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Effects of Symmetric Multi-Vibration Absorbers on the Nonlinear Vibration Control of Carbon Nanotube-Reinforced Composite Marine Panels

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Abstract

In this paper, the nonlinear vibration (NV) response of carbon nanotube-reinforced composite (CNTRC) marine panels (MPs) fitted with symmetric multi-vibration absorbers (MVAs) subjected to steady, velocity-dependent hydrodynamic loads is investigated. To model actual marine conditions more realistically, the lift and drag forces varying with flow velocity were taken into account using experimentally supported Matveev-based formulations for a specific ship. Within the shell, three carbon nanotube (CNT) distribution schemes are considered: one uniformly distributed (UD) CNT configuration and two functionally graded (FG) CNT patterns, namely FG-V and FG-X. The analytical framework is further constructed using classical shell theory (CST) by incorporating geometric nonlinear terms, and the Galerkin technique is employed to obtain a reduced-order model. Thereafter, the NV response of the CNTRC-MPs is predicted through the P-T method, which relies on the joint application of the piecewise constant argument and Taylor series expansion. The results indicate that symmetric MVAs can effectively suppress NV behavior and significantly decrease the maximum NV amplitude of the panel. Moreover, the effectiveness of the proposed configuration is shown to depend on both the absorber characteristics and the reinforcement pattern of CNTs. The study demonstrates that the use of symmetric absorber systems offers a practical and efficient passive vibration-control solution for advanced marine composite panels.

Keywords: nonlinear vibration; CNTRC marine panels; multi-vibration absorbers; P-T method; hydrodynamic loading

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1. Introduction

Cylindrical panels fabricated from advanced materials have significant potential for use in a wide range of engineering and industrial applications, particularly in the aerospace, marine, mechanical, civil, and energy sectors. They are commonly employed in complex structures including ship hulls, offshore platforms, submarines, satellite systems, and aircraft components. Because of their high structural load resistance, geometrical efficiency, and compatibility with lightweight design requirements, cylindrical panels are regarded as one of the most important structural configurations in modern engineering. To further improve their mechanical performance, CNTs are increasingly incorporated

into composite cylindrical panels as reinforcing materials because of their exceptional stiffness and tensile strength. However, these lightweight structures may be exposed to severe dynamic loads and undesirable vibrations, especially in marine and hydrodynamic environments. In such cases, nonlinear vibration absorbers can be used as an effective passive vibration-control solution. nonlinear vibration absorbers reduce unwanted vibrations by transferring and dissipating vibrational energy via nonlinear dynamic mechanisms. Their passive configuration and capability to reduce large-amplitude responses make them attractive for many engineering applications. In particular, nonlinear vibration absorber devices can improve the serviceability and lifetime of marine structures exposed to hydrodynamic loading. By mitigating high-energy vibrations, they can help prevent structural damage or failure, reduce maintenance requirements, and enhance the long-term operational reliability of structures operating in harsh marine environments.

A considerable number of studies have examined the vibration response of FG plate and shell structures to provide deeper insight into their behavior under vibration-induced excitation [1–7]. In addition, several researchers have investigated the vibrational characteristics of FG panels in particular [8–11]. To mitigate undesirable vibrations, various vibration-control approaches have also been proposed for FG shells [12–14] and FG panels [15,16].

Although FG structures, particularly FG-CNT-reinforced structures, have broad applications in various industries, their dynamic and vibration characteristics have not been comprehensively examined in the aforementioned studies. Several researchers have investigated the dynamic and vibrational behavior of FG-CNT shells [17–19]. Recently, Sun et al. [20] investigated the active vibration suppression in rotating CNTRC cylindrical shells by employing piezoelectric patches. Moreover, the vibration response of cylindrical panel shells made of FG-CNT materials has received attention in a number of studies [21–24]. Quoc et al. [25] addressed the active vibration control of a piezoelectric FG-CNT-reinforced spherical shell panel.

Although nonlinear vibration absorbers have been widely used in different engineering applications, their potential for controlling the vibration of FG-CNT structures has not yet been sufficiently explored. Some studies have used such absorbers for vibration suppression purposes [26–28], whereas others have investigated the vibration response of FG shells equipped with vibration absorbers [29,30]. For example, Wang et al. [27] applied nonlinear energy sink (NES) cells for low-frequency multimode vibration suppression of a floating raft system, while Hu et al. [30] investigated an FG-GPLRC cylindrical shell equipped with periodically embedded dynamic vibration absorbers, showing that regularly spaced absorbers can create localized attenuation zones along the host structure. Recently, Foroutan and Torabi [31] examined the effect of a single nonlinear energy sink in mitigating the NV of stiffened FG panels. However, it must be pointed out that the material properties, geometry, number of absorbers, and absorber positions considered in [31] differ from those in the current study. Furthermore, a cross-scale integrated optimization framework combining genetic-algorithm-based layout, sizing, and material-parameter optimization of multiple magnetorheological (MR) dampers was recently proposed for mitigating torsional vibration in spatially irregular civil structures [32]. While this and related studies demonstrate the effectiveness of distributing several active or semi-active control devices throughout a structure, they generally require numerical layout optimization, real-time control hardware, and external power, and are formulated for civil/building-type torsional response rather than marine panel flexural vibration. In contrast, the present study proposes a fully passive, symmetric MVA arrangement in which absorber placement is determined directly by the inherent geometric and modal symmetry of the CNTRC marine panel, requiring no external power, control algorithm, or numerical layout optimization, and is coupled with a hydrodynamic loading model based on experi-

mentally supported Matveev-based lift and drag relations together with three distinct CNT distribution patterns (UD, FG-V, FG-X). To the authors' knowledge, this combination of passive symmetric multi-absorber placement, realistic hydrodynamic loading, and CNT-grading effects has not been jointly investigated in the existing multi-absorber/multi-damper literature, constituting the principal novelty and practical advantage (simplicity, no power requirement, ease of retrofit) of the present configuration for marine applications.

Although previous studies have examined the vibrational behavior of FG-CNTRC panels, the NV control of FG-CNTRC marine panels fitted with symmetric MVAs under hydro-dynamic loading has not yet been investigated. This issue is important for improving the durability and sustainability of marine structures, as time-dependent hydrodynamic pressures can produce severe vibrations, accelerate material degradation, and increase maintenance costs. In this study, multiple nonlinear vibration absorbers, each consisting of linear and nonlinear springs and a damper, are symmetrically arranged to reduce the NV response of FG-CNTRC marine panels subjected to hydrodynamic loads. Hydrodynamic loads, including velocity-dependent lift and drag forces, were evaluated using Matveev's experimentally validated equations. These relations provide realistic loading conditions for a specific vessel and enable the effectiveness of the MVAs to be assessed under practical marine operating conditions. In addition, three CNT distribution patterns are considered through the shell thickness: FG-V, FG-X, and a UD configuration. The governing equations are established based on CST with geometric nonlinearity and are reduced using the Galerkin technique to predict the NV behavior of the panel-absorber system. This combined formulation provides a reliable framework for evaluating the response of FG-CNTRC marine panels under different hydrodynamic conditions. Finally, the P-T technique is applied to obtain the NV responses, and the findings offer valuable insight into the design of more durable, vibration-resistant, and sustainable marine-based structural systems. The main contributions of this study can be summarized as follows:

- (1) A NV model is developed for CNTRC marine panels equipped with symmetric MVAs.
- (2) Realistic hydrodynamic excitations are incorporated via velocity-dependent lift and drag forces based on Matveev's experimentally validated formulations for a specific vessel.
- (3) The combined effects of three CNT distribution patterns (UD, FG-V, FG-X) and the symmetric MVA configuration are systematically examined using the Galerkin technique with the P-T method, offering design guidance for passive vibration control of marine composite panels.

2. Materials and Methods

2.1. CNTRC-MPs Fitted with Symmetric MVAs Under Hydrodynamic Loading

The configuration of the CNTRC-MPs equipped with J (number of vibration absorber) vibration absorbers and subjected to hydrodynamic loading, including drag (F_1) and lift (F_2), is illustrated in Figure 1. The geometrical characteristics of the CNTRC-MPs include the thickness (h), width (b), length (a), and radius of curvature (R). The MVAs, each consisting of a mass (M), damping coefficient (C), and linear (k_L) and nonlinear (k_{NL}) springs with stiffnesses, are attached to the structure. Figure 2 illustrates the three CNT distribution patterns examined in this study: FG-V, FG-X, and UD. In the UD model, CNTs are uniformly dispersed along the thickness of the shell. For the FG-V distribution, the CNT volume fraction is lowest at the inner surface and gradually increases toward the outer surface. By contrast, the FG-X pattern is characterized by higher CNT concentrations near both bounding surfaces. It should be emphasized that, for a specified flow velocity

V , the hydrodynamic lift (F_2) and drag (F_1) forces obtained from Matveev's formulation are steady, velocity-dependent quantities and do not vary periodically with time. Consequently, the resulting vibration of the coupled CNTRC-MPs-MVAs system is not driven by a continuously oscillating dynamic force; rather, F_1 and F_2 act as a constant load applied to the panel starting from its undeformed, at-rest configuration. The dynamic (forced) response of the coupled system therefore corresponds to the classical suddenly applied load problem: the inertia, geometric nonlinearity, and absorber coupling of the system produce a transient oscillatory response that evolves from the initial rest state toward a new nonlinear static equilibrium position governed by $F_1 + F_2$.

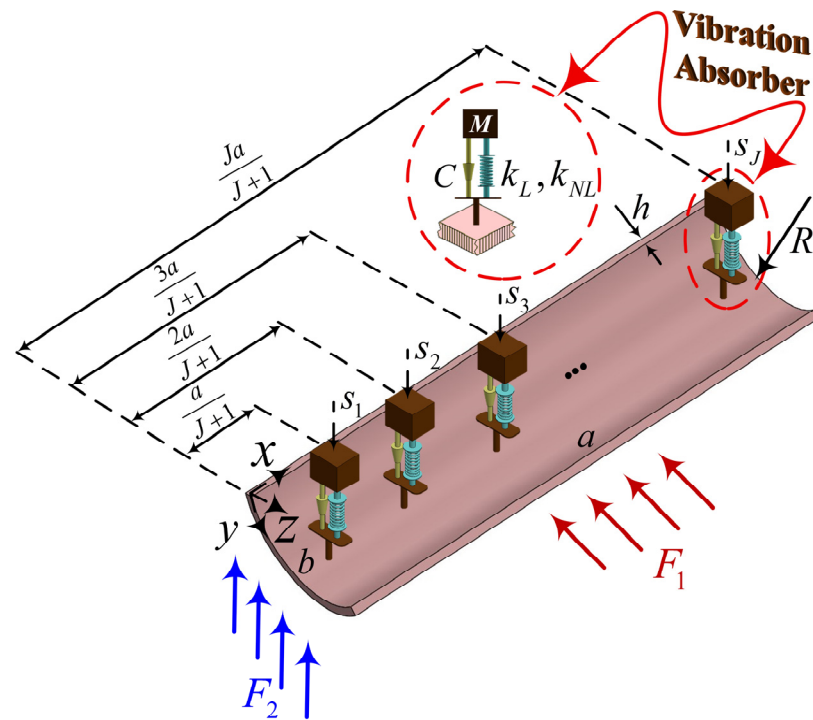


Figure 1. Schematic of CNTRC-MPs fitted with symmetric MVAs under hydrodynamic loading.

