

Article

Forecasting Renewable Energy Consumption Using a Novel Fractional Grey Reverse Accumulation Model

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Abstract: The accumulation operation is the most fundamental method for processing data in grey models, playing a decisive role in the accuracy of model predictions. However, the traditional forward accumulation method does not adhere to the principle of prioritizing new information. Therefore, we propose a novel fractional reverse accumulation, which increases the accumulation coefficient for new data to fully utilize the new information carried by the latest data. This led to the development of a novel grey model, termed the FGRM(1,1). This model was validated using renewable energy consumption data from France, Spain, the UK, and Europe, and the results demonstrated that the FGRM(1,1) outperformed other models in terms of simulation error, prediction error, and comprehensive error. The predictions indicated significant growth in renewable energy consumption for France and Spain, moderate growth for the UK, and robust growth for Europe overall. These findings highlight the effectiveness of the proposed model in utilizing new information and provide insights into energy transition and emission reduction potential in Europe.

Keywords: forecasting; renewable energy consumption; fractional reverse accumulation; grey model



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1. Introduction

In the face of the dual challenges posed by global climate change and the energy crisis, the development and utilization of renewable energy, as a clean and sustainable form of energy, has become a consensus within the international community. Particularly against the backdrop of turbulent international situations, countries heavily reliant on traditional energy sources have no choice but to prioritize the development of renewable energy on their critical agendas [1]. With continuous technological advancements and persistent policy promotion, renewable energy has witnessed rapid global development, exerting a profound impact on energy consumption structures [2]. However, predicting renewable energy consumption (REC) presents a complex and challenging task. The consumption of renewable energy is influenced by a multitude of factors, including natural resource conditions, technological progress, policy environments, market demand, and economic situations [3]. These factors intertwine, resulting in nonlinear and nonstationary characteristics in the variations of REC. Traditional forecasting methods, such as time-series analysis and regression analysis, often struggle to achieve satisfactory prediction results when dealing with such complex data [4]. Therefore, exploring more accurate and reliable methods for predicting REC is of paramount importance. This not only aids governments and businesses in formulating scientific and reasonable energy development

plans and optimizing energy consumption structures but provides decision-making support for investors.

In current research, numerous models have been proposed and applied in the field of energy consumption forecasting, demonstrating diverse methodological perspectives. These models include multiple linear regression analysis [5], the autoregressive integrated moving average (ARIMA) method [6], machine learning [7,8], Bayesian neural networks [9], support vector machine [10], deep neural networks [11], the intelligent-selection-based self-adaptive hybrid (ISSH) method [12], grey models [13,14], and other various methods [15–17]. However, the limitations of these models are also evident. For instance, linear regression models struggle to predict nonlinear sequences and require support from large sample data. Similarly, machine learning models necessitate a sufficient amount of training data to ensure modeling accuracy, and their structures are often complex. In practice, because of challenges in data statistics, accuracy issues, and considerations of collection costs, it is often difficult to obtain a large amount of raw sample data. This results in the inability of the aforementioned models to guarantee prediction accuracy. The grey model has garnered significant attention in academia owing to its simplicity in modeling, minimal data requirements, and excellent prediction performance [18]. The theoretical foundation of this model is rooted in grey system theory, proposed by Professor Deng in 1982 [19], which provides a powerful tool for addressing issues with scarce data or incomplete information. Compared with statistical methods that rely on large datasets, the grey prediction model demonstrates unique advantages in small-sample or information-poor scenarios. Despite its streamlined model structure and minimal parameter settings, it achieves high-precision prediction results. Consequently, it has been widely applied in various fields such as per capita energy consumption [20], electricity consumption [21], solar energy generation [22], spread of a pandemic [23], market clearing price [24], battery incremental capacity [25], aircraft remaining useful life [26], carbon emissions [27], output of high-tech industry [28], short-term queue length [29], petroleum product consumption [30], photovoltaic power generation [31], and so on. In summary, the grey prediction model, with its unique advantages, effectively solves numerous complex problems in real life and has become an indispensable forecasting tool in economic, social, and other domains.

As a cornerstone of grey system theory, the GM(1,1) model, despite its groundbreaking significance, faces challenges in practical applications, including large performance fluctuations and restricted flexibility, with the prediction errors sometimes failing to meet high-precision requirements [32]. To overcome these limitations and enhance prediction efficacy, the academic community has conducted extensive and in-depth optimization and improvements on the GM(1,1) model. These efforts include optimizing the background value [33,34], seeking optimal parameters to better align with data characteristics [35,36], expanding the model structure to enhance its expressive capabilities [37–39], and integrating the advantages of different models by constructing combination models [40–42]. These endeavors have not only led to the development of a series of novel grey models but significantly improved the prediction accuracy and adaptability of grey models at both theoretical and practical levels.

In system modeling, new information is highly valued for its ability to more accurately reflect system characteristics; this is a core principle of grey prediction models. The traditional GM(1,1), which relies on positive accumulation, has a notable limitation in that it does not adequately account for the significance of new incoming information. This oversight can potentially lead to inaccuracies and limitations in the model's predictive capabilities, as it fails to incorporate the dynamic nature and evolving relevance of fresh datapoints in its calculations. Therefore, researchers have focused on studies surrounding the priority of new information, mainly including initial condition optimization [43,44],

adopting a rolling mode to dynamically update data [45,46], and introducing reverse accumulation and subtraction, particularly with fractional accumulation strategies [47–49]. Reverse accumulation, by improving traditional accumulation methods, effectively utilizes new information and enhances prediction accuracy [50,51]. The majority of existing research on reverse accumulation in the current literature primarily concentrates on fitting models to available data rather than on extrapolation predictions. This emphasis is largely attributed to the inherent limitations in the accuracy of extrapolation predictions when utilizing reverse accumulation, as its characteristics often lead to less reliable and precise forecasts when attempting to project beyond the observed data range. To tackle this challenge, we constructed a novel fractional grey reverse accumulation model (FGRM (1,1)) and applied it to REC predictions for France, the United Kingdom, Spain, and Europe. The primary contributions of this study are outlined below:

- (1) We introduced a novel reverse accumulation operation method, accompanied by its corresponding reverse operation and a detailed examination of its various characteristics. Building upon this foundation, we constructed the FGRM(1,1). To optimize the model's parameters and enhance its predictive accuracy, we employed particle swarm optimization (PSO), which effectively searches for the best parameter values.
- (2) The forecasting performance of the FGRM(1,1) was rigorously evaluated through four comprehensive case studies conducted in France, Spain, the United Kingdom, and Europe. In comparison with four other models, the FGRM(1,1) consistently demonstrated superior prediction capabilities, as evidenced by its lower prediction errors, higher accuracy rates, and better overall fit to the observed data.
- (3) The FGRM(1,1) was employed to predict REC for France, the United Kingdom, Spain, and Europe from 2023 to 2030, providing insights for future renewable energy development strategies in these economies.

