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Bayesian Estimation of the Difference and Ratio of the Coefficients of Variation Under the Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

The coefficient of variation (CV) is a widely used unit-free measure for comparing variability across two or more populations, even when the data are measured on different scales. In this study, we investigate confidence and credible interval estimation for the difference and ratio of CVs from the zero-inflated two-parameter Rayleigh (ZITR) distribution, which accommodates excess zeros and positively skewed observations through a two-component mixture structure. Several interval estimation methods are proposed, including the Bayesian Credible Interval (BCI), Bayesian Highest Posterior Density (HPD), and Approximate Normal (AN) approaches. Their performances are compared with established methods, namely the Generalized Confidence Interval (GCI), Method of Variance Estimates Recovery (MOVER), Percentile Bootstrap (PB), and Bootstrap with Standard Error (BS). A Monte Carlo simulation study is conducted to evaluate the methods in terms of coverage probability (CP), Monte Carlo standard error (MCSE) of the estimated coverage probability, and expected length (EL). The simulation results indicate that the HPD method provides the best overall performance, achieving coverage probabilities close to the nominal level with small MCSE values, indicating stable simulation estimates, while maintaining relatively short interval lengths. The proposed confidence interval methods were further applied to district-level road traffic fatality rates (per 100,000 population) in Uttaradit and Mae Hong Son provinces, Thailand. The road traffic fatality data included fatalities occurring in March 2026. The fatality rates were calculated based on the district-level population size to enable comparisons across districts with different population scales.

Keywords: Bayesian method; gibbs sampling; highest posterior density; prior distribution; delta method; pivotal quantities; resampling method



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1. Introduction

Parameter estimation is a fundamental task in statistical inference as it allows researchers to make inferences about the characteristics of a population from random samples, when the underlying parameters are not known [1]. Estimation methods are used to summarize and infer important characteristics of a distribution, such as measures of central tendency and variability, from observed data. Therefore, selecting an appropriate probability distribution and determining suitable estimation methods are essential tasks, especially in fields where high accuracy is required, such as insurance, medicine, engineering, and risk analysis.

The two-parameter Rayleigh distribution is a positively skewed continuous distribution that is a generalization of the classical one-parameter Rayleigh distribution [2] by adding a location parameter to the scale parameter. The generalization provides more flexibility in modeling real phenomena and makes the distribution fit a wider variety of data patterns. The two-parameter Rayleigh distribution was first introduced by Johnson et al. [3]. Since then, some works have studied methods for estimating its unknown parameters. For example, Seo et al. [4] proposed the exact confidence intervals for the parameters of the two-parameter Rayleigh distribution based on the upper record values and studied the performance of the proposed confidence intervals using the Monte Carlo simulation and real data application. Similarly, El-Faheem [5] proposed point estimation procedures for the location and scale parameters of the two-parameter Rayleigh distribution and evaluated their performance through simulation studies under the mean squared error criterion. In addition, Liu et al. [6] studied the parameter estimation for the two-parameter Rayleigh distribution under adaptive Type-II progressive hybrid censored data. They developed maximum likelihood estimators and established their existence and uniqueness. In addition, Bayesian estimation methods under various loss functions were investigated and compared.

In many practical applications, datasets contain an excessive number of zero observations. Ignoring these excess zeros may lead to inaccurate statistical inference and inadequate model fitting. To solve this problem, zero-inflated models have been widely used as a flexible framework for modeling data with a large proportion of zeros. The basic idea originated from Aitchison [7], who suggested a mixture model consisting of a degenerate distribution that assigns positive probability to zero and a continuous distribution for the positive observations. Based on this concept, Lambert [8] introduced the zero-inflated Poisson model, which established a general framework for modeling excess-zero data. This framework has subsequently been extended to many probability distributions, including the zero-inflated Rayleigh distribution proposed by Li et al. [9], which combines a point mass at zero with the standard one-parameter Rayleigh distribution for modeling the positive part of the data.

Many real-world datasets have a large proportion of zero observations and positively skewed continuous positive values. Such data are common in road traffic safety, where many districts report no fatalities during a certain time period, while the remaining districts report different levels of fatality. Such features are also seen in environmental monitoring, reliability engineering, insurance claims, and public health studies. In these applications, researchers are often not only interested in modeling the distribution of the data but also interested in comparing the relative variability of different populations or regions. There is therefore a need for statistical procedures that work well in the presence of excess zeros and positively skewed observations to guarantee reliable statistical inference.

The coefficient of variation (CV) is defined as the ratio of the standard deviation to the mean and is commonly used to measure the dispersion of data. Since the CV is a unit-free parameter, its value is typically expressed as a percentage. A higher CV indicates greater relative variability, whereas a lower CV suggests less relative variability. Therefore, the CV can be applied in a wide range of fields. For example, Billings et al. [10] estimated the CV to examine the impact of socioeconomic status on hospitals. Romano et al. [11] analyzed the shear and tensile bond strength of adhesion to tooth structures, while Saelee et al. [12] investigated the variability of agricultural production using approximate confidence intervals for the CV. In addition, comparisons of coefficients of variation have been widely used because the CV provides an effective way to compare variability between two or more datasets. The most common approaches for comparing two CVs are based on either the difference or the ratio of the CVs from two independent samples. For example, Verrill and Johnson [13] introduced

confidence bounds for the ratio of two CVs under normal distributions using simulation-based procedures. Sangnawakij and Niwitpong [14] proposed confidence intervals for the single coefficient of variation and the difference between coefficients of variation in two-parameter exponential distributions.

Furthermore, many researchers have focused on studying the difference and ratio of CVs for datasets containing excess zeros. For instance, Yosboonruang et al. [15] proposed confidence intervals for the difference between CVs of delta-lognormal distributions. Later, Yosboonruang et al. [16] demonstrated that the ratio of CVs can be used to compare rainfall variability between two regions. More recently, Janthasuwan et al. [17] presented confidence intervals for the ratio of CVs under the Delta-Birnbaum–Saunders distribution and applied the proposed methods to compare wind speed data in Thailand.

This study aims to investigate the estimation of confidence intervals for the difference and ratio of coefficients of variation (CVs) under the zero-inflated two-parameter Rayleigh (ZITR) distribution. Several inferential approaches are considered, including Bayesian Credible Intervals (BCIs), Bayesian Highest Posterior Density (HPD) credible intervals, the Approximate Normal (AN) method, Generalized Confidence Intervals (GCIs), the Method of Variance Estimates Recovery (MOVER), Percentile Bootstrap (PB), and bootstrap based on standard errors (BS). The performances of these methods are evaluated through simulation studies under various scenarios using coverage probability (CP) and expected length (EL) as the evaluation criteria. Furthermore, the proposed confidence interval methods are illustrated through an empirical analysis of district-level road traffic fatality rates (per 100,000 population) from Uttaradit and Mae Hong Son provinces, Thailand, recorded in March 2026. The fatality rates were calculated based on district-level population sizes, allowing comparisons across districts with different population scales. The data contain a substantial proportion of zero observations, corresponding to districts with no reported fatalities, making them well suited for assessing the performance and practical applicability of the proposed methods under zero-inflated settings.

2. Materials and Methods

Consider two independent samples, where the j -th observation from the (i) -th population, denoted by X_{ij} for $j = 1, 2, \dots, n_i$ and $i = 1, 2$, follows a zero-inflated two-parameter Rayleigh distribution (ZITR) with parameters μ_i , λ_i , and δ_i . This distribution is denoted as $X_{ij} \sim \text{ZITR}(\mu_i, \lambda_i, \delta_i)$. The parameter δ_i is the proportion of zero observations, while μ_i and λ_i determine the location and scale properties of the positive Rayleigh component, respectively. Therefore, the probability density function of the ZITR distribution can be expressed as

$$f(x_{ij}; \mu_i, \lambda_i, \delta_i) = \begin{cases} \delta_i & ; x_{ij} = 0 \\ (1 - \delta_i) 2\lambda_i(x_{ij} - \mu_i)e^{-\lambda_i(x_{ij} - \mu_i)^2} & ; x_{ij} > \mu_i, \end{cases} \quad (1)$$

where $P(X_{ij} = 0) = \delta_i$. For each group, the sample of size n_i consists of $n_{i(0)}$ zero observations and $n_{i(1)}$ positive observations, satisfying $n_i = n_{i(0)} + n_{i(1)}$. Under this formulation, the number of zero observations follows a binomial distribution, $n_{i(0)} \sim \text{Bin}(n_i, \delta_i)$, which leads directly to the maximum likelihood estimator (MLE) of the zero-inflation parameter, given by $\hat{\delta}_i = n_{i(0)}/n_i$.

The parameters μ_i and λ_i are estimated via the MLEs. However, closed-form solutions for these estimators are not available [18,19]; therefore, numerical optimization techniques are employed. The estimator of the location parameter μ_i is obtained by solving

$$\frac{2n^2_{i(1)}(\bar{x} - \mu)}{\sum_{j=1}^{n_{i(1)}} (x_{ij} - \mu_i)^2} = \sum_{j=1}^{n_{i(1)}} (x_{ij} - \mu_i)^{-1},$$

This is resolved numerically. Upon obtaining $\hat{\mu}_i$, the maximum likelihood estimator for the scale parameter is

$$\hat{\lambda}_i = \frac{n_{i(1)}}{\sum_{j=1}^{n_{i(1)}} (x_{ij} - \hat{\mu}_i)^2}.$$

The profile log-likelihood is optimized, and the root-finding procedure for μ_i is performed using the uniroot function in R, with a convergence threshold of 10^{-5} . The coefficient of variation of ZITR is expressed as

$$CV(X_i) = \kappa_i = \frac{\sqrt{V(X_i)}}{E(X_i)} = \frac{\sqrt{\frac{4(1-\delta_i)-(1-\delta_i)^2\pi}{4\lambda_i} + \frac{\mu_i}{\lambda_i^{\frac{1}{2}}} \sqrt{\pi\delta_i(1-\delta_i)} + \mu_i^2\delta_i(1-\delta_i)}}{(1-\delta_i)\left(\mu_i + \frac{1}{\sqrt{\lambda_i}}\Gamma\left(\frac{3}{2}\right)\right)}$$

Let x_1 and x_2 represent two independent samples of sizes n_1 and n_2 extracted from populations characterized by characteristics $(\mu_1, \lambda_1, \delta_1)$ and $(\mu_2, \lambda_2, \delta_2)$, respectively. The parameters of interest are the difference and ratio of the coefficients of variation, which are specified as

$$\theta = \kappa_1 - \kappa_2 = \left(\frac{\sqrt{\frac{4(1-\delta_1)-(1-\delta_1)^2\pi}{4\lambda_1} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi\delta_1(1-\delta_1)} + \mu_1^2\delta_1(1-\delta_1)}}{(1-\delta_1)\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)} - \frac{\sqrt{\frac{4(1-\delta_2)-(1-\delta_2)^2\pi}{4\lambda_2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi\delta_2(1-\delta_2)} + \mu_2^2\delta_2(1-\delta_2)}}{(1-\delta_2)\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)} \right), \quad (2)$$

and

$$\tau = \kappa_1 / \kappa_2 = \left(\frac{\sqrt{\frac{4(1-\delta_1)-(1-\delta_1)^2\pi}{4\lambda_1} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi\delta_1(1-\delta_1)} + \mu_1^2\delta_1(1-\delta_1)}}{(1-\delta_1)\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)} \right) / \left(\frac{\sqrt{\frac{4(1-\delta_2)-(1-\delta_2)^2\pi}{4\lambda_2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi\delta_2(1-\delta_2)} + \mu_2^2\delta_2(1-\delta_2)}}{(1-\delta_2)\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)} \right). \quad (3)$$

These statistics are used to derive confidence intervals and credible intervals for the difference and ratio of CVs under the proposed and standard methods.

2.1. Proposed Method

This study proposes confidence interval and credible interval procedures for estimating the difference and ratio of coefficients of variation under the ZITR distribution. The proposed methods include the Bayesian Credible Interval (BCI), Bayesian Highest Posterior Density (HPD), and Approximate Normal (AN) approaches, selected due to their flexibility and computational efficiency in complex distributional settings. The theoretical derivation of the AN method is provided in Appendix B.

2.1.1. Bayesian Credible Interval (BCI)

BCI combines prior knowledge with observed data to revise uncertainty around model parameters via the posterior distribution. Let $X_{ij} = (X_{i1}, \dots, X_{in_i})$, for $i = 1, 2$, denote two independent samples from the ZITR distribution. This study focuses on Bayesian inference for the difference and ratio of coefficients of variation. Since the posterior distributions do not have closed-form expressions, simulation-based inference is required. A Markov Chain Monte Carlo (MCMC) algorithm implemented in OpenBUGS version 3.2.3 is employed to generate posterior samples.

The Bayesian computation is performed using a Metropolis-within-Gibbs algorithm. For the zero-inflation component, the parameter δ is updated using Gibbs sampling due to the conjugate Beta prior distribution. For the positive component, the likelihood is implemented using the ones trick, resulting in non-conjugate full conditional distributions for μ and λ . Therefore, these parameters are updated using the Metropolis step within the Gibbs sampling framework.

The prior distributions are specified as Gamma(1, 2), Uniform(0, 3), and Beta(8, 2) for the parameters λ , μ , and δ , respectively. These prior choices ensure that the parameter constraints are satisfied and improve numerical stability during computation. All analyses are implemented in R version 4.4.1 using the R2OpenBUGS package. The MCMC procedure is performed using one Markov chain with specified initial values. The chain is run for 20,000 iterations, with the first 5000 iterations discarded as burn-in. The convergence and mixing performance are assessed using trace plots and autocorrelation plots, and the corresponding MCMC convergence diagnostics, together with the prior sensitivity analysis, are reported in Appendix A.

The Bayesian Credible Intervals (BCIs) for the difference in CVs, denoted by θ , and the ratio of CVs, denoted by τ , are constructed based on the corresponding posterior distributions as follows:

$$CI_{\theta(\text{BCI})} = (L_{\theta(\text{BCI})}, U_{\theta(\text{BCI})}) = (\theta_{\text{BCI}}(\alpha/2), \theta_{\text{BCI}}(1 - \alpha/2)), \quad (4)$$

and

$$CI_{\tau(\text{BCI})} = (L_{\tau(\text{BCI})}, U_{\tau(\text{BCI})}) = (\tau_{\text{BCI}}(\alpha/2), \tau_{\text{BCI}}(1 - \alpha/2)). \quad (5)$$

where $\theta_{\text{BCI}}(p)$ and $\tau_{\text{BCI}}(p)$ denote the p -th posterior quantiles of θ and τ , respectively, obtained from the MCMC samples. Therefore, $\theta_{\text{BCI}}(\alpha/2)$ and $\theta_{\text{BCI}}(1 - \alpha/2)$ represent the lower and upper equal-tailed posterior quantiles, respectively (Algorithm 1).

Algorithm 1 Bayesian credible interval.

1. Assume that $x_i = (x_{i1}, \dots, x_{in_i})$; $i = 1, 2$ is observed from a $ZITR(\mu_i, \lambda_i, \delta_i)$ distribution. Independent prior specifications are assigned as follows: $\delta \sim \text{Beta}(8, 2)$, $\lambda \sim \text{Gamma}(1, 2)$, $\mu \sim \text{Uniform}(0, 3)$.
 2. Posterior results are obtained via a Gibbs sampling technique used in OpenBUGS.
 3. At iteration t , evaluate $\hat{\theta}^{(t)}$, $\hat{\tau}^{(t)}$ and discard the first draws as burn-in, and keep the remaining draws for inference.
 4. The $(1 - \alpha)100\%$ equal-tailed credible intervals for θ and τ are obtained using the empirical $\alpha/2$ and $1 - \alpha/2$ quantiles of the posterior samples.
-

2.1.2. Bayesian Highest Posterior Density (HPD)

The Bayesian HPD method establishes an interval based on the posterior distribution that contains the most probable parameter values while minimizing the interval length [20]. Unlike the equal-tailed Bayesian Credible Interval, which is obtained from posterior quantiles, the HPD credible interval is defined as the shortest interval containing the specified

posterior probability. The HDInterval package in R is employed to explicitly calculate the HPD credible intervals for the difference and ratio of the coefficients of variation, denoted by θ and τ , from the retained posterior samples in this study. Consequently, the two-sided HPD credible intervals for θ and τ are denoted by

$$CI_{\theta(\text{HPD})} = (L_{\theta(\text{HPD})}, U_{\theta(\text{HPD})}), \quad (6)$$

and

$$CI_{\tau(\text{HPD})} = (L_{\tau(\text{HPD})}, U_{\tau(\text{HPD})}). \quad (7)$$

The methodology for deriving Bayesian HPD credible intervals for the differences and ratios of CVs under the ZITR is included in Algorithm 2.

2.1.3. Approximate Normal (AN)

The estimation of confidence intervals for the difference and ratio of CVs under the normal approximation framework is conducted using the Delta method. This approach begins by deriving the approximate means and variances of θ and τ , while assuming that the estimators are asymptotically normally distributed under large-sample conditions. Let $\hat{\delta}$, $\hat{\mu}$, and $\hat{\lambda}$ denote MLEs of the parameters δ , μ , and λ , respectively. For ZITR distribution, the estimator of $\hat{V}(\hat{\theta})$ and $\hat{V}(\hat{\tau})$ are provided in Appendix B and can be expressed as follows:

$$\begin{aligned} \hat{V}(\hat{\theta}) = & \left(\frac{\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{2\pi(1-\hat{\delta}_1)-4}{4\hat{\lambda}_1} + \frac{\hat{\mu}_1}{\hat{\lambda}_1^{\frac{3}{2}}} \sqrt{\pi} [1-2\hat{\delta}_1] + \hat{\mu}_1^2 [1-2\hat{\delta}_1] \right) - \left(\sqrt{\hat{\xi}_1} \right) \left(-1 \left(\left(\hat{\mu}_1 + \frac{1}{\sqrt{\hat{\lambda}_1}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\hat{\psi}_1^2} \right)^2 \left(\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1} \right) \\ & + \left(\frac{\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{\hat{\lambda}_1^{\frac{3}{2}}} + 2\hat{\mu}_1\hat{\delta}_1(1-\hat{\delta}_1) \right) - \left(\sqrt{\hat{\xi}_1} \right) (1-\hat{\delta}_1)}{\hat{\psi}_1^2} \right)^2 \left(\frac{1}{2\hat{\mu}_1 + \frac{1}{n_{1(1)}} \sum_{i=1}^{n_{1(1)}} (x_{1j(1)} - \hat{\mu}_1)^{-2}} \right) \\ & + \left(\frac{\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{(1-\hat{\delta}_1)^2\pi-4(1-\hat{\delta}_1)}{4\hat{\lambda}_1^2} - \frac{\hat{\mu}_1\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{2\hat{\lambda}_1^{\frac{3}{2}}} \right) - \sqrt{\hat{\xi}_1} \left((1-\hat{\delta}_1) \left(-\frac{1}{2}\hat{\lambda}_1^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right)}{\hat{\psi}_1^2} \right)^2 \frac{\hat{\lambda}_1^2}{n_{1(1)}} \\ & + \left(\frac{\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{2\pi(1-\hat{\delta}_2)-4}{4\hat{\lambda}_2} + \frac{\hat{\mu}_2}{\hat{\lambda}_2^{\frac{3}{2}}} \sqrt{\pi} [1-2\hat{\delta}_2] + \hat{\mu}_2^2 [1-2\hat{\delta}_2] \right) - \left(\sqrt{\hat{\xi}_2} \right) \left(-1 \left(\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\hat{\psi}_2^2} \right)^2 \left(\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2} \right) \\ & + \left(\frac{\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{\hat{\lambda}_2^{\frac{3}{2}}} + 2\hat{\mu}_2\hat{\delta}_2(1-\hat{\delta}_2) \right) - \left(\sqrt{\hat{\xi}_2} \right) (1-\hat{\delta}_2)}{\hat{\psi}_2^2} \right)^2 \left(\frac{1}{2\hat{\mu}_2 + \frac{1}{n_{2(1)}} \sum_{j=1}^{n_{2(1)}} (x_{2j(1)} - \hat{\mu}_2)^{-2}} \right) \\ & + \left(\frac{\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{(1-\hat{\delta}_2)^2\pi-4(1-\hat{\delta}_2)}{4\hat{\lambda}_2^2} - \frac{\hat{\mu}_2\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{2\hat{\lambda}_2^{\frac{3}{2}}} \right) - \sqrt{\hat{\xi}_2} \left((1-\hat{\delta}_2) \left(-\frac{1}{2}\hat{\lambda}_2^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right)}{\hat{\psi}_2^2} \right)^2 \frac{\hat{\lambda}_2^2}{n_{2(1)}}, \end{aligned} \quad (8)$$

and

$$\begin{aligned}
\hat{V}(\hat{\tau}) = & \left(\left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{2\pi(1-\hat{\delta}_2)-4}{4\hat{\lambda}_2^2} + \frac{\hat{\mu}_2}{\hat{\lambda}_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\hat{\delta}_2] + \hat{\mu}_2^2 [1-2\hat{\delta}_2] \right) - (\sqrt{\hat{\xi}_2}) \left(-1 \left(\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \left(\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1} \right) \\
& + \left(\frac{\hat{\kappa}_2 \left(\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{\hat{\lambda}_1^{\frac{1}{2}}} + 2\hat{\mu}_1\hat{\delta}_1(1-\hat{\delta}_1) \right) - (\sqrt{\hat{\xi}_1})(1-\hat{\delta}_1) \right)}{\frac{\hat{\psi}_1^2}{\hat{\kappa}_2^2}} \right)^2 \left(\frac{1}{2\hat{\mu}_1 + \frac{1}{n_{1(1)}} \sum_{i=1}^{n_{1(1)}} (x_{1j(1)} - \hat{\mu}_1)^{-2}} \right) \\
& + \left(\frac{\hat{\kappa}_2 \left(\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{(1-\hat{\delta}_1)^2\pi-4(1-\hat{\delta}_1)}{4\hat{\lambda}_1^2} - \frac{\hat{\mu}_1\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{2\hat{\lambda}_1^{\frac{3}{2}}} \right) - \sqrt{\hat{\xi}_1} \left((1-\hat{\delta}_1) \left(-\frac{1}{2}\hat{\lambda}_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\frac{\hat{\psi}_1^2}{\hat{\kappa}_2^2}} \right)^2 \frac{\hat{\lambda}_1^2}{n_{1(1)}} \\
& + \left(\left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{2\pi(1-\hat{\delta}_2)-4}{4\hat{\lambda}_2^2} + \frac{\hat{\mu}_2}{\hat{\lambda}_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\hat{\delta}_2] + \hat{\mu}_2^2 [1-2\hat{\delta}_2] \right) - (\sqrt{\hat{\xi}_2}) \left(-1 \left(\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \left(\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2} \right) \\
& + \left(\left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{\hat{\lambda}_2^{\frac{1}{2}}} + 2\hat{\mu}_2\hat{\delta}_2(1-\hat{\delta}_2) \right) - (\sqrt{\hat{\xi}_2})(1-\hat{\delta}_2) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \left(\frac{1}{2\hat{\mu}_2 + \frac{1}{n_{2(1)}} \sum_{j=1}^{n_{2(1)}} (x_{2j(1)} - \hat{\mu}_2)^{-2}} \right) \\
& + \left(\left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{(1-\hat{\delta}_2)^2\pi-4(1-\hat{\delta}_2)}{4\hat{\lambda}_2^2} - \frac{\hat{\mu}_2\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{2\hat{\lambda}_2^{\frac{3}{2}}} \right) - \sqrt{\hat{\xi}_2} \left((1-\hat{\delta}_2) \left(-\frac{1}{2}\hat{\lambda}_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \frac{\hat{\lambda}_2^2}{n_{2(1)}},
\end{aligned} \tag{9}$$

The estimators $\hat{V}(\hat{\theta})$ and $\hat{V}(\hat{\tau})$ for ZITR distribution are detailed in Appendix B. Assuming asymptotic normality of the estimators, approximate $(1 - \alpha)100\%$ confidence intervals for θ and τ under ZITR distribution are constructed as follows:

$$CI_{\theta(AN)} = \left(\hat{\theta} - z_{\alpha/2} \sqrt{\hat{V}(\hat{\theta})}, \hat{\theta} + z_{\alpha/2} \sqrt{\hat{V}(\hat{\theta})} \right), \tag{10}$$

and

$$CI_{\tau(AN)} = \left(\hat{\tau} - z_{\alpha/2} \sqrt{\hat{V}(\hat{\tau})}, \hat{\tau} + z_{\alpha/2} \sqrt{\hat{V}(\hat{\tau})} \right), \tag{11}$$

respectively, where denotes the upper $\alpha/2$ quantile of the standard normal distribution. The explicit formulations of $\hat{V}(\hat{\theta})$ and $\hat{V}(\hat{\tau})$ are derived by the delta technique and are

provided in Appendix B. Confidence intervals for the difference and ratio of CVs for the ZITR are calculated based on these variance estimates for CVs, following the process outlined in Algorithm 3.

Algorithm 2 Bayesian highest posterior density.

1. Assume that $x = (x_1, \dots, x_n)$ is observed from $ZITR(\mu_i, \lambda_i, \delta_i)$ distribution. Independent prior specifications are assigned as follows: $\delta \sim \text{Beta}(8, 2)$, $\lambda \sim \text{Gamma}(1, 2)$, $\mu \sim \text{Uniform}(0, 3)$.
 2. Posterior results are obtained via a Gibbs sampling technique used in OpenBUGS.
 3. At iteration t , evaluate $\hat{\theta}^{(t)}$, $\hat{\tau}^{(t)}$ and discard the first draws as burn-in, and keep the remaining draws for inference.
 4. Obtain the 95% HPD credible interval for θ and τ using the R package HDInterval, which identifies the shortest interval containing 95% of the posterior probability based on the retained posterior samples.
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Algorithm 3 Approximate normal method.

1. Assume that $x_i = (x_{i1}, \dots, x_{in_i})$; $i = 1, 2$ is observed from a $ZITR(\mu_i, \lambda_i, \delta_i)$ distribution.
 2. Find the MLEs of $\hat{\mu}_i$, $\hat{\lambda}_i$ and $\hat{\delta}_i$ where $i = 1, 2$
 3. Compute confidence intervals based on the Approximate Normal distribution from Equations (10) and (11).
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2.2. Standard Method

The three proposed methods were compared with four established approaches, namely the Generalized Confidence Interval (GCI), the Method of Variance Estimates Recovery (MOVER), the Percentile Bootstrap (PB), and the Bootstrap with Standard Error (BS).

2.2.1. Generalized Confidence Interval (GCI)

Krishnamoorthy et al. [19] derived critical values for the two-parameter Rayleigh distribution as an alternative approach for estimating parameters. Several authors have proposed the GCI method for interval estimation, and Weerahandi [21] suggested it as a general method for constructing confidence intervals in the presence of nuisance parameters. Let $X_i = (X_{i1}, \dots, X_{in_i})$, for $i = 1, 2$, denote a random sample from a distribution with pdf $f(x; \theta, \zeta)$, where ζ denotes a vector of nuisance parameters and θ represents the parameter of interest. A function $R(X, x, \theta, \zeta)$ is defined as a generalized pivotal quantity (GPQ) for interval estimation if it satisfies the following conditions:

- (i) Unknown parameters do not affect the sampling distribution of $R(X; x, \theta, \zeta)$.
- (ii) This formulation of $R(X; x, \theta, \zeta)$ is derived by substituting the random variables with their observed values. The parameter of interest θ is present in the aforementioned formulation, and the nuisance parameters have been efficiently eliminated.

Wu and Hsieh [22] further expanded the approach by using variance-stabilizing transforms (VST) to develop GPQ for the fraction of zero data. Based on this approach, GPQ for difference and ratio CVs is articulated as follows:

$$Q_\theta = Q_{\kappa_1} - Q_{\kappa_2}, \quad (12)$$

and

$$Q_\tau = \frac{Q_{\kappa_1}}{Q_{\kappa_2}}, \quad (13)$$

respectively, where

$$Q_{\kappa_i} = \frac{\sqrt{\frac{4(1-Q_{\delta_i})-(1-Q_{\delta_i})^2\pi}{4Q_{\lambda_i}} + \frac{Q_{\mu_i}}{Q_{\lambda_i}^{1/2}}\sqrt{\pi}Q_{\delta_i}(1-Q_{\delta_i}) + Q_{\mu_i}^2Q_{\delta_i}(1-Q_{\delta_i})}}{(1-Q_{\delta_i})\left(Q_{\mu_i} + \frac{1}{\sqrt{Q_{\lambda_i}}}\Gamma\left(\frac{3}{2}\right)\right)}$$

The empirical distributions of Q_θ and Q_τ , which do not depend on unknown factors, may be derived by Monte Carlo simulation. The $(1 - \alpha)$ GCI are generated using the empirical quantiles of Q_θ and Q_τ . In particular,

$$CI_{\theta(\text{GCI})} = (Q_\theta(\alpha/2), Q_\theta(1 - \alpha/2)), \quad (14)$$

and

$$CI_{\tau(\text{GCI})} = (Q_\tau(\alpha/2), Q_\tau(1 - \alpha/2)). \quad (15)$$

$Q_\theta(\alpha/2)$ and $Q_\tau(\alpha/2)$ represent the $100(\alpha/2)$ th percentiles of Q_θ and Q_τ , respectively. The methodology for establishing these intervals inside the zero-inflated two-parameter Rayleigh distribution is encapsulated in Algorithm 4.

Algorithm 4 The generalized confidence interval.

1. Generate two mutually independent random samples, $X_{1j} \sim \text{ZITR}(\mu_1, \lambda_1, \delta_1)$ and $X_{2j} \sim \text{ZITR}(\mu_2, \lambda_2, \delta_2)$.
2. Find the MLEs of $\hat{a}_i$, $\hat{b}_i$ and $\hat{\delta}_i$.
3. Using the distributions of the Rayleigh parameter estimators, construct the crucial quantities $\hat{a}_i^*$ and $\hat{b}_i^*$.
4. For the zero-inflation parameter, we use a variance-stabilizing transformation and take

$$T = 2\sqrt{n}\left(\arcsin \sqrt{\hat{\delta}} - \arcsin \sqrt{\delta}\right),$$

where $T \sim N(0, 1)$. The associated GPQ is

$$Q_\delta = \sin^2\left(\arcsin \sqrt{\hat{\delta}} - \frac{T}{2\sqrt{n}}\right).$$

For the two-parameter Rayleigh parameters, construct GPQ as

$$Q_{a_i} = \hat{a}_i - \hat{b}_i \frac{\hat{a}_i^*}{\hat{b}_i^*}, \quad Q_{b_i} = \frac{\hat{b}_i}{\hat{b}_i^*}.$$

Then define

$$Q_{\mu_i} = Q_{a_i}, \quad Q_{\lambda_i} = \frac{1}{2Q_{b_i}^2}.$$

Substitute these quantities into the expression of the difference and ratio of CV to obtain the GPQ for the parameter of interest.

