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Modeling a Multimodal Routing Problem with Flexible Time Window in a Multi-Uncertainty Environment

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Abstract: In this study, we extend the research on the multimodal routing problem by considering flexible time window and multi-uncertainty environment. A multi-uncertainty environment includes uncertainty regarding the demand for goods, the travel speed of the transportation mode, and the transfer time between different transportation modes. This environment further results in uncertainty regarding the delivery time of goods at their destination and the earliness and lateness caused by time window violations. This study adopts triangular fuzzy numbers to model the uncertain parameters and the resulting uncertain variables. Then, a fuzzy mixed integer nonlinear programming model is established to formulate the specific problem, including both fuzzy parameters and fuzzy variables. To make the problem easily solvable, this study employs chance-constrained programming and linearization to process the proposed model to obtain an equivalent credibilistic chance-constrained linear programming reformulation with an attainable global optimum solution. A numerical case study based on a commonly used multimodal network structure is presented to demonstrate the feasibility of the proposed method. Compared to hard and soft time windows, the numerical case analysis reveals the advantages of the flexible time window in reducing the total costs, avoiding low reliability regarding timeliness, and providing confidence level-sensitive route schemes to achieve flexible routing decision-making under uncertainty. Furthermore, the numerical case analysis verifies that it is necessary to model the multi-uncertainty environment to satisfy the improved customer requirements for timeliness and enhance the flexibility of the routing, and multimodal transportation is better than unimodal transportation when routing goods in an uncertain environment. The sensitivity analysis in the numerical case study shows the conflicting relationship between the economic objective and the reliability regarding the timeliness of the routing, and the result provides a reference for the customer to find a balance between them.

Keywords: multimodal routing; flexible time window; multi-uncertainty environment; fuzzy programming; credibilistic chance-constrained programming



Citation: Ge, Y.; Sun, Y.; Zhang, C. Modeling a Multimodal Routing Problem with Flexible Time Window in a Multi-Uncertainty Environment. *Systems* **2024**, *12*, 212. <https://doi.org/10.3390/systems12060212>

Academic Editor: Mahyar Amirgholy

Received: 5 May 2024

Revised: 31 May 2024

Accepted: 13 June 2024

Published: 15 June 2024



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1. Introduction

Multimodal transportation utilizes various transportation modes (e.g., rail, road, and water) to perform the “door-to-door” transportation of goods, in which the respective advantages of different transportation modes can be integrated into combined transportation [1]. Multimodal transportation shows a great economic advantage compared to unimodal transportation, especially in long-haul transportation [2]. However, the organization of multimodal transportation is more complicated than that of unimodal transportation [3]. The multimodal transportation operator (MTO) should use infrastructure and services to travel and transfer the goods to accomplish the transportation orders proposed by the customers [4]. It is difficult for the MTO to maintain timeliness and reliability when organizing complex transportation in a long-haul setting. In response to this challenge, MTOs are paying close attention to the multimodal routing problem (MRP), which aims to design

the best route for the origin-to-destination transportation of the goods. By applying the best route, which is planned in advance, the economy, timeliness, reliability, etc., of the transportation, which are considered the objectives of the MRP, can be improved [5], which significantly enhances the service level of the transportation provided by the MTO.

Currently, the MRP is a highlighted research field in multimodal network planning [6]. An economic advantage is the most important factor influencing the selection of a transportation service [7]. This can be fully reinforced by the MRP, which minimizes the costs related to multimodal transportation. By lowering transportation related costs, the companies' total production costs can be reduced, and they can become more competitive, meaning that more profits can be made. Therefore, almost all relevant studies formulate the minimization of costs as their objective, not only in single-objective MRP models but also in multi-objective MRP models.

In addition to the economic objective, improving timeliness is another important consideration in the MRP, as customers pay great attention to whether delivery takes place at the right time. This can be achieved by minimizing the inventory and adopting the "Just-in-Time" strategy [8]. Under this condition, using an MRP with time window has been widely discussed in the literature. There are two main kinds of time windows formulated in the relevant studies. A hard time window (HTW) requires the delivery of the goods to be accomplished within a given time interval expected by the customer. An HTW provides strict bound constraints and reduces the flexibility of the MRP; therefore, it has been modeled in only a few studies [9–12]. Most MRP studies focusing on time windows prefer to model a soft time window (STW), where violations of the expected time window are allowed and penalized, providing customers with potential economic benefits [2,13–19].

The STW accepts the delivery of goods at any time within the planning horizon [20]. However, customers usually tolerate time window violations to a certain extent. Therefore, besides the expected time window, the customer has an endurable time window, which has the same concept as the HTW, and the endurable time window covers the interval of the expected time window. In this case, the concept of a flexible time window (FlexTW), proposed by Taş et al. [20] in the vehicle routing problem, is more suitable to represent the customer's timeliness requirements. Moreover, the FlexTW can be easily converted into the HTW or STW by modifying the bounds of the endurable time window. However, to the best of our knowledge, no relevant studies have been conducted using a FlexTW. Furthermore, there is no existing literature analyzing and comparing the feasibility of the HTW, STW, and FlexTW in the MRP, as well as an MRP under uncertainty.

Reliability is also a key influencing factor that determines whether the route planned in advance can accomplish the transportation order according to the customer's requirements in real-world transportation. Modeling uncertainty plays a promising role in improving the reliability of not only transportation planning [21] but also the entire process systems engineering [22]. The MRP should be performed before real-world transportation. The working conditions of the multimodal network are influenced by various factors, as summarized by Zhang et al. [16], and are, therefore, constantly changing, which makes the prediction of the real-time values of the network parameters impossible [23]. Therefore, the MRP should deal with uncertainty to enhance its reliability and realize the successful transportation of the transportation order. The sources of uncertainty in the MRP derive from the customer and the multimodal network.

As for the customer, the demand for goods fluctuates and is difficult to accurately predict [16]. Currently, demand uncertainty has been explored in MRP research [13,15,16,24–27], in which Li et al. [25] and Zhao et al. [26] adopted stochastic programming, while other researchers used fuzzy programming, which takes a fuzzy number to model the uncertain demand. For the multimodal network, the uncertainty includes the travel speed of the transportation mode and the transfer time between different transportation modes. Speed uncertainty has attracted limited attention in the MRP literature. Quite a few studies on the MRP [28] and vehicle routing [29,30] discuss the speed uncertainty in their modeling. In contrast, the existing MRP articles attach great importance to the travel time uncertainty

that actually results from speed uncertainty, in which stochastic programming is commonly utilized to establish optimization models [27,31–36], and some are found to employ fuzzy programming [37–39]. Compared to the investigation of demand uncertainty and travel time uncertainty, studies of transfer time uncertainty are quite limited. Some works can be found in this regard [27,34,37].

Although uncertainty has been discussed in relation to the MRP, the combination of uncertainty of the demand, speed, and time to achieve improved reliability has not been given full consideration in the literature. To the best of our knowledge, there are a few studies that cover the uncertainty of demand, travel time, and transfer time [16,27]. In these works, the authors use stochastic programming to separately model the MRP, considering the due-date constraint and STW in such a multi-uncertainty environment. However, according to the literature, stochastic programming has the following limitations that reduce its feasibility:

- (1) There are not enough data in most decision-making cases to fit the probability distribution of uncertain parameters [40,41].
- (2) Taking a certain distribution (e.g., normal distribution) to formulate the uncertainty is applicable in theory; however, it might not match the real world [23].
- (3) Using a large number of scenarios to represent the uncertainty is widely employed in stochastic programming, which challenges the computational efficiency of problem-solving [15].

As an alternative, fuzzy programming can be employed to formulate the problem under uncertainty, in which the uncertainty is modeled by fuzzy numbers. The fuzzy number representation of an uncertain parameter can be determined by combining available data and decision-makers' expert knowledge [42]. Establishing a fuzzy number is thus easier and more feasible than modeling the stochasticity and does not lead to computationally challenging problems. Moreover, a fuzzy programming model can be effectively solved via many well-studied approaches, e.g., chance-constrained programming and robust possibilistic programming. As a result, this study employs fuzzy programming to deal with the problem. Although fuzzy numbers are easier to construct, their accuracy in representing uncertain parameters relies heavily on available data and expert knowledge, and decision-makers should emphasize high-quality data and expert input when applying fuzzy numbers in practice.

