

Article

A Nonlinear Hoek–Brown Criterion for Bedded Rock with Brittle–Ductile Transition

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Abstract

The conventional Hoek–Brown (H–B) failure criterion does not explicitly account for the combined effects of anisotropy and high confining pressure on rock strength. In bedded rocks, anisotropic mechanical behavior arises from the orientation-dependent weakening induced by bedding planes, which breaks the stress symmetry relative to the loading direction. To address this limitation, this study proposes a modified H–B criterion that incorporates a dip-angle-dependent anisotropy parameter and a high-confinement correction term to describe the brittle–ductile transition of bedded rocks. The proposed criterion was implemented in the discrete element software 3DEC through secondary development of the built-in H–B module using the FISH language. Triaxial compression tests were conducted on bedded marble and sandstone specimens with different bedding dip angles under uniaxial compression and confining pressures of 40, 80, and 100 MPa to examine their stress–strain responses and failure characteristics. Comparisons between experimental results and predictions from existing failure criteria show that the proposed criterion provides improved prediction accuracy and effectively captures the variation in strength with bedding dip angle and confining pressure. Numerical triaxial tests were further performed on Martinsburg Slate specimens with bedding dip angles of 0°, 45°, and 90° under confining pressures of 50, 100, and 200 MPa. The results indicate a clear transition from elastoplastic softening to ductile deformation with increasing confining pressure. The compressive resistance follows the order $\alpha = 0^\circ > \alpha = 45^\circ > \alpha = 90^\circ$, highlighting the role of bedding-induced strength asymmetry in the brittle–ductile transition of bedded rocks.

Keywords: bedded rock; Hoek–Brown criterion; brittle–ductile transition; symmetry breaking; discrete element method



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1. Introduction

Bedded rock masses are common natural geological bodies containing macroscopic bedding planes [1–3]. With increasing burial depth, bedded rock masses are commonly

subjected to complex and extreme geological conditions, including high confining pressure and high in situ stress. As inherent weak structural planes in deep rock masses, bedding planes not only induce pronounced anisotropy but also exert a significant control on the brittle–ductile transition of rocks [4,5].

Among the various strength criteria used in rock mechanics, the Hoek–Brown (H–B) criterion is one of the most widely applied and has been extensively validated in rock engineering practice [6–9]. However, its prediction accuracy is often reduced when it is applied to naturally bedded rocks [10,11]. Therefore, numerous studies have attempted to modify the H–B criterion for bedded rocks by introducing bedding-orientation-dependent anisotropy parameters or by allowing the empirical parameters in the criterion to vary with the orientation of weak planes. Saroglou and Tsiambaos [12] introduced an anisotropy parameter k_β to describe the angle β between the bedding plane and the principal loading direction, and incorporated this parameter into the H–B criterion to account for the influence of bedding on rock strength. However, the ability of k_β to fully characterize the strength variation induced by bedding orientation remains limited. Shi et al. [13] extended this approach by establishing a predictive expression for the uniaxial compressive strength as a function of β . Zhang et al. [14] further considered the effects of bedding planes and pore water pressure on shale strength and proposed an improved H–B criterion incorporating k_β . Colak and Unlu [15] identified a significant correlation between the dimensionless empirical parameter m_i and β and proposed a modified H–B criterion. Lee and Pietruszczak [16] modeled the rock mass disturbance parameter s and m_i as exponential scalar functions of bedding-plane orientation and established the corresponding fitting expressions. Wen et al. [17] further expressed m_i as a bivariate second-order polynomial function of the minor principal stress and the angle β between the major principal stress and the weak plane.

Rock deformation and failure under high confining pressures differ fundamentally from those under low-pressure conditions [18,19]. Under high confinement, the standard H–B criterion commonly overestimates rock strength, indicating the need for further modification to improve its predictive capability. Existing modifications of the H–B criterion for high-confinement conditions can be broadly classified into coefficient-based corrections and correction-term-based modifications. The first category introduces a confining-pressure-dependent coefficient or weighting function into the original H–B criterion to account for the influence of high confinement. For example, Yang et al. [20] replaced the confining pressure term σ_3 in the original H–B criterion with $A\sigma_3$ to evaluate triaxial strength under high confining pressure. Liu et al. [21] proposed a nonlinear strength criterion incorporating the effect of bedding dip angle based on triaxial tests conducted at a confining pressure of 40 MPa. The second category introduces an additional error term to correct the deviation of the original H–B criterion under high-confinement conditions. Li et al. [10,11] investigated the brittle–ductile transition characteristics of granite specimens through high-confinement triaxial tests and constructed a high-confinement error term to improve the H–B criterion. However, their work was mainly limited to homogeneous granite. Therefore, further extending high-confinement correction methods to bedded rocks is essential for improving the generality and applicability of the modified H–B criterion.

In addition to modifications of the H–B criterion, the failure mechanisms of brittle and layered rocks have recently received increasing attention from the perspectives of instability, damage and energy evolution. Li et al. [22] applied fold catastrophe theory to analyze coal-wall spalling in longwall faces with large mining heights, showing that sudden instability is governed by multi-source energy coupling and can be quantified using an energy-threshold-based spalling criterion. Their study indicates that catastrophe theory is useful for describing abrupt brittle failure when accumulated energy exceeds a critical threshold. Xue et al. [23] developed a brittleness evaluation method for gas-

bearing coal by combining a statistical damage model with energy evolution analysis, and established a brittleness index capable of characterizing the reduction in brittleness under increasing gas pressure. These studies suggest that the brittle–ductile transition is closely related to damage accumulation, energy evolution, and critical instability. However, such mechanisms have not yet been fully incorporated into an anisotropic H–B framework for bedded rocks under high confining pressures.

Although previous modifications of the H–B criterion have improved strength prediction by considering anisotropy, high confining pressure, or critical transition behavior, these factors have generally been treated independently. Consequently, a unified strength criterion capable of simultaneously describing bedding-induced anisotropy, high-confinement strength correction, and the brittle–ductile transition remains lacking. To address this gap, this study develops a modified triaxial H–B criterion with physically interpretable parameters. The proposed criterion describes the progressive transition of bedded rocks from bedding-controlled anisotropic failure under low confinement to nearly isotropic ductile behavior under high confinement.

In this study, a modified triaxial H–B criterion is proposed for brittle bedded rocks containing a single set of bedding planes. The criterion introduces a dip-angle-dependent anisotropy parameter, together with the concept of critical confining pressure and a high-confinement correction term, into the conventional H–B framework. This formulation describes the strength response of bedded rocks from bedding-controlled anisotropic failure under low confinement to nearly isotropic ductile behavior under high confinement. The anisotropy parameter quantifies the orientation-dependent strength weakening induced by bedding planes and the resulting asymmetry of the failure surface. As confinement increases and the rock progressively enters the ductile regime, the controlling effect of bedding planes diminishes, and the strength response tends to become isotropic. The proposed criterion was implemented in 3DEC through FISH scripting by embedding the modified yield surface into the built-in H–B module. This numerical implementation provides a practical tool for stability assessment and support design of bedded rock masses under high in situ stress conditions, particularly in deep tunnels, mining excavations, and underground caverns.

2. A Modified Hoek–Brown Failure Criterion for Bedded Rocks

2.1. Anisotropic Modification of the Hoek–Brown Criterion

The original Hoek–Brown (H–B) criterion provides a nonlinear relationship between the major and minor principal stresses at failure and has been widely used to describe the strength of isotropic or equivalent rock masses [6–9]. Its general form is expressed as Equation (1).

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(m_i \frac{\sigma_3}{\sigma_{ci}} + s \right)^\zeta \quad (1)$$

where σ_1 and σ_3 are the major and minor principal stresses, respectively; σ_{ci} is the uniaxial compressive strength of intact rock; m_i is a dimensionless empirical parameter; and s and ζ are material constants.

For bedded rocks, the strength response is strongly affected by the orientation of bedding planes relative to the loading direction. To account for this directional dependence, Saroglou and Tsiambaos [12] introduced an anisotropy parameter into the H–B criterion, where β denotes the angle between the loading direction and the plane of anisotropy [13]. The corresponding anisotropic form is given by Equation (2).

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(k_\beta m_i \frac{\sigma_3}{\sigma_{ci}} + 1 \right)^{0.5} \quad (2)$$

The parameter k_β can be determined from the ratio between the maximum and minimum uniaxial compressive strengths measured at different values of β , as expressed in Equation (3).

$$k_\beta = \frac{\sigma_{ci,\beta\max}}{\sigma_{ci,\beta\min}} \quad (3)$$

where $\sigma_{ci,\beta\max}$ and $\sigma_{ci,\beta\min}$ denote the maximum and minimum uniaxial compressive strengths associated with the bedding orientation, respectively.

