

Article

Stage-Dependent Responses in Sequential Dual-Well Supercritical CO₂ Fracturing: A Sparse Second-Order Choquet Fracability Framework with Monte Carlo Uncertainty

Feng Guo ¹, Xiaozhong Zhang ^{1,*} , Qing Qiao ², Pengfei Hao ² and Guolong Zhang ³

¹ School of Civil Engineering, Guizhou University of Engineering Science, Bijie 551700, China

² School of Mechanics and Civil Engineering, China University of Mining and Technology-Beijing, D11 Xueyuan Road, Beijing 100083, China

³ College of Resources, Environment and Safety Engineering, Jining University, Jining 273155, China

* Correspondence: zhangxiaozhong@126.com

Abstract

Deep tight-reservoir fracability depends on temperature, fluid transport, in situ stress, fracture development, and injection energy, so no single response can provide a defensible ranking of fracturing conditions. This study develops a sparse second-order Choquet–Monte Carlo fracability framework. Additionally, the CRITIC method is used independently to quantify the information contrast of the five evaluation indicators. This was done using true triaxial sequential dual-well experiments conducted at 25–200 °C, horizontal stress differences of 3–9 MPa, and with either supercritical CO₂ (SC-CO₂) or water. CT scanning, U-Net segmentation, and three-dimensional reconstruction supplied fracture volume and fractal dimension; two-stage breakdown pressures and hydraulic injection work described initiation accessibility and energy demand. The CRITIC information weights were 0.261 for stress compatibility, 0.235 for injection-work demand, 0.182 for fluid mobility, 0.172 for specimen temperature, and 0.150 for pressure accessibility. The fitted second-order model identified positive temperature–fluid (0.122) and temperature–stress (0.071) non-additive coefficients, while the pressure–work coefficient was close to zero (−0.008). Sequential injection produced a consistent stage-dependent difference in breakdown pressure, with the second-stage value exceeding the first-stage value in every test. This observation is interpreted descriptively within the fixed sequential injection protocol. Condition No. 5 (SC-CO₂, 200 °C, 3 MPa) ranked first with a fracability index of 0.935 and a 95% Monte Carlo interval of 0.908–0.945. Leave-one-out cross-validation gave a Spearman coefficient of 0.830 and $R^2 = 0.860$. Within the tested domain, temperature was associated with improved fracability, particularly under the low stress-difference condition where enhanced SC-CO₂ mobility and thermal effects acted together. The framework therefore distinguishes coupled benefits, redundancies, and uncertainty that an additive score would conceal.



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Keywords: supercritical CO₂; dual-well fracturing; stage-dependent response; fracability evaluation; CRITIC; Choquet integral; indicator interaction; Monte Carlo analysis

1. Introduction

Unconventional hydrocarbon, hot-dry-rock, and deep coal-measure development increasingly targets high-temperature, high-stress, low-permeability formations. Supercritical CO₂ (SC-CO₂) combines low viscosity, high diffusivity, and low interfacial tension with the possibility of geological carbon storage. These properties can reduce water-sensitive

damage and improve access to small fractures, making SC-CO₂ a candidate for waterless or reduced-water stimulation [1–9]. Recent true triaxial experiments, CT-based fracture characterization, and coupled thermo-hydro-mechanical simulations show that SC-CO₂ can generate more branched, multiscale fracture networks than water, although the outcome depends on temperature, stress difference, bedding, leakoff, and phase behavior [2–18].

Dual-well and multi-well fracturing can enlarge the stimulated volume through fracture interaction, but the interaction is not necessarily beneficial. In dual-well and multi-well fracturing, sequential activation can introduce stage-dependent differences because the first-stage fracture modifies the mechanical and hydraulic environment encountered during the subsequent stage [10,19–21]. In high-temperature rock, cold-fluid injection also produces thermal tensile stress and shear disturbance, while the temperature- and pressure-dependent viscosity of SC-CO₂ changes penetration and energy transfer [11–16,22–26]. A condition may therefore combine a low breakdown pressure with a complex fracture network but relatively high injection energy. Brittleness, breakdown pressure, or fracture volume alone cannot represent these competing responses.

Most fracability indices use linear weighting, the analytic hierarchy process (AHP), entropy weighting, or machine learning prediction. Li et al. [1] proposed an AHP–entropy model for SC-CO₂ dual-well fracturing, and risk-based or machine learning methods have been used for tight-reservoir evaluation and initiation-pressure prediction [27–30]. Linear aggregation, however, assumes that indicators contribute independently and can fully compensate for one another. Under that assumption, the combined benefit of high temperature and low fluid viscosity is split into two unrelated terms, while correlated penalties such as high pressure and high energy demand may be counted twice. A Choquet integral relaxes additivity and represents synergy or redundancy explicitly [31,32], but this capability has rarely been used in experimental SC-CO₂ fracability assessment.

The limited size of true triaxial test programs creates a second problem. High-temperature sequential fracturing tests are costly, and a deterministic score can hide sensitivity to image digitization, measurement error, weighting, and model calibration. This study therefore combines a five-dimensional indicator system with CRITIC-based information-weight analysis and, separately, a physically restricted second-order Choquet model, together with leave-one-out cross-validation (LOOCV) and Monte Carlo perturbation. The objectives are to (i) rank the tested conditions without introducing expert scores that were not collected, (ii) separate the marginal importance of each indicator from pairwise interaction effects, (iii) quantify the stability of the ranking, and (iv) characterize stage-dependent responses in sequential dual-well fracturing under thermal, fluid, and stress variations. Section 2 describes the experiments and image-based fracture measurements. Section 3 defines the indicators and evaluation model. Section 4 reports the experimental responses, rankings, and validation results. Section 5 discusses the physical interpretation, methodological implications, and limitations, followed by the conclusions in Section 6.

2. Experimental Basis and Data Acquisition

2.1. Specimen Preparation and Basic Mechanical Parameters

Artificial tight-sandstone cubes (100 mm × 100 mm × 100 mm) were prepared from quartz sand and Portland cement at a water-to-cement ratio of 0.20. The specimen formulation and preparation procedure followed Ref. [18]. Temperature-dependent mechanical properties of the same artificial tight-sandstone system had been characterized previously by our group and are therefore referenced from Ref. [18] for the thermo-mechanical interpretation in the present study. The quartz sand particle-size range was 0.15–0.30 mm, and ordinary Portland cement of grade 42.5 was used. After casting, the specimens were

demolded after 24 h and cured for 28 days at 20 °C and 95% relative humidity. All specimens were prepared using the same material batch, mixing procedure, casting method, and curing protocol to minimize specimen-to-specimen variability. The controlled mixture reduced specimen-to-specimen heterogeneity and allowed the same dual-well spacing to be reproduced. Tables 1 and 2 summarize the mixture and mechanical properties; the measured properties lie within the ranges reported for natural tight sandstone. Two parallel wellbores were embedded during casting to maintain sealing and geometric consistency (Figure 1). The two wellbores were parallel, but their perforation positions were vertically offset as shown in Figure 1; therefore, the specimen geometry was not strictly invariant under interchange of Well A and Well B.