STWs are widely incorporated into MRP in a fuzzy environment. The fuzziness of demand, speed, and time leads to the fuzziness of the delivery time of the goods and, accordingly, results in the fuzziness of the earliness and lateness caused by the time window violations. Such fuzziness is not well formulated in the literature. Only Sun et al. [13,28] considered the fuzziness related to the STW. However, the studies by Sun et al. [13,28] have not fully covered the uncertainty of demand, speed, and time. Moreover, the FlexTW has not been considered in the studies of MRP under fuzziness.

Above all, existing studies have explored MRP with time windows in an uncertain environment and achieved solid research progress in this field. However, their work still has the following weaknesses that need further studying to improve the feasibility of MRP optimization and support real-world transportation:

- (1) Both HTW and STW have their respective disadvantages that reduce their feasibility in the MRP that emphasizes the timeliness of transportation, while a FlexTW, which might yield a better performance, has not been tried in the MRP literature.
- (2) Although the uncertain environment has been considered in the MRP literature, the modeling of uncertainty in these studies does not fully cover its multiple sources, which include demand, time, and speed.
- (3) There are no existing articles that combine the FlexTW and an uncertain environment in the MRP. The combination of the uncertain delivery time of goods caused by the uncertain environment with FlexTW is thus not modeled in the relevant studies.

- (4) The feasibility of formulating FlexTW and a multi-uncertainty environment in the MRP has not been verified in the existing articles. Such a verification should be conducted to ensure that such a consideration is feasible.

To overcome the research weaknesses of the existing literature on the time window formulation and uncertainty modeling indicated above, this study aims to extend the MRP from the FlexTW and multi-uncertainty environment. The following contributions are made by this study:

- (1) A FlexTW is integrated into MRP under uncertainty, considering its potential benefits in reducing costs, handling uncertainty, and enabling flexible decision-making.
- (2) A multi-uncertainty environment is modeled in the MRP to obtain an improved reliability of transportation, in which the demand for goods, the travel speed of the transportation mode, and the transfer time between different transportation modes are considered uncertain.
- (3) Based on the use of triangular fuzzy numbers (TFNs) [43] to model the uncertainty, a fuzzy nonlinear optimization model containing both fuzzy parameters and fuzzy variables and its equivalent credibilistic chance-constrained linear reformulations are established to address the problem in which we model the fuzziness of the delivery time of the goods and its earliness and lateness regarding the FlexTW.
- (4) A systematic numerical case study is conducted to verify the feasibility of the MRP with FlexTW in a multi-uncertainty environment, in which the following questions are addressed:
 - Does FlexTW show better performance in the MRP in a multi-uncertainty environment? To answer this question, we need to compare the FlexTW with the HTW and STW in the problem optimization.
 - Does considering the multi-uncertainty environment improve the optimization of the MRP? To answer this question, we need to compare the optimization results of the uncertain modeling with those of the deterministic modeling.
 - Is multimodal transportation a better choice for the customer to route the goods with FlexTW in a multi-uncertainty environment? To answer this question, we need to compare the performance of MRP with the unimodal routing problem (URP) in goods transportation.
 - How does the confidence level introduced by chance-constrained programming influence the optimization results? To answer this question, we need to conduct a sensitivity analysis to reveal the change in the best route with the variation in the confidence level values.

The rest of the sections in this study are organized as follows. In Section 2, we describe the specific MRP, formulate the FlexTW, and model the multi-uncertainty environment. An optimization model is then established to deal with the problem under the above considerations. In Section 3, we process the proposed model using chance-constrained programming and linearization to obtain an equivalent crisp linear reformulation to make the problem easily solvable. In Section 4, a numerical case study is systematically presented to answer the four questions proposed above, in which some managerial insights are summarized to help the MTO and customer better organize multimodal transportation in a multi-uncertainty environment to satisfy customer demands. Finally, the conclusions of this study are given in Section 5.

2. Problem Modeling

2.1. Proposed MRP

2.1.1. Problem Description

In this study, we consider multimodal transportation that uses rail, road, and water. A multimodal network consists of nodes (origin, destination, and transfer nodes), links, and transportation modes on the links. Let us take an illustrative multimodal network shown in Figure 1 to show the transportation process of the goods. In this network, the transportation

of goods from the origin to the destination can be divided into the travel stage of goods on the link and the transfer stage of goods at the transfer node when transportation modes e and f are different. In this case, the transfer of goods from transportation mode e to transportation f can realize the combination of the two transportation modes in the transportation process. However, it will consume time and incur costs to carry out the operations. When transportation modes e and f are the same, there are no transfer operations for the goods at the transfer node, in which the goods actually use unimodal transportation and thus avoid increased transfer times and costs. Consequently, there are unimodal and multimodal routes in the multimodal network. The MTO can select a unimodal route if it is better than the multimodal route.

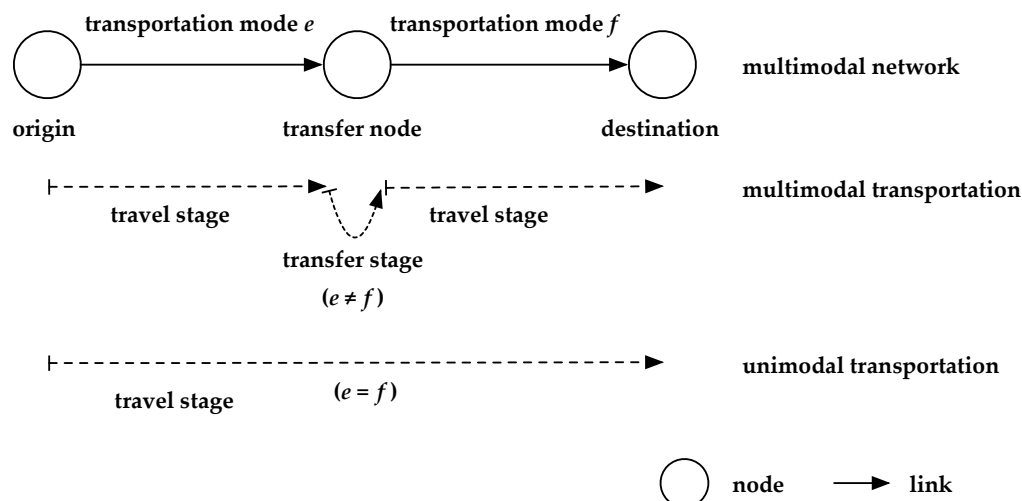


Figure 1. An illustrative multimodal network with 3 nodes.

The customer proposes the transportation order in which the origin, destination, fuzzy demand, release time, and FlexTW are provided. After receiving the transportation order, the MTO represents the customer to use the infrastructure and services in the multimodal network to travel and transfer the goods from the origin to the destination, in which the MTO should carry out the MRP according to the customer's requirement. An MRP aims to find the best route for the customer to improve the service level of transportation.

According to Chang [44], logistics costs account for approximately 30–50% of the total production costs of companies. In multimodal transportation, lowering transportation-related costs helps the customer reduce the total costs of the products, improve competition in the market, and make more profits. Therefore, costs are the most important consideration when selecting transportation services. In this situation, modeling MRP should take the minimization of the total costs paid for accomplishing the transportation order as its optimization objective.

2.1.2. Flexible Time Window Formulation

To improve the timeliness of the delivery service of multimodal transportation, this study considers the customer's requirement for on-time transportation, which means that the delivery of the goods should be neither too early nor too late. As claimed in Section 1, the customer has an expected time window to receive the goods; however, they also accept certain time window violations on the condition that the violations are tolerable and benefits can be obtained through the violations. In such a situation, this study formulates a FlexTW (i.e., $[l', l, u, u']$) in the MRP by referring to Taş et al. [20].

In the FlexTW, l represents the earliest allowable time for delivery, and u is the latest allowable time for delivery. l' and u' are the flexible earliest and latest allowable times for delivery and reflect the customer's tolerance for early and delayed delivery in that the delivery should be neither earlier than l' nor later than u' . In the FlexTW, $[l, u]$ is the expected time window of the customer for delivery, and the customer is satisfied with the

delivery service if the goods are delivered within $[l, u]$. However, a delivery accomplished within $[l', l]$ or $[u, u']$ affects customer satisfaction and should be penalized based on the resulting earliness or lateness. Therefore, the total costs to accomplish the transportation order include travel, transfer, and penalty costs.