5. Repeat steps 1–4 many times, say q times, to approximate the distribution of the GPQ.
 6. Order the simulated values of Q_θ and Q_τ and find their corresponding percentiles to construct the 95% GCI method as given by Equations (14) and (15).
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2.2.2. The Method of Variance Estimates Recovery (MOVER)

Donner and Zou [23] introduced a methodology for calculating confidence intervals for both the difference and the ratio of two distribution parameters. This method is especially advantageous when obtaining analytical formulas for the coefficients of variation

of the estimators proves challenging, hence providing a pragmatic option for interval estimation of both metrics. This research presents the confidence ranges for the differences in coefficients of variation and the coefficients of variation ratio, represented by θ and τ , as

$$CI_{\theta(\text{MOVER})} = \left(L_{\theta(\text{MOVER})}, U_{\theta(\text{MOVER})} \right), \quad (16)$$

and

$$CI_{\tau(\text{MOVER})} = \left(L_{\tau(\text{MOVER})}, U_{\tau(\text{MOVER})} \right). \quad (17)$$

The boundaries for θ are specified as follows

$$L_{\theta(\text{MOVER})} = (\hat{\kappa}_1 - \hat{\kappa}_2) - \sqrt{(\hat{\kappa}_1 - l_1)^2 - (u_2 - \hat{\kappa}_2)^2}, \quad (18)$$

and

$$U_{\theta(\text{MOVER})} = (\hat{\kappa}_1 - \hat{\kappa}_2) + \sqrt{(u_1 - \hat{\kappa}_1)^2 - (\hat{\kappa}_2 - l_2)^2}. \quad (19)$$

The associated limits for the coefficient of variation for ratio τ are specified by

$$L_{\tau(\text{MOVER})} = \frac{\hat{\kappa}_1 \hat{\kappa}_2 - \sqrt{(\hat{\kappa}_1 \hat{\kappa}_2)^2 - l_1 u_2 (2\hat{\kappa}_1 - l_1)(2\hat{\kappa}_2 - u_2)}}{u_2 (2\hat{\kappa}_2 - u_2)}, \quad (20)$$

and

$$U_{\tau(\text{MOVER})} = \frac{\hat{\kappa}_1 \hat{\kappa}_2 + \sqrt{(\hat{\kappa}_1 \hat{\kappa}_2)^2 - u_1 l_2 (2\hat{\kappa}_1 - u_1)(2\hat{\kappa}_2 - l_2)}}{l_2 (2\hat{\kappa}_2 - l_2)}. \quad (21)$$

In this context, (l_1, u_1) and (l_2, u_2) represent the confidence intervals for the respective coefficients of variation parameters, derived by the Generalized Confidence Interval (GCI) methodology. The MOVER framework integrates these limitations to formulate the confidence intervals for θ and τ , as detailed in Algorithm 5.

Algorithm 5 The method of variance estimates recovery.

1. Generate two mutually independent random samples, $X_{1j} \sim ZITR(\mu_1, \lambda_1, \delta_1)$ and $X_{2j} \sim ZITR(\mu_2, \lambda_2, \delta_2)$.
 2. Estimate the parameters $\hat{\kappa}_1$ and $\hat{\kappa}_2$ using the Generalized Confidence Interval (GCI) method.
 3. Calculate the confidence intervals for κ_1 and κ_2 using the GCI methodology.
 4. Obtain the confidence intervals for θ and τ according to Equations (16) and (17), respectively.
-

2.2.3. Percentile Bootstrap (PB)

The Percentile Bootstrap procedure is a nonparametric resampling technique that requires no strict distributional assumptions and was originally introduced by Efron and Tibshirani [24]. Consider a random sample $X_i = (X_{i1}, X_{i2}, \dots, X_{in_i})$ drawn from the *ZITR* distribution. Bootstrap replications are obtained by drawing observations with replacement from the observed data, resulting in $X_i^{*b} = (X_{i1}^{*b}, X_{i2}^{*b}, \dots, X_{in_i}^{*b})$, and the process is repeated B times.

In this framework, let θ and τ denote the difference and the ratio of the coefficients of variation for the two populations, respectively. The corresponding bootstrap statistics, $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$, are computed from each resample for $b = 1, 2, \dots, B$. The empirical distributions of these bootstrap replicates are then utilized to construct confidence intervals. Accordingly, the two-sided Percentile Bootstrap confidence interval for the difference in CVs is defined as follows:

$$CI_{\theta(PB)} = (L_{\theta(PB)}, U_{\theta(PB)}) = (\hat{\theta}^*(\frac{\alpha}{2}), \hat{\theta}^*(1 - \frac{\alpha}{2})), \quad (22)$$

Here, $\hat{\theta}^*(\frac{\alpha}{2})$ and $\hat{\theta}^*(1 - \frac{\alpha}{2})$ denote the empirical $\frac{\alpha}{2}$ and $1 - \frac{\alpha}{2}$ quantiles of the ordered bootstrap estimates $\hat{\theta}^{*b}$. The two-sided Percentile Bootstrap confidence interval for the ratio in CVs is

$$CI_{\tau(PB)} = (L_{\tau(PB)}, U_{\tau(PB)}) = (\hat{\tau}^*(\frac{\alpha}{2}), \hat{\tau}^*(1 - \frac{\alpha}{2})), \quad (23)$$

where $\hat{\tau}^*(\frac{\alpha}{2})$ and $\hat{\tau}^*(1 - \frac{\alpha}{2})$ represent the empirical $\frac{\alpha}{2}$ and $1 - \frac{\alpha}{2}$ quantiles of the ordered bootstrap estimates $\hat{\tau}^{*b}$. Consequently, the Percentile Bootstrap yields an estimated confidence range for both the difference and the ratio of CVs inside the zero-inflated two-parameter Rayleigh model. Algorithm 6 provides a comprehensive summary of the detailed implementation steps.

Algorithm 6 The percentile bootstrap.

1. Let $x_i = (x_{i1}, \dots, x_{in_i})$, for $i = 1, 2$, be an observed sample from the $ZITR(\mu_i, \lambda_i, \delta_i)$ distribution.
 2. For each sample, construct a bootstrap resample $X_i^{*b} = (X_{i1}^{*b}, \dots, X_{in_i}^{*b})$ by sampling with replacement from x_i .
 3. Based on the b th bootstrap sample, obtain the maximum likelihood estimators $\hat{\mu}_i^{*b}$, $\hat{\lambda}_i^{*b}$, and $\hat{\delta}_i^{*b}$ for $i = 1, 2$.
 4. Compute the bootstrap statistics $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$ corresponding to the difference and ratio of CVs using the estimates obtained in Step 3.
 5. For $b = 1, 2, \dots, B$, where B represents the total number of bootstrap replications, repeat Steps 2–4.
 6. Sort the bootstrap replicates $\{\hat{\theta}^{*1}, \dots, \hat{\theta}^{*B}\}$ and $\{\hat{\tau}^{*1}, \dots, \hat{\tau}^{*B}\}$ in ascending order.
 7. Construct the $100(1 - \alpha)\%$ PB for θ and τ using the empirical quantiles as defined in Equations (22) and (23).
-

The Bootstrap with Standard Error (BS)

The bootstrap-based standard error approach approximates the variability of an estimator through repeated resampling from the observed data, thereby avoiding the need for explicit analytical derivation of variance formulas. This makes the method particularly useful in situations where the underlying distribution is complex or intractable, as the dispersion of the resampled statistics can be directly used to quantify uncertainty.

In this framework, let θ and τ denote the difference and the ratio of CVs under the $ZITR$ model, respectively. For $b = 1, 2, \dots, B$, let $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$ be the corresponding bootstrap estimates obtained from the resamples. The bootstrap sample means are then defined as

$$\bar{\theta}^* = \frac{1}{B} \sum_{b=1}^B \hat{\theta}^{*b}, \quad \bar{\tau}^* = \frac{1}{B} \sum_{b=1}^B \hat{\tau}^{*b}.$$

The standard errors are obtained from the variability of the bootstrap replicates and are calculated using the sample standard deviations of the corresponding bootstrap distributions:

$$SE(\hat{\theta}) = \sqrt{\frac{1}{B-1} \sum_{b=1}^B (\hat{\theta}^{*b} - \bar{\theta}^*)^2},$$

$$SE(\hat{\tau}) = \sqrt{\frac{1}{B-1} \sum_{b=1}^B (\hat{\tau}^{*b} - \bar{\tau}^*)^2}.$$

Thus, the $100(1 - \alpha)\%$ confidence range on both sides for the difference between CVs is given by

$$CI_{\theta(BS)} = (L_{\theta(BS)}, U_{\theta(BS)}) = (\bar{\theta}^* - Z_{\alpha/2}SE(\hat{\theta}), \bar{\theta}^* + Z_{\alpha/2}SE(\hat{\theta})), \quad (24)$$

and the confidence interval for the ratio of CVs is

$$CI_{\tau(BS)} = (L_{\tau(BS)}, U_{\tau(BS)}) = (\bar{\tau}^* - Z_{\alpha/2}SE(\hat{\tau}), \bar{\tau}^* + Z_{\alpha/2}SE(\hat{\tau})), \quad (25)$$

where $Z_{\alpha/2}$ represents the higher $\alpha/2$ quantile of the ordinary normal distribution. The computational procedures for generating these bootstrap confidence intervals are included in Algorithm 7.

Algorithm 7 The bootstrap with standard error.

1. Let $x_i = (x_{i1}, \dots, x_{in_i})$, for $i = 1, 2$, denote a random sample drawn from the $ZITR(\mu_i, \lambda_i, \delta_i)$ distribution.
 2. For each sample, generate a bootstrap resample $X_i^{*b} = (X_{i1}^{*b}, X_{i2}^{*b}, \dots, X_{in_i}^{*b})$ by sampling with replacement from the observed data.
 3. Based on the b th bootstrap sample, compute the bootstrap estimates of the coefficient of variation difference $\hat{\theta}^{*b}$ and the coefficient of variation ratio $\hat{\tau}^{*b}$.
 4. Repeat Steps 2–3 for $b = 1, 2, \dots, B$, where B denotes the total number of bootstrap replications.
 5. Compute the bootstrap means $\bar{\theta}^* = \frac{1}{B} \sum_{b=1}^B \hat{\theta}^{*b}$ and $\bar{\tau}^* = \frac{1}{B} \sum_{b=1}^B \hat{\tau}^{*b}$, and derive the associated standard errors defined by $SE(\hat{\theta}) = \sqrt{\frac{1}{B-1} \sum_{b=1}^B (\hat{\theta}^{*b} - \bar{\theta}^*)^2}$ and $SE(\hat{\tau}) = \sqrt{\frac{1}{B-1} \sum_{b=1}^B (\hat{\tau}^{*b} - \bar{\tau}^*)^2}$.
 6. Formulate the two-sided $100(1 - \alpha)\%$ confidence intervals for θ and τ using the bootstrap method as delineated in Equations (24) and (25).
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3. Results

The purpose of this study is to examine the construction of confidence intervals for the ratio and difference of CVs under the ZITR distribution. A Monte Carlo simulation study is conducted to evaluate the performance of the proposed methods. The proposed methods include the BCI, HPD, and AN approaches. These are compared with standard methods, namely GCI, MOVER, PB, and BS.

Random samples from the ZITR distribution were generated using a two-step mixture procedure. First, a Bernoulli random variable B_i with success probability $1 - \delta$ was generated, i.e., $P_i \sim \text{Bernoulli}(1 - \delta)$. If $B_i = 0$, then $X_i = 0$. Otherwise, if $P_i = 1$, a positive observation was generated from the two-parameter Rayleigh distribution with parameters μ_i and λ_i . Thus,

$$X_i = \begin{cases} 0, & \text{with probability } \delta, \\ Y_i, & \text{with probability } 1 - \delta, \end{cases} \quad (26)$$

where Y_i follows the two-parameter Rayleigh (TR) distribution with parameters μ_i and λ_i . This process was repeated independently until a sample of size n_i was obtained.

Positive observations from TR were generated using the new.dist package in R. The resulting positive observations were then combined with structural zeros according to the Bernoulli mixture mechanism described above to obtain random samples from the ZITR distribution.

The simulation study was carried out using the R statistical software. For each simulation scenario, 1000 replications were generated. In the Bayesian approaches, posterior

inference was obtained using a Metropolis-within-Gibbs algorithm implemented through OpenBUGS. Each replication included the generation of 20,000 MCMC iterations to estimate the joint posterior distributions of the model parameters. To reduce the influence of initial values and improve the convergence of the Markov chains, the first 5000 iterations were discarded as burn-in samples. The remaining posterior samples were used to construct Bayesian Credible Intervals. Each simulation iteration for the GCI methodology generated 2500 generalized critical quantities, whereas the bootstrap-based techniques yielded 1000 bootstrap resamples.

The interval estimators were assessed based on three criteria: coverage probability (CP), expected length (EL), and the Monte Carlo standard error (MCSE) of the estimated coverage probability. The coverage probability (CP) was estimated as the proportion of simulation replications in which the constructed interval contained the true parameter value and was computed as

$$CP = \frac{1}{M} \sum_{m=1}^M I(L_m \leq \theta \leq U_m), \quad (27)$$

where L_m and U_m denote the lower and upper bounds of the interval estimate in the m th simulation replication, θ is the true parameter value, $I(\cdot)$ is the indicator function, and M is the number of simulation replications.

The expected length (EL) was computed as the average interval length across all simulation replications:

$$EL = \frac{1}{M} \sum_{m=1}^M (U_m - L_m). \quad (28)$$

The MCSE was reported to quantify the simulation uncertainty associated with the estimated CP and was computed as

$$MCSE(CP) = \sqrt{\frac{CP(1-CP)}{M}}. \quad (29)$$

where $M = 1000$ denotes the number of Monte Carlo simulation replications. The MCSE measures the variability of the estimated CP resulting from the finite number of simulation replications. A small MCSE indicates that the estimated CP is stable and has low simulation error, thereby supporting the reliability of the simulation results. The interval estimation methods were evaluated primarily based on the coverage probability (CP) and expected length (EL). The MCSE was additionally reported to assess the precision of the estimated CP values obtained from the Monte Carlo simulation.

The techniques were deemed adequate if their CP approached or surpassed the nominal level of 0.95 while preserving a very short EL. These criteria provide a comprehensive evaluation of dependability and accuracy. The examined sample size pairings were $(n_1, n_2) = (20, 20), (20, 50), (20, 70), (20, 100), (50, 50), (50, 70), (50, 100), (70, 70), (70, 100), (100, 100), (200, 200)$. Representative parameter combinations were chosen to preserve clarity while guaranteeing practical significance. The scale parameters were delineated as $(\lambda_1, \lambda_2) = (0.2, 0.2), (0.2, 1), (1, 1)$, while the location parameters were fixed at $(\mu_1, \mu_2) = (0.2, 0.2)$ and $(1, 0.2)$. The zero-inflation probabilities were specified as $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$. The simulation settings covered scenarios with small to moderate zero inflation and both equal and unequal scale parameters, providing a comprehensive framework for comparing the interval estimation methods for the difference and ratio of the CVs under the ZITR distribution. Since increasing the proportion of excess zeros results in a more highly skewed distribution, interval estimation becomes more challenging. As a consequence, the empirical coverage probability

may differ from the nominal 95% confidence level. These simulation scenarios therefore facilitate a comprehensive comparison of the competing interval estimation methods in terms of empirical coverage probability and expected interval length across different levels of zero inflation.

The performance of the confidence intervals for the difference of the coefficients of variation was evaluated using coverage probability (CP), Monte Carlo standard error (MCSE) of the estimated coverage probability, and expected length (EL), as presented in Tables 1–6 and Figures 1 and 2. The primary criterion for comparing the methods was whether the coverage probability was sufficiently close to the nominal confidence level. Expected length was subsequently used to compare interval precision only among methods with acceptable coverage performance. The reported MCSE values were uniformly small across all simulation settings, indicating that the estimated coverage probabilities were obtained with satisfactory simulation precision. In terms of CP, the BCI, HPD, AN, GCI, and MOVER methods generally produced coverage probabilities close to or exceeding the nominal confidence level of 0.95 across most sample sizes. Meanwhile, the PB method showed better performance with the increase in sample sizes, leading to more satisfactory coverage behavior for larger samples.

The HPD method consistently produced the narrowest confidence intervals across all scenarios for interval precision as measured by EL. However, in some simulation settings, the HPD method produced coverage probabilities slightly below the nominal level of 0.95, although the deviations were generally small. Therefore, its shorter interval lengths should be interpreted together with its coverage performance rather than considered in isolation. This discrepancy may be attributable to the finite number of simulation replications and the variability arising from the data generation process.

Similarly, the confidence intervals for the ratio of the coefficients of variation under the ZITR distribution were evaluated using coverage probability (CP), Monte Carlo standard error (MCSE) of the estimated coverage probability, and expected length (EL), as presented in Tables 7–12 and Figures 3 and 4. The results exhibited patterns similar to those observed for the difference of the coefficients of variation, and the main conclusions remained consistent across both comparison settings. The reported MCSE values were uniformly small across all simulation settings, indicating that the estimated coverage probabilities were obtained with satisfactory simulation precision.

Table 1. CP and EL for 95% confidence intervals for the difference of two CVs for ZITR when $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.930	0.850	0.957	0.957	0.983	0.989	0.983
		EL	(0.54611)	(0.56021)	(0.62506)	(0.61977)	(0.95328)	(0.41789)	(0.41630)
		MCSE	[0.00807]	[0.01129]	[0.00641]	[0.00641]	[0.00409]	[0.00330]	[0.00409]
	20:50	CP	0.904	0.805	0.958	0.958	0.989	0.986	0.989
		EL	(0.44910)	(0.44855)	(0.50021)	(0.49716)	(0.88093)	(0.40481)	(0.40191)
		MCSE	[0.00932]	[0.01253]	[0.00634]	[0.00634]	[0.00330]	[0.00372]	[0.00330]
	20:70	CP	0.901	0.817	0.963	0.960	0.994	0.990	0.991
		EL	(0.43359)	(0.43220)	(0.48440)	(0.48172)	(0.87665)	(0.38863)	(0.38463)
		MCSE	[0.00944]	[0.01223]	[0.00597]	[0.00620]	[0.00244]	[0.00315]	[0.00299]

Table 1. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:100	CP	0.881	0.787	0.944	0.945	0.991	0.983	0.988
		EL	(0.41279)	(0.40753)	(0.46165)	(0.45960)	(0.85112)	(0.37187)	(0.36691)
		MCSE	[0.01024]	[0.01295]	[0.00727]	[0.00721]	[0.00299]	[0.00409]	[0.00344]
	50:50	CP	0.936	0.852	0.945	0.942	0.999	0.962	0.962
		EL	(0.33836)	(0.34912)	(0.35583)	(0.35447)	(0.83334)	(0.28961)	(0.28871)
		MCSE	[0.00774]	[0.01123]	[0.00721]	[0.00739]	[0.00100]	[0.00605]	[0.00605]
	50:70	CP	0.948	0.825	0.955	0.955	0.999	0.972	0.973
		EL	(0.31292)	(0.31857)	(0.32708)	(0.32597)	(0.81744)	(0.29519)	(0.29419)
		MCSE	[0.00702]	[0.01202]	[0.00656]	[0.00656]	[0.00100]	[0.00522]	[0.00513]
	50:100	CP	0.926	0.810	0.940	0.942	0.999	0.965	0.967
		EL	(0.29165)	(0.28371)	(0.30347)	(0.30259)	(0.80576)	(0.27509)	(0.27378)
		MCSE	[0.00828]	[0.01241]	[0.00751]	[0.00739]	[0.00100]	[0.00581]	[0.00565]
	70:70	CP	0.947	0.824	0.955	0.955	0.998	0.955	0.954
		EL	(0.28265)	(0.28214)	(0.29314)	(0.29230)	(0.80921)	(0.24691)	(0.24615)
		MCSE	[0.00708]	[0.01204]	[0.00656]	[0.00656]	[0.00141]	[0.00656]	[0.00662]
	70:100	CP	0.940	0.819	0.951	0.952	1.000	0.969	0.969
		EL	(0.26122)	(0.26383)	(0.26970)	(0.26908)	(0.80112)	(0.24925)	(0.24845)
		MCSE	[0.00751]	[0.01218]	[0.00683]	[0.00676]	[0.00000]	[0.00548]	[0.00548]
	100:100	CP	0.948	0.842	0.950	0.948	0.999	0.954	0.955
		EL	(0.23646)	(0.23534)	(0.24269)	(0.24209)	(0.78546)	(0.21409)	(0.21349)
		MCSE	[0.00702]	[0.01153]	[0.00689]	[0.00702]	[0.00100]	[0.00662]	[0.00656]
	200:200	CP	0.933	0.807	0.947	0.946	1.000	0.951	0.945
		EL	(0.16532)	(0.16327)	(0.16799)	(0.16788)	(0.75730)	(0.15703)	(0.15662)
		MCSE	[0.00791]	[0.01248]	[0.00708]	[0.00715]	[0.00000]	[0.00683]	[0.00721]
0.1:0.3	20:20	CP	0.951	0.843	0.960	0.959	0.949	0.924	0.921
		EL	(0.75722)	(0.76189)	(0.84406)	(0.83974)	(1.04946)	(0.56725)	(0.56432)
		MCSE	[0.00683]	[0.01150]	[0.00620]	[0.00627]	[0.00696]	[0.00838]	[0.00853]
	20:50	CP	0.936	0.834	0.946	0.948	0.973	0.948	0.955
		EL	(0.54886)	(0.55395)	(0.59922)	(0.59556)	(0.89237)	(0.48056)	(0.47879)
		MCSE	[0.00774]	[0.01177]	[0.00715]	[0.00702]	[0.00513]	[0.00702]	[0.00656]
	20:70	CP	0.917	0.829	0.958	0.960	0.976	0.971	0.977
		EL	(0.50690)	(0.50292)	(0.55190)	(0.54954)	(0.85351)	(0.45200)	(0.44990)
		MCSE	[0.00872]	[0.01191]	[0.00634]	[0.00620]	[0.00484]	[0.00531]	[0.00474]
	20:100	CP	0.904	0.813	0.949	0.948	0.970	0.965	0.978
		EL	(0.47379)	(0.46728)	(0.51901)	(0.51680)	(0.83224)	(0.42355)	(0.42053)
		MCSE	[0.00932]	[0.01233]	[0.00696]	[0.00702]	[0.00539]	[0.00581]	[0.00464]

Table 1. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	50:50	CP	0.945	0.833	0.953	0.951	0.984	0.950	0.947
		EL	(0.46108)	(0.45153)	(0.47911)	(0.47815)	(0.83221)	(0.40711)	(0.40533)
		MCSE	[0.00721]	[0.01179]	[0.00669]	[0.00683]	[0.00397]	[0.00689]	[0.00708]
	50:70	CP	0.937	0.823	0.940	0.940	0.995	0.941	0.942
		EL	(0.40939)	(0.41348)	(0.42170)	(0.42109)	(0.80018)	(0.37252)	(0.37137)
		MCSE	[0.00768]	[0.01207]	[0.00751]	[0.00751]	[0.00223]	[0.00745]	[0.00739]
	50:100	CP	0.936	0.827	0.940	0.940	0.992	0.944	0.950
		EL	(0.36364)	(0.36420)	(0.37519)	(0.37400)	(0.78039)	(0.33756)	(0.33659)
		MCSE	[0.00774]	[0.01196]	[0.00751]	[0.00751]	[0.00282]	[0.00727]	[0.00689]
	70:70	CP	0.952	0.820	0.954	0.955	0.997	0.947	0.946
		EL	(0.38605)	(0.39015)	(0.39664)	(0.39580)	(0.79547)	(0.35190)	(0.35062)
		MCSE	[0.00676]	[0.01215]	[0.00662]	[0.00656]	[0.00173]	[0.00708]	[0.00715]
	70:100	CP	0.939	0.849	0.941	0.944	0.998	0.949	0.951
		EL	(0.34093)	(0.34726)	(0.34914)	(0.34858)	(0.77323)	(0.31720)	(0.31628)
		MCSE	[0.00757]	[0.01132]	[0.00745]	[0.00727]	[0.00141]	[0.00696]	[0.00683]
	100:100	CP	0.948	0.847	0.952	0.952	0.999	0.949	0.950
		EL	(0.32188)	(0.32223)	(0.32765)	(0.32727)	(0.75423)	(0.30026)	(0.29924)
		MCSE	[0.00702]	[0.01138]	[0.00676]	[0.00676]	[0.00100]	[0.00696]	[0.00689]
	200:200	CP	0.940	0.832	0.939	0.941	1.000	0.936	0.933
		EL	(0.22498)	(0.22511)	(0.22707)	(0.22678)	(0.70655)	(0.21708)	(0.21643)
		MCSE	[0.00751]	[0.01182]	[0.00757]	[0.00745]	[0.00000]	[0.00774]	[0.00791]
0.3:0.3	20:20	CP	0.934	0.870	0.944	0.940	0.926	0.970	0.970
		EL	(0.93437)	(0.93813)	(1.03460)	(1.02971)	(1.13382)	(0.60492)	(0.60269)
		MCSE	[0.00785]	[0.01063]	[0.00727]	[0.00751]	[0.00830]	[0.00539]	[0.00539]
	20:50	CP	0.940	0.850	0.947	0.950	0.931	0.960	0.963
		EL	(0.76232)	(0.75888)	(0.82534)	(0.82232)	(0.99256)	(0.58632)	(0.58301)
		MCSE	[0.00751]	[0.01129]	[0.00708]	[0.00689]	[0.00801]	[0.00620]	[0.00597]
	20:70	CP	0.935	0.849	0.950	0.952	0.944	0.962	0.959
		EL	(0.72833)	(0.71723)	(0.78941)	(0.78781)	(0.97195)	(0.55925)	(0.55485)
		MCSE	[0.00780]	[0.01132]	[0.00689]	[0.00676]	[0.00727]	[0.00605]	[0.00627]
	20:100	CP	0.931	0.835	0.953	0.952	0.943	0.955	0.949
		EL	(0.70921)	(0.70550)	(0.76962)	(0.76767)	(0.94830)	(0.54073)	(0.53525)
		MCSE	[0.00801]	[0.01174]	[0.00669]	[0.00676]	[0.00733]	[0.00656]	[0.00696]
	50:50	CP	0.941	0.827	0.948	0.945	0.951	0.954	0.952
		EL	(0.55988)	(0.56168)	(0.57729)	(0.57630)	(0.85220)	(0.44920)	(0.44788)
		MCSE	[0.00745]	[0.01196]	[0.00702]	[0.00721]	[0.00683]	[0.00662]	[0.00676]

Table 1. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	50:70	CP	0.932	0.827	0.936	0.931	0.974	0.948	0.951
		EL	(0.51791)	(0.51138)	(0.53267)	(0.53160)	(0.81589)	(0.45413)	(0.45266)
		MCSE	[0.00796]	[0.01196]	[0.00774]	[0.00801]	[0.00503]	[0.00702]	[0.00683]
	50:100	CP	0.951	0.844	0.951	0.952	0.973	0.964	0.968
		EL	(0.48431)	(0.48938)	(0.49690)	(0.49647)	(0.78451)	(0.42902)	(0.42723)
		MCSE	[0.00683]	[0.01147]	[0.00683]	[0.00676]	[0.00513]	[0.00589]	[0.00557]
	70:70	CP	0.936	0.832	0.936	0.940	0.973	0.950	0.948
		EL	(0.46632)	(0.46422)	(0.47885)	(0.47774)	(0.78479)	(0.39030)	(0.38925)
		MCSE	[0.00774]	[0.01182]	[0.00774]	[0.00751]	[0.00513]	[0.00689]	[0.00702]
	70:100	CP	0.951	0.860	0.957	0.960	0.985	0.962	0.960
		EL	(0.43140)	(0.43071)	(0.44013)	(0.43964)	(0.75285)	(0.39168)	(0.39051)
		MCSE	[0.00683]	[0.01097]	[0.00641]	[0.00620]	[0.00384]	[0.00605]	[0.00620]
	100:100	CP	0.939	0.832	0.945	0.943	0.988	0.947	0.947
		EL	(0.38713)	(0.38834)	(0.39454)	(0.39441)	(0.73106)	(0.33777)	(0.33683)
		MCSE	[0.00757]	[0.01182]	[0.00721]	[0.00733]	[0.00344]	[0.00708]	[0.00708]
	200:200	CP	0.938	0.832	0.940	0.938	1.000	0.940	0.943
		EL	(0.27191)	(0.27437)	(0.27447)	(0.27429)	(0.66483)	(0.25174)	(0.25107)
		MCSE	[0.00763]	[0.01182]	[0.00751]	[0.00763]	[0.00000]	[0.00751]	[0.00733]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 2. CP and EL for 95% confidence intervals for the difference of two CVs for *ZITR* when $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.923	0.825	0.944	0.947	0.977	0.968	0.966
		EL	(0.53846)	(0.52584)	(0.60303)	(0.59789)	(0.90611)	(0.42196)	(0.42041)
		MCSE	[0.00843]	[0.01202]	[0.00727]	[0.00708]	[0.00474]	[0.00557]	[0.00573]
	20:50	CP	0.919	0.832	0.950	0.954	0.989	0.985	0.987
		EL	(0.45476)	(0.45805)	(0.50195)	(0.49928)	(0.85147)	(0.41263)	(0.40989)
		MCSE	[0.00863]	[0.01182]	[0.00689]	[0.00662]	[0.00330]	[0.00384]	[0.00358]
	20:70	CP	0.897	0.783	0.945	0.945	0.984	0.989	0.991
		EL	(0.43145)	(0.42330)	(0.47812)	(0.47510)	(0.83035)	(0.39193)	(0.38830)
		MCSE	[0.00961]	[0.01303]	[0.00721]	[0.00721]	[0.00397]	[0.00330]	[0.00299]
	20:100	CP	0.883	0.797	0.953	0.954	0.990	0.987	0.990
		EL	(0.41342)	(0.41654)	(0.46261)	(0.46098)	(0.83046)	(0.37500)	(0.37026)
		MCSE	[0.01016]	[0.01272]	[0.00669]	[0.00662]	[0.00315]	[0.00358]	[0.00315]
	50:50	CP	0.931	0.806	0.940	0.944	1.000	0.969	0.971
		EL	(0.33727)	(0.32428)	(0.35206)	(0.35076)	(0.80417)	(0.29395)	(0.29298)
		MCSE	[0.00801]	[0.01250]	[0.00751]	[0.00727]	[0.00000]	[0.00548]	[0.00531]