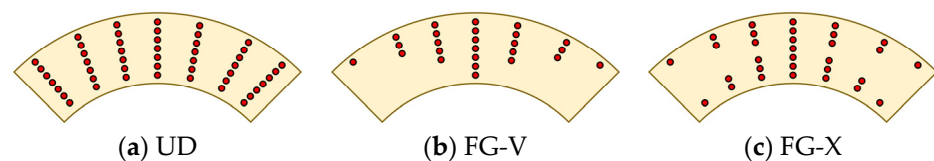


Figure 2. Three representative CNT distribution schemes for CNTRC-MPs.

The elastic properties of CNTRC-MPs are commonly determined through a modified mixture rule that includes efficiency parameters, as expressed below [33]:

$$\begin{Bmatrix} E_{11} \\ \lambda_2/E_{22} \\ \lambda_3/G_{12} \end{Bmatrix} = \begin{bmatrix} \lambda_1 E_{11}^{CNT} & E^M \\ 1/E_{22}^{CNT} & 1/E^M \\ 1/G_{12}^{CNT} & 1/G^M \end{bmatrix} \begin{Bmatrix} V^{CNT} \\ V^M \end{Bmatrix} \quad (1)$$

Here, E_{11}^{CNT} , E_{22}^{CNT} , and G_{12}^{CNT} denote the CNT Young's moduli and shear modulus, respectively, while E^M and G^M correspond to the related properties of the isotropic matrix. The effective properties of the CNTRC-MPs are denoted by E_{11} , E_{22} , and G_{12} , respectively. The parameters λ_i ($i = 1, 2, 3$) indicate the efficiency factors associated with the CNTs. Furthermore, V^{CNT} and V^M denote the CNT and matrix volume fractions, respectively, satisfying the relation $V^{CNT} + V^M = 1$. As stated earlier, three distinct CNT distribution schemes are considered across the shell thickness, as given below:

$$V_{CNT}(z) = \begin{cases} \text{UD:} & V_{CNT}^* \\ \text{FG-V:} & V_{CNT}^* \left(1 + \frac{2z}{h}\right) \\ \text{FG-X:} & 4V_{CNT}^* \frac{|z|}{h} \end{cases} ; -\frac{h}{2} \leq z \leq \frac{h}{2} \quad (2)$$

where V_{CNT}^* denotes the total CNT volume fraction, as reported in Table 1. In addition, Poisson's ratio of the CNTRC-MPs may be evaluated using the rule of mixtures as: $\nu_{12} = V_{CNT}^* \nu_{12}^{CNT} + V^M \nu^M$, where ν_{12}^{CNT} and ν^M indicate the Poisson's ratios corresponding to the CNT and matrix, respectively.

2.2. Theoretical Formulation

Herein, CST is adopted to establish the nonlinear motion equations required for analyzing the NV responses of the structural system. CST, based on the Kirchhoff–Love assumption that normals to the mid-surface remain straight and normal after deformation, does not explicitly account for transverse shear deformation and rotary inertia, and is therefore well suited to geometrically thin shells and panels. The CNTRC-MP considered in this study has a radius-to-thickness ratio of $R/h = 250$, which lies well within the thin-shell regime commonly adopted in the literature (typically taken as $R/h > 20$), so that transverse shear strains remain negligible relative to the membrane and bending strains governing the structural response. Accordingly, CST provides a physically consistent and computationally efficient framework for capturing the geometrically nonlinear (von Kármán-type) behavior that drives the large-amplitude NV response investigated herein, while keeping the reduced-order model compatible with the Galerkin and P-T solution procedures adopted in Section 2.3. This modeling choice is also consistent with an established body of literature on the nonlinear vibration and dynamic response of thin CNTRC and FG-CNT cylindrical shells and panels, where CST has been routinely and successfully adopted to capture geometrically nonlinear behavior under comparable thin-shell conditions [34,35]. The consistent use of CST across these studies reflects a broader consensus that, for panels and shells falling within the thin-shell regime, the governing nonlinear vibration response is dominated by membrane and bending effects rather than transverse shear, making CST both an accurate and a computationally efficient choice for this class of problems. The strain components together with the nonlinear strain–displacement relationships are expressed according to the geometric nonlinearity assumptions of von Kármán [36]:

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{pmatrix} - z \begin{pmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{pmatrix}; \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{pmatrix} = \begin{pmatrix} u_{,x} + w_{,x}^2/2 \\ v_{,y} - w/R + w_{,y}^2/2 \\ u_{,y} + v_{,x} + w_{,x}w_{,y} \end{pmatrix} \quad (3)$$

In this formulation, the displacement components are expressed in the three coordinate directions, where u , v , and w denote the displacement components along the x -, y -, and z -axes, respectively. Using Equation (3), the compatibility relation for CNTRC-MPs can be expressed as $\varepsilon_{x,yy}^0 + \varepsilon_{y,xx}^0 - \gamma_{xy,xy}^0 = w_{,xy}^2 - w_{,xx}/R - w_{,xx}w_{,yy}$, where γ_{xy}^0 represents the mid-plane shear strain, while ε_x^0 and ε_y^0 denote the mid-plane normal strains in the x - and y -directions, respectively. Also, the constitutive relations for the CNTRC-MPs are defined as follows:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}; \begin{pmatrix} Q_{11} \\ Q_{12} \\ Q_{22} \\ Q_{66} \end{pmatrix} = \begin{pmatrix} \frac{E_{11}}{\nu} \\ \frac{\nu_{21}E_{11}}{\nu} \\ \frac{E_{22}}{\nu} \\ \frac{G_{12}}{\nu} \end{pmatrix}; \nu = 1 - \nu_{12}\nu_{21} \quad (4)$$

Here, σ_x and σ_y are the normal stresses, and τ_{xy} is the shear stress. Also, $\nu_{21} = (E_{22}/E_{11})\nu_{12}$. In this regard, by integrating the stress–strain relations (Equation (4)) and

the associated stress moments through the panel thickness, the force (N_x , N_y , N_{xy}) and moment (M_x , M_y , M_{xy}) resultants of the CNTRC-MPs can be obtained.

The nonlinear equilibrium equations of the CNTRC-MPs, which are developed based on CST, can be written as follows [37,38]:

$$N_{x,x} + N_{xy,y} = 0 \quad (5a)$$

$$N_{xy,x} + N_{y,y} = 0 \quad (5b)$$

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} + N_y/R + F_1 + F_2 = \rho_1 w_{,tt} \quad (5c)$$

where $\rho_1 = \int_{-h/2}^{h/2} \rho \, dz$. In the present work, the MVAs are incorporated to suppress the NV response of the CNTRC-MPs. Therefore, by extending Equation (5), the coupled governing equations of the CNTRC-MPs fitted with symmetric MVAs under hydrodynamic loading are obtained as:

$$N_{x,x} + N_{xy,y} = 0 \quad (6a)$$

$$N_{xy,x} + N_{y,y} = 0 \quad (6b)$$

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} + N_y/R + F_1 + F_2 + \sum_{i=1}^J [C(w_{,t} - s_{i,t}) - k_L(s_i - w) - k_{NL}(s_i - w)^3] \delta_k \left(x - \frac{ia}{J+1}, y - R\pi/4 \right) = \rho_1 w_{,tt} \quad (6c)$$

$$Ms_{i,tt} + [k_L(s_i - w) + C(s_{i,t} - w_{,t}) + k_{NL}(s_i - w)^3] \delta_k \left(x - \frac{ia}{J+1}, y - R\pi/4 \right) = 0 \quad (6d)$$

This system has $J + 1$ degrees of freedom (DOF), represented by $w, s_1, s_2, \dots, s_J$. Here, w denotes the degree of freedom of the CNTRC-MPs, while $s_i (i = 1, 2, \dots, J)$ represents the degrees of freedom associated with the J vibration absorbers. Also, the term δ_k represents the Dirac delta function, which is used to define the attachment location of the MVAs on the CNTRC-MPs. It is worth clarifying explicitly what is meant by the “symmetric” arrangement of the MVAs referred to throughout this study. The attachment locations $x_i = ia/(J + 1)$, $i = 1, 2, \dots, J$, used in the Dirac-delta terms of Equation (6c,d) satisfy the mirror condition $x_i + x_{J+1-i} = a$, $i = 1, 2, \dots, J$, for any number of absorbers J . This means that the absorbers are always attached in pairs located at equal distances from the panel mid-length $x = a/2$ (for odd J , the middle absorber coincides with $x = a/2$ itself), while the circumferential coordinate $y = R\pi/4$ is kept the same for every absorber, i.e., all absorbers are attached along a single generator (the centerline of the panel in the width direction). This is the precise sense in which the MVA configuration is termed “symmetric” in this study, and it is this mirror-image placement about $x = a/2$ that is examined and validated quantitatively in Section 3.3.

Considering Equation (6a,b), the in-plane force resultants can be represented through the Airy stress function ζ as:

$$[N_x \quad N_y \quad N_{xy}] = [\zeta_{,yy} \quad \zeta_{,xx} \quad -\zeta_{,xy}] \quad (7)$$

Using the force and moment resultants together with compatibility relation, Equations (3) and (6c,d), the final governing equations of the coupled CNTRC-MPs-MVAs system are obtained as:

$$\Gamma_{11}\zeta_{,xxxx} + (\Gamma_{66} - 2\Gamma_{12})\zeta_{,xxyy} + \Gamma_{22}\zeta_{,yyyy} + \Gamma_{21}^*\zeta_{,xxxx} + (\Gamma_{11}^* + \Gamma_{22}^* - 2\Gamma_{66}^*)w_{,xxyy} + \Gamma_{12}^*w_{,yyyy} + w_{,xx}/R + [w_{,xy}^2 - w_{,xx}w_{,yy}] = 0 \quad (8a)$$

$$\begin{aligned} \rho_1 w_{,tt} + \Gamma_{11}^{**} w_{,xxx} + (\Gamma_{12}^{**} + \Gamma_{21}^{**} + 4\Gamma_{66}^{**}) w_{,xxyy} + \Gamma_{22}^{**} w_{,yyyy} - \Gamma_{21}^{*} \zeta_{,xxx} - (\Gamma_{11}^{*} + \Gamma_{22}^{*} - 2\Gamma_{66}^{*}) \zeta_{,xxyy} - \Gamma_{12}^{*} \zeta_{,yyyy} \\ - \zeta_{,xx}/R - \zeta_{,yy} w_{,xx} + 2\zeta_{,xy} w_{,xy} - \zeta_{,xx} w_{,yy} - F_1 - F_2 \\ - \sum_{i=1}^J [C(w_{,t} - s_{i,t}) - k_L(s_i - w) - k_{NL}(s_i - w)^3] \delta_k \left(x - \frac{ia}{J+1}, y - \frac{R\pi}{4} \right) = 0 \end{aligned} \quad (8b)$$

$$Ms_{i,tt} + \left[k_L(s_i - w) + C(s_{i,t} - w_{,t}) + k_{NL}(s_i - w)^3 \right] \delta_k \left(x - \frac{ia}{J+1}, y - R\pi/4 \right) = 0 \quad (8c)$$

In these equations, Γ_{ij} , Γ_{ij}^{*} , and Γ_{ij}^{**} are stiffness-related coefficients presented in the Appendix A.