2.1.3. Multi-Uncertainty Environment Modeling

In this study, we consider that the demand for goods, the travel speed of the transportation mode, and the transfer time between different transportation modes are uncertain. To model uncertainty in a feasible way, we use TFNs to represent the uncertain parameters. This type of fuzzy number has been widely used not only in the multimodal routing problem but also in the network design problem [45,46], vehicle routing problem [47,48], transportation problem [49,50], etc., when the problems were studied under uncertainty.

A non-negative TFN can be represented by $\tilde{r} = (r^1, r^2, r^3)$, where $r^3 \geq r^2 \geq r^1 \geq 0$. For a fuzzy demand, $\tilde{r}$ can help the advanced MRP cover all possible demand conditions that emerge in real-world transportation, in which r^1 represents an insufficient demand case, r^3 means a high demand case, and r^2 indicates the demand in most cases. For the fuzzy travel speed and transfer time, $\tilde{r}$ includes the best, most likely, and worst working conditions of the equipment and facilities that could possibly occur in real-world transportation and influence the speed and time of the transportation process. r^1 shows that the travel and transfer stages are in extremely good condition and yield the highest efficiency, while r^3 means that the two stages are in extremely bad condition, where bad weather, traffic congestion, accidents, or breakdowns occur, and are of the lowest efficiency. r^2 expresses that the travel and transfer stages are under the most likely conditions and yield an efficiency between the two extreme conditions. For $\tilde{r} = (r^1, r^2, r^3)$, r^1 and r^3 occur in the real world with extremely low possibility; however, r^2 is most likely to appear in practice, which can be reflected by its fuzzy membership degree, illustrated in Figure 2.

Fuzzy membership degree

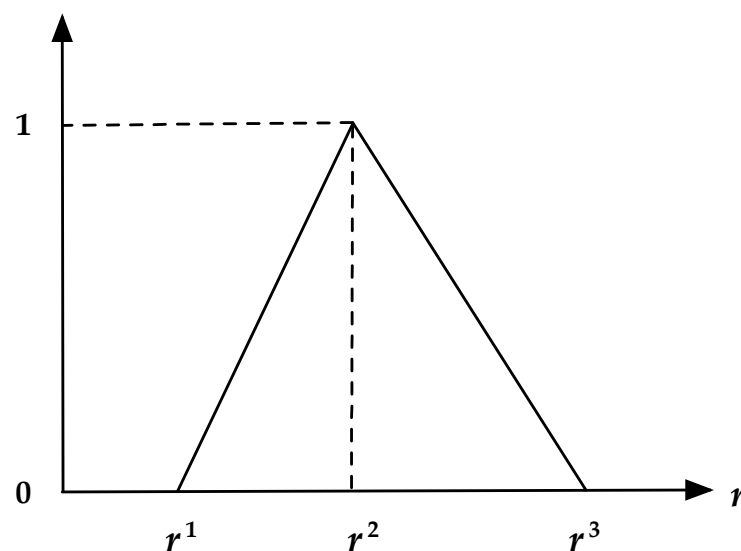


Figure 2. Fuzzy membership degree of a TFN.

Consequently, the use of TFNs helps the MRP to consider all possible conditions of real-world transportation that cannot be exactly determined in advance, improving the MRP's reliability. Demand, travel speed, and transfer time are all associated with the delivery time of the goods at the destination. Therefore, the delivery time of the goods is also uncertain and is actually a TFN. The earliness and lateness regarding the FlexTW depend on the delivery time and expected time window. Therefore, the earliness and lateness that determine the penalty costs are also fuzzy. Under this condition, the MRP

model contains both fuzzy parameters and fuzzy variables. The modeling of the problem under fuzziness should follow some basic fuzzy arithmetic operations. These are shown as Equations (1)–(4), where (r^1, r^2, r^3) is a non-negative TFN that follows $r^3 \geq r^2 \geq r^1 \geq 0$ in Equations (1)–(3) and a positive TFN satisfying $r^3 \geq r^2 \geq r^1 > 0$ in Equation (4), and r is a non-negative deterministic number [51–53].

$$(r^1, r^2, r^3) + (r'^1, r'^2, r'^3) = (r^1 + r'^1, r^2 + r'^2, r^3 + r'^3) \quad (1)$$

$$(r^1, r^2, r^3) \cdot (r'^1, r'^2, r'^3) = (r^1 \cdot r'^1, r^2 \cdot r'^2, r^3 \cdot r'^3) \quad (2)$$

$$r \cdot (r^1, r^2, r^3) = (r \cdot r^1, r \cdot r^2, r \cdot r^3) \quad (3)$$

$$r / (r'^1, r'^2, r'^3) = (r / r'^3, r / r'^2, r / r'^1) \quad (4)$$

2.2. Optimization Model

2.2.1. Symbol Definition

The symbols used to establish the optimization model are given in Table 1, in which they are classified into sets, indices, parameters, and variables. Using these symbols, a fuzzy mixed integer nonlinear programming model (FMINLPM) is constructed based on the modeling structure of Li et al. [2] and Ge and Sun [23] in Section 2.2.2 to address the specific MRP.

Table 1. Symbols used to establish the FMINLPM.

Sets and Indices	Definitions
N	Sets of the nodes in the multimodal network
L	Set of the links connecting the nodes in the multimodal network
M	Set of transportation modes running on the links in the multimodal network
h, i, j	Indices of the nodes, $h, i, j \in N$
n^-	Index of the origin of the transportation order, $n^- \in N$
n^+	Index of the destination of the transportation order, $n^+ \in N$
N_i	Set of the predecessor nodes to node i , $N_i \subseteq N$
N'_i	Set of the successor nodes to node i , $N'_i \subseteq N$
(i, j)	Index of the link from node i to node j , $(i, j) \in L$
M_{ij}	Set of the transportation modes on link (i, j) , $M_{ij} \subseteq M$
M_i	Set of the transportation modes connecting node i , $M_i \subseteq M$
e, f	Indices of the transportation modes, $e, f \in M$
p	Index of the prominent points of the TFN, $p \in \{1, 2, 3\}$
Parameters	Definitions
d_{ij}^f	Travel distance in km of transportation mode f on link (i, j)
$\tilde{s}_{ij}^f$	Fuzzy travel speed in km/hr of transportation mode f on link (i, j) , $\tilde{s}_{ij}^f = (s_{ij}^{f-1}, s_{ij}^{f-2}, s_{ij}^{f-3})$
$\tilde{t}_i^{ef}$	Fuzzy time in hr/TEU to transfer the goods from transportation mode e to transportation mode f at node i , $\tilde{t}_i^{ef} = (t_i^{ef-1}, t_i^{ef-2}, t_i^{ef-3})$
c_{ij}^f	Travel costs in CNY/TEU of transportation mode f on link (i, j)
$c_{ij}^{f'}$	Travel costs in CNY/TEU/km of transportation mode f on link (i, j)
c_i^{ef}	Costs in CNY/TEU to transfer the goods from transportation mode e to transportation mode f at node i
c_{early}	Penalty costs in CNY/TEU/hr for earliness of delivery
c_{late}	Penalty costs in CNY/TEU/hr for lateness of delivery
$\tilde{q}$	Fuzzy demand in TEU for the goods of the transportation order, $\tilde{q} = (q^1, q^2, q^3)$
t_0	Release time of the transportation order at the origin
$[l', l, u, u']$	FlexTW of delivering the goods at the destination

Table 1. Cont.