In this study, the bedding dip angle α is used to describe the orientation of the bedding plane in triaxial compression. As shown in Figure 1, when the loading direction is perpendicular to the specimen axis, α and β are geometrically complementary. Therefore, k_β can be reformulated as a dip-angle-dependent anisotropy parameter, k_α , as shown in Equation (4).

$$k_\alpha = \frac{\sigma_{ci,\alpha\max}}{\sigma_{ci,\alpha\min}} \quad (4)$$

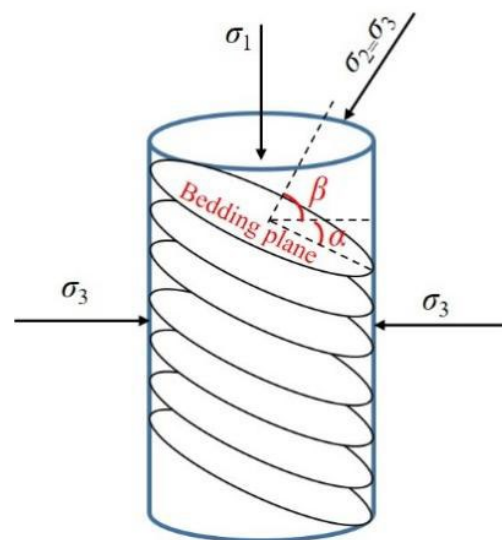


Figure 1. The bedding plane in rock under triaxial compression.

The parameter k_α characterizes the inherent anisotropy induced by bedding planes and represents the orientation-dependent weakening effect under a given bedding configuration. It is treated as a material constant that reflects the relative weakening effect of bedding planes, independent of confining pressure. By incorporating k_α into the anisotropic H-B formulation, Equation (2) can be rewritten as Equation (5).

$$\sigma_1 = \sigma_3 + \sigma_{ci,\alpha} \left(k_\alpha m_i \frac{\sigma_3}{\sigma_{ci,\alpha}} + 1 \right)^{0.5} \quad (5)$$

where $\sigma_{ci,\alpha}$ is the uniaxial compressive strength of the anisotropic rock at a bedding dip angle α .

In addition to the anisotropy parameter k_α , the empirical parameter m_i should also reflect the influence of bedding orientation. Following Lee and Pietruszczak [16], the orientation-dependent parameter m_p is introduced, as expressed in Equation (6).

$$m_p = m_i \left[1 - k_\alpha \sin^2(\alpha - \alpha_0) \right] \quad (6)$$

where α_0 denotes the bedding dip angle corresponding to the extremum of m_i . This angle is commonly associated with the most favorable orientation for sliding along the weak plane and is generally taken as 45° .

2.2. High-Confinement Correction and Brittle–Ductile Transition

The original H-B criterion and Saroglou's modified criterion may overestimate rock strength under high confining pressures. To improve the applicability of the H-B criterion under high-confinement conditions, a high-confinement correction term is introduced into Equation (5), leading to Equation (7).

$$\sigma_1 = \sigma_3 + \sigma_{ci,\alpha} \left(k_\alpha m_p \frac{\sigma_3}{\sigma_{ci,\alpha}} + 1 \right)^{0.5} - M\sigma_3^2 \quad (7)$$

where $M\sigma_3^2$ denotes the high-confinement correction term, and M is the corresponding correction coefficient.

The brittle–ductile transition is described using the concept of critical confining pressure, σ_{ccp} . The critical confining pressure is assumed to be related to the uniaxial compressive strength of bedded rock at a given dip angle, as expressed in Equation (8). When the confining pressure reaches this critical value, the stress envelope approaches a limiting state, at which the slope of the deviatoric stress with respect to σ_3 becomes zero [11,24]. This limiting condition is expressed by Equation (9).

$$\sigma_{ccp} = n\sigma_{ci,\alpha} \quad (8)$$

where n is the rock confining pressure coefficient, σ_{ccp} is the rock critical confining pressure.

$$\frac{\partial(\sigma_1 - \sigma_3)}{\partial\sigma_3} = 0 \quad (9)$$

By substituting Equation (7) into Equation (9), and setting $\sigma_3 = \sigma_{ccp}$, the coefficient M can be derived, as shown in Equation (10).

$$M = \frac{m_p k_\alpha}{4n\sigma_{ci,\alpha} \sqrt{k_\alpha m_p n + 1}} \quad (10)$$

The Byerlee's friction law is adopted to describe the relationship between the shear and normal stresses acting on the bedding plane. According to this law, the frictional resistance along the weak plane is primarily governed by the normal stress, as expressed in Equation (11).

$$\tau = c_j + \mu_j \sigma_n \quad (11)$$

where τ is the shear stress on the joint surface, c_j is the cohesive force on the joint surface, and μ_j is the friction coefficient of the joint surface.

For intermediate to high normal stress levels, the effect of surface roughness and rock type on the friction coefficient is commonly considered negligible [25], and the shear resistance can be approximated by Equation (12).

$$\tau = 0.85\sigma_n \quad (12)$$

Using the stress transformation relations for a plane inclined to the principal stress direction, the normal and shear stresses acting on the bedding plane can be expressed by Equations (13) and (14).

$$\sigma_n = \frac{1}{2}(\sigma_3 + \sigma_1) - \frac{1}{2}(\sigma_3 - \sigma_1) \cos 2\beta \quad (13)$$

$$\tau = \frac{1}{2}(\sigma_3 - \sigma_1) \sin 2\beta \quad (14)$$

where β is defined as the angle between the joint surface and the principal stress axis, and is given by $\frac{\pi}{4} - \frac{\varphi}{2}$, φ is the friction angle of joint surface.

Substituting these relations into Byerlee's friction law [25] yields the limiting major principal stress on the weak plane, as given in Equation (15).

$$\sigma_{1j} = \sigma_3 \cot \beta \tan(\varphi_j + \beta) + \frac{c_j}{\sin \beta (\cos \beta - \sin \beta \tan \varphi_j)} \quad (15)$$

where σ_{1j} is the limiting major principal stress corresponding to shear failure along the weak plane.

With $c_j = 0$, $\mu_j = 0.85$, and $\varphi_j = 40.385^\circ$, and $\beta = (\frac{\pi}{4} - \frac{\varphi_j}{2})$, Equation (15) reduces to Equation (16). Combining the weak-plane limiting condition in Equation (16) with the anisotropic H-B expression in Equation (5) gives the inequality in Equation (17). By solving this inequality for σ_3 , the critical condition for the onset of the brittle–ductile transition is obtained as Equation (18), where only the positive root is retained because $\sigma_3 \geq 0$.

$$\sigma_{1j} = 4.68\sigma_3 \quad (16)$$

$$4.68\sigma_3 \geq \sigma_3 + \sigma_{ci,\alpha} \left(k_\alpha m_p \frac{\sigma_3}{\sigma_{ci,\alpha}} + 1 \right)^{0.5} \quad (17)$$

$$\sigma_3 \geq \frac{m_p k_\alpha + \sqrt{(m_p k_\alpha)^2 + 54}}{27} \sigma_{ci,\alpha} \quad (18)$$

For the critical confining pressure state, $\sigma_3 = \sigma_{ccp}$, Equation (18) becomes Equation (19). Accordingly, the confining-pressure coefficient n , which links the critical confining pressure to the dip-angle-dependent uniaxial compressive strength, is obtained as Equation (20).

$$\sigma_{ccp} = n\sigma_{ci,\alpha} = \frac{m_p k_\alpha + \sqrt{(m_p k_\alpha)^2 + 54}}{27} \sigma_{ci,\alpha} \quad (19)$$

$$n = \frac{m_p k_\alpha + \sqrt{(m_p k_\alpha)^2 + 54}}{27} \quad (20)$$

Substituting the derived correction coefficient (M) into Equation (7) yields the modified H-B criterion for the brittle regime, as shown in Equation (21).

$$\sigma_1 = \sigma_3 + \sigma_{ci,\alpha} \left(k_\alpha m_p \frac{\sigma_3}{\sigma_{ci,\alpha}} + 1 \right)^{0.5} - \frac{m_p k_\alpha}{4n\sigma_{ci,\alpha} \sqrt{k_\alpha m_p n + 1}} \sigma_3^2 \quad (0 \leq \sigma_3 \leq \sigma_{ccp}) \quad (21)$$

When the confining pressure exceeds σ_{ccp} , the rock response enters the ductile regime. In this stage, the deviatoric stress at failure is assumed to remain approximately constant, and the post-critical strength is described by Equation (22).

$$(\sigma_1 - \sigma_3)_{\max} = \frac{3k_\alpha m_p n + 4}{4\sqrt{k_\alpha m_p n + 1}} \sigma_{ci,\alpha} \quad (\sigma_3 > \sigma_{ccp}) \quad (22)$$

Therefore, the proposed criterion is expressed in a piecewise form by Equations (21) and (22), enabling the transition from brittle to ductile behavior to be described within a unified anisotropic H-B framework, as illustrated in Figure 2.

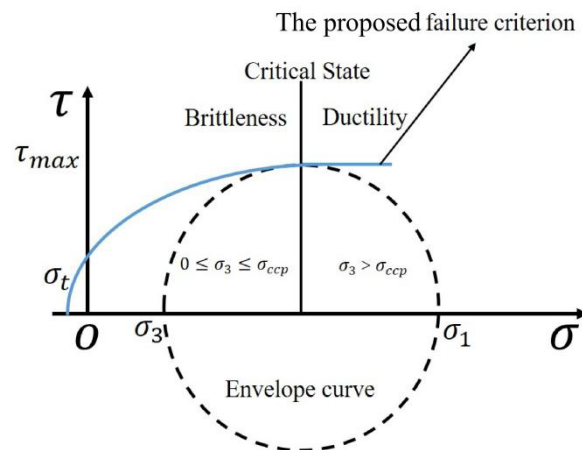


Figure 2. The modified criterion showing the evolution of rock behavior from brittle to ductile.

