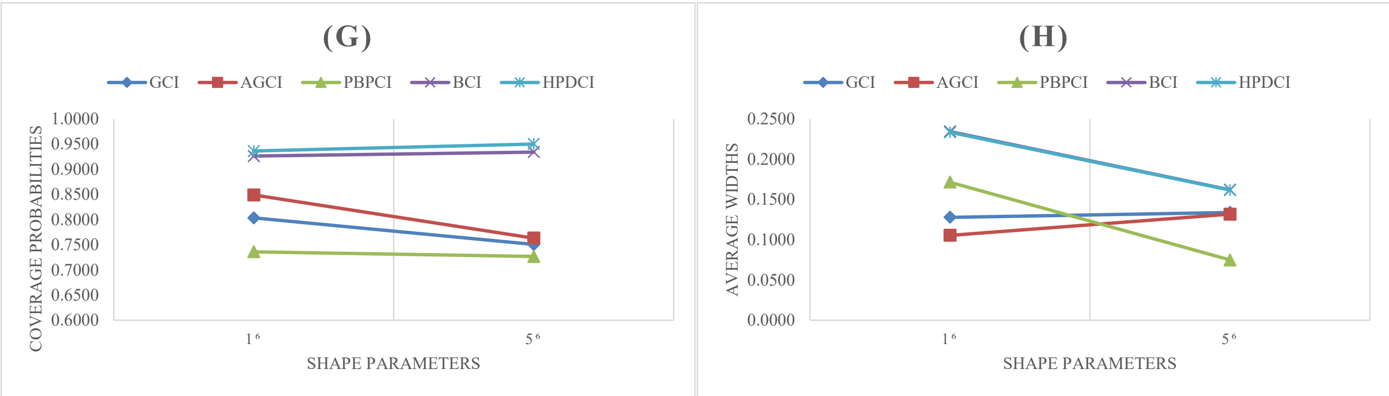


Figure 2. Comparison of the performance of confidence intervals for the common coefficient of variation for several delta-inverse Gaussian distributions for $k = 6$. The performance is evaluated using CP and AW for five methods: GCI, AGCI, PBPCI, BCI, and HPDCI. Panels (**A**,**C**,**E**,**G**) display CP with respect to sample sizes, zero-inflation probabilities δ, mean parameters μ, and shape parameters λ, respectively, while panels (**B**,**D**,**F**,**H**) present the corresponding AW results.

Table 1. Coverage probabilities and average widths of the 95% confidence intervals for the common coefficient of variation under multiple delta-IG distributions in the case of $k = 3$, with various sample sizes and parameter configurations (μ, λ, and δ) based on the GCI, AGCI, PBPCI, BCI, and HPDCI methods.

n_i^3	μ_i^3	λ_i^3	δ_i^3	Coverage Probabilities					Average Widths				
				GCI	AGCI	PBPCI	BCI	HPDCI	GCI	AGCI	PBPCI	BCI	HPDCI
30^3	0.5^3	1^3	0.1^3	0.8405	0.8948	0.8048	0.9421	**0.9561**	0.9774	0.5649	0.2236	0.3252	*0.3221*
			0.3^3	0.8305	0.8826	0.8369	0.9421	**0.9521**	1.2971	0.9239	0.2333	0.4592	*0.4542*
			0.5^3	0.8485	0.9265	0.8461	0.9361	**0.9561**	1.7717	1.3932	0.2693	0.7011	*0.6889*
			0.1, 0.3, 0.5	0.8864	0.9399	0.8228	0.9254	**0.9472**	1.3343	1.0448	0.3066	0.4686	*0.4623*
		5^3	0.1^3	0.9142	0.7884	0.8271	**0.9541**	**0.9701**	0.5174	0.3281	0.0840	0.2141	*0.2126*
			0.3^3	**0.9461**	0.9082	0.8575	0.9221	0.9381	*0.9246*	0.5026	0.0634	0.3141	0.3124
			0.5^3	**0.9401**	0.9202	0.8194	0.8842	0.8902	*1.2274*	0.8146	0.0686	0.4509	*0.4482*
			0.1, 0.3, 0.5	0.8727	0.8944	0.8271	0.9201	**0.9282**	0.9474	0.5835	0.0982	0.3419	*0.3391*
	1^3	1^3	0.1^3	0.8126	0.8291	0.8589	**0.9501**	**0.9581**	1.6129	1.0290	0.3700	0.5234	*0.5119*
			0.3^3	0.8589	0.8729	0.8499	**0.9481**	**0.9501**	2.2880	1.5339	0.3939	0.7283	*0.7089*
			0.5^3	0.8212	0.5908	0.8443	0.9381	**0.9501**	3.7561	2.2028	0.4698	1.1726	*1.1261*
			0.1, 0.3, 0.5	0.8050	0.8453	0.8275	0.8846	**0.9028**	2.9338	1.7207	0.4663	0.7555	*0.7325*
		5^3	0.1^3	0.8603	0.8770	0.8591	0.9321	**0.9541**	0.6030	0.4001	0.1246	0.2278	*0.2260*
			0.3^3	0.9102	0.9209	0.8425	0.9401	**0.9521**	0.8927	0.5355	0.1142	0.3350	*0.3331*
			0.5^3	0.9102	0.9202	0.8134	0.9121	**0.9142**	1.4204	0.7561	0.1282	0.5023	*0.4989*
			0.1, 0.3, 0.5	**0.9381**	0.9168	0.8242	0.8363	0.8512	*0.9795*	0.5812	0.1731	0.3529	0.3497
50^3	0.5^3	1^3	0.1^3	0.8405	0.8749	0.7699	0.9281	**0.9461**	0.6302	0.3895	0.1781	0.2369	*0.2352*
			0.3^3	0.8648	0.8765	0.7647	0.9401	**0.9581**	0.9266	0.5825	0.1901	0.3311	*0.3290*
			0.5^3	0.8589	0.9009	0.7553	0.9361	**0.9461**	1.2610	0.8736	0.2243	0.4912	*0.4875*
			0.1, 0.3, 0.5	0.8806	0.8981	0.8210	0.8287	**0.8461**	1.1224	0.6479	0.2581	0.3562	*0.3535*
		5^3	0.1^3	0.9281	0.9362	0.8487	0.9381	**0.9521**	0.3917	0.2920	0.0619	0.1629	*0.1620*
			0.3^3	0.9061	0.9301	0.7293	0.9201	**0.9361**	0.7223	0.3728	0.0505	0.2404	*0.2393*
			0.5^3	0.9441	0.9222	0.7240	0.9402	**0.9516**	0.9762	0.5717	0.0564	0.3518	*0.3501*
			0.1, 0.3, 0.5	0.8870	**0.9044**	0.7236	0.8722	0.8703	0.7399	*0.4013*	0.0814	0.2711	0.2694
	1^3	1^3	0.1^3	0.8226	0.8423	0.7667	0.9401	**0.9681**	1.0367	0.6299	0.2906	0.3633	*0.3589*
			0.3^3	0.8792	0.8890	0.7631	0.9261	**0.9481**	1.5211	0.9598	0.3267	0.4935	*0.4874*
			0.5^3	0.8156	0.8313	0.7543	0.9301	**0.9401**	2.3653	1.5727	0.4012	0.7714	*0.7544*
			0.1, 0.3, 0.5	0.8174	0.8341	0.8253	0.8850	**0.9068**	1.5693	1.0820	0.3867	0.5184	*0.5110*
		5^3	0.1^3	0.8942	0.9092	0.8617	0.9281	**0.9461**	0.4898	0.2956	0.0969	0.1708	*0.1699*