Table 2. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	50:70	CP	0.934	0.821	0.949	0.949	0.995	0.963	0.967
		EL	(0.31316)	(0.31383)	(0.32508)	(0.32384)	(0.78449)	(0.30041)	(0.29944)
		MCSE	[0.00785]	[0.01212]	[0.00696]	[0.00696]	[0.00223]	[0.00597]	[0.00565]
	50:100	CP	0.921	0.813	0.936	0.937	0.998	0.966	0.963
		EL	(0.29133)	(0.28717)	(0.30213)	(0.30132)	(0.76919)	(0.27900)	(0.27779)
		MCSE	[0.00853]	[0.01233]	[0.00774]	[0.00768]	[0.00141]	[0.00573]	[0.00597]
	70:70	CP	0.940	0.822	0.947	0.946	1.000	0.953	0.953
		EL	(0.28341)	(0.27868)	(0.29162)	(0.29084)	(0.77505)	(0.25667)	(0.25591)
		MCSE	[0.00751]	[0.01210]	[0.00708]	[0.00715]	[0.00000]	[0.00669]	[0.00669]
	70:100	CP	0.938	0.823	0.948	0.950	1.000	0.959	0.960
		EL	(0.26216)	(0.26441)	(0.26977)	(0.26897)	(0.75747)	(0.25418)	(0.25339)
		MCSE	[0.00763]	[0.01207]	[0.00702]	[0.00689]	[0.00000]	[0.00627]	[0.00620]
	100:100	CP	0.942	0.841	0.955	0.953	1.000	0.965	0.967
		EL	(0.23784)	(0.23891)	(0.24214)	(0.24172)	(0.74867)	(0.21974)	(0.21911)
		MCSE	[0.00739]	[0.01156]	[0.00656]	[0.00669]	[0.00000]	[0.00581]	[0.00565]
	200:200	CP	0.948	0.835	0.959	0.960	1.000	0.964	0.963
		EL	(0.16613)	(0.16483)	(0.16790)	(0.16781)	(0.71350)	(0.15748)	(0.15705)
		MCSE	[0.00702]	[0.01174]	[0.00627]	[0.00620]	[0.00000]	[0.00589]	[0.00597]
0.1:0.3	20:20	CP	0.936	0.844	0.948	0.946	0.949	0.922	0.925
		EL	(0.76652)	(0.78374)	(0.83298)	(0.82878)	(1.06451)	(0.57611)	(0.57314)
		MCSE	[0.00774]	[0.01147]	[0.00702]	[0.00715]	[0.00696]	[0.00848]	[0.00833]
	20:50	CP	0.932	0.824	0.952	0.951	0.969	0.974	0.976
		EL	(0.54726)	(0.54106)	(0.59241)	(0.58934)	(0.86517)	(0.48500)	(0.48331)
		MCSE	[0.00796]	[0.01204]	[0.00676]	[0.00683]	[0.00548]	[0.00503]	[0.00484]
	20:70	CP	0.913	0.820	0.944	0.942	0.963	0.956	0.963
		EL	(0.50937)	(0.51642)	(0.55303)	(0.54967)	(0.82445)	(0.45662)	(0.45442)
		MCSE	[0.00891]	[0.01215]	[0.00727]	[0.00739]	[0.00597]	[0.00649]	[0.00597]
	20:100	CP	0.903	0.789	0.949	0.949	0.970	0.969	0.976
		EL	(0.47173)	(0.46674)	(0.51556)	(0.51260)	(0.79562)	(0.42628)	(0.42340)
		MCSE	[0.00936]	[0.01290]	[0.00696]	[0.00696]	[0.00539]	[0.00548]	[0.00484]
	50:50	CP	0.932	0.834	0.934	0.935	0.981	0.947	0.947
		EL	(0.46214)	(0.45543)	(0.47496)	(0.47415)	(0.82628)	(0.41367)	(0.41191)
		MCSE	[0.00796]	[0.01177]	[0.00785]	[0.00780]	[0.00432]	[0.00708]	[0.00708]
	50:70	CP	0.950	0.821	0.953	0.955	0.990	0.957	0.958
		EL	(0.40790)	(0.40653)	(0.41917)	(0.41837)	(0.77904)	(0.37658)	(0.37548)
		MCSE	[0.00689]	[0.01212]	[0.00669]	[0.00656]	[0.00315]	[0.00641]	[0.00634]

Table 2. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	50:100	CP	0.954	0.822	0.942	0.942	0.997	0.956	0.957
		EL	(0.36249)	(0.35211)	(0.37244)	(0.37158)	(0.74770)	(0.33965)	(0.33865)
		MCSE	[0.00662]	[0.01210]	[0.00739]	[0.00739]	[0.00173]	[0.00649]	[0.00641]
	70:70	CP	0.937	0.828	0.944	0.944	0.993	0.950	0.949
		EL	(0.38629)	(0.38769)	(0.39406)	(0.39358)	(0.76485)	(0.35705)	(0.35574)
		MCSE	[0.00768]	[0.01193]	[0.00727]	[0.00727]	[0.00264]	[0.00689]	[0.00696]
	70:100	CP	0.949	0.842	0.953	0.952	0.996	0.956	0.955
		EL	(0.33952)	(0.34380)	(0.34618)	(0.34565)	(0.73545)	(0.31974)	(0.31881)
		MCSE	[0.00696]	[0.01153]	[0.00669]	[0.00676]	[0.00200]	[0.00649]	[0.00656]
	100:100	CP	0.949	0.841	0.949	0.950	0.996	0.955	0.954
		EL	(0.32037)	(0.31817)	(0.32465)	(0.32445)	(0.72390)	(0.30265)	(0.30162)
		MCSE	[0.00696]	[0.01156]	[0.00696]	[0.00689]	[0.00200]	[0.00656]	[0.00662]
	200:200	CP	0.941	0.819	0.938	0.942	0.999	0.935	0.932
		EL	(0.22443)	(0.22514)	(0.22612)	(0.22602)	(0.66100)	(0.21810)	(0.21745)
		MCSE	[0.00745]	[0.01218]	[0.00763]	[0.00739]	[0.00100]	[0.00780]	[0.00796]
0.3:0.3	20:20	CP	0.943	0.854	0.950	0.951	0.935	0.967	0.970
		EL	(0.93295)	(0.93010)	(1.01796)	(1.01378)	(1.20351)	(0.60965)	(0.60756)
		MCSE	[0.00733]	[0.01117]	[0.00689]	[0.00683]	[0.00780]	[0.00565]	[0.00539]
	20:50	CP	0.938	0.823	0.945	0.945	0.947	0.967	0.969
		EL	(0.76185)	(0.73234)	(0.81886)	(0.81765)	(0.98687)	(0.58838)	(0.58504)
		MCSE	[0.00763]	[0.01207]	[0.00721]	[0.00721]	[0.00708]	[0.00565]	[0.00548]
	20:70	CP	0.935	0.828	0.945	0.945	0.936	0.949	0.944
		EL	(0.73437)	(0.72166)	(0.79300)	(0.79087)	(0.95722)	(0.56500)	(0.56056)
		MCSE	[0.00780]	[0.01193]	[0.00721]	[0.00721]	[0.00774]	[0.00696]	[0.00727]
	20:100	CP	0.927	0.832	0.952	0.954	0.938	0.941	0.929
		EL	(0.70312)	(0.70936)	(0.76310)	(0.76106)	(0.92375)	(0.53871)	(0.53341)
		MCSE	[0.00823]	[0.01182]	[0.00676]	[0.00662]	[0.00763]	[0.00745]	[0.00812]
	50:50	CP	0.945	0.835	0.949	0.950	0.972	0.954	0.960
		EL	(0.55816)	(0.55238)	(0.57311)	(0.57220)	(0.84303)	(0.45761)	(0.45620)
		MCSE	[0.00721]	[0.01174]	[0.00696]	[0.00689]	[0.00522]	[0.00662]	[0.00620]
	50:70	CP	0.955	0.837	0.959	0.961	0.970	0.961	0.964
		EL	(0.51327)	(0.51126)	(0.52675)	(0.52643)	(0.79819)	(0.45540)	(0.45398)
		MCSE	[0.00656]	[0.01168]	[0.00627]	[0.00612]	[0.00539]	[0.00612]	[0.00589]
	50:100	CP	0.941	0.831	0.945	0.942	0.963	0.951	0.952
		EL	(0.48080)	(0.48669)	(0.49344)	(0.49247)	(0.75607)	(0.42892)	(0.42713)
		MCSE	[0.00745]	[0.01185]	[0.00721]	[0.00739]	[0.00597]	[0.00683]	[0.00676]

Table 2. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	70:70	CP	0.942	0.833	0.942	0.942	0.972	0.946	0.950
		EL	(0.46550)	(0.46656)	(0.47440)	(0.47392)	(0.76157)	(0.39795)	(0.39682)
		MCSE	[0.00739]	[0.01179]	[0.00739]	[0.00739]	[0.00522]	[0.00715]	[0.00689]
	70:100	CP	0.953	0.843	0.954	0.954	0.973	0.954	0.956
		EL	(0.42949)	(0.42529)	(0.43790)	(0.43693)	(0.72644)	(0.39307)	(0.39188)
		MCSE	[0.00669]	[0.01150]	[0.00662]	[0.00662]	[0.00513]	[0.00662]	[0.00649]
	100:100	CP	0.944	0.834	0.950	0.950	0.983	0.950	0.955
		EL	(0.38785)	(0.39225)	(0.39365)	(0.39302)	(0.69665)	(0.34405)	(0.34312)
		MCSE	[0.00727]	[0.01177]	[0.00689]	[0.00689]	[0.00409]	[0.00689]	[0.00656]
	200:200	CP	0.957	0.827	0.961	0.961	0.998	0.960	0.959
		EL	(0.27046)	(0.26835)	(0.27297)	(0.27291)	(0.61600)	(0.25142)	(0.25076)
		MCSE	[0.00641]	[0.01196]	[0.00612]	[0.00612]	[0.00141]	[0.00620]	[0.00627]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 3. CP and EL for 95% confidence intervals for the difference of two CVs for *ZITR* when $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (1, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.923	0.832	0.947	0.947	0.973	0.992	0.993
		EL	(0.55167)	(0.54289)	(0.60164)	(0.59583)	(0.85541)	(0.43394)	(0.43235)
		MCSE	[0.00843]	[0.01182]	[0.00708]	[0.00708]	[0.00513]	[0.00282]	[0.00264]
	20:50	CP	0.901	0.815	0.946	0.948	0.982	0.989	0.993
		EL	(0.45070)	(0.45808)	(0.48359)	(0.48077)	(0.79935)	(0.41971)	(0.41703)
		MCSE	[0.00944]	[0.01228]	[0.00715]	[0.00702]	[0.00420]	[0.00330]	[0.00264]
	20:70	CP	0.887	0.797	0.947	0.951	0.989	0.984	0.989
		EL	(0.42773)	(0.42851)	(0.46146)	(0.45912)	(0.79274)	(0.39959)	(0.39588)
		MCSE	[0.01001]	[0.01272]	[0.00708]	[0.00683]	[0.00330]	[0.00397]	[0.00330]
	20:100	CP	0.884	0.798	0.958	0.958	0.987	0.987	0.997
		EL	(0.41129)	(0.41484)	(0.44402)	(0.44211)	(0.78377)	(0.38251)	(0.37791)
		MCSE	[0.01013]	[0.01270]	[0.00634]	[0.00634]	[0.00358]	[0.00358]	[0.00173]
	50:50	CP	0.933	0.837	0.936	0.938	0.995	0.960	0.963
		EL	(0.34023)	(0.33965)	(0.35018)	(0.34889)	(0.74496)	(0.30391)	(0.30299)
		MCSE	[0.00791]	[0.01168]	[0.00774]	[0.00763]	[0.00223]	[0.00620]	[0.00597]
	50:70	CP	0.936	0.822	0.944	0.945	0.998	0.969	0.970
		EL	(0.31247)	(0.30797)	(0.32208)	(0.32094)	(0.73565)	(0.30745)	(0.30647)
		MCSE	[0.00774]	[0.01210]	[0.00727]	[0.00721]	[0.00141]	[0.00548]	[0.00539]
	50:100	CP	0.923	0.819	0.941	0.942	0.999	0.960	0.962
		EL	(0.29014)	(0.29246)	(0.29781)	(0.29688)	(0.71848)	(0.28580)	(0.28456)
		MCSE	[0.00843]	[0.01218]	[0.00745]	[0.00739]	[0.00100]	[0.00620]	[0.00605]

Table 3. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	70:70	CP	0.956	0.855	0.963	0.960	0.995	0.978	0.981
		EL	(0.28554)	(0.28499)	(0.29147)	(0.29096)	(0.71848)	(0.26376)	(0.26301)
		MCSE	[0.00649]	[0.01113]	[0.00597]	[0.00620]	[0.00223]	[0.00464]	[0.00432]
	70:100	CP	0.942	0.819	0.947	0.949	0.998	0.967	0.971
		EL	(0.26108)	(0.26184)	(0.26624)	(0.26544)	(0.70122)	(0.25897)	(0.25815)
		MCSE	[0.00739]	[0.01218]	[0.00708]	[0.00696]	[0.00141]	[0.00565]	[0.00531]
	100:100	CP	0.924	0.815	0.930	0.928	1.000	0.949	0.949
		EL	(0.23698)	(0.23863)	(0.24075)	(0.24031)	(0.69297)	(0.22367)	(0.22305)
		MCSE	[0.00838]	[0.01228]	[0.00807]	[0.00817]	[0.00000]	[0.00696]	[0.00696]
	200:200	CP	0.946	0.831	0.947	0.946	1.000	0.948	0.952
		EL	(0.16668)	(0.16669)	(0.16790)	(0.16774)	(0.65717)	(0.15923)	(0.15882)
		MCSE	[0.00715]	[0.01185]	[0.00708]	[0.00715]	[0.00000]	[0.00702]	[0.00676]
0.1:0.3	20:20	CP	0.923	0.841	0.937	0.938	0.937	0.916	0.915
		EL	(0.76389)	(0.77342)	(0.82502)	(0.82155)	(0.98795)	(0.58230)	(0.57937)
		MCSE	[0.00843]	[0.01156]	[0.00768]	[0.00763]	[0.00768]	[0.00877]	[0.00882]
	20:50	CP	0.923	0.812	0.937	0.939	0.934	0.947	0.951
		EL	(0.54973)	(0.54720)	(0.58022)	(0.57742)	(0.80217)	(0.49197)	(0.49031)
		MCSE	[0.00843]	[0.01236]	[0.00768]	[0.00757]	[0.00785]	[0.00708]	[0.00683]
	20:70	CP	0.900	0.817	0.934	0.938	0.970	0.938	0.949
		EL	(0.50327)	(0.50027)	(0.53367)	(0.53097)	(0.76881)	(0.46054)	(0.45839)
		MCSE	[0.00949]	[0.01223]	[0.00785]	[0.00763]	[0.00539]	[0.00763]	[0.00696]
	20:100	CP	0.887	0.787	0.933	0.935	0.958	0.943	0.952
		EL	(0.47113)	(0.47042)	(0.50156)	(0.49857)	(0.74614)	(0.43313)	(0.43024)
		MCSE	[0.01001]	[0.01295]	[0.00791]	[0.00780]	[0.00634]	[0.00733]	[0.00676]
	50:50	CP	0.935	0.814	0.942	0.941	0.972	0.937	0.933
		EL	(0.45770)	(0.45442)	(0.46816)	(0.46742)	(0.74424)	(0.41584)	(0.41417)
		MCSE	[0.00780]	[0.01230]	[0.00739]	[0.00745]	[0.00522]	[0.00768]	[0.00791]
	50:70	CP	0.936	0.814	0.943	0.944	0.984	0.946	0.945
		EL	(0.40805)	(0.40026)	(0.41678)	(0.41595)	(0.70818)	(0.38231)	(0.38122)
		MCSE	[0.00774]	[0.01230]	[0.00733]	[0.00727]	[0.00397]	[0.00715]	[0.00721]
	50:100	CP	0.939	0.826	0.951	0.950	0.984	0.950	0.952
		EL	(0.36362)	(0.36317)	(0.37046)	(0.36959)	(0.67887)	(0.34686)	(0.34589)
		MCSE	[0.00757]	[0.01199]	[0.00683]	[0.00689]	[0.00397]	[0.00689]	[0.00676]
	70:70	CP	0.956	0.825	0.954	0.950	0.988	0.958	0.952
		EL	(0.38845)	(0.38572)	(0.39473)	(0.39381)	(0.69771)	(0.36296)	(0.36172)
		MCSE	[0.00649]	[0.01202]	[0.00662]	[0.00689]	[0.00344]	[0.00634]	[0.00676]

Table 3. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	70:100	CP	0.938	0.835	0.940	0.941	0.990	0.951	0.950
		EL	(0.33866)	(0.34037)	(0.34411)	(0.34365)	(0.66760)	(0.32388)	(0.32297)
		MCSE	[0.00763]	[0.01174]	[0.00751]	[0.00745]	[0.00315]	[0.00683]	[0.00689]
	100:100	CP	0.946	0.814	0.950	0.948	0.994	0.945	0.944
		EL	(0.32055)	(0.31104)	(0.32473)	(0.32431)	(0.65376)	(0.30611)	(0.30514)
		MCSE	[0.00715]	[0.01230]	[0.00689]	[0.00702]	[0.00244]	[0.00721]	[0.00727]
	200:200	CP	0.931	0.833	0.934	0.932	1.000	0.933	0.933
		EL	(0.22417)	(0.22168)	(0.22572)	(0.22566)	(0.59771)	(0.21885)	(0.21822)
		MCSE	[0.00801]	[0.01179]	[0.00785]	[0.00796]	[0.00000]	[0.00791]	[0.00791]
0.3:0.3	20:20	CP	0.942	0.864	0.947	0.945	0.907	0.977	0.979
		EL	(0.93711)	(0.94655)	(0.99934)	(0.99492)	(1.08285)	(0.61472)	(0.61260)
		MCSE	[0.00739]	[0.01084]	[0.00708]	[0.00721]	[0.00918]	[0.00474]	[0.00453]
	20:50	CP	0.942	0.838	0.954	0.955	0.928	0.971	0.973
		EL	(0.75309)	(0.75982)	(0.79258)	(0.79123)	(0.91182)	(0.59042)	(0.58714)
		MCSE	[0.00739]	[0.01165]	[0.00662]	[0.00656]	[0.00817]	[0.00531]	[0.00513]
	20:70	CP	0.934	0.814	0.944	0.947	0.917	0.963	0.964
		EL	(0.72832)	(0.72581)	(0.76862)	(0.76735)	(0.89505)	(0.56746)	(0.56301)
		MCSE	[0.00785]	[0.01230]	[0.00727]	[0.00708]	[0.00872]	[0.00597]	[0.00589]
	20:100	CP	0.939	0.823	0.950	0.949	0.909	0.968	0.966
		EL	(0.70561)	(0.70561)	(0.74308)	(0.74191)	(0.86919)	(0.54553)	(0.54029)
		MCSE	[0.00757]	[0.01207]	[0.00689]	[0.00696]	[0.00909]	[0.00557]	[0.00573]
	50:50	CP	0.945	0.827	0.946	0.943	0.950	0.959	0.962
		EL	(0.55240)	(0.54335)	(0.56452)	(0.56322)	(0.74526)	(0.45585)	(0.45451)
		MCSE	[0.00721]	[0.01196]	[0.00715]	[0.00733]	[0.00689]	[0.00627]	[0.00605]
	50:70	CP	0.945	0.838	0.946	0.946	0.935	0.963	0.966
		EL	(0.51136)	(0.51149)	(0.52068)	(0.51969)	(0.71264)	(0.45847)	(0.45704)
		MCSE	[0.00721]	[0.01165]	[0.00715]	[0.00715]	[0.00780]	[0.00597]	[0.00573]
	50:100	CP	0.947	0.840	0.955	0.956	0.958	0.964	0.963
		EL	(0.48008)	(0.47703)	(0.48837)	(0.48768)	(0.68397)	(0.43277)	(0.43099)
		MCSE	[0.00708]	[0.01159]	[0.00656]	[0.00649]	[0.00634]	[0.00589]	[0.00597]
	70:70	CP	0.943	0.839	0.945	0.943	0.948	0.953	0.956
		EL	(0.46640)	(0.46759)	(0.47282)	(0.47250)	(0.67787)	(0.40723)	(0.40609)
		MCSE	[0.00733]	[0.01162]	[0.00721]	[0.00733]	[0.00702]	[0.00669]	[0.00649]
	70:100	CP	0.941	0.844	0.942	0.944	0.962	0.957	0.956
		EL	(0.42715)	(0.42630)	(0.43269)	(0.43258)	(0.64463)	(0.39535)	(0.39421)
		MCSE	[0.00745]	[0.01147]	[0.00739]	[0.00727]	[0.00605]	[0.00641]	[0.00649]

Table 3. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	100:100	CP	0.946	0.841	0.944	0.945	0.965	0.950	0.948
		EL	(0.38686)	(0.38911)	(0.39091)	(0.39095)	(0.61166)	(0.34544)	(0.34453)
		MCSE	[0.00715]	[0.01156]	[0.00727]	[0.00721]	[0.00581]	[0.00689]	[0.00702]
	200:200	CP	0.944	0.840	0.946	0.947	0.990	0.947	0.947
		EL	(0.27118)	(0.27354)	(0.27272)	(0.27256)	(0.52692)	(0.25270)	(0.25204)
		MCSE	[0.00727]	[0.01159]	[0.00715]	[0.00708]	[0.00315]	[0.00708]	[0.00708]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 4. CP and EL for 95% confidence intervals for the difference of two CVs for *ZITR* when $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.913	0.827	0.936	0.938	0.961	0.944	0.952
		EL	(0.55807)	(0.55677)	(0.60519)	(0.60124)	(0.84880)	(0.42414)	(0.42227)
		MCSE	[0.00891]	[0.01196]	[0.00774]	[0.00763]	[0.00613]	[0.00727]	[0.00676]
	20:50	CP	0.901	0.810	0.945	0.948	0.977	0.957	0.969
		EL	(0.44921)	(0.44718)	(0.46845)	(0.46604)	(0.74744)	(0.41245)	(0.40944)
		MCSE	[0.00944]	[0.01241]	[0.00721]	[0.00702]	[0.00474]	[0.00641]	[0.00548]
	20:70	CP	0.896	0.788	0.931	0.930	0.986	0.946	0.959
		EL	(0.43659)	(0.43993)	(0.45139)	(0.44958)	(0.73464)	(0.39745)	(0.39309)
		MCSE	[0.00965]	[0.01293]	[0.00801]	[0.00807]	[0.00372]	[0.00715]	[0.00627]
	20:100	CP	0.879	0.792	0.931	0.933	0.981	0.950	0.964
		EL	(0.42181)	(0.42413)	(0.43560)	(0.43435)	(0.72656)	(0.38386)	(0.37853)
		MCSE	[0.01031]	[0.01283]	[0.00801]	[0.00791]	[0.00432]	[0.00689]	[0.00589]
	50:50	CP	0.934	0.855	0.943	0.945	0.993	0.949	0.950
		EL	(0.34191)	(0.34493)	(0.35130)	(0.34999)	(0.68076)	(0.29345)	(0.29245)
		MCSE	[0.00785]	[0.01113]	[0.00733]	[0.00721]	[0.00264]	[0.00696]	[0.00689]
	50:70	CP	0.944	0.820	0.946	0.948	0.996	0.963	0.965
		EL	(0.31900)	(0.31135)	(0.32425)	(0.32325)	(0.66750)	(0.30101)	(0.29989)
		MCSE	[0.00727]	[0.01215]	[0.00715]	[0.00702]	[0.00200]	[0.00597]	[0.00581]
	50:100	CP	0.932	0.834	0.934	0.934	0.997	0.941	0.951
		EL	(0.29725)	(0.29867)	(0.30108)	(0.30042)	(0.65995)	(0.28185)	(0.28036)
		MCSE	[0.00796]	[0.01177]	[0.00785]	[0.00785]	[0.00173]	[0.00745]	[0.00683]
	70:70	CP	0.944	0.833	0.945	0.945	0.995	0.961	0.965
		EL	(0.28897)	(0.28816)	(0.29500)	(0.29401)	(0.65243)	(0.25419)	(0.25338)
		MCSE	[0.00727]	[0.01179]	[0.00721]	[0.00721]	[0.00223]	[0.00612]	[0.00581]
	70:100	CP	0.940	0.821	0.940	0.946	0.999	0.947	0.951
		EL	(0.26641)	(0.26502)	(0.26955)	(0.26907)	(0.64031)	(0.25408)	(0.25316)
		MCSE	[0.00751]	[0.01212]	[0.00751]	[0.00715]	[0.00100]	[0.00708]	[0.00683]

Table 4. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	100:100	CP	0.948	0.832	0.955	0.955	1.000	0.953	0.959
		EL	(0.24046)	(0.24095)	(0.24381)	(0.24326)	(0.63529)	(0.21668)	(0.21607)
		MCSE	[0.00702]	[0.01182]	[0.00656]	[0.00656]	[0.00000]	[0.00669]	[0.00627]
	200:200	CP	0.943	0.826	0.940	0.941	1.000	0.944	0.943
		EL	(0.16823)	(0.16732)	(0.16993)	(0.16984)	(0.60724)	(0.15629)	(0.15588)
		MCSE	[0.00733]	[0.01199]	[0.00751]	[0.00745]	[0.00000]	[0.00727]	[0.00733]
0.1:0.3	20:20	CP	0.942	0.859	0.951	0.952	0.927	0.884	0.882
		EL	(0.77029)	(0.77101)	(0.83416)	(0.83084)	(0.95705)	(0.57705)	(0.57428)
		MCSE	[0.00739]	[0.01101]	[0.00683]	[0.00676]	[0.00824]	[0.01013]	[0.01020]
	20:50	CP	0.911	0.822	0.946	0.948	0.950	0.891	0.910
		EL	(0.55337)	(0.55097)	(0.57178)	(0.56880)	(0.75292)	(0.48918)	(0.48748)
		MCSE	[0.00900]	[0.01210]	[0.00715]	[0.00702]	[0.00689]	[0.00985]	[0.00905]
	20:70	CP	0.906	0.814	0.937	0.937	0.946	0.887	0.907
		EL	(0.51152)	(0.51258)	(0.52624)	(0.52422)	(0.72091)	(0.45915)	(0.45686)
		MCSE	[0.00923]	[0.01230]	[0.00768]	[0.00768]	[0.00715]	[0.01001]	[0.00918]
	20:100	CP	0.908	0.838	0.946	0.950	0.952	0.891	0.916
		EL	(0.47537)	(0.48521)	(0.48845)	(0.48578)	(0.68800)	(0.43017)	(0.42711)
		MCSE	[0.00914]	[0.01165]	[0.00715]	[0.00689]	[0.00676]	[0.00985]	[0.00877]
	50:50	CP	0.941	0.837	0.949	0.949	0.966	0.913	0.913
		EL	(0.46390)	(0.46898)	(0.47561)	(0.47443)	(0.69437)	(0.41132)	(0.40974)
		MCSE	[0.00745]	[0.01168]	[0.00696]	[0.00696]	[0.00573]	[0.00891]	[0.00891]
	50:70	CP	0.942	0.845	0.950	0.951	0.970	0.925	0.922
		EL	(0.41420)	(0.41208)	(0.42117)	(0.42021)	(0.65838)	(0.37838)	(0.37727)
		MCSE	[0.00739]	[0.01144]	[0.00689]	[0.00683]	[0.00539]	[0.00833]	[0.00848]
	50:100	CP	0.949	0.834	0.949	0.948	0.979	0.930	0.933
		EL	(0.36843)	(0.36186)	(0.37309)	(0.37235)	(0.63004)	(0.34304)	(0.34206)
		MCSE	[0.00696]	[0.01177]	[0.00696]	[0.00702]	[0.00453]	[0.00807]	[0.00791]
	70:70	CP	0.965	0.836	0.962	0.965	0.980	0.935	0.931
		EL	(0.38948)	(0.38320)	(0.39645)	(0.39581)	(0.64739)	(0.35579)	(0.35453)
		MCSE	[0.00581]	[0.01171]	[0.00605]	[0.00581]	[0.00443]	[0.00780]	[0.00801]
	70:100	CP	0.934	0.842	0.939	0.939	0.983	0.914	0.914
		EL	(0.34359)	(0.34351)	(0.34719)	(0.34693)	(0.61408)	(0.31950)	(0.31861)
		MCSE	[0.00785]	[0.01153]	[0.00757]	[0.00757]	[0.00409]	[0.00887]	[0.00887]
	100:100	CP	0.952	0.830	0.952	0.957	0.987	0.911	0.909
		EL	(0.32487)	(0.32343)	(0.32931)	(0.32889)	(0.60382)	(0.30273)	(0.30176)
		MCSE	[0.00676]	[0.01188]	[0.00676]	[0.00641]	[0.00358]	[0.00900]	[0.00909]