Variables	Definitions
x_{ij}^f	0–1 variable. When the goods are moved via transportation mode f on link (i, j) , $x_{ij}^f = 1$; otherwise, $x_{ij}^f = 0$.
y_i^{ef}	0–1 variable. When the goods are transferred from transportation mode e to transportation mode f at node i , $y_i^{ef} = 1$; otherwise, $y_i^{ek} = 0$.
$\tilde{z}$	Non-negative fuzzy variable indicating the delivery time of the goods at the destination, $\tilde{z} = (z^1, z^2, z^3)$
$\tilde{v}$	Non-negative fuzzy variable in hr indicating the earliness caused by early delivery, $\tilde{v} = (v^1, v^2, v^3)$
$\tilde{w}$	Non-negative fuzzy variable in hr indicating the lateness caused by delayed delivery, $\tilde{w} = (w^1, w^2, w^3)$

2.2.2. Model Formulation

$$\begin{aligned} \text{minimize } & \sum_{(i,j) \in L} \sum_{f \in M_{ij}} \left(c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^f \right) \cdot \tilde{q} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot \tilde{q} \cdot y_i^{ef} \\ & + c_{early} \cdot \tilde{q} \cdot \tilde{v} + c_{late} \cdot \tilde{q} \cdot \tilde{w} \end{aligned} \quad (5)$$

such that

$$\sum_{h \in N_i} \sum_{e \in M_{hi}} x_{hi}^e - \sum_{j \in N_i'} \sum_{f \in M_{ij}} x_{ij}^f = \begin{cases} -1 & i = n \\ 0 & \forall i \in N \setminus \{n^-, n^+\} \\ 1 & i = n' \end{cases} \quad (6)$$

$$\sum_{f \in M_{ij}} x_{ij}^f \leq 1 \quad \forall (i, j) \in L \quad (7)$$

$$\sum_{e \in M_i} \sum_{f \in M_i} y_i^{ef} \leq 1 \quad \forall i \in N \setminus \{n^-, n^+\} \quad (8)$$

$$\sum_{h \in N_i} x_{hi}^e = \sum_{f \in M_i} y_i^{ef} \quad \forall i \in N \setminus \{n^-, n^+\} \quad \forall e \in M_i \quad (9)$$

$$\sum_{e \in M_i} y_i^{ef} = \sum_{j \in N_i'} x_{ij}^f \quad \forall i \in N \setminus \{n^-, n^+\} \quad \forall f \in M_i \quad (10)$$

$$z^p = t_0 + \sum_{(i,j) \in L} \sum_{f \in M_{ij}} \frac{d_{ijk}}{s_{ij}^{f-4-p}} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} \left(t_i^{ef-p} \cdot q^p \right) \cdot y_i^{ef} \quad \forall p \in \{1, 2, 3\} \quad (11)$$

$$\tilde{z} = (z^1, z^2, z^3) \quad (12)$$

$$\tilde{z} \geq l' \quad (13)$$

$$\tilde{z} \leq u' \quad (14)$$

$$v^p = \max\{0, l - z^{4-p}\} \quad \forall p \in \{1, 2, 3\} \quad (15)$$

$$\tilde{v} = (v^1, v^2, v^3) \quad (16)$$

$$w^p = \max\{0, z^p - u\} \quad \forall p \in \{1, 2, 3\} \quad (17)$$

$$\tilde{w} = (w^1, w^2, w^3) \quad (18)$$

$$x_{ij}^f \in \{0, 1\} \quad \forall (i, j) \in L \quad \forall f \in M_{ij} \quad (19)$$

$$y_i^{ef} \in \{0, 1\} \quad \forall i \in N \setminus \{n^-, n^+\} \quad \forall e \in M_i \quad \forall f \in M_i \quad (20)$$

$$z^3 \geq z^2 \geq z^1 \geq 0 \quad (21)$$

$$v^3 \geq v^2 \geq v^1 \geq 0 \quad (22)$$

$$w^3 \geq w^2 \geq w^1 \geq 0 \quad (23)$$

The optimization objective Equation (5) minimizes the total costs of accomplishing the transportation order. The total costs consist of the travel costs paid for moving goods by transportation modes on the links, the transfer costs paid for transferring goods at the transfer nodes, and the penalty costs caused by time window violations at the destination. Equation (6) is the flow equilibrium constraint of the transportation order. Equation (7) means that only one transportation mode can be used to move goods on the selected link. Equation (8) requires that only one transfer type can be used to transfer goods at the selected transfer node. The combination of Equations (7) and (8) ensures that the goods are unsplittable in multimodal transportation. Equations (9) and (10) make the travel and transfer stages connect at the transfer nodes to generate a feasible multimodal route in space. Equations (11) and (12) determine the fuzzy delivery time of the goods at the destination, in which its prominent points are calculated via Equation (11) based on the fuzzy arithmetic operations. Equations (13) and (14) are the FlexTW constraints to guarantee that the delivery time of the goods falls into the time interval from the flexible earliest allowable time to the flexible latest allowable time. Equations (15) and (16) give the earliness that is fuzzy caused by the fuzziness of the delivery time of the goods, in which its prominent points are presented by Equation (15), which is a piecewise linear function and thus nonlinear. Equations (17) and (18) calculate the fuzzy lateness using functions similar to Equations (15) and (16). Equations (19)–(23) indicate the domains of the variables based on their definitions.

3. Model Reformulation

The proposed FMINLPM in Section 2.2.2 is fuzzy and nonlinear. Compared to the linear optimization model, which is easily solvable, it is impossible to solve the fuzzy nonlinear optimization model straightforwardly, which makes the best route unattainable for the MTO and the customer. In this case, we should first reformulate the initial FMINLPM in Section 2.2.2 to realize its defuzzification and make the MRP solvable. Although the crisp model obtained via defuzzification is solvable, it is nonlinear, which makes its global optimum solution difficult to find in an acceptable computational time [54]. Although heuristic algorithms can be developed to solve the nonlinear optimization model, it is difficult to identify whether the solution obtained via these algorithms is a local or global optimum. Therefore, linearization should be conducted to construct an equivalent linear model for the MRP with FlexTW in a multi-uncertainty environment, enabling the global optimum solution to be attainable using exact solution algorithms. Consequently, this section aims to reformulate the FMINLPM to present its equivalent crisp linear form. After the simplification of the FMINLPM, the specific MRP can be easily solved to provide the MTO and the customer with the global optimum solution as the route scheme.

3.1. Model Defuzzification

The FMINLPM contains fuzzy information in Equations (5), (13) and (14). In this study, we adopt the commonly used chance-constrained programming (CCP) [55,56] to deal with the proposed model to obtain its crisp reformulation.

3.1.1. Defuzzification of the Fuzzy Optimization Objective

The CCP can employ the fuzzy expected value operator (FEVO) to address the fuzzy optimization objective. Taking into account the linearity of the FEVO and the objective function, we can reformulate Equation (5) as Equation (24), which aims to minimize the expected value of the fuzzy total costs to accomplish the transportation order.

$$\begin{aligned} \text{minimize} \quad & \sum_{(i,j) \in L} \sum_{f \in M_{ij}} \left(c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^f \right) \cdot E[\tilde{q}] \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot E[\tilde{q}] \cdot y_i^{ef} \\ & + c_{early} \cdot E[\tilde{q} \cdot \tilde{v}] + c_{late} \cdot E[\tilde{q} \cdot \tilde{w}] \end{aligned} \quad (24)$$

For a non-negative TFN $\tilde{r} = (r^1, r^2, r^3)$, its fuzzy expected value is determined by Equation (25) [55].

$$E[\tilde{r}] = \frac{r^1 + 2r^2 + r^3}{4} \quad (25)$$

Based on Equation (25), Equation (24) can be rewritten as Equation (26), which removes the fuzziness of the optimization objective of the model.

$$\begin{aligned} \text{minimize} \quad & \sum_{(i,j) \in L} \sum_{f \in M_{ij}} \left(c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^f \right) \cdot \frac{q^1 + 2q^2 + q^3}{4} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot \frac{q^1 + 2q^2 + q^3}{4} \cdot y_i^{ef} \\ & + c_{early} \cdot \frac{q^1 \cdot v^1 + 2q^2 \cdot v^2 + q^3 \cdot v^3}{4} + c_{late} \cdot \frac{q^1 \cdot w^1 + 2q^2 \cdot w^2 + q^3 \cdot w^3}{4} \end{aligned} \quad (26)$$

Besides the FEVO, according to Ji et al. [57], the defuzzification of the optimization objective can also be achieved via Equations (27) and (28), where $\varphi \geq 0$ is an auxiliary variable.

$$\text{minimize } \varphi \quad (27)$$

$$\sum_{(i,j) \in L} \sum_{f \in M_{ij}} \left(c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^f \right) \cdot \tilde{q} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot \tilde{q} \cdot y_i^{ef} + c_{early} \cdot \tilde{q} \cdot \tilde{v} + c_{late} \cdot \tilde{q} \cdot \tilde{w} \leq \varphi \quad (28)$$

Then, Equation (28) can be converted into a chance constraint based on the fuzzy measure. For example, when the credibility measure (Cr) is considered, the chance constraint of Equation (28) is shown as Equation (29), where η is the confidence level and belongs to the interval $[0, 1.0]$.