https://doi.org/10.3390/sym18071194

			0.3^3	0.8802	0.8987	0.8431	0.9241	**0.9361**	0.6573	0.4294	0.0903	0.2510	*0.2498*
			0.5^3	0.8822	0.9026	0.8417	0.9281	**0.9361**	1.0631	0.5968	0.1052	0.3741	*0.3723*
			0.1, 0.3, 0.5	0.9182	0.9256	0.8194	0.9040	**0.9285**	0.8287	0.4176	0.1441	0.2758	*0.2741*
100^3	0.5^3	1^3	0.1^3	0.8128	0.8788	0.7794	**0.9521**	**0.9741**	0.4732	0.2786	0.1278	0.1594	*0.1586*
			0.3^3	0.8774	0.8936	0.7689	0.9421	**0.9561**	0.6639	0.3700	0.1362	0.2192	*0.2182*
			0.5^3	0.8836	0.9017	0.7653	0.9181	**0.9401**	0.8848	0.5399	0.1685	0.3266	*0.3249*
			0.1, 0.3, 0.5	0.8431	0.8629	0.8096	0.9369	**0.9373**	0.6063	0.4295	0.1956	0.2500	*0.2487*
		5^3	0.1^3	0.9122	0.9102	0.8489	**0.9501**	**0.9621**	0.3389	0.2035	0.0416	0.1112	*0.1108*
			0.3^3	0.9062	0.8802	0.8309	0.9201	**0.9401**	0.4383	0.2875	0.0363	0.1648	*0.1642*
			0.5^3	0.8942	0.8483	0.8285	0.9341	**0.9461**	0.6934	0.3972	0.0415	0.2428	*0.2419*
			0.1, 0.3, 0.5	0.8617	0.8204	0.8180	0.8788	**0.8884**	0.5178	0.3150	0.0586	0.1991	*0.1983*
	1^3	1^3	0.1^3	0.8888	0.8943	0.7774	0.9381	**0.9581**	0.7275	0.4169	0.2090	0.2402	*0.2385*
			0.3^3	0.8838	0.9016	0.7186	0.9301	**0.9541**	0.9918	0.5705	0.2370	0.3183	*0.3161*
			0.5^3	0.8060	0.8206	0.7689	0.9161	**0.9341**	1.3759	0.7884	0.3000	0.4670	*0.4633*
			0.1, 0.3, 0.5	0.8639	0.8743	0.8216	0.8555	**0.8753**	1.0621	0.6262	0.2957	0.3552	*0.3525*
		5^3	0.1^3	0.8263	0.8379	0.7725	0.9421	**0.9561**	0.3576	0.1832	0.0683	0.1166	*0.1160*
			0.3^3	0.7764	0.7926	0.8505	0.9301	**0.9401**	0.4203	0.2894	0.0656	0.1723	*0.1715*
			0.5^3	0.7445	0.7669	0.8395	0.9341	**0.9501**	0.6657	0.4035	0.0769	0.2567	*0.2556*
			0.1, 0.3, 0.5	0.9182	0.9207	0.8096	**0.9455**	**0.9461**	0.5349	0.3124	0.1079	0.1973	*0.1965*
30, 50, 100	0.5^3	1^3	0.1^3	0.8265	0.8495	0.7717	0.9361	**0.9521**	0.6778	0.4275	0.1679	0.2146	*0.2133*
			0.3^3	0.8607	0.8868	0.7671	0.9421	**0.9601**	1.0893	0.6528	0.1742	0.2970	*0.2953*
			0.5^3	0.8545	0.8653	0.8577	0.9201	**0.9361**	1.5720	0.9593	0.2131	0.4452	*0.4423*
			0.1, 0.3, 0.5	0.8417	0.8612	0.8158	0.9166	**0.9267**	0.9650	0.5534	0.3032	0.4211	*0.4188*
		5^3	0.1^3	0.9381	0.8743	0.7441	0.9361	**0.9541**	0.4778	0.2738	0.0560	0.1481	*0.1474*
			0.3^3	0.9361	0.9142	0.8315	0.9441	**0.9641**	0.6506	0.4085	0.0469	0.2179	*0.2170*
			0.5^3	0.9242	0.8982	0.8261	0.9281	**0.9461**	0.8505	0.6013	0.0522	0.3190	*0.3177*
			0.1, 0.3, 0.5	0.9381	0.8904	0.8188	0.9387	**0.9488**	0.5705	0.3649	0.0941	0.3396	*0.3379*
	1^3	1^3	0.1^3	0.8986	0.8914	0.7752	0.9401	**0.9561**	1.0967	0.7844	0.2780	0.3271	*0.3240*
			0.3^3	0.8972	0.9152	0.7669	0.9381	**0.9561**	1.6887	1.0832	0.3046	0.4403	*0.4360*
			0.5^3	0.8415	0.8794	0.7603	0.9361	**0.9441**	2.8200	1.6292	0.3810	0.6679	*0.6582*
			0.1, 0.3, 0.5	0.8758	0.8953	0.8214	0.9377	**0.9476**	1.4042	0.8791	0.4499	0.6012	*0.5953*
		5^3	0.1^3	0.8922	0.8924	0.8601	**0.9481**	**0.9661**	0.4255	0.3013	0.0909	0.1540	*0.1531*
			0.3^3	0.8902	0.9036	0.7477	0.9361	**0.9521**	0.7474	0.4335	0.0840	0.2278	*0.2268*
			0.5^3	0.8623	0.8852	0.8399	0.9341	**0.9521**	0.9889	0.6357	0.0975	0.3429	*0.3412*
			0.1, 0.3, 0.5	0.8365	0.8680	0.8168	**0.9670**	**0.9745**	0.6482	0.3794	0.1690	0.3383	*0.3366*

Note: Bold values indicate CPs that exceed the nominal confidence level of 0.95 (values greater than 0.945 are rounded to 0.95) or achieved the highest CP among all methods. Italicized values denote the smallest AW among methods that achieved satisfactory coverage probability.

Table 2. Coverage probabilities and average widths of the 95% confidence intervals for the common coefficient of variation under multiple delta-IG distributions in the case of $k = 6$, with various sample sizes and parameter configurations (μ, λ, and δ) based on the GCI, AGCI, PBPCI, BCI, and HPDCI methods.

n_i^6	μ_i^6	λ_i^6	δ_i^6	Coverage Probabilities					Average Widths				
				GCI	AGCI	PBPCI	BCI	HPDCI	GCI	AGCI	PBPCI	BCI	HPDCI
30^6	0.5^6	1^6	0.1^6	0.8034	0.8489	0.7359	0.9261	**0.9361**	0.1278	0.1054	0.1715	0.2343	*0.2333*
			0.3^6	0.8178	0.8358	0.7315	0.8982	**0.9142**	0.2173	0.2242	0.1698	0.3240	*0.3227*
			0.5^6	0.8419	0.8584	0.7242	0.8602	**0.8603**	0.3332	0.4092	0.1953	0.4892	*0.4866*
			$0.1^2, 0.3^2, 0.5^2$	0.8060	0.8190	0.7030	0.8852	**0.9070**	0.1916	0.1692	0.2429	0.3452	*0.3436*
		5^6	0.1^6	0.7505	0.7631	0.7265	0.9341	**0.9501**	0.1338	0.1316	0.0747	0.1621	*0.1616*
			0.3^6	0.8388	0.8503	0.7212	0.8942	**0.9142**	0.2356	0.2345	0.0470	0.2322	*0.2316*
			0.5^6	0.8623	0.8922	0.7144	0.9024	**0.9224**	0.3353	0.3496	0.0505	0.3354	*0.3345*
			$0.1^2, 0.3^2, 0.5^2$	0.8571	0.8621	0.7126	0.9323	**0.9423**	0.2369	0.2366	0.0854	0.2623	*0.2614*
	1^6	1^6	0.1^6	0.8275	0.8331	0.7319	0.8982	**0.9122**	0.1747	0.1097	0.2810	0.3649	*0.3624*