2.3. Dynamic Galerkin Method

CNTRC-MPs having simply supported boundary edges and the attached symmetric MVAs are considered under hydrodynamic loading. The corresponding boundary conditions can be written as

$$\begin{aligned} w = 0, M_x = 0, N_{xy} = 0, \text{ at } x = 0; a \\ w = 0, M_y = 0, N_{xy} = 0, \text{ at } y = 0; b \end{aligned} \quad (9)$$

Based on the boundary conditions given in Equation (9), the transverse deflection of the panel is assumed in the following form:

$$w = W(t) \sin \frac{n\pi}{b} x \sin \frac{m\pi}{a} y \quad (10)$$

Here, n and m denote the number of half-waves in the y - and x -directions, respectively. It is worth clarifying the rationale behind the single-mode representation of the panel deflection adopted in Equation (10). First, for each set of geometric and material parameters examined, the half-wave numbers (m , n) are selected as those corresponding to the lowest natural frequency of the associated linear panel (see Section 2.4), i.e., the fundamental (critical) mode, which is expected to dominate the dynamic response under the spatially smooth hydrodynamic lift and drag excitation (F_1 , F_2) considered in this study. Second, the linear natural-frequency spectrum of the panel was examined over the parameter ranges considered, and no low-order internal resonance (near-integer frequency ratio) was identified between the fundamental mode and its neighboring modes; the associated energy transfer to higher-order modes through the geometric (cubic) nonlinearity is therefore expected to remain small. Third, although the MVAs exert spatially localized (pointwise) reaction forces on the panel, these forces enter the reduced-order model only through their projection onto the assumed mode shape by the Galerkin procedure (Equations (6c), (8b) and (12a)); because the J absorbers are arranged symmetrically about the panel centerline (Figure 1), the resultant excitation preserves the spatial symmetry of the fundamental mode and does not efficiently project onto antisymmetric higher-order modes. Fourth, this single-mode reduction is consistent with, and supported by, the benchmark comparisons of the panel's natural frequencies performed in Section 3.1. It should be noted that at markedly larger vibration amplitudes or for strongly asymmetric absorber arrangements, coupling with higher-order modes may become non-negligible; extending the present model to a multi-mode reduced-order formulation is identified as a direction for future work. After inserting Equation (10) into Equation (8a) and solving the resulting equation, ζ is obtained as:

$$\zeta = \zeta_1 \sin \left(\frac{m\pi}{a} x \right) \sin \left(\frac{n\pi}{b} y \right) + \zeta_2 \cos \left(2 \frac{m\pi}{a} x \right) + \zeta_3 \cos \left(2 \frac{n\pi}{b} y \right) \quad (11)$$

where ζ_i ($i = 1, 2, 3$) are provided in the Appendix A. Next, Equations (10) and (11) are substituted into Equation (8b,c). The Galerkin procedure is then employed to reduce the governing partial differential equations into a system of ordinary differential equations, leading to the following coupled nonlinear equations of motion:

$$\ddot{W} + a_1 W + a_2 W^2 + a_3 W^3 + C \sum_{i=1}^J (a_{4i} \dot{W} - a_{5i} \dot{S}_i) + k_L \sum_{i=1}^J (a_{4i} W - a_{5i} S_i) + k_{NL} \sum_{i=1}^J (a_{6i} W^3 - a_{5i} S_i^3 + 3a_{4i} S_i^2 W - a_{7i} W^2 S_i) + a_4 F_1 + a_5 F_2 = 0 \quad (12a)$$

$$\ddot{S}_i + C(b_{2i} \dot{S}_i - b_{1i} \dot{W}) + k_L(b_{2i} S_i - b_{1i} W) + k_{NL}(a_{2i} S_i^3 - b_{3i} W^3 + 3b_{4i} W^2 S_i - b_{1i} S_i^2 W) = 0; i = 1, 2, \dots, J \quad (12b)$$

where a_j ($j = 1, 2, \dots, 5$), a_{ki} ($k = 4, 5, \dots, 7$ & $i = 1, 2, \dots, J$), and b_{li} ($l = 1, 2, \dots, 4$ & $i = 1, 2, \dots, J$) are provided in the Appendix B.

2.4. Analysis of Free Vibrations

To carry out the linear free vibration investigation for the CNTRC-MPs, the effects of the MVAs and the hydrodynamic loading terms in Equation (12a) are ignored. Accordingly, the governing equation reduces to:

$$\dot{W} + a_1 W = 0 \quad (13)$$

Based on Equation (13), the natural frequency (NF) corresponding to the CNTRC-MPs can be expressed as:

$$\omega_{mn} = \sqrt{a_1} \quad (14)$$

2.5. P-T Technique

The P-T technique is adopted in this study to numerically solve Equation (12), which represents a system with $J + 1$ DOF: one DOF for the CNTRC-MP and J DOF for the MVAs. This technique is used to determine the NV responses of the system. Also, it should be explained that this method has been widely used in previous studies and has been shown to provide high accuracy and reliable numerical results compared with many conventional numerical techniques [39]. The P-T technique is formulated using Taylor series expansion combined with a piecewise constant approximation strategy. Therefore, Equation (12) can be expressed in the following equivalent form:

$$\ddot{W} + 2C_1 \dot{W} + \omega_1^2 W = \psi_1(W, S_i, \dot{S}_i, t) \quad (15a)$$

$$\ddot{S}_i + 2C_{2i} \dot{S}_i + \omega_{2i}^2 S_i = \psi_{2i}(S_i, W, \dot{W}, t) \quad (15b)$$

where

$$C_1 = \frac{C}{2} \sum_{i=1}^J a_{4i}; \quad C_{2i} = -\frac{Cb_{2i}}{2}; \quad \omega_1^2 = a_1 + k_L \sum_{i=1}^J a_{4i}; \quad \omega_{2i}^2 = -k_L b_{2i}$$

$$\psi_1(W, S_i, \dot{S}_i, t) = -\left\{ a_2 W^2 + a_3 W^3 - C \sum_{i=1}^J a_{5i} \dot{S}_i - k_L \sum_{i=1}^J a_{5i} S_i + k_{NL} \sum_{i=1}^J (a_{6i} W^3 - a_{5i} S_i^3 + 3a_{4i} S_i^2 W - a_{7i} W^2 S_i) + a_4 F_1 + a_5 F_2 \right\} \quad (16)$$

$$\psi_{2i}(S_i, W, \dot{W}, t) = -\{ -Cb_{1i} \dot{W} - k_L b_{1i} W + k_{NL}(a_{2i} S_i^3 - b_{3i} W^3 + 3b_{4i} W^2 S_i - b_{1i} S_i^2 W) \}$$

To obtain an approximate numerical solution of Equation (15) over the full time domain, the governing equations are first linearized within small time intervals. This linearization is necessary because it allows the response to be expressed analytically over each limited interval using Taylor series expansion. In the P-T procedure, the nonlinear functions ψ_1 and ψ_{2i} ($i = 1, 2, \dots, J$) are approximated using Taylor series expansion within the time interval $\frac{[Nt]}{N} \leq t < \frac{[Nt]+1}{N}$. Thus, the governing equations can be expressed as:

$$\ddot{W} + 2C_1\dot{W} + \omega_1^2 W = \psi_{1[Nt]/N} + \psi'_{1[Nt]/N} \left(t - \frac{[Nt]}{N} \right) + \frac{1}{2!} \psi''_{1[Nt]/N} \left(t - \frac{[Nt]}{N} \right)^2 + \frac{1}{3!} \psi'''_{1[Nt]/N} \left(t - \frac{[Nt]}{N} \right)^3 + \dots \quad (17a)$$

$$\ddot{S}_i + 2C_{2i}\dot{S}_i + \omega_{2i}^2 S_i = \psi_{2i[Nt]/N} + \psi'_{2i[Nt]/N} \left(t - \frac{[Nt]}{N} \right) + \frac{1}{2!} \psi''_{2i[Nt]/N} \left(t - \frac{[Nt]}{N} \right)^2 + \frac{1}{3!} \psi'''_{2i[Nt]/N} \left(t - \frac{[Nt]}{N} \right)^3 + \dots \quad (17b)$$

For the fourth-order implementation of the P-T technique, the first four Taylor expansion terms of ψ_1 and ψ_{2i} ($i = 1, 2, \dots, J$) are retained. Then, the numerical response within each interval $\frac{[Nt]}{N} \leq t < \frac{[Nt]+1}{N}$, can be obtained as follows:

$$W(t) = e^{-c_1 \left(t - \frac{[Nt]}{N} \right)} \left\{ \Lambda_{11} \cos \left[\Pi_1 \left(t - \frac{[Nt]}{N} \right) \right] + \Lambda_{12} \sin \left[\Pi_1 \left(t - \frac{[Nt]}{N} \right) \right] \right\} + \Sigma_{11} + \Sigma_{12} \left(t - \frac{[Nt]}{N} \right) + \Sigma_{13} \left(t - \frac{[Nt]}{N} \right)^2 + \Sigma_{14} \left(t - \frac{[Nt]}{N} \right)^3 \quad (18a)$$

$$S_i(t) = e^{-c_{2i} \left(t - \frac{[Nt]}{N} \right)} \left\{ \Lambda_{i21} \cos \left[\Pi_{i2} \left(t - \frac{[Nt]}{N} \right) \right] + \Lambda_{i22} \sin \left[\Pi_{i2} \left(t - \frac{[Nt]}{N} \right) \right] \right\} + \Sigma_{i21} + \Sigma_{i22} \left(t - \frac{[Nt]}{N} \right) + \Sigma_{i23} \left(t - \frac{[Nt]}{N} \right)^2 + \Sigma_{i24} \left(t - \frac{[Nt]}{N} \right)^3 \quad (18b)$$

where the coefficients involved in Equation (18) are presented in the Appendix C. It should be clarified that solving Equation (18) enables the NV response of CNTRC-MPs equipped with symmetric MVAs under hydrodynamic loading to be obtained.

Table 1. Hydrodynamic lift and drag forces under different flow velocities [40].

V (m/s)	Lift (N)	Drag (N)
10	110,537	5550.8
10.4	114,204	6165.6
10.9	121,090	5130.6
12	129,529	4416.1

3. Numerical Results

In the present section, the present results are first verified against available results in the literature. Then, the impacts of the MVAs on reducing the NV of CNTRC-MPs subjected to hydrodynamic loads are addressed. Although the present study is mainly based on numerical simulations, the simulation parameters were carefully selected to closely represent realistic marine operating conditions. In particular, the velocity-dependent hydrodynamic forces, including lift and drag, were determined through Matveev's experimentally based equations [37] for a particular ship with a width of 3.4 m, a length of 16.4 m, and a displaced water mass of 12,000 kg [40]. The corresponding hydrodynamic force values are reported in Table 1. Therefore, the proposed model provides a reliable basis for evaluating the performance of the MVAs in marine structures subjected to dynamic hydrodynamic loading.