$$Cr \left\{ \sum_{(i,j) \in L} \sum_{f \in M_{ij}} \left(c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^f \right) \cdot \tilde{q} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot \tilde{q} \cdot y_i^{ef} + c_{early} \cdot \tilde{q} \cdot \tilde{v} + c_{late} \cdot \tilde{q} \cdot \tilde{w} \leq \varphi \right\} \geq \eta \quad (29)$$

The above method proposed by Ji et al. [57] also removes the fuzziness of the optimization objective. However, compared to the FEVO, this method is more complicated and improves the computational complexity of problem-solving. In this method, the change in the objective value might only be caused by the variation in the confidence level value, in which the optimum solution to the problem remains the same. Therefore, we cannot reveal the sensitivity of the optimization result with respect to the confidence level. On the contrary, the change in the value of Equation (26) resulting from the FEVO clearly indicates the change in the optimum solution. Furthermore, Sun and Li [58] verified that the two methods above actually yield the same optimization results. Therefore, this study uses FEVO to handle the fuzzy optimization objective to make the problem easier to solve and the sensitivity analysis more convenient. Finally, the use of Equation (26) as the optimization objective enables the nonlinear constraints (Equations (15) and (17)) to be easily linearized. As a result, the optimization objective of the CCP reformulation of the FMINLPM is Equation (26).

3.1.2. Defuzzification of the Fuzzy Constraints

After the defuzzification of the optimization objective, we build the chance constraints of Equations (13) and (14) based on the credibility measure. Compared to the possibility measure (Pos), which represents an extremely optimistic attitude, and the necessity measure (Nec), which represents an extremely pessimistic attitude, this measure expresses a compromised attitude and better fits the real-world decision-making situation [23]. Moreover, only the credibility measure is self-dual, which means that the fuzzy event must hold or fail when its credibility is 1 or 0 [40]. Therefore, the credibilistic chance constraints of Equations (13) and (14) are Equations (30) and (31), respectively.

$$Cr \{ \tilde{z} \geq l' \} \geq \eta \quad (30)$$

$$Cr\{\tilde{z} \leq u'\} \geq \eta \quad (31)$$

Equations (30) and (31) mean that the credibility that the fuzzy delivery time of the goods complies with the FlexTW requirements should not be less than the minimum confidence level denoted by η , which belongs to the interval $[0, 1.0]$.

For a non-negative TFN $\tilde{r} = (r^1, r^2, r^3)$ and a deterministic number r , the distributions of $Cr\{\tilde{r} \geq r\}$ and $Cr\{\tilde{r} \leq r\}$ are Equations (32) and (33) [55].

$$Cr\{\tilde{r} \geq r\} = \begin{cases} 1 & \text{if } r^1 \geq r \\ \frac{2r^2 - r^1 - r}{2(r^2 - r^1)} & \text{if } r^1 \leq r \leq r^2 \\ \frac{r^3 - r}{2(r^3 - r^2)} & \text{if } r^2 \leq r \leq r^3 \\ 0 & \text{if } r^3 \leq r \end{cases} \quad (32)$$

$$Cr\{\tilde{r} \leq r\} = \begin{cases} 0 & \text{if } r^1 \geq r \\ \frac{r - r^1}{2(r^2 - r^1)} & \text{if } r^1 \leq r \leq r^2 \\ \frac{r + r^3 - 2r^2}{2(r^3 - r^2)} & \text{if } r^2 \leq r \leq r^3 \\ 1 & \text{if } r^3 \leq r \end{cases} \quad (33)$$

Therefore, we have the following formula derivations:

$$Cr\{\tilde{r} \geq r\} \geq \eta \iff \begin{cases} 2\eta \cdot r^2 + (1 - 2\eta) \cdot r^3 \geq r & \text{if } 0 \leq \eta \leq 0.5 \\ (2\eta - 1) \cdot r^1 + 2(1 - \eta) \cdot r^2 \geq r & \text{if } 0.5 \leq \eta \leq 1.0 \end{cases} \quad (34)$$

$$Cr\{\tilde{r} \leq r\} \geq \eta \iff \begin{cases} (1 - 2\eta) \cdot r^1 + 2\eta \cdot r^2 \leq r & \text{if } 0 \leq \eta \leq 0.5 \\ 2(1 - \eta) \cdot r^2 + (2\eta - 1) \cdot r^3 \leq r & \text{if } 0.5 \leq \eta \leq 1.0 \end{cases} \quad (35)$$

According to Equations (34) and (35), the crisp reformulations of Equations (30) and (31) are Equations (36) and (37), respectively.

$$\begin{cases} 2\eta \cdot z^2 + (1 - 2\eta) \cdot z^3 \geq l' & \text{if } 0 \leq \eta \leq 0.5 \\ (2\eta - 1) \cdot z^1 + 2(1 - \eta) \cdot z^2 \geq l' & \text{if } 0.5 \leq \eta \leq 1.0 \end{cases} \quad (36)$$

$$\begin{cases} (1 - 2\eta) \cdot z^1 + 2\eta \cdot z^2 \leq u' & \text{if } 0 \leq \eta \leq 0.5 \\ 2(1 - \eta) \cdot z^2 + (2\eta - 1) \cdot z^3 \leq u' & \text{if } 0.5 \leq \eta \leq 1.0 \end{cases} \quad (37)$$

Above all, we obtain a credibilistic chance-constrained nonlinear programming model (CCCNLPM) for which the optimization objective is Equation (26); the constraints include Equations (6)–(11), (15)–(17), (19)–(23), (36), and (37). It is a parametric optimization model in which the value of the confidence level in the chance constraints should be determined by the decision-makers in advance. Although the confidence level belongs to the interval $[0, 1.0]$, the customers usually do not accept a low credibility that is indicated by a low confidence level as they demand reliable transportation under a flexible time window to ensure that the delivery of goods at the destination should not be too early or too late. Therefore, real-world decision-making prefers a interval of $[0.5, 1.0]$ to set the confidence level value [5,23].

3.2. Model Linearization

After defuzzification, we obtain the CCCNLPM, which is crisp and thus solvable. However, this model is nonlinear due to Equations (15) and (17), which involve piecewise linear functions. To avoid the difficulty of obtaining the global optimum solution to the CCCNLPM within an acceptable computational time, model linearization should be performed to build its equivalent linear form, i.e., a credibilistic chance-constrained linear programming model (CCCLPM). After linearization, we can find such a solution to

CCCLPM, in which mathematical programming software can be utilized to run the exact solution algorithm to solve the specific MRP.

Based on the linearization developed by Sun et al. [13,28], under the control of Equation (26), which minimizes v^p and w^p for $p \in \{1, 2, 3\}$ and their domain constraints (Equations (22) and (23)), nonlinear Equations (25) and (17) can be linearized via Equations (38) and (39), respectively.

$$v^p \geq l - z^{4-p} \quad \forall p \in \{1, 2, 3\} \quad (38)$$

$$w^p \geq z^p - u \quad \forall p \in \{1, 2, 3\} \quad (39)$$

By replacing Equations (15) and (17) in the CCCNLPM with Equations (38) and (39), we finally establish a CCCLPM for which the global optimum solution is attainable.

4. Numerical Case Study

In this section, we present a numerical case study to demonstrate the feasibility of the proposed CCCLPM and answer the four questions proposed in Section 1, which determine the research feasibility of the MRP with FlexTW in a multi-uncertainty environment. Alongside the verification above, we draw some managerial insights from the optimization results. After describing the numerical case, we will analyze the sensitivity of the optimization results with respect to the confidence level and then carry out a series of comparisons: FlexTW vs. HTW/STW, uncertain modeling vs. deterministic modeling, and MRP vs. URP.

4.1. Numerical Case Description

This study presents a rail–road–water multimodal transportation case to verify the feasibility of the proposed model. The structure of the multimodal network in the numerical case study is based on a commonly used one proposed by Sun and Lang [59], in which there are 35 nodes, 69 links, and 136 transportation modes on the links in the multimodal network. It is a generalized structure to ensure that the conclusions and insights drawn from the optimization results are representative.

The travel distances in km of these transportation modes are listed in Table S1 in the supplementary file. The travel costs of the transportation modes have different structures in the Chinese scenario and refer to Li et al. [2] and Ge and Sun [23]. Table 2 presents the travel costs and assumed fuzzy speeds of the transportation modes, in which the most likely speeds of the transportation modes refer to References [2,16,23], and the fuzzy travel speeds are determined using such values as benchmarks.