Table 1. Mix proportions of the artificial tight-sandstone specimens.

Size (mm)	Water Cement Ratio	Cement (g)	Sand (g)	Water (g)	Water Reducer (g)
100 × 100 × 100	0.2	991.3	111	198.2	24.7

Table 2. Mechanical properties of natural sandstone and the artificial specimen.

Categories	Elasticity Modulus (GPa)	Uniaxial Compressive Strength (MPa)	Tensile Strength (MPa)	Density (g/cm ³)	Porosity (%)	Permeability (mD)
Natural sandstone	16.2–21.1	66.0–75.0	3.0–4.5	2.3–2.4	4.0–21.1	0.01–0.1
Artificial specimen	18.8 ± 1.3	69.3 ± 1.6	4.1 ± 0.7	2.4 ± 0.05	7.2 ± 0.08	0.09 ± 0.004

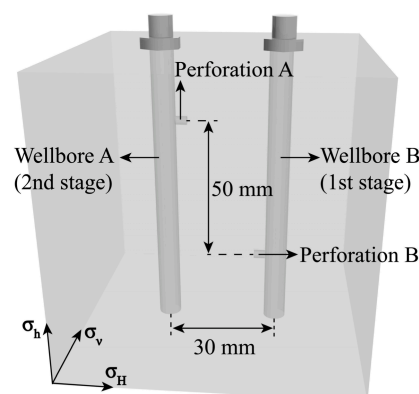


Figure 1. Structure of the dual-well artificial tight-sandstone specimen.

2.2. Experimental Equipment

The apparatus comprised a true triaxial loading frame, two independent injection channels, a CO₂ pressurization and temperature-control module, a specimen heater, and dynamic pressure acquisition (Figure 2). Each specimen was heated at 0.1 °C·min^{−1} to the target temperature and held for 1 h. Before injection, CO₂ was maintained at 40 °C and 7.5 MPa to ensure a supercritical state at the supply end. Water was injected at room temperature (approximately 25 °C). The injection rate was 10 mL·min^{−1} for both fluids. After the first well showed a pronounced pressure drop, injection was transferred to the second well. Tungsten and lead powders were used as tracers for the first and second stages, respectively. Each wellbore had a diameter of 5 mm. The perforations were 1 mm in diameter and 3 mm in length. The wellbore annulus was sealed with epoxy resin to prevent fluid leakage along the wellbore–specimen interface. Tungsten and lead tracer powders with a particle size of 100 nm were used. During sequential fracturing, Well B was injected first. After the Stage-1 breakdown event was identified, injection was stopped, and the injection line was immediately switched to Well A. Stage 2 was then initiated under the same nominal injection rate. Breakdown pressure was defined as the maximum injection

pressure immediately preceding the first abrupt pressure drop associated with macroscopic fracture initiation.

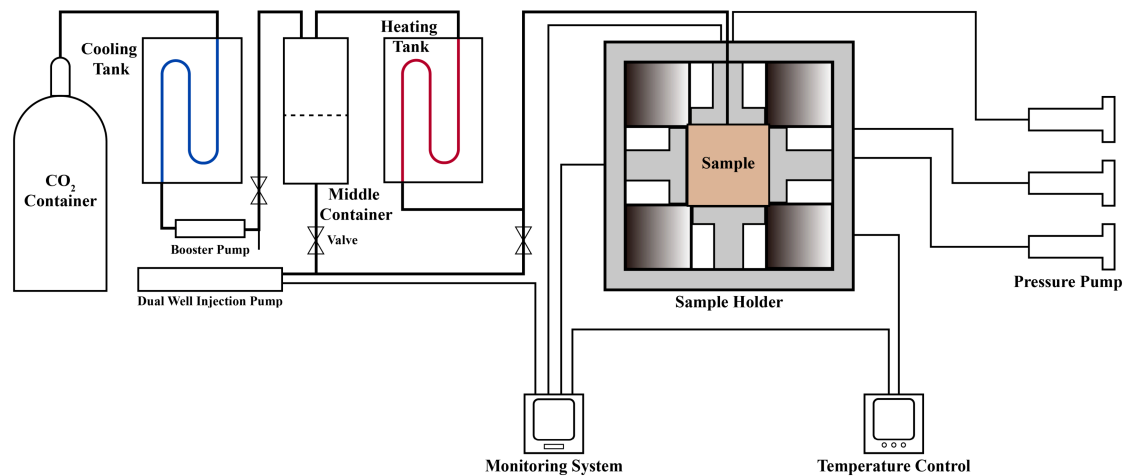


Figure 2. True triaxial dual-well fracturing test system.

2.3. Experimental Condition Design

Conditions No. 1–No. 5 used SC-CO₂ at 25, 50, 100, 150, and 200 °C with a horizontal stress difference of 3 MPa. Conditions No. 6 and No. 7 retained 200 °C but increased the stress difference to 6 and 9 MPa. Conditions No. 8–No. 10 were water-fracturing controls at 25, 100, and 200 °C. Table 3 lists the complete loading and fluid conditions. Each experimental condition was independently repeated three times. The mean values and standard deviations of quantitative measurements were calculated from the three replicates. A representative three-dimensional fracture reconstruction was selected only for visualization, whereas all quantitative analyses were based on averaged experimental results.

Table 3. True triaxial dual-well fracturing experimental conditions, replication: $n = 3$.