In the work, Poly(m-phenylenevinylene)-co-[(2,5-dioctoxy-p-phenylene) vinylene] (PmPV) is selected as the isotropic matrix material. The matrix has a Poisson's ratio of $\nu^M = 0.34$ and an elastic modulus of $E^M = 2.1$ GPa. As mentioned previously, three CNT distribution schemes, including FG-V, FG-X, and the UD case, are examined. The CNTs are taken to be armchair (10, 10) single-walled CNTs, with a thickness, average radius,

and length of 0.067 nm, 0.68 nm, and 9.26 nm, respectively [41]. Their mechanical properties are taken as $E_{11}^{CNT} = 5.6466$ TPa, $E_{22}^{CNT} = 7.08$ TPa, and $G_{12}^{CNT} = 1.9445$ TPa [41]. All material properties of the CNTs and the polymer matrix are considered at room temperature, $T = 300$ K. The CNT efficiency parameters $\eta_i (i = 1, 2, 3)$, associated with the considered CNT volume fractions, are listed in Table 2. Unless mentioned otherwise, the following material and geometrical parameters are used in numerical analysis:

$$R/h = 250; a/b = 1; a/R = 1.5; m = 1; n = 7 \quad (19)$$

$$M = 0.5 \text{ kg}; C = 20 \text{ Ns/m}; k_L = 3 \times 10^2 \text{ N/m}; k_{NL} = 4 \times 10^3 \text{ N/m}^5$$

To quantify the added-mass effect of the multiple vibration absorbers (MVAs), the total absorber-to-panel mass ratio is defined as:

$$\mu = \frac{J M}{M_{panel}} \times 100\%, \quad M_{panel} = \rho_1 a b \quad (20)$$

where M_{panel} is the total mass of the CNTRC panel, obtained from the area mass density ρ_1 (Equation (5c)) and the in-plane panel dimensions a and b . For the geometric and material parameters considered in this study, $M_{panel} = 4.0732$ kg. Therefore, the absorber-to-panel mass ratios for $J = 1$ to 6 absorbers are obtained as $\mu_1 = 12.28\%$, $\mu_2 = 24.56\%$, $\mu_3 = 36.83\%$, $\mu_4 = 49.11\%$, $\mu_5 = 61.39\%$, and $\mu_6 = 73.65\%$, respectively. Although increasing the number of absorbers enhances vibration suppression capability, the associated increase in structural mass should be considered in practical applications. Therefore, an appropriate trade-off between vibration reduction performance and added absorber mass is required for the optimal design of MVA systems.

Table 2. Efficiency factors for different total CNT volume fractions.

V_{CNT}^*	η_3	η_2	η_1
0.12	0.715	1.022	0.137
0.17	1.138	1.626	0.142
0.28	1.109	1.585	0.141

3.1. Verification of the Present Research

The reliability of the present formulation is first assessed based on a comparison of the calculated dimensionless NFs with benchmark data available in previous studies. As reported in Table 3, the current results are in close agreement with the data provided by Matsunaga [42], Chorfi and Houmat [43], and Alijani et al. [44]. In this study, Young's modulus is obtained as $E = E_m + (E_c - E_m)((2z + h)/2h)^k$, and the dimensionless NF is expressed as $\omega_{mn} = \omega_{mn} h \sqrt{\frac{\rho_c}{E_c}}$. To further verify the reliability of the model, an additional validation is conducted for isotropic cylindrical panels. The NFs listed in Table 4 are compared with the published results of Deb Nath [45], Soedel [46], Bardell and Mead [47], Chern and Chao [48], and Shen and Xiang [49]. The close correspondence observed in these comparisons confirms the reliability and precision of the proposed approach.

Table 3. Comparison of dimensionless NFs of FG cylindrical panel ($h/a = 0.1, a/b = 1, a/R = 0.5, E_m = 7 \times 10^{10} \text{ N/m}^2, E_c = \rho_m = 2796 \text{ kg/m}^3, 38 \times 10^{10} \text{ N/m}^2, \rho_c = 3800 \text{ kg/m}^3$).

k^*	Present	Matsunaga [42]	Chorfi and Houmat [43]	Alijani et al. [44]
0	0.065	0.062	0.063	0.065
1	0.050	0.049	0.049	0.050
10	0.041	0.039	0.040	0.041

* Volume-fraction index.

Table 4. Comparison of NFs (Hz) of isotropic cylindrical panels ($a/b = 1.22$, $h = 1.2190$ mm, $\rho = 2800$ kg/m³, $E = 7.2 \times 10^{10}$ N/m², $m = 1$, $\nu = 1/3$).

$R(m)$	Present	Deb Nath [45]	Soedel [46]	Bardell and Mead [47]	Chern and Chao [48]	Shen and Xiang [49]
1.2192	280.6 (2 *)	282.3 (2)	282.3 (2)	281.8 (2)	278.1 (2)	281.9 (2)
1.8288	199.9 (1)	201.7 (1)	201.4 (1)	200.8 (1)	198.5 (1)	200.8 (1)
2.4384	162.2 (1)	163.9 (1)	163.7 (1)	163.1 (1)	161.4 (1)	163.2 (1)

* Number of half-waves (n).

As an additional validation of the numerical solution procedure, the NV responses of the CNTRC-MPs obtained using the P-T method are further verified against those obtained through the fourth-order Runge–Kutta (R-K) method. Figure 3 presents a comparison of the NV time-history responses predicted by both approaches. In this comparison, the P-T method is implemented with a time step of 0.01 s, whereas the R-K method, owing to its explicit nature, requires a considerably smaller time step of 0.001 s to maintain numerical stability and accuracy. The results obtained using the P-T method with the larger time step of 0.01 s are in close agreement with the more refined R-K solution obtained using the smaller time step of 0.001 s. It is worth noting that, because of the significantly smaller time step required by the R-K method, the P-T method may exhibit slightly superior computational efficiency even while employing a considerably larger time step, further confirming its suitability for solving the nonlinear governing equations of the present model.

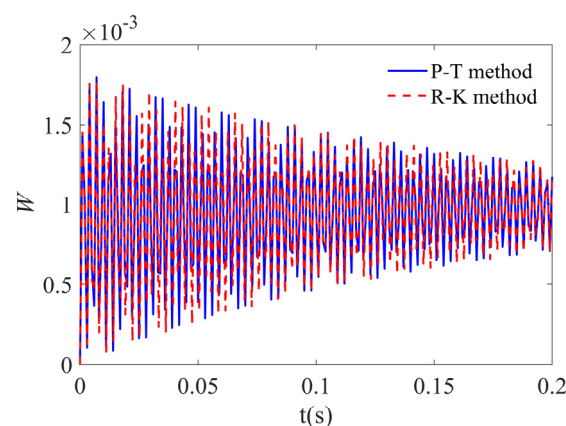


Figure 3. Comparison of NV responses of CNTRC-MPs obtained using the P-T and R-K methods.

3.2. NV Responses of CNTRC-MPs Fitted with Symmetric MVAs Under Hydrodynamic Loading

In this subsection, the effects of MVAs on reducing the NV of CNTRC-MPs subjected to hydrodynamic loads are investigated. In this regard, Equation (12) is solved subject to the initial conditions $W(0) = 0$, $\dot{W}(0) = 0$, and $S_i(0) = 0$, $\dot{S}_i(0) = 0$ for $i = 1, 2, \dots, J$, i.e., the panel and the absorbers are assumed to be undeformed and at rest prior to the onset of the hydrodynamic loading. Because F_1 and F_2 appear explicitly on the right-hand side of Equation (12a) as externally applied forcing terms, the resulting time-domain response constitutes the forced transient response of the coupled CNTRC-MPs-MVAs system to a suddenly applied hydrodynamic load, rather than a free-vibration decay from an arbitrary initial displacement. The MVAs are symmetrically distributed along the centerline of the panel. This symmetric arrangement is advantageous because it takes advantage of the panel's dynamic behavior and improves the absorbers' capacity for absorbing and dissipating vibrational energy. Consequently, the largest deflection of the CNTRC-MP caused by hydrodynamic loading is considerably reduced, leading to improved structural

performance, durability, and service life in marine applications. Unless otherwise specified, the flow velocity in this study is considered to be $V = 10.9$ m/s.

The vibration-suppression mechanism of the MVAs can be understood directly from the coupled equations of motion, Equation (12a,b). The panel and each absorber are connected only through the relative displacement ($W - S_i$) and relative velocity ($\dot{W} - \dot{S}_i$) terms, weighted by the linear stiffness k_L , the nonlinear (cubic) stiffness k_{NL} , and the damping coefficient C . Physically, this means that vibrational energy is not removed from the system directly; rather, it is transferred from the panel to the absorbers whenever the panel and an absorber move out of phase, i.e., whenever $(W - S_i) \neq 0$. Once this energy has migrated into the absorber degree of freedom (S_i), it is progressively dissipated through the damping term $C(\dot{S}_i - \dot{W})$, rather than being retained within the panel itself. This is analogous to the operating principle of a nonlinear tuned-mass/energy-sink system: the absorber acts as an auxiliary energy reservoir that intermittently exchanges energy with the primary structure and then dissipates it, thereby lowering the net vibration amplitude experienced by the panel. The efficiency of this exchange depends on how strongly the panel and absorber are coupled, which is controlled by k_L , k_{NL} , and C , and on how many independent energy-exchange pathways exist, which is controlled by the number of absorbers (J). This mechanism underlies all of the parametric trends discussed in the remainder of this subsection.

It is worth clarifying the physical interpretation of the time-domain responses presented in Figures 4–11. Since the hydrodynamic lift and drag forces are steady for a given flow velocity, the panel and MVAs start from rest and are subjected to this load as a step-type input; the plotted responses therefore show the transient forced oscillation of the coupled CNTRC-MPs-MVAs system as it moves from its undeformed state toward a new nonlinear equilibrium deflection under the applied hydrodynamic load, with the vibration absorbers acting to suppress the transient overshoot and amplitude of this forced response.

Figure 4 presents the influence of the vibration absorber on reducing the NV response of CNTRC-MPs made of UD-CNTs and subjected to hydrodynamic lift and drag forces. Without the absorber, the panel response in Figure 4 does not decay toward the unloaded position $W = 0$; instead, it exhibits a sustained oscillation centered on a nonzero deflection level, corresponding to the static equilibrium produced by the hydrodynamic load $F_1 + F_2$. This behavior, together with the initial conditions stated above, confirms that the response is a forced vibration and not a free-vibration decay, since a free response would necessarily oscillate about, or decay toward, the unloaded equilibrium. With the absorber attached, this same load-shifted oscillation is considerably reduced, so applying a vibration absorber to CNTRC-MPs can effectively dissipate vibrational energy, which may ultimately enhance the durability and long-term serviceability of marine structures. As explained above, this reduction occurs because the coupling terms $k_L(S_i - W)$, $k_{NL}(S_i - W)^3$, and $C(\dot{S}_i - \dot{W})$ in Equation (12a,b) transfer part of the vibrational energy from the panel's degree of freedom W into the absorber's degree of freedom S_i , where it is dissipated; without the absorber, this pathway does not exist, and the input energy must be dissipated by the panel's own inherent damping alone.

Figure 5 demonstrates the influence of the number of vibration absorbers (J) on the NV response of CNTRC-MPs made of UD-CNTs. As shown in the figure, increasing the number of vibration absorbers enhances the vibration suppression performance of the system. In particular, when more absorbers are attached to the CNTRC-MPs, the maximum deflection decreases noticeably. The figure also presents the time-domain response curves of the CNTRC-MPs as well as those of each vibration absorber. This reduction in deflection occurs because the absorbers transfer and dissipate part of the vibrational energy from the main CNTRC-MPs, thereby reducing the overall vibration amplitude.

Therefore, employing a larger number of vibration absorbers can be an effective strategy for controlling the nonlinear deflection response of CNTRC-MPs. Based on this observation, the subsequent results consider the effect of six vibration absorbers ($J = 6$) on the NV response of the CNTRC-MPs. This trend is consistent with the summation structure of the coupling terms in Equation (12a), $\sum_{i=1}^J [\mathcal{C}(\dot{W} - \dot{S}_i) + k_L(W - S_i) + k_{NL}(\dots)]$, in which each additional absorber contributes an independent energy-exchange pathway between the panel and its own degree of freedom S_i . Increasing J therefore increases the number of simultaneous channels through which vibrational energy can leave the panel, so that a larger share of the total energy input from the hydrodynamic forces is redistributed among the absorbers and dissipated, rather than remaining concentrated in the panel's own response. This explains why the maximum deflection decreases progressively as more absorbers are added, up to the six-absorber configuration considered in the remainder of the study.