3. Experimental Validation and Comparative Assessment

3.1. Triaxial Compression Tests and Stress–Strain Responses

Triaxial compression tests were conducted using an MTS815.03 rock testing system at the Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, as shown in Figure 3. The confining pressure was applied under stress control, whereas the axial load was applied under displacement control. Axial strain was measured using both a linear variable differential transformer (LVDT) and strain gauges.

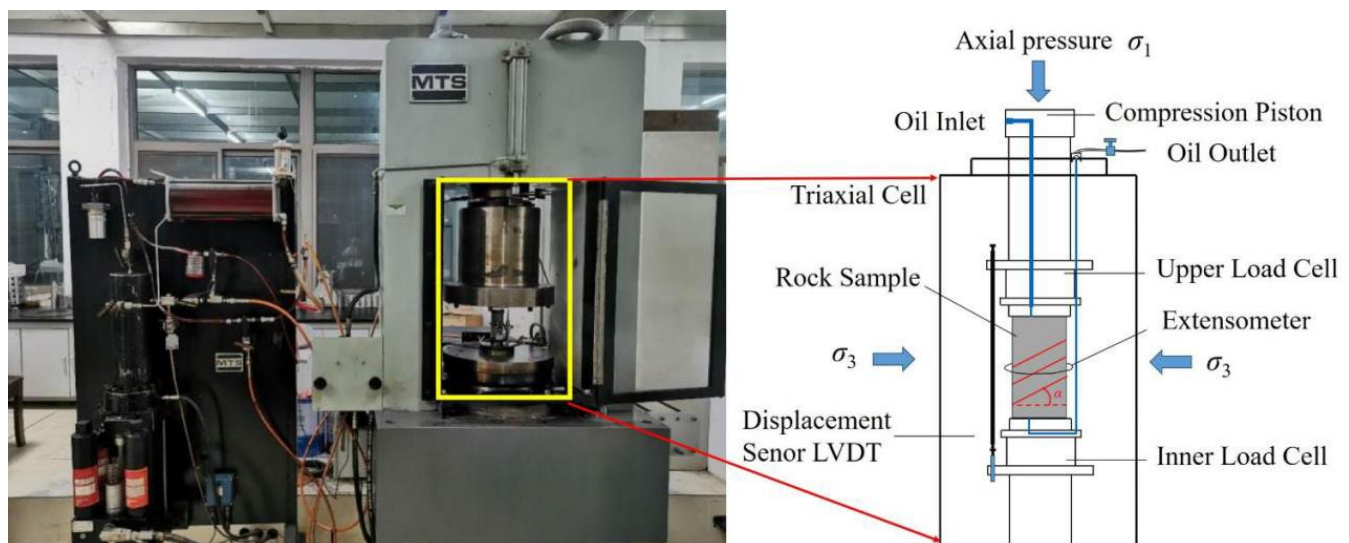


Figure 3. MTS rock testing system. Note: The yellow box indicates the pressure chamber of the triaxial apparatus, and the red arrows show its internal structure; σ_3 denotes the confining pressure.

Two representative anisotropic rocks were selected for experimental validation. The sandstone was collected from Zigong, Sichuan Province, and marble was collected from Heping, Guangxi Zhuang Autonomous Region. Both rock types contain well-developed bedding or foliation planes and are therefore suitable for investigating bedding-induced strength anisotropy under different confining pressures. Representative specimens with a bedding dip angle of 45° are shown in Figure 4.

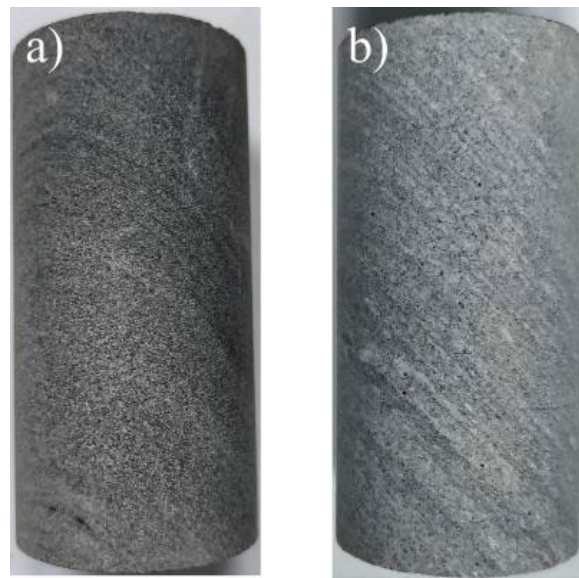


Figure 4. Bedding rock specimens, (a) sandstone and (b) marble.

All specimens were prepared as standard cylinders with dimensions of Φ 50 mm $\times$ 100 mm. Five bedding dip angles, namely 0° , 30° , 45° , 75° and 90° , were considered. Triaxial tests were performed under confining pressures of 0, 40, 80, and 100 MPa. The resulting axial stress–strain curves for sandstone and marble are shown in Figures 5 and 6, respectively.

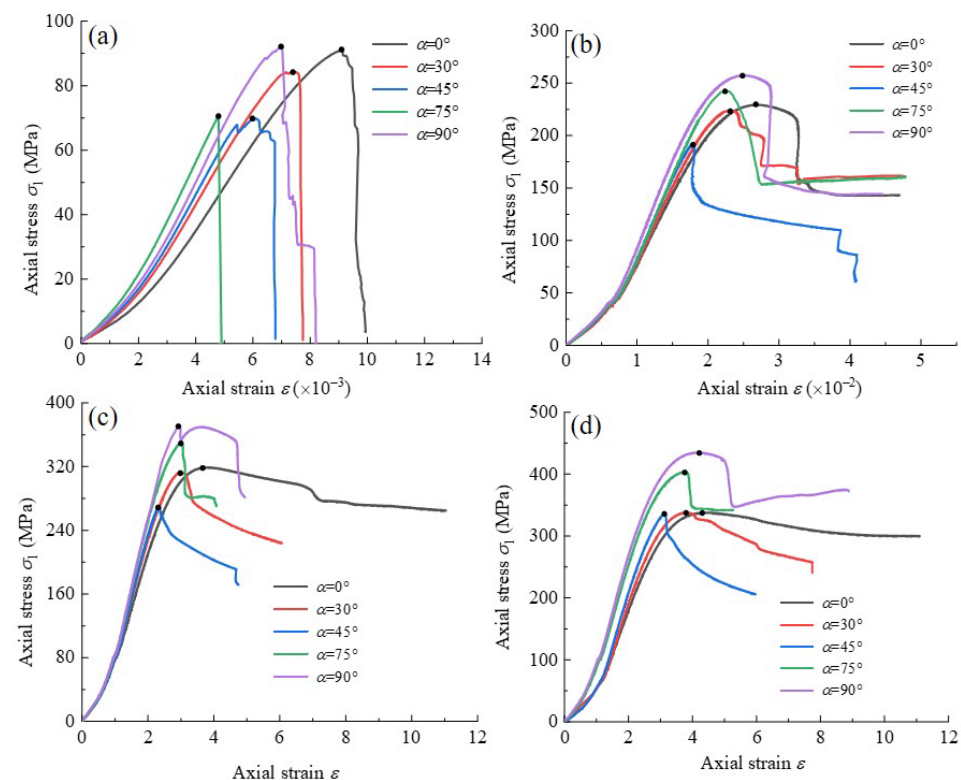


Figure 5. Axial stress–strain curves of bedded sandstone under different confining pressures: (a) $\sigma_3 = 0$ MPa, (b) $\sigma_3 = 40$ MPa, (c) $\sigma_3 = 80$ MPa, and (d) $\sigma_3 = 100$ MPa. Solid circles (●) denote peak stress. The peak stress represents the maximum load-bearing capacity.

