

Mathematics: Feature Papers 2025

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This Special Issue of the journal *Symmetry* is dedicated to recent mathematics investigations, showcasing a collection of twenty research works and four reviews in different fields of mathematics. This collection features a balanced selection of new results in pure mathematics and applied mathematics, as well as applications of mathematics in real-life processes. A special emphasis is placed on the solution of model problems, but also on a deeper understanding of evolutionary networks in complex systems. We identify several actual research directions that are currently being continuously improved, from consolidated theories to numerical methods. Matrix equations, fractional calculus, fuzzy optimization and stochastic analysis, specialized operators and advanced framework spaces and settings are explored in detail to assess their impact on the understanding and performance of complex systems.

We now provide some detail regarding the collected works to highlight the salient points of this Special Issue.

Edoardo Ballico [1] computes the dimensions of secant varieties of the Veronese embeddings of hypersurfaces, including singular ones, focusing on projective spaces that have at most five dimensions. The strategy is based on the Differential Horace Lemma.

Qing-Wen Wang et al. [2] review several solution methods for classical systems of matrix equations by involving generalized inverse methods and matrix decomposition techniques in the forms of Hermitian, extreme rank, reflexive, and conjugate solutions. They also show applications in color images.

Shu-Ting Zhuang et al. [3] propose an advanced robust principal component analysis model that integrates bidirectional graph Laplacian constraints along with the anchor point technique. The anchor point technique serves to substantially reduce computational complexity, making the model more efficient and scalable.

Jacek Dziok [4] establishes coefficient formulas, distortion theorems, partial sums, convexity, and extreme points for a class of harmonic functions with Montel's normalization associated with the generalized hypergeometric function. The radii of convexity and starlikeness are also considered.

Kan Fang et al. [5] consider a modified Lotka–Volterra commensalism system that incorporates the weak Allee effect in prey and symmetric non-selective harvesting, addressing a critical gap in ecological modeling. Using eigenvalue analysis and the Dulac–Bendixson criterion, they obtain sufficient conditions for the existence and stability of equilibria.

Jocelyn Sabatier [6] deals with distributed time delay models that may exhibit fractional behaviors, consequently providing a compelling alternative to fractional models. The demonstration is done both in discrete time and in continuous time.

Salma Aljawi et al. [7] show the necessary and sufficient conditions to attain the maximum for the operator norm of the sum of two orthogonal projections onto closed subspaces of a complex Hilbert space by using the Dixmier angle between such subspaces.



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Wail Zita et al. [8] develop an approach which combines Bayesian classification with advanced regression techniques to jointly address job title identification and salary prediction. The model is designed to capture the inherent complexity and variability of real-world job market data.

Qing-Wen Wang et al. [9] explore general solutions, symmetric solutions, Hermitian solutions, and other special types of solutions to systems of equations over different algebraic structures, such as number fields, the real field, the complex field, the quaternion division ring, principal ideal domains, regular rings, and operators on Banach spaces.

Jaume de Haro and Emilio Elizalde [10] consider derivation of the spacetime metric generated by matter, without invoking Einstein's field equations. A Lorentz transformation to the static metric is then used to extend the description to moving matter. The approach is grounded in a new dynamical interpretation of the Equivalence Principle.

Sergey K. Sekatskii [11], referring to the Nyman–Beurling theorem, proves that if the closure of linear manifolds of the same functions under suitable conditions is dense in $L_p(0,1)$, then some combinations of Riemann zeta-functions are free from zeroes in the half-plane.

Yao Lu et al. [12] study the p th moment exponential stability of random impulsive delayed nonlinear systems with multiple periodic delayed impulses. The continuous dynamics are described by random delay differential equations whose random disturbances are driven by second-order moment processes.

Antonio F. Miguel [13] deals with a cohesive physical model that predicts lava flow morphology by establishing a quantitative link between a lava's yield strength and its geometric complexity. It incorporates the temporal evolution of complexity as the lava cools.

Valery V. Obukhov [14] focuses on a classification of exact solutions to Maxwell vacuum equations for pseudo-Riemannian spaces with spatial symmetry in the presence of electromagnetic fields being invariant with respect to the action of the group of space motions.

Egils Ginters and Patriks Voldemars Ginters [15] develop a comparative mathematical analysis of electrode configurations used in active fog water harvesting systems based on electrostatic ionization. They solve the Laplace equation for various electrode geometries to compute electrostatic field distributions and analyze field line density patterns.

Zhili Ge et al. [16] focus on a NP-hard problem of solving the rank of a matrix in Robust Principal Component Analysis by proposing a nonconvex fractional regularization approximation. This model effectively avoids parameter selection while preserving scale invariance.

Rakib Feyruz Efendiev et al. [17] study the inverse spectral problem for a PT-symmetric Dirac operator with discontinuity conditions imposed along the entire real axis. They construct fundamental solutions via recursive series expansions and prove their linear independence through a constant Wronskian.

Barry C. Arnold et al. [18] develop the framework of doubly generalized transmutation models via a repeated application of rank transmutation maps and their generalizations. They highlight that several flexible families in the literature can be reinterpreted as instances of double or multiple transmutation.

Qing-Wen Wang and Jiale Gao [19] develop a survey on a generalized Sylvester equation by focusing on research methods, constrained solutions, various generalizations, iterative algorithms, and applications. Some insights and observations for future research trends are also given.

Xiaoping Shi et al. [20] assess new statistical methods by testing the homogeneity of n independent Poisson means when only one observation per population is available. This study is relevant to determine whether different groups share the same underlying event rate or mean.

Iqbol Ergashevich Niyozov et al. [21] establish a suitable analytic framework to obtain the solution of a Cauchy problem of polyharmonic equations in $\mathbb{R}^n$; attention is paid to the role of symmetry in the structure, stability, and solvability of solutions.

Junhyeok Lee et al. [22] review how mathematical foundations support and enhance key Virtual Reality components. They consider the differential equations and numerical integration methods used to simulate dynamic environments, as well as geometric transformations and coordinate systems that enable seamless motion and viewpoint control.

Mohammad Mahdi Mohammad et al. [23] establish a symmetry-aware optimized fuzzy deep reinforcement learning-gated recurrent unit model that investigates both the symmetry and asymmetry in demand-generation behavior to reach stable and adaptive load balancing.

Tianying Li and Lu Niu [24] provide a semiparametric matrix-variate differential network model to assess how biological networks change across different conditions or external stimuli. They use a rank-based D-trace loss with an ℓ_1 penalty to estimate the spatial differential partial correlation matrix.

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