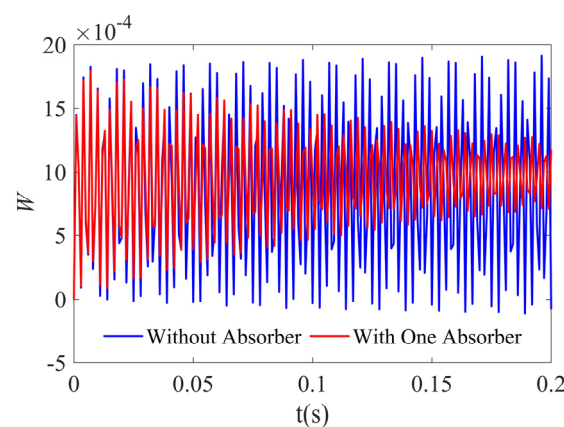


Figure 4. Influence of vibration absorber on the NV response of CNTRC-MPs.

Figure 6 demonstrates the impact of MVAs on reducing the NV behavior of CNTRC-MPs made with three different CNT distribution patterns and subjected to hydrodynamic lift and drag forces. As observed, the use of MVAs leads to a considerable reduction in the NV amplitude of the CNTRC-MPs, especially for the panels reinforced with FG-V-CNTs. This indicates that the interaction between the absorber system and the FG-V-CNT distribution is more effective in suppressing the vibration response. In addition, the results show that the MVAs reduce the NV amplitude of the CNTRC-MPs with UD-CNTs more effectively than those with FG-X-CNTs. This difference can be related to the way CNTs are distributed through the panel thickness, which directly affects the stiffness, mass distribution, and energy transfer mechanism of the structure. The relatively high vibration amplitude observed in the CNTRC-MPs can be attributed to the combined effects of dynamic hydrodynamic loading, material inhomogeneity caused by the nonuniform distribution of CNTs and functional grading, and geometric nonlinearity. The coupling of these effects produces a complex NV behavior, particularly under varying flow-induced loading conditions.

The influence of MVAs on mitigating the NV behavior of CNTRC-MPs with various total CNT volume fractions (V_{CNT}^*) under hydrodynamic lift and drag forces is presented in Figure 7. As can be observed, increasing V_{CNT}^* enhances the effectiveness of the MVAs in reducing the NV amplitude of the CNTRC-MPs. In other words, the vibration amplitude is suppressed more rapidly when $V_{CNT}^* = 0.28$ compared with $V_{CNT}^* = 0.17$, and the reduction for $V_{CNT}^* = 0.17$ is faster than that for $V_{CNT}^* = 0.12$. This indicates that a higher CNT content improves the vibration-control performance of the MVAs under hydrodynamic loading.

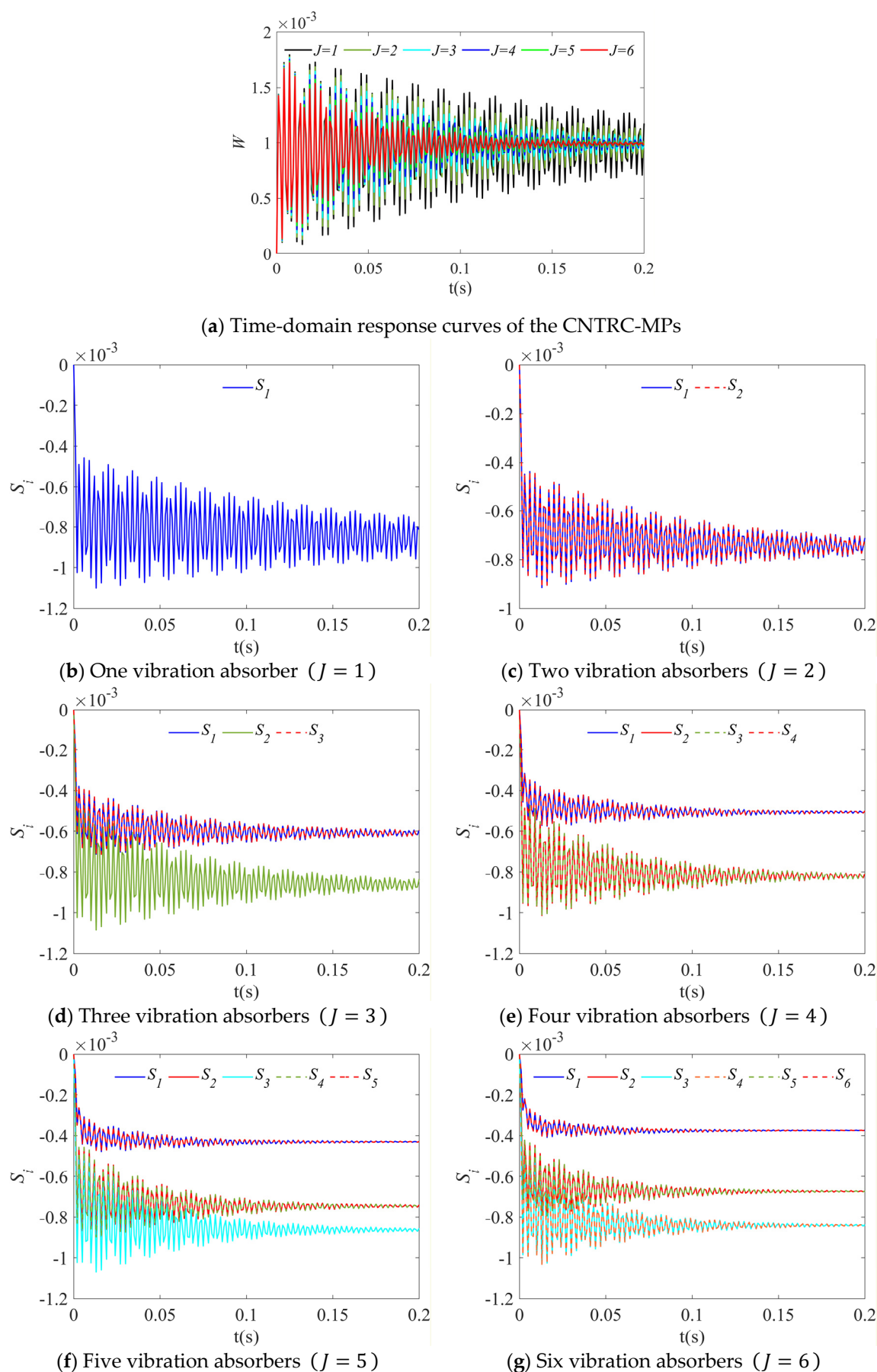


Figure 5. Effect of number of vibration absorbers on the NV response of CNTRC-MPs.

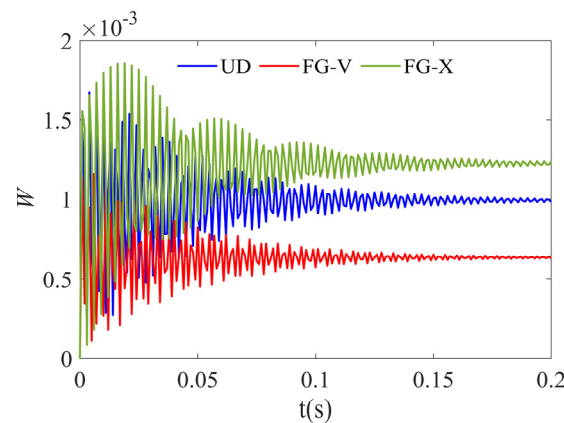


Figure 6. Impact of MVAs on the NV behavior of CNTRC-MPs with various CNT distribution patterns.

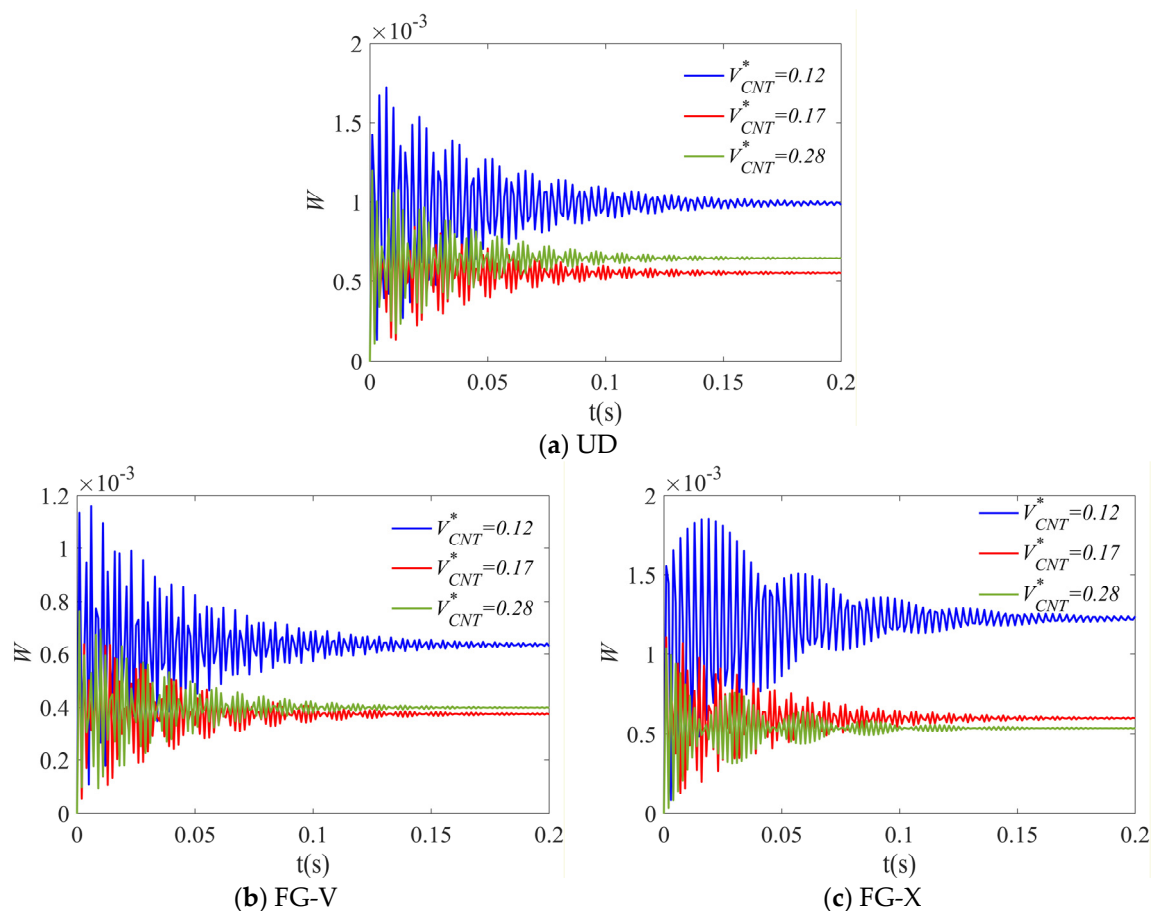


Figure 7. Impact of MVAs on the NV response of CNTRC-MPs for various CNT volume fractions.

Figure 8 illustrates the impact of the MVAs on reducing the NV response of CNTRC-MPs subjected to hydrodynamic lift and drag loads at different flow velocities. As observed from the results, the vibration amplitude of the CNTRC-MPs is noticeably decreased when the MVAs are incorporated into the system. This indicates that the MVAs can absorb and dissipate a significant portion of the vibrational energy generated by fluid-induced forces, leading to a more stable dynamic response. Such vibration reduction is particularly important for marine structures, where panels are continuously exposed to hydrodynamic loading during operation. Therefore, the present findings confirm that the MVAs can serve as an effective passive vibration control device for CNTRC-MPs, helping

to improve structural durability, extend service life, reduce fatigue-related damage, and decrease the need for frequent inspection and maintenance.