			0.3^6	0.8038	0.8068	0.7259	0.8822	**0.8902**	0.2538	0.2466	0.2931	0.5007	*0.4963*
			0.5^6	0.8020	0.8367	0.7196	**0.8882**	0.8862	0.3675	0.4794	0.3418	0.7675	*0.7561*
			$0.1^2, 0.3^2, 0.5^2$	0.8060	0.8190	0.7028	**0.9459**	**0.9591**	0.2385	0.1737	0.3599	0.5084	*0.5030*
		5^6	0.1^6	0.8449	0.8657	0.7335	0.9441	**0.9641**	0.1175	0.1123	0.1006	0.1690	*0.1684*
			0.3^6	0.8212	0.8523	0.7319	**0.9488**	**0.9502**	0.2203	0.2226	0.0850	0.2462	*0.2456*
			0.5^6	0.9134	0.9142	0.7238	0.9437	**0.9514**	0.3310	0.3628	0.0931	0.3640	*0.3629*
			$0.1^2, 0.3^2, 0.5^2$	0.9088	0.9033	0.7080	0.9090	**0.9203**	0.1997	0.1969	0.1407	0.2659	*0.2649*
50^6	0.5^6	1^6	0.1^6	0.8029	0.8068	0.7503	0.9421	**0.9561**	0.0974	0.0797	0.1308	0.1701	*0.1695*
			0.3^6	0.8583	0.8631	0.7445	0.9061	**0.9261**	0.1645	0.1690	0.1357	0.2358	*0.2350*
			0.5^6	0.8200	0.8531	0.7393	0.8702	**0.8922**	0.2498	0.2997	0.1635	0.3541	*0.3528*
			$0.1^2, 0.3^2, 0.5^2$	0.8320	0.8756	0.8016	0.9303	**0.9541**	0.1493	0.1368	0.1944	0.2597	*0.2587*
		5^6	0.1^6	0.7948	0.7904	0.7329	0.9401	**0.9541**	0.1100	0.1076	0.0463	0.1206	*0.1203*
			0.3^6	0.8643	0.9142	0.7255	0.9348	**0.9466**	0.1728	0.1713	0.0369	0.1733	*0.1728*
			0.5^6	0.8527	0.9182	0.7180	**0.9503**	**0.9564**	0.2500	0.2589	0.0413	0.2563	*0.2557*
			$0.1^2, 0.3^2, 0.5^2$	0.8192	0.8645	0.7130	0.9365	**0.9545**	0.1974	0.1991	0.0629	0.2064	*0.2059*
	1^6	1^6	0.1^6	0.8234	0.8461	0.7469	0.9241	**0.9381**	0.1339	0.0819	0.2145	0.2579	*0.2566*
			0.3^6	0.8028	0.8311	0.7357	0.9161	**0.9202**	0.1923	0.1864	0.2365	0.3475	*0.3457*
			0.5^6	0.8134	0.8515	0.7289	0.8802	**0.8882**	0.2754	0.3541	0.2880	0.5186	*0.5153*
			$0.1^2, 0.3^2, 0.5^2$	0.8904	0.9198	0.8030	0.9293	**0.9381**	0.1836	0.1361	0.2931	0.3699	*0.3678*
		5^6	0.1^6	0.7497	0.7824	0.7495	0.9305	**0.9406**	0.0932	0.0889	0.0698	0.1251	*0.1248*
			0.3^6	0.8401	0.8483	0.7376	0.8810	**0.9010**	0.1647	0.1657	0.0657	0.1844	*0.1840*
			0.5^6	0.8171	0.8283	0.7267	0.8909	**0.8911**	0.2476	0.2676	0.0742	0.2716	*0.2709*
			$0.1^2, 0.3^2, 0.5^2$	0.8824	0.9034	0.8020	0.9430	**0.9517**	0.1638	0.1648	0.1075	0.2061	*0.2055*
100^6	0.5^6	1^6	0.1^6	0.8765	0.8853	0.7675	0.9002	**0.9162**	0.0677	0.0557	0.0915	0.1132	*0.1128*
			0.3^6	0.9012	0.9122	0.7603	0.9281	**0.9461**	0.1123	0.1139	0.0990	0.1578	*0.1573*
			0.5^6	0.9015	0.9166	0.7527	0.8982	**0.9102**	0.1723	0.2058	0.1199	0.2336	*0.2329*
			$0.1^2, 0.3^2, 0.5^2$	0.8523	0.8858	0.8006	**0.9456**	**0.9699**	0.1036	0.0956	0.1429	0.1795	*0.1790*
		5^6	0.1^6	0.8447	0.8842	0.7515	0.9404	**0.9505**	0.0760	0.0743	0.0293	0.0807	*0.0805*
			0.3^6	0.8509	0.8703	0.7307	0.9008	**0.9208**	0.1161	0.1149	0.0254	0.1182	*0.1179*
			0.5^6	0.8505	0.8683	0.7267	0.9008	**0.9210**	0.1706	0.1762	0.0296	0.1737	*0.1732*
			$0.1^2, 0.3^2, 0.5^2$	0.8214	0.8549	0.8079	0.9354	**0.9574**	0.1416	0.1430	0.0429	0.1479	*0.1475*
	1^6	1^6	0.1^6	0.8192	0.8417	0.7733	0.9404	**0.9604**	0.0937	0.0565	0.1520	0.1721	*0.1714*
			0.3^6	0.8020	0.8898	0.7574	0.9206	**0.9208**	0.1307	0.1250	0.1700	0.2259	*0.2251*
			0.5^6	0.8567	0.8702	0.7564	0.9305	**0.9505**	0.1888	0.2419	0.2219	0.3375	*0.3361*
			$0.1^2, 0.3^2, 0.5^2$	0.8753	0.9140	0.7825	0.9099	**0.9119**	0.1280	0.0942	0.0784	0.1445	*0.1442*
		5^6	0.1^6	0.7810	0.8124	0.7030	0.9305	**0.9505**	0.0655	0.0628	0.0488	0.0838	*0.0836*
			0.3^6	0.8253	0.8527	0.7554	0.9206	**0.9406**	0.1122	0.1119	0.0464	0.1230	*0.1227*
			0.5^6	0.8142	0.8467	0.7366	0.9008	**0.9109**	0.1699	0.1829	0.0554	0.1831	*0.1826*
			$0.1^2, 0.3^2, 0.5^2$	0.8062	0.8637	0.7894	0.9099	**0.9119**	0.1170	0.1181	0.0784	0.1445	*0.1442*
$30^2, 50^2, 100^2$	0.5^6	1^6	0.1^6	0.7914	0.8059	0.7554	0.8909	**0.9010**	0.0898	0.0739	0.1204	0.1514	*0.1510*
			0.3^6	0.8399	0.8561	0.7475	0.9206	**0.9307**	0.1496	0.1537	0.1286	0.2132	*0.2126*
			0.5^6	0.8454	0.8734	0.7406	0.9216	**0.9416**	0.2290	0.2800	0.1512	0.3147	*0.3136*
			$0.1^2, 0.3^2, 0.5^2$	0.7004	0.7232	0.8013	0.9356	**0.9366**	0.1754	0.1735	0.2389	0.3154	*0.3144*
		5^6	0.1^6	0.8058	0.8144	0.7396	0.9404	**0.9505**	0.1019	0.0994	0.0476	0.1113	*0.1110*
			0.3^6	0.8808	0.9162	0.7277	0.9385	**0.9486**	0.1598	0.1593	0.0334	0.1596	*0.1591*
			0.5^6	0.8547	0.9321	0.7228	0.9281	**0.9483**	0.2296	0.2387	0.0377	0.2356	*0.2349*
			$0.1^2, 0.3^2, 0.5^2$	0.8349	0.8601	0.7089	0.9279	**0.9402**	0.2579	0.2624	0.0783	0.2763	*0.2753*
	1^6	1^6	0.1^6	0.8188	0.8425	0.7436	0.9107	**0.9208**	0.1232	0.0759	0.2040	0.2307	*0.2296*
			0.3^6	0.8006	0.8359	0.7366	0.9305	**0.9604**	0.1753	0.1677	0.2171	0.3114	*0.3099*
			0.5^6	0.7934	0.8016	0.7416	0.8513	**0.8614**	0.2528	0.3321	0.2751	0.4710	*0.4689*
			$0.1^2, 0.3^2, 0.5^2$	0.8343	0.8715	0.7141	**0.9485**	**0.9495**	0.2006	0.1740	0.3625	0.4448	*0.4431*
		5^6	0.1^6	0.7885	0.8044	0.7465	**0.9503**	**0.9703**	0.0864	0.0822	0.0715	0.1128	*0.1124*
			0.3^6	0.8341	0.8723	0.7465	0.9305	**0.9505**	0.1512	0.1527	0.0611	0.1641	*0.1637*
			0.5^6	0.8246	0.8788	0.7376	0.9008	**0.9109**	0.2273	0.2482	0.0690	0.2487	*0.2480*
			$0.1^2, 0.3^2, 0.5^2$	0.9026	0.9333	0.8020	0.9327	**0.9337**	0.2045	0.2109	0.1411	0.2661	*0.2654*

Note: Bold values indicate CPs that exceed the nominal confidence level of 0.95 (values greater than 0.945 are rounded to 0.95) or achieved the highest CP among all methods. Italicized values denote the smallest AW among methods that achieved satisfactory coverage probability.

4. Empirical Applications

Statistical analysis of environmental and meteorological data plays a fundamental role in fields such as climatology, agriculture, renewable energy planning, and disaster risk management. Such data frequently exhibit a substantial proportion of zero observations together with marked right-skewness in the nonzero measurements, reflecting the intermittent and highly variable nature of natural phenomena such as rainfall and wind. These characteristics limit the applicability of conventional statistical models and motivate the use of zero-inflated distributions that jointly accommodate excess zeros and substantial variability in the nonzero component.

This section applies the proposed confidence interval procedures for the common coefficient of variation under multiple delta-inverse Gaussian distributions to two independent empirical settings: wind speed data and rainfall amount data, each collected from three geographically distinct meteorological stations in Thailand. Rather than focusing on variability within a single population or pairwise comparisons between locations, the common CV framework assumes a shared relative variability across multiple stations and aims to estimate this common measure while accounting for heterogeneity in the underlying location-specific parameters. As a scale-free measure of dispersion, the CV enables meaningful comparisons across stations with differing average wind speeds or rainfall amounts. In meteorological and environmental studies, estimating a common CV across multiple monitoring stations provides critical insights that separate group-specific averages from regional structural traits. For the wind speed application, establishing a common CV allows wind energy developers to evaluate the shared relative volatility and stability of wind resources; even when individual stations exhibit different mean wind speeds due to distinct local topographies, knowing the common relative dispersion is essential for grid stability planning and optimizing regional wind power backup systems. For rainfall-amount application, the common CV is a crucial scale-free metric for characterizing the regional homogeneity and relative variability of precipitation, especially when dealing with zero-inflated data. This shared measure of variability is highly valuable for joint reservoir design, collective water resource management, and regional agricultural risk assessment against droughts or floods in adjacent areas.

Importantly, the three stations selected for each application are situated in geographically distinct provinces governed by distinct local topography and weather systems, providing a reasonable basis for treating the resulting measurements as independent populations. The two applications are presented in turn: Section 4.1 considers wind speed data from three coastal weather stations, and Section 4.2 considers rainfall amount data from three stations in northern Thailand. For each application, we first identify an appropriate distributional model among several candidate asymmetric distributions using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), and then construct and compare the 95% confidence intervals for the common CV obtained from the five methods considered in this study: GCI, AGCI, PBPCI, BCI, and HPDCI.

4.1. Wind Speed Data

Wind speed is a fundamental atmospheric quantity driven by the movement of air from areas of high to low pressure, usually due to temperature variations. Wind speed and direction significantly affect various factors, including evaporation rates, sea surface turbulence, the formation of ocean waves and storms, and water quality, as well as weather forecasting, climatology, renewable energy, environmental monitoring, aviation, and agriculture. Therefore, a comprehensive understanding of wind speed variability is essential for managing its potential impacts across these fields.

The CV is particularly useful for assessing wind speed variability, as it measures the dispersion of data relative to the mean regardless of the unit of measurement. A high CV

https://doi.org/10.3390/sym18071194

indicates that wind speed is highly variable, making wind conditions more difficult to predict and potentially leading to inconsistent energy production in wind farms. When analyzing wind variability across multiple weather stations simultaneously, the common CV serves as a single representative indicator of overall wind speed variability, without requiring adjustments for differences in average wind speed at each location. This makes it especially valuable for planning wind energy projects, designing wind turbines, and examining the relationship between wind variability and long-term climate changes, as well as recurring events such as tropical storms and shifts in seasonal wind patterns.

