

Three-Dimensional Dynamical Systems and Symmetry

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Dynamical systems are used to describe processes arising in a wide range of fields. They offer a natural way to understand how interactions evolve over time and how complex behavior can emerge [1].

The qualitative behavior of one-dimensional and planar continuous systems is largely well understood. This changes when moving to higher-dimensional cases. In particular, three-dimensional systems already allow for phenomena that do not appear in lower dimensions, such as deterministic chaos, more complex geometric structures, and a wider variety of bifurcations. In this context, three-dimensional models can be seen as a natural extension of planar systems and as a first step toward higher-dimensional dynamics [2,3].

This Special Issue is devoted to exploring this transition, with a particular emphasis on the interplay between symmetry and the behavior of three-dimensional dynamical systems and their applications. The volume also includes the complex dynamics of discrete-time planar systems, where rich behavior may already appear, as well as the additional degrees of freedom inherent in four-dimensional dynamical systems and their symmetries. The contributions collected here reflect both the diversity of applications and the variety of methods currently in use.

A first group of papers addresses bifurcation theory and complex dynamics. One study presents a detailed analysis of an asymmetric discrete predator–prey model, where codimension-one and codimension-two bifurcations are thoroughly characterized and the existence of chaos is investigated. Complementing this perspective, another contribution investigates bifurcation structures in a symmetric three-dimensional T -system, revealing the remarkable emergence of infinitely many equilibria and invariant foliations at critical parameter values, as well as fast-slow oscillations.

A second group of papers explores applications in life sciences, particularly epidemiology and oncology. The SEIR model presented here incorporates several interacting effects, such as the quality of health care services and the impact of fear. These ingredients lead to nontrivial dynamical features, including backward bifurcation and sustained oscillations. Another contribution considers a model describing the interaction between tumor cells and components of the immune system, namely NK cells and cytotoxic T lymphocytes. By analyzing both scenarios, with and without chemotherapy, the study shows how treatment outcomes depend sensitively on parameter interactions. These studies underline how higher-dimensional frameworks can capture biological interactions that are inaccessible to low-dimensional models, and they offer quantitative tools for optimizing intervention strategies.

Another direction explored in this issue is given by the fractional-order systems. The analysis of a novel fractional chaotic map shows how commensurate and incommensurate derivative orders influence the emergence of hidden attractors, symmetry breaking, and complexity. The incorporation of a three-dimensional nonlinear controller suggests possible ways to stabilize such dynamics.



Received: 15 July 2026

Accepted: 16 July 2026

Published: 23 July 2026

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The issue also includes advances in the qualitative theory of polynomial dynamical systems. One contribution extends the discriminant criterion from two-dimensional to three-dimensional quadratic systems. Using this criterion, a classification of symmetry relations inherent to the coefficient matrices is given. The identification of twisted fold bifurcations leads to a clearer understanding of how algebraic structure influences dynamical behavior.

From a geometric perspective, another paper introduces a novel framework based on generalized spheres in $\mathbb{R}^3$, where the Euclidean norm is replaced with an inhomogeneous functional. This leads to a three-dimensional dynamic complex structure. Results about invariant probability densities, generalized uniform distributions on generalized spheres, geometric measure representations, and dynamic ball numbers associated with this structure are reported.

Finally, using symmetries and first integrals, closed-form solutions of the Rabinovich system are discussed. Using the Optimal Auxiliary Functions Method, approximate analytic solutions are derived.

We hope that this collection will serve as a source of inspiration for further research at the intersection of theory and applications in dynamical systems.

Acknowledgments: We would like to thank all the authors for their contributions and the reviewers for their time, care, and valuable feedback.

Conflicts of Interest: The author declares no conflicts of interest.

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