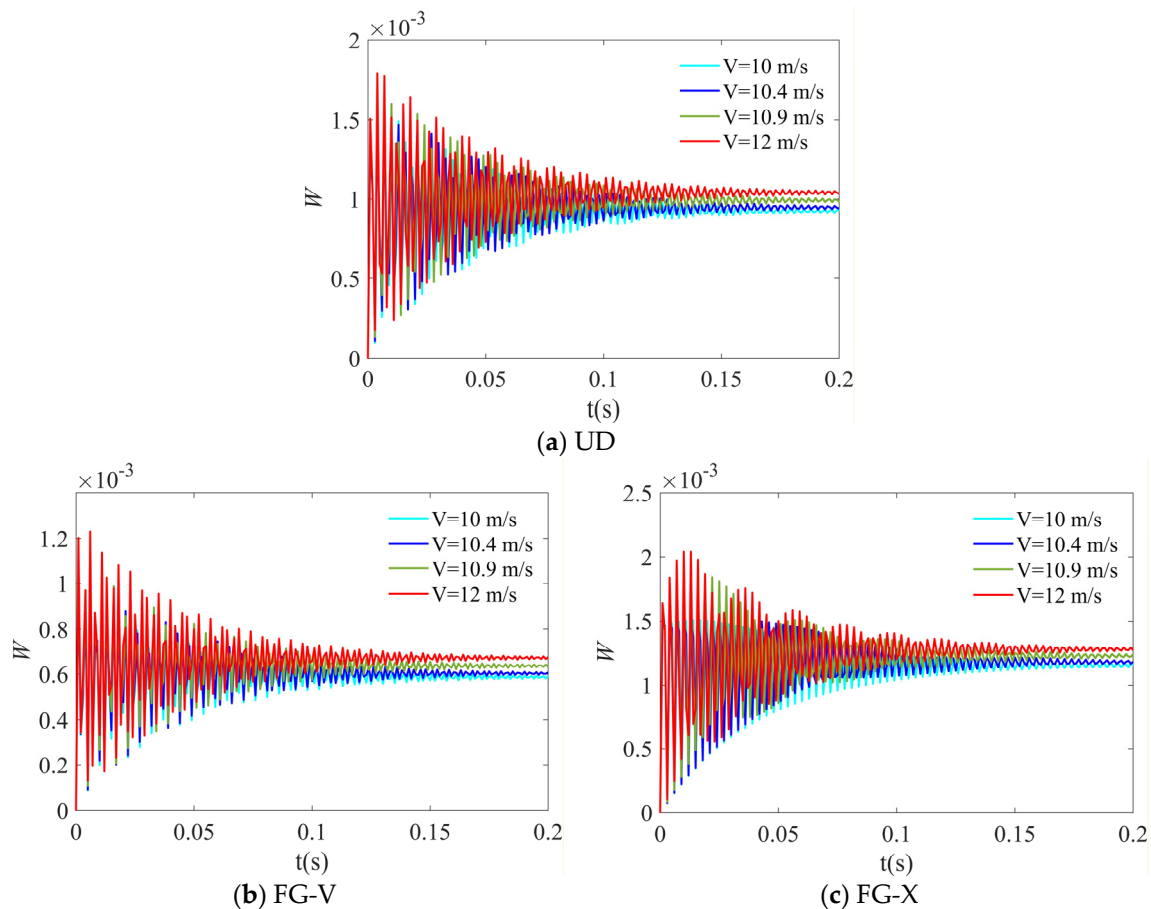


Figure 8. Impact of the MVAs on the vibration reduction of CNTRC-MPs at different flow velocities.

The impact of different linear spring stiffness values (k_L) on the NV response of CNTRC-MPs subjected to hydrodynamic lift and drag loading is analyzed in Figure 9. The findings reveal that the maximum deflection of the CNTRC-MPs depends strongly on the selected value of the linear spring stiffness. When the spring stiffness is either lower or higher than $k_L = 3 \times 10^4$ N/m, the maximum deflection of the system increases. This indicates that both very soft and very stiff linear springs are less effective in controlling the vibration amplitude. Therefore, among the examined stiffness values, $k_L = 3 \times 10^4$ N/m can be considered the most effective case. At this value, the linear spring provides a better balance between flexibility and resistance, leading to a reduction in the largest deflection of the CNTRC-MPs exposed to lift and drag hydrodynamic loads. Thus, this stiffness value is selected as the optimum case for improving the vibration performance of the system. This non-monotonic trend can be explained through the coupling term $k_L(S_i - W)$ in Equation (12a,b). When k_L is too small, the panel and absorbers are only weakly coupled, so little energy is transferred from the panel into the absorber DOF, and the absorber essentially fails to engage with the panel's motion. Conversely, when k_L is too large, the absorber is forced to move almost synchronously with the panel, driving the relative displacement ($W - S_i$) toward zero; since energy transfer depends precisely on this relative motion, an overly stiff coupling suppresses the absorber's ability to draw energy away from the panel. The intermediate value $k_L = 3 \times 10^4$ N/m therefore represents the coupling strength at which the relative motion between panel and absorber, and hence the associated energy transfer, is maximized.

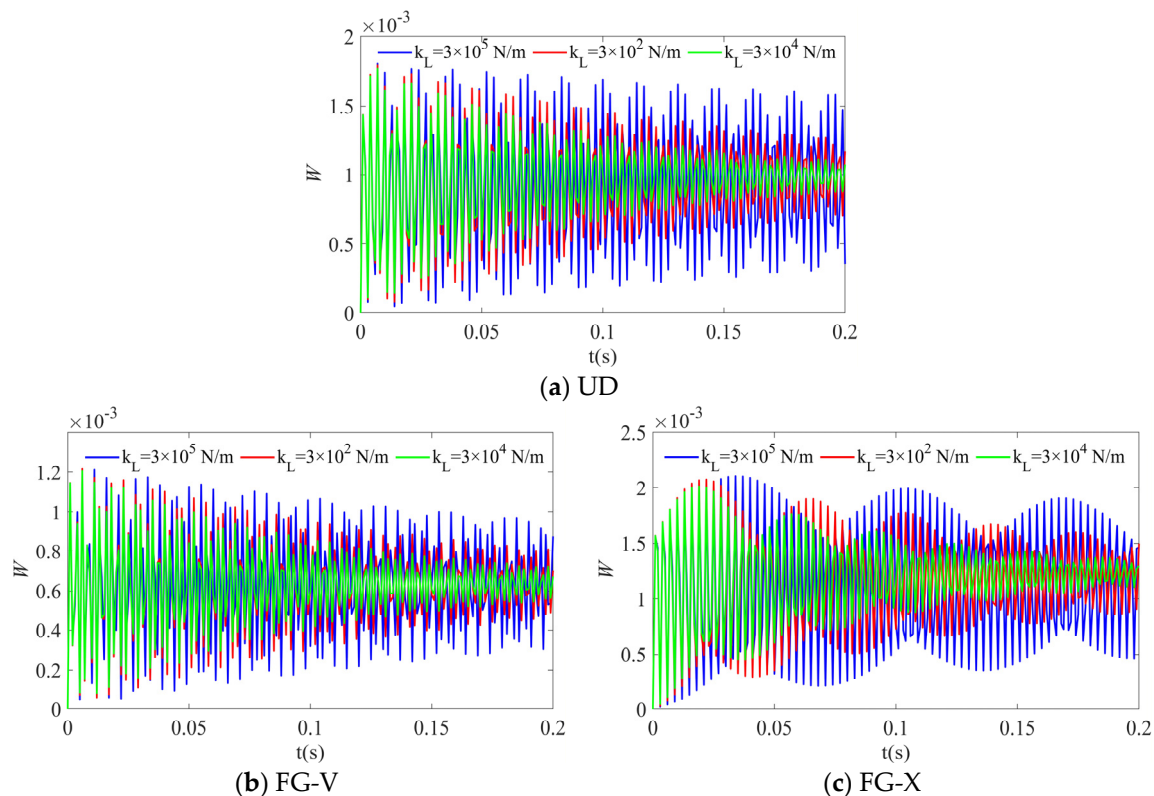


Figure 9. Impact of linear spring stiffness on the NV response of CNTRC-MPs.

The impact of different damping coefficient values (C) on the NV response of CNTRC-MPs subjected to lift and drag hydrodynamic loads is studied in Figure 10. The findings reveal that the damping coefficient has an important role in controlling the vibration amplitude of the system. By increasing or decreasing the damping coefficient from the effective range, the maximum deflection of the CNTRC-MPs may not be reduced efficiently. Among the examined values, a damping coefficient of approximately $C = 20$ Ns/m is identified as the most effective case. At this value, the damping mechanism can dissipate the vibration energy more efficiently, leading to a noticeable reduction in the largest deflection of the CNTRC-MPs exposed to lift and drag hydrodynamic loads. Therefore, $C = 20$ Ns/m can be considered the best damping coefficient for improving the NV performance of the system, consistent with the trend discussed in the previous paragraph. This behavior follows from the dissipative term $C(\dot{S}_i - \dot{W})$ in the coupled equations of motion. If C is too small, energy that has already been transferred into the absorber DOF is dissipated slowly, allowing it to be re-transferred back into the panel and sustaining the vibration. If C is too large, the absorber's relative velocity ($\dot{S}_i - \dot{W}$) is strongly suppressed, which restricts the absorber's motion and limits how much energy can be extracted from the panel in the first place. The identified optimal value, $C \approx 20$ Ns/m, therefore reflects the damping level at which energy is both effectively extracted from the panel and efficiently dissipated within the absorber, rather than simply the level that maximizes damping force.

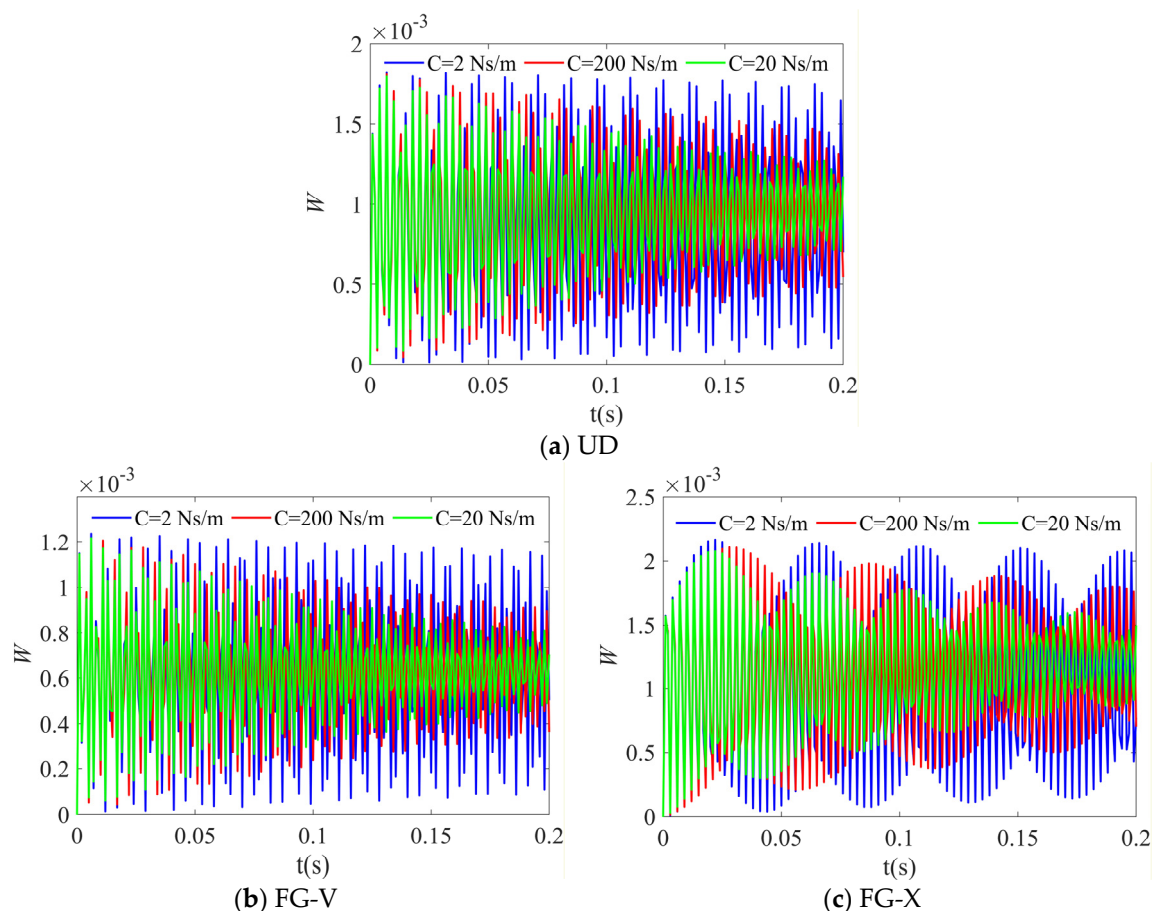


Figure 10. Impact of damping coefficient on the NV response of CNTRC-MPs.