The remainder of this paper is organized as follows. Section 2 introduces the traditional reverse accumulation method and the newly proposed reverse accumulation method, elaborates on several characteristics of this reverse accumulation, constructs the FGRM(1,1), and presents its solution. Section 3 assesses the effectiveness of the FGRM(1,1) through four cases. Section 4 applies the FGRM(1,1) to predict REC for France, the United Kingdom, Spain, and Europe from 2023 to 2030. Section 5 concludes the paper with a discussion of the findings and limitations.

2. Methodology

2.1. Traditional Accumulation Generation

Definition 1. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ is a non-negative sequence and T denotes transposition. $G^r, r \in R^+$ represents the r -th accumulation generation operator (AGO).

$$\begin{aligned} G^r X^{(0)} &= X^{(r)} = (g^r x_1^{(0)}, g^r x_2^{(0)}, \dots, g^r x_n^{(0)})^T \\ &= (x_1^{(r)}, x_2^{(r)}, \dots, x_n^{(r)})^T \end{aligned} \quad (1)$$

$G^r X^{(0)}$ is the r -th accumulation generation sequence, where

$$g^r x_k^{(0)} = x_k^{(r)} = \sum_{i=1}^k \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x_i^{(0)}, k \in N^+ \quad (2)$$

G^r represents the r -AGO matrix and is expressed as follows:

$$G^r = \begin{bmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix} & 0 & 0 & \cdots & 0 \\ \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 0 \end{bmatrix} & 0 & \cdots & 0 \\ \begin{bmatrix} r \\ 2 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 0 \end{bmatrix} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \begin{bmatrix} r \\ n-1 \end{bmatrix} & \begin{bmatrix} r \\ n-2 \end{bmatrix} & \begin{bmatrix} r \\ n-3 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ 0 \end{bmatrix} \end{bmatrix}_{n \times n} \quad (3)$$

where

$$\begin{bmatrix} r \\ i \end{bmatrix} = \begin{cases} \frac{r(r+1)\cdots(r+i-1)}{i!} = \binom{r+i-1}{i} = \frac{(r+i-1)!}{i!(r-1)!}, r \in \mathbb{Z} \\ \frac{\Gamma(r+i)}{\Gamma(i+1)\Gamma(r)}, r \notin \mathbb{Z} \end{cases}, \begin{bmatrix} 0 \\ i \end{bmatrix} = 0, \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 1 \quad (4)$$

When $r = 1$, 1-AGO is $x_k^{(r)} = \sum_{i=1}^k x_i^{(0)}, k \in \mathbb{N}^+$,

$$G^1 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}_{n \times n} \quad (5)$$

Definition 2. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ is a non-negative sequence. $O^r, r \in \mathbb{R}^+$ represents the r -th reverse accumulation generation operator (RAGO).

$$\begin{aligned} O^r X^{(0)} &= X^{(r)} = (o^r x_1^{(0)}, o^r x_2^{(0)}, \dots, o^r x_n^{(0)})^T \\ &= (x_1^{(r)}, x_2^{(r)}, \dots, x_n^{(r)})^T \end{aligned} \quad (6)$$

$O^r X^{(0)}$ represents the r -th reverse accumulation generation sequence.

$$o^r x_k^{(0)} = x_k^{(r)} = \sum_{i=k}^n \frac{\Gamma(r+i-k)}{\Gamma(i-k+1)\Gamma(r)} x_i^{(0)}, k \in \mathbb{N}^+ \quad (7)$$

O^r represents the r -OAGO matrix, which satisfies $X^{(r)} = O^r X^{(0)}$ and is expressed as follows:

$$O^r = \begin{bmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 2 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-1 \end{bmatrix} \\ 0 & \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-2 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} r \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-3 \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \begin{bmatrix} r \\ 0 \end{bmatrix} \end{bmatrix}_{n \times n} \quad (8)$$

when $r = 1$, 1-RAGO is $x_k^{(1)} = \sum_{i=k}^n x_i^{(0)}, k \in N^+$

$$O^1 = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{n \times n} \quad (9)$$

Property 1. Based on the aforementioned accumulation operations, it has been established that entities G^r and O^r satisfy the equality condition, namely $O^r_{i,j} = G^r_{n-i+1,n-j+1}$. This means that the two entities exhibit identical characteristics or values when subjected to the defined accumulation operations, indicating a strong correspondence or equivalence between them.

Based on the explanation of accumulation operations provided above, reverse accumulation can be seen as initiating the accumulation process from a specific term to generate series of progressively diminishing values, thereby emphasizing the importance of new information. While this method yields satisfactory outcomes in practical modeling simulations, it may adversely affect the prediction results of the model. In grey model theory, predictions are typically formulated through processes of accumulation and subtraction, which constitute a defining characteristic of the grey model and significantly enhance its accuracy. However, the reverse accumulate subtract can lead to a mismatch between the accumulation of the original data and the inverse accumulation of the modeling results. This is primarily due to the fact that during the accumulation process, random errors in the original data are amplified, causing the accumulated sequence to deviate from the true trend. When performing accumulate subtract, these accumulated errors are further propagated into the prediction results. Additionally, during the prediction phase, the model needs to extrapolate future data, and the accuracy of the inverse accumulation operation in the extrapolation process is difficult to guarantee, especially when the data exhibit significant fluctuations, making the mismatch phenomenon more pronounced.