In this study, we apply wind speed data from multiple directions to construct confidence intervals for the common coefficient of variation of several delta-IG distributions. The data were collected between 1 January 2024 and 7 January 2024 from three weather stations operated by the Thai Meteorological Department: the Chanthaburi Weather Observing Station in Chanthaburi Province, the Chumphon Weather Observing Station in Chumphon Province, and the Songkhla Weather Observing Station in Songkhla Province (Thai Meteorological Department Automatic Weather System, https://www.tmd.go.th/service/tmdData (accessed on 28 June 2026)). These three stations were selected for their proximity to the Gulf of Thailand, where they are directly influenced by sea breezes and tropical storms, resulting in pronounced wind-speed fluctuations that significantly affect the livelihoods, economy, and environment of surrounding communities.

The wind speed data from all three stations contain a mixture of zero values (representing calm conditions with no wind) and positive values, as shown in Table 3. Histograms of the wind speed data from each station are shown in Figure 3, clearly illustrating both zero and positive observations. Summary statistics for each station are provided in Table 4, showing estimated coefficients of variation for wind speed of 5.4360, 4.4281, and 5.5299 at the Chanthaburi, Chumphon, and Songkhla Weather Observing Stations, respectively.

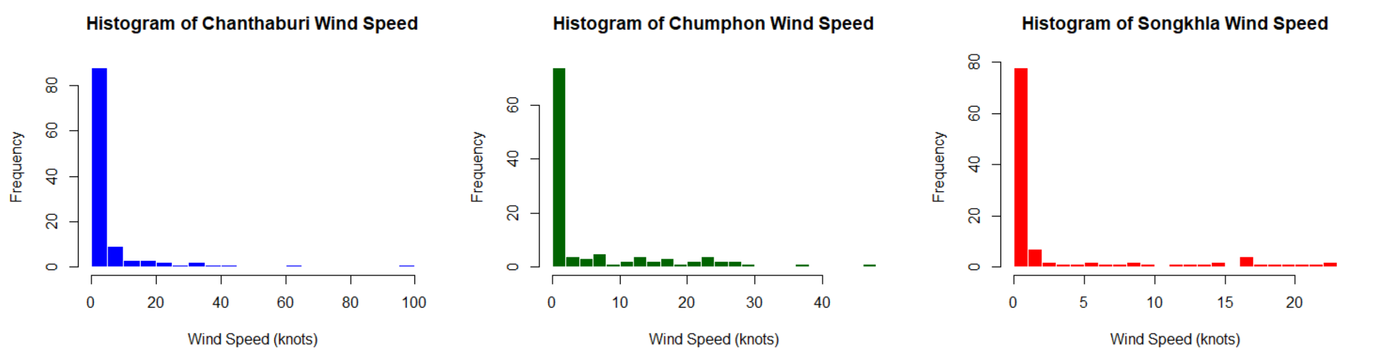


Figure 3. Histograms of wind speed for each weather-observing station.

Table 3. Wind speed (knots) data recorded from the Chanthaburi Weather Observing Station, Chumphon Weather Observing Station, and Songkhla Weather Observing Station, Thailand.

Provinces	Wind Speed (Knots)																	
Chanthaburi	4.7	19.4	25.7	10.2	1.7	0.9	0.3	2.2	0.7	0.3	1.5	1.5	0.3	1.8	9.2	18.2	20.6	8.2
	0.7	0.1	0.7	1.9	0.4	0.6	1.7	0.3	0.3	2.2	7.4	9.8	31.8	36.5	5.6	0.8	0.1	1.2
	5.4	8.3	30.4	43.9	5.6	0.1	0.1	2.6	2.3	25	60.7	11.3	0.4	0.2	100	19.2	9.9	1.2
	0.3	0.9	13.3								0 (55 times)							
Chumphon	0.8	21.4	14.6	2.8	7.6	11	13.2	6.3	1.5	0.5	17.6	13.9	3.5	0.9	1.5	26.5	18.1	0.3
	0.2	15.6	23.5	5.9	0.9	4.6	21	14	0.4	11.5	25.2	7.9	0.6	1.9	16.1	29.2	0.3	17.8
	36.2	1.9	0.3	2	22.2	13.1	0.1	8.7	26.6	6.2	1	2.9	22.6	24.2	0.2	0.3	5.4	2.3

	1.8	7.1	46.9	23	0.1	0 (53 times)												
Songkhla	0.9	4.8	12.2	17	11.3	0.8	0.1	0.4	0.3	5.8	19.2	16.9	7.2	1.5	0.2	3.7	13.8	21
	14.9	2.8	0.2	0.1	0.4	0.1	0.1	1.4	6.9	17.6	16.3	5.3	0.3	0.1	1.5	8.3	21.9	17
	1.5	0.1	0.1	0.4	1.8	9.4	22.1	14.6	2.1	0.1	0.3	0.1	0.3	1.3	8.8	22.2	18.8	1.7
	0.5	0.2	0 (56 times)															

Table 4. Fundamental statistics for the wind speed data in Thailand.

Statistics	Provinces		
	Chanthaburi	Chumphon	Songkhla
n_i	112	112	112
$n_{i(0)}$	55	53	56
$n_{i(1)}$	57	59	56
$\hat{\mu}_i$	10.0094	10.3978	6.4027
$\hat{\lambda}_i$	0.6880	1.0550	0.4329
$\hat{\delta}_i$	0.4911	0.4732	0.5000
$\hat{\theta}_{ij}$	5.4360	4.4281	5.5299

To identify an appropriate model for real-data analysis, model selection is conducted using AIC and BIC, both widely used likelihood-based measures for comparing competing models. The AIC, proposed by Akaike [35], is defined as

$$AIC = -2\ln L + 2p, \tag{28}$$

while the BIC, introduced by Schwarz [36], is given by

$$BIC = -2\ln L + p\ln(n), \tag{29}$$

respectively. Here, L denotes the likelihood function, p is the number of estimated parameters, and n is the sample size. As shown in Table 5, the Inverse Gaussian distribution yields the lowest AIC and BIC values among all candidate distributions considered, indicating that it provides the best fit for the positive wind speed observations. Since the wind speed data consist of both zero and positive values, with the positive component well described by the Inverse Gaussian distribution, the delta-IG distribution is therefore appropriate for modeling this dataset.

Accordingly, the proposed confidence interval methods were applied to the wind speed data from all three weather stations. Table 6 presents the 95% confidence intervals for the common coefficient of variation obtained using the GCI, AGCI, PBPCI, BCI, and HPDCI methods. The results show that the 95% confidence interval for the common coefficient of variation of wind speed data using the PBPCI method is the narrowest (width = 1.3889), indicating the highest efficiency, followed by AGCI (width = 2.5148), HPDCI (width = 2.9668), BCI (width = 3.0061), and GCI (width = 5.2497), which produced the widest interval among all methods.

From a methodological perspective, the narrower interval produced by PBPCI suggests greater efficiency but may come at the cost of undercoverage, as observed in the simulation results. The notably wider interval obtained from GCI in this application can be attributed to its reliance on a normal-approximation pivotal quantity for μ that does not explicitly account for the additional uncertainty in estimating λ. When the effective sample size for the nonzero component is reduced, as is the case here due to the substantial proportion of zero observations across all three stations ($\hat{\delta}$ ranging from 0.4732 to 0.5000), this pivotal quantity becomes more prone to producing extreme values in the tails of its sampling distribution, which propagate through the nonlinear CV transformation and inflate the resulting interval width. In contrast, AGCI incorporates a chi-square-adjusted pivotal quantity that better reflects the joint uncertainty of μ and λ in finite samples, yielding a substantially narrower interval than GCI while still maintaining closer-to-nominal coverage than PBPCI, based on the simulation results. Although AGCI does not achieve the narrowest width among all methods, it offers a more favorable trade-off between interval efficiency and coverage accuracy than PBPCI, which tends to undercover due to its reliance on the empirical bootstrap distribution without theoretical correction for finite-sample bias. The AGCI method therefore provides a balance between width and stability, while the Bayesian-based methods yield wider intervals that may offer improved reliability by better accounting for parameter uncertainty, skewness, and excess zeros inherent in the delta-IG distribution.

Table 5. AIC and BIC values for the fitting of asymmetric distributions.

Criteria	Provinces	Distributions						
		Normal	Lognormal	Weibull	Gamma	Exponential	Logistic	Inverse Gaussian
AIC	Chanthaburi	490.5147	341.5048	345.8316	350.1323	378.6146	466.0012	336.8198
	Chumphon	450.8045	400.6373	391.7504	390.3980	396.3523	450.8063	389.5093
	Songkhla	388.2634	301.9090	300.2085	299.5289	321.9991	391.0419	297.6235
BIC	Chanthaburi	494.6008	345.5909	349.9177	354.2184	380.6577	470.0873	340.9059
	Chumphon	454.9596	404.7924	395.9055	394.5531	398.4299	454.9614	393.6644
	Songkhla	392.3141	305.9597	304.2592	303.5796	324.0244	395.0926	301.6742

Table 6. The 95% CIs for the common CV from the wind speed data in Thailand.