Table 2. Travel costs and assumed fuzzy speeds of the transportation modes.

Parameters	Rail	Road	Water	Units
c_{ij}^f /Travel costs	500	15	950	CNY/TEU
$c_{ij}^{f'}$ /Travel costs	2.03	18	0	CNY/TEU/km
$\tilde{s}_{ij}^f$ /Fuzzy travel speeds	(50, 60, 70)	(60, 80, 100)	(25, 30, 35)	km/hr

The costs and assumed fuzzy time to transfer goods between different transportation modes are given in Table 3, in which the costs and the most likely time related to the transfer stage are set, as in References [2,23,59], and the fuzzy transfer time is determined in the same way that we set the fuzzy speeds.

In this numerical case study, we consider a three-day planning horizon. The time parameters are converted into real numbers before solving the problem. For example, 6:00 on the second day is converted to 30, and 2:00 on the third day is converted to 50. The transportation order has the information indicated in Table 4.

Table 3. Costs and assumed fuzzy time to transfer goods between transportation modes.

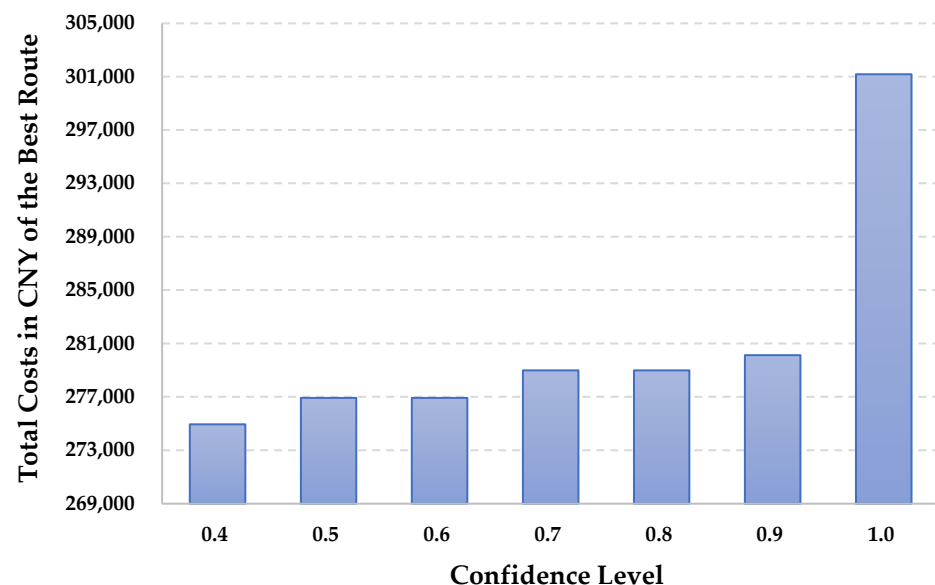
Parameters	Rail–Road	Rail–Water	Road–Water	Units
c_i^{ef} / Transfer costs	5	7	10	CNY/TEU
$\tilde{t}_i^{ef}$ / Fuzzy transfer time	(2.4, 4.0, 6.0)	(4.8, 8.0, 12.0)	(3.6, 6.0, 9.0)	min/TEU

Table 4. Information on transportation order.

Parameters	Settings	Units
n^- / Origin	Node 1	-
n^+ / Destination	Node 35	-
$\tilde{q}$ / Fuzzy demand	(40, 48, 53)	TEU
t_0 / Release time	8	-
$[l', l, u, u']$ / FlexTW	[30, 36, 46, 50]	-
c_{early} / Penalty for earliness	10	CNY/TEU/hr
c_{late} / Penalty for lateness	20	CNY/TEU/hr

4.2. Sensitivity Analysis

In this study, we use Lingo, a well-known mathematical programming software package, to run the branch-and-bound algorithm embedded in it to solve the problem and obtain its global optimum solution. The solution can be found efficiently due to the good performance of this software in solving linear programming models. In the CCCLPM, the confidence level value should be subjectively prescribed within the interval [0.5, 1.0]. To discuss the influence of the confidence level on the optimization result of the problem, we improve its value from 0.4 to 1.0, with a step of 0.1, and obtain the total costs of the best route under different confidence level values. The sensitivity is illustrated in Figure 3. In Figure 3, the optimization result is also presented under a confidence level of 0.4 since it will be used in Section 4.3 to compare the FlexTW with the STW in the MRP in a multi-uncertainty environment.

**Figure 3.** Sensitivity of the total costs of the best route with respect to the confidence level.

As we can see in Figure 3, with the improvement in the confidence level, the total costs of the best route increase. This change is very sensitive when the confidence level improves from 0.9 to 1.0. Improving the confidence level makes Equations (36) and (37) stricter, and the solution space of the MRP, hence, tends to be smaller. A smaller solution space worsens

the best solution to the MRP and further increases the objective value (i.e., the total costs) of the best solution (i.e., the best route) found in it. Therefore, we can obtain a sensitivity illustrated in Figure 3.

An improvement in the confidence level indicates an enhancement in the credibility that the delivery of goods using the best route satisfies the lower and upper bound constraints of the FlexTW in real-world transportation. This means that the reliability regarding time of the best route is improved by using a higher confidence level. Therefore, we can conclude that the reliability and economy of the routing are conflicting, and the optimization of one objective worsens the other.

In this case, the customer should balance lowering the total costs and improving the reliability of the transportation to determine a suitable confidence level. Then, the MTO can plan the best route to satisfy the customer requirements using the proposed CCCLPM. Table 5 shows the best routes selected under two different customer requirements.

Table 5. Best routes selected under different customer requirements.

Customer Requirements	Preferred Confidence Level Values	Best Route	Total Costs (CNY)	Travel and Transfer Costs (CNY)	Penalty Costs (CNY)
On-time transportation is required to implement the Just-in-Time strategy Violating the FlexTW to a certain degree is acceptable for reduced costs	1.0	Rail–water multimodal route: 1—water→3—water→5—water→12— rail→16—rail→21—rail→27— rail→28—rail→35	301,181	29,406	775
	0.7 or 0.8	Rail–water–road multimodal route: 1—road→4—rail→5—water→12— rail→16—rail→21—rail→27— rail→28—rail→35	278,984	275,871	3113

4.3. Comparison between FlexTW and HTW/STW

In this section, we compare the FlexTW with the widely used HTW and STW. When the MRP uses the HTW, it requires that the fuzzy delivery time $\tilde{z}$ must fall into the time window $[l, u]$. Moreover, in the HTW setting, no penalty costs are paid for early and delayed deliveries. For the CCCLPM with HTW, the optimization objective is Equation (40), the time window constraints are Equations (41) and (42), and the other constraints remain unchanged.

$$\text{minimize } \sum_{(i,j) \in L} \sum_{f \in M_{ij}} (c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^f) \cdot \frac{q^1 + 2q^2 + q^3}{4} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot \frac{q^1 + 2q^2 + q^3}{4} \cdot y_i^{ef} \quad (40)$$

$$\begin{cases} 2\eta \cdot z^2 + (1 - 2\eta) \cdot z^3 \geq l & \text{if } 0 \leq \eta \leq 0.5 \\ (2\eta - 1) \cdot z^1 + 2(1 - \eta) \cdot z^2 \geq l & \text{if } 0.5 \leq \eta \leq 1.0 \end{cases} \quad (41)$$

$$\begin{cases} (1 - 2\eta) \cdot z^1 + 2\eta \cdot z^2 \leq u & \text{if } 0 \leq \eta \leq 0.5 \\ 2(1 - \eta) \cdot z^2 + (2\eta - 1) \cdot z^3 \leq u & \text{if } 0.5 \leq \eta \leq 1.0 \end{cases} \quad (42)$$

The total costs of the best route with HTW under different confidence level values are shown in Figure 4. It should be noted that there is no feasible solution to the problem with HTW when the confidence level value is 1.0.