2.2. Novel Fractional Reverse Accumulation

The novel method shares a similarity with the previous method in that it also involves the accumulation of values along the sequence. However, the key distinction lies in the direction of accumulation. While the traditional method accumulates values in a forward direction, the novel accumulation operates in reverse, starting from a specific term and moving backwards, thereby generating a unique sequence of progressively diminishing

values. This reversed approach allows for a different perspective on the data and can potentially highlight different patterns or trends that might not be as evident in the forward accumulation method. It distinguishes itself by prioritizing new information, treating $x_k^{(0)}$ as the starting term for accumulation in the calculation of $x_k^{(r)}$. Below, we present the detailed definition of this novel reverse accumulation operation.

Definition 3. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ is a non-negative sequence. $P^r, r \in R^+$ represents the novel r -th RAGO.

$$\begin{aligned} P^r X^{(0)} &= X^{(r)} = (p^r x_1^{(0)}, p^r x_2^{(0)}, \dots, p^r x_n^{(0)})^T \\ &= (x_1^{(r)}, x_2^{(r)}, \dots, x_n^{(r)})^T \end{aligned} \quad (10)$$

$P^r X^{(0)}$ is the novel r -th reverse accumulation generation sequence, where

$$p^r x_k^{(0)} = x_k^{(r)} = g^r y_k^{(0)}, k \in N^+ \quad (11)$$

$g^r y_k^{(0)}$ represents the r -th accumulation of $y_k^{(0)}$, and $y_k^{(0)}$ is described by the following relationships:

$$y_k^{(0)} = x_1^{(0)}, y_{k-1}^{(0)} = x_2^{(0)}, \dots, y_1^{(0)} = x_k^{(0)} \quad (12)$$

Then, we obtain:

$$Y^{(0)} = (y_1^{(0)}, y_2^{(0)}, \dots, y_k^{(0)})^T = (x_k^{(0)}, x_{k-1}^{(0)}, \dots, x_1^{(0)})^T \quad (13)$$

Note that when k varies, $Y^{(0)}$ also changes accordingly. Then,

$$x_k^{(r)} = \sum_{i=1}^k \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} y_i^{(0)} = \sum_{i=1}^k \frac{\Gamma(r+i-1)}{\Gamma(i)\Gamma(r)} x_i^{(0)}, k \in N^+ \quad (14)$$

P^r represents the r -RAGO matrix and satisfies $P^r X^{(0)} = X^{(r)}$.

$$P^r = \begin{bmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix} & 0 & 0 & \cdots & 0 \\ \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & 0 & \cdots & 0 \\ \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 2 \end{bmatrix} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \begin{bmatrix} r \\ 0 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 2 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ n-1 \end{bmatrix} \end{bmatrix} \quad (15)$$

Property 2. The novel reverse accumulation matrix is equivalent to reversing the order of rows in the original accumulation matrix. Based on the aforementioned definition, the following mathematical relationships between matrix P^r and matrix G^r can be derived:

$$P^r_{i,j} = G^r_{i,i-j+1} \quad (16)$$

Definition 4. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ is a non-negative sequence. Then, P^{-r} , $r \in R^+$ represents the novel r -th reverse subtraction generation operator (RSGO).

$$\begin{aligned} P^{-r}X^{(0)} &= X^{(-r)} = (p^{-r}x_1^{(0)}, p^{-r}x_2^{(0)}, \dots, p^{-r}x_n^{(0)})^T \\ &= (x_1^{(-r)}, x_2^{(-r)}, \dots, x_n^{(-r)})^T \end{aligned} \quad (17)$$

$P^{-r}X^{(0)}$ is a novel r -th reverse subtraction generation sequence, where

$$\begin{aligned} p^{-r}x_1^{(0)} &= x_1^{(-r)} = x_1^{(0)} \\ p^{-r}x_k^{(0)} &= x_k^{(-r)} = \frac{\Gamma(k)\Gamma(r)}{\Gamma(r+k-1)}(x_k^{(0)} - x_{k-1}^{(0)}) \end{aligned} \quad (18)$$

P^{-r} denotes the r -RSGO matrix, satisfying $P^{-r}X^{(0)} = X^{(-r)}$, $P^{-r}X^{(r)} = X^{(0)}$.

$$P^{-r} = \begin{bmatrix} \frac{1}{\Gamma(r)} & 0 & 0 & \cdots & 0 \\ \frac{-1}{\Gamma(r)} & \frac{1}{\Gamma(r)} & 0 & \cdots & 0 \\ 0 & \frac{-1}{\Gamma(r)} & \frac{1}{\Gamma(r)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \frac{1}{\Gamma(r)} \end{bmatrix} \quad (19)$$

Property 3. Based on the description provided above, P^r and P^{-r} satisfy the following relationship:

$$X^{(0)} = P^{-r}P^rX^{(0)} \quad (20)$$

Proof. Based on Definition 3, $X^{(0)} = (P^r)^{-1}X^{(r)}$ is inferred from $P^rX^{(0)} = X^{(r)}$. $P^{-r}X^{(r)} = X^{(0)}$ is derived from Equation (18); consequently, we arrive at the conclusion $(P^r)^{-1} = P^{-r}$, thus completing the proof. $\square$

Property 4. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ is a non-negative sequence, and P^ξ , P^δ and $P^{\xi+\delta}$ are the novel RAGOs. The novel operators do not adhere to the exponential law and only satisfy the commutative law when $\xi = \delta$.

$$\begin{aligned} P^\xi P^\delta &\neq P^\delta P^\xi \neq P^{\xi+\delta}, \xi \neq \delta \\ P^\xi P^\delta &= P^\delta P^\xi \neq P^{\xi+\delta}, \xi = \delta \end{aligned} \quad (21)$$