Specimen	Temperature (°C)	In Situ Stress ($\sigma_v/\sigma_H/\sigma_h$) (MPa)	Stress Difference σ_{H-h} (MPa)	Fracturing Fluid
1	25	40/33/30	3	SC-CO ₂
2	50	40/33/30	3	SC-CO ₂
3	100	40/33/30	3	SC-CO ₂
4	150	40/33/30	3	SC-CO ₂
5	200	40/33/30	3	SC-CO ₂
6	200	40/36/30	6	SC-CO ₂
7	200	40/39/30	9	SC-CO ₂
8	25	40/33/30	3	Water
9	100	40/33/30	3	Water
10	200	40/33/30	3	Water

2.4. CT Scanning, U-Net Fracture Segmentation, and 3D Reconstruction

After fracturing, each specimen was scanned using a ProCon high-resolution CT scanner at a tube voltage of 150 kV and a tube current of 120 μ A/mA. The reconstructed voxel size was 20 μ m $\times$ 20 μ m $\times$ 20 μ m, the slice thickness was 20 μ m, and the FBP reconstruction filter was used. The same scanning and reconstruction parameters were applied to all specimens. The segmented volumes were imported into Mimics for three-dimensional reconstruction. Compared with fixed-threshold segmentation, the U-Net result contained fewer isolated segmented regions and showed better spatial continuity along the main fracture paths (Figure 3). Because a larger segmented white area does not necessarily

correspond to more true fractures, the segmentation quality was evaluated quantitatively against manual annotations rather than judged from the apparent number of white pixels. The U-Net was trained using 200 manually annotated CT slices selected from 20 specimens. Fractures were manually delineated according to grayscale discontinuity and spatial continuity across adjacent slices, while isolated pores, wellbore regions, tracer particles, and reconstruction artifacts were excluded. The dataset was divided into training, validation, and test subsets at a ratio of 7:2:1. Segmentation performance was evaluated against manual annotations using the Dice coefficient, intersection over union (IoU), precision, and recall. The resulting values were 0.87, 0.78, 0.85, and 0.89, respectively, indicating acceptable agreement between the U-Net predictions and manual labels. Figure 4 shows the reconstructed networks. Fracture volume was obtained by voxel counting, and the fractal dimension D was calculated by three-dimensional box counting. The validated U-Net segmentation masks were used for subsequent fracture-volume and fractal-dimension calculations.

$$D = \lim_{(\varepsilon \rightarrow 0)} (\log N(\varepsilon)) / (\log(1/\varepsilon)) \quad (1)$$

where $N(\varepsilon)$ is the minimum number of cubes with side length ε needed to cover the fracture voxels. The stage-specific tracer distributions and CT reconstructions were also examined to determine whether fractures generated during the two stages intersected or connected. Stage-1 and Stage-2 fractures showed condition-dependent connection in the reconstructed networks.

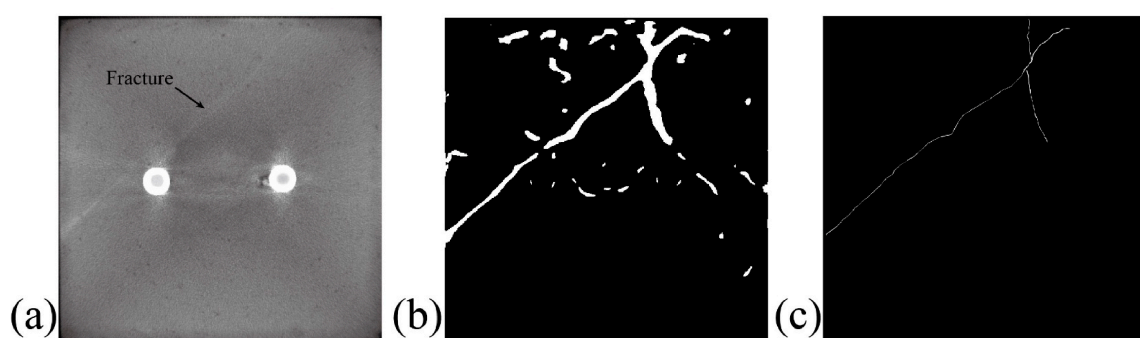


Figure 3. CT scanning and U-Net fracture segmentation: (a) original CT slice; (b) fixed-threshold segmentation; (c) U-Net segmentation result.

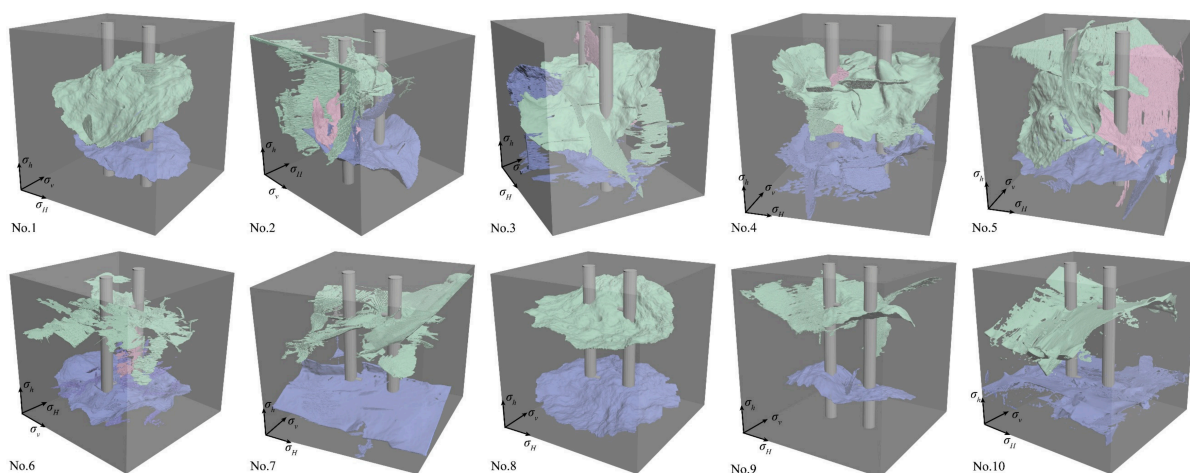


Figure 4. Three-dimensional fracture network reconstruction for all 10 specimens.

3. Fracability Evaluation Method

3.1. Five-Dimensional Fracability Indicator System

The five indicators were defined from the measured conditions and responses as follows.