Table 4. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	200:200	CP	0.953	0.848	0.959	0.957	0.999	0.941	0.939
		EL	(0.22833)	(0.23135)	(0.22944)	(0.22923)	(0.54072)	(0.21872)	(0.21810)
		MCSE	[0.00669]	[0.01135]	[0.00627]	[0.00641]	[0.00100]	[0.00745]	[0.00757]
0.3:0.3	20:20	CP	0.937	0.859	0.946	0.944	0.911	0.945	0.954
		EL	(0.93556)	(0.92652)	(0.99252)	(0.98762)	(1.10503)	(0.61588)	(0.61355)
		MCSE	[0.00768]	[0.01101]	[0.00715]	[0.00727]	[0.00900]	[0.00721]	[0.00662]
	20:50	CP	0.927	0.822	0.932	0.932	0.898	0.965	0.968
		EL	(0.76727)	(0.74933)	(0.78388)	(0.78258)	(0.93061)	(0.59126)	(0.58795)
		MCSE	[0.00823]	[0.01210]	[0.00796]	[0.00796]	[0.00957]	[0.00581]	[0.00557]
	20:70	CP	0.946	0.869	0.945	0.945	0.911	0.972	0.976
		EL	(0.73315)	(0.73973)	(0.74689)	(0.74568)	(0.89838)	(0.56523)	(0.56076)
		MCSE	[0.00715]	[0.01067]	[0.00721]	[0.00721]	[0.00900]	[0.00522]	[0.00484]
	20:100	CP	0.925	0.835	0.929	0.928	0.919	0.975	0.970
		EL	(0.71216)	(0.70345)	(0.72422)	(0.72307)	(0.88285)	(0.54430)	(0.53881)
		MCSE	[0.00833]	[0.01174]	[0.00812]	[0.00817]	[0.00863]	[0.00494]	[0.00539]
	50:50	CP	0.949	0.836	0.952	0.953	0.949	0.953	0.955
		EL	(0.55933)	(0.56336)	(0.56962)	(0.56909)	(0.76297)	(0.45200)	(0.45056)
		MCSE	[0.00696]	[0.01171]	[0.00676]	[0.00669]	[0.00696]	[0.00669]	[0.00656]
	50:70	CP	0.949	0.826	0.952	0.951	0.945	0.957	0.959
		EL	(0.51331)	(0.50460)	(0.51968)	(0.51954)	(0.72704)	(0.45408)	(0.45258)
		MCSE	[0.00696]	[0.01199]	[0.00676]	[0.00683]	[0.00721]	[0.00641]	[0.00627]
	50:100	CP	0.938	0.838	0.940	0.941	0.952	0.956	0.956
		EL	(0.48066)	(0.47590)	(0.48489)	(0.48425)	(0.69076)	(0.42810)	(0.42635)
		MCSE	[0.00763]	[0.01165]	[0.00751]	[0.00745]	[0.00676]	[0.00649]	[0.00649]
	70:70	CP	0.944	0.843	0.948	0.945	0.949	0.948	0.951
		EL	(0.46718)	(0.46834)	(0.47371)	(0.47326)	(0.67752)	(0.40121)	(0.40006)
		MCSE	[0.00727]	[0.01150]	[0.00702]	[0.00721]	[0.00696]	[0.00702]	[0.00683]
	70:100	CP	0.945	0.830	0.945	0.947	0.957	0.958	0.960
		EL	(0.43008)	(0.43105)	(0.43332)	(0.43330)	(0.64354)	(0.39177)	(0.39059)
		MCSE	[0.00721]	[0.01188]	[0.00721]	[0.00708]	[0.00641]	[0.00634]	[0.00620]
	100:100	CP	0.940	0.827	0.942	0.943	0.967	0.947	0.948
		EL	(0.38652)	(0.38615)	(0.39009)	(0.39006)	(0.61877)	(0.33898)	(0.33805)
		MCSE	[0.00751]	[0.01196]	[0.00739]	[0.00733]	[0.00565]	[0.00708]	[0.00702]
	200:200	CP	0.950	0.822	0.947	0.951	0.987	0.954	0.957
		EL	(0.27170)	(0.26964)	(0.27279)	(0.27301)	(0.53027)	(0.25176)	(0.25110)
		MCSE	[0.00689]	[0.01210]	[0.00708]	[0.00683]	[0.00358]	[0.00662]	[0.00641]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 5. CP and EL for 95% confidence intervals for the difference of two CVs for ZITR when $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.925	0.842	0.948	0.948	0.972	0.982	0.983
		EL	(0.54269)	(0.54221)	(0.57996)	(0.57554)	(0.81006)	(0.43041)	(0.42872)
		MCSE	[0.00833]	[0.01153]	[0.00702]	[0.00702]	[0.00522]	[0.00420]	[0.00409]
	20:50	CP	0.893	0.817	0.940	0.939	0.968	0.962	0.973
		EL	(0.45796)	(0.45731)	(0.47192)	(0.46977)	(0.72255)	(0.42086)	(0.41784)
		MCSE	[0.00978]	[0.01223]	[0.00751]	[0.00757]	[0.00557]	[0.00605]	[0.00513]
	20:70	CP	0.884	0.803	0.938	0.940	0.977	0.962	0.973
		EL	(0.43200)	(0.42893)	(0.44499)	(0.44276)	(0.69603)	(0.39883)	(0.39492)
		MCSE	[0.01013]	[0.01258]	[0.00763]	[0.00751]	[0.00474]	[0.00605]	[0.00513]
	20:100	CP	0.881	0.794	0.940	0.938	0.974	0.956	0.973
		EL	(0.42152)	(0.42471)	(0.43385)	(0.43203)	(0.68526)	(0.38556)	(0.38048)
		MCSE	[0.01024]	[0.01279]	[0.00751]	[0.00763]	[0.00503]	[0.00649]	[0.00513]
	50:50	CP	0.955	0.830	0.959	0.960	0.996	0.974	0.973
		EL	(0.34035)	(0.33913)	(0.34708)	(0.34590)	(0.65569)	(0.29947)	(0.29856)
		MCSE	[0.00656]	[0.01188]	[0.00627]	[0.00620]	[0.00200]	[0.00503]	[0.00513]
	50:70	CP	0.935	0.823	0.942	0.944	0.991	0.966	0.969
		EL	(0.31879)	(0.31815)	(0.32255)	(0.32114)	(0.63647)	(0.30606)	(0.30499)
		MCSE	[0.00780]	[0.01207]	[0.00739]	[0.00727]	[0.00299]	[0.00573]	[0.00548]
	50:100	CP	0.935	0.835	0.945	0.947	0.998	0.966	0.969
		EL	(0.29519)	(0.29224)	(0.29833)	(0.29722)	(0.61653)	(0.28376)	(0.28236)
		MCSE	[0.00780]	[0.01174]	[0.00721]	[0.00708]	[0.00141]	[0.00573]	[0.00548]
	70:70	CP	0.947	0.838	0.951	0.950	0.997	0.964	0.967
		EL	(0.28801)	(0.28442)	(0.29181)	(0.29079)	(0.62096)	(0.26023)	(0.25946)
		MCSE	[0.00708]	[0.01165]	[0.00683]	[0.00689]	[0.00173]	[0.00589]	[0.00565]
	70:100	CP	0.934	0.818	0.935	0.939	0.996	0.953	0.958
		EL	(0.26710)	(0.26440)	(0.26907)	(0.26835)	(0.60437)	(0.25799)	(0.25715)
		MCSE	[0.00785]	[0.01220]	[0.00780]	[0.00757]	[0.00200]	[0.00669]	[0.00634]
	100:100	CP	0.940	0.824	0.942	0.941	0.997	0.947	0.949
		EL	(0.24060)	(0.24342)	(0.24285)	(0.24216)	(0.59380)	(0.22120)	(0.22056)
		MCSE	[0.00751]	[0.01204]	[0.00739]	[0.00745]	[0.00173]	[0.00708]	[0.00696]
	200:200	CP	0.946	0.819	0.949	0.951	1.000	0.945	0.946
		EL	(0.16895)	(0.16749)	(0.16974)	(0.16945)	(0.54967)	(0.15755)	(0.15713)
		MCSE	[0.00715]	[0.01218]	[0.00696]	[0.00683]	[0.00000]	[0.00721]	[0.00715]

Table 5. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.931	0.831	0.939	0.938	0.942	0.904	0.903
		EL	(0.76387)	(0.75431)	(0.80536)	(0.80171)	(0.99053)	(0.58043)	(0.57770)
		MCSE	[0.00801]	[0.01185]	[0.00757]	[0.00763]	[0.00739]	[0.00932]	[0.00936]
	20:50	CP	0.920	0.837	0.947	0.945	0.947	0.924	0.931
		EL	(0.55309)	(0.55113)	(0.56885)	(0.56643)	(0.74413)	(0.49407)	(0.49238)
		MCSE	[0.00858]	[0.01168]	[0.00708]	[0.00721]	[0.00708]	[0.00838]	[0.00801]
	20:70	CP	0.896	0.814	0.935	0.935	0.929	0.923	0.932
		EL	(0.50667)	(0.50090)	(0.52125)	(0.51850)	(0.69955)	(0.46135)	(0.45909)
		MCSE	[0.00965]	[0.01230]	[0.00780]	[0.00780]	[0.00812]	[0.00843]	[0.00796]
	20:100	CP	0.884	0.786	0.933	0.934	0.945	0.914	0.933
		EL	(0.47475)	(0.48006)	(0.48602)	(0.48391)	(0.65560)	(0.43310)	(0.43003)
		MCSE	[0.01013]	[0.01297]	[0.00791]	[0.00785]	[0.00721]	[0.00887]	[0.00791]
	50:50	CP	0.945	0.836	0.950	0.948	0.966	0.923	0.919
		EL	(0.46136)	(0.45588)	(0.46862)	(0.46774)	(0.69040)	(0.41473)	(0.41314)
		MCSE	[0.00721]	[0.01171]	[0.00689]	[0.00702]	[0.00573]	[0.00843]	[0.00863]
	50:70	CP	0.936	0.818	0.940	0.942	0.974	0.925	0.924
		EL	(0.40833)	(0.40965)	(0.41293)	(0.41209)	(0.63471)	(0.37852)	(0.37741)
		MCSE	[0.00774]	[0.01220]	[0.00751]	[0.00739]	[0.00503]	[0.00833]	[0.00838]
	50:100	CP	0.934	0.834	0.944	0.944	0.963	0.921	0.925
		EL	(0.36656)	(0.36296)	(0.37026)	(0.36931)	(0.59526)	(0.34585)	(0.34487)
		MCSE	[0.00785]	[0.01177]	[0.00727]	[0.00727]	[0.00597]	[0.00853]	[0.00833]
	70:70	CP	0.951	0.834	0.955	0.957	0.968	0.935	0.931
		EL	(0.38813)	(0.38700)	(0.39194)	(0.39174)	(0.62287)	(0.35885)	(0.35762)
		MCSE	[0.00683]	[0.01177]	[0.00656]	[0.00641]	[0.00557]	[0.00780]	[0.00801]
	70:100	CP	0.937	0.839	0.938	0.937	0.981	0.939	0.938
		EL	(0.34383)	(0.34214)	(0.34676)	(0.34621)	(0.57513)	(0.32373)	(0.32285)
		MCSE	[0.00768]	[0.01162]	[0.00763]	[0.00768]	[0.00432]	[0.00757]	[0.00763]
	100:100	CP	0.945	0.820	0.945	0.945	0.984	0.945	0.943
		EL	(0.32278)	(0.31448)	(0.32539)	(0.32491)	(0.56551)	(0.30457)	(0.30362)
		MCSE	[0.00721]	[0.01215]	[0.00721]	[0.00721]	[0.00397]	[0.00721]	[0.00733]
	200:200	CP	0.955	0.834	0.958	0.961	0.995	0.941	0.936
		EL	(0.22613)	(0.22306)	(0.22746)	(0.22721)	(0.48619)	(0.21881)	(0.21816)
		MCSE	[0.00656]	[0.01177]	[0.00634]	[0.00612]	[0.00223]	[0.00745]	[0.00774]

Table 5. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.948	0.837	0.950	0.950	0.943	0.980	0.984
		EL	(0.93754)	(0.92908)	(0.97775)	(0.97428)	(1.22114)	(0.61732)	(0.61508)
		MCSE	[0.00702]	[0.01168]	[0.00689]	[0.00689]	[0.00733]	[0.00443]	[0.00397]
	20:50	CP	0.941	0.859	0.952	0.951	0.920	0.974	0.980
		EL	(0.77190)	(0.77723)	(0.78465)	(0.78303)	(0.95566)	(0.59632)	(0.59279)
		MCSE	[0.00745]	[0.01101]	[0.00676]	[0.00683]	[0.00858]	[0.00503]	[0.00443]
	20:70	CP	0.932	0.837	0.938	0.939	0.919	0.974	0.976
		EL	(0.73129)	(0.71714)	(0.74349)	(0.74233)	(0.90435)	(0.56758)	(0.56324)
		MCSE	[0.00796]	[0.01168]	[0.00763]	[0.00757]	[0.00863]	[0.00503]	[0.00484]
	20:100	CP	0.938	0.830	0.943	0.941	0.903	0.971	0.971
		EL	(0.70811)	(0.70697)	(0.71919)	(0.71842)	(0.85603)	(0.54456)	(0.53919)
		MCSE	[0.00763]	[0.01188]	[0.00733]	[0.00745]	[0.00936]	[0.00531]	[0.00531]
	50:50	CP	0.942	0.826	0.950	0.951	0.961	0.954	0.957
		EL	(0.55642)	(0.54336)	(0.56418)	(0.56322)	(0.79343)	(0.45341)	(0.45207)
		MCSE	[0.00739]	[0.01199]	[0.00689]	[0.00683]	[0.00612]	[0.00662]	[0.00641]
	50:70	CP	0.953	0.823	0.953	0.950	0.943	0.968	0.970
		EL	(0.51291)	(0.50378)	(0.51765)	(0.51705)	(0.72659)	(0.45712)	(0.45569)
		MCSE	[0.00669]	[0.01207]	[0.00669]	[0.00689]	[0.00733]	[0.00557]	[0.00539]
	50:100	CP	0.945	0.811	0.949	0.945	0.954	0.960	0.967
		EL	(0.47994)	(0.47200)	(0.48306)	(0.48270)	(0.67792)	(0.43028)	(0.42850)
		MCSE	[0.00721]	[0.01238]	[0.00696]	[0.00721]	[0.00662]	[0.00620]	[0.00565]
	70:70	CP	0.952	0.842	0.949	0.950	0.963	0.965	0.967
		EL	(0.46727)	(0.46959)	(0.47186)	(0.47122)	(0.69361)	(0.40274)	(0.40162)
		MCSE	[0.00676]	[0.01153]	[0.00696]	[0.00689]	[0.00597]	[0.00581]	[0.00565]
	70:100	CP	0.938	0.823	0.941	0.937	0.958	0.944	0.944
		EL	(0.42689)	(0.42173)	(0.43008)	(0.42981)	(0.63335)	(0.39270)	(0.39152)
		MCSE	[0.00763]	[0.01207]	[0.00745]	[0.00768]	[0.00634]	[0.00727]	[0.00727]
	100:100	CP	0.956	0.861	0.957	0.957	0.981	0.960	0.962
		EL	(0.38636)	(0.39499)	(0.38916)	(0.38850)	(0.60571)	(0.34031)	(0.33942)
		MCSE	[0.00649]	[0.01094]	[0.00641]	[0.00641]	[0.00432]	[0.00620]	[0.00605]
	200:200	CP	0.956	0.861	0.957	0.957	0.981	0.960	0.962
		EL	(0.38636)	(0.39499)	(0.38916)	(0.38850)	(0.60571)	(0.34031)	(0.33942)
		MCSE	[0.00649]	[0.01094]	[0.00641]	[0.00641]	[0.00432]	[0.00620]	[0.00605]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 6. CP and EL for 95% confidence intervals for the difference of two CVs for ZITR when $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (1, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.908	0.848	0.928	0.928	0.936	0.981	0.986
		EL	(0.56405)	(0.56385)	(0.58646)	(0.58204)	(0.72991)	(0.43331)	(0.43173)
		MCSE	[0.00914]	[0.01135]	[0.00817]	[0.00817]	[0.00774]	[0.00432]	[0.00372]
	20:50	CP	0.879	0.798	0.924	0.930	0.948	0.975	0.981
		EL	(0.47324)	(0.47190)	(0.47196)	(0.46962)	(0.66313)	(0.43303)	(0.43004)
		MCSE	[0.01031]	[0.01270]	[0.00838]	[0.00807]	[0.00702]	[0.00494]	[0.00432]
	20:70	CP	0.874	0.772	0.910	0.910	0.955	0.970	0.979
		EL	(0.45532)	(0.46305)	(0.45181)	(0.44948)	(0.65181)	(0.41549)	(0.41133)
		MCSE	[0.01049]	[0.01327]	[0.00905]	[0.00905]	[0.00656]	[0.00539]	[0.00453]
	20:100	CP	0.885	0.792	0.910	0.910	0.946	0.954	0.970
		EL	(0.44578)	(0.44870)	(0.43899)	(0.43740)	(0.63882)	(0.40099)	(0.39576)
		MCSE	[0.01009]	[0.01283]	[0.00905]	[0.00905]	[0.00715]	[0.00662]	[0.00539]
	50:50	CP	0.939	0.819	0.944	0.940	0.976	0.965	0.965
		EL	(0.35446)	(0.35655)	(0.35704)	(0.35563)	(0.59195)	(0.31002)	(0.30905)
		MCSE	[0.00757]	[0.01218]	[0.00727]	[0.00751]	[0.00484]	[0.00581]	[0.00581]
	50:70	CP	0.946	0.833	0.954	0.952	0.984	0.974	0.978
		EL	(0.33074)	(0.33699)	(0.33110)	(0.32977)	(0.57951)	(0.31559)	(0.31444)
		MCSE	[0.00715]	[0.01179]	[0.00662]	[0.00676]	[0.00397]	[0.00503]	[0.00464]
	50:100	CP	0.930	0.832	0.943	0.943	0.987	0.960	0.966
		EL	(0.31340)	(0.32075)	(0.31101)	(0.31020)	(0.55952)	(0.29661)	(0.29515)
		MCSE	[0.00807]	[0.01182]	[0.00733]	[0.00733]	[0.00358]	[0.00620]	[0.00573]
	70:70	CP	0.946	0.842	0.946	0.949	0.988	0.964	0.969
		EL	(0.30013)	(0.30309)	(0.30159)	(0.30081)	(0.56790)	(0.26782)	(0.26698)
		MCSE	[0.00715]	[0.01153]	[0.00715]	[0.00696]	[0.00344]	[0.00589]	[0.00548]
	70:100	CP	0.932	0.838	0.940	0.938	0.994	0.966	0.969
		EL	(0.27829)	(0.28055)	(0.27820)	(0.27737)	(0.55202)	(0.26759)	(0.26666)
		MCSE	[0.00796]	[0.01165]	[0.00751]	[0.00763]	[0.00244]	[0.00573]	[0.00548]
	100:100	CP	0.950	0.843	0.950	0.951	0.997	0.961	0.959
		EL	(0.25062)	(0.24892)	(0.25105)	(0.25061)	(0.53271)	(0.22836)	(0.22769)
		MCSE	[0.00689]	[0.01150]	[0.00689]	[0.00683]	[0.00173]	[0.00612]	[0.00627]
	200:200	CP	0.949	0.846	0.954	0.955	0.999	0.954	0.956
		EL	(0.17566)	(0.17388)	(0.17569)	(0.17553)	(0.49384)	(0.16321)	(0.16274)
		MCSE	[0.00696]	[0.01141]	[0.00662]	[0.00656]	[0.00100]	[0.00662]	[0.00649]

Table 6. Cont.

$\delta_1 : \delta_2$	Coverage Probability								
	$n_1 : n_2$	Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.935	0.845	0.948	0.952	0.904	0.902	0.903
		EL	(0.77686)	(0.78084)	(0.80988)	(0.80675)	(0.87352)	(0.58949)	(0.58691)
		MCSE	[0.00780]	[0.01144]	[0.00702]	[0.00676]	[0.00932]	[0.00940]	[0.00936]
	20:50	CP	0.903	0.815	0.925	0.929	0.901	0.914	0.921
		EL	(0.57201)	(0.56693)	(0.57326)	(0.57061)	(0.66997)	(0.50625)	(0.50446)
		MCSE	[0.00936]	[0.01228]	[0.00833]	[0.00812]	[0.00944]	[0.00887]	[0.00853]
	20:70	CP	0.892	0.807	0.929	0.927	0.903	0.922	0.935
		EL	(0.52721)	(0.53608)	(0.52544)	(0.52284)	(0.62188)	(0.47424)	(0.47185)
		MCSE	[0.00982]	[0.01248]	[0.00812]	[0.00823]	[0.00936]	[0.00848]	[0.00780]
	20:100	CP	0.900	0.801	0.932	0.933	0.898	0.919	0.937
		EL	(0.49996)	(0.49210)	(0.49416)	(0.49230)	(0.59233)	(0.44796)	(0.44480)
		MCSE	[0.00949]	[0.01263]	[0.00796]	[0.00791]	[0.00957]	[0.00863]	[0.00768]
	50:50	CP	0.939	0.829	0.942	0.943	0.925	0.939	0.938
		EL	(0.47398)	(0.46427)	(0.47836)	(0.47785)	(0.60325)	(0.42364)	(0.42210)
		MCSE	[0.00757]	[0.01191]	[0.00739]	[0.00733]	[0.00833]	[0.00757]	[0.00763]
	50:70	CP	0.940	0.835	0.940	0.945	0.942	0.938	0.939
		EL	(0.42142)	(0.42289)	(0.42269)	(0.42215)	(0.55316)	(0.38875)	(0.38768)
		MCSE	[0.00751]	[0.01174]	[0.00751]	[0.00721]	[0.00739]	[0.00763]	[0.00757]
	50:100	CP	0.943	0.816	0.943	0.939	0.936	0.927	0.931
		EL	(0.38081)	(0.37295)	(0.38011)	(0.37911)	(0.51365)	(0.35554)	(0.35447)
		MCSE	[0.00733]	[0.01225]	[0.00733]	[0.00757]	[0.00774]	[0.00823]	[0.00801]
	70:70	CP	0.942	0.839	0.948	0.947	0.945	0.939	0.937
		EL	(0.39779)	(0.39740)	(0.40120)	(0.40046)	(0.53697)	(0.36792)	(0.36674)
		MCSE	[0.00739]	[0.01162]	[0.00702]	[0.00708]	[0.00721]	[0.00757]	[0.00768]
	70:100	CP	0.949	0.828	0.952	0.950	0.938	0.941	0.940
		EL	(0.35336)	(0.35119)	(0.35383)	(0.35366)	(0.49777)	(0.33203)	(0.33115)
		MCSE	[0.00696]	[0.01193]	[0.00676]	[0.00689]	[0.00763]	[0.00745]	[0.00751]
	100:100	CP	0.943	0.832	0.950	0.950	0.958	0.935	0.931
		EL	(0.32971)	(0.32639)	(0.33108)	(0.33096)	(0.48401)	(0.31136)	(0.31040)
		MCSE	[0.00733]	[0.01182]	[0.00689]	[0.00689]	[0.00634]	[0.00780]	[0.00801]
	200:200	CP	0.944	0.839	0.943	0.944	0.982	0.943	0.939
		EL	(0.23244)	(0.23252)	(0.23318)	(0.23294)	(0.41202)	(0.22503)	(0.22439)
		MCSE	[0.00727]	[0.01162]	[0.00733]	[0.00727]	[0.00420]	[0.00733]	[0.00757]

Table 6. Cont.

$\delta_1 : \delta_2$	Coverage Probability								
	$n_1 : n_2$	Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.933	0.861	0.946	0.947	0.907	0.975	0.976
		EL	(0.93767)	(0.94050)	(0.96455)	(0.96148)	(1.07664)	(0.62586)	(0.62353)
		MCSE	[0.00791]	[0.01094]	[0.00715]	[0.00708]	[0.00918]	[0.00494]	[0.00484]
	20:50	CP	0.954	0.831	0.952	0.953	0.899	0.976	0.979
		EL	(0.77000)	(0.74421)	(0.76851)	(0.76736)	(0.90044)	(0.59931)	(0.59602)
		MCSE	[0.00662]	[0.01185]	[0.00676]	[0.00669]	[0.00953]	[0.00484]	[0.00453]
	20:70	CP	0.930	0.834	0.937	0.938	0.912	0.968	0.970
		EL	(0.74489)	(0.73419)	(0.73707)	(0.73671)	(0.86387)	(0.57463)	(0.57035)
		MCSE	[0.00807]	[0.01177]	[0.00768]	[0.00763]	[0.00896]	[0.00557]	[0.00539]
	20:100	CP	0.953	0.853	0.952	0.951	0.891	0.970	0.970
		EL	(0.72321)	(0.72412)	(0.71641)	(0.71549)	(0.85214)	(0.55412)	(0.54902)
		MCSE	[0.00669]	[0.01120]	[0.00676]	[0.00683]	[0.00985]	[0.00539]	[0.00539]
	50:50	CP	0.956	0.855	0.959	0.957	0.938	0.968	0.972
		EL	(0.55880)	(0.56189)	(0.56171)	(0.56195)	(0.71425)	(0.45198)	(0.45067)
		MCSE	[0.00649]	[0.01113]	[0.00627]	[0.00641]	[0.00763]	[0.00557]	[0.00522]
	50:70	CP	0.952	0.860	0.958	0.954	0.925	0.962	0.964
		EL	(0.51897)	(0.51560)	(0.52132)	(0.52053)	(0.66759)	(0.46195)	(0.46055)
		MCSE	[0.00676]	[0.01097]	[0.00634]	[0.00662]	[0.00833]	[0.00605]	[0.00589]
	50:100	CP	0.954	0.849	0.958	0.958	0.928	0.963	0.962
		EL	(0.48418)	(0.48928)	(0.48387)	(0.48344)	(0.63228)	(0.43348)	(0.43178)
		MCSE	[0.00662]	[0.01132]	[0.00634]	[0.00634]	[0.00817]	[0.00597]	[0.00605]
	70:70	CP	0.938	0.829	0.934	0.934	0.922	0.944	0.944
		EL	(0.46860)	(0.46153)	(0.47136)	(0.47091)	(0.61902)	(0.40138)	(0.40019)
		MCSE	[0.00763]	[0.01191]	[0.00785]	[0.00785]	[0.00848]	[0.00727]	[0.00727]
	70:100	CP	0.938	0.826	0.941	0.944	0.937	0.947	0.951
		EL	(0.43257)	(0.42980)	(0.43363)	(0.43360)	(0.58952)	(0.39709)	(0.39594)
		MCSE	[0.00763]	[0.01199]	[0.00745]	[0.00727]	[0.00768]	[0.00708]	[0.00683]
	100:100	CP	0.944	0.834	0.943	0.942	0.945	0.948	0.951
		EL	(0.39091)	(0.39118)	(0.39169)	(0.39149)	(0.54732)	(0.34802)	(0.34708)
		MCSE	[0.00727]	[0.01177]	[0.00733]	[0.00739]	[0.00721]	[0.00702]	[0.00683]
	200:200	CP	0.945	0.850	0.947	0.949	0.972	0.947	0.945
		EL	(0.27270)	(0.27460)	(0.27330)	(0.27353)	(0.43888)	(0.24921)	(0.24853)
		MCSE	[0.00721]	[0.01129]	[0.00708]	[0.00696]	[0.00522]	[0.00708]	[0.00721]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 7. CP and EL of 95% confidence intervals for the ratio of two CVs for ZITR when $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.930	0.855	0.957	0.957	0.999	0.989	0.981
		EL	(1.04826)	(1.08818)	(0.99795)	(0.99152)	(7.44508)	(0.67670)	(0.66355)
		MCSE	[0.00807]	[0.01113]	[0.00641]	[0.00641]	[0.00108]	[0.00330]	[0.00432]
	20:50	CP	0.904	0.798	0.958	0.958	1.000	0.986	0.986
		EL	(0.77829)	(0.78033)	(0.82669)	(0.82519)	(6.80031)	(0.66126)	(0.64678)
		MCSE	[0.00932]	[0.01270]	[0.00634]	[0.00634]	[0.00000]	[0.00372]	[0.00372]
	20:70	CP	0.901	0.807	0.963	0.960	1.000	0.990	0.992
		EL	(0.73981)	(0.74031)	(0.80175)	(0.80069)	(6.50132)	(0.63673)	(0.62203)
		MCSE	[0.00944]	[0.01248]	[0.00597]	[0.00620]	[0.00000]	[0.00315]	[0.00282]
	20:100	CP	0.881	0.777	0.944	0.945	1.000	0.983	0.987
		EL	(0.69735)	(0.68897)	(0.76516)	(0.76463)	(6.05151)	(0.61059)	(0.59571)
		MCSE	[0.01024]	[0.01316]	[0.00727]	[0.00721]	[0.00000]	[0.00409]	[0.00358]
	50:50	CP	0.936	0.856	0.945	0.942	1.000	0.962	0.959
		EL	(0.59699)	(0.61499)	(0.58408)	(0.58294)	(7.33827)	(0.47320)	(0.46826)
		MCSE	[0.00774]	[0.01110]	[0.00721]	[0.00739]	[0.00000]	[0.00605]	[0.00627]
	50:70	CP	0.948	0.822	0.955	0.955	1.000	0.972	0.971
		EL	(0.53852)	(0.55186)	(0.53769)	(0.53667)	(6.19673)	(0.48107)	(0.47543)
		MCSE	[0.00702]	[0.01210]	[0.00656]	[0.00656]	[0.00000]	[0.00522]	[0.00531]
	50:100	CP	0.926	0.807	0.940	0.942	1.000	0.965	0.964
		EL	(0.49740)	(0.48707)	(0.50353)	(0.50304)	(6.59437)	(0.45223)	(0.44651)
		MCSE	[0.00828]	[0.01248]	[0.00751]	[0.00739]	[0.00000]	[0.00581]	[0.00589]
	70:70	CP	0.947	0.830	0.955	0.955	1.000	0.955	0.953
		EL	(0.48883)	(0.48823)	(0.48100)	(0.48006)	(6.20446)	(0.40283)	(0.39943)
		MCSE	[0.00708]	[0.01188]	[0.00656]	[0.00656]	[0.00000]	[0.00656]	[0.00669]
	70:100	CP	0.940	0.825	0.951	0.952	1.000	0.969	0.964
		EL	(0.44341)	(0.44751)	(0.44355)	(0.44311)	(6.03022)	(0.40665)	(0.40283)
		MCSE	[0.00751]	[0.01202]	[0.00683]	[0.00676]	[0.00000]	[0.00548]	[0.00589]
	100:100	CP	0.948	0.840	0.950	0.948	1.000	0.954	0.952
		EL	(0.40384)	(0.40259)	(0.39925)	(0.39868)	(5.85993)	(0.34955)	(0.34715)
		MCSE	[0.00702]	[0.01159]	[0.00689]	[0.00702]	[0.00000]	[0.00662]	[0.00676]
	200:200	CP	0.933	0.807	0.947	0.946	1.000	0.951	0.951
		EL	(0.27867)	(0.27591)	(0.27786)	(0.27782)	(5.55541)	(0.25793)	(0.25667)
		MCSE	[0.00791]	[0.01248]	[0.00708]	[0.00715]	[0.00000]	[0.00683]	[0.00683]