The reasons leading to Figure 4 are the same as those resulting in Figure 3. The combination of Figures 3 and 4 indicates that when the confidence level is low, the use of the HTW can reduce the total costs of the best route compared to the FlexTW. However, the reduction is only 0.38%, which is extremely slight. Under a high confidence level, indicating that the customer prefers reliable transportation for the goods, the HTW considerably increases the total costs. The increase in the total cost is 1.3%, 32.3%, and 58.5% when the confidence

level is 0.7, 0.8, and 0.9, respectively. Under a high confidence level, Equations (41) and (42) in the MRP with the HTW and Equations (36) and (37) in the MRP with the FlexTW become very strict and considerably narrow the solution spaces of the two problems. Obviously, Equations (41) and (42) are much stricter than Equations (36) and (37), and the MRP with the FlexTW can achieve a balance between travel and transfer costs and penalty costs to reduce the total costs of the best route when a high confidence level is selected. Therefore, the MRP with the FlexTW yields a better solution than the MRP with the HTW. In conclusion, the FlexTW is more suitable to help the MRP deal with the uncertain environment since it can make the best route attainable, reduce its total costs, and meanwhile maintain high reliability.

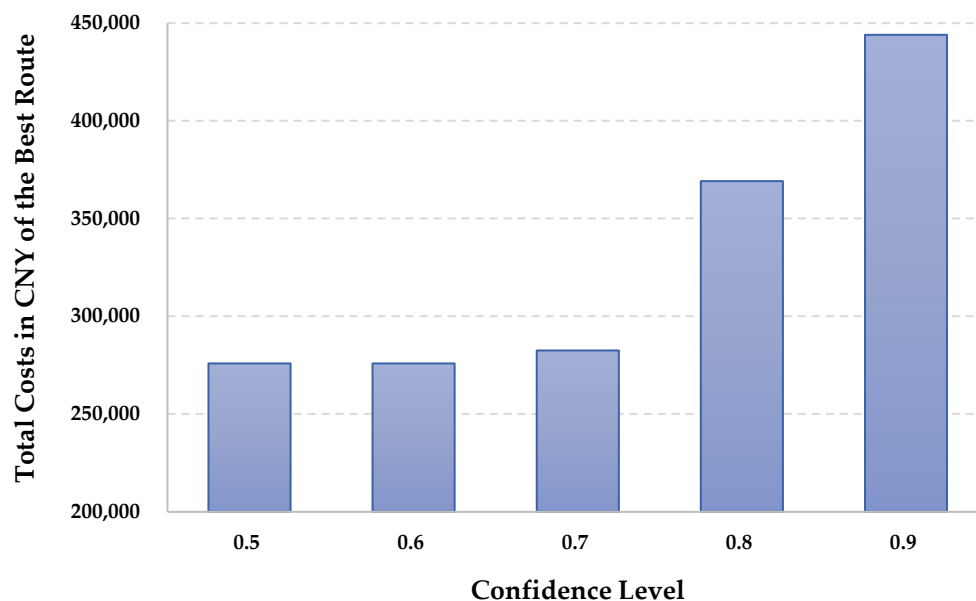


Figure 4. Total costs of the best route with HTW under different confidence level values.

Then, we compare the FlexTW with the STW. The CCCLPM removes Equations (36) and (37) and keeps the optimization objective and other constraints unchanged to formulate the linear routing model with the STW. In this case, the routing model is not associated with the confidence level and is, thus, nonparametric. Therefore, we can obtain a single solution for a specific case. When using the STW, the best route yields the total costs, which is CNY 268,543. As we can see in Figure 3, the best route based on the STW is identical to the one based on the FlexTW when the confidence level is 0.4. Without the restrictions of Equations (36) and (37), the MRP with the STW obtains a larger solution space compared to the MRP with the FlexTW. As a result, the MRP with the STW can find the best solution that is not worse than the MRP with the FlexTW in a larger solution space.

However, the use of the STW leads to a delivery time that has a great chance of violating the customer's endurable earliest and latest time requirements in real-world transportation since the credibility that the fuzzy delivery time satisfies the endurable time window is only 0.4. Consequently, the best route associated with the STW is of low reliability regarding time and, hence, not applicable in real-world transportation when the customer emphasizes timeliness. Compared to the STW, the FlexTW can effectively balance reliability and economy, provide diverse route schemes, and thus make routing decision-making more flexible.

We calculate the penalty costs of the best route with the FlexTW under different confidence level values, and the results are shown in Figure 5.

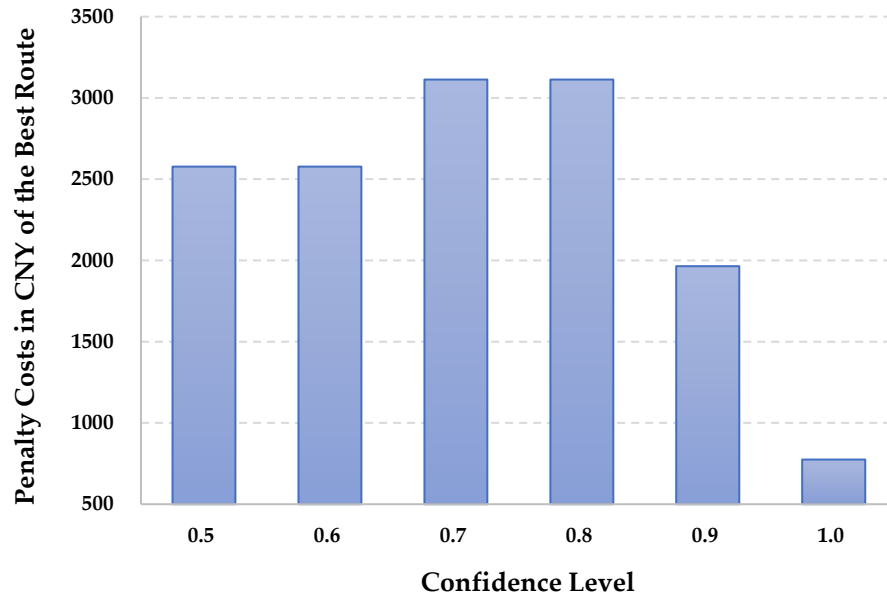


Figure 5. Penalty costs of the best route with FlexTW under different confidence level values.

As illustrated in Figure 5, the change in penalty costs shows that CCCLPM using the FlexTW is able to balance the travel and transfer costs that represent the economy of the routing and the penalty costs that indicate the timeliness of the routing to achieve the best performance in the two objectives under the customer's different requirements on reliability. This enables the FlexTW to help the CCCLPM to effectively handle the economy, timeliness, and reliability of transportation, which makes it more feasible than the HTW and STW when addressing the MRP in a multi-uncertainty environment.

4.4. Comparison between Uncertain Modeling and Deterministic Modeling

Currently, most relevant studies still focus on the MRP under certainty. The existing literature prefers to use the most likely values of the parameters in the deterministic modeling of the MRP [19,23]. In this situation, the optimization objective of the deterministic model should be formulated as Equation (43) and cover Equations (6)–(10), (19), (20) and (44)–(51) as its constraints.

$$\begin{aligned} \text{minimize } \sum_{(i,j) \in L} \sum_{f \in M_{ij}} & \left(c_{ij}^f + c_{ij}^{f'} \cdot d_{ij}^{f'} \right) \cdot q^2 \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} c_i^{ef} \cdot q^2 \cdot y_i^{ef} \\ & + c_{early} \cdot q^2 \cdot v^2 + c_{late} \cdot q^2 \cdot w^2 \end{aligned} \quad (43)$$

$$z^2 = t_0 + \sum_{(i,j) \in L} \sum_{f \in M_{ij}} \frac{d_{ijk}}{s_{ij}^{f-2}} \cdot x_{ij}^f + \sum_{i \in N} \sum_{e \in M_i} \sum_{f \in M_i} \left(t_i^{ef-2} \cdot q^2 \right) \cdot y_i^{ef} \quad (44)$$

$$z^2 \geq l' \quad (45)$$

$$z^2 \leq u' \quad (46)$$

$$v^2 \geq l - z^2 \quad (47)$$

$$w^2 \geq z^2 - u \quad (48)$$

$$z^2 \geq 0 \quad (49)$$

$$v^2 \geq 0 \quad (50)$$

$$w^2 \geq 0 \quad (51)$$

The deterministic model is also a linear formulation and, therefore, can be solved via the branch-and-bound algorithm to obtain its global optimum solution. Based on the

optimization result of the deterministic model, the best route obtained via deterministic modeling for the case is identical to the one under uncertainty when the confidence level is 0.7 and 0.8. As shown in Figure 5, this route produces the highest penalty costs, indicating that its timeliness is the worst. Therefore, when the customer needs a higher confidence level (e.g., 0.9 and 1.0) to ensure reliable transportation to match the “Just-in-Time” production, this route could be infeasible in real-world transportation.