- (1) Specimen temperature (I_1) was represented by the normalized target specimen temperature, $T/T_{\max}$. This indicator characterizes the imposed thermal condition of the specimen within the experimental design and is not intended to represent the actual transient thermal stress during fluid injection. The latter additionally depends on fluid temperature, transient heat exchange, local temperature gradients, and mechanical constraints.
- (2) Fluid mobility (I_2) combined the SC-CO₂ viscosity–temperature relation with a fluid-type score. Lower viscosity corresponds to easier penetration, because SC-CO₂ viscosity varies with temperature. The fluid-mobility indicator used temperature-dependent CO₂ viscosity values based on the imposed supply and thermal conditions [18]. Accordingly, their non-additive coefficient is not interpreted as an independently identified physical interaction; after direction conversion, larger I_2 values denote greater mobility. The water controls were assigned lower mobility scores than SC-CO₂ under otherwise comparable conditions.
- (3) Stress compatibility (I_3) was the complement of the normalized horizontal stress difference. Larger values denote weaker directional constraint on fracture deflection and branching.
- (4) Pressure accessibility (I_4) was the complement of the normalized two-stage breakdown pressure. Larger values denote easier fracture initiation.
- (5) Injection-work demand (I_5) was defined from the hydraulic work delivered during fluid injection. For each fracturing stage s , the hydraulic injection work was calculated as:

$$E_s = \int_{t_{s,0}}^{t_{s,1}} P(t)Q(t)dt \quad (2)$$

where $P(t)$ is the measured injection pressure and $Q(t)$ is the injection flow rate. Because the nominal flow rate was maintained at $10 \text{ mL} \cdot \text{min}^{-1}$, the work was obtained by numerical integration of the recorded pressure–time curve. The total injection work was $E = E_1 + E_2$. After complement normalization, larger I_5 values denote lower hydraulic injection work within the laboratory test, rather than lower total engineering energy consumption or economic cost.

3.2. CRITIC-Based Reference Weighting

CRITIC assigns greater weight to an indicator when it has both high contrast across the samples and low correlation with the other indicators. The weight of indicator j was calculated as [1]:

$$w_j = \left(\sigma_j \sum_{k=1}^n (1 - r_{jk}) \right) / \left(\sum_{l=1}^n \left[\sigma_l \sum_{k=1}^n (1 - r_{lk}) \right] \right) \quad (3)$$

where σ_j is the standard deviation of indicator j , r_{jk} is the Pearson correlation coefficient between indicators j and k , and $n = 5$. The resulting weights describe information contrast within this sample; they should not be interpreted as causal effect sizes. In the present framework, the CRITIC weights are used as an independent reference describing the information contrast and inter-indicator conflict in the experimental dataset. They are not imposed as constraints, priors, or penalty terms in the subsequent Choquet-capacity calibration and therefore do not enter the final least-squares objective function. Their role is

to provide a benchmark against which the fitted Choquet–Shapley marginal importance can be compared.

3.3. Sparse Second-Order Choquet Integral Model

The Choquet integral permits the contribution of one indicator to depend on another [31,32]. Let μ be a normalized monotone capacity on $N = \{1, 2, 3, 4, 5\}$, with $\mu(\emptyset) = 0$ and $\mu(N) = 1$. In the second-order Möbius representation used here, the fracability index for condition i is

$$C_{\mu}(x_i) = \sum_{(j=1)}^n m_j x_{ij} + \sum_{(j < k)} m_{jk} \min(x_{ij}, x_{ik}) \quad (4)$$

where m_j is the singleton Möbius coefficient and m_{jk} is the pairwise interaction coefficient. A positive m_{jk} represents positive non-additivity, whereas a negative value represents negative non-additivity within the fitted aggregation model. The sign is interpreted physically only when it is stable under uncertainty analysis and independently supported by the experimental design. Considering the limited number of experimental conditions, a fully parameterized second-order Choquet model containing all ten pairwise interactions would introduce excessive degrees of freedom. Therefore, a sparse interaction structure was adopted. Candidate interactions were screened according to two criteria: (1) consistency with the coupled thermo-hydro-mechanical mechanisms of SC-CO₂ fracturing; and (2) improvement in model performance during leave-one-out cross-validation. Based on these criteria, three interactions, namely specimen temperature–fluid mobility, specimen temperature–stress compatibility, and pressure accessibility–injection work demand, were retained. It should be emphasized that the experimental matrix was not a balanced factorial design of temperature, fluid type, and stress difference. Therefore, the thermal–fluid and thermal–stress coefficients quantify non-additivity within the fitted indicator structure only and cannot be interpreted as independently identified main-effect interactions.

The calibration targets combined fracture volume and fractal dimension:

$$R_i = 0.55(\log(V_i/V_{min}))/(\log(V_{max}/V_{min})) + 0.45(D_i - D_{min})/(D_{max} - D_{min}) \quad (5)$$

where V_i and D_i are the measured fracture volume and fractal dimension for condition i . The logarithmic transformation reduces the leverage of the fracture-volume range. The fixed 0.55/0.45 balance gives slightly greater emphasis to stimulated volume than to geometric complexity. R_i is used only for capacity calibration and does not enter the CRITIC calculation. The Choquet coefficients were estimated by constrained least squares subject to capacity normalization, monotonicity, and the predefined sparse interaction structure. Importantly, the CRITIC weights w_j do not appear in this objective function.

3.4. Importance, Validation, and Uncertainty Analysis

For a second-order Möbius capacity, the Shapley importance of indicator j is

$$\varphi_j = m_j + 1/2 \sum_{(k \neq j)} m_{jk} \quad (6)$$

The Shapley value allocates half of each pairwise interaction to each participating indicator. It therefore provides a marginal-importance measure that can differ from the CRITIC information weight.

LOOCV was used for internal validation. In each round, the coefficients were calibrated on nine conditions and used to predict the omitted condition. Performance was summarized by the root mean square error (RMSE), coefficient of determination (R^2), and Spearman rank correlation (ρ).

The Monte Carlo analysis comprised 12,000 repetitions. Fracture volume, fractal dimension, and energy were perturbed within their digitization-error ranges, and breakdown

pressures were perturbed within measurement accuracy; normalization, CRITIC weights, and Choquet coefficients were then recalculated. A further 6000 runs applied $\pm 10\%$ perturbations to the fitted capacity parameters to estimate rank probabilities and the probability that each condition ranked first.

4. Results

4.1. Breakdown Pressure and Inter-Stage Stress Interference

The second-stage breakdown pressure exceeded the first-stage value in every condition (Figure 5), indicating a consistent stage-dependent difference under the adopted sequential injection protocol. Because the injection order was fixed and no reverse-sequence, simultaneous-injection, or independent single-well controls were included, this difference is interpreted descriptively and is not taken as direct evidence of formal symmetry breaking or uniquely attributed to stress redistribution from the first fracture. At a 3 MPa stress difference, the first-stage SC-CO₂ breakdown pressure decreased from 29.95 MPa at 25 °C to 23.85 MPa at 200 °C (20.4%), while the second-stage pressure decreased from 33.82 to 28.47 MPa (15.8%). Thermal weakening and near-wellbore thermal cracking, together with greater SC-CO₂ mobility, can account for this reduction [2,13,15,17,18]. At 200 °C, increasing the stress difference from 3 to 9 MPa reduced the two pressures further, to 19.65 and 24.01 MPa, but did not improve the fracture network (Figure 6). Lower breakdown pressure is therefore not equivalent to greater fracability. The water controls also showed lower pressure at higher temperature, but produced smaller and less complex fracture networks overall (Figures 5c and 6a). The complete pressure–time histories for all experimental conditions are provided in Figure S1, while Figure 5 summarizes the extracted Stage-1 and Stage-2 breakdown pressures.