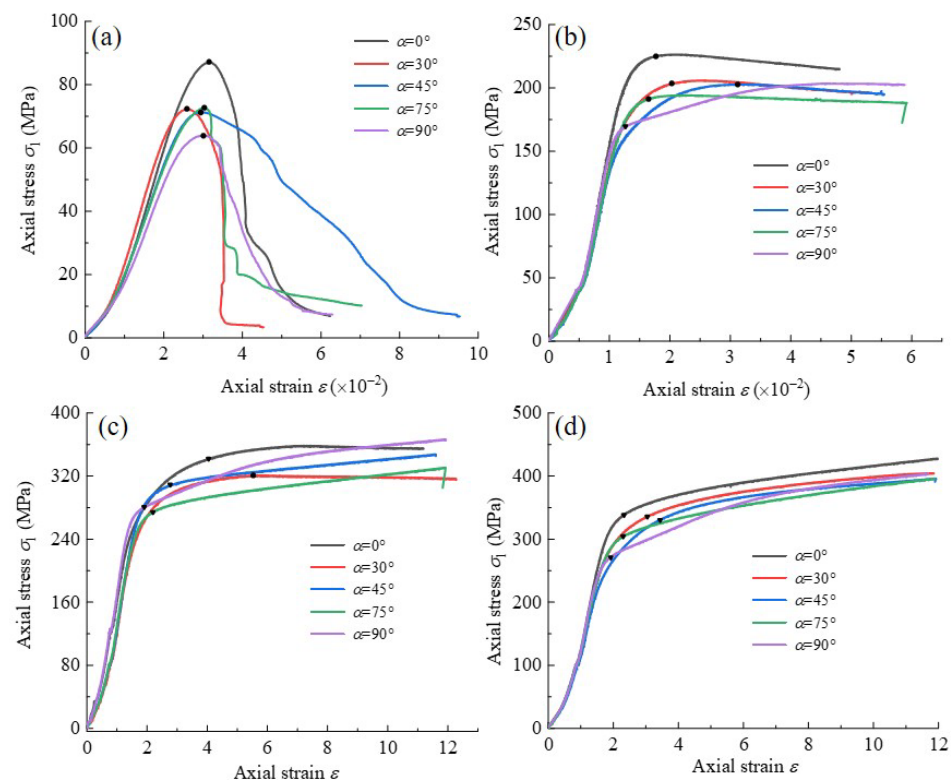


Figure 6. Axial stress–strain curves of bedded marble under different confining pressures: (a) $\sigma_3 = 0$ MPa, (b) $\sigma_3 = 40$ MPa, (c) $\sigma_3 = 80$ MPa, and (d) $\sigma_3 = 100$ MPa. Solid circles (●) denote peak stress; triangles (▼) denote yield point. The yield point marks the onset of plastic deformation, and the peak stress represents the maximum load-bearing capacity.

3.2. Prediction Accuracy of the Proposed Criterion

The anisotropy parameter was calculated using Equation (4) based on the uniaxial compressive strengths obtained for specimens with different bedding dip angles. The orientation-dependent empirical parameter m_p was then determined using Equation (6). The resulting mechanical parameters for bedded sandstone and marble are summarized in Table 1.

Table 1. The mechanical parameters of anisotropic rocks.

Rock Type	α	$\sigma_{ci,\alpha}$	k_α	m_p
Sandstone	0	90.97	0.99	8.59
	30	84.31	0.92	15.96
	45	69.91	0.76	17
	75	70.31	0.76	13.76
	90	92.13	1	8.5
Marble	0	87.12	1	3.57
	30	72.28	0.83	6.74
	45	71.29	0.82	7.14
	75	72.55	0.89	5.54
	90	63.92	0.74	4.51

Note: α is the dip angle and expressed in degrees; $\sigma_{ci,\alpha}$ is the UCS (MPa); k_α is the anisotropic parameter and dimensionless; m_p is the dimensionless empirical parameter considering dip angles.

From the stress–strain curves, we obtained test results under varying confining pressures. For specimens lacking a distinct brittle peak but showing a yield plateau, the yield strength during the brittle–ductile transition was taken as the experimental strength. The

strengths predicted by the proposed criterion, namely Equations (21) and (22), are listed in Table 2.

Table 2. Comparison of experimental and predicted results.

Rock Type	α	σ_3	σ_1^{exp}	σ_1^{cal}	σ_3	σ_1^{exp}	σ_1^{cal}	σ_3	σ_1^{exp}	σ_1^{cal}
Sandstone	0	40	203.125	217.339	80	338.778	350.154	100	335.227	345.228
	30		204.545	218.331		329.83	339.413		336.648	368.952
	45		201.705	219.857		282.528	319.82		335.227	358.674
	75		207.386	218.004		277.415	297.001		318.182	339.608
	90		226.563	238.076		393.75	412.189		437.5	455.137
Marble	0	40	258.239	268.973	80	317.958	358.207	100	338.068	400.119
	30		223.295	248.056		297.496	329.92		318.182	352.603
	45		190.057	201.784		295.278	337.116		286.932	322.845
	75		163.636	172.159		296.391	329.753		306.818	348.991
	90		228.409	245.886		274.867	310.412		269.886	312.307

Note: α is expressed in degrees; σ_3 is the confining pressure (MPa); σ_1^{exp} is the experimentally measured strength (MPa); σ_1^{cal} is the proposed model-predicted strength (MPa).

The relative prediction error, ω , between the theoretical prediction and the experimental result was calculated using Equation (23) [26], and the results are summarized in Table 3.

$$\omega = \frac{\sigma_{c,\text{pred}} - \sigma_{c,\text{exp}}}{\sigma_{c,\text{exp}}} \times 100\% \quad (23)$$

where $\sigma_{c,\text{exp}}$ is the stress value determined by experiments, $\sigma_{c,\text{pred}}$ is the stress value calculated by the theoretical model.

Table 3. Comparison of errors between prediction and experimental results.

Rock Type	α	σ_3	ω	σ_3	ω	σ_3	ω
Sandstone	0	40	5.08	80	4.70	100	4.03
	30		6.74		2.91		9.60
	45		8.99		13.20		6.99
	75		5.12		7.06		6.37
	90		6.99		3.36		2.98
Marble	0	40	4.15	80	12.66	100	18.35
	30		11.09		10.90		10.82
	45		6.17		14.17		12.52
	75		5.20		11.26		13.75
	90		7.65		12.94		15.72

Note: α is expressed in degrees; σ_3 is the confining pressure (MPa); ω is the relative error (%); σ_1^{cal} is the proposed model-predicted strength (MPa).

As shown in Tables 2 and 3, the proposed criterion provides reasonable predictions for both sandstone and marble. For sandstone, the prediction error generally remains within an acceptable range and tends to decrease with increasing confining pressure, indicating that the proposed criterion effectively captures the high-confinement strength response. For marble, the prediction accuracy is relatively high at 40 MPa, whereas larger errors occur under 80 and 100 MPa. This may be attributed to the fact that marble progressively approaches the brittle–ductile transition at high confinement, during which bedding-induced anisotropy weakens and the mechanical response tends toward isotropy. Consequently, the critical confining pressure parameter in the proposed criterion may have limited accuracy when the rock enters a more developed ductile regime. Nevertheless, the predicted strength

varies systematically with bedding dip angle, confirming that the proposed criterion can capture the influence of bedding orientation on rock strength.

3.3. Comparative Assessment with Existing Failure Criteria

To further evaluate the applicability of the proposed criterion, its prediction performance was compared with that of existing strength criteria for bedded or high-confinement rocks. The five criteria considered in this study include the original H-B criterion, Lee and Pietruszczak's criterion, Saroglou and Tsiambaos's criterion, Li et al.'s high-confinement modified criterion, and the proposed criterion.

Lee and Pietruszczak [16] proposed the failure criterion in Equation (24).

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(m_\theta \frac{\sigma_3}{\sigma_{ci}} + s_\theta \right)^{0.5} \quad (24)$$

where m_θ is the dimensionless empirical parameter under the angle θ between the loading direction and the bedding plane, s_θ is the disturbance parameter of the rock at the same angle θ . These parameters are obtained from the following equation:

$$\begin{aligned} m_\theta &= a_1^m + a_2^m \exp[\Omega_0^m (1 - 3 \cos^2 \theta)] \\ s_\theta &= a_1^s + a_2^s \exp[\Omega_0^s (1 - 3 \cos^2 \theta)] \end{aligned} \quad (25)$$

where a_1^m , a_2^m , a_1^s , and a_2^s are the direction-independent coefficients, and Ω_0^s and Ω_0^m are second-order tensors quantifying the preferred orientation of strength parameters. The coefficients $a_{1,2}^m$ and $a_{1,2}^s$ are given by:

$$\begin{aligned} a_1^m &= \frac{m_\theta \exp(-2\Omega_0^m) - m_0 \exp(\lambda\Omega_0^m)}{\exp(-2\Omega_0^m) - \exp(\lambda\Omega_0^m)} \\ a_2^m &= \frac{m_0 - m_\theta}{\exp(-2\Omega_0^m) - \exp(\lambda\Omega_0^m)} \\ a_1^s &= \frac{s_\theta \exp(-2\Omega_0^s) - s_0 \exp(\lambda\Omega_0^s)}{\exp(-2\Omega_0^s) - \exp(\lambda\Omega_0^s)} \\ a_2^s &= \frac{s_0 - s_\theta}{\exp(-2\Omega_0^s) - \exp(\lambda\Omega_0^s)} \end{aligned} \quad (26)$$

Based on the H-B criterion and the concept of critical confining pressure, Li et al. [11] developed a modified H-B criterion applicable to rock strength prediction under high confining pressures.

$$\begin{cases} (\sigma_1 - \sigma_3) = (m_i \sigma_{ci} \sigma_3 + \sigma_{ci}^2)^{0.5} - \frac{m_i}{4n\sigma_{ci}\sqrt{m_i n + 1}} (\sigma_3)^2 & (\sigma_3 \leq \sigma_{rc}) \\ (\sigma_1 - \sigma_3)_{\max} = \frac{3m_i n + 4}{4\sqrt{m_i n + 1}} \sigma_{ci} & (\sigma_3 \geq \sigma_{rc}) \end{cases} \quad (27)$$

where σ_{ci} is the uniaxial compressive strength; m_i is the dimensionless empirical parameter; n is the rock critical confining pressure coefficient; σ_{rc} is the rock critical confining pressure.