Point Estimations	Methods	CI of the Common CV	
		Interval	Width
$\hat{\theta} = 4.9447$	GCI	(1.0022, 6.2519)	5.2497
	AGCI	(3.7644, 6.2793)	2.5148
	PBPCI	(4.4141, 5.8029)	1.3889
	BCI	(4.1317, 7.1378)	3.0061
	HPDCI	(4.0980, 7.0648)	2.9668

4.2. Rainfall Amount Data

Rainfall is one of the most critical climatic variables influencing water resource management, agriculture, and disaster risk planning. As a highly variable atmospheric phenomenon governed by complex interactions among topography, seasonal monsoon systems, and local convective processes, rainfall amounts often fluctuate substantially over time and across locations, with many days recording no precipitation. Understanding rainfall variability is therefore essential for assessing drought risk, planning irrigation systems, and forecasting potential flooding, particularly in regions where livelihoods depend heavily on agricultural water supply.

The CV is a widely used measure for quantifying rainfall variability, as it expresses dispersion relative to the mean rainfall amount, thereby allowing comparisons across locations with substantially different average rainfall levels. A high CV indicates highly erratic rainfall patterns, which complicate agricultural planning and increase the risk of both drought and flash flooding, while a low CV reflects more consistent and predictable precipitation. When examining rainfall variability across multiple meteorological stations simultaneously, the common CV provides a single representative measure of overall rainfall dispersion across locations, without requiring separate adjustment for differing mean rainfall levels at each station. This makes the common CV particularly valuable for regional water resource planning, climate risk assessment, and the study of broader hydrological patterns such as monsoon variability and long-term climatic shifts.

In this study, we apply daily rainfall amount data to construct confidence intervals for the common CV of several delta-IG distributions. The data were daily collected in April to June of 2025 from three weather stations operated by the Northern Meteorological Center of Thai Meteorological Department: the Mae Hong Son Weather Observing Station in Mae Hong Son Province, the Phayao Weather Observing Station in Phayao Province, and the Lamphun Weather Observing Station in Lamphun Province (Northern Meteorological Center of Thai Meteorological Department, https://cmmet.tmd.go.th/index1.php (accessed on 28 June 2026)). These three stations were selected because they are situated in geographically distinct provinces across northern Thailand, each with distinct local topography and monsoon exposure, resulting in rainfall patterns that exhibit considerable regional variability while remaining reasonably independent of one another.

The rainfall data from all three stations contain a mixture of zero values (representing days with no measurable rainfall) and positive values, as shown in Table 7. Histograms of the rainfall data from each station are displayed in Figure 4, clearly illustrating the presence of both zero and positive observations together with pronounced right-skewness among the nonzero amounts. Summary statistics for each station are provided in Table 8, showing that each dataset comprises 91 daily observations, with zero-rainfall proportions of 41.76%, 56.04%, and 51.65% at the Mae Hong Son, Phayao, and Lamphun Weather Observing Stations, respectively. The estimated coefficients of variation of rainfall are 2.2322, 3.5180, and 4.7973 for the Mae Hong Son, Phayao, and Lamphun stations, respectively. These values indicate considerable relative variability in rainfall, although they are generally lower than those observed for the wind speed data presented in Section 4.1.

Table 7. Rainfall amount (millimeters) data recorded from the Mae Hong Son, Phayao, and Lamphun Weather Observing Stations, Thailand.

Provinces	**Rainfall Amount (Millimeters)**																
Mae Hong Son	3.7	9.5	4.6	2.4	2.6	0.3	9.9	9.8	3.7	0.5	3.4	0.6	18.0	5.3	2.4	16.2	0.6
	12.6	1.1	7.9	0.4	4.4	2.0	0.6	5.1	7.8	3.6	31.9	9.0	3.5	20.2	3.0	2.1	3.3
	8.0	8.3	0.8	6.6	8.5	11.5	0.9	4.2	3.2	1.2	0.3	0.8	2.7	2.0	6.5	3.6	26.5
	9.6	1.8								0 (38 times)							
Phayao	11.5	7.6	25.9	1.3	0.4	42.3	5.4	3.0	6.5	3.0	0.6	16.4	8.0	14.1	8.3	5.0	16.2
	33.0	60.5	3.6	7.8	3.1	10.9	0.4	3.9	3.2	6.9	2.7	11.0	30.9	0.9	16.2	1.0	2.6
	0.2	0.6	1.6	2.4	7.2	0.4						0 (51 times)					
Lamphun	3.6	19.1	0.4	0.6	0.2	31.2	6.0	2.3	7.7	0.1	2.3	6.1	10.6	1.9	72.4	26.0	14.6
	8.4	1.2	10.2	1.2	0.2	71.5	38.9	13.1	21.7	1.5	3.5	5.2	0.6	3.3	18.8	5.9	1.8
	15.2	2.8	0.4	0.1	1.0	1.0	16.6	5.7	1.6	7.4				0 (47 times)			

Table 8. Fundamental statistics for the rainfall amount data in Thailand.

Statistics	**Provinces**		
	Mae Hong Son	**Phayao**	**Lamphun**
n_i	91	91	91
$n_{i(0)}$	38	51	47
$n_{i(1)}$	53	40	44
$\hat{\mu}_i$	6.0195	9.6649	10.5493
$\hat{\lambda}_i$	2.4222	1.9802	0.99345
$\hat{\delta}_i$	0.4176	0.5604	0.5165
$\hat{\theta}_{ij}$	2.2324	3.5184	4.7989

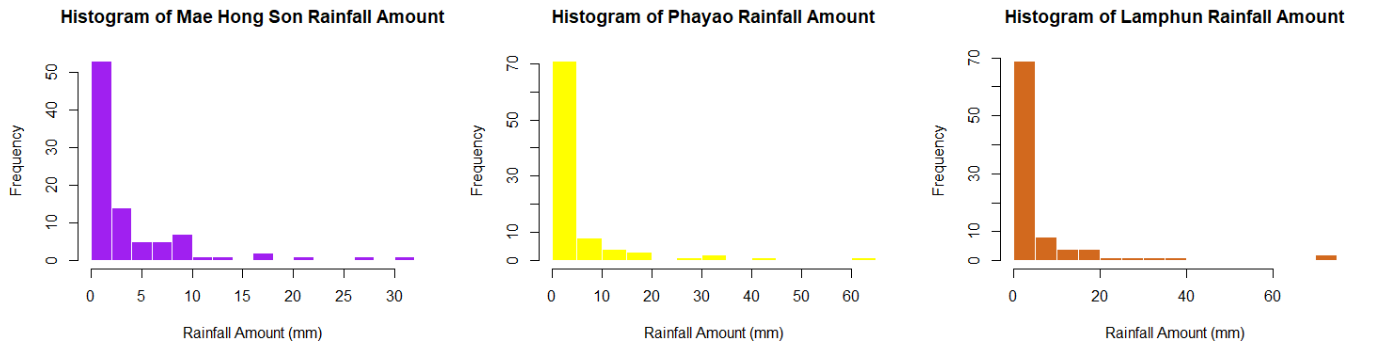


Figure 4. Histograms of rainfall amounts for each weather-observing station.

To identify an appropriate model for real-data analysis, model selection is performed using the AIC and BIC to compare competing models. As shown in Table 9, the Inverse Gaussian distribution yields the lowest AIC and BIC values among all candidate distributions considered, indicating that it provides the best fit for the positive rainfall amount observations. Since the rainfall amount data consist of both zero and positive values, with the positive component well described by the Inverse Gaussian distribution, the delta-IG distribution is therefore appropriate for modeling this dataset.

Then, the proposed confidence interval methods were applied to the rainfall amount data from all three weather stations. Table 10 presents the 95% confidence intervals for the common CV obtained using the GCI, AGCI, PBPCI, BCI, and HPDCI methods. The results show that the 95% confidence interval for the common CV of rainfall amount data using the HPDCI method is the narrowest (width = 0.9787), followed by BCI (width = 0.9873), AGCI (width = 1.1109), GCI (width = 1.2901), and PBPCI (width = 1.4313), which produced the widest interval among all methods.

From a methodological perspective, the pattern observed for the rainfall data differs notably from that of the wind speed data. Rather than GCI producing the widest interval as observed previously in the wind speed data, PBPCI yielded the widest average width here, while the Bayesian-based methods achieved the narrowest widths. This reversal can be attributed to differences in the underlying shape parameter estimates between the two datasets. The rainfall data exhibit comparatively larger estimated shape parameters ($\hat{\lambda}$ = 2.4222, 1.9802, and 0.9935) than those of the wind speed data ($\hat{\lambda}$ = 0.6880, 1.0550, and 0.4329), despite both datasets showing similar proportions of zero observations ($\hat{\delta}$ averaging approximately 0.50 for rainfall and 0.49 for wind speed). Since the variance of the inverse Gaussian component is inversely related to λ, the larger $\hat{\lambda}$ values in the rainfall data correspond to lower variability among the nonzero observations, thereby stabilizing the normal-approximation pivotal quantity underlying GCI and preventing the extreme tail behavior observed in the wind-speed application. As a result, GCI performs comparably to AGCI for the rainfall data rather than producing a disproportionately wide interval.