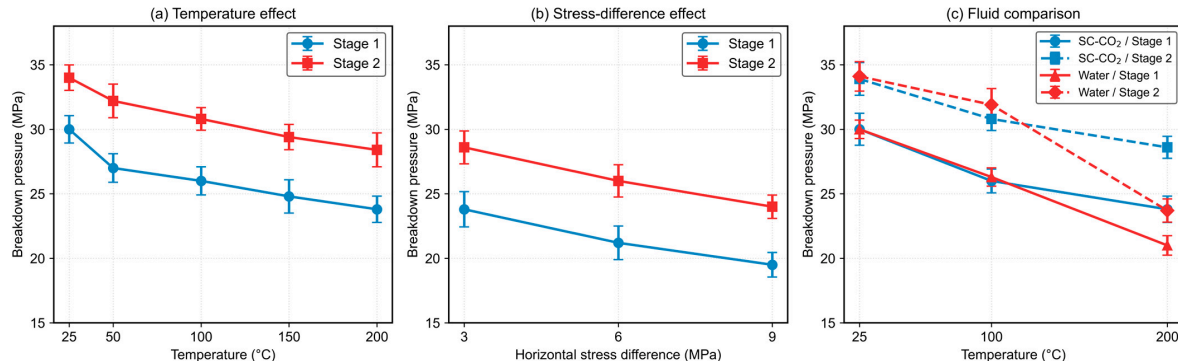


Figure 5. Two-stage breakdown pressure variation across experimental conditions: (a) effect of temperature on SC-CO₂ fracturing; (b) effect of stress difference on SC-CO₂ fracturing; (c) fluid comparison (SC-CO₂ vs. water fracturing).

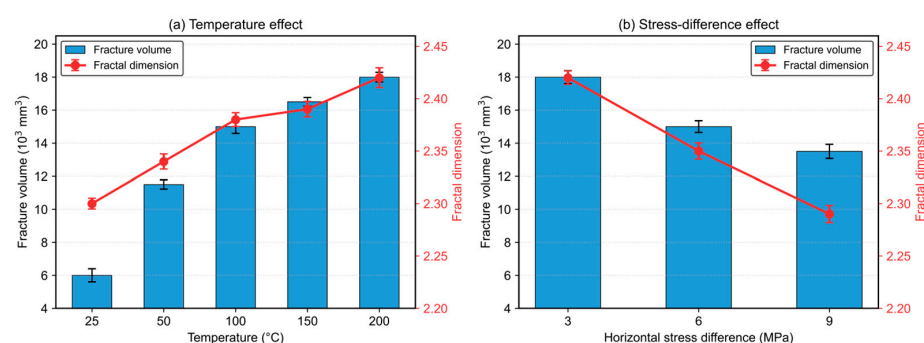


Figure 6. Fracture volume and fractal dimension for SC-CO₂ fracturing: (a) temperature effect (SC-CO₂, 3 MPa stress difference); (b) stress difference effect (SC-CO₂, 200 °C).

4.2. Fracture Geometry and Injection Energy

At a 3 MPa stress difference, SC-CO₂ fracture volume increased from $6.1 \times 10^3 \text{ mm}^3$ at 25 °C to $1.8 \times 10^4 \text{ mm}^3$ at 200 °C, while the fractal dimension increased from 2.295 to 2.420 (Figure 6). Heating and the near-wellbore thermal gradient activate microcracks, and low-viscosity SC-CO₂ can enter these paths and promote connection and branching [3,7,9,14,23,26]. At 200 °C, raising the stress difference from 3 to 9 MPa reduced fracture volume to $1.32 \times 10^4 \text{ mm}^3$ and fractal dimension to 2.290. The observed reduction suggests that increased principal-stress constraint may limit fracture deflection and branching within the tested conditions. At corresponding temperatures, water fracturing produced 4.6×10^3 – $7.0 \times 10^3 \text{ mm}^3$ of fracture volume, below the SC-CO₂ values.

Total hydraulic injection work followed a different trend (Figure 7). For SC-CO₂ at a 3 MPa stress difference, the measured work decreased from 965 J at 25 °C to 880 J at 200 °C, while at 200 °C and a 9 MPa stress difference it decreased to 780 J. Water fracturing showed lower hydraulic injection work (320–490 J), but also produced substantially smaller fracture volumes. The lower measured work for water may reflect differences in pressure evolution, injection duration, fluid compressibility, and injected volume.

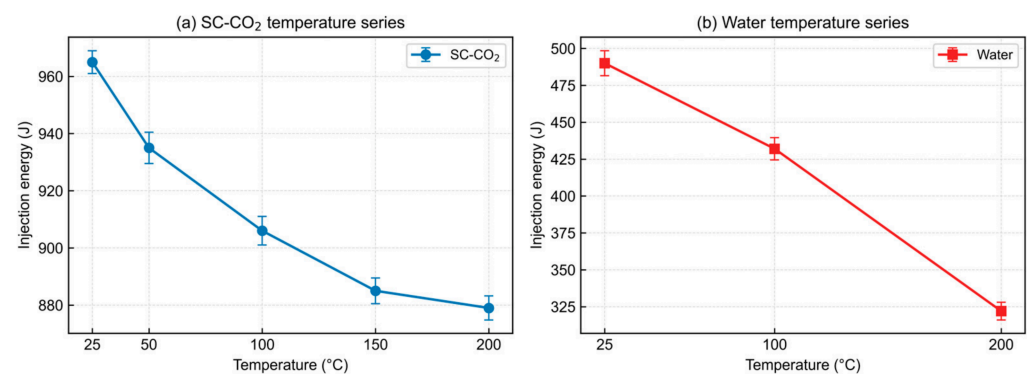


Figure 7. Total hydraulic injection work ($E_1 + E_2$) under different conditions: (a) SC-CO₂ fracturing at various temperatures (3 MPa stress difference); (b) fluid comparison (SC-CO₂ vs. water fracturing).