Table 7. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.940	0.839	0.959	0.955	0.994	0.923	0.937
		EL	(0.74183)	(0.75364)	(0.74951)	(0.74903)	(3.73535)	(0.60827)	(0.59471)
		MCSE	[0.00751]	[0.01162]	[0.00627]	[0.00656]	[0.00241]	[0.00843]	[0.00768]
	20:50	CP	0.909	0.807	0.949	0.947	1.000	0.961	0.977
		EL	(0.55448)	(0.56230)	(0.60287)	(0.60320)	(3.67821)	(0.50905)	(0.49697)
		MCSE	[0.00909]	[0.01248]	[0.00696]	[0.00708]	[0.00000]	[0.00612]	[0.00474]
	20:70	CP	0.883	0.779	0.959	0.961	1.000	0.974	0.979
		EL	(0.51614)	(0.51134)	(0.56432)	(0.56562)	(3.27652)	(0.47571)	(0.46445)
		MCSE	[0.01016]	[0.01312]	[0.00627]	[0.00612]	[0.00000]	[0.00503]	[0.00453]
	20:100	CP	0.892	0.783	0.951	0.948	1.000	0.972	0.981
		EL	(0.49956)	(0.49441)	(0.55035)	(0.55143)	(3.27634)	(0.45412)	(0.44299)
		MCSE	[0.00982]	[0.01303]	[0.00683]	[0.00702]	[0.00000]	[0.00522]	[0.00432]
	50:50	CP	0.938	0.827	0.948	0.946	1.000	0.950	0.958
		EL	(0.43321)	(0.42390)	(0.43546)	(0.43513)	(3.26381)	(0.40207)	(0.39704)
		MCSE	[0.00763]	[0.01196]	[0.00702]	[0.00715]	[0.00000]	[0.00689]	[0.00634]
	50:70	CP	0.928	0.817	0.944	0.941	1.000	0.942	0.962
		EL	(0.39326)	(0.39697)	(0.39765)	(0.39814)	(3.15251)	(0.37108)	(0.36628)
		MCSE	[0.00817]	[0.01223]	[0.00727]	[0.00745]	[0.00000]	[0.00739]	[0.00605]
	50:100	CP	0.924	0.813	0.940	0.940	1.000	0.949	0.959
		EL	(0.35235)	(0.35302)	(0.36100)	(0.36061)	(3.11204)	(0.33601)	(0.33145)
		MCSE	[0.00838]	[0.01233]	[0.00751]	[0.00751]	[0.00000]	[0.00696]	[0.00627]
	70:70	CP	0.951	0.818	0.956	0.954	1.000	0.945	0.958
		EL	(0.36267)	(0.36638)	(0.36303)	(0.36294)	(3.19366)	(0.34275)	(0.33914)
		MCSE	[0.00683]	[0.01220]	[0.00649]	[0.00662]	[0.00000]	[0.00721]	[0.00634]
	70:100	CP	0.938	0.833	0.934	0.935	1.000	0.946	0.944
		EL	(0.32340)	(0.33200)	(0.32687)	(0.32689)	(3.06683)	(0.30907)	(0.30588)
		MCSE	[0.00763]	[0.01179]	[0.00785]	[0.00780]	[0.00000]	[0.00715]	[0.00727]
	100:100	CP	0.944	0.840	0.950	0.949	1.000	0.950	0.959
		EL	(0.29800)	(0.29916)	(0.29825)	(0.29825)	(2.95615)	(0.28638)	(0.28399)
		MCSE	[0.00727]	[0.01159]	[0.00689]	[0.00696]	[0.00000]	[0.00689]	[0.00627]
	200:200	CP	0.939	0.816	0.941	0.940	1.000	0.939	0.944
		EL	(0.20796)	(0.20861)	(0.20809)	(0.20798)	(2.79270)	(0.20353)	(0.20235)
		MCSE	[0.00757]	[0.01225]	[0.00745]	[0.00751]	[0.00000]	[0.00757]	[0.00727]

Table 7. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.934	0.875	0.944	0.940	0.983	0.970	0.972
		EL	(1.20345)	(1.22728)	(1.18231)	(1.18272)	(3.82819)	(0.77453)	(0.75674)
		MCSE	[0.00785]	[0.01046]	[0.00727]	[0.00751]	[0.00411]	[0.00539]	[0.00522]
	20:50	CP	0.940	0.843	0.947	0.950	1.000	0.960	0.947
		EL	(0.89712)	(0.89787)	(0.94534)	(0.94642)	(3.19903)	(0.68824)	(0.67316)
		MCSE	[0.00751]	[0.01150]	[0.00708]	[0.00689]	[0.00000]	[0.00620]	[0.00708]
	20:70	CP	0.935	0.834	0.950	0.952	1.000	0.962	0.949
		EL	(0.85635)	(0.84529)	(0.91108)	(0.91431)	(3.14375)	(0.65360)	(0.63902)
		MCSE	[0.00780]	[0.01177]	[0.00689]	[0.00676]	[0.00000]	[0.00605]	[0.00696]
	20:100	CP	0.931	0.834	0.953	0.952	0.998	0.955	0.943
		EL	(0.82493)	(0.82419)	(0.88476)	(0.88596)	(3.14070)	(0.62395)	(0.61001)
		MCSE	[0.00801]	[0.01177]	[0.00669]	[0.00676]	[0.00147]	[0.00656]	[0.00733]
	50:50	CP	0.941	0.822	0.948	0.945	0.996	0.954	0.947
		EL	(0.66504)	(0.67228)	(0.65747)	(0.65777)	(3.00758)	(0.53700)	(0.53002)
		MCSE	[0.00745]	[0.01210]	[0.00702]	[0.00721]	[0.00207]	[0.00662]	[0.00708]
	50:70	CP	0.932	0.825	0.936	0.931	1.000	0.948	0.947
		EL	(0.61408)	(0.61150)	(0.61512)	(0.61486)	(2.92412)	(0.53763)	(0.53068)
		MCSE	[0.00796]	[0.01202]	[0.00774]	[0.00801]	[0.00000]	[0.00702]	[0.00708]
	50:100	CP	0.951	0.848	0.951	0.952	1.000	0.964	0.961
		EL	(0.56711)	(0.57550)	(0.57243)	(0.57276)	(2.69026)	(0.50074)	(0.49449)
		MCSE	[0.00683]	[0.01135]	[0.00683]	[0.00676]	[0.00000]	[0.00589]	[0.00612]
	70:70	CP	0.936	0.829	0.936	0.940	1.000	0.950	0.947
		EL	(0.55050)	(0.55149)	(0.54885)	(0.54822)	(2.90048)	(0.46278)	(0.45811)
		MCSE	[0.00774]	[0.01191]	[0.00774]	[0.00751]	[0.00000]	[0.00689]	[0.00708]
	70:100	CP	0.951	0.861	0.957	0.960	1.000	0.962	0.960
		EL	(0.50846)	(0.50923)	(0.50915)	(0.50935)	(2.78277)	(0.46108)	(0.45660)
		MCSE	[0.00683]	[0.01094]	[0.00641]	[0.00620]	[0.00000]	[0.00605]	[0.00620]
	100:100	CP	0.939	0.831	0.945	0.943	1.000	0.947	0.946
		EL	(0.45420)	(0.45676)	(0.45265)	(0.45284)	(2.70876)	(0.39691)	(0.39369)
		MCSE	[0.00757]	[0.01185]	[0.00721]	[0.00733]	[0.00000]	[0.00708]	[0.00715]
	200:200	CP	0.938	0.839	0.940	0.938	1.000	0.940	0.937
		EL	(0.31596)	(0.31945)	(0.31517)	(0.31514)	(2.44075)	(0.29222)	(0.29060)
		MCSE	[0.00763]	[0.01162]	[0.00751]	[0.00763]	[0.00000]	[0.00751]	[0.00768]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 8. CP and EL of 95% confidence intervals for the ratio of CVs for *ZITR* when $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.917	0.826	0.942	0.945	0.999	0.968	0.949
		EL	(1.16750)	(1.14676)	(1.09358)	(1.08646)	(11.51422)	(0.71775)	(0.70280)
		MCSE	[0.00872]	[0.01199]	[0.00739]	[0.00721]	[0.00103]	[0.00557]	[0.00696]
	20:50	CP	0.922	0.826	0.951	0.954	1.000	0.984	0.986
		EL	(0.87160)	(0.88326)	(0.91837)	(0.91718)	(10.01841)	(0.72341)	(0.70752)
		MCSE	[0.00848]	[0.01199]	[0.00683]	[0.00662]	[0.00000]	[0.00397]	[0.00372]
	20:70	CP	0.904	0.779	0.947	0.946	1.000	0.989	0.987
		EL	(0.80337)	(0.79184)	(0.86515)	(0.86369)	(9.93983)	(0.68852)	(0.67266)
		MCSE	[0.00932]	[0.01312]	[0.00708]	[0.00715]	[0.00000]	[0.00330]	[0.00358]
	20:100	CP	0.885	0.792	0.952	0.953	1.000	0.987	0.990
		EL	(0.76435)	(0.77190)	(0.83838)	(0.83840)	(10.26779)	(0.66534)	(0.64948)
		MCSE	[0.01009]	[0.01283]	[0.00676]	[0.00669]	[0.00000]	[0.00358]	[0.00315]
	50:50	CP	0.930	0.809	0.939	0.944	1.000	0.967	0.954
		EL	(0.67045)	(0.64900)	(0.65241)	(0.65077)	(10.65238)	(0.52090)	(0.51531)
		MCSE	[0.00807]	[0.01243]	[0.00757]	[0.00727]	[0.00000]	[0.00565]	[0.00662]
	50:70	CP	0.934	0.811	0.950	0.948	1.000	0.962	0.955
		EL	(0.59344)	(0.59740)	(0.59038)	(0.58905)	(9.82943)	(0.52733)	(0.52106)
		MCSE	[0.00785]	[0.01238]	[0.00689]	[0.00702]	[0.00000]	[0.00605]	[0.00656]
	50:100	CP	0.923	0.808	0.937	0.936	1.000	0.964	0.960
		EL	(0.54603)	(0.53740)	(0.55080)	(0.55038)	(9.74916)	(0.49615)	(0.49008)
		MCSE	[0.00843]	[0.01246]	[0.00768]	[0.00774]	[0.00000]	[0.00589]	[0.00620]
	70:70	CP	0.939	0.821	0.946	0.947	1.000	0.953	0.940
		EL	(0.54666)	(0.53973)	(0.53493)	(0.53439)	(10.10730)	(0.45469)	(0.45076)
		MCSE	[0.00757]	[0.01212]	[0.00715]	[0.00708]	[0.00000]	[0.00669]	[0.00751]
	70:100	CP	0.936	0.824	0.946	0.946	1.000	0.960	0.956
		EL	(0.49329)	(0.49869)	(0.49218)	(0.49161)	(9.57202)	(0.45209)	(0.44785)
		MCSE	[0.00774]	[0.01204]	[0.00715]	[0.00715]	[0.00000]	[0.00620]	[0.00649]
	100:100	CP	0.944	0.848	0.954	0.951	1.000	0.966	0.959
		EL	(0.45410)	(0.45584)	(0.44607)	(0.44579)	(9.66435)	(0.39395)	(0.39127)
		MCSE	[0.00727]	[0.01135]	[0.00662]	[0.00683]	[0.00000]	[0.00573]	[0.00627]
	200:200	CP	0.952	0.836	0.961	0.963	1.000	0.964	0.958
		EL	(0.31262)	(0.31053)	(0.31028)	(0.31027)	(9.31447)	(0.28682)	(0.28541)
		MCSE	[0.00676]	[0.01171]	[0.00612]	[0.00597]	[0.00000]	[0.00589]	[0.00634]

Table 8. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.930	0.820	0.950	0.945	0.999	0.925	0.938
		EL	(0.80958)	(0.83053)	(0.81063)	(0.81059)	(6.23656)	(0.63441)	(0.61945)
		MCSE	[0.00807]	[0.01215]	[0.00689]	[0.00721]	[0.00102]	[0.00833]	[0.00763]
	20:50	CP	0.913	0.798	0.951	0.950	1.000	0.981	0.985
		EL	(0.58020)	(0.57394)	(0.62974)	(0.63059)	(5.12277)	(0.53011)	(0.51750)
		MCSE	[0.00891]	[0.01270]	[0.00683]	[0.00689]	[0.00000]	[0.00432]	[0.00384]
	20:70	CP	0.899	0.803	0.950	0.949	1.000	0.962	0.977
		EL	(0.55284)	(0.56195)	(0.60377)	(0.60438)	(5.16143)	(0.50317)	(0.49106)
		MCSE	[0.00953]	[0.01258]	[0.00689]	[0.00696]	[0.00000]	[0.00605]	[0.00474]
	20:100	CP	0.872	0.780	0.941	0.939	1.000	0.973	0.979
		EL	(0.52087)	(0.51712)	(0.57506)	(0.57494)	(4.96249)	(0.47569)	(0.46409)
		MCSE	[0.01056]	[0.01310]	[0.00745]	[0.00757]	[0.00000]	[0.00513]	[0.00453]
	50:50	CP	0.938	0.834	0.946	0.944	1.000	0.953	0.966
		EL	(0.46443)	(0.45783)	(0.46421)	(0.46434)	(5.11965)	(0.42666)	(0.42090)
		MCSE	[0.00763]	[0.01177]	[0.00715]	[0.00727]	[0.00000]	[0.00669]	[0.00573]
	50:70	CP	0.940	0.813	0.950	0.948	1.000	0.961	0.970
		EL	(0.41518)	(0.41568)	(0.42158)	(0.42173)	(4.95559)	(0.39112)	(0.38589)
		MCSE	[0.00751]	[0.01233]	[0.00689]	[0.00702]	[0.00000]	[0.00612]	[0.00539]
	50:100	CP	0.951	0.815	0.944	0.942	1.000	0.957	0.960
		EL	(0.37976)	(0.36972)	(0.38861)	(0.38881)	(4.98458)	(0.36036)	(0.35546)
		MCSE	[0.00683]	[0.01228]	[0.00727]	[0.00739]	[0.00000]	[0.00641]	[0.00620]
	70:70	CP	0.929	0.824	0.944	0.941	1.000	0.950	0.957
		EL	(0.38315)	(0.38403)	(0.38349)	(0.38360)	(4.85726)	(0.36123)	(0.35727)
		MCSE	[0.00812]	[0.01204]	[0.00727]	[0.00745]	[0.00000]	[0.00689]	[0.00641]
	70:100	CP	0.948	0.844	0.955	0.953	1.000	0.955	0.960
		EL	(0.34612)	(0.35175)	(0.35010)	(0.35015)	(4.82487)	(0.33061)	(0.32712)
		MCSE	[0.00702]	[0.01147]	[0.00656]	[0.00669]	[0.00000]	[0.00656]	[0.00620]
	100:100	CP	0.946	0.845	0.949	0.947	1.000	0.954	0.955
		EL	(0.31725)	(0.31694)	(0.31725)	(0.31744)	(4.81504)	(0.30440)	(0.30173)
		MCSE	[0.00715]	[0.01144]	[0.00696]	[0.00708]	[0.00000]	[0.00662]	[0.00656]
	200:200	CP	0.942	0.818	0.942	0.943	1.000	0.930	0.940
		EL	(0.22109)	(0.22203)	(0.22122)	(0.22135)	(4.70490)	(0.21699)	(0.21573)
		MCSE	[0.00739]	[0.01220]	[0.00739]	[0.00733]	[0.00000]	[0.00807]	[0.00751]

Table 8. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.945	0.849	0.950	0.948	0.994	0.965	0.961
		EL	(1.25539)	(1.26866)	(1.22903)	(1.23106)	(5.22800)	(0.78770)	(0.76858)
		MCSE	[0.00721]	[0.01132]	[0.00689]	[0.00702]	[0.00252]	[0.00581]	[0.00612]
	20:50	CP	0.936	0.816	0.944	0.945	0.998	0.968	0.945
		EL	(0.97217)	(0.94227)	(1.01863)	(1.02316)	(4.61521)	(0.73283)	(0.71640)
		MCSE	[0.00774]	[0.01225]	[0.00727]	[0.00721]	[0.00145]	[0.00557]	[0.00721]
	20:70	CP	0.937	0.814	0.946	0.946	0.998	0.949	0.928
		EL	(0.90711)	(0.89864)	(0.96554)	(0.96713)	(4.43234)	(0.68551)	(0.67004)
		MCSE	[0.00768]	[0.01230]	[0.00715]	[0.00715]	[0.00146]	[0.00696]	[0.00817]
	20:100	CP	0.928	0.830	0.953	0.955	0.998	0.940	0.918
		EL	(0.86028)	(0.87063)	(0.92426)	(0.92642)	(4.25453)	(0.64950)	(0.63531)
		MCSE	[0.00817]	[0.01188]	[0.00669]	[0.00656]	[0.00147]	[0.00751]	[0.00868]
	50:50	CP	0.944	0.831	0.951	0.949	1.000	0.953	0.954
		EL	(0.70855)	(0.70594)	(0.70024)	(0.70077)	(4.37985)	(0.56999)	(0.56227)
		MCSE	[0.00727]	[0.01185]	[0.00683]	[0.00696]	[0.00000]	[0.00669]	[0.00662]
	50:70	CP	0.953	0.838	0.960	0.961	0.999	0.961	0.960
		EL	(0.64388)	(0.64238)	(0.64506)	(0.64639)	(4.14893)	(0.56208)	(0.55467)
		MCSE	[0.00669]	[0.01165]	[0.00620]	[0.00612]	[0.00102]	[0.00612]	[0.00620]
	50:100	CP	0.938	0.818	0.944	0.942	1.000	0.951	0.945
		EL	(0.59712)	(0.60851)	(0.60432)	(0.60468)	(4.03200)	(0.52659)	(0.51988)
		MCSE	[0.00763]	[0.01220]	[0.00727]	[0.00739]	[0.00000]	[0.00683]	[0.00721]
	70:70	CP	0.944	0.830	0.943	0.946	1.000	0.948	0.948
		EL	(0.58498)	(0.58823)	(0.58025)	(0.58031)	(4.05447)	(0.49315)	(0.48793)
		MCSE	[0.00727]	[0.01188]	[0.00733]	[0.00715]	[0.00000]	[0.00702]	[0.00702]
	70:100	CP	0.954	0.840	0.955	0.954	1.000	0.956	0.951
		EL	(0.54054)	(0.53770)	(0.54199)	(0.54152)	(4.01196)	(0.48903)	(0.48404)
		MCSE	[0.00662]	[0.01159]	[0.00656]	[0.00662]	[0.00000]	[0.00649]	[0.00683]
	100:100	CP	0.948	0.839	0.950	0.950	1.000	0.952	0.952
		EL	(0.48562)	(0.49263)	(0.48358)	(0.48345)	(3.84935)	(0.42742)	(0.42390)
		MCSE	[0.00702]	[0.01162]	[0.00689]	[0.00689]	[0.00000]	[0.00676]	[0.00676]
	200:200	CP	0.956	0.828	0.962	0.961	1.000	0.961	0.958
		EL	(0.33756)	(0.33534)	(0.33695)	(0.33708)	(3.77093)	(0.31234)	(0.31052)
		MCSE	[0.00649]	[0.01193]	[0.00605]	[0.00612]	[0.00000]	[0.00612]	[0.00634]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 9. CP and EL of 95% confidence intervals for the ratio of two CVs for ZITR when $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (1, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	Coverage Probability								
	$n_1 : n_2$	Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.923	0.828	0.947	0.947	1.000	0.992	0.987
		EL	(1.13431)	(1.12781)	(1.03694)	(1.02902)	(14.45970)	(0.72146)	(0.70622)
		MCSE	[0.00843]	[0.01193]	[0.00708]	[0.00708]	[0.00000]	[0.00282]	[0.00358]
	20:50	CP	0.901	0.806	0.946	0.948	1.000	0.989	0.991
		EL	(0.83982)	(0.85520)	(0.86160)	(0.85955)	(12.39183)	(0.72583)	(0.70995)
		MCSE	[0.00944]	[0.01250]	[0.00715]	[0.00702]	[0.00000]	[0.00330]	[0.00299]
	20:70	CP	0.887	0.780	0.947	0.951	1.000	0.984	0.992
		EL	(0.78237)	(0.78587)	(0.82160)	(0.82039)	(12.23276)	(0.69670)	(0.68100)
		MCSE	[0.01001]	[0.01310]	[0.00708]	[0.00683]	[0.00000]	[0.00397]	[0.00282]
	20:100	CP	0.884	0.792	0.958	0.958	1.000	0.987	0.997
		EL	(0.74554)	(0.75371)	(0.79095)	(0.79041)	(11.99445)	(0.67169)	(0.65639)
		MCSE	[0.01013]	[0.01283]	[0.00634]	[0.00634]	[0.00000]	[0.00358]	[0.00173]
	50:50	CP	0.933	0.842	0.936	0.938	1.000	0.960	0.960
		EL	(0.63405)	(0.63584)	(0.61136)	(0.61001)	(12.16727)	(0.51494)	(0.50883)
		MCSE	[0.00791]	[0.01153]	[0.00774]	[0.00763]	[0.00000]	[0.00620]	[0.00620]
	50:70	CP	0.937	0.825	0.944	0.945	1.000	0.969	0.968
		EL	(0.57900)	(0.57240)	(0.57204)	(0.57081)	(12.21059)	(0.53264)	(0.52592)
		MCSE	[0.00768]	[0.01202]	[0.00727]	[0.00721]	[0.00000]	[0.00548]	[0.00557]
	50:100	CP	0.923	0.811	0.941	0.942	1.000	0.960	0.961
		EL	(0.53277)	(0.53726)	(0.53221)	(0.53148)	(11.92583)	(0.50164)	(0.49532)
		MCSE	[0.00843]	[0.01238]	[0.00745]	[0.00739]	[0.00000]	[0.00620]	[0.00612]
	70:70	CP	0.956	0.847	0.963	0.960	1.000	0.978	0.974
		EL	(0.52985)	(0.53064)	(0.51469)	(0.51419)	(12.18341)	(0.45357)	(0.44917)
		MCSE	[0.00649]	[0.01138]	[0.00597]	[0.00620]	[0.00000]	[0.00464]	[0.00503]
	70:100	CP	0.942	0.809	0.947	0.949	1.000	0.967	0.970
		EL	(0.47802)	(0.48083)	(0.47291)	(0.47204)	(11.73854)	(0.45100)	(0.44634)
		MCSE	[0.00739]	[0.01243]	[0.00708]	[0.00696]	[0.00000]	[0.00565]	[0.00539]
	100:100	CP	0.924	0.817	0.930	0.928	1.000	0.949	0.945
		EL	(0.43561)	(0.43910)	(0.42741)	(0.42681)	(11.96882)	(0.38838)	(0.38540)
		MCSE	[0.00838]	[0.01223]	[0.00807]	[0.00817]	[0.00000]	[0.00696]	[0.00721]
	200:200	CP	0.946	0.830	0.947	0.946	1.000	0.948	0.948
		EL	(0.30278)	(0.30314)	(0.29932)	(0.29908)	(11.68953)	(0.28013)	(0.27868)
		MCSE	[0.00715]	[0.01188]	[0.00708]	[0.00715]	[0.00000]	[0.00702]	[0.00702]

Table 9. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.921	0.818	0.940	0.940	1.000	0.916	0.930
		EL	(0.75931)	(0.77329)	(0.74858)	(0.74895)	(7.16678)	(0.62911)	(0.61424)
		MCSE	[0.00853]	[0.01220]	[0.00751]	[0.00751]	[0.00000]	[0.00877]	[0.00807]
	20:50	CP	0.897	0.791	0.939	0.941	1.000	0.953	0.970
		EL	(0.57525)	(0.57461)	(0.60536)	(0.60638)	(6.11925)	(0.54021)	(0.52768)
		MCSE	[0.00961]	[0.01286]	[0.00757]	[0.00745]	[0.00000]	[0.00669]	[0.00539]
	20:70	CP	0.877	0.774	0.934	0.934	1.000	0.947	0.966
		EL	(0.53556)	(0.53362)	(0.56920)	(0.56979)	(5.99314)	(0.50719)	(0.49510)
		MCSE	[0.01039]	[0.01323]	[0.00785]	[0.00785]	[0.00000]	[0.00708]	[0.00573]
	20:100	CP	0.871	0.766	0.934	0.935	1.000	0.946	0.969
		EL	(0.51318)	(0.51200)	(0.54962)	(0.54940)	(5.91088)	(0.48289)	(0.47163)
		MCSE	[0.01060]	[0.01339]	[0.00785]	[0.00780]	[0.00000]	[0.00715]	[0.00548]
	50:50	CP	0.931	0.807	0.942	0.941	1.000	0.940	0.959
		EL	(0.44531)	(0.44363)	(0.44255)	(0.44248)	(5.95666)	(0.42338)	(0.41741)
		MCSE	[0.00801]	[0.01248]	[0.00739]	[0.00745]	[0.00000]	[0.00751]	[0.00627]
	50:70	CP	0.933	0.807	0.944	0.945	1.000	0.946	0.958
		EL	(0.39854)	(0.39475)	(0.40072)	(0.40065)	(5.78744)	(0.38815)	(0.38257)
		MCSE	[0.00791]	[0.01248]	[0.00727]	[0.00721]	[0.00000]	[0.00715]	[0.00634]
	50:100	CP	0.927	0.820	0.946	0.946	1.000	0.948	0.960
		EL	(0.36824)	(0.36864)	(0.37178)	(0.37190)	(5.64887)	(0.36129)	(0.35626)
		MCSE	[0.00823]	[0.01215]	[0.00715]	[0.00715]	[0.00000]	[0.00702]	[0.00620]
	70:70	CP	0.952	0.818	0.952	0.952	1.000	0.956	0.968
		EL	(0.36905)	(0.36427)	(0.36713)	(0.36668)	(5.69126)	(0.35750)	(0.35330)
		MCSE	[0.00676]	[0.01220]	[0.00676]	[0.00676]	[0.00000]	[0.00649]	[0.00557]
	70:100	CP	0.936	0.821	0.945	0.947	1.000	0.952	0.961
		EL	(0.33178)	(0.33385)	(0.33338)	(0.33323)	(5.65410)	(0.32655)	(0.32284)
		MCSE	[0.00774]	[0.01212]	[0.00721]	[0.00708]	[0.00000]	[0.00676]	[0.00612]
	100:100	CP	0.942	0.812	0.945	0.947	1.000	0.946	0.953
		EL	(0.30414)	(0.29709)	(0.30374)	(0.30356)	(5.71246)	(0.29876)	(0.29592)
		MCSE	[0.00739]	[0.01236]	[0.00721]	[0.00708]	[0.00000]	[0.00715]	[0.00669]
	200:200	CP	0.935	0.831	0.937	0.939	1.000	0.933	0.939
		EL	(0.21297)	(0.21085)	(0.21295)	(0.21287)	(5.59169)	(0.21078)	(0.20947)
		MCSE	[0.00780]	[0.01185]	[0.00768]	[0.00757]	[0.00000]	[0.00791]	[0.00757]

Table 9. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.942	0.857	0.947	0.945	0.999	0.977	0.973
		EL	(1.25869)	(1.28483)	(1.20479)	(1.20566)	(6.82400)	(0.80076)	(0.78137)
		MCSE	[0.00739]	[0.01107]	[0.00708]	[0.00721]	[0.00101]	[0.00474]	[0.00513]
	20:50	CP	0.942	0.842	0.954	0.955	0.999	0.971	0.962
		EL	(0.94320)	(0.96404)	(0.96785)	(0.97096)	(5.56953)	(0.72774)	(0.71167)
		MCSE	[0.00739]	[0.01153]	[0.00662]	[0.00656]	[0.00100]	[0.00531]	[0.00605]
	20:70	CP	0.934	0.819	0.944	0.947	1.000	0.963	0.960
		EL	(0.89823)	(0.90164)	(0.93315)	(0.93577)	(5.38876)	(0.68998)	(0.67471)
		MCSE	[0.00785]	[0.01218]	[0.00727]	[0.00708]	[0.00000]	[0.00597]	[0.00620]
	20:100	CP	0.939	0.822	0.950	0.949	1.000	0.968	0.963
		EL	(0.86371)	(0.86613)	(0.89906)	(0.90120)	(5.15010)	(0.65991)	(0.64592)
		MCSE	[0.00757]	[0.01210]	[0.00689]	[0.00696]	[0.00000]	[0.00557]	[0.00597]
	50:50	CP	0.945	0.826	0.946	0.943	1.000	0.959	0.959
		EL	(0.69606)	(0.68986)	(0.68469)	(0.68398)	(5.24756)	(0.56715)	(0.55950)
		MCSE	[0.00721]	[0.01199]	[0.00715]	[0.00733]	[0.00000]	[0.00627]	[0.00627]
	50:70	CP	0.945	0.826	0.946	0.946	1.000	0.963	0.956
		EL	(0.62957)	(0.63072)	(0.62511)	(0.62511)	(5.06683)	(0.55874)	(0.55119)
		MCSE	[0.00721]	[0.01199]	[0.00715]	[0.00715]	[0.00000]	[0.00597]	[0.00649]
	50:100	CP	0.947	0.840	0.955	0.956	1.000	0.964	0.958
		EL	(0.58925)	(0.58798)	(0.59069)	(0.59082)	(4.95057)	(0.52708)	(0.52038)
		MCSE	[0.00708]	[0.01159]	[0.00656]	[0.00649]	[0.00000]	[0.00589]	[0.00634]
	70:70	CP	0.943	0.843	0.945	0.943	1.000	0.953	0.951
		EL	(0.58409)	(0.58722)	(0.57567)	(0.57602)	(5.09755)	(0.50519)	(0.49969)
		MCSE	[0.00733]	[0.01150]	[0.00721]	[0.00733]	[0.00000]	[0.00669]	[0.00683]
	70:100	CP	0.941	0.835	0.942	0.944	1.000	0.957	0.951
		EL	(0.52252)	(0.52128)	(0.52052)	(0.52112)	(4.92762)	(0.48086)	(0.47587)
		MCSE	[0.00745]	[0.01174]	[0.00739]	[0.00727]	[0.00000]	[0.00641]	[0.00683]
	100:100	CP	0.946	0.836	0.944	0.945	1.000	0.950	0.954
		EL	(0.47965)	(0.48525)	(0.47512)	(0.47541)	(5.00236)	(0.42530)	(0.42162)
		MCSE	[0.00715]	[0.01171]	[0.00727]	[0.00721]	[0.00000]	[0.00689]	[0.00662]
	200:200	CP	0.944	0.838	0.946	0.947	1.000	0.947	0.941
		EL	(0.32981)	(0.33200)	(0.32843)	(0.32833)	(4.73595)	(0.30644)	(0.30462)
		MCSE	[0.00727]	[0.01165]	[0.00715]	[0.00708]	[0.00000]	[0.00708]	[0.00745]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 10. CP and EL of 95% confidence intervals for the ratio of CVs for *ZITR* when $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.917	0.822	0.939	0.939	1.000	0.931	0.958
		EL	(0.98523)	(0.99497)	(0.88363)	(0.87655)	(5.54751)	(0.66488)	(0.64980)
		MCSE	[0.00872]	[0.01210]	[0.00757]	[0.00757]	[0.00000]	[0.00801]	[0.00634]
	20:50	CP	0.887	0.783	0.943	0.946	1.000	0.959	0.977
		EL	(0.74005)	(0.73777)	(0.73377)	(0.73255)	(5.32177)	(0.65779)	(0.64150)
		MCSE	[0.01001]	[0.01303]	[0.00733]	[0.00715]	[0.00000]	[0.00627]	[0.00474]
	20:70	CP	0.886	0.780	0.928	0.930	1.000	0.943	0.970
		EL	(0.72327)	(0.73122)	(0.72308)	(0.72251)	(5.09508)	(0.64300)	(0.62633)
		MCSE	[0.01005]	[0.01310]	[0.00817]	[0.00807]	[0.00000]	[0.00733]	[0.00539]
	20:100	CP	0.872	0.773	0.927	0.927	1.000	0.947	0.970
		EL	(0.69356)	(0.69868)	(0.70123)	(0.70167)	(4.97333)	(0.62154)	(0.60497)
		MCSE	[0.01056]	[0.01325]	[0.00823]	[0.00823]	[0.00000]	[0.00708]	[0.00539]
	50:50	CP	0.933	0.838	0.944	0.943	1.000	0.947	0.951
		EL	(0.54968)	(0.55549)	(0.52514)	(0.52369)	(4.84343)	(0.44888)	(0.44283)
		MCSE	[0.00791]	[0.01165]	[0.00727]	[0.00733]	[0.00000]	[0.00708]	[0.00683]
	50:70	CP	0.942	0.820	0.948	0.948	1.000	0.961	0.970
		EL	(0.51081)	(0.50228)	(0.49580)	(0.49461)	(4.61162)	(0.46578)	(0.45876)
		MCSE	[0.00739]	[0.01215]	[0.00702]	[0.00702]	[0.00000]	[0.00612]	[0.00539]
	50:100	CP	0.924	0.831	0.933	0.937	1.000	0.936	0.951
		EL	(0.47866)	(0.48336)	(0.46985)	(0.46960)	(4.56794)	(0.44274)	(0.43581)
		MCSE	[0.00838]	[0.01185]	[0.00791]	[0.00768]	[0.00000]	[0.00774]	[0.00683]
	70:70	CP	0.940	0.824	0.943	0.946	1.000	0.956	0.966
		EL	(0.45496)	(0.45386)	(0.44050)	(0.43919)	(4.65332)	(0.38580)	(0.38147)
		MCSE	[0.00751]	[0.01204]	[0.00733]	[0.00715]	[0.00000]	[0.00649]	[0.00573]
	70:100	CP	0.933	0.818	0.941	0.943	1.000	0.941	0.958
		EL	(0.42427)	(0.42254)	(0.41458)	(0.41402)	(4.44729)	(0.39302)	(0.38817)
		MCSE	[0.00791]	[0.01220]	[0.00745]	[0.00733]	[0.00000]	[0.00745]	[0.00634]
	100:100	CP	0.948	0.825	0.955	0.955	1.000	0.951	0.958
		EL	(0.37942)	(0.38174)	(0.36981)	(0.36943)	(4.40467)	(0.33071)	(0.32781)
		MCSE	[0.00702]	[0.01202]	[0.00656]	[0.00656]	[0.00000]	[0.00683]	[0.00634]
	200:200	CP	0.936	0.823	0.945	0.946	1.000	0.944	0.949
		EL	(0.26144)	(0.25946)	(0.25862)	(0.25848)	(4.25558)	(0.23795)	(0.23658)
		MCSE	[0.00774]	[0.01207]	[0.00721]	[0.00715]	[0.00000]	[0.00727]	[0.00696]