The relatively wider interval from PBPCI in the rainfall application, by contrast, likely reflects the heavier-tailed and more skewed nature of rainfall measurements, in which extreme precipitation events in the resampled bootstrap datasets can produce highly variable CV estimates across replications. Because PBPCI relies entirely on the empirical distribution of the resampled statistic without any theoretical adjustment for skewness or finite-sample bias, it is more susceptible to this source of variability than the pivotal-based or Bayesian approaches. The Bayesian-based methods, which incorporate prior information and yield posterior distributions that smooth over sampling irregularities, instead produced the most stable and narrowest intervals for this dataset. Thus, the Bayesian approach may offer particular advantages when the underlying nonzero component exhibits stronger asymmetry or extreme observations, as is characteristic of rainfall data.

Taken together, these contrasting results across the two applications indicate that the relative performance of the confidence interval methods is not uniform across datasets but depends critically on the magnitude and stability of the estimated shape parameter $\hat{\lambda}$, as well as on the degree of skewness in the nonzero component of the data. GCI tends to underperform when $\hat{\lambda}$ is small, while PBPCI tends to underperform when the data exhibit pronounced skewness or extreme values, even when the proportion of zero observations is comparable across datasets. Although AGCI provided a reasonable balance between efficiency and stability in both applications, the simulation results indicated that it suffers from undercoverage relative to the nominal 95% level. Considering the empirical application results and the coverage performance observed in the simulation study, HPDCI is therefore recommended as the preferred method for constructing confidence intervals for the common CV under the delta-IG model, as it consistently achieved CPs closer to the nominal level while maintaining reasonably stable interval widths across both applications.

Table 9. AIC and BIC values for the fitting of asymmetric distributions.

Criteria	Provinces	Distributions						
		Normal	Lognormal	Weibull	Gamma	Exponential	Logistic	Inverse Gaussian
AIC	Mae Hong Son	352.4592	300.0902	300.2422	300.2401	298.2593	341.0109	297.5276
	Phayao	320.1124	261.6382	262.1174	263.0323	263.4602	308.9811	260.6721
	Lamphun	373.1772	286.3943	286.4132	288.3389	297.2822	356.5758	285.6182
BIC	Mae Hong Son	356.3998	304.0308	304.1828	304.1807	300.2296	344.9515	302.4682
	Phayao	323.4902	265.0160	265.4952	266.4101	265.1491	312.3588	264.0499
	Lamphun	376.7456	289.9627	289.9816	291.9073	299.0663	360.1442	287.1866

Table 10. The 95% CIs for the common CV from the rainfall amount data in Thailand.

Point Estimations	Methods	CI of the Common CV	
		Interval	Width
$\hat{\theta} = 2.5898$	GCI	(1.9796, 3.2697)	1.2901
	AGCI	(2.1285, 3.2394)	1.1109
	PBPCI	(2.7802, 4.2115)	1.4313
	BCI	(1.8984, 2.8857)	0.9873
	HPDCI	(1.8893, 2.8680)	0.9787

5. Conclusions

This study investigated five confidence interval procedures: GCI, AGCI, PBPCI, BCI, and HPDCI for the common coefficient of variation of multiple delta-inverse Gaussian distributions, addressing a gap in the literature where existing methods do not adequately account for the combined effects of skewness, zero inflation, and multiple nuisance parameters across several populations.

The Monte Carlo simulation results for k = 3 and k = 6 populations showed that none of the methods consistently attained the nominal 0.95 confidence level across all parameter configurations, revealing a persistent trade-off between coverage accuracy and interval efficiency. Among the competing methods, the HPDCI approach consistently achieved the highest CP across a wide range of parameter settings. As noted in Section 3, its average coverage probability of approximately 0.944 remained statistically below the nominal level of 0.95. Although this was the smallest degree of undercoverage among the methods examined, it is still not negligible. Nevertheless, it outperformed the competing methods in terms of coverage accuracy. The BCI method also demonstrated relatively stable performance, though its coverage similarly remained below the nominal target. By contrast, GCI and AGCI exhibited noticeable undercoverage across most settings, indicating that the bias correction mechanism in AGCI was insufficient to fully offset the combined effects of skewness and excess zeros. PBPCI produced the narrowest intervals overall but suffered from severe undercoverage, limiting its reliability despite its apparent efficiency.

As interval efficiency should not be evaluated independently of coverage probability, HPDCI is recommended as the preferred method for general use, as it offered the most favorable overall compromise between coverage accuracy and interval width, even though it did not attain the nominal level in every scenario considered. Nonetheless, certain conditions may justify prioritizing PBPCI despite its undercoverage: when sample sizes are sufficiently large for asymptotic properties to take hold, in exploratory analyses where only a plausible range of values is sought rather than a formally guaranteed interval, or in applications where narrower intervals are valued and some undercoverage can be tolerated. Conversely, when sample sizes are small to moderate, when the data exhibit strong skewness and high zero inflation, or when the number of populations is large, coverage accuracy becomes a more critical consideration, and HPDCI is strongly recommended.

The Bayesian-based methods, particularly HPDCI, achieved a more desirable balance between coverage and width by effectively incorporating parameter uncertainty. As the number of populations increased from $k = 3$ to $k = 6$, the relative ranking of the methods remained unchanged, but performance differences became more pronounced. Undercoverage issues in PBPCI, GCI, and AGCI were exacerbated, while HPDCI and BCI maintained relatively stable coverage, demonstrating greater robustness to increasing model complexity.

The two empirical applications—wind speed data from three coastal stations and rainfall amount data from three northern stations in Thailand—were both well described by the delta-IG distribution based on AIC and BIC, corroborating and extending the simulation findings. For the wind speed data, PBPCI produced the narrowest interval (width = 1.3889), followed by AGCI (2.5148), HPDCI (2.9668), BCI (3.0061), and GCI, which produced the widest interval (5.2497). The unusually wide GCI interval was attributable to the relatively small estimated shape parameters across the three stations ($\hat{\lambda}$ ranging from 0.4329 to 1.0550), which amplified the sensitivity of the pivotal quantity for μ in the normal approximation, despite a substantial proportion of zero observations ($\hat{\delta}$ averaging approximately 0.49) common to all three series. For the rainfall data, the pattern reversed: HPDCI produced the narrowest interval (width = 0.9787), followed by BCI (0.9873), AGCI (1.1109), GCI (1.2901), and PBPCI, which produced the widest interval (1.4313). Here, the comparatively larger shape parameter estimates ($\hat{\lambda}$ ranging from 0.9935 to 2.4222) stabilized the GCI pivotal quantity, bringing its performance close to that of AGCI, while the heavier-tailed and more skewed nature of the rainfall measurements appeared to inflate the bootstrap-based PBPCI interval instead. Relative efficiency calculations using HPDCI as the reference method confirmed these patterns in both applications, with PBPCI attaining the highest relative efficiency for wind speed (RE = 2.1361) and the lowest for rainfall (RE = 0.6838).

Taken together, these contrasting empirical results demonstrate that the relative performance of the confidence interval methods is not uniform across datasets but depends critically on the magnitude and stability of the estimated shape parameter $\hat{\lambda}$, as well as on the degree of skewness in the nonzero component of the data. GCI tends to underperform when $\hat{\lambda}$ is small, while PBPCI tends to underperform when the data exhibit pronounced skewness or extreme values, even under comparable levels of zero inflation. Although

AGCI provided a reasonable balance between efficiency and stability in both applications, the simulation results indicated that it remains prone to undercoverage relative to the nominal level. Considering the simulation and empirical findings jointly, HPDCI is recommended as the preferred method for constructing confidence intervals for the common CV under the delta-IG model, as it consistently achieved CPs closer to the nominal level while maintaining reasonably stable interval widths across both sample sizes, parameter configurations, and the two distinct empirical applications considered in this study.

Despite its superior overall performance, HPDCI did not achieve the nominal coverage level of 0.95 in all simulation settings. This residual undercoverage indicates that constructing confidence intervals for the common coefficient of variation under delta-IG distributions remains challenging, particularly in the presence of substantial skewness and zero inflation.

Overall, the findings emphasize that properly accounting for skewness and zero inflation is essential when constructing confidence intervals for the common CV under delta-IG distributions. Methods that fail to address these characteristics tend to exhibit poor coverage performance. The HPDCI method is recommended because it consistently achieved the highest coverage probability among the competing methods while maintaining reasonably short interval widths. Although this undercoverage is statistically significant rather than negligible, HPDCI provided the most favorable compromise between coverage accuracy and interval efficiency across the simulation settings considered. The BCI provides a reasonable alternative method. In contrast, PBPCI, GCI, and AGCI do not offer a satisfactory balance between accuracy and efficiency in this setting. These results highlight the importance of selecting appropriate methods for reliable inference in complex distributional frameworks involving zero-inflated and highly skewed data.

6. Discussion

The results provide clear, practically relevant insights into interval estimation for the common coefficient of variation under delta-IG distributions in the presence of skewness and zero inflation. Performance is strongly influenced by the degree of zero inflation, parameter heterogeneity, and the number of populations k: interval widths expand as dimensionality increases. This reflects the added uncertainty in estimating a shared parameter across more groups; hence, methods relying on simplified assumptions or asymptotic approximations become correspondingly less reliable.