4.3. Objective Weights and Indicator Interaction Structure

The CRITIC information weights decreased in the order stress compatibility (0.261), injection-work demand (0.235), fluid mobility (0.182), specimen temperature (0.172), and pressure accessibility (0.150) (Figure 8a). Stress compatibility varied strongly across the 3–9 MPa tests and separated conditions with directional fracture growth from those with greater network development. Injection-work demand also distinguished SC-CO₂ from water. These CRITIC weights describe contrast and conflict in the measured data and are reported as an independent reference. The Choquet–Shapley importance, in contrast, is derived from the separately calibrated non-additive capacity and represents marginal contribution within that fitted model.

The fitted singleton Möbius coefficients for specimen temperature, fluid mobility, stress compatibility, pressure accessibility, and injection-work demand were 0.123, 0.349, 0.190, 0.137, and 0.016. The temperature–fluid and temperature–stress non-additive coefficients were positive (0.122 and 0.071), whereas the pressure accessibility–injection work coefficient was close to zero (−0.008) (Figure 8b). Given the limited sample size and high parameter-to-sample ratio, these coefficients should be interpreted as model-dependent non-additivity parameters rather than stable physical interaction effects. In particular, the magnitude of the pressure–work coefficient is very small and is therefore not interpreted as evidence of a stable redundancy effect. Given the non-factorial experimental matrix, the first two coefficients should be regarded as model-dependent associations rather than

direct evidence of physical synergy. After interaction allocation, the Shapley importances were 0.410 for fluid mobility, 0.225 for stress compatibility, 0.220 for specimen temperature, 0.133 for pressure accessibility, and 0.012 for injection-work demand. The small marginal importance of injection-work demand does not mean that energy is unimportant; much of its information overlaps with pressure, fluid type, and fracture response. The marked difference between the CRITIC weight of injection-work demand (0.235) and its fitted singleton coefficient (0.016) is expected because these quantities have different definitions: the former measures information contrast in the raw indicator matrix, whereas the latter is a parameter of the independently calibrated non-additive capacity.

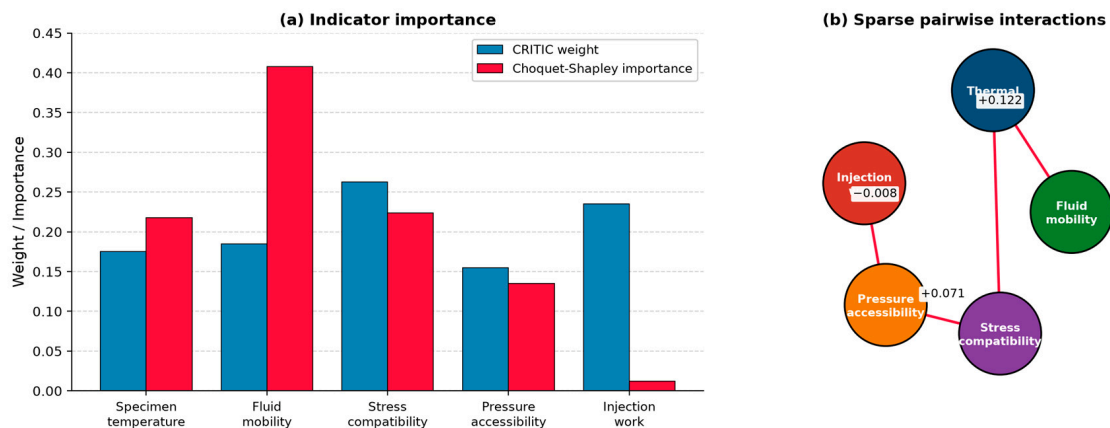


Figure 8. CRITIC weights, Choquet-Shapley importance, and sparse non-additive structure: (a) comparison of indicator weights and importance; (b) retained pairwise non-additive coefficients.

4.4. Comprehensive Fracability Index and Condition Ranking

Condition No. 5 (200 °C, 3 MPa, SC-CO₂) ranked first with an index of 0.935 and a 95% Monte Carlo interval of 0.908–0.945 (Figure 9). Conditions No. 6 (200 °C, 6 MPa, SC-CO₂) and No. 4 (150 °C, 3 MPa, SC-CO₂) followed, with indices of 0.832 and 0.828. No. 6 had weaker fracture indicators than No. 4 but lower breakdown pressure and energy demand, so the two overall scores were similar. No. 10 (200 °C, 3 MPa, water; 0.728) ranked above No. 7 (200 °C, 9 MPa, SC-CO₂; 0.716), showing that a large stress difference can offset the mobility advantage of SC-CO₂. No. 8 (25 °C, water) ranked last at 0.201.

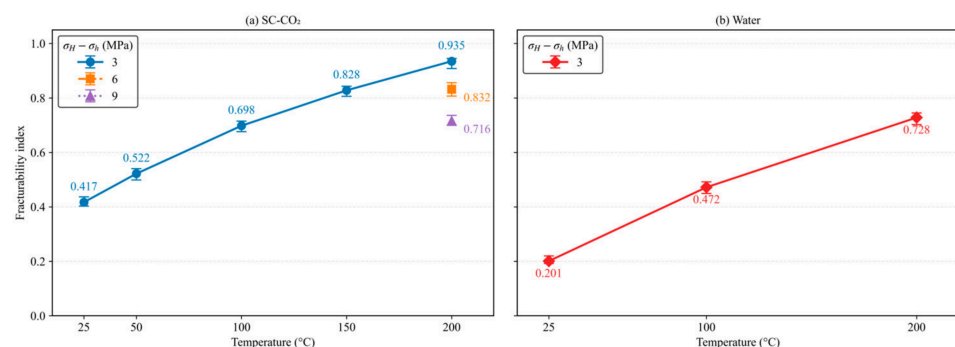


Figure 9. Fracability index as a function of temperature with 95% Monte Carlo intervals.

4.5. Leave-One-out Validation and Ranking Robustness

LOOCV predictions followed the observed response reasonably well (Figure 10). Spearman $\rho = 0.830$ indicates that most of the superior–inferior ordering was retained, while $R^2 = 0.860$ and $RMSE = 0.080$ summarize agreement in score magnitude.