The strength criterion proposed by Li et al. [11] lacks a definitive method for determining the critical confining pressure coefficient n . To enable its application to strength prediction under triaxial conditions, we have adopted $n = 0.85$ for marble [27] and $n = 0.47$ for sandstone [28], based on a comprehensive review of the available literature.

Using the parameters in Table 1, the H-B criterion (Equation (1)), Lee's criterion, Saroglou and Tsiambaos's (Equation (2)), Li et al.'s criterion, and the proposed criterion were applied to predict the strength of sandstone and marble under different confining pressures and dip angles. The results are summarized in Tables 4 and 5, and the relative errors between the experimental data and the predictions are shown in Figures 7 and 8.

Table 4. Comparison of predicted strengths from several strength criteria with experimental values ($\sigma_3 = 40$ MPa).

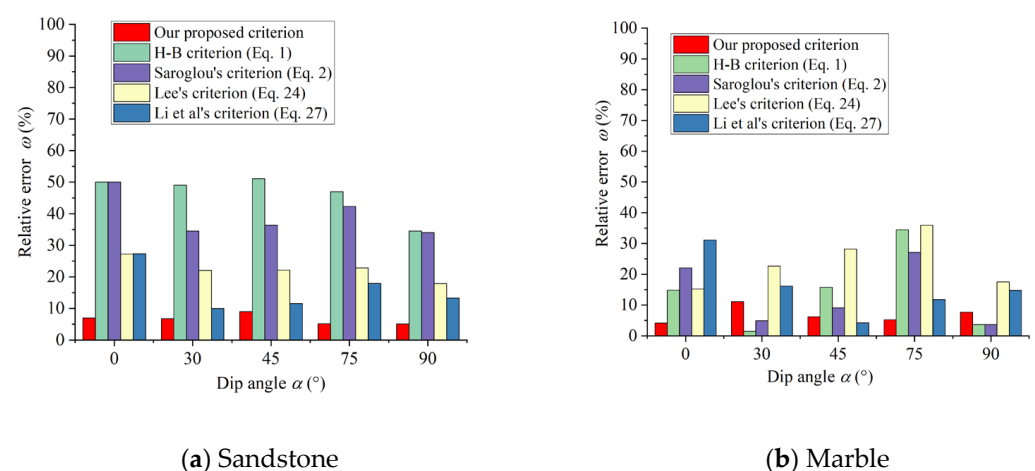
Rock Type	α	σ_1^{exp}	Our Proposed Criterion (σ_1)	H-B Criterion (σ_1)	Saroglou's Criterion (σ_1)	Lee's Criterion (σ_1)	Li et al.'s Criterion (σ_1)
Sandstone	0	203.125	217.339	304.8	304.8	258.4	258.6
	30	204.545	218.331	304.8	275.1	249.6	225.1
	45	201.705	219.857	304.8	275.1	246.3	225.1
	75	207.386	218.004	304.8	295.2	254.8	244.6
	90	226.563	238.076	304.8	303.6	267.1	256.6
Marble	0	258.239	268.973	220.0	201.3	297.5	177.9
	30	223.295	248.056	220.0	212.4	274.0	187.2
	45	190.057	201.784	220.0	207.3	243.7	182.1
	75	163.636	172.159	220.0	208.0	222.4	183.0
	90	228.409	245.886	220.0	220.1	268.5	194.6

Note: σ_1 is the major principal stress (MPa); σ_1^{exp} is the experimentally measured strength (MPa); σ_1^{cal} is the proposed model-predicted strength (MPa).

Table 5. Comparison of predicted strengths from several strength criteria with experimental values ($\sigma_3 = 80$ MPa).

Rock Type	α	σ_1^{exp}	Our Proposed Criterion (σ_1)	H-B Criterion (σ_1)	Saroglou's Criterion (σ_1)	Lee's Criterion (σ_1)	Li et al.'s Criterion (σ_1)
Sandstone	0	338.778	350.154	443.2	443.2	423.7	292.1
	30	329.83	339.413	443.2	396.7	387.5	270.0
	45	282.528	319.82	443.2	396.7	357.2	270.0
	75	277.415	297.001	443.2	428.5	321.5	285.0
	90	393.75	412.189	443.2	441.5	441.0	291.2
Marble	0	317.958	358.207	319.4	285.9	387.4	242.4
	30	297.496	329.92	319.4	305.8	364.3	253.9
	45	295.278	337.116	319.4	296.7	341.6	248.7
	75	296.391	329.753	319.4	298.0	343.3	249.4
	90	274.867	310.412	319.4	319.4	327.8	261.9

Note: σ_1 is the major principal stress (MPa); σ_1^{exp} is the experimentally measured strength (MPa).

**Figure 7.** Comparison of prediction errors among different strength criteria under a confining pressure of 40 MPa.

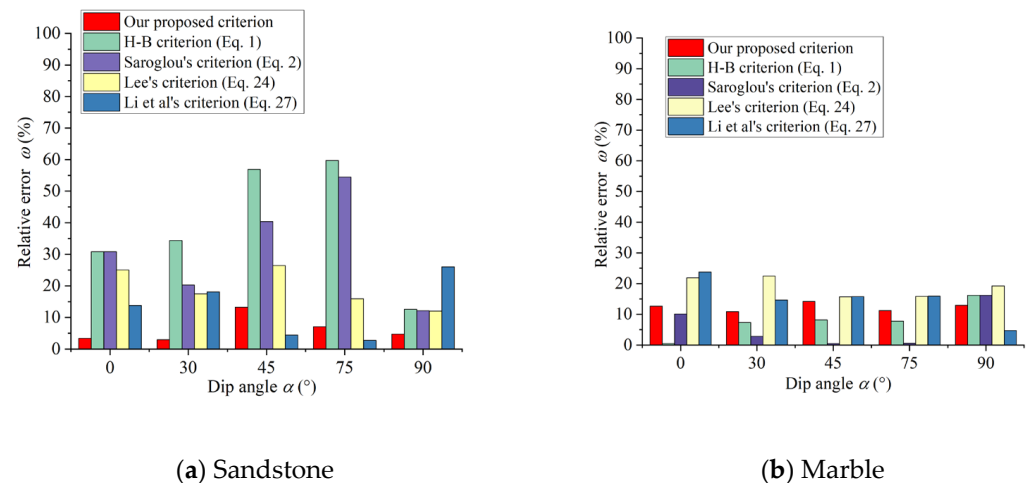


Figure 8. Comparison of prediction errors among different strength criteria under a confining pressure of 80 MPa.

As shown in Figures 7 and 8, the proposed criterion yields strength predictions that are in good agreement with the experimental data over a wide range of confining pressures and dip angles. The prediction errors remain within an acceptable range, demonstrating the reliability and applicability of the proposed criterion for strength prediction of bedded rocks. The relatively large errors for marble at $\sigma_3 = 80$ MPa may be associated with the sensitivity of the predicted strength to the empirical parameter m_p , which was determined from values reported in the literature. Nevertheless, the proposed criterion captures strength variations with dip angle and confining pressure more effectively than others, indicating its improved predictive capability. Tables 4 and 5 indicate that the H-B criterion overestimates strength with increasing confining pressure. Saroglou's criterion improves strength prediction with increasing confining pressure but lacks a high-confinement correction, leading to errors. Lee's criterion introduces the direction-dependent parameters m_θ and s_θ to account for the angle θ (the angle between the loading direction and the bedding plane), but the complexity of these parameters may reduce prediction accuracy. Li et al.'s criterion addresses the effect of high confining pressures but neglects dip angle dependence, and therefore fails to predict the strength variation associated with bedding orientation.

4. Application of the Proposed Criterion in Numerical Simulations

4.1. Numerical Model Construction and Boundary Conditions

Numerical triaxial compression tests were performed using the distinct-element code 3DEC to evaluate the numerical implementation of the proposed criterion. The built-in Hoek–Brown (H–B) module was modified through FISH scripting so that the proposed yield function could be embedded into the numerical framework. Martinsburg Slate was selected as a representative bedded rock to investigate the effects of bedding dip angle and confining pressure on triaxial strength and failure mode.

The numerical specimens were modeled as rectangular prisms with dimensions of 25 mm × 25 mm × 50 mm, as shown in Figure 9. Three bedding dip angles, $\alpha = 0^\circ$, 45° , and 90° , were considered. The boundary constraints and axial loading conditions are illustrated in Figure 10.