3.3. Effect of Symmetric vs. Asymmetric MVA Arrangement

To quantitatively justify the symmetric placement of the MVAs adopted in this study, the NV response of the CNTRC-MP is compared for two configurations with the same number of absorbers ($J = 4$) and identical absorber parameters (M , C , k_L , k_{NL}): (i) the symmetric configuration used throughout this paper, with attachment locations $x_i = ia/(J + 1)$ satisfying $x_i + x_{J+1-i} = a$; and (ii) an asymmetric configuration in which the same four absorbers are shifted toward one end of the panel (e.g., $x_1, x_2, x_3, x_4 = 0.10a, 0.20a, 0.30a, 0.40a$), so that the mirror condition above is no longer satisfied, while the total absorber mass and all other system parameters remain unchanged.

Figure 11 presents the time-domain deflection response of the CNTRC-MP for both configurations. The results show that the symmetric arrangement reduces the maximum NV amplitude of the panel by 43.6% relative to the panel without absorbers, whereas the asymmetric arrangement achieves only a 41.0% reduction under the same hydrodynamic loading conditions. This confirms that redistributing the same absorbers asymmetrically noticeably degrades the vibration-suppression performance.

This behavior can be explained physically as follows. Because the panel deflection is represented in the Galerkin reduction by a single dominant mode shape that is itself symmetric about the panel mid-length $x = a/2$ (i.e., it attains its maximum amplitude at the center and decays symmetrically toward the supported edges), attaching the absorbers in mirror-image pairs about $x = a/2$ ensures that each pair samples an equal modal displacement amplitude. Consequently, the absorbers act in a balanced, coordinated manner to extract and dissipate vibrational energy from the panel over its entire span. In contrast, when the absorbers are clustered asymmetrically (e.g., toward one end), the absorbers sample unequal, generally smaller modal amplitudes at their respective attachment

points, so their combined energy-dissipation capacity is reduced, and the panel exhibits a larger residual vibration amplitude, particularly near the end away from the absorber cluster.

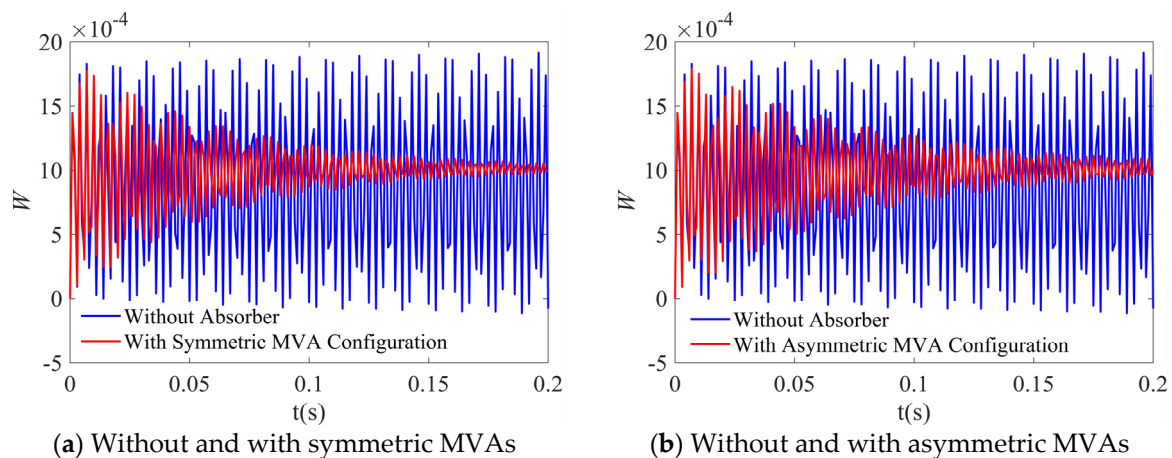


Figure 11. Effect of symmetric vs. asymmetric MVA arrangement on the NV response of CNTRC-MPs ($J = 4$).

3.4. Practical Implications: Cost and Manufacturing Feasibility

Beyond the numerical performance of the proposed symmetric MVA configuration, it is important to consider the practical implications of implementing such a system in real marine structures. From a manufacturing standpoint, CNTRC panels with UD, FG-V, or FG-X through-thickness CNT distributions can be produced using established composite-fabrication routes, such as vacuum-assisted resin transfer molding or filament winding, combined with layer-wise or centrifugal-casting deposition techniques to achieve the required CNT gradation. These processes are already used industrially for fiber-reinforced marine composites, indicating that the functionally graded CNT configurations examined in this study are manufacturable at scale rather than being purely idealized constructs.

With respect to the MVAs themselves, each unit consists only of a lumped mass together with linear and nonlinear springs and a damper, all of which can be assembled from commercially available mechanical components rather than custom-engineered parts. This keeps unit fabrication cost, lead time, and design complexity low relative to active vibration-control alternatives that require piezoelectric actuators, sensors, and control electronics. Because the MVA is a purely passive device, it also eliminates the recurring costs, power requirements, and potential points of electrical failure associated with active systems, an important consideration for equipment operating continuously in a marine environment.

The symmetric centerline attachment pattern adopted in this study is also compatible with surface-mounted installation, meaning the absorbers can, in principle, be retrofitted onto existing panels without extensive structural modification. Finally, since the results in Section 3.2 show that the MVAs substantially reduce the maximum nonlinear vibration amplitude of the CNTRC-MPs, a corresponding reduction in fatigue accumulation and inspection/maintenance frequency can be expected over the service life of the structure. Although the absorbers introduce a modest upfront material and installation cost, this is expected to be offset over time by reduced maintenance and extended service life, giving the proposed configuration a favorable life-cycle cost profile for marine applications.

4. Conclusions

This study examines how MVAs can reduce the NV response of CNTRC-MPs subjected to hydrodynamic lift and drag forces. The nonlinear governing equations are derived using CST together with the Galerkin technique. Then, the P-T technique is applied to obtain the NV response of CNTRC-MPs fitted with symmetric MVAs under hydrodynamic loading. The results highlight the capability of the MVAs to effectively reduce vibration effects in marine structures. These findings also suggest that vibration absorber systems could be practically used in real marine applications. Incorporating MVAs into marine panel designs can help engineers improve structural durability and extend the service life of such systems. Furthermore, this work offers a useful foundation for future research on different vibration absorber configurations and materials, which may lead to more efficient and adaptable vibration control strategies. From a sustainability perspective, the use of MVAs may also help reduce maintenance costs and improve energy efficiency in marine structures. While the accuracy of the proposed model has been confirmed through comparison with benchmark numerical and analytical results available in the literature, experimental validation data for CNTRC marine panels integrated with multiple vibration absorbers are not yet available. As part of future work, the authors intend to conduct experimental testing on fabricated CNTRC panel specimens equipped with tuned vibration absorbers under controlled hydrodynamic loading conditions in order to further validate the natural frequency predictions and nonlinear vibration responses obtained in this study. The key findings of the study are summarized as follows:

- The incorporation of vibration absorbers significantly reduces the NV amplitude of CNTRC marine panels subjected to hydrodynamic lift and drag forces. This confirms that the proposed absorber system can effectively dissipate vibration energy and improve the dynamic stability, durability, and operational lifetime of marine structures.
- Increasing the number of multiple vibration absorbers enhances the vibration suppression performance of the CNTRC-MPs. The results show that using six absorbers provides an effective configuration for reducing the maximum nonlinear deflection by transferring part of the vibrational energy from the main panel to the absorbers.
- The vibration-mitigation performance of the MVAs is strongly affected by the CNT distribution pattern and CNT volume fraction. Among the considered patterns, the FG-V-CNT distribution shows the most effective vibration suppression, while higher CNT volume fractions lead to faster and greater reductions in the nonlinear vibration amplitude.
- The absorber parameters play a key role in controlling the nonlinear response of the system. Among the examined cases, the linear spring stiffness of $k_L = 3 \times 10^4$ N/m and damping coefficient of $C = 20$ Ns/m provide the best vibration mitigation performance, indicating that proper tuning of the absorber is essential for achieving optimal vibration control.

While the accuracy of the proposed model has been confirmed through comparison with benchmark numerical and analytical results available in the literature, experimental validation data for CNTRC marine panels integrated with multiple vibration absorbers are not yet available.

It should also be noted that the present formulation employs a single-mode Galerkin reduction for the panel deflection, justified in Section 2.3 on the basis of the dominance of the fundamental mode under the hydrodynamic loading considered, the absence of low-order internal resonance, and the symmetric arrangement of the MVAs. For larger-amplitude vibrations or asymmetric absorber configurations, higher-order modal interactions may become significant, and extending the present approach to a multi-mode reduced-order model represents a promising direction for future research. In addition, the present

formulation is based on CST, which is well suited to the thin CNTRC marine panel considered in this study ($R/h = 250$), where membrane and bending effects dominate the nonlinear vibration response. The present MVA-based control strategy, together with the CST-based reduced-order formulation, thus provides an accurate and computationally efficient framework for this class of thin-shell panels, and can be readily extended to other CNTRC panel and shell configurations, hydrodynamic loading scenarios, and absorber arrangements in future research.

Two further modeling simplifications should also be acknowledged. First, the hydrodynamic loading was evaluated under a steady flow velocity assumption, following Matveev's experimentally supported formulation; unsteady effects such as gusts, vortex shedding, or maneuvering-induced velocity fluctuations were not considered and may alter the transient nonlinear vibration response in real sea states. Second, the analysis was restricted to simply supported boundary conditions, which, although representative of many practical panel supports, do not capture the influence of alternative edge conditions, such as clamped or elastically restrained supports, on the coupled panel-absorber dynamics. Extending the present framework to unsteady/stochastic hydrodynamic loading and to a broader range of boundary conditions therefore represents an important direction for future research, complementing the multi-mode extension and experimental validation already noted above.