Proof. If the novel RAGOs satisfy the exponential law and commutative law, then the sufficient and necessary conditions are: $p^\xi p^\delta x^{(0)}(k) = p^\delta p^\xi x^{(0)}(k) = p^{\xi+\delta} x^{(0)}(k)$. According to Equation (14), it can be concluded that:

$$p^\xi x^{(0)}(k) = \sum_{i=1}^k \frac{\Gamma(\xi+i-1)}{\Gamma(i)\Gamma(\xi)} x^{(0)}(i) \quad (22)$$

$$P^{\xi+\delta}x^{(0)}(k) = \sum_{i=1}^k \frac{\Gamma(\xi + \delta + i - 1)}{\Gamma(i)\Gamma(\xi + \delta)} x^{(0)}(i) \quad (23)$$

$$\begin{aligned} P^{\delta}P^{\xi}x^{(0)}(k) &= P^{\delta}x^{(\xi)}(k) = \sum_{i=1}^k \frac{\Gamma(\delta + i - 1)}{\Gamma(i)\Gamma(\delta)} x^{(\xi)}(i) \\ &= \sum_{i=1}^k \frac{\Gamma(\xi + i - 1)}{\Gamma(i)\Gamma(\xi)} \sum_{j=1}^i \frac{\Gamma(\delta + j - 1)}{\Gamma(j)\Gamma(\delta)} x^{(0)}(j) \end{aligned} \quad (24)$$

$$\begin{aligned} P^{\xi}P^{\delta}x^{(0)}(k) &= P^{\xi}x^{(\delta)}(k) = \sum_{i=1}^k \frac{\Gamma(\xi + i - 1)}{\Gamma(i)\Gamma(\xi)} x^{(\delta)}(i) \\ &= \sum_{i=1}^k \frac{\Gamma(\xi + i - 1)}{\Gamma(i)\Gamma(\xi)} \sum_{j=1}^i \frac{\Gamma(\delta + j - 1)}{\Gamma(j)\Gamma(\delta)} x^{(0)}(j) \end{aligned} \quad (25)$$

It is easy to see that when $\xi = \delta$, $P^{\xi}P^{\delta}x^{(0)}(k) = P^{\delta}P^{\xi}x^{(0)}(k)$ and $P^{\xi}P^{\delta} = P^{\delta}P^{\xi}$. When $k = 2$, we can get:

$$\begin{aligned} P^{\xi+\delta}x^{(0)}(2) &= \frac{\Gamma(\xi + \delta + 1)}{\Gamma(2)\Gamma(\xi + \delta)} x^{(0)}(2) + x^{(0)}(1) \\ &= (\xi + \delta)x^{(0)}(2) + x^{(0)}(1) \end{aligned} \quad (26)$$

$$\begin{aligned} P^{\delta}P^{\xi}x^{(0)}(2) &= P^{\delta}x^{(\xi)}(2) = \frac{\Gamma(\delta + 1)}{\Gamma(2)\Gamma(\delta)} x^{(\xi)}(2) + x^{(\xi)}(1) \\ &= \frac{\Gamma(\delta + 1)}{\Gamma(2)\Gamma(\delta)} \left(\frac{\Gamma(\xi + 1)}{\Gamma(2)\Gamma(\xi)} x^{(0)}(2) + x^{(0)}(1) \right) + x^{(0)}(1) \\ &= \delta\xi x^{(0)}(2) + (\delta + 1)x^{(0)}(1) \end{aligned} \quad (27)$$

$$\begin{aligned} P^{\xi}P^{\delta}x^{(0)}(2) &= P^{\xi}x^{(\delta)}(2) = \frac{\Gamma(\xi + 1)}{\Gamma(2)\Gamma(\xi)} x^{(\delta)}(2) + x^{(\delta)}(1) \\ &= \frac{\Gamma(\xi + 1)}{\Gamma(2)\Gamma(\xi)} \left(\frac{\Gamma(\delta + 1)}{\Gamma(2)\Gamma(\delta)} x^{(0)}(2) + x^{(0)}(1) \right) + x^{(0)}(1) \\ &= \xi\delta x^{(0)}(2) + (\xi + 1)x^{(0)}(1) \end{aligned} \quad (28)$$

It is easy to see that when $\xi \neq \delta$, $P^{\xi}P^{\delta}x^{(0)}(2) \neq P^{\delta}P^{\xi}x^{(0)}(2) \neq P^{\xi+\delta}x^{(0)}(2)$, so $P^{\xi}P^{\delta} \neq P^{\delta}P^{\xi} \neq P^{\xi+\delta}$. When $\xi = \delta$, $P^{\xi}P^{\delta}x^{(0)}(2) = P^{\delta}P^{\xi}x^{(0)}(2) \neq P^{\xi+\delta}x^{(0)}(2)$, so $P^{\xi}P^{\delta} = P^{\delta}P^{\xi} \neq P^{\xi+\delta}$.

When $k = 3$, we can get:

$$\begin{aligned} P^{\xi+\delta}x^{(0)}(3) &= \frac{\Gamma(\xi + \delta + 2)}{\Gamma(3)\Gamma(\xi + \delta)} x^{(0)}(3) + \frac{\Gamma(\xi + \delta + 1)}{\Gamma(2)\Gamma(\xi + \delta)} x^{(0)}(2) + x^{(0)}(1) \\ &= \frac{(\xi + \delta + 1)(\xi + \delta)}{2} x^{(0)}(3) + (\xi + \delta)x^{(0)}(2) + x^{(0)}(1) \end{aligned} \quad (29)$$