Table 10. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.926	0.836	0.952	0.953	0.997	0.873	0.919
		EL	(0.67535)	(0.69057)	(0.64979)	(0.64819)	(3.37618)	(0.59190)	(0.57685)
		MCSE	[0.00828]	[0.01171]	[0.00676]	[0.00669]	[0.00181]	[0.01053]	[0.00863]
	20:50	CP	0.876	0.761	0.936	0.938	1.000	0.882	0.932
		EL	(0.52066)	(0.51943)	(0.52625)	(0.52581)	(3.17833)	(0.50116)	(0.48813)
		MCSE	[0.01042]	[0.01349]	[0.00774]	[0.00763]	[0.00000]	[0.01020]	[0.00796]
	20:70	CP	0.865	0.761	0.928	0.927	1.000	0.891	0.934
		EL	(0.50075)	(0.50542)	(0.50871)	(0.50952)	(3.07141)	(0.47612)	(0.46346)
		MCSE	[0.01081]	[0.01349]	[0.00817]	[0.00823]	[0.00000]	[0.00985]	[0.00785]
	20:100	CP	0.886	0.798	0.936	0.939	1.000	0.902	0.941
		EL	(0.48338)	(0.49488)	(0.49307)	(0.49304)	(2.99740)	(0.45359)	(0.44137)
		MCSE	[0.01005]	[0.01270]	[0.00774]	[0.00757]	[0.00000]	[0.00940]	[0.00745]
	50:50	CP	0.935	0.820	0.948	0.946	1.000	0.904	0.934
		EL	(0.39848)	(0.40425)	(0.39043)	(0.38991)	(2.88818)	(0.38198)	(0.37594)
		MCSE	[0.00780]	[0.01215]	[0.00702]	[0.00715]	[0.00000]	[0.00932]	[0.00785]
	50:70	CP	0.935	0.820	0.947	0.949	1.000	0.918	0.934
		EL	(0.36120)	(0.36338)	(0.35689)	(0.35670)	(2.85675)	(0.35076)	(0.34513)
		MCSE	[0.00780]	[0.01215]	[0.00708]	[0.00696]	[0.00000]	[0.00868]	[0.00785]
	50:100	CP	0.931	0.812	0.948	0.949	1.000	0.925	0.946
		EL	(0.34007)	(0.33573)	(0.33791)	(0.33803)	(2.78032)	(0.33002)	(0.32467)
		MCSE	[0.00801]	[0.01236]	[0.00702]	[0.00696]	[0.00000]	[0.00833]	[0.00715]
	70:70	CP	0.953	0.841	0.953	0.952	1.000	0.934	0.945
		EL	(0.32976)	(0.32908)	(0.32457)	(0.32420)	(2.82368)	(0.31904)	(0.31500)
		MCSE	[0.00669]	[0.01156]	[0.00669]	[0.00676]	[0.00000]	[0.00785]	[0.00721]
	70:100	CP	0.929	0.834	0.935	0.935	1.000	0.911	0.926
		EL	(0.30450)	(0.30548)	(0.30143)	(0.30126)	(2.69638)	(0.29474)	(0.29101)
		MCSE	[0.00812]	[0.01177]	[0.00780]	[0.00780]	[0.00000]	[0.00900]	[0.00828]
	100:100	CP	0.945	0.830	0.953	0.953	1.000	0.910	0.930
		EL	(0.27336)	(0.27361)	(0.27053)	(0.27047)	(2.63929)	(0.26574)	(0.26302)
		MCSE	[0.00721]	[0.01188]	[0.00669]	[0.00669]	[0.00000]	[0.00905]	[0.00807]
	200:200	CP	0.951	0.834	0.953	0.956	1.000	0.933	0.944
		EL	(0.19139)	(0.19478)	(0.18992)	(0.18981)	(2.43269)	(0.18656)	(0.18535)
		MCSE	[0.00683]	[0.01177]	[0.00669]	[0.00649]	[0.00000]	[0.00791]	[0.00727]

Table 10. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.939	0.857	0.941	0.940	0.992	0.953	0.969
		EL	(1.13728)	(1.15022)	(1.06045)	(1.05817)	(3.26471)	(0.76649)	(0.74775)
		MCSE	[0.00757]	[0.01107]	[0.00745]	[0.00751]	[0.00274]	[0.00669]	[0.00548]
	20:50	CP	0.926	0.820	0.935	0.935	1.000	0.968	0.968
		EL	(0.89488)	(0.88341)	(0.88214)	(0.88447)	(3.14324)	(0.69200)	(0.67634)
		MCSE	[0.00828]	[0.01215]	[0.00780]	[0.00780]	[0.00000]	[0.00557]	[0.00557]
	20:70	CP	0.940	0.862	0.942	0.943	1.000	0.972	0.978
		EL	(0.84968)	(0.85863)	(0.84492)	(0.84688)	(2.97728)	(0.65486)	(0.64002)
		MCSE	[0.00751]	[0.01091]	[0.00739]	[0.00733]	[0.00000]	[0.00522]	[0.00464]
	20:100	CP	0.927	0.833	0.927	0.925	1.000	0.976	0.967
		EL	(0.82978)	(0.82412)	(0.82889)	(0.83051)	(2.85394)	(0.63064)	(0.61661)
		MCSE	[0.00823]	[0.01179]	[0.00823]	[0.00833]	[0.00000]	[0.00484]	[0.00565]
	50:50	CP	0.949	0.833	0.952	0.952	1.000	0.955	0.965
		EL	(0.64246)	(0.65069)	(0.62413)	(0.62407)	(2.47024)	(0.52865)	(0.52151)
		MCSE	[0.00696]	[0.01179]	[0.00676]	[0.00676]	[0.00000]	[0.00656]	[0.00581]
	50:70	CP	0.950	0.830	0.951	0.949	1.000	0.955	0.958
		EL	(0.58735)	(0.58082)	(0.57684)	(0.57740)	(2.48267)	(0.52398)	(0.51681)
		MCSE	[0.00689]	[0.01188]	[0.00683]	[0.00696]	[0.00000]	[0.00656]	[0.00634]
	50:100	CP	0.937	0.840	0.942	0.945	1.000	0.954	0.953
		EL	(0.54602)	(0.54342)	(0.53956)	(0.53993)	(2.33437)	(0.48774)	(0.48139)
		MCSE	[0.00768]	[0.01159]	[0.00739]	[0.00721]	[0.00000]	[0.00662]	[0.00669]
	70:70	CP	0.945	0.845	0.946	0.946	1.000	0.949	0.955
		EL	(0.53300)	(0.53438)	(0.52164)	(0.52165)	(2.19088)	(0.46186)	(0.45697)
		MCSE	[0.00721]	[0.01144]	[0.00715]	[0.00715]	[0.00000]	[0.00696]	[0.00656]
	70:100	CP	0.940	0.832	0.945	0.947	1.000	0.956	0.957
		EL	(0.48471)	(0.48783)	(0.47809)	(0.47861)	(2.15924)	(0.44421)	(0.43954)
		MCSE	[0.00751]	[0.01182]	[0.00721]	[0.00708]	[0.00000]	[0.00649]	[0.00641]
	100:100	CP	0.940	0.823	0.943	0.943	1.000	0.951	0.947
		EL	(0.43465)	(0.43566)	(0.42787)	(0.42815)	(2.15295)	(0.38473)	(0.38139)
		MCSE	[0.00751]	[0.01207]	[0.00733]	[0.00733]	[0.00000]	[0.00683]	[0.00708]
	200:200	CP	0.951	0.820	0.948	0.948	1.000	0.956	0.955
		EL	(0.30034)	(0.29848)	(0.29761)	(0.29784)	(1.94487)	(0.27905)	(0.27747)
		MCSE	[0.00683]	[0.01215]	[0.00702]	[0.00702]	[0.00000]	[0.00649]	[0.00656]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 11. CP and EL of 95% confidence intervals for the ratio of CVs for *ZITR* when $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.917	0.827	0.941	0.944	1.000	0.981	0.989
		EL	(1.06715)	(1.08167)	(0.95781)	(0.95009)	(10.18142)	(0.70394)	(0.68734)
		MCSE	[0.00872]	[0.01196]	[0.00745]	[0.00727]	[0.00000]	[0.00432]	[0.00330]
	20:50	CP	0.882	0.799	0.935	0.936	1.000	0.963	0.977
		EL	(0.83128)	(0.83190)	(0.81644)	(0.81536)	(9.32031)	(0.71665)	(0.69858)
		MCSE	[0.01020]	[0.01267]	[0.00780]	[0.00774]	[0.00000]	[0.00597]	[0.00474]
	20:70	CP	0.880	0.785	0.936	0.939	1.000	0.961	0.977
		EL	(0.77342)	(0.77331)	(0.77295)	(0.77212)	(9.14646)	(0.68565)	(0.66794)
		MCSE	[0.01028]	[0.01299]	[0.00774]	[0.00757]	[0.00000]	[0.00612]	[0.00474]
	20:100	CP	0.881	0.771	0.938	0.936	1.000	0.955	0.974
		EL	(0.74751)	(0.75716)	(0.75518)	(0.75486)	(9.01403)	(0.66649)	(0.64864)
		MCSE	[0.01024]	[0.01329]	[0.00763]	[0.00774]	[0.00000]	[0.00656]	[0.00503]
	50:50	CP	0.954	0.844	0.959	0.957	1.000	0.971	0.972
		EL	(0.61352)	(0.61548)	(0.58437)	(0.58263)	(8.97940)	(0.49362)	(0.48696)
		MCSE	[0.00662]	[0.01147]	[0.00627]	[0.00641]	[0.00000]	[0.00531]	[0.00522]
	50:70	CP	0.933	0.811	0.944	0.941	1.000	0.963	0.971
		EL	(0.56353)	(0.56351)	(0.54602)	(0.54455)	(8.75846)	(0.51006)	(0.50256)
		MCSE	[0.00791]	[0.01238]	[0.00727]	[0.00745]	[0.00000]	[0.00597]	[0.00531]
	50:100	CP	0.929	0.831	0.947	0.946	1.000	0.965	0.972
		EL	(0.52100)	(0.51751)	(0.51179)	(0.51040)	(8.80094)	(0.48117)	(0.47381)
		MCSE	[0.00812]	[0.01185]	[0.00708]	[0.00715]	[0.00000]	[0.00581]	[0.00522]
	70:70	CP	0.945	0.839	0.947	0.949	1.000	0.963	0.970
		EL	(0.50570)	(0.50146)	(0.48671)	(0.48559)	(8.88094)	(0.42697)	(0.42218)
		MCSE	[0.00721]	[0.01162]	[0.00708]	[0.00696]	[0.00000]	[0.00597]	[0.00539]
	70:100	CP	0.930	0.823	0.935	0.939	1.000	0.954	0.959
		EL	(0.46529)	(0.46119)	(0.45407)	(0.45341)	(8.69342)	(0.43008)	(0.42495)
		MCSE	[0.00807]	[0.01207]	[0.00780]	[0.00757]	[0.00000]	[0.00662]	[0.00627]
	100:100	CP	0.934	0.822	0.941	0.941	1.000	0.947	0.952
		EL	(0.41834)	(0.42495)	(0.40749)	(0.40682)	(8.68934)	(0.36586)	(0.36266)
		MCSE	[0.00785]	[0.01210]	[0.00745]	[0.00745]	[0.00000]	[0.00708]	[0.00676]
	200:200	CP	0.949	0.818	0.946	0.949	1.000	0.948	0.947
		EL	(0.29119)	(0.28902)	(0.28719)	(0.28680)	(8.58164)	(0.26447)	(0.26291)
		MCSE	[0.00696]	[0.01220]	[0.00715]	[0.00696]	[0.00000]	[0.00702]	[0.00708]

Table 11. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.909	0.815	0.938	0.935	1.000	0.906	0.924
		EL	(0.72177)	(0.72093)	(0.68863)	(0.68755)	(5.90382)	(0.61065)	(0.59462)
		MCSE	[0.00909]	[0.01228]	[0.00763]	[0.00780]	[0.00000]	[0.00923]	[0.00838]
	20:50	CP	0.877	0.777	0.930	0.929	1.000	0.924	0.956
		EL	(0.54310)	(0.54690)	(0.55135)	(0.55153)	(5.21853)	(0.51977)	(0.50599)
		MCSE	[0.01039]	[0.01316]	[0.00807]	[0.00812]	[0.00000]	[0.00838]	[0.00649]
	20:70	CP	0.863	0.763	0.927	0.927	1.000	0.919	0.952
		EL	(0.51732)	(0.51472)	(0.52967)	(0.52954)	(5.22866)	(0.49399)	(0.48061)
		MCSE	[0.01087]	[0.01345]	[0.00823]	[0.00823]	[0.00000]	[0.00863]	[0.00676]
	20:100	CP	0.846	0.753	0.919	0.925	1.000	0.910	0.949
		EL	(0.50473)	(0.51335)	(0.51565)	(0.51646)	(5.16800)	(0.47525)	(0.46231)
		MCSE	[0.01141]	[0.01364]	[0.00863]	[0.00833]	[0.00000]	[0.00905]	[0.00696]
	50:50	CP	0.948	0.844	0.946	0.945	1.000	0.920	0.941
		EL	(0.42680)	(0.42396)	(0.41805)	(0.41764)	(4.96777)	(0.40396)	(0.39742)
		MCSE	[0.00702]	[0.01147]	[0.00715]	[0.00721]	[0.00000]	[0.00858]	[0.00745]
	50:70	CP	0.932	0.808	0.933	0.932	1.000	0.920	0.942
		EL	(0.38657)	(0.38867)	(0.38201)	(0.38182)	(5.00262)	(0.37317)	(0.36718)
		MCSE	[0.00796]	[0.01246]	[0.00791]	[0.00796]	[0.00000]	[0.00858]	[0.00739]
	50:100	CP	0.930	0.808	0.942	0.941	1.000	0.916	0.942
		EL	(0.35851)	(0.35605)	(0.35723)	(0.35707)	(4.91902)	(0.34825)	(0.34254)
		MCSE	[0.00807]	[0.01246]	[0.00739]	[0.00745]	[0.00000]	[0.00877]	[0.00739]
	70:70	CP	0.953	0.838	0.950	0.952	1.000	0.933	0.957
		EL	(0.35127)	(0.35200)	(0.34531)	(0.34550)	(4.95755)	(0.33754)	(0.33309)
		MCSE	[0.00669]	[0.01165]	[0.00689]	[0.00676]	[0.00000]	[0.00791]	[0.00641]
	70:100	CP	0.934	0.833	0.937	0.941	1.000	0.928	0.945
		EL	(0.32264)	(0.32240)	(0.31983)	(0.31985)	(4.89001)	(0.31248)	(0.30846)
		MCSE	[0.00785]	[0.01179]	[0.00768]	[0.00745]	[0.00000]	[0.00817]	[0.00721]
	100:100	CP	0.943	0.820	0.948	0.948	1.000	0.942	0.955
		EL	(0.29038)	(0.28412)	(0.28710)	(0.28705)	(4.88004)	(0.28182)	(0.27882)
		MCSE	[0.00733]	[0.01215]	[0.00702]	[0.00702]	[0.00000]	[0.00739]	[0.00656]
	200:200	CP	0.952	0.830	0.955	0.956	1.000	0.940	0.948
		EL	(0.20227)	(0.20068)	(0.20144)	(0.20125)	(4.80422)	(0.19817)	(0.19684)
		MCSE	[0.00676]	[0.01188]	[0.00656]	[0.00649]	[0.00000]	[0.00751]	[0.00702]

Table 11. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.947	0.830	0.950	0.950	0.998	0.980	0.979
		EL	(1.19786)	(1.21006)	(1.11541)	(1.11493)	(4.98623)	(0.77804)	(0.75843)
		MCSE	[0.00708]	[0.01188]	[0.00689]	[0.00689]	[0.00142]	[0.00443]	[0.00453]
	20:50	CP	0.943	0.840	0.952	0.949	1.000	0.973	0.974
		EL	(0.95651)	(0.96947)	(0.94457)	(0.94682)	(4.45349)	(0.72987)	(0.71295)
		MCSE	[0.00733]	[0.01159]	[0.00676]	[0.00696]	[0.00000]	[0.00513]	[0.00503]
	20:70	CP	0.929	0.814	0.941	0.942	1.000	0.976	0.964
		EL	(0.88807)	(0.87471)	(0.88458)	(0.88728)	(4.24022)	(0.68168)	(0.66584)
		MCSE	[0.00812]	[0.01230]	[0.00745]	[0.00739]	[0.00000]	[0.00484]	[0.00589]
	20:100	CP	0.937	0.837	0.946	0.944	1.000	0.973	0.966
		EL	(0.86243)	(0.86638)	(0.86346)	(0.86501)	(4.30506)	(0.65545)	(0.64074)
		MCSE	[0.00768]	[0.01168]	[0.00715]	[0.00727]	[0.00000]	[0.00513]	[0.00573]
	50:50	CP	0.943	0.823	0.949	0.950	1.000	0.956	0.957
		EL	(0.68342)	(0.67108)	(0.66409)	(0.66393)	(4.00062)	(0.55238)	(0.54470)
		MCSE	[0.00733]	[0.01207]	[0.00696]	[0.00689]	[0.00000]	[0.00649]	[0.00641]
	50:70	CP	0.953	0.824	0.952	0.950	1.000	0.967	0.967
		EL	(0.61691)	(0.60869)	(0.60588)	(0.60613)	(3.84093)	(0.54621)	(0.53861)
		MCSE	[0.00669]	[0.01204]	[0.00676]	[0.00689]	[0.00000]	[0.00565]	[0.00565]
	50:100	CP	0.945	0.808	0.948	0.946	1.000	0.960	0.962
		EL	(0.58038)	(0.57424)	(0.57401)	(0.57446)	(3.77062)	(0.51675)	(0.50971)
		MCSE	[0.00721]	[0.01246]	[0.00702]	[0.00715]	[0.00000]	[0.00620]	[0.00605]
	70:70	CP	0.954	0.831	0.949	0.945	1.000	0.964	0.964
		EL	(0.56136)	(0.56505)	(0.54989)	(0.54971)	(3.77425)	(0.48170)	(0.47623)
		MCSE	[0.00662]	[0.01185]	[0.00696]	[0.00721]	[0.00000]	[0.00589]	[0.00589]
	70:100	CP	0.937	0.823	0.940	0.940	1.000	0.944	0.947
		EL	(0.50814)	(0.50373)	(0.50263)	(0.50283)	(3.71780)	(0.46603)	(0.46100)
		MCSE	[0.00768]	[0.01207]	[0.00751]	[0.00751]	[0.00000]	[0.00727]	[0.00708]
	100:100	CP	0.957	0.861	0.958	0.957	1.000	0.961	0.963
		EL	(0.46012)	(0.46982)	(0.45305)	(0.45283)	(3.67152)	(0.40385)	(0.40026)
		MCSE	[0.00641]	[0.01094]	[0.00634]	[0.00641]	[0.00000]	[0.00612]	[0.00597]
	200:200	CP	0.957	0.861	0.958	0.957	1.000	0.961	0.963
		EL	(0.46012)	(0.46982)	(0.45305)	(0.45283)	(3.67152)	(0.40385)	(0.40026)
		MCSE	[0.00641]	[0.01094]	[0.00634]	[0.00641]	[0.00000]	[0.00612]	[0.00597]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

Table 12. CP and EL of 95% confidence intervals for the ratio of two CVs for ZITR when $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (1, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.1	20:20	CP	0.883	0.819	0.911	0.912	1.000	0.980	0.986
		EL	(1.03421)	(1.04767)	(0.89940)	(0.89171)	(9.93262)	(0.67047)	(0.65259)
		MCSE	[0.01016]	[0.01218]	[0.00900]	[0.00896]	[0.00000]	[0.00443]	[0.00372]
	20:50	CP	0.875	0.770	0.903	0.906	1.000	0.970	0.981
		EL	(0.82465)	(0.83038)	(0.78263)	(0.78142)	(9.14507)	(0.70865)	(0.68907)
		MCSE	[0.01046]	[0.01331]	[0.00936]	[0.00923]	[0.00000]	[0.00539]	[0.00432]
	20:70	CP	0.871	0.761	0.900	0.900	1.000	0.963	0.978
		EL	(0.79546)	(0.81177)	(0.76460)	(0.76329)	(9.02480)	(0.69547)	(0.67624)
		MCSE	[0.01060]	[0.01349]	[0.00949]	[0.00949]	[0.00000]	[0.00597]	[0.00464]
	20:100	CP	0.883	0.780	0.906	0.905	1.000	0.955	0.972
		EL	(0.77947)	(0.78884)	(0.75206)	(0.75197)	(8.93392)	(0.68073)	(0.66220)
		MCSE	[0.01016]	[0.01310]	[0.00923]	[0.00927]	[0.00000]	[0.00656]	[0.00522]
	50:50	CP	0.938	0.818	0.940	0.939	1.000	0.962	0.965
		EL	(0.60902)	(0.61270)	(0.56906)	(0.56694)	(9.16581)	(0.48613)	(0.47816)
		MCSE	[0.00763]	[0.01220]	[0.00751]	[0.00757]	[0.00000]	[0.00605]	[0.00581]
	50:70	CP	0.945	0.823	0.956	0.957	1.000	0.975	0.980
		EL	(0.56119)	(0.57088)	(0.53460)	(0.53305)	(9.03713)	(0.50040)	(0.49175)
		MCSE	[0.00721]	[0.01207]	[0.00649]	[0.00641]	[0.00000]	[0.00494]	[0.00443]
	50:100	CP	0.928	0.822	0.934	0.938	1.000	0.956	0.968
		EL	(0.54212)	(0.55554)	(0.52034)	(0.51936)	(8.89645)	(0.48829)	(0.47998)
		MCSE	[0.00817]	[0.01210]	[0.00785]	[0.00763]	[0.00000]	[0.00649]	[0.00557]
	70:70	CP	0.949	0.840	0.951	0.949	1.000	0.967	0.968
		EL	(0.50178)	(0.50704)	(0.47859)	(0.47744)	(9.06557)	(0.41799)	(0.41242)
		MCSE	[0.00696]	[0.01159]	[0.00683]	[0.00696]	[0.00000]	[0.00565]	[0.00557]
	70:100	CP	0.933	0.823	0.935	0.936	1.000	0.963	0.966
		EL	(0.46606)	(0.47032)	(0.44950)	(0.44834)	(8.93967)	(0.42536)	(0.41954)
		MCSE	[0.00791]	[0.01207]	[0.00780]	[0.00774]	[0.00000]	[0.00597]	[0.00573]
	100:100	CP	0.947	0.842	0.949	0.946	1.000	0.960	0.961
		EL	(0.41613)	(0.41318)	(0.40144)	(0.40064)	(8.85206)	(0.36008)	(0.35635)
		MCSE	[0.00708]	[0.01153]	[0.00696]	[0.00715]	[0.00000]	[0.00620]	[0.00612]
	200:200	CP	0.946	0.846	0.953	0.953	1.000	0.948	0.959
		EL	(0.28883)	(0.28511)	(0.28282)	(0.28269)	(8.77195)	(0.26143)	(0.25965)
		MCSE	[0.00715]	[0.01141]	[0.00669]	[0.00669]	[0.00000]	[0.00702]	[0.00627]

Table 12. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.1:0.3	20:20	CP	0.895	0.802	0.924	0.928	1.000	0.905	0.920
		EL	(0.70250)	(0.70941)	(0.64721)	(0.64551)	(5.80609)	(0.59093)	(0.57326)
		MCSE	[0.00969]	[0.01260]	[0.00838]	[0.00817]	[0.00000]	[0.00927]	[0.00858]
	20:50	CP	0.865	0.754	0.901	0.905	1.000	0.908	0.943
		EL	(0.55708)	(0.55498)	(0.54201)	(0.54230)	(5.32051)	(0.52182)	(0.50671)
		MCSE	[0.01081]	[0.01362]	[0.00944]	[0.00927]	[0.00000]	[0.00914]	[0.00733]
	20:70	CP	0.854	0.748	0.900	0.900	1.000	0.930	0.953
		EL	(0.52888)	(0.54221)	(0.51914)	(0.51928)	(5.26468)	(0.49672)	(0.48263)
		MCSE	[0.01117]	[0.01373]	[0.00949]	[0.00949]	[0.00000]	[0.00807]	[0.00669]
	20:100	CP	0.891	0.767	0.905	0.910	1.000	0.919	0.946
		EL	(0.52816)	(0.52155)	(0.51410)	(0.51498)	(5.20100)	(0.48208)	(0.46873)
		MCSE	[0.00985]	[0.01337]	[0.00927]	[0.00905]	[0.00000]	[0.00863]	[0.00715]
	50:50	CP	0.932	0.808	0.938	0.942	1.000	0.934	0.954
		EL	(0.41475)	(0.40523)	(0.39830)	(0.39809)	(5.15768)	(0.38608)	(0.37891)
		MCSE	[0.00796]	[0.01246]	[0.00763]	[0.00739]	[0.00000]	[0.00785]	[0.00662]
	50:70	CP	0.932	0.823	0.937	0.941	1.000	0.928	0.949
		EL	(0.38758)	(0.39037)	(0.37595)	(0.37567)	(5.17766)	(0.36700)	(0.36037)
		MCSE	[0.00796]	[0.01207]	[0.00768]	[0.00745]	[0.00000]	[0.00817]	[0.00696]
	50:100	CP	0.937	0.804	0.943	0.939	1.000	0.929	0.950
		EL	(0.36646)	(0.35969)	(0.35597)	(0.35577)	(5.17025)	(0.34647)	(0.34036)
		MCSE	[0.00768]	[0.01255]	[0.00733]	[0.00757]	[0.00000]	[0.00812]	[0.00689]
	70:70	CP	0.951	0.834	0.953	0.953	1.000	0.931	0.949
		EL	(0.34471)	(0.34392)	(0.33566)	(0.33518)	(5.13867)	(0.32813)	(0.32326)
		MCSE	[0.00683]	[0.01177]	[0.00669]	[0.00669]	[0.00000]	[0.00801]	[0.00696]
	70:100	CP	0.929	0.832	0.943	0.945	1.000	0.938	0.953
		EL	(0.32123)	(0.32185)	(0.31372)	(0.31363)	(5.08335)	(0.30694)	(0.30254)
		MCSE	[0.00812]	[0.01182]	[0.00733]	[0.00721]	[0.00000]	[0.00763]	[0.00669]
	100:100	CP	0.940	0.829	0.948	0.944	1.000	0.931	0.940
		EL	(0.28384)	(0.28014)	(0.27803)	(0.27787)	(5.12306)	(0.27368)	(0.27044)
		MCSE	[0.00751]	[0.01191]	[0.00702]	[0.00727]	[0.00000]	[0.00801]	[0.00751]
	200:200	CP	0.938	0.833	0.938	0.940	1.000	0.939	0.944
		EL	(0.19889)	(0.20002)	(0.19655)	(0.19656)	(5.03722)	(0.19405)	(0.19267)
		MCSE	[0.00763]	[0.01179]	[0.00763]	[0.00751]	[0.00000]	[0.00757]	[0.00727]

Table 12. Cont.