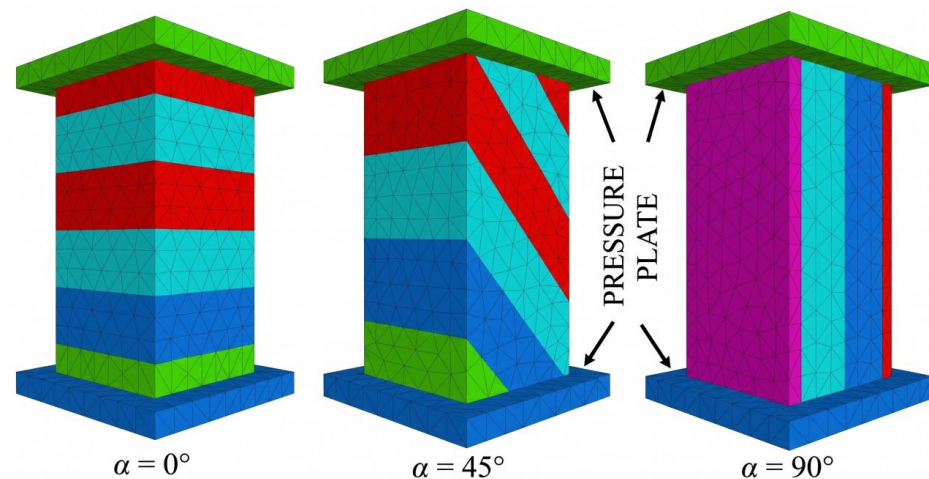


Figure 9. Triaxial compression model for Martinsburg Slate with various dip angles (0°, 45°, and 90°).

Axial Loading Rate Control

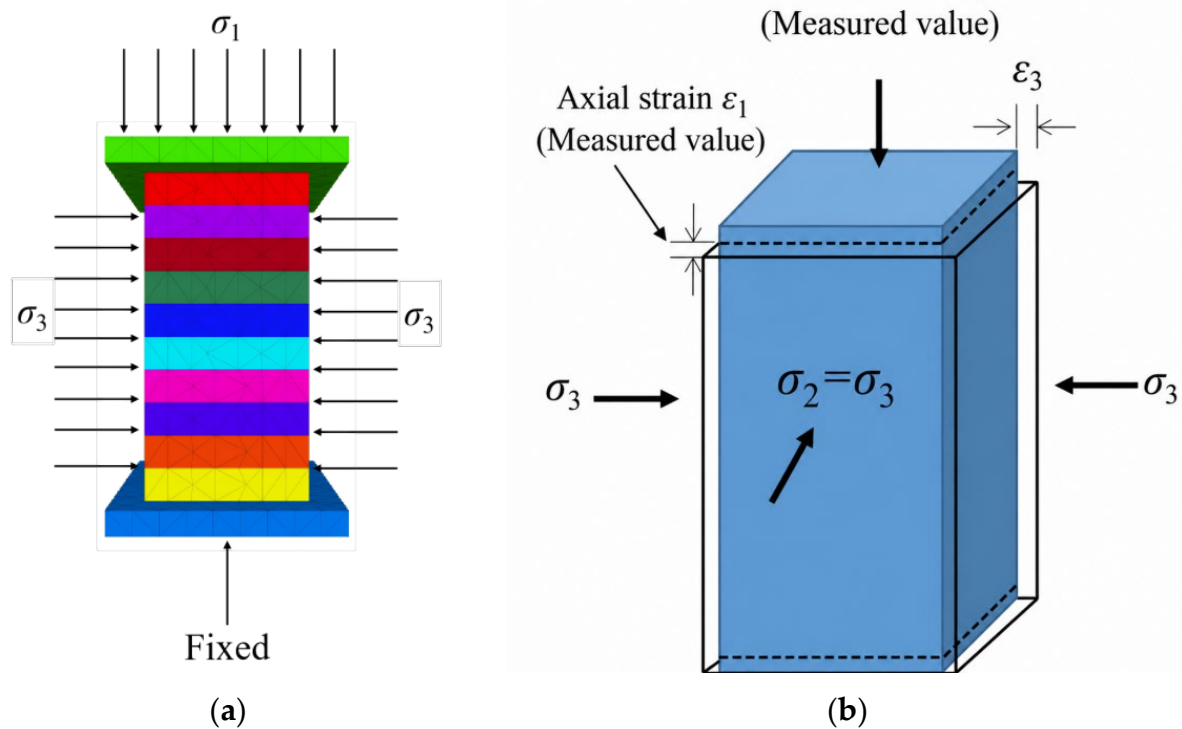


Figure 10. Schematic representation of the boundary constraints and axial loading conditions. (a) loading and fixed bottom with stresses, (b) Strain evolution of the specimen.

4.2. Implementation of the Yield Criterion and Plastic Correction Procedure

To simulate the brittle–ductile transition of bedded rocks, the built-in H–B constitutive module in 3DEC v5.2 was modified through secondary development. The proposed criterion was incorporated into the core FISH scripts as a modified yield surface. To ensure numerical convergence and stability, the maximum number of iterations was set to 20,000.

Within the incremental stress-update scheme, the plastic strain increments in the maximum and minimum principal stress directions satisfy the flow relation given in Equation (28), where γ is the plastic multiplier.

$$\Delta e_1^p = \gamma \Delta e_3^p \quad (28)$$

The current and initial principal stresses are updated incrementally according to Equation (29) [29].

$$\sigma_i = \sigma_i^t + E_1 \Delta e_i + E_2 (\Delta e_j + \Delta e_k) \quad (29)$$

where $E_1 = K + 4G/3$ and $E_2 = K - 2G/3$, K and G are the bulk and shear moduli, respectively, and the strain increments are decomposed into elastic and plastic components when the stress state reaches the yield surface, as shown in Equation (30) [30].

$$\Delta e_i = \Delta e_i^e + \Delta e_i^p \quad (30)$$

No plastic flow is assumed in the intermediate principal stress direction. Therefore, the final stresses obtained from the model are related only to the elastic components of the strain increments, as expressed in Equation (31).

$$\begin{aligned} \sigma_1^f - \sigma_1 &= E_1 (\Delta e_1 - \Delta e_1^p) + E_2 (\Delta e_2 + \Delta e_3 - \Delta e_3^p) \\ \sigma_2^f - \sigma_2 &= E_2 \Delta e_2 + E_2 (\Delta e_1 - \Delta e_1^p + \Delta e_3 - \Delta e_3^p) \\ \sigma_3^f - \sigma_3 &= E_1 (\Delta e_1 - \Delta e_1^p) + E_2 (\Delta e_1 + \Delta e_2 - \Delta e_1^p) \end{aligned} \quad (31)$$

Combining Equations (28) and (31) yields Equation (32).

$$\begin{aligned} \sigma_1^f &= \sigma_1^i - \Delta e_3^p (\gamma E_1 + E_2) \\ \sigma_2^f &= \sigma_2^i - \Delta e_3^p E_2 (\gamma + 1) \\ \sigma_3^f &= \sigma_3^i - \Delta e_3^p (\gamma E_2 + E_1) \end{aligned} \quad (32)$$

Based on the original H-B module, the dip-angle-dependent anisotropy parameter, k_α the orientation-dependent empirical parameter m_p , and the high-confinement correction term were incorporated into the computational framework. The final stress state is constrained by the proposed yield function, as expressed in Equation (33).

$$F = \sigma_1^f - \sigma_3^f - \frac{4[\Omega] \times \left\{ \left[k_\alpha m_p^2 + \left((k_\alpha m_p)^4 + 2R k_\alpha m_p^2 \right)^{0.5} \right] + R \right\}^{0.5} \sigma_{ci,\alpha}^2 (k_\alpha m_p \frac{\sigma_3^f}{\sigma_{ci,\alpha}} + 1)^{0.5} + k_\alpha m_p}{\frac{4[\Omega] \times \left\{ \left[k_\alpha m_p^2 + \left((k_\alpha m_p)^4 + 2R k_\alpha m_p^2 \right)^{0.5} \right] + R \right\}^{0.5}}{R^{0.5}} \sigma_{ci,\alpha}} = 0 \quad (33)$$

where $\Omega = k_\alpha m_p + \left((k_\alpha m_p)^2 + 2R \right)^{0.5}$. The parameter R is taken as 27.

The critical confining pressure parameter in the numerical model was set equal to σ_{ccp} in the proposed criterion. When the confining pressure is greater than or equal to σ_{ccp} , yielding is determined using Equation (34); otherwise, Equation (33) is applied.

$$F = \sigma_1^f - \sigma_3^f - \frac{\left[k_\alpha m_p^2 + \left((k_\alpha m_p)^4 + 54 k_\alpha m_p^2 \right)^{0.5} \right] - 36}{\left\{ 6 \left[k_\alpha m_p^2 + \left((k_\alpha m_p)^4 + 54 k_\alpha m_p^2 \right)^{0.5} + 162 \right] \right\}^{0.5}} = 0 \quad (34)$$

After substituting σ_{f1} and σ_{f3} from Equation (33) into Equation (34), the unknown variable can be solved iteratively using the Newton–Raphson method or the bisection method. The obtained solution is then substituted into Equation (32) to update the final stress state.