$$\begin{aligned}
P^\delta P^\xi x^{(0)}(3) &= P^\delta x^{(\xi)}(3) = \frac{\Gamma(\delta+2)}{\Gamma(3)\Gamma(\delta)} x^{(\xi)}(3) + \frac{\Gamma(\delta+1)}{\Gamma(2)\Gamma(\delta)} x^{(\xi)}(2) + x^{(\xi)}(1) \\
&= \frac{\Gamma(\delta+2)}{\Gamma(3)\Gamma(\delta)} \left(\frac{\Gamma(\xi+2)}{\Gamma(3)\Gamma(\xi)} x^{(0)}(3) + \frac{\Gamma(\xi+1)}{\Gamma(2)\Gamma(\xi)} x^{(0)}(2) + x^{(0)}(1) \right) \\
&\quad + \frac{\Gamma(\delta+1)}{\Gamma(2)\Gamma(\delta)} \left(\frac{\Gamma(\xi+1)}{\Gamma(2)\Gamma(\xi)} x^{(0)}(2) + x^{(0)}(1) \right) + x^{(0)}(1) \\
&= \frac{\delta\xi(\delta+1)(\xi+1)}{4} x^{(0)}(3) + \frac{\delta\xi(\delta+1)}{2} x^{(0)}(2) + \frac{\delta(\delta+1)}{2} x^{(0)}(1) \\
&\quad + \delta\xi x^{(0)}(2) + \delta x^{(0)}(1) + x^{(0)}(1) \\
&= \frac{\delta\xi(\delta+1)(\xi+1)}{4} x^{(0)}(3) + \frac{\delta\xi(\delta+3)}{2} x^{(0)}(2) + \frac{\delta(\delta+3)+2}{2} x^{(0)}(1)
\end{aligned} \tag{30}$$

$$\begin{aligned}
P^\xi P^\delta x^{(0)}(3) &= P^\xi x^{(\delta)}(3) = \frac{\Gamma(\xi+2)}{\Gamma(3)\Gamma(\xi)} x^{(\delta)}(3) + \frac{\Gamma(\xi+1)}{\Gamma(2)\Gamma(\xi)} x^{(\delta)}(2) + x^{(\delta)}(1) \\
&= \frac{\Gamma(\xi+2)}{\Gamma(3)\Gamma(\xi)} \left(\frac{\Gamma(\delta+2)}{\Gamma(3)\Gamma(\delta)} x^{(0)}(3) + \frac{\Gamma(\delta+1)}{\Gamma(2)\Gamma(\delta)} x^{(0)}(2) + x^{(0)}(1) \right) \\
&\quad + \frac{\Gamma(\xi+1)}{\Gamma(2)\Gamma(\xi)} \left(\frac{\Gamma(\delta+1)}{\Gamma(2)\Gamma(\delta)} x^{(0)}(2) + x^{(0)}(1) \right) + x^{(0)}(1) \\
&= \frac{\delta\xi(\delta+1)(\xi+1)}{4} x^{(0)}(3) + \frac{\delta\xi(\xi+1)}{2} x^{(0)}(2) + \frac{\xi(\xi+1)}{2} x^{(0)}(1) \\
&\quad + \delta\xi x^{(0)}(2) + \xi x^{(0)}(1) + x^{(0)}(1) \\
&= \frac{\delta\xi(\delta+1)(\xi+1)}{4} x^{(0)}(3) + \frac{\delta\xi(\xi+3)}{2} x^{(0)}(2) + \frac{\xi(\xi+3)+2}{2} x^{(0)}(1)
\end{aligned} \tag{31}$$

It is easy to see that when $\xi \neq \delta$, $P^\xi P^\delta x^{(0)}(3) \neq P^\delta P^\xi x^{(0)}(3) \neq P^{\xi+\delta} x^{(0)}(3)$, so $P^\xi P^\delta \neq P^\delta P^\xi \neq P^{\xi+\delta}$. When $\xi = \delta$, $P^\xi P^\delta x^{(0)}(3) = P^\delta P^\xi x^{(0)}(3) \neq P^{\xi+\delta} x^{(0)}(3)$, so $P^\xi P^\delta = P^\delta P^\xi \neq P^{\xi+\delta}$. $\square$

2.3. Development of the FGRM (1,1)

Definition 5. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ is a non-negative sequence. The novel r -th fractional RAGO is $x_k^{(r)} = \sum_{i=1}^k \frac{\Gamma(r+i-1)}{\Gamma(i)\Gamma(r)} x_i^{(0)}$, $k \in N^+$; therefore,

$$\begin{aligned}
x_k^{(r)} - x_{k-1}^{(r)} + a z_k^r &= b \\
z_k^{(r)} &= \frac{x_k^{(r)} + x_{k-1}^{(r)}}{2}
\end{aligned} \tag{32}$$

is denoted as the FGRM (1,1), and the time response function is:

$$\hat{x}_k^{(r)} = \left(x_1^{(0)} - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{r}(k-1)} + \frac{\hat{b}}{\hat{a}} \tag{33}$$

Parameters a, b are calculated by least squares, as following:

$$\begin{aligned}
\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} &= (\phi^T \phi)^{-1} \phi^T Y \\
\phi &= \begin{bmatrix} -z_2^{(r)} & 1 \\ -z_3^{(r)} & 1 \\ \vdots & \vdots \\ -z_n^{(r)} & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x_2^{(r)} - x_1^{(r)} \\ x_3^{(r)} - x_2^{(r)} \\ \vdots \\ x_n^{(r)} - x_{n-1}^{(r)} \end{bmatrix}
\end{aligned} \tag{34}$$

Property 5. The forecasting values generated by the FGRM (1,1) remain consistent and unaltered, regardless of any fluctuations in the initial value. In simpler terms, altering the initial value, denoted as $x_1^{(0)}$, does not exert any influence on the subsequent outputs or predictions, referred to as $\hat{x}_k^{(r)}$, produced by the model. This robustness of the FGRM (1,1) to initial value variations underscores its stability and reliability in generating accurate forecasts.