$\delta_1 : \delta_2$	$n_1 : n_2$	Coverage Probability							
		Metric	PB	BS	GCI	MOVER	AN	BCI	HPD
0.3:0.3	20:20	CP	0.943	0.848	0.946	0.944	0.999	0.973	0.972
		EL	(1.16330)	(1.17436)	(1.05306)	(1.05256)	(4.78796)	(0.75739)	(0.73756)
		MCSE	[0.00733]	[0.01135]	[0.00715]	[0.00727]	[0.00100]	[0.00513]	[0.00522]
	20:50	CP	0.953	0.834	0.953	0.956	1.000	0.975	0.970
		EL	(0.93642)	(0.91201)	(0.89868)	(0.90088)	(4.10481)	(0.71050)	(0.69392)
		MCSE	[0.00669]	[0.01177]	[0.00669]	[0.00649]	[0.00000]	[0.00494]	[0.00539]
	20:70	CP	0.932	0.841	0.938	0.938	1.000	0.968	0.962
		EL	(0.90192)	(0.89615)	(0.86978)	(0.87144)	(4.09191)	(0.68009)	(0.66454)
		MCSE	[0.00796]	[0.01156]	[0.00763]	[0.00763]	[0.00000]	[0.00557]	[0.00605]
	20:100	CP	0.949	0.850	0.955	0.950	1.000	0.970	0.971
		EL	(0.87548)	(0.87915)	(0.85095)	(0.85276)	(4.02727)	(0.65778)	(0.64331)
		MCSE	[0.00696]	[0.01129]	[0.00656]	[0.00689]	[0.00000]	[0.00539]	[0.00531]
	50:50	CP	0.956	0.862	0.957	0.957	1.000	0.966	0.961
		EL	(0.65958)	(0.66833)	(0.63377)	(0.63438)	(4.09313)	(0.52767)	(0.51999)
		MCSE	[0.00649]	[0.01091]	[0.00641]	[0.00641]	[0.00000]	[0.00573]	[0.00612]
	50:70	CP	0.950	0.851	0.956	0.953	1.000	0.965	0.963
		EL	(0.61006)	(0.61197)	(0.59430)	(0.59397)	(3.87166)	(0.53493)	(0.52739)
		MCSE	[0.00689]	[0.01126]	[0.00649]	[0.00669]	[0.00000]	[0.00581]	[0.00597]
	50:100	CP	0.953	0.846	0.954	0.954	1.000	0.963	0.958
		EL	(0.57092)	(0.57941)	(0.55986)	(0.55970)	(3.83622)	(0.50468)	(0.49796)
		MCSE	[0.00669]	[0.01141]	[0.00662]	[0.00662]	[0.00000]	[0.00597]	[0.00634]
	70:70	CP	0.938	0.833	0.935	0.935	1.000	0.944	0.946
		EL	(0.54849)	(0.54044)	(0.53353)	(0.53333)	(3.96700)	(0.46416)	(0.45875)
		MCSE	[0.00763]	[0.01179]	[0.00780]	[0.00780]	[0.00000]	[0.00727]	[0.00715]
	70:100	CP	0.939	0.825	0.944	0.941	1.000	0.949	0.948
		EL	(0.50512)	(0.50399)	(0.49582)	(0.49572)	(3.89454)	(0.45877)	(0.45375)
		MCSE	[0.00757]	[0.01202]	[0.00727]	[0.00745]	[0.00000]	[0.00696]	[0.00702]
	100:100	CP	0.944	0.848	0.944	0.944	1.000	0.948	0.949
		EL	(0.45133)	(0.45303)	(0.44147)	(0.44148)	(3.83052)	(0.39909)	(0.39546)
		MCSE	[0.00727]	[0.01135]	[0.00727]	[0.00727]	[0.00000]	[0.00702]	[0.00696]
	200:200	CP	0.945	0.851	0.950	0.951	1.000	0.953	0.948
		EL	(0.31173)	(0.31389)	(0.30866)	(0.30886)	(3.79956)	(0.28396)	(0.28228)
		MCSE	[0.00721]	[0.01126]	[0.00689]	[0.00683]	[0.00000]	[0.00669]	[0.00702]

Bold values indicate the shortest expected lengths among the methods with coverage probabilities greater than or equal to the nominal confidence level.

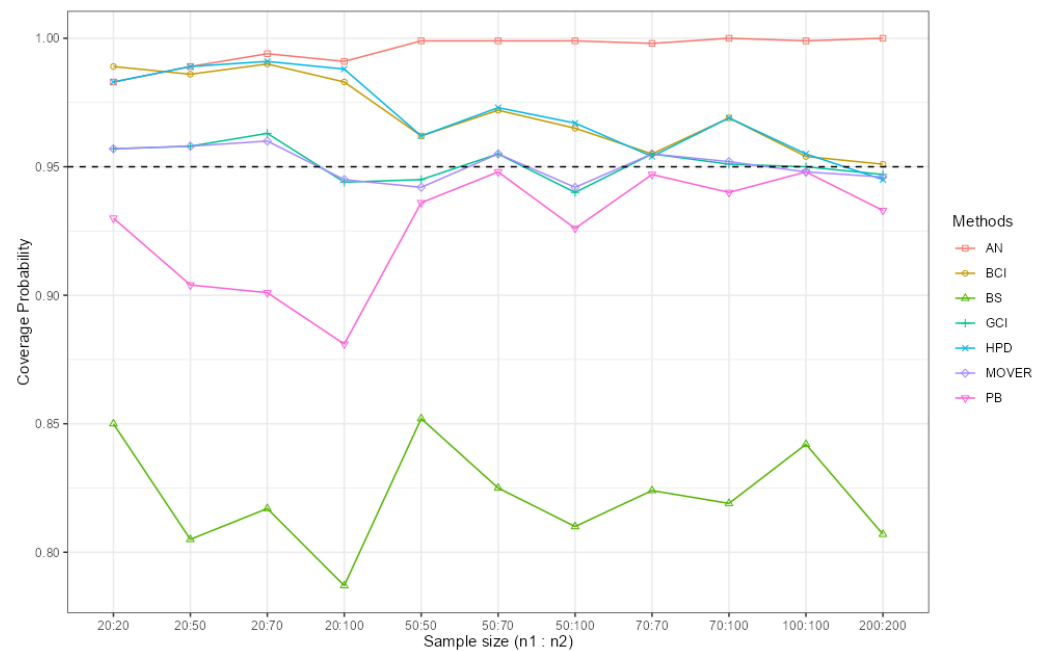


Figure 1. Coverage probabilities (CPs) for confidence intervals of the difference in coefficients of variation under the ZITR distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1)$. The AN, BCI, and HPD methods achieve CP values close to or exceeding the nominal confidence level of 0.95.

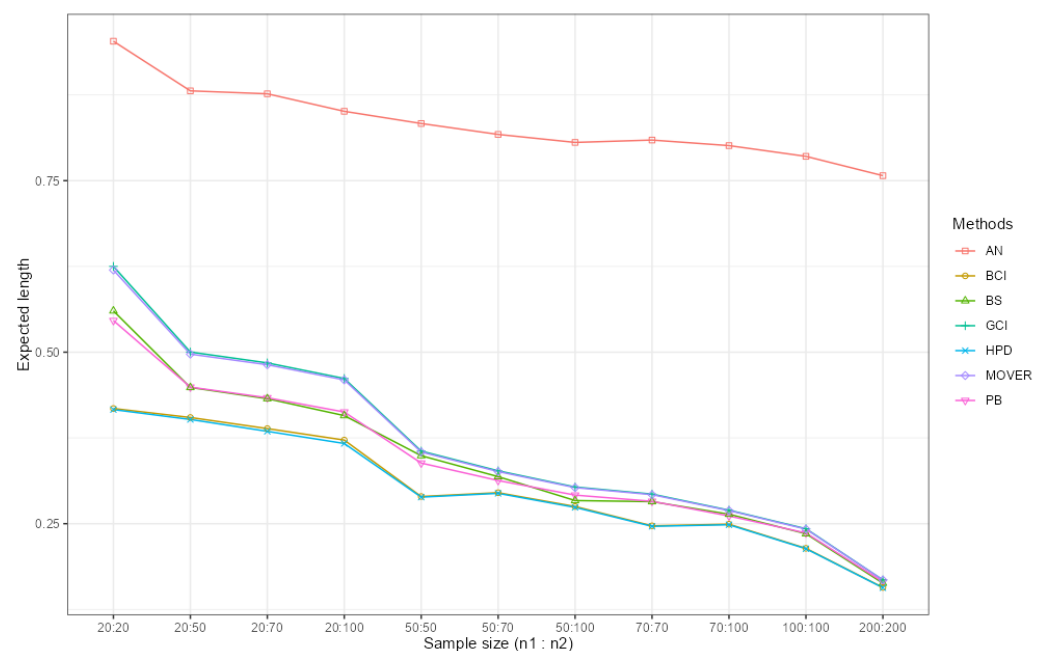


Figure 2. Expected lengths (ELs) of confidence intervals for the difference in coefficients of variation under the ZITR distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1)$. Although the PB and BS methods produce relatively short intervals, they fail to attain the nominal CP level of 0.95. The BCI, HPD, MOVER, GCI, and AN methods are therefore preferred due to their adequate coverage probabilities and comparatively short interval lengths.

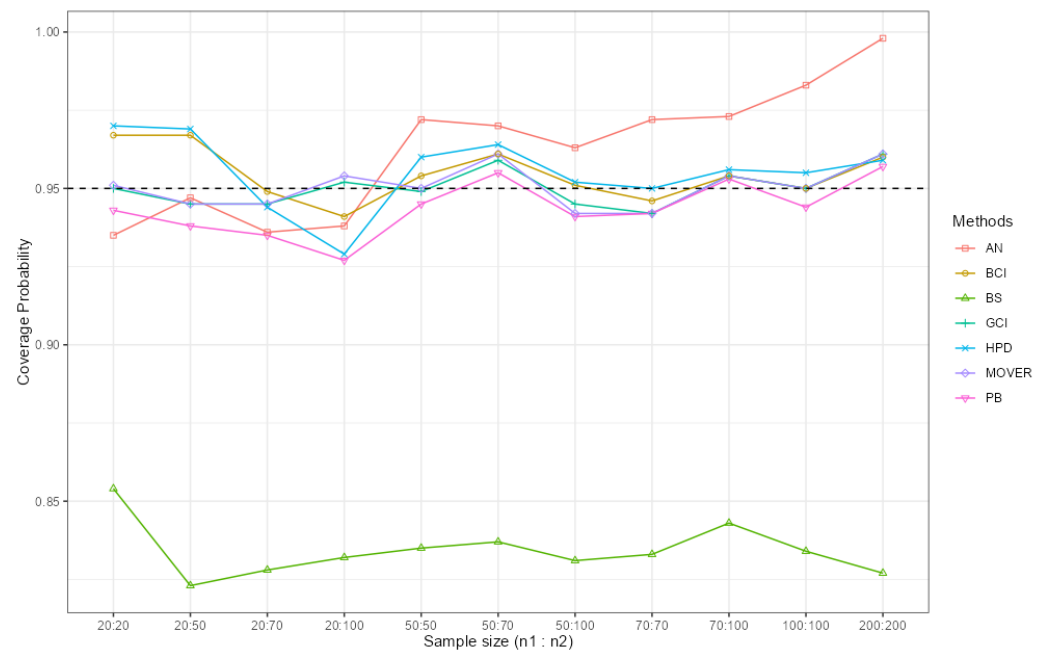


Figure 3. Coverage probabilities (CPs) of confidence intervals for the ratio of coefficients of variation under the ZITR distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.3, 0.3)$. The GCI, MOVER, AN, BCI, and HPD methods achieve CP values close to or exceeding the nominal confidence level of 0.95, indicating satisfactory coverage performance.

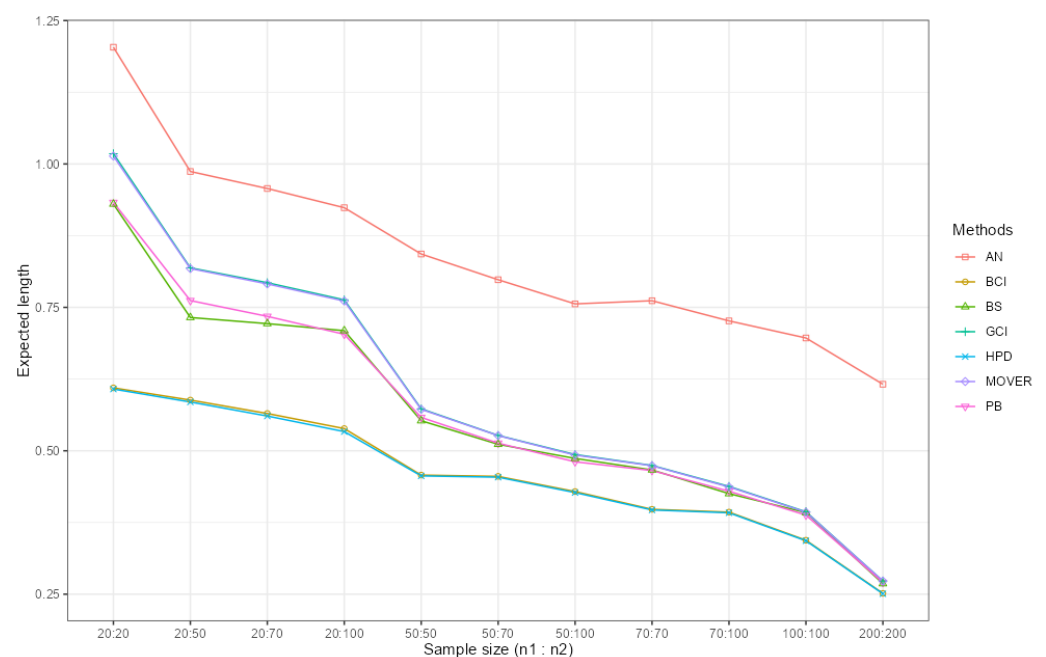


Figure 4. Expected lengths (ELs) of confidence intervals for the difference in coefficients of variation under the ZITR distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.3, 0.3)$. Although the PB and BS methods produce relatively short intervals, they fail to attain the nominal CP level of 0.95. The BCI, HPD, MOVER, GCI, and AN methods are therefore preferred because they provide adequate coverage probabilities while maintaining comparatively short interval lengths.

In terms of CP, the BCI, HPD, AN, GCI, and MOVER methods generally achieved coverage probabilities close to or exceeding the nominal level of 0.95. In addition, all methods tended to produce shorter expected lengths as the sample sizes increased.

Overall, the HPD method generally performed well in terms of both coverage probability and expected length under most simulation settings. Its performance should be

interpreted jointly with other competing methods, particularly in scenarios where slight undercoverage was observed.

4. Application

The proposed confidence interval methods were applied to the road traffic fatality rates at district level (per 100,000 population) in Uttaradit and Mae Hong Son provinces of Thailand. Data on road traffic fatalities were obtained from the Thai Road Safety Collaboration (ThaiRSC) database (<https://www.thairsc.com/>), and included those occurring in March 2026. These data were linked with district level population data obtained from Bureau of Registration Administration, Department of Provincial Administration <https://stat.bora.dopa.go.th/stat/statnew/statMenu/newStat/home.php> (accessed on 28 April 2026). The district level road traffic fatality rate was calculated as the number of road traffic fatalities divided by the corresponding district population and multiplied by 100,000.

Specifically, if F_i and N_i denote the number of road traffic fatalities and the population of district i , respectively, then the fatality rate is given by

$$R_i = \frac{F_i}{N_i} \times 100,000, \quad (30)$$

where R_i denotes the road traffic fatality rate (per 100,000 population) for district i . The resulting data are presented in Table 13. Uttaradit and Mae Hong Son provinces were selected because their district-level fatality rates exhibited a substantial proportion of zero observations, making them appropriate for illustrating the proposed methods under a zero-inflated setting. Before constructing the confidence intervals, several candidate distributions were fitted to the data. Based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the zero-inflated two-parameter Rayleigh (ZITR) distribution provided the best fit and was therefore adopted for the subsequent analysis. Then, we constructed confidence intervals for the difference and ratio of coefficients of variation (CVs) based on the proposed methods.

Table 13. District-level road traffic fatality rates per 100,000 population in Uttaradit and Mae Hong Son provinces.

Province	Road Traffic Fatality Rates per 100,000 Population						
Uttaradit	2.23	0	3.553	0	0	0	4.325
	2.807	0					
Mae Hong Son	3.528	0	8.353	3.774	6.211	0	0

Both datasets have a significant quantity of zero observations, reflecting districts with no recorded incidents throughout the research period. The positive numbers fluctuate across districts, reflecting variations in accident severity within each province.

The study starts by scrutinizing the distribution of the positive observations, considering these properties. The appropriateness of candidate models for the non-zero component is evaluated using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The findings (refer to Tables 14 and 15) demonstrate that the two-parameter Rayleigh distribution adequately fits the positive data for both provinces.

The existence of excess zeros and a positively skewed distribution of the positive fatality rates suggests that the ZITR model is appropriate for these data. The model structure allows the districts to be represented by two distinct components: those with zero recorded road traffic fatality rates and those with positive fatality rates. These structural dif-

ferences may reflect variations in traffic density, road infrastructure, enforcement measures, healthcare accessibility, or other local characteristics across districts.

The above datasets were used to demonstrate the proposed confidence interval procedures for the difference and ratio of the coefficients of variation using observed road traffic fatality rates. The confidence intervals were constructed according to the procedures described in this paper, allowing a direct comparison of the relative variability in road traffic fatality rates between the two provinces under the assumed ZITR model.

Table 14. AIC and BIC values for candidate distributions fitted to March 2026 road traffic fatality rates in Uttaradit province.

Criterion	Normal	Rayleigh	Weibull	Lognormal	Exponential	Gamma
AIC	13.44253	12.98544	13.41914	13.35213	18.37715	13.33844
BIC	12.21512	11.75803	12.19172	12.12472	18.76345	12.11103

Table 15. AIC and BIC values for candidate distributions fitted to March 2026 road traffic fatality rates in Mae Hong Son province.

Criterion	Normal	Rayleigh	Weibull	Lognormal	Exponential	Gamma
AIC	20.77278	19.98839	20.56379	20.17872	22.58955	20.30145
BIC	19.54537	18.76097	19.33638	18.95131	22.97584	19.07404

The resulting confidence intervals and the associated performance measures are presented in Tables 16 and 17 for the difference and ratio of the coefficients of variation, respectively. The results illustrate that the proposed methodologies are practical and applicable for analyzing real-world road traffic fatality rate data.

Table 16. Confidence interval estimation for the difference of CVs under the ZITR distribution: An application to March 2026 district-level road traffic fatality rates in Uttaradit and Mae Hong Son provinces.

Method	Lower	Upper	Length
BCI	−0.4352	0.6096	1.0448
HPD	−0.4224	0.6202	1.0426
AN	−0.1697	1.4464	1.6161
GCI	−1.8364	1.8547	3.6911
MOVER	−1.9269	1.7130	3.6399
PB	−0.6897	1.1797	1.8694
BS	−0.1864	1.9134	2.0998

The analysis of district-level road traffic fatality rates (per 100,000 population) from Uttaradit and Mae Hong Son provinces, Thailand, recorded in March 2026, reveals noticeable differences in the performance of the confidence interval methods under the ZITR model for the difference between coefficients of variation. The HPD method produces the shortest confidence interval, which is consistent with the findings from the simulation study. The remaining methods can be ranked in ascending order of interval length as BCI, AN, BS, PB, GCI, and MOVER. The substantially wider intervals produced by the latter methods reflect a greater degree of uncertainty in the estimation process. Furthermore, the simulation results demonstrate that the HPD method consistently achieves CP close to the nominal confidence level while maintaining relatively short interval lengths. Therefore,

the favorable performance of the HPD method observed in the real-data application is consistent with the simulation results, suggesting that it provides a competitive approach for interval estimation under the ZITR distribution.

Table 17. Confidence interval estimation for the ratio of CVs under the ZITR distribution: An application to March 2026 district-level road traffic fatality rates in Uttaradit and Mae Hong Son provinces.

Method	Lower	Upper	Length
BCI	0.5928	2.0387	1.4459
HPD	0.5172	1.8911	1.3739
AN	−3.4951	7.8910	11.3861
GCI	0.3391	3.3510	3.0119
MOVER	0.3176	3.1546	2.837
PB	0.4816	2.8947	2.4131
BS	−0.0823	2.8909	2.9732

A similar pattern is observed for the ratio of coefficients of variation. The HPD method again produces the shortest confidence interval among all competing methods. The remaining methods can be ranked in ascending order of interval length as BCI, PB, GCI, MOVER, BS, and AN. The wider intervals produced by these methods indicate a greater degree of uncertainty in the estimation process. It is also noteworthy that the AN and BS methods produced negative lower confidence limits for the ratio of the coefficients of variation. Since the ratio of two positive coefficients of variation is itself strictly positive, these intervals extend beyond the admissible parameter space. This limitation arises from the additive construction of these intervals on the original scale and should be taken into account when interpreting the results.

To address this issue, log-transformed versions of the AN and BS methods were additionally considered. Let

$$\eta = \log\left(\frac{CV_1}{CV_2}\right).$$

The confidence intervals were first constructed on the logarithmic scale and subsequently transformed back to the original scale according to

$$CI_{\log} = (L_{\eta}, U_{\eta}),$$

and

$$CI_R = (\exp(L_{\eta}), \exp(U_{\eta})).$$

To address the issue of negative lower confidence limits produced by the AN and BS methods, log-transformed versions of these confidence intervals were additionally considered. The resulting confidence intervals are presented in Table 18. By constructing the intervals on the logarithmic scale and transforming them back to the original scale, both confidence limits remain strictly positive and therefore lie within the admissible parameter space for the ratio of coefficients of variation. As expected, the log-transformed intervals successfully eliminated the negative lower confidence limits observed for the original AN and BS methods. However, this improvement was accompanied by an increase in interval length, particularly for the Log-AN method, illustrating the trade-off between respecting the parameter space and maintaining interval precision. Nevertheless, the log-ratio intervals provide more appropriate interval estimates for a strictly positive parameter.

Table 18. Log-transformed confidence intervals based on the AN and BS methods for the ratio of coefficients of variation.

Method	Lower	Upper	Length
log-ratio AN	0.16485	29.30384	29.13899
log ratio BS	1.61106	10.29579	8.68473

Furthermore, the simulation study demonstrates that the HPD method consistently provides satisfactory coverage probabilities while maintaining relatively short interval lengths. Therefore, the agreement between the simulation results and the real-data analysis suggests that the HPD method is the most suitable approach for constructing confidence intervals for the ratio of coefficients of variation in district-level road traffic fatality rate data exhibiting the characteristics of the ZITR distribution.

5. Discussion

While studying confidence intervals for the difference and ratio of coefficients of variation in the zero-inflated two-parameter Rayleigh distribution, the results showed that the BCI, HPD, AN, GCI, and MOVER methods generally achieved coverage probability (CP) values close to or above the nominal level. These results indicate that the proposed interval estimation methods are able to adequately capture the true parameter values of interest.

A key aspect of interval estimation is the trade-off between CP and EL. The HPD method generally produced shorter confidence intervals for both the difference and ratio of CVs across different sample sizes, while maintaining coverage probabilities close to the nominal level. This is particularly relevant for small and moderate sample sizes, where efficient interval estimation is important due to data limitations and inference uncertainty.

Nevertheless, although the BCI, AN, GCI, and MOVER methods also provided reliable coverage performance, all methods demonstrated comparable behavior in terms of coverage probability under most settings, with differences mainly observed in interval length.

For the ratio of coefficients of variation, an additional consideration is that the AN and BS methods may produce negative lower confidence limits because these intervals are constructed additively on the original scale. Since the ratio of two positive coefficients of variation is itself strictly positive, such intervals extend beyond the admissible parameter space. To address this limitation, log-transformed versions of the AN and BS methods were additionally examined. The log-transformed intervals successfully eliminated negative lower confidence limits while preserving the positivity of the parameter space. However, this improvement came at the expense of larger interval lengths, indicating the trade-off between respecting the parameter space and interval precision. Therefore, although the log-transformed intervals provide a theoretically more appropriate inference for strictly positive ratio parameters, the HPD method remained the most competitive approach because it maintained satisfactory coverage probabilities together with relatively short interval lengths.

These findings are consistent with previous studies on confidence interval construction for the difference and ratio of means, as well as the difference and ratio of variances [25,26] under the ZITR distribution, which also reported that the HPD method performs competitively in terms of interval efficiency while maintaining adequate coverage.

Future research may extend the proposed confidence interval estimation methods to other zero-inflated lifetime distributions. Another promising direction is to develop interval estimation procedures for the differences and ratios of the coefficients of variation involving more than two populations. Such an extension would provide useful tools for

simultaneous comparisons in practical applications with multiple groups. Future studies may also evaluate the performance of the proposed methods under model misspecification, varying sample sizes, and different levels of zero inflation.

6. Conclusions

In this study, confidence intervals for the difference and ratio of the coefficients of variation under the ZITR distribution were developed and investigated. The proposed approaches, including the BCI, HPD, and AN methods, were compared with several well-established interval estimation methods, namely GCI, MOVER, PB, and BS. The performance of these methods was evaluated through a Monte Carlo simulation study using coverage probability (CP), Monte Carlo standard error (MCSE) of the estimated CP, and expected length (EL) as evaluation criteria.

The simulation results for both the difference and ratio of coefficients of variation indicate that the HPD method generally performs well, achieving coverage probabilities close to the nominal level while producing relatively short interval lengths. In addition, the MCSE values of the estimated coverage probabilities were small across all simulation settings, indicating that the reported CP values were obtained with sufficient Monte Carlo precision.

For the difference of CVs, the remaining methods can be ranked in terms of expected length performance as BCI, MOVER, GCI, and AN, respectively. Although the PB and BS methods sometimes produce shorter intervals, their coverage probabilities are not consistently close to the nominal level.

A similar pattern is observed for the ratio of CVs, where the BCI, GCI, MOVER, and AN methods show comparable performance after HPD. The PB and BS methods again exhibit lower coverage probabilities in some settings.

A real-world application using road traffic fatality rate data at the district level (per 100,000 population) from Uttaradit and Mae Hong Son provinces, Thailand, in March 2026 was conducted to demonstrate the practical applicability of the proposed approaches. The empirical results are consistent with the simulation findings. The HPD method consistently produces relatively short confidence intervals while maintaining coverage close to the nominal level, suggesting its competitive performance for interval estimation under the ZITR distribution.

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Data Availability Statement: The data on road traffic fatalities were obtained from the Thai Road Safety Collaboration (ThaiRSC) database (<https://www.thairsc.com/>). The variable of interest was the district-level road traffic fatality rate (per 100,000 population). The analysis was restricted to Uttaradit and Mae Hong Son provinces, Thailand, using road traffic fatalities that occurred in March 2026. Fatality data were linked with district-level population data to calculate the fatality rates. No observations were excluded from the analysis. Data were retrieved on 28 April 2026.

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suggestions, which greatly enhanced the quality and completeness of the manuscript. In preparing this manuscript, the authors used ChatGPT (OpenAI, GPT-5.5) and Grammarly (v.1.2.279.1925) solely to assist with English language editing and to improve the clarity, grammar, and readability of the text. The authors critically reviewed and revised all AI-generated suggestions and accept full responsibility for the content of the published work.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

BCI	Bayesian Credible Interval
HPD	Bayesian Highest Posterior Density
GCI	Generalized Confidence Interval method
MOVER	Method of Variance Estimates Recovery
AN	Approximate Normal based on delta method
PB	Percentile Bootstrap confidence interval
BS	Bootstrap method with standard error
CP	Coverage probability
EL	Expected length
AIC	Akaike information criterion
BIC	Bayesian information criterion

Appendix A

Before conducting the prior sensitivity analysis, the convergence and mixing performance of the Markov chain Monte Carlo (MCMC) algorithm were assessed under the baseline prior specification adopted throughout this study, namely $\lambda \sim \text{Gamma}(1, 2)$, $\mu \sim \text{Uniform}(0, 3)$, and $\delta \sim \text{Beta}(8, 2)$. The MCMC algorithm was implemented using a single Markov chain with 20,000 iterations, of which the first 5000 iterations were discarded as burn-in, yielding 15,000 posterior samples for inference. Figures A1 and A2 present the trace plots and autocorrelation function (ACF) plots for the Bayesian estimation of the difference and ratio of the coefficients of variation, respectively. The trace plots indicate that the Markov chains fluctuated stably around their posterior means without exhibiting apparent trends or non-stationary behavior. Furthermore, the ACF plots show that the autocorrelations decreased rapidly toward zero within the first several lags, indicating weak serial dependence and satisfactory mixing of the chains. These diagnostic results suggest that the posterior samples were adequate for subsequent Bayesian inference.

To further assess the robustness of the Bayesian interval estimators with respect to the choice of prior hyperparameters, an additional prior sensitivity analysis was performed. Three prior specifications with different levels of prior concentration were considered to investigate whether the performance of the Bayesian methods, in terms of coverage probability (CP) and expected length (EL), was sensitive to the prior assumptions. The analysis was conducted using a subset of the simulation scenarios with sample sizes $n_1 = n_2 = 20$. The parameter settings considered were $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$ and $(0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1)$, $(0.1, 0.3)$, and $(0.3, 0.3)$. The prior distributions for each specification are summarized in Table A1.

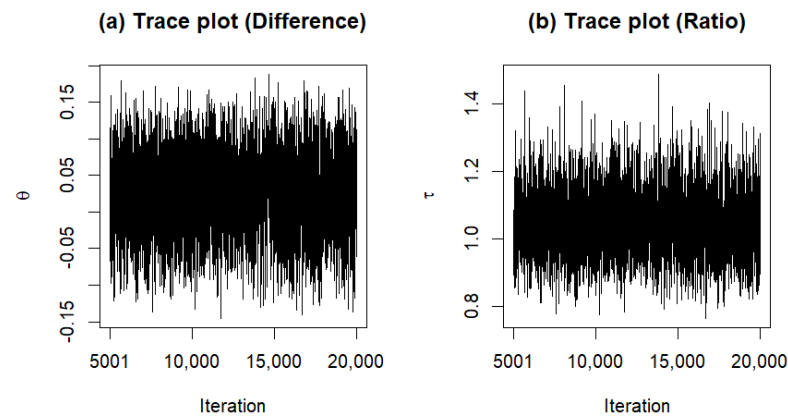


Figure A1. Trace plots of the MCMC samples for the difference and ratio of the coefficients of variation.

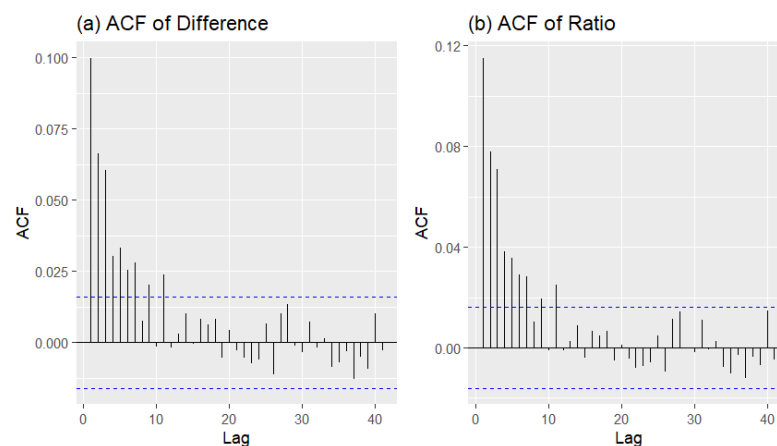


Figure A2. Autocorrelation function (ACF) plots of the MCMC samples for the Bayesian estimates of (a) the difference and (b) the ratio of the coefficients of variation.

Table A1. Prior hyperparameter specifications used in the sensitivity analysis.

Prior	δ_i	λ_i	μ_i
S1	Beta(8,2)	Gamma(1,2)	Uniform(0,M)
S2	Beta(2,2)	Gamma(1,1)	Uniform(0,M)
S3	Beta(1,1)	Gamma(0.5,0.5)	Uniform(0,M)

The prior specification S1, $\delta_i \sim \text{Beta}(8, 2)$ and $\lambda_i \sim \text{Gamma}(1, 2)$, corresponds to the baseline prior used in the main simulation study and real-data applications. The prior specification S2, with $\delta_i \sim \text{Beta}(2, 2)$ and $\lambda_i \sim \text{Gamma}(1, 1)$, represents a less concentrated prior specification, whereas S3, with $\delta_i \sim \text{Beta}(1, 1)$ and $\lambda_i \sim \text{Gamma}(0.5, 0.5)$, provides the least concentrated prior among the three settings. The prior for μ_i was kept unchanged as $\text{Uniform}(0, M)$ across all scenarios to focus the sensitivity analysis on the zero-inflation and scale parameters. For each prior specification, the coverage probability (CP) and expected length (EL) of the resulting 95% Bayesian Credible Interval (BCI) and highest posterior density (HPD) interval were calculated under the selected simulation scenarios. The corresponding results are reported in Table A2.