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Abbreviations

The following abbreviations are used in this manuscript:

NV	Nonlinear vibration
CNTRC	Carbon nanotube-reinforced composite
MPs	Marine panels
MVAs	Multi-vibration absorbers
CNTs	Carbon nanotubes
UD	Uniform distribution
CST	Classical shell theory
DOF	Degrees of freedom

Appendix A

The coefficients Γ_{ij} , Γ_{ij}^* , and Γ_{ij}^{**} in Equation (8) are as follows:

$$\begin{aligned}\Gamma_{ij} &= \frac{A_{ij}}{A_{11}A_{22} - A_{12}^2} \quad (i, j = 1, 2); \quad \Gamma_{66} = \frac{1}{A_{66}} \\ \Gamma_{11}^* &= \Gamma_{22}B_{11} - \Gamma_{12}B_{12}; \quad \Gamma_{22}^* = \Gamma_{11}B_{22} - \Gamma_{12}B_{12}; \quad \Gamma_{12}^* = \Gamma_{22}B_{12} - \Gamma_{12}B_{22} \\ \Gamma_{21}^* &= \Gamma_{11}B_{12} - \Gamma_{12}B_{11}; \quad \Gamma_{66}^* = \frac{B_{66}}{A_{66}}; \quad \Gamma_{11}^{**} = D_{11} - B_{11}\Gamma_{11}^* - B_{12}\Gamma_{21}^*\end{aligned}\tag{A1}$$

$$\begin{aligned}\Gamma_{22}^{**} &= D_{22} - B_{12}\Gamma_{12}^* - B_{22}\Gamma_{22}^*; \Gamma_{12}^{**} = D_{12} - B_{11}\Gamma_{12}^* - B_{12}\Gamma_{22}^* \\ \Gamma_{21}^{**} &= D_{12} - B_{12}\Gamma_{11}^* - B_{22}\Gamma_{21}^*; \Gamma_{66}^{**} = D_{66} - B_{66}\Gamma_{66}^*\end{aligned}$$

in which

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2) dz; i, j = 1, 2, 6 \quad (\text{A2})$$

The coefficients ζ_i ($i = 1, 2, 3$) in Equation (11) are as follows:

$$\zeta_1 = \frac{a^2 n^2}{32\Gamma_{11} b^2 m^2} W^2; \quad \zeta_2 = \frac{b^2 m^2}{32\Gamma_{22} a^2 n^2} W^2; \quad \zeta_3 = \frac{\Gamma_1}{\Gamma_2} W \quad (\text{A3})$$

in which

$$\begin{aligned}\Gamma_1 &= \Gamma_{21}^* \left(\frac{m\pi}{a}\right)^4 + (\Gamma_{11}^* + \Gamma_{22}^* - 2\Gamma_{66}^*) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + \Gamma_{12}^* \left(\frac{n\pi}{b}\right)^4 - \left(\frac{m\pi}{a}\right)^2 / R \\ \Gamma_2 &= \Gamma_{11} \left(\frac{m\pi}{a}\right)^4 + (\Gamma_{66} - 2\Gamma_{12}) \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + \Gamma_{22} \left(\frac{n\pi}{b}\right)^4\end{aligned} \quad (\text{A4})$$

Appendix B

The coefficients a_j ($j = 1, 2, \dots, 5$), a_{ki} ($k = 4, 5, \dots, 7$ & $i = 1, 2, \dots, J$), and b_{li} ($l = 1, 2, \dots, 4$ & $i = 1, 2, \dots, J$) in Equation (12) are as follows:

$$\begin{aligned}a_1 &= \frac{\pi^2}{a^4 b^4 R \Gamma_{22} \rho_1} \{ m^2 (m^2 R (\Gamma_{21}^{**} + \Gamma_{11}^*) \pi^2 - \Gamma_1 a^2) b^4 \\ &\quad + m^2 R n^2 a^2 \pi^2 ((\Gamma_{12}^{**} + \Gamma_{21}^{**} + 4\Gamma_{66}^{**}) \Gamma_2 + \Gamma_1 (\Gamma_{11}^* + \Gamma_{22}^* - 2\Gamma_{66}^*)) b^2 + n^4 \pi^2 R a^4 (\Gamma_{22}^{**} + \Gamma_{12}^*) \} \\ a_2 &= \frac{1}{a^4 b^4 R m \pi^2 n \Gamma_{11} \Gamma_{22} \rho_1} \left\{ \frac{2a^2 n^2 \left(m^2 R ((\Gamma_{21}^* + 4\Gamma_{11}) \Gamma_{22} + \Gamma_2 \Gamma_{11} \Gamma_{12}^*) \pi^2 - \frac{\Gamma_2 \Gamma_{22} a^2}{4} \right) \pi^2 b^2 (-1)^{m+n}}{3} \right. \\ &\quad - \frac{2a^2 n^2 \left(m^2 R ((\Gamma_{21}^* + 4\Gamma_{11}) \Gamma_{22} + \Gamma_2 \Gamma_{11} \Gamma_{12}^*) \pi^2 - \frac{\Gamma_2 \Gamma_{22} a^2}{4} \right) \pi^2 b^2 (-1)^m}{3} \\ &\quad - \frac{2a^2 n^2 \left(m^2 R ((\Gamma_{21}^* + 4\Gamma_{11}) \Gamma_{22} + \Gamma_2 \Gamma_{11} \Gamma_{12}^*) \pi^2 - \frac{\Gamma_2 \Gamma_{22} a^2}{4} \right) \pi^2 b^2 (-1)^n}{3} \\ &\quad \left. + \frac{2a^2 \pi^2 \left(m^2 R ((\Gamma_{21}^* + 4\Gamma_{11}) \Gamma_{22} + \Gamma_2 \Gamma_{11} \Gamma_{12}^*) \pi^2 - \frac{\Gamma_2 \Gamma_{22} a^2}{4} \right) n^2 b^2}{3} \right\} \\ a_3 &= \frac{\pi^4 (\Gamma_{11} b^4 m^4 + \Gamma_{22} a^4 n^4)}{16 a^4 b^4 \Gamma_{11} \Gamma_{22} \rho_1} \\ a_{4i} &= \frac{4 \text{Heaviside} \left(b - \frac{\pi R}{4} \right) \left(\sin^2 \left(\frac{im\pi a}{a(J+1)} \right) \right) \left(\sin^2 \left(\frac{nR\pi^2}{4b} \right) \right)}{ab\rho_1}; i = 1, 2, \dots, J \\ a_{5i} &= - \frac{4 \text{Heaviside} \left(b - \frac{\pi R}{4} \right) \sin \left(\frac{im\pi a}{a(J+1)} \right) \sin \left(\frac{nR\pi^2}{4b} \right)}{ab\rho_1} \\ a_{6i} &= \frac{4 \text{Heaviside} \left(b - \frac{\pi R}{4} \right) \left(\sin^4 \left(\frac{im\pi a}{a(J+1)} \right) \right) \left(\sin^4 \left(\frac{nR\pi^2}{4b} \right) \right)}{ab\rho_1} \\ a_{7i} &= - \frac{12 \text{Heaviside} \left(b - \frac{\pi R}{4} \right) \left(\sin^3 \left(\frac{im\pi a}{a(J+1)} \right) \right) \left(\sin^3 \left(\frac{nR\pi^2}{4b} \right) \right)}{ab\rho_1}\end{aligned} \quad (\text{A5})$$

$$\begin{aligned}
a_4 = a_5 &= \frac{1}{a^4 b^4 R m \pi^2 n \Gamma_{11} \Gamma_{22} \rho_1} \{4a^4 \Gamma_2 \Gamma_{11} \Gamma_{22} b^4 R (-1)^{m+n} - 4a^4 \Gamma_2 \Gamma_{11} \Gamma_{22} b^4 R (-1)^m - 4a^4 \Gamma_2 \Gamma_{11} \Gamma_{22} b^4 R (-1)^n \\
&\quad + 4\Gamma_2 \Gamma_{11} \Gamma_{22} b^4 R a^4\} \\
b_{1i} &= -\frac{nm \left(\sin^2 \left(\frac{im\pi a}{a(J+1)} \right) \right) \pi^2 \text{Heaviside} \left(b - \frac{\pi R}{4} \right) \left(\sin^2 \left(\frac{nR\pi^2}{4b} \right) \right)}{Mab((-1)^m - 1 + (-1)^n - (-1)^{m+n})} \\
b_{2i} &= -\frac{nmsin \left(\frac{im\pi a}{a(J+1)} \right) \pi^2 \text{Heaviside} \left(b - \frac{\pi R}{4} \right) \sin \left(\frac{nR\pi^2}{4b} \right)}{Mab((-1)^m - 1 + (-1)^n - (-1)^{m+n})} \\
b_{3i} &= -\frac{\pi^2 \left(\sin^4 \left(\frac{im\pi a}{a(J+1)} \right) \right) \left(\sin^4 \left(\frac{nR\pi^2}{4b} \right) \right) \text{Heaviside} \left(b - \frac{\pi R}{4} \right) mn}{Mab((-1)^m - 1 + (-1)^n - (-1)^{m+n})} \\
b_{4i} &= \frac{\pi^2 \left(\sin^3 \left(\frac{im\pi a}{a(J+1)} \right) \right) \left(\sin^3 \left(\frac{nR\pi^2}{4b} \right) \right) \text{Heaviside} \left(b - \frac{\pi R}{4} \right) mn}{Mab((-1)^m - 1 + (-1)^n - (-1)^{m+n})}
\end{aligned} \tag{A6}$$

Appendix C

The coefficients appearing in Equation (18), for $i = 1, 2, \dots, J$, are given as follows:

$$\begin{aligned}
\Pi_1 &= \sqrt{\omega_1^2 - C_1^2}; \Sigma_{14} = \frac{1}{6\omega_1^2} \psi_{1[Nt]/N}'''; \Sigma_{13} = \frac{1}{\omega_1^2} \left(\frac{1}{2} \psi_{1[Nt]/N}'' - 6C_1 \Sigma_{14} \right) \\
\Sigma_{12} &= \frac{1}{\omega_1^2} \left[\frac{1}{2} \psi_{1[Nt]/N}' - 4C_1 \Pi_{i3} - 6\Sigma_{14} \right]; \Sigma_{11} = \frac{1}{\omega_1^2} \left[\psi_{1[Nt]/N} - 2C_1 \Sigma_{12} - 2\Sigma_{13} \right] \\
\Lambda_{11} &= W - \Sigma_{11}; \Lambda_{12} = \frac{1}{\Pi_1} (\dot{W} - C_1 \Lambda_{11} - \Sigma_{12}) \\
\Pi_{i2} &= \sqrt{\omega_{2i}^2 - C_{2i}^2}; \Sigma_{i24} = \frac{1}{6\omega_{2i}^2} \psi_{2i[Nt]/N}'''; \Sigma_{i23} = \frac{1}{\omega_{2i}^2} \left(\frac{1}{2} \psi_{2i[Nt]/N}'' - 6C_{2i} \Sigma_{i24} \right) \\
\Sigma_{i22} &= \frac{1}{\omega_{2i}^2} \left[\frac{1}{2} \psi_{2i[Nt]/N}' - 4C_{2i} \Sigma_{i23} - 6\Sigma_{i24} \right] \\
\Sigma_{i21} &= \frac{1}{\omega_{2i}^2} \left[\psi_{2i[Nt]/N} - 2C_{2i} \Sigma_{i22} - 2\Sigma_{i23} \right] \\
\Lambda_{i21} &= S - \Pi_{i2}; \Lambda_{i22} = \frac{1}{\Pi_{i2}} (\dot{S} - C_{2i} \Lambda_{i21} - \Sigma_{i21})
\end{aligned} \tag{A7}$$

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