Brittle rocks commonly exhibit significant volumetric dilation during yielding under low confinement, which is associated with axial splitting and wedging effects. In this study,

an associated flow rule is adopted to describe the volumetric response during yielding, as expressed in Equation (35).

$$\Delta e_i^p = -\gamma \frac{\partial F}{\partial \sigma_i} \quad (35)$$

Substituting Equation (33) into Equation (35) and differentiating it with respect to σ_3^f gives the plastic multiplier, as shown in Equation (36).

$$\gamma = \frac{\partial F / \partial \sigma_3^f}{\partial F / \partial \sigma_1^f} = - \frac{1}{1 + 0.5 \sigma_{ci,\alpha} (k_\alpha m_p \frac{\sigma_3}{\sigma_{ci,\alpha}} + 1)^{-0.5} \left(\frac{k_\alpha m_p}{\sigma_{ci,\alpha}} \right)} \quad (36)$$

4.3. Numerical Validation and Analysis

In the developed yield criterion module, the mechanical response of bedded rocks can be defined using a limited set of key parameters, including the critical confining pressure σ_{ccp} , the anisotropy parameter k_α , the orientation-dependent empirical parameter m_p , and the uniaxial compressive strength. An ideal plastic Coulomb slip model was used to describe the mechanical behavior of the bedding planes. The physical and mechanical parameters of the Martinsburg Slate matrix and the associated bedding planes are summarized in Tables 6 and 7, respectively. The numerical simulations were benchmarked against Donath's triaxial experimental data [31], which include the effects of bedding orientation and high confining pressure.

Table 6. Physical and mechanical parameters of Martinsburg Slate [31].

ρ (g/cm ³)	E (GPa)	μ	σ_{ci} (MPa)
2.64	24.7	0.286	68.4

Table 7. Physical and mechanical parameters of bedding plane [31].

c_j (MPa)	φ_j (°)	jkn (GPa)	jks (GPa)
9	21	1	2

The peak stresses obtained from nine numerical simulation cases were compared with the experimental data reported by Donath [32]. As shown in Figure 11, the numerical peak stresses show good agreement with the laboratory measurements over the examined range of confining pressures. The proposed criterion, as implemented through secondary development, reduces the tendency of traditional strength criteria to overestimate rock strength under high-confinement conditions. The remaining discrepancies may be attributed to the idealization of bedding planes as smooth surfaces governed by Coulomb slip, whereas natural bedding planes usually exhibit roughness and asperity interlocking that can provide additional shear resistance under high confining pressure. The deviation is most pronounced at a bedding dip angle of 45°, which represents an unfavorable orientation for shear sliding along bedding planes and makes the predicted strength particularly sensitive to the idealized representation of bedding-plane properties.

The simulated axial stress–strain and lateral strain curves are shown in Figures 12 and 13, respectively. As shown in Figure 12, increasing confining pressure promotes a transition from elastoplastic softening to ductile behavior for specimens with bedding dip angles of 0°, 45°, and 90°. This result indicates that the modified numerical model can capture the pressure-dependent brittle–ductile transition described by the proposed criterion. The lateral strain curves in Figure 13 further show that the specimen with a bedding dip angle of 45° exhibits the most pronounced lateral deformation, indicating that

deformation and failure are strongly controlled by bedding orientation. The compressive resistance follows the order $\alpha = 0^\circ > \alpha = 45^\circ > \alpha = 90^\circ$, which is consistent with the anisotropic mechanical response observed in laboratory tests.

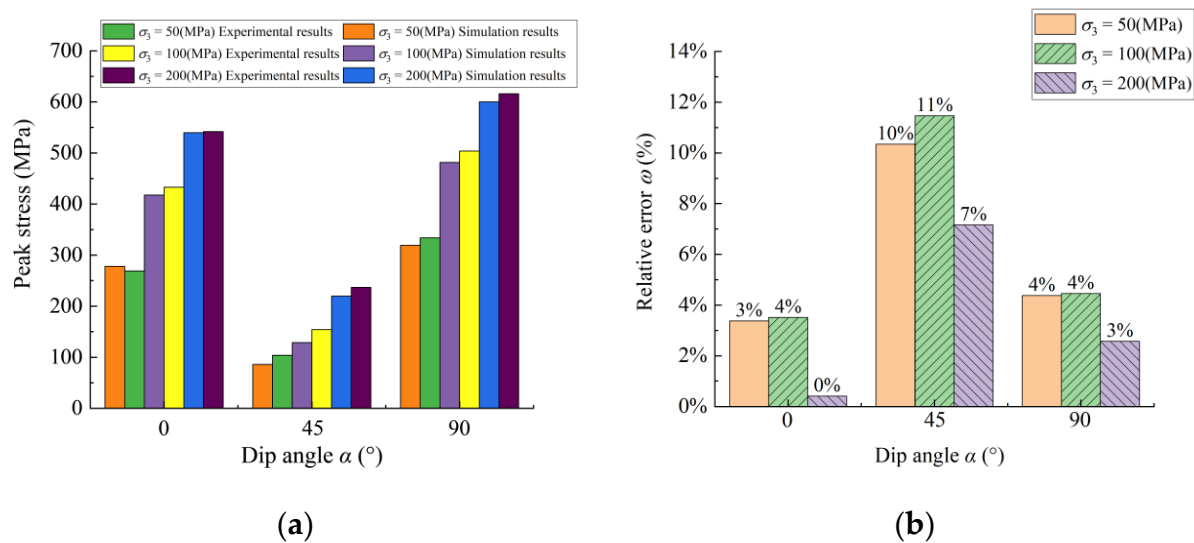


Figure 11. Correlation analysis between numerical peak stresses and experimental triaxial data for Martinsburg Slate. (a) Peak stress versus bedding dip angle and confining pressure, (b) Error comparison under different dip angles and confining pressures.

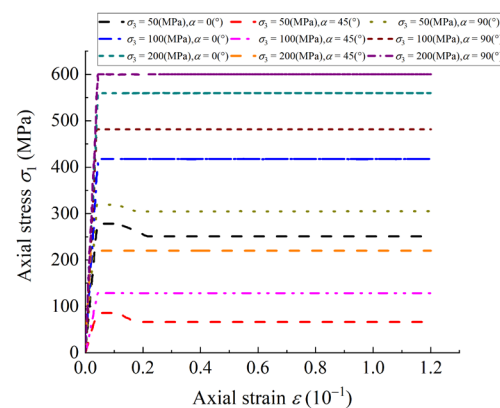


Figure 12. Axial stress–strain relationships.

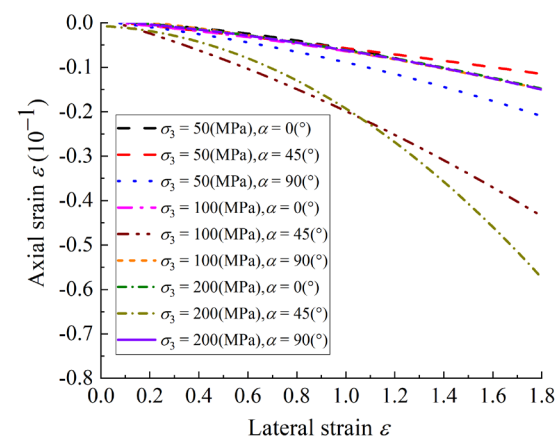


Figure 13. Lateral stress–strain relationships.

4.4. Failure Modes and Crack Propagation

The simulated failure modes and crack propagation patterns under different bedding dip angles and confining pressures are shown in Figure 14. The results indicate that both bedding orientation and confining pressure strongly influence the failure mechanism of bedded rocks.