Proof. Based on Equation (34), we can obtain:

$$\begin{aligned} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} &= (\phi^T \phi)^{-1} \phi^T Y = \left(\begin{bmatrix} -z_2^{(r)} & 1 \\ -z_3^{(r)} & 1 \\ \vdots & \vdots \\ -z_n^{(r)} & 1 \end{bmatrix}^T \begin{bmatrix} -z_2^{(r)} & 1 \\ -z_3^{(r)} & 1 \\ \vdots & \vdots \\ -z_n^{(r)} & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} -z_2^{(r)} & 1 \\ -z_3^{(r)} & 1 \\ \vdots & \vdots \\ -z_n^{(r)} & 1 \end{bmatrix}^T \begin{bmatrix} x_2^{(r)} - x_1^{(r)} \\ x_3^{(r)} - x_2^{(r)} \\ \vdots \\ x_n^{(r)} - x_{n-1}^{(r)} \end{bmatrix} \\ &= \frac{1}{(n-1) \sum_{k=2}^n [z_k^r]^2 - \left[\sum_{k=2}^n z_k^r \right]^2} \begin{bmatrix} n-1 & \sum_{k=2}^n z_k^r \\ \sum_{k=2}^n z_k^r & \sum_{k=2}^n [z_k^r]^2 \end{bmatrix} \begin{bmatrix} -\sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] x_k^r \\ \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] \end{bmatrix} \\ &= \begin{bmatrix} \frac{\sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] \sum_{k=2}^n z_k^r - (n-1) \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] z_k^r}{(n-1) \sum_{k=2}^n [z_k^r]^2 - \left[\sum_{k=2}^n z_k^r \right]^2} \\ \frac{\frac{1}{n-1} \left[\sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] + \hat{a} \sum_{k=2}^n z_k^r \right]}{\frac{1}{n-1} \left[\sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] + \hat{a} \sum_{k=2}^n z_k^r \right]} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] \sum_{k=2}^n z_k^r - (n-1) \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] z_k^r}{(n-1) \sum_{k=2}^n [z_k^r]^2 - \left[\sum_{k=2}^n z_k^r \right]^2} \\ \frac{1}{n-1} \left[\sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] + \hat{a} \sum_{k=2}^n z_k^r \right] \end{bmatrix} \end{aligned} \quad (35)$$

This leads to:

$$\hat{a} = \frac{[x_n^{(r)} - x_1^{(r)}] \sum_{k=2}^n z_k^r - (n-1) \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] z_k^r}{(n-1) \sum_{k=2}^n [z_k^r]^2 - \left[\sum_{k=2}^n z_k^r \right]^2} \quad (36)$$

$$\hat{b} = \frac{1}{n-1} \left[[x_n^{(r)} - x_1^{(r)}] + \hat{a} \sum_{k=2}^n z_k^r \right] \quad (37)$$

If $x_1^{(0)}$ undergoes a disturbance, that is, $\hat{x}_1^{(0)} = x_1^{(0)} + \varepsilon$, we can obtain:

$$X^{(r)} = \left(x_1^{(r)} + \varepsilon, x_2^{(r)} + \varepsilon, \dots, x_n^{(r)} + \varepsilon \right)^T \quad (38)$$

$$\begin{aligned}
 \hat{a} &= \frac{[x_n^{(r)} + \varepsilon - x_1^{(r)} - \varepsilon] \sum_{k=2}^n [z_k^r + \varepsilon] - (n-1) \sum_{k=2}^n [x_k^{(r)} + \varepsilon - s_{k-1}^{(r)} - \varepsilon][z_k^r + \varepsilon]}{(n-1) \sum_{k=2}^n [z_k^r + \varepsilon]^2 - \left[\sum_{k=2}^n z_k^r + (n-1)\varepsilon \right]^2} \\
 &= \frac{[x_n^{(r)} - x_1^{(r)}] \sum_{k=2}^n z_k^r + (n-1)\varepsilon[x_n^{(r)} - x_1^{(r)}] - (n-1) \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}]z_k^r - (n-1)\varepsilon[x_n^{(r)} - x_1^{(r)}]}{(n-1) \sum_{k=2}^n [z_k^r]^2 + 2(n-1)\varepsilon \sum_{k=2}^n z_k^r + (n-1)^2\varepsilon^2 - \left[\sum_{k=2}^n z_k^r \right]^2 - 2(n-1)\varepsilon \sum_{k=2}^n z_k^r - (n-1)^2\varepsilon^2} \\
 &= \frac{[x_n^{(r)} - x_1^{(r)}] \sum_{k=2}^n z_k^r - (n-1) \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}]z_k^r}{(n-1) \sum_{k=2}^n [z_k^r]^2 - \left[\sum_{k=2}^n z_k^r \right]^2} \\
 \hat{b} &= \frac{1}{n-1} \left[[x_n^{(r)} + \varepsilon - x_1^{(r)} + \varepsilon] + \hat{a} \sum_{k=2}^n [z_k^r + \varepsilon] \right] \\
 &= \frac{1}{n-1} \left[[x_n^{(r)} - x_1^{(r)}] + \hat{a} \sum_{k=2}^n z_k^r \right] + \hat{a}\varepsilon
 \end{aligned} \tag{39}$$

$$\begin{aligned}
 \hat{b} &= \frac{1}{n-1} \left[[x_n^{(r)} + \varepsilon - x_1^{(r)} + \varepsilon] + \hat{a} \sum_{k=2}^n [z_k^r + \varepsilon] \right] \\
 &= \frac{1}{n-1} \left[[x_n^{(r)} - x_1^{(r)}] + \hat{a} \sum_{k=2}^n z_k^r \right] + \hat{a}\varepsilon
 \end{aligned} \tag{40}$$

Consequently, the value of $\hat{a}$ remains unchanged, whereas the value of $\hat{b}$ experiences an increment of $\hat{a}\varepsilon$. Based on the time response function, we obtain:

$$\begin{aligned}
 \hat{x}_k^{(r)} &= \left(x_1^{(0)} + \varepsilon - \frac{\hat{b} + \hat{a}\varepsilon}{\hat{a}} \right) e^{-\hat{r}(k-1)} + \frac{\hat{b}}{\hat{a}} \\
 &= \left(x_1^{(0)} - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{r}(k-1)} + \frac{\hat{b}}{\hat{a}}
 \end{aligned} \tag{41}$$

Therefore, the value of $\hat{x}_k^{(r)}$ remains unchanged.
This completes the proof. $\square$

Property 6. Suppose $X^{(0)} = (x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)})^T$ and $Y^{(0)} = (y_1^{(0)}, y_2^{(0)}, \dots, y_n^{(0)})^T$ are non-negative sequences, where $y_k^{(0)} = \theta x_k^{(0)}$, and θ are multiple conversion values. We utilized sequences $X^{(0)}$ and $Y^{(0)}$ to develop the FGRM (1,1), ensuring that both the fitting and predicted values meet

$$\hat{y}_k^{(0)} = \theta \hat{x}_k^{(0)} \tag{42}$$

In other words, if the original sequence undergoes a uniform multiplication, the corresponding forecasting results by the FGRM (1,1) also scale proportionally while maintaining the same accumulation order.