Table A2. Coverage probabilities (CP) and Expected lengths (EL) of BCI and HPD under different prior specifications.

λ_1, λ_2	δ_1, δ_2	Prior	Diff		Ratio	
			BCI	HPD	BCI	HPD
0.2, 0.2	0.1, 0.1	S1	0.988	0.993	0.988	0.989
			(0.41773)	(0.32166)	(0.67174)	(0.65872)
		S2	0.958	0.967	0.958	0.958
			(0.48365)	(0.37150)	(0.77819)	(0.75973)
		S3	0.987	0.987	0.987	0.983
			(0.50912)	(0.50717)	(0.77028)	(0.75144)
	0.1, 0.3	S1	0.909	0.908	0.915	0.931
			(0.56787)	(0.56499)	(0.62122)	(0.60734)
		S2	0.928	0.935	0.935	0.929
			(0.72676)	(0.72049)	(0.65435)	(0.63501)
		S3	0.927	0.930	0.926	0.931
			(0.72589)	(0.71230)	(0.67150)	(0.65206)
	0.3, 0.3	S1	0.972	0.970	0.972	0.973
			(0.61387)	(0.61163)	(0.78535)	(0.76708)
		S2	0.955	0.969	0.955	0.951
			(0.80333)	(0.79911)	(0.91026)	(0.88206)
		S3	0.973	0.975	0.973	0.972
			(0.77953)	(0.77631)	(0.87904)	(0.85398)
0.2, 1	0.1, 0.1	S1	0.977	0.977	0.975	0.954
			(0.41490)	(0.41335)	(0.69688)	(0.68254)
		S2	0.966	0.968	0.965	0.958
			(0.49074)	(0.48851)	(0.84462)	(0.82342)
		S3	0.973	0.972	0.972	0.966
			(0.50167)	(0.49971)	(0.79790)	(0.77747)
	0.1, 0.3	S1	0.917	0.916	0.914	0.925
			(0.57142)	(0.56848)	(0.63583)	(0.62080)
		S2	0.914	0.925	0.916	0.910
			(0.73465)	(0.72839)	(0.69115)	(0.66985)
		S3	0.943	0.947	0.943	0.950
			(0.73275)	(0.72841)	(0.70607)	(0.68469)
	0.3, 0.3	S1	0.952	0.954	0.953	0.947
			(0.62055)	(0.61835)	(0.81090)	(0.79112)
		S2	0.950	0.960	0.950	0.944
			(0.80612)	(0.80190)	(0.95898)	(0.92845)
		S3	0.947	0.962	0.946	0.938
			(0.79956)	(0.79614)	(0.92719)	(0.89903)

The results indicate that the CP and EL of both BCI and HPD remained highly stable across different prior specifications for both the difference and ratio of CVs. In particular, the HPD method consistently produced narrower intervals than the BCI method, indicating higher interval efficiency of the HPD approach. These findings suggest that the interval estimation performance of the Bayesian methods was not substantially affected by the choice of prior hyperparameters under the considered scenarios.

The differences in CP and EL among the three prior specifications were relatively small, suggesting that the posterior inference was reasonably robust to alternative prior

assumptions. This stability is particularly important for the proposed Bayesian interval estimators because it indicates that the resulting credible intervals do not depend strongly on a single prior specification.

Based on this sensitivity analysis, the prior specification S1 used in the main simulation study and real-data applications remains a reasonable baseline choice. Although alternative prior specifications were considered, the resulting CP and EL of the Bayesian Credible Intervals for both the difference and ratio of CVs were not substantially affected. Therefore, the use of S1 provides stable and reliable interval estimation performance within the investigated scenarios.

Appendix B

This section derives variance approximations for the estimators of the difference and ratio of CVs, denoted by $\hat{\theta}$ and $\hat{\tau}$, respectively, using the delta method. Let

$$\hat{\kappa}_i = g(\hat{\delta}_i, \hat{\mu}_i, \hat{\lambda}_i),$$

be the estimator of CV for the ZITR distribution, where

$$g(\delta_i, \mu_i, \lambda_i) = \frac{\sqrt{\frac{4(1-\delta_i)-(1-\delta_i)^2\pi}{4\lambda_i} + \frac{\mu_i}{\lambda_i^{1/2}}\sqrt{\pi}\delta_i(1-\delta_i) + \mu_i^2\delta_i(1-\delta_i)}}{(1-\delta_i)\left(\mu_i + \frac{1}{\sqrt{\lambda_i}}\Gamma\left(\frac{3}{2}\right)\right)}.$$

To obtain approximate variances for $\hat{\theta}$ and $\hat{\tau}$, the delta method is employed. This approach provides a convenient way to approximate the sampling distribution of a smooth function of estimators through linearization. Let $G = g(\hat{U}_1, \hat{U}_2, \hat{U}_3, \hat{U}_4, \hat{U}_5, \hat{U}_6)$ denote an estimator of the continuously differentiable function $g(U_1, U_2, U_3, U_4, U_5, U_6)$. A first-order Taylor series expansion of $g(\cdot)$ about the true parameter vector $(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6)$, corresponding to $(U_1, U_2, U_3, U_4, U_5, U_6)$, yields an asymptotic approximation to the distribution of G . Under regularity conditions and for sufficiently large sample sizes, G is approximately normally distributed. Consequently, approximate expressions for the variance of G can be obtained and subsequently used to construct confidence intervals for the parameters θ and τ .

$$g(U_1, U_2, U_3, U_4, U_5, U_6) \approx g((\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6)) + \sum_{i=1}^6 \frac{\partial g((\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6))}{\partial u_i} (U_i - \omega_i).$$

In this setting, define the vector of basic statistics as $U_1 = \hat{\delta}_1, U_2 = \hat{\mu}_1, U_3 = \hat{\lambda}_1, U_4 = \hat{\delta}_2, U_5 = \hat{\mu}_2, U_6 = \hat{\lambda}_2$, where $\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2$ are the maximum likelihood estimators (MLEs) of $\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2$, respectively. Substituting these estimators into the function $g(\cdot)$ yields the estimator of θ as

$$\hat{\theta} = g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2) = \frac{\sqrt{\frac{4(1-\hat{\delta}_1)-(1-\hat{\delta}_1)^2\pi}{4\hat{\lambda}_1} + \frac{\hat{\mu}_1}{\hat{\lambda}_1^{1/2}}\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1) + \hat{\mu}_1^2\hat{\delta}_1(1-\hat{\delta}_1)}}{(1-\hat{\delta}_1)\left(\hat{\mu}_1 + \frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)} - \frac{\sqrt{\frac{4(1-\hat{\delta}_2)-(1-\hat{\delta}_2)^2\pi}{4\hat{\lambda}_2} + \frac{\hat{\mu}_2}{\hat{\lambda}_2^{1/2}}\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2) + \hat{\mu}_2^2\hat{\delta}_2(1-\hat{\delta}_2)}}{(1-\hat{\delta}_2)\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}$$

and the estimator of τ as

$$\hat{\tau} = g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2) = \frac{\sqrt{\frac{4(1-\hat{\delta}_1)-(1-\hat{\delta}_1)^2\pi}{4\lambda_1} + \frac{\hat{\mu}_1}{\lambda_1^{\frac{1}{2}}}\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1) + \hat{\mu}_1^2\hat{\delta}_1(1-\hat{\delta}_1)}}{(1-\hat{\delta}_1)\left(\hat{\mu}_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)} \cdot \frac{\sqrt{\frac{4(1-\hat{\delta}_2)-(1-\hat{\delta}_2)^2\pi}{4\lambda_2} + \frac{\hat{\mu}_2}{\lambda_2^{\frac{1}{2}}}\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2) + \hat{\mu}_2^2\hat{\delta}_2(1-\hat{\delta}_2)}}{(1-\hat{\delta}_2)\left(\hat{\mu}_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}$$

The partial derivatives of $\hat{\theta}$ up to the first order with regard to the components of the parameter vector are derived as follows:

$$\frac{\partial}{\partial \delta_1} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = \frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{2\pi(1-\delta_1)-4}{4\lambda_1} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}}\sqrt{\pi}[1-2\delta_1] + \mu_1^2[1-2\delta_1]\right) - (\sqrt{\xi_1})\left(-1\left(\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_1^2}$$

$$\frac{\partial}{\partial \mu_1} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = \frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\sqrt{\pi}\delta_1(1-\delta_1)}{\lambda_1^{\frac{1}{2}}} + 2\mu_1\delta_1(1-\delta_1)\right) - (\sqrt{\xi_1})(1-\delta_1)}{\psi_1^2}$$

$$\frac{\partial}{\partial \lambda_1} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = \frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{(1-\delta_1)^2\pi-4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\sqrt{\pi}\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}}\right) - \sqrt{\xi_1}\left((1-\delta_1)\left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_1^2}$$

$$\frac{\partial}{\partial \delta_2} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = -\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{2\pi(1-\delta_2)-4}{4\lambda_2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}}\sqrt{\pi}[1-2\delta_2] + \mu_2^2[1-2\delta_2]\right) - (\sqrt{\xi_2})\left(-1\left(\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_2^2}$$

$$\frac{\partial}{\partial \mu_2} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = -\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\sqrt{\pi}\delta_2(1-\delta_2)}{\lambda_2^{\frac{1}{2}}} + 2\mu_2\delta_2(1-\delta_2)\right) - (\sqrt{\xi_2})(1-\delta_2)}{\psi_2^2}$$

$$\frac{\partial}{\partial \lambda_2} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = -\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{(1-\delta_2)^2\pi-4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2\sqrt{\pi}\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}}\right) - \sqrt{\xi_2}\left((1-\delta_2)\left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_2^2}$$

Therefore

$$\begin{aligned} \hat{\theta} \approx & g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\delta}_1}(\hat{\delta}_1 - \delta_1) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\mu}_{11}}(\hat{\mu}_1 - \mu_1) \\ & + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\lambda}_1}(\hat{\lambda}_1 - \lambda_1) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\delta}_2}(\hat{\delta}_2 - \delta_2) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\mu}_{21}}(\hat{\mu}_2 - \mu_2) \\ & + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\lambda}_2}(\hat{\lambda}_2 - \lambda_2) \end{aligned}$$

$$\begin{aligned}
V(\hat{\theta}) = & \left(\frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{2\pi(1-\delta_1)-4}{4\lambda_1} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_1] + \mu_1^2 [1-2\delta_1] \right) - (\sqrt{\xi_1}) \left(-1 \left(\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}} \Gamma \left(\frac{3}{2} \right) \right) \right) \right)}{\psi_1^2} \right)^2 V(\hat{\delta}_1 - \delta_1) \\
& + \left(\frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{\sqrt{\pi}\delta_1(1-\delta_1)}{\lambda_1^{\frac{1}{2}}} + 2\mu_1\delta_1(1-\delta_1) \right) - (\sqrt{\xi_1})(1-\delta_1)}{\psi_1^2} \right)^2 V(\hat{\mu}_1 - \mu_1) \\
& + \left(\frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{(1-\delta_1)^2\pi-4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\sqrt{\pi}\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}} \right) - \sqrt{\xi_1} \left((1-\delta_1) \left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}} \Gamma \left(\frac{3}{2} \right) \right) \right)}{\psi_1^2} \right)^2 V(\hat{\lambda}_1 - \lambda_1) \\
& + \left(\frac{\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{2\pi(1-\delta_2)-4}{4\lambda_2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_2] + \mu_2^2 [1-2\delta_2] \right) - (\sqrt{\xi_2}) \left(-1 \left(\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}} \Gamma \left(\frac{3}{2} \right) \right) \right) \right)}{\psi_2^2} \right)^2 V(\hat{\delta}_2 - \delta_2) \\
& + \left(\frac{\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{\sqrt{\pi}\delta_2(1-\delta_2)}{\lambda_2^{\frac{1}{2}}} + 2\mu_2\delta_2(1-\delta_2) \right) - (\sqrt{\xi_2})(1-\delta_2)}{\psi_2^2} \right)^2 V(\hat{\mu}_2 - \mu_2) \\
& + \left(\frac{\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{(1-\delta_2)^2\pi-4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2\sqrt{\pi}\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}} \right) - \sqrt{\xi_2} \left((1-\delta_2) \left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}} \Gamma \left(\frac{3}{2} \right) \right) \right)}{\psi_2^2} \right)^2 V(\hat{\lambda}_2 - \lambda_2) \\
V(\hat{\theta}) = & \left(\frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{2\pi(1-\delta_1)-4}{4\lambda_1} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_1] + \mu_1^2 [1-2\delta_1] \right) - (\sqrt{\xi_1}) \left(-1 \left(\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}} \Gamma \left(\frac{3}{2} \right) \right) \right) \right)}{\psi_1^2} \right)^2 \left(\frac{\delta_1(1-\delta_1)}{n_1} \right) \\
& + \left(\frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{\sqrt{\pi}\delta_1(1-\delta_1)}{\lambda_1^{\frac{1}{2}}} + 2\mu_1\delta_1(1-\delta_1) \right) - (\sqrt{\xi_1})(1-\delta_1)}{\psi_1^2} \right)^2 \left(\frac{1}{2\mu_1 + \frac{1}{n_{1(1)}} \sum_{i=1}^{n_{1(1)}} (x_{1j(1)} - \mu_1)^{-2}} \right) \\
& + \left(\frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{(1-\delta_1)^2\pi-4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\sqrt{\pi}\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}} \right) - \sqrt{\xi_1} \left((1-\delta_1) \left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}} \Gamma \left(\frac{3}{2} \right) \right) \right)}{\psi_1^2} \right)^2 \frac{\lambda_1^2}{n_{1(1)}} \\
& + \left(\frac{\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{2\pi(1-\delta_2)-4}{4\lambda_2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_2] + \mu_2^2 [1-2\delta_2] \right) - (\sqrt{\xi_2}) \left(-1 \left(\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}} \Gamma \left(\frac{3}{2} \right) \right) \right) \right)}{\psi_2^2} \right)^2 \left(\frac{\delta_2(1-\delta_2)}{n_2} \right) \\
& + \left(\frac{\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{\sqrt{\pi}\delta_2(1-\delta_2)}{\lambda_2^{\frac{1}{2}}} + 2\mu_2\delta_2(1-\delta_2) \right) - (\sqrt{\xi_2})(1-\delta_2)}{\psi_2^2} \right)^2 \left(\frac{1}{2\mu_2 + \frac{1}{n_{2(1)}} \sum_{j=1}^{n_{2(1)}} (x_{2j(1)} - \mu_2)^{-2}} \right) \\
& + \left(\frac{\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{(1-\delta_2)^2\pi-4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2\sqrt{\pi}\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}} \right) - \sqrt{\xi_2} \left((1-\delta_2) \left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}} \Gamma \left(\frac{3}{2} \right) \right) \right)}{\psi_2^2} \right)^2 \frac{\lambda_2^2}{n_{2(1)}}
\end{aligned}$$

So estimate of $V(\hat{\theta})$ defined $\hat{V}(\hat{\theta})$ expressed as

$$\begin{aligned}
V(\hat{\theta}) = & \left(\frac{\hat{\psi}_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{2\pi(1-\hat{\delta}_1)-4}{4\lambda_1} + \frac{\hat{\mu}_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi} [1-2\hat{\delta}_1] + \hat{\mu}_1^2 [1-2\hat{\delta}_1] \right) - \left(\sqrt{\xi_1} \right) \left(-1 \left(\left(\hat{\mu}_1 + \frac{1}{\sqrt{\lambda_1}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\hat{\psi}_1^2} \right)^2 \left(\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1} \right) \\
& + \left(\frac{\hat{\psi}_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{\lambda_1^{\frac{1}{2}}} + 2\hat{\mu}_1\hat{\delta}_1(1-\hat{\delta}_1) \right) - \left(\sqrt{\xi_1} \right) (1-\hat{\delta}_1)}{\hat{\psi}_1^2} \right)^2 \left(\frac{1}{2\hat{\mu}_1 + \frac{1}{n_{1(1)}} \sum_{i=1}^{n_{1(1)}} (x_{1j(1)} - \hat{\mu}_1)^{-2}} \right) \\
& + \left(\frac{\hat{\psi}_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{(1-\hat{\delta}_1)^2\pi-4(1-\hat{\delta}_1)}{4\lambda_1^2} - \frac{\hat{\mu}_1\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{2\lambda_1^{\frac{3}{2}}} \right) - \sqrt{\xi_1} \left((1-\hat{\delta}_1) \left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right)}{\hat{\psi}_1^2} \right)^2 \frac{\lambda_1^2}{n_{1(1)}} \\
& + \left(\frac{\hat{\psi}_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{2\pi(1-\hat{\delta}_2)-4}{4\lambda_2} + \frac{\hat{\mu}_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\hat{\delta}_2] + \hat{\mu}_2^2 [1-2\hat{\delta}_2] \right) - \left(\sqrt{\xi_2} \right) \left(-1 \left(\left(\hat{\mu}_2 + \frac{1}{\sqrt{\lambda_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\hat{\psi}_2^2} \right)^2 \left(\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2} \right) \\
& + \left(\frac{\hat{\psi}_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{\lambda_2^{\frac{1}{2}}} + 2\hat{\mu}_2\hat{\delta}_2(1-\hat{\delta}_2) \right) - \left(\sqrt{\xi_2} \right) (1-\hat{\delta}_2)}{\hat{\psi}_2^2} \right)^2 \left(\frac{1}{2\hat{\mu}_2 + \frac{1}{n_{2(1)}} \sum_{j=1}^{n_{2(1)}} (x_{2j(1)} - \hat{\mu}_2)^{-2}} \right) \\
& + \left(\frac{\hat{\psi}_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{(1-\hat{\delta}_2)^2\pi-4(1-\hat{\delta}_2)}{4\lambda_2^2} - \frac{\hat{\mu}_2\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{2\lambda_2^{\frac{3}{2}}} \right) - \sqrt{\xi_2} \left((1-\hat{\delta}_2) \left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right)}{\hat{\psi}_2^2} \right)^2 \frac{\lambda_2^2}{n_{2(1)}}
\end{aligned}$$

For ratio of CVs. The first-order partial derivatives with regard to the parts of the parameter vector are derived as follows:

$$\frac{\partial}{\partial \delta_1} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = \frac{\kappa_2 \frac{\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{2\pi(1-\delta_1)-4}{4\lambda_1} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_1] + \mu_1^2 [1-2\delta_1] \right) - \left(\sqrt{\xi_1} \right) \left(-1 \left(\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_1^2}}{(\kappa_2)^2}$$

$$\frac{\partial}{\partial \mu_1} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = \frac{\kappa_2 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{\sqrt{\pi}\delta_1(1-\delta_1)}{\lambda_1^{\frac{1}{2}}} + 2\mu_1\delta_1(1-\delta_1) \right) - \left(\sqrt{\xi_1} \right) (1-\delta_1) \right)}{\psi_1^2 \kappa_2^2}$$

$$\frac{\partial}{\partial \lambda_1} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = \frac{\kappa_2 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{(1-\delta_1)^2\pi-4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\sqrt{\pi}\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}} \right) - \sqrt{\xi_1} \left((1-\delta_1) \left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_1^2 \kappa_2^2}$$

$$\frac{\partial}{\partial \delta_2} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = - \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{2\pi(1-\delta_2)-4}{4\lambda_2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_2] + \mu_2^2 [1-2\delta_2] \right) - \left(\sqrt{\xi_2} \right) \left(-1 \left(\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\psi_2^2 \kappa_2^2} \right)$$

$$\frac{\partial}{\partial \mu_2} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = - \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{\sqrt{\pi} \delta_2 (1-\delta_2)}{\lambda_2^{\frac{1}{2}}} + 2\mu_2 \delta_2 (1-\delta_2) \right) - (\sqrt{\xi_2})(1-\delta_2) \right)}{\psi_2^2 \kappa_2^2} \right)$$

$$\frac{\partial}{\partial \lambda_2} g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) = - \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{(1-\delta_2)^2 \pi - 4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2 \sqrt{\pi} \delta_2 (1-\delta_2)}{2\lambda_2^{\frac{3}{2}}} \right) - \sqrt{\xi_2} \left((1-\delta_2) \left(-\frac{1}{2} \lambda_2^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_2^2 \kappa_2^2} \right)$$

Therefore

$$\begin{aligned} \hat{\tau} &\approx g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\delta}_1} (\hat{\delta}_1 - \delta_1) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\mu}_{11}} (\hat{\mu}_1 - \mu_1) \\ &+ \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\lambda}_1} (\hat{\lambda}_1 - \lambda_1) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\delta}_2} (\hat{\delta}_2 - \delta_2) + \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\mu}_{21}} (\hat{\mu}_2 - \mu_2) \\ &+ \frac{\partial g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2)}{\partial \hat{\lambda}_2} (\hat{\lambda}_2 - \lambda_2) \\ V(\hat{\tau}) &= \left(- \left(\frac{\kappa_1 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{2\pi(1-\delta_1)-4}{4\lambda_1^2} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_1] + \mu_1^2 [1-2\delta_1] \right) - (\sqrt{\xi_1}) \left(-1 \left(\mu_1 + \frac{1}{\sqrt{\lambda_1}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_1^2 \kappa_2^2} \right) \right)^2 V(\hat{\delta}_1 - \delta_1) \\ &+ \left(\frac{\kappa_2 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{\sqrt{\pi} \delta_1 (1-\delta_1)}{\lambda_1^{\frac{1}{2}}} + 2\mu_1 \delta_1 (1-\delta_1) \right) - (\sqrt{\xi_1})(1-\delta_1) \right)}{\psi_1^2 \kappa_2^2} \right)^2 V(\hat{\mu}_1 - \mu_1) \\ &+ \left(\frac{\kappa_2 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{(1-\delta_1)^2 \pi - 4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1 \sqrt{\pi} \delta_1 (1-\delta_1)}{2\lambda_1^{\frac{3}{2}}} \right) - \sqrt{\xi_1} \left((1-\delta_1) \left(-\frac{1}{2} \lambda_1^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_1^2 \kappa_2^2} \right)^2 V(\hat{\lambda}_1 - \lambda_1) \\ &+ \left(- \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{2\pi(1-\delta_2)-4}{4\lambda_2^2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi} [1-2\delta_2] + \mu_2^2 [1-2\delta_2] \right) - (\sqrt{\xi_2}) \left(-1 \left(\mu_2 + \frac{1}{\sqrt{\lambda_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_2^2 \kappa_2^2} \right) \right)^2 V(\hat{\delta}_2 - \delta_2) \\ &+ \left(- \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{\sqrt{\pi} \delta_2 (1-\delta_2)}{\lambda_2^{\frac{1}{2}}} + 2\mu_2 \delta_2 (1-\delta_2) \right) - (\sqrt{\xi_2})(1-\delta_2) \right)}{\psi_2^2 \kappa_2^2} \right) \right)^2 V(\hat{\mu}_2 - \mu_2) \\ &+ \left(- \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{(1-\delta_2)^2 \pi - 4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2 \sqrt{\pi} \delta_2 (1-\delta_2)}{2\lambda_2^{\frac{3}{2}}} \right) - \sqrt{\xi_2} \left((1-\delta_2) \left(-\frac{1}{2} \lambda_2^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\psi_2^2 \kappa_2^2} \right) \right)^2 V(\hat{\lambda}_2 - \lambda_2) \end{aligned}$$

$$\begin{aligned}
V(\hat{\tau}) = & \left(- \left(\frac{\kappa_1 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{2\pi(1-\delta_1)-4}{4\lambda_1^2} + \frac{\mu_1}{\lambda_1^{\frac{1}{2}}} \sqrt{\pi}[1-2\delta_1] + \mu_1^2[1-2\delta_1] \right) - (\sqrt{\xi_1}) \left(-1 \left(\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\frac{\psi_1^2}{\kappa_2^2}} \right) \right)^2 \left(\frac{\delta_1(1-\delta_1)}{n_1} \right) \\
& + \left(\frac{\kappa_2 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{\sqrt{\pi}\delta_1(1-\delta_1)}{\lambda_1^{\frac{1}{2}}} + 2\mu_1\delta_1(1-\delta_1) \right) - (\sqrt{\xi_1})(1-\delta_1) \right)}{\frac{\psi_1^2}{\kappa_2^2}} \right)^2 \left(\frac{1}{2\hat{\mu}_1 + \frac{1}{n_{1(1)}} \sum_{i=1}^{n_{1(1)}} (x_{1j(1)} - \hat{\mu}_1)^{-2}} \right) \\
& + \left(\frac{\kappa_2 \left(\psi_1 \left(\frac{1}{2\sqrt{\xi_1}} \right) \left(\frac{(1-\delta_1)^2\pi-4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\sqrt{\pi}\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}} \right) - \sqrt{\xi_1} \left((1-\delta_1) \left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\frac{\psi_1^2}{\kappa_2^2}} \right)^2 \frac{\lambda_1^2}{n_{1(1)}} \\
& + \left(- \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{2\pi(1-\delta_2)-4}{4\lambda_2^2} + \frac{\mu_2}{\lambda_2^{\frac{1}{2}}} \sqrt{\pi}[1-2\delta_2] + \mu_2^2[1-2\delta_2] \right) - (\sqrt{\xi_2}) \left(-1 \left(\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\frac{\psi_2^2}{\kappa_2^2}} \right) \right)^2 \left(\frac{\delta_2(1-\delta_2)}{n_2} \right) \\
& + \left(- \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{\sqrt{\pi}\delta_2(1-\delta_2)}{\lambda_2^{\frac{1}{2}}} + 2\mu_2\delta_2(1-\delta_2) \right) - (\sqrt{\xi_2})(1-\delta_2) \right)}{\frac{\psi_2^2}{\kappa_2^2}} \right) \right)^2 \left(\frac{1}{2\mu_2 + \frac{1}{n_{2(1)}} \sum_{j=1}^{n_{2(1)}} (x_{2j(1)} - \mu_2)^{-2}} \right) \\
& + \left(- \left(\frac{\kappa_1 \left(\psi_2 \left(\frac{1}{2\sqrt{\xi_2}} \right) \left(\frac{(1-\delta_2)^2\pi-4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2\sqrt{\pi}\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}} \right) - \sqrt{\xi_2} \left((1-\delta_2) \left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\frac{\psi_2^2}{\kappa_2^2}} \right) \right)^2 \frac{\lambda_2^2}{n_{2(1)}}
\end{aligned}$$

The estimated value of $\hat{V}(\hat{\tau})$ is defined as

$$\begin{aligned}
\hat{V}(\hat{\tau}) = & \left(\left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{2\pi(1-\hat{\delta}_2)-4}{4\hat{\lambda}_2^2} + \frac{\hat{\mu}_2}{\hat{\lambda}_2^2} \sqrt{\pi} [1-2\hat{\delta}_2] + \hat{\mu}_2^2 [1-2\hat{\delta}_2] \right) - (\sqrt{\hat{\xi}_2}) \left(-1 \left(\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \left(\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1} \right) \\
& + \left(\frac{\hat{\kappa}_2 \left(\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{\hat{\lambda}_1^2} + 2\hat{\mu}_1\hat{\delta}_1(1-\hat{\delta}_1) \right) - (\sqrt{\hat{\xi}_1})(1-\hat{\delta}_1) \right)}{\frac{\hat{\psi}_1^2}{\hat{\kappa}_2^2}} \right)^2 \left(\frac{1}{2\hat{\mu}_1 + \frac{1}{n_{1(1)}} \sum_{i=1}^{n_{1(1)}} (x_{1j(1)} - \hat{\mu}_1)^{-2}} \right) \\
& + \left(\frac{\hat{\kappa}_2 \left(\hat{\psi}_1 \left(\frac{1}{2\sqrt{\hat{\xi}_1}} \right) \left(\frac{(1-\hat{\delta}_1)^2\pi-4(1-\hat{\delta}_1)}{4\hat{\lambda}_1^2} - \frac{\hat{\mu}_1\sqrt{\pi}\hat{\delta}_1(1-\hat{\delta}_1)}{2\hat{\lambda}_1^2} \right) - \sqrt{\hat{\xi}_1} \left((1-\hat{\delta}_1) \left(-\frac{1}{2}\hat{\lambda}_1^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\frac{\hat{\psi}_1^2}{\hat{\kappa}_2^2}} \right)^2 \frac{\hat{\lambda}_1^2}{n_{1(1)}} \\
& + \left(- \left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{2\pi(1-\hat{\delta}_2)-4}{4\hat{\lambda}_2^2} + \frac{\hat{\mu}_2}{\hat{\lambda}_2^2} \sqrt{\pi} [1-2\hat{\delta}_2] + \hat{\mu}_2^2 [1-2\hat{\delta}_2] \right) - (\sqrt{\hat{\xi}_2}) \left(-1 \left(\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \left(\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2} \right) \\
& + \left(- \left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{\hat{\lambda}_2^2} + 2\hat{\mu}_2\hat{\delta}_2(1-\hat{\delta}_2) \right) - (\sqrt{\hat{\xi}_2})(1-\hat{\delta}_2) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \left(\frac{1}{2\hat{\mu}_2 + \frac{1}{n_{2(1)}} \sum_{j=1}^{n_{2(1)}} (x_{2j(1)} - \hat{\mu}_2)^{-2}} \right) \\
& + \left(- \left(\frac{\hat{\kappa}_1 \left(\hat{\psi}_2 \left(\frac{1}{2\sqrt{\hat{\xi}_2}} \right) \left(\frac{(1-\hat{\delta}_2)^2\pi-4(1-\hat{\delta}_2)}{4\hat{\lambda}_2^2} - \frac{\hat{\mu}_2\sqrt{\pi}\hat{\delta}_2(1-\hat{\delta}_2)}{2\hat{\lambda}_2^2} \right) - \sqrt{\hat{\xi}_2} \left((1-\hat{\delta}_2) \left(-\frac{1}{2}\hat{\lambda}_2^{-\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \right) \right) \right)}{\frac{\hat{\psi}_2^2}{\hat{\kappa}_2^2}} \right)^2 \right) \frac{\hat{\lambda}_2^2}{n_{2(1)}}
\end{aligned}$$

The variance estimators $\hat{V}(\hat{\theta})$ and $\hat{V}(\hat{\tau})$ are obtained by substituting estimated partial derivatives and asymptotic variances of $\hat{\delta}$, $\hat{\mu}$ and $\hat{\lambda}$ into the delta method. The variance estimates are used in the construction of confidence ranges for the difference and the ratio of the coefficients of variation, see Section 2.

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