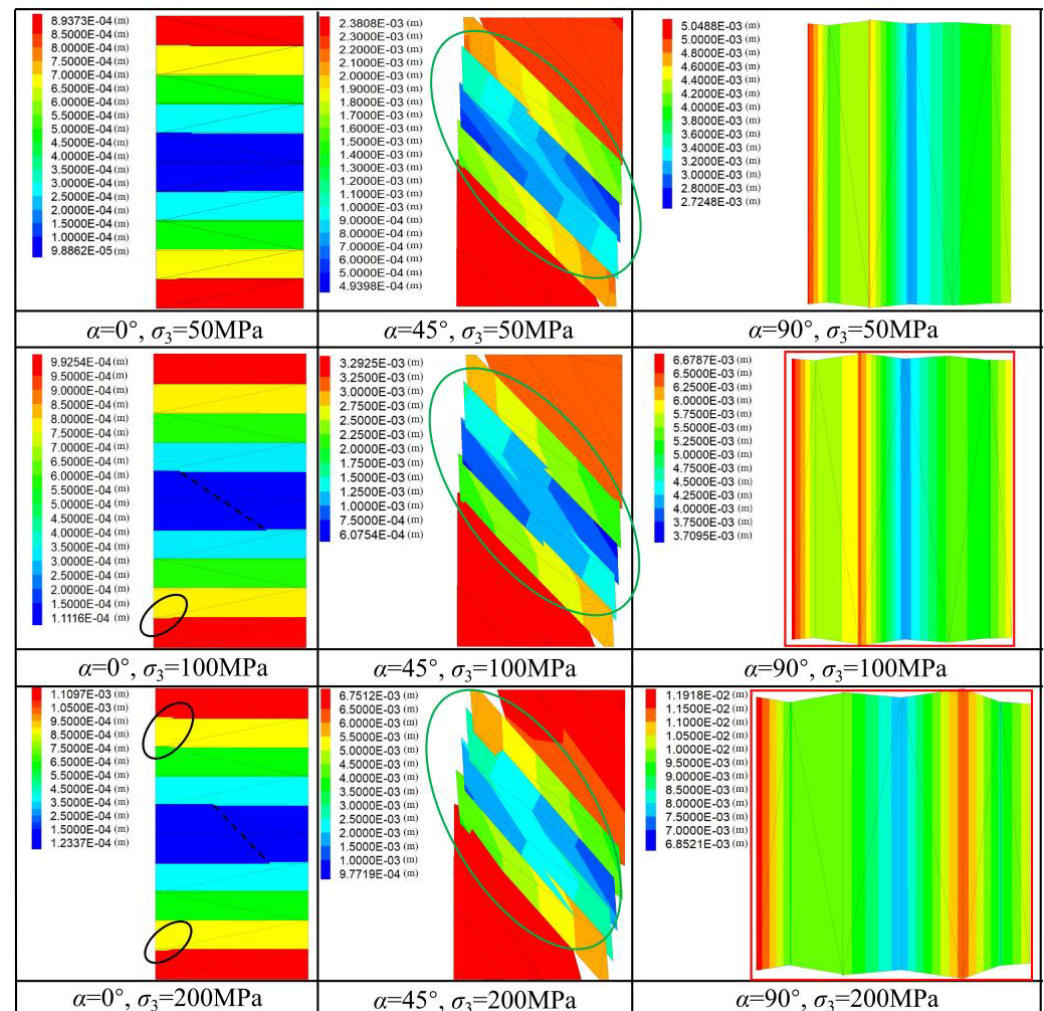


Figure 14. The failure modes of the numerical models under different dip angles and confining pressures: Black circles indicate shear cracks propagating through the rock matrix; green circles denote a combination of shear along bedding planes and matrix shear; red boxes highlight lateral dilation.

For specimens with $\alpha = 0^\circ$, failure is mainly governed by the intact rock matrix under all confining pressures. At a confining pressure of 50 MPa, shear cracks initiate near the specimen ends and propagate diagonally toward the central region. With increasing confining pressure, a localized shear band gradually develops in the specimen center, accompanied by increasingly pronounced compressive deformation along the loading direction. At 200 MPa, more shear cracks initiate near the specimen ends, indicating matrix-controlled failure in which the external load is primarily transmitted through the intact rock matrix rather than along bedding planes.

For specimens with $\alpha = 45^\circ$, failure is strongly affected by the bedding planes. At 50 MPa, shear slip mainly occurs along the bedding surfaces. As the confining pressure increases, tensile cracking is progressively suppressed, whereas shear deformation becomes increasingly dominant. The failure mode gradually evolves into a mixed pattern controlled

by both bedding-plane slip and matrix shear, with cracks propagating both along and across the bedding planes.

For specimens with $\alpha = 90^\circ$, cracks initiate and propagate preferentially along the bedding planes under low confining pressure, leading to longitudinal splitting. With increasing confining pressure, tensile splitting is suppressed and the failure mode changes toward compressive shear, with shear bands cutting across the bedding structure. Under high confining pressure, the specimen exhibits more distributed cracking and lateral deformation, indicating a transition toward ductile behavior.

5. Discussion and Limitations

The proposed criterion describes the brittle–ductile transition of bedded rocks by introducing the concept of critical confining pressure. In this framework, the deviatoric stress at failure is assumed to remain approximately constant once the confining pressure exceeds the critical value. This assumption is inherited from the classical critical-confining-pressure framework and provides a clear theoretical threshold for identifying the transition from brittle failure to ductile behavior. However, the actual brittle–ductile transition of rocks is usually gradual and may be affected by loading path, bedding orientation, and lithology. In many cases, rocks do not exhibit an ideal perfectly plastic plateau after reaching the critical state; instead, their strength may continue to increase gradually in the ductile regime. Therefore, the constant-deviatoric-stress assumption should be regarded as a simplified representation of the post-critical response. Future improvements may incorporate a post-critical hardening function calibrated against broader high-confinement experimental datasets to better describe the progressive strength evolution of bedded rocks after the critical state. Moreover, the proposed criterion is primarily formulated as an empirical extension of the original H-B criterion, consistent with its empirical tradition. However, a more rigorous theoretical foundation would further strengthen the formulation. We will focus on reformulating the proposed criterion within a thermodynamic framework and defining a plastic potential function for the yield surface to enable a non-associated flow rule, thereby providing a more unified and physically consistent constitutive framework for bedded rocks.

The critical confining pressure coefficient n represents the normalized confining pressure level at which the rock transitions from brittle to ductile behavior. This angle-dependence reflects the fact that the critical confining pressure is not a universal constant but varies with bedding plane, which is consistent with experimental observations. In future work, we will derive n by coupling Byerlee's friction law with the anisotropy-to-isotropy transition criterion. The key concept is that as confinement rises, the frictional resistance along bedding planes tends toward the intact rock strength.

The experimental validation in this study is also subject to limitations in terms of bedding dip angles and confining pressure range. The laboratory tests considered five bedding dip angles, namely 0° , 30° , 45° , 75° , and 90° , while additional bedding dip angles, such as 15° and 60° , may provide further information on the transition of anisotropic strength behavior. In addition, the maximum confining pressure in the physical tests was 100 MPa. Although this range is sufficient to reveal the main pressure-dependent strength characteristics of the tested rocks, higher confining pressures would be useful for further constraining the brittle–ductile transition and the post-critical strength response. Future experimental work should include a wider range of bedding orientations, higher confining pressures, and more lithologies to improve the generality of the proposed criterion.

In the numerical simulations, bedding planes were represented using an ideal plastic Coulomb slip model. This treatment provides a simple and computationally efficient representation of bedding-controlled slip, but it does not fully capture progressive stiffness

degradation, strength degradation, or nonlinear shear behavior along natural bedding planes. Natural bedding surfaces may exhibit roughness, asperity interlocking, and progressive damage, particularly under high confining pressure. Future numerical work should therefore incorporate strain-softening or damage-based joint models in which shear stiffness and strength parameters evolve with plastic shear displacement. Introducing a nonlinear shear constitutive relationship would also improve the representation of pre-peak nonlinearity and post-peak softening in bedding-controlled failure.

The proposed criterion is currently formulated for monotonic conventional triaxial loading and does not explicitly consider the intermediate principal stress, cyclic loading, or progressive damage evolution of bedding planes. Therefore, further extensions are required for true triaxial stress states, cyclic loading conditions, and more complex engineering environments. Nevertheless, the criterion provides a practical framework for preliminary strength evaluation and numerical simulation of bedded rock masses under high in situ stress, including deep tunnels, underground caverns, and high-slope engineering. The required parameters can be obtained from conventional laboratory tests, including uniaxial compression tests, triaxial compression tests under different confining pressures, and tests on specimens with different bedding dip angles. Since the criterion has been implemented in 3DEC through secondary development, a similar implementation strategy may also be extended to other numerical platforms, such as FLAC3D or ABAQUS, through user-defined constitutive models.

6. Conclusions

The conventional Hoek–Brown (H–B) criterion has limited applicability to bedded rocks under high confining pressure because it does not explicitly account for bedding-induced anisotropy and brittle–ductile transition behavior. To address this limitation, this study proposed a modified H–B criterion for bedded rocks by incorporating bedding-orientation effects, high-confinement correction, and the concept of critical confining pressure. The main conclusions are summarized as follows.

- (1) A modified H–B criterion was developed by introducing a bedding-dip-angle-dependent anisotropy parameter, a high-confinement correction term, and a critical confining pressure parameter into the conventional H–B framework. The proposed criterion describes the strength response of bedded rocks from bedding-controlled anisotropic failure under low confinement to nearly isotropic ductile behavior under high confinement. Validation using triaxial compression data for bedded sandstone and marble shows that the proposed criterion provides reasonable predictions of strength variation with both bedding dip angle and confining pressure. Some deviations remain for marble under high confinement, indicating that further validation is required for fully ductile responses and broader lithologies.
- (2) The proposed criterion was implemented in the distinct-element code 3DEC through FISH scripting by embedding the modified yield surface into the built-in H–B module. A plastic correction procedure based on the associated flow rule was established, and the corresponding plastic multiplier was derived and incorporated into the stress-update algorithm. This implementation enables the proposed criterion to be used for numerical analysis of bedded rock masses under high in situ stress conditions.
- (3) Numerical simulations of bedded slate show that increasing confining pressure promotes a transition from elastoplastic softening to ductile behavior for specimens with bedding dip angles of 0° , 45° , and 90° . The compressive resistance follows the order $\alpha = 0^\circ > \alpha = 45^\circ > \alpha = 90^\circ$, indicating a clear bedding-orientation effect. The simulated failure modes further show that specimens with $\alpha = 0^\circ$ are mainly controlled by matrix shear failure, those with $\alpha = 45^\circ$ exhibit mixed failure governed by

bedding-plane slip and matrix shear, and those with $\alpha = 90^\circ$ tend to develop bedding-parallel splitting under low confinement and more distributed shear deformation under high confinement.

Author Contributions: Z.W.: Conceptualization, Methodology, Formal analysis, Investigation, Writing—original draft. S.O.: Writing—reviewing. X.L.: Editing, Funding acquisition, Resources, Supervision. C.W.: Writing—original draft. Y.F.: Writing—original draft. T.W.: Writing—original draft, Writing—reviewing. Z.H.: Formal analysis. L.L.: Writing—original draft. All authors have read and agreed to the published version of the manuscript.

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