A key finding is that coverage accuracy and interval efficiency are driven by different mechanisms, and no method optimizes both simultaneously. HPDCI consistently gives coverage probabilities closest to the nominal level, and BCI performs similarly but slightly weaker; both benefit from incorporating parameter uncertainty directly into the posterior, which matters most when the CV estimator's sampling distribution is strongly skewed by zero inflation. PBPCI achieves the narrowest intervals but at the cost of severe undercoverage, reflecting the general risk of prioritizing precision without adequately modeling distributional asymmetry. GCI and AGCI produce wider intervals than PBPCI, yet still fail to achieve satisfactory coverage; the bias correction in AGCI is insufficient to offset the combined effects of skewness and excess zeros. This coverage–efficiency trade-off cannot be eliminated; it can only be managed: HPDCI is preferable when reliable inference is the priority, whereas narrower but less conservative methods, such as PBPCI, may be acceptable when some undercoverage can be tolerated.

The two empirical applications reinforce this picture while adding an important qualification: method performance is not uniform across datasets, but depends on the magnitude of the estimated shape parameter $\hat{\lambda}$ and the degree of skewness in the nonzero component. In the wind speed data, comparatively small $\hat{\lambda}$ values amplified the sensitivity of GCI's normal-approximation pivotal quantity, producing an unusually wide interval; in the rainfall data, larger $\hat{\lambda}$ values stabilized GCI, so it performed comparably to AGCI, whereas the heavier-tailed rainfall measurements inflated the bootstrap-based PBPCI interval. HPDCI and BCI remained comparatively stable across both datasets. Practically, this suggests examining the estimated shape parameter and the skewness of the nonzero component before relying on GCI or PBPCI, since their relative performance can reverse depending on these characteristics.

Two further considerations qualify the proposed framework. First, the three weather stations in each application were treated as independent because they lie in geographically separated provinces with distinct topography and weather systems; however, meteorological variables can still exhibit broader spatial or temporal dependence (shared synoptic systems, monsoon patterns, within-station serial correlation) that the i.i.d. delta-IG assumption does not capture, and ignoring this may affect validity to some degree. Second, a prior sensitivity analysis (Appendix A) showed that the interval width for BCI and HPDCI is highly robust to the choice of hyperparameters, but coverage probability is more sensitive, particularly under small effective sample sizes or high zero-inflation probabilities; more informative priors tended to improve coverage in most, but not all, configurations. The non-informative prior used in the main analysis remains a reasonable default, but practitioners working with small or strongly zero-inflated samples may benefit from testing mildly informative priors and conducting a similar sensitivity check.

In summary, the findings underscore that reliable interval estimation for the common CV in non-normal settings requires methods that explicitly address skewness and zero inflation. Among the methods considered, HPDCI emerges as the most robust in terms of coverage, while other approaches present clear limitations in balancing accuracy and efficiency. These results provide a practical foundation for selecting appropriate methods for applications involving complex and heterogeneous data. This study presents a systematic comparison of CI procedures for the common CV within a flexible distributional framework, complementing prior pairwise-comparison approaches; however, its scope is restricted to delta-IG distributions, with extremely small-sample scenarios and potential model misspecification left unexplored.

Finally, several promising directions exist for future research to expand upon this work. While this study investigated five approach frameworks for independent delta-IG populations, with a specific focus on the common coefficient of variation, future studies could shift the inferential focus to other critical population parameters, such as the median or specific upper percentiles (e.g., the 90th or 95th percentiles) of delta-IG distributions. Because these location-based parameters are highly robust against extreme right skewness, developing interval estimation procedures for them would provide indispensable tools for risk assessment in meteorology. Additionally, future research could extend these procedures to other skewed and zero-inflated distributions, such as zero-inflated Rayleigh models, to accommodate varying degrees of heavy-tailedness. Crucially, because meteorological data often possess inherent geographical and temporal structures, the proposed framework could be extended to accommodate spatially or temporally dependent data. This could be achieved by incorporating spatial random effects, conditional autoregressive (CAR) models, time-series structures, or copula-based dependence structures, thereby providing a more realistic and statistically rigorous framework.

Author Contributions: Conceptualization, W.K., S.-A.N., and S.N.; Methodology, W.K. and S.-A.N.; Software, W.K.; Validation, S.-A.N. and S.N.; Formal Analysis, S.N.; Investigation, S.-A.N. and S.N.; Resources, W.K.; Data Curation, W.K.; Writing—Original Draft, W.K.; Preparation, W.K.; Writing—Review and Editing, S.-A.N. and S.N.; Visualization, W.K.; Supervision, S.-A.N. and S.N.; Project Administration, S.-A.N.; Funding Acquisition, S.-A.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research budget was allocated by the National Science Research and Innovation Fund (NSRF) and King Mongkut's University of Technology North Bangkok (KMUTNB-FF-69-B-07).

Data Availability Statement: Data are contained within the article.

Acknowledgments: The authors express gratitude to King Mongkut's University of Technology North Bangkok for providing the opportunity and resources for research. During the preparation of this manuscript, the authors used ChatGPT (OpenAI, GPT-5.5) for the purposes of English language editing and improving the clarity, grammar, and readability of the manuscript. The authors have reviewed and edited all AI-assisted outputs and take full responsibility for the content of this publication.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Prior Sensitivity Analysis

To assess whether the coverage and width performance of the Bayesian-based methods reported in this study are sensitive to the choice of prior hyperparameters, rather than artifacts of the specific non-informative prior adopted in the main analysis, a supplementary sensitivity analysis was conducted using three prior configurations that represent increasing degrees of prior informativeness. These configurations were applied to the first eight parameter settings considered in the k = 3 simulation study (Table 1) and are summarized in Table A1.

Table A1. Prior hyperparameter configurations used in sensitivity analysis.

Prior Set	μ_i	λ_i	δ_i
S1 (Main, non-informative)	Uniform(0, M)	Gamma(0.001, 0.001)	Beta(1,1)
S2 (Less vague)	Uniform(0, M)	Gamma(0.01, 0.01)	Beta(1,1)
S3 (Weakly informative)	Uniform(0, M)	Gamma(0.1, 0.1)	Beta(2,2)

For each prior configuration, the CP and AW of the resulting 95% credible/HPD intervals were computed for both BCI and HPDCI under the same eight parameter settings and replication scheme used in the main simulation study. The results are presented in Table A2.

Table A2. Coverage probabilities and average widths of BCI and HPDCI under three prior configurations (S1, S2, S3).

S1: Beta(1,1), Gamma(0.001,0.001) (Main)				**S2: Beta(1,1), Gamma(0.01,0.01)**				**S3: Beta(2,2), Gamma(0.1,0.1)**			
CP		**AW**		**CP**		**AW**		**CP**		**AW**	
BCI	**HPDCI**	**BCI**	**HPDCI**	**BCI**	**HPDCI**	**BCI**	**HPDCI**	**BCI**	**HPDCI**	**BCI**	**HPDCI**
0.9421	0.9561	0.3252	0.3221	0.9709	0.9612	0.3240	0.3209	0.9806	0.9903	0.3244	0.3213
0.9421	0.9521	0.4592	0.4542	0.9806	0.9612	0.4537	0.4492	0.9903	0.9903	0.4537	0.4492
0.9361	0.9561	0.7011	0.6889	0.9417	0.9320	0.7020	0.6897	0.9417	0.9417	0.6862	0.6742
0.9254	0.9472	0.4686	0.4623	0.8350	0.7476	0.4684	0.4621	0.8252	0.8058	0.4813	0.4753
0.9541	0.9701	0.2141	0.2126	0.9612	0.9612	0.2155	0.2138	0.9903	0.9903	0.2156	0.2141
0.9221	0.9381	0.3141	0.3124	0.9806	0.9806	0.3177	0.3161	0.9903	0.9903	0.3156	0.3140
0.8842	0.8902	0.4509	0.4482	0.8641	0.8447	0.4572	0.4548	0.9417	0.9223	0.4408	0.4385
0.9201	0.9282	0.3419	0.3391	0.9029	0.8544	0.3410	0.3377	0.9903	0.9806	0.3426	0.3401

Note: Cases 1–8 correspond to the first eight parameter configurations of Table 1 (k = 3) in the main text. CP and AW are reported for both BCI and HPDCI under each prior configuration.

The AWs of both BCI and HPDCI remained highly stable across all three prior specifications, with differences generally below 0.02 across the eight configurations examined (e.g., Case 1: AW of HPDCI = 0.3221, 0.3209, and 0.3213 under S1, S2, and S3, respectively). This indicates that the interval efficiency of the Bayesian-based methods is largely insensitive to the choice of prior, even when the prior shifts from a strongly non-informative specification (S1) to a weakly informative one (S3).

In contrast to interval width, coverage probability exhibited greater sensitivity to prior specification than initially anticipated, particularly in configurations associated with small effective sample sizes or high zero-inflation probabilities. Differences in CP between prior settings reached approximately 0.10–0.14 in some cases (e.g., Case 4: CP of HPDCI = 0.9472 under S1 versus 0.7476 under S2, a difference of approximately 0.20; Case 8: CP of HPDCI = 0.9282 under S1 versus 0.8544 under S2). In most configurations, however, the more informative priors (S2 and S3) tended to yield CPs closer to or exceeding the nominal level of 0.95 relative to the non-informative prior (S1), with the notable exception of Cases 4 and 7–8, where S2 instead produced lower coverage than S1 before recovering or exceeding the nominal level under S3.