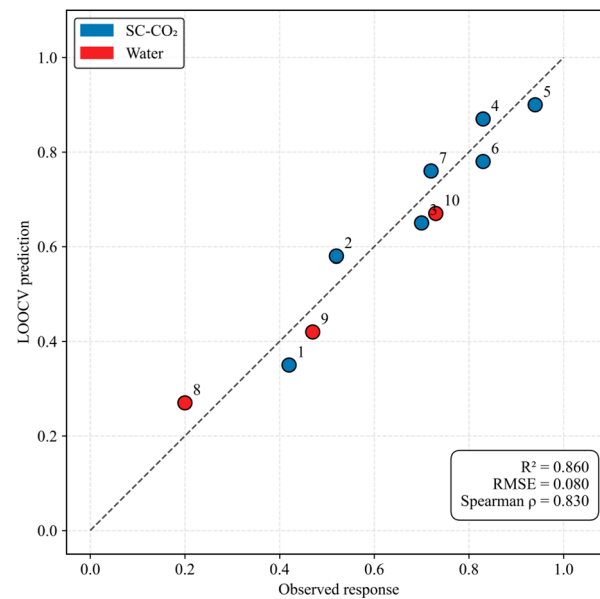


Figure 10. Leave-one-out cross-validation: predicted vs. observed responses.

The 95% interval for condition No. 5 remained separated from those of the other conditions, and No. 5 retained first place throughout the 6000 parameter-perturbation runs (Figure 11). The intervals for No. 6 and No. 4 overlapped substantially, and their order changed with the relative emphasis on operational economy and fracture complexity. They are therefore better treated as a common second tier. Figure 12 provides a model-based visualization of the fitted fracability index over the nominal temperature–stress-difference range. Because the experimental matrix does not contain factorial temperature $\times$ stress-difference combinations, this surface is illustrative only and should not be interpreted as an experimentally validated response surface.

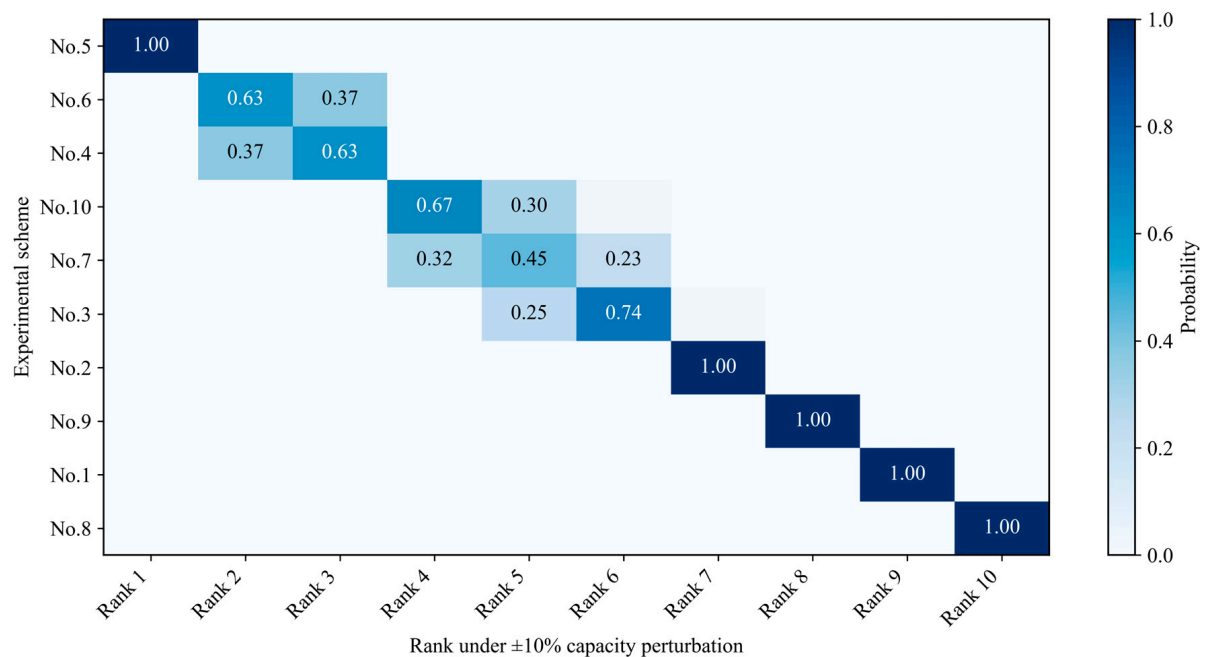


Figure 11. Condition ranking robustness (probability matrix under $\pm 10\%$ capacity perturbation).

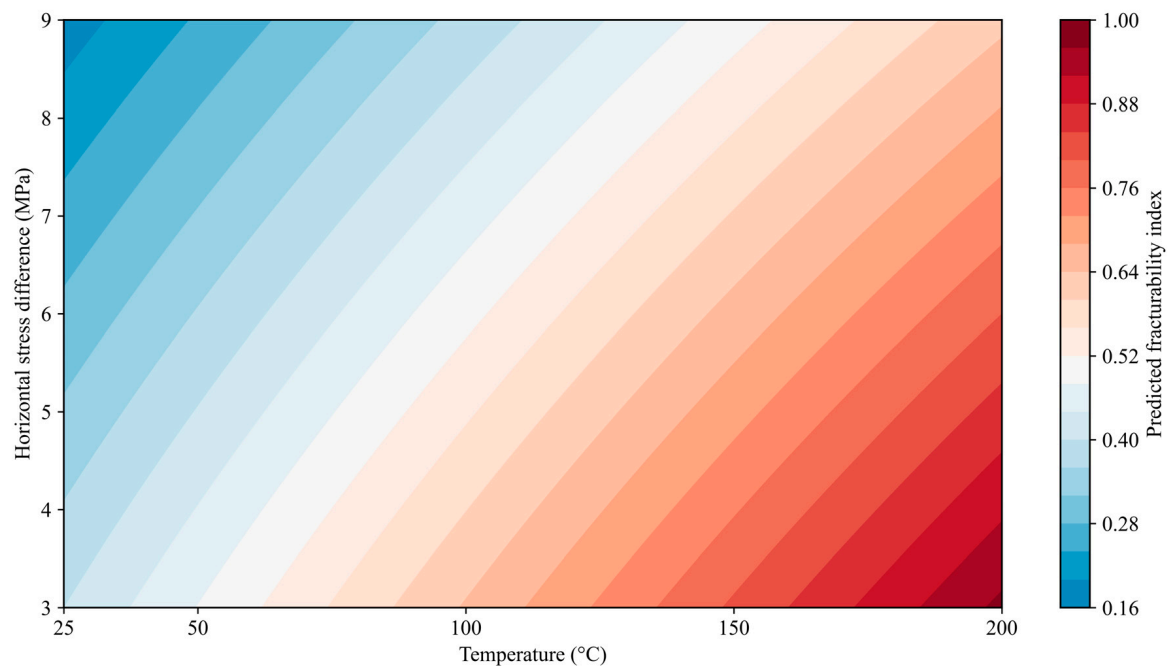


Figure 12. Illustrative model-based visualization of the SC-CO₂ fracability index over the nominal temperature–stress-difference range; the surface is not an experimentally validated factorial response surface.