Compared to deterministic modeling, considering uncertainty makes routing decision-making more flexible by generating diverse confidence level-sensitive route schemes for the customer to make the decision according to his/her attitude on the reliability of transportation. Moreover, the route schemes provided by the CCCLPM include those given by deterministic modeling. The flexibility enables the MTO and the customer to balance between lowering total costs and improving reliability. Consequently, considering the multi-uncertainty environment is necessary for the MRP.

4.5. Comparison between MRP and URP

In this section, we compare the MRP and URP in a multi-uncertainty environment and with the FlexTW. In real-world transportation, road transportation is the most representative of unimodal transportation [60,61]. Therefore, the URP uses road-only transportation. Under different confidence level values, the total costs of the best routes given by the URP are shown in Figure 6.

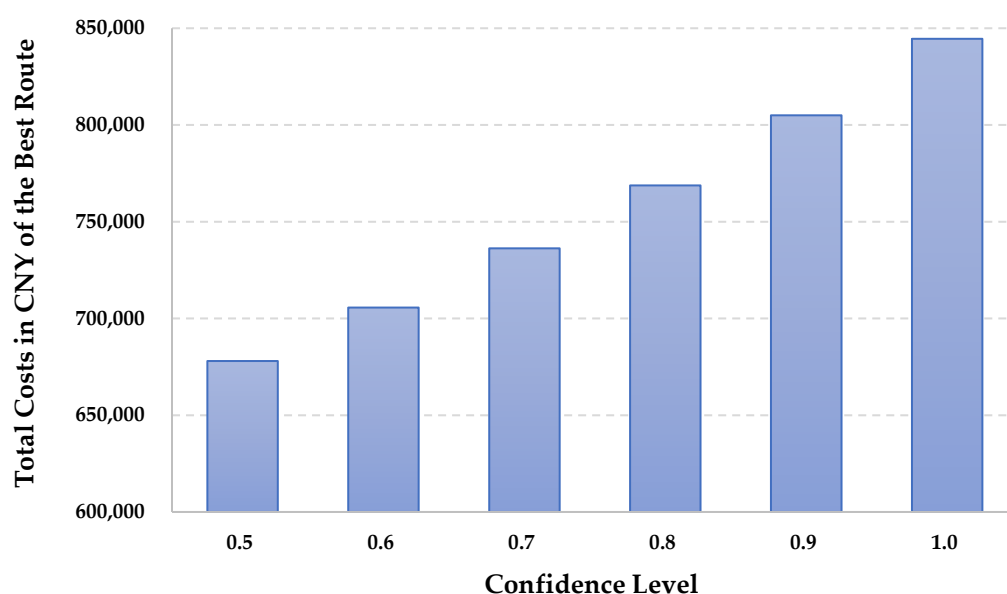


Figure 6. Sensitivity of the best route using road-only transportation with respect to the confidence level.

By comparing Figures 3 and 6, we find that improving the confidence level also increases the total costs of the best route given by the URP. The reasons leading to Figure 6 are the same as those resulting in Figure 3. Under the same confidence level, although there are no transfer costs when using road-only transportation, the total costs of the best route using the URP are, on average, 2.7 times higher than those using the MRP. When we only use road transportation in the multimodal network in the same case, the solution space of the URP is actually the subspace of the MRP’s. Therefore, the URP yields a considerably narrowed solution space compared to the MRP, which extensively increases the objective values of the best route under different confidence level values.

Consequently, multimodal transportation is more applicable than unimodal transportation for the MTO to accomplish the transportation order in an uncertain environment since multimodal transportation can reduce the total costs in any circumstance where the customer has different attitudes on reliability.

5. Conclusions

This study discussed an MRP with FlexTW in a multi-uncertainty environment. We employed TFNs to model the uncertainty of the demand for goods, the travel speed of the transportation mode, and the transfer time between different transportation modes. The resulting fuzziness of the delivery time of goods and the penalty regarding the FlexTW was formulated on the basis of fuzzy arithmetic operations. This study constructed an FMINLPM to address the specific problem and further reformulated its equivalent CCCLPM, for which the global optimum solution is easily attainable. A numerical case study based on a commonly used multimodal network structure verified that the proposed model is feasible and can be used to perform the numerical case analysis. The findings of the numerical case study are presented as follows, which verifies that our study on the MRP with the FlexTW in a multi-uncertainty environment is meaningful:

- (1) The FlexTW is more feasible than the HTW and STW for the MRP to deal with the multi-uncertainty environment. Compared to the HTW, the FlexTW enables the best route under a high confidence level that enables reliable transportation to be attainable and reduces the total costs. Compared to the STW, in addition to avoiding low reliability by setting a high confidence level, the FlexTW results in flexible routing decision-making in which the customer can use the confidence level-sensitive route schemes to make a balance between the economy and reliability of the routing.
- (2) It is necessary for the MRP to consider the multi-uncertainty environment. Compared to deterministic modeling, this consideration enables the MRP to better satisfy the customer's requirements on the timeliness of delivery by selecting a suitable confidence level, in which the worst timeliness can be avoided. It also improves the flexibility of routing decision-making by providing confidence level-sensitive route schemes.
- (3) Multimodal transportation is better than unimodal transportation in routing the goods from the origin to the destination in the multi-uncertainty environment since multimodal transportation can always help the customer to remarkably lower the transportation budget no matter what attitude the customer holds on the reliability of transportation.
- (4) The economy and reliability of the MRP are conflicting. In real-world transportation, the customer needs to balance the two objectives by considering his/her requirements regarding the timeliness of delivery and the transportation budget prepared for the transportation order. The MTO can then use the proposed model to help the customer plan the best route using multimodal transportation.

In real-world transportation, the customer should first consider the use of multimodal transportation to accomplish the transportation order, find a MTO to organize the transportation, and propose the transportation order with a fuzzy demand and FlexTW in advance for the MTO to plan the route. The MTO should establish a multimodal network for transportation orders by coordinating the infrastructure and service operators. These operators should provide the MTO with information on the deterministic and fuzzy parameters of the CCCLPM. Then, based on the FlexTW and the customer's fuzzy demand, the MTO can use the proposed CCCLPM to provide the customer with confident level-sensitive routes. The customer can accordingly select the one that satisfies his/her specific requirements on costs and reliability. This route can finally be applied in real-world transportation by the MTO to accomplish the transportation order.

In future work, we will consider multiple transportation orders in the MRP with FlexTW in a multi-uncertainty environment, in which capacity constraints and capacity uncertainty in the travel and transfer stage in multimodal transportation should be modeled. This consideration will improve the computational complexity of the MRP. Therefore, heuristic algorithms might be suitable and can be developed to effectively solve the problem, and the exact solution method proposed in this study can be used to test their efficiency and accuracy by providing the benchmarks. Furthermore, for the multi-uncertainty environment, we will consider the use of other types of fuzzy numbers (e.g., trapezoidal fuzzy numbers and interval fuzzy numbers) to model the uncertainty and compare the

optimization results with this study. We will also discuss methods for improving the data collection and validation to formulate the uncertainty as fuzzy numbers more accurately.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/systems12060212/s1>, Table S1: Travel distances in km of the three transportation modes on the links [59].

Author Contributions: Conceptualization, Y.G., Y.S. and C.Z.; Data curation, Y.G. and Y.S.; Formal analysis, Y.G., Y.S. and C.Z.; Funding acquisition, Y.S.; Investigation, Y.S.; Methodology, Y.G., Y.S. and C.Z.; Resources, Y.S.; Software, Y.G., Y.S. and C.Z.; Supervision, Y.S.; Validation, Y.G., Y.S. and C.Z.; Writing—original draft, Y.G. and Y.S.; Writing—review and editing, Y.S. and C.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This study was funded by the Shandong Provincial Natural Science Foundation of China under grant no. ZR2023MG020 and the University–Industry Collaborative Education Program of the Ministry of Education of China under grant nos. 230711255407320 and 220601369102256.

Data Availability Statement: All data generated or analyzed during this study are included in this paper and its Supplementary Materials.

Conflicts of Interest: The authors declare no conflicts of interest.

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