These results are reported transparently rather than as evidence of uniform robustness. The general pattern indicates that, in small-sample and high-zero-inflation regimes, the regularizing effect of a more informative prior can meaningfully improve coverage beyond what the likelihood alone provides, since the limited information available in the nonzero component is supplemented by stronger prior structure on λ_i and δ_i. At the same time, the non-monotonic behavior observed in a subset of cases (e.g., Case 4) indicates that the relationship between prior informativeness and coverage is not strictly monotonic across all parameter regimes, and caution is warranted before generalizing this benefit to all small-sample settings.

Based on this analysis, the non-informative prior (S1) used throughout the main simulation and empirical applications remains a reasonable default when no strong prior information is available, as it does not materially compromise interval efficiency relative to S2 or S3. However, practitioners working with small samples or strongly zero-inflated data may benefit from considering mildly informative priors (such as S2 or S3) to improve coverage probability and from conducting a similar prior sensitivity analysis, as standard practice, when applying BCI or HPDCI to data with comparable characteristics.

References

1. Hsieh, H.K. Inferences on the coefficient of variation of an inverse Gaussian distribution. *Commun. Stat.—Theory Methods* **1990**, *19*, 1589–1605. https://doi.org/10.1080/03610929008830279.
2. Gupta, R.C.; Akman, O. Estimation of coefficient of variation in a weighted inverse Gaussian model. *Appl. Stoch. Models Data Anal.* **1996**, *12*, 255–263. https://doi.org/10.1002/(SICI)1099-0747(199612)12:4<255::AID-ASM287>3.0.CO;2-R.
3. Borenstein, M.; Hedges, L.V.; Higgins, J.P.T.; Rothstein, H.R. *Introduction to Meta-Analysis*; Wiley: Hoboken, NJ, USA, 2009. https://doi.org/10.1002/9780470743386.
4. Gelman, A.; Hill, J. *Data Analysis Using Regression and Multilevel/Hierarchical Models*; Cambridge University Press: Cambridge, UK, 2007. https://doi.org/10.1017/CBO9780511790942.
5. Graybill, F.A.; Deal, R.B. Combining unbiased estimators. *Biometrics* **1959**, *15*, 543–550. https://doi.org/10.2307/2527652.
6. Feltz, C.J.; Miller, G.E. An asymptotic test for the equality of coefficients of variation from k populations. *Stat. Med.* **1996**, *15*, 647–658. https://doi.org/ 10.1002/(sici)1097-0258(19960330)15:6<647::aid-sim184>3.0.co;2-p.
7. Forkman, J. Estimator and tests for common coefficients of variation in normal distributions. *Commun. Stat. Theory Methods* **2008**, *38*, 233–251. https://doi.org/10.1080/03610920802187448.
8. Ng, C.K. Inference on the common coefficient of variation when populations are lognormal: A simulation-based approach. *J. Stat. Adv. Theory Appl.* **2014**, *11*, 117–134.
9. Tian, L. Inferences on the common coefficient of variation. *Stat. Med.* **2005**, *24*, 2213–2220. https://doi.org/10.1002/sim.2088.

10. Puggard, W.; Niwitpong, S.A.; Niwitpong, S. Confidence intervals for common coefficient of variation of several Birnbaum–Saunders distributions. *Symmetry* **2022**, *14*, 2133. https://doi.org/10.3390/sym14102101.
11. Chhikara, R.S.; Folks, J.L. *The Inverse Gaussian Distribution: Theory, Methodology, and Applications*; Marcel Dekker: New York, NY, USA, 1989.
12. Chaubey, Y.P.; Singh, M.; Sen, D. Symmetrizing and variance stabilizing transformations of sample coefficient of variation from inverse Gaussian distribution. *Sankhya B* **2017**, *79*, 217–246. https://doi.org/10.1007/s13571-017-0136-z.
13. Singh, M.; Chaubey, Y.P.; Sen, D.; Sarker, A. Estimation and Testing of a Common Coefficient of Variation from Inverse Gaussian Distributions. In *Applied Statistics and Data Science*; Springer Proceedings in Mathematics & Statistics; Chaubey, Y.P., Lahmiri, S., Nebebe, F., Sen, A., Eds.; Springer: Cham, Switzerland, 2021; Volume 375, pp. 69–85. https://doi.org/10.1007/978-3-030-86133-9_5.
14. Bardsley, W.E. An alternative distribution for describing the instantaneous unit hydrograph. *J. Hydrol*. **1983**, *62*, 375–378. https://doi.org/10.1016/0022-1694(83)90115-4.
15. Punzo, A. A new look at the inverse Gaussian distribution with applications to insurance and economic data. *J. Appl. Stat.* **2019**, *46*, 1260–1287.
16. Jana, N.; Gautam, M. Confidence intervals of difference and ratio of means for zero-adjusted inverse Gaussian distributions. *Commun. Stat. Simul. Comput.* **2024**, *53*, 3302–3326. https://doi.org/10.1080/03610918.2022.2102652.
17. Khumpasee, W.; Niwitpong, S.A.; Niwitpong, S. Confidence Intervals for the Coefficient of Variation in Delta Inverse Gaussian Distributions. *Symmetry* **2024**, *16*, 1488. https://doi.org/10.3390/sym16111488.
18. Khumpasee, W.; Niwitpong, S.-A.; Niwitpong, S. Simultaneous Confidence Intervals for All Pairwise Differences of Coefficients of Variation of Delta-Inverse Gaussian Distributions. *Symmetry* **2026**, *18*, 604. https://doi.org/10.3390/sym18040604.
19. Weerahandi, S. Generalized confidence intervals. *J. Am. Stat. Assoc.* **1993**, *88*, 899–905.
20. Hannig, J.; Iyer, H.; Patterson, P. Fiducial Generalized Confidence Intervals. *J. Am. Stat. Assoc.* **2006**, *101*, 254–269. https://doi.org/10.1198/016214505000000736.
21. Efron, B.; Tibshirani, R. Bootstrap methods for standard errors, confidence intervals, and other measures of statistical accuracy. *Stat. Sci.* **1986**, *1*, 54–77.
22. Zou, G.Y.; Taleban, J.; Huo, C.Y. Confidence interval estimation for lognormal data with application to health economics. Comput. *Stat. Data Anal.* **2009**, *53*, 3755–3764. https://doi.org/10.1016/j.csda.2009.03.016.
23. Yosboonruang, N.; Niwitpong, S.-A.; Niwitpong, S. Bayesian Computation for the Common Coefficient of Variation of Delta-Lognormal Distributions with Application to Common Rainfall Dispersion in Thailand. *PeerJ* **2022**, *10*, e12858. https://doi.org/10.7717/peerj.12858.
24. Ye, R.D.; Ma, T.F.; Wang, S.G. Inferences on the common mean of several inverse Gaussian populations. *Comput. Stat. Data Anal.* **2010**, *54*, 906–915.
25. Bera, S.; Jana, N. On estimating common mean of several inverse Gaussian distributions. *Metrika* **2022**, *85*, 115–139. https://doi.org/10.1007/s00184-021-00829-y.
26. Slud, E.V.; Vonta, I.; Kagan, A.M. Combining estimators of a common parameter across samples. *Stat. Theory Relat. Fields* **2018**, *2*, 158–171. https://doi.org/10.1080/24754269.2018.1530903.
27. Folks, J.L.; Chhikara, R.S. The Inverse Gaussian Distribution and its Statistical Application—A Review. *J. R. Stat. Soc. Ser. B Methodol.* **1978**, *40*, 263–275. https://doi.org/10.1111/j.2517-6161.1978.tb01039.x.
28. Khumpasee, W.; Niwitpong, S.; Niwitpong, S. Confidence intervals for the difference and ratio of two coefficients of variation of delta-inverse Gaussian distributions. *Comput. J. Math. Stat. Sci.* **2026**, *5*, 331–353. https://doi.org/10.21608/cjmss.2026.438871.1306.
29. Weerahandi, S. Generalized Confidence Intervals. In *Exact Statistical Methods for Data Analysis. Springer Series in Statistics*; Springer: New York, NY, USA, 1995. https://doi.org/10.1007/978-1-4612-0825-9_6.
30. Krishnamoorthy, K.; Mathew, T. *Statistical Tolerance Regions: Theory, Applications, and Computation*, 1st ed.; Wiley: Abingdon, UK, 2009.
31. Khumpasee, W.; Niwitpong, S.A.; Niwitpong, S. Confidence Intervals for the Variance and Standard Deviation of Delta-Inverse Gaussian Distributions with Application to Traffic Mortality Count. *Symmetry* **2025**, *17*, 387. https://doi.org/10.3390/sym17030387.
32. Dasgupta, A. *Asymptotic Theory of Statistics and Probability*; Springer: New York, NY, USA, 2008.
33. Wu, W.H.; Hsieh, H.N. Generalized confidence interval estimation for the mean of delta-lognormal distribution: An application to New Zealand trawl survey data. *J. Appl. Stat.* **2014**, *41*, 1471–1485.

34. Krishnamoorthy, K.; Tian, L. Inferences on the difference and ratio of the means of two inverse Gaussian distributions. *J. Stat. Plan. Inference* **2008**, *138*, 2082–2089.
35. Akaike, H. A new look at the statistical model identification. *IEEE Trans. Autom. Control* **1974**, *19*, 716–723. https://doi.org/10.1109/TAC.1974.1100705.
36. Schwarz, G. Estimating the dimension of a model. *Ann. Stat.* **1978**, *6*, 461–464. https://doi.org/10.1214/aos/1176344136.