5. Discussion

5.1. Physical Mechanism Interpretation of Key Findings

Within the tested conditions, higher temperature was associated with improved composite fracability under the 3 MPa stress-difference series. Liu et al. [2] likewise reported that temperature can reduce breakdown pressure through tensile-strength loss and near-wellbore thermal cracking, with the effect constrained by the in situ stress field. Hu et al. [13] found that thermal-shock damage increased with temperature difference but was moderated by stress conditions.

For an elastic approximation, the local thermal tensile stress induced by cooling of a hot fracture wall is

$$\sigma_T = (E\alpha\Delta T)/(1 - \nu) \quad (7)$$

The temperature-dependent mechanical parameters used for this interpretation were taken from the previous measurements of the same artificial tight-sandstone system reported in Ref. [18]. Because these properties were not remeasured for every specimen in the present test series, they are used as reference material parameters rather than specimen-specific measurements. This increment is not sufficient to initiate fracture independently, but it can assist SC-CO₂ penetration into thermally activated microcracks and reduce the effective resistance at the fracture tip. The fitted positive thermal-stress coefficient indicates non-additivity within the selected sparse model structure; however, because temperature and stress difference were not varied factorially, this coefficient cannot be interpreted as an independently established thermo-mechanical interaction. Phase-field simulations have also linked SC-CO₂ pressure and phase behavior to concentrated damage near the wellbore [26,33–36].

Stress difference affected fracture-network morphology in the opposite direction. At 3 MPa, increasing temperature from 25 to 200 °C increased fracture volume by 195% and fractal dimension by 5.4%. At 200 °C, increasing the stress difference to 9 MPa reduced fracture volume by 27% relative to the 3 MPa condition. Ju et al. [18] reported a similar

loss of SC-CO₂ fracture complexity with increasing stress difference. Within the tested stress-difference range (3–9 MPa), higher stress difference was associated with reduced fracture complexity. This observation is consistent with the concept that stronger stress constraints may promote more directionally dominated propagation.

5.2. Stage-Dependent Responses in Sequential Dual-Well Fracturing

The two wellbores are parallel but are not strictly equivalent under spatial interchange because the perforation positions are vertically offset and the two stages use different tracer materials. The sequential injection protocol further imposes an ordered loading path. Under this fixed sequence, the second-stage breakdown pressure was consistently higher than the first-stage value, demonstrating a reproducible stage-dependent response difference. However, the present design does not distinguish this difference uniquely from other possible contributions, including wellbore or perforation differences, pipeline dead volume, tracer effects, the shut-in state of the first well, and residual specimen heterogeneity. Therefore, the observed pressure difference is treated as a stage-dependent response difference under the adopted protocol rather than as evidence of symmetry breaking. No exact spatial symmetry transformation between Well A and Well B is assumed in the present analysis. Reverse-sequence, simultaneous-injection, and independent single-well controls would be required to isolate the origin of this asymmetry. Stress redistribution after first-stage fracturing may contribute to this behavior, as suggested by previous multi-well studies [37–39], but it cannot be isolated from the present experimental design.

It should be noted that the identified non-additive coefficients are conditional on the selected sparse model structure and represent relationships within the tested experimental domain rather than universal pairwise effects [40–44]. The Choquet integral is used here solely as a non-additive aggregation method that allows the contribution of one evaluation indicator to depend on another. This statistical non-additivity is conceptually independent of the experimentally observed stage-dependent response difference. Accordingly, no mathematical equivalence or mechanistic linkage between Choquet non-additivity and spatial or stage-wise symmetry is assumed in this study.

6. Conclusions

This study combined true triaxial sequential dual-well tests, CT-U-Net fracture reconstruction, CRITIC weighting, a sparse second-order Choquet integral, and Monte Carlo perturbation. The analysis focused on stage-dependent responses in sequential dual-well fracturing and on how non-additive indicator relationships affect fracability ranking.

- (1) The second-stage breakdown pressure exceeded the first-stage value in every test, demonstrating a consistent stage-dependent response difference under the fixed sequential injection protocol. Because reverse-sequence, simultaneous-injection, and independent single-well controls were not included, this result is interpreted descriptively rather than as direct evidence of formal symmetry breaking or uniquely attributed to first-stage stress redistribution. CRITIC assigned the largest information weights to stress compatibility (0.261) and injection-work demand (0.235).
- (2) Under the selected sparse second-order Choquet specification, the model yielded temperature–fluid (0.122) and temperature–stress (0.071) non-additive coefficients, whereas the pressure–work coefficient was close to zero (−0.008). Given the small sample size and coefficient uncertainty, these values are treated as model-dependent non-additivity parameters rather than independently established physical interactions; in particular, the −0.008 coefficient is not interpreted as a stable redundancy effect. Because the experimental matrix was non-factorial and fluid mobility partly incorporates temperature-dependent viscosity, the temperature-related coefficients

should be interpreted as model-dependent associations rather than independently identified physical interactions. The corresponding Shapley values differed substantially from the CRITIC weights, confirming that information contrast and marginal contribution are not interchangeable.

- (3) Based on the averaged experimental responses, condition No. 5 (SC-CO₂, 200 °C, 3 MPa) ranked first with an index of 0.935 and a 95% Monte Carlo interval of 0.908–0.945. Conditions No. 6 and No. 4 formed a second tier because their intervals and rankings were sensitive to the balance between fracture development and operational economy. The Monte Carlo analysis quantified measurement and parameter uncertainty after experimental averaging; it was not intended to replace independent experimental replication.
- (4) LOOCV gave Spearman $\rho = 0.830$, $R^2 = 0.860$, and RMSE = 0.080. These results support ranking within the tested design space, but the small, nonorthogonal dataset and artificial specimens limit external generalization.

The model is best used as a transparent screening tool for the tested laboratory domain. Validation with natural rock, a fuller experimental design, and field-scale thermo-hydro-mechanical data is required before transferring the numerical ranking to reservoir operations. Direct measurements of fluid-injection temperature and transient wellbore-wall temperature are required in future experiments to distinguish imposed specimen temperature from the actual thermo-mechanical loading history.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/sym18091527/s1>. Figure S1. Representative complete pressure–time curves are shown for each condition.

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