Proof. Based on Equation (14), the following equation can be obtained:

$$y_k^{(r)} = \theta x_k^{(r)}, \quad k \in N^+ \tag{43}$$

Combined with Equation (32), we can obtain:

$$z_{ky}^{(r)} = \frac{y_k^{(r)} + y_{k-1}^{(r)}}{2} = \theta z_{kx}^{(r)} \tag{44}$$

Therefore, the calculation formulas for a_y, b_y are as follows:

$$\begin{aligned}\hat{a}_y &= \frac{[y_n^{(r)} - y_1^{(r)}] \sum_{k=2}^n z_{ky}^r - (n-1) \sum_{k=2}^n [y_k^{(r)} - y_{k-1}^{(r)}] z_{ky}^r}{(n-1) \sum_{k=2}^n [z_{ky}^r]^2 - [\sum_{k=2}^n z_{ky}^r]^2} \\ &= \frac{\delta^2 [x_n^{(r)} - x_1^{(r)}] \sum_{k=2}^n z_{kx}^r - (n-1) \theta^2 \sum_{k=2}^n [x_k^{(r)} - x_{k-1}^{(r)}] z_{kx}^r}{(n-1) \theta^2 \sum_{k=2}^n [z_{kx}^r]^2 - \theta^2 [\sum_{k=2}^n z_{kx}^r]^2} \\ &= \hat{a}_x\end{aligned}\quad (45)$$

$$\begin{aligned}\hat{b}_y &= \frac{1}{n-1} \left[[y_n^{(r)} - y_1^{(r)}] + \hat{a}_y \sum_{k=2}^n z_{ky}^r \right] \\ &= \frac{\theta}{n-1} \left[[x_n^{(r)} - x_1^{(r)}] + \hat{a}_x \sum_{k=2}^n z_{ky}^r \right] \\ &= \theta \hat{b}_x\end{aligned}\quad (46)$$

Based on the Equation (18), we obtain:

$$\begin{aligned}\hat{y}_k^{(r)} &= \left(y_1^{(0)} - \frac{\hat{b}_y}{\hat{a}_y} \right) e^{-\hat{r}_y(k-1)} + \frac{\hat{b}_y}{\hat{a}_y} \\ &= \left(\theta x_1^{(0)} - \theta \frac{\hat{b}_x}{\hat{a}_x} \right) e^{-\hat{r}_x(k-1)} + \theta \frac{\hat{b}_x}{\hat{a}_x} \\ &= \theta \hat{x}_k^{(r)}\end{aligned}\quad (47)$$

Then, $\hat{y}_k^{(0)} = \theta \hat{x}_k^{(0)}$ is obtained.

This completes the proof. $\square$

2.4. Solution of the Accumulation Order r

In the FGRM (1,1), the accumulation order r plays a pivotal role as a critical parameter that has a direct impact on the determination of parameters a and b . Consequently, the selection of r significantly influences the model's prediction accuracy and overall performance. Given its importance, it is imperative to carefully choose a reliable and appropriate accumulation order to ensure high modeling accuracy. To achieve this goal, this paper adopted a systematic approach by leveraging the PSO, which was employed to select an optimal value of r that minimized the mean relative simulation percentage error (MRSPE) of the modeling data. This approach ensured that the chosen r not only aligned with the inherent characteristics of the data but maximized the model's predictive power. The formulas are provided below:

$$\min \text{MRSPE} = \frac{1}{n-1} \sum_{i=1}^n (\hat{x}_i^{(0)} - x_i^{(0)}) / x_i^{(0)} \times 100\% \quad (48)$$

$$\text{s.t.} \begin{cases} \begin{bmatrix} \hat{a} & \hat{b} \end{bmatrix}^T = (\phi^T \phi)^{-1} \phi^T Y \\ \hat{x}_k^{(r)} = \left(x_1^{(0)} - \frac{\hat{b}}{\hat{a}} \right) e^{-\hat{r}(k-1)} + \frac{\hat{b}}{\hat{a}} \\ x_{k+1}^{(0)} = \frac{\Gamma(k+1)\Gamma(r)}{\Gamma(r+k)} (x_{k+1}^{(r)} - x_k^{(r)}) \\ k \in N^+ \end{cases} \quad (49)$$

2.5. Flowchart of the FGRM (1,1)

Based on the analysis results presented above, the modeling process of the FGRM (1,1) is illustrated in Figure 1. The primary steps were as follows. First, the original data were collected and divided into two groups, the simulation group and the prediction group. Next, the simulation data were utilized to construct the FGRM (1,1), FGM (1,1), CFGM (1,1), and ARIMA model, and the PSO was employed to identify the optimal parameters for each grey model. Subsequently, the performance of the various models was compared to validate the superiority of the FGRM(1,1). Ultimately, the FGRM(1,1) was employed to predict the future trends of the system.

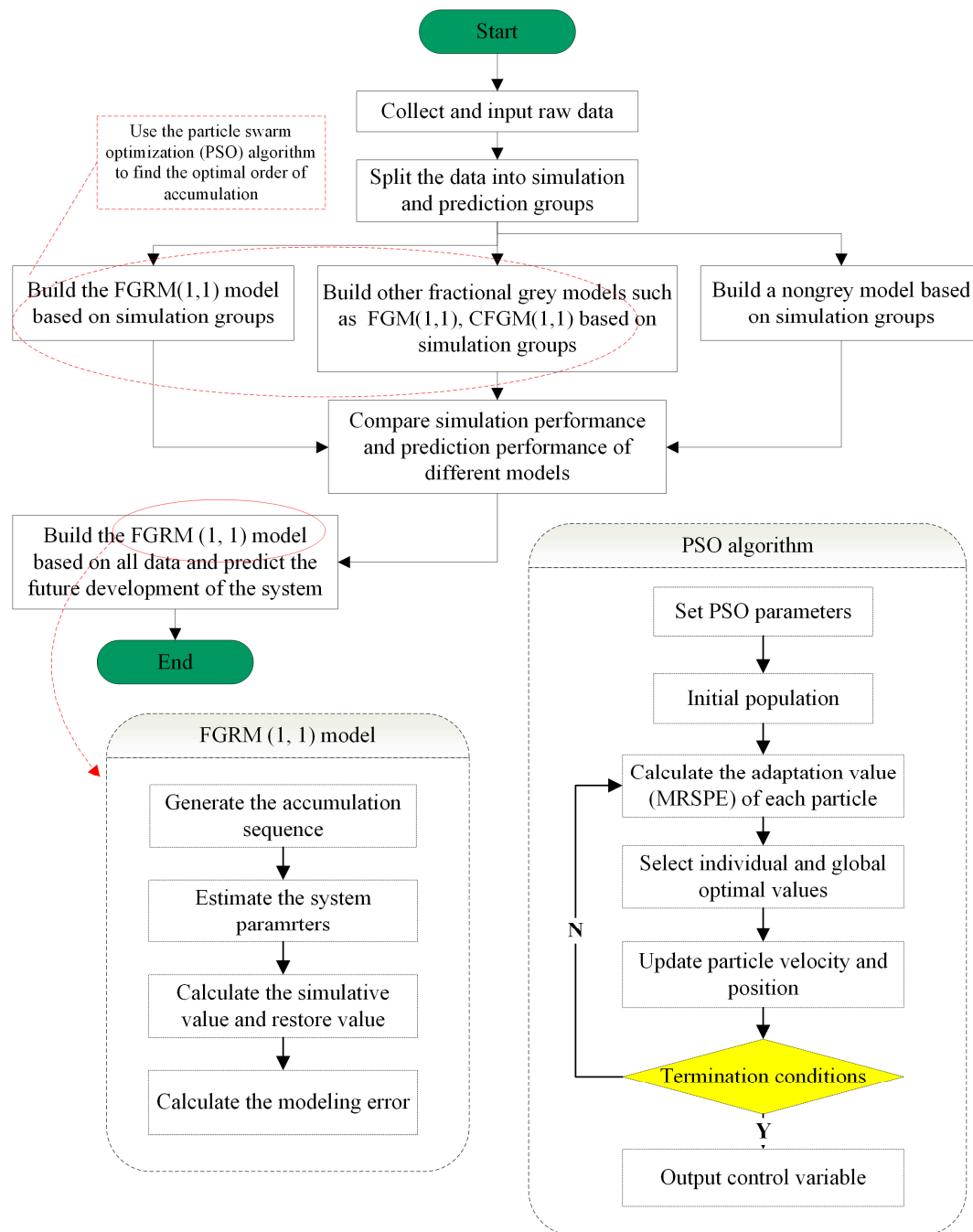


Figure 1. Flowchart of the FGRM (1,1).

2.6. Error Index

To more accurately assess the performance of the FGRM (1,1), we employed three error indexes to gauge the accuracy: MRSPE, MRFPE and CMRPE. The MRSPE evaluates the simulating precision, the MRFPE assesses the predicting precision, and the CMRPE reflects the total precision. The formulas for these indexes are as follows:

$$MRSPE = \frac{1}{n} \sum_{k=1}^n \left| \frac{x_k^{(0)} - \hat{x}_k^{(0)}}{x_k^{(0)}} \right| \times 100\% \quad (50)$$

$$MRFPE = \frac{1}{t} \sum_{k=n+1}^{n+t} \left| \frac{x_k^{(0)} - \hat{x}_k^{(0)}}{x_k^{(0)}} \right| \times 100\% \quad (51)$$

$$CMRPE = \frac{(n) \times MRSPE + t \times MRFPE}{n + t} \quad (52)$$

3. Validity Verification of the FGRM (1,1)

In this section, REC data from France, Spain, the United Kingdom, and Europe were employed to test the effectiveness of the FGRM (1,1), with the REC data sourced from the Statistical Review of World Energy. Given that fractional accumulation was applied in many grey models, such as the FGM (1,1) [47], CFGM (1,1) [48], and CFGOM (1,1) [52], these three models were chosen for comparison in this study. For the sake of clarity and convenience, the models discussed in the upcoming section are referred to using the abbreviations FGRM, FGM, CFGM, and CFGOM. Furthermore, to validate the effectiveness of our findings, the ARIMA model was selected as a benchmark for comparison in this study.

3.1. Case 1: REC in France

Although nuclear energy dominates France's energy mix, the government also recognizes the importance of renewable energy in the energy transition. By 2023, France had added 3135 MW of new photovoltaic installed capacity, bringing the cumulative photovoltaic installed capacity to 19 GW. Furthermore, France plans to establish 50 offshore wind farms by 2050, achieving a capacity target of 40 GW, and to double its onshore wind capacity. According to the latest plans, France aims to increase the share of renewable energy in total energy consumption to 40% by 2040. Therefore, we validated the effectiveness of the FGRM using France's REC. The models were built using data from 2015 to 2021 and applied to predict data for 2022. Table 1 presents the modeling results for each model. The optimal accumulation orders for the four grey models were 0.864, 0.504, 0.812, and 1 for the FGRM, FGM, CFGM, and CFGOM, respectively. Figure 2 shows the PSO iterations for the accumulation order, Figure 3 illustrates the fitting and prediction curves for each model, Figure 4 displays the deviations of each model's modeling results from the original system at various timepoints, and Figure 5 shows the simulation and prediction errors under other error indexes.

It can be observed that the modeling results of the FGRM were relatively close to the original data, particularly from 2015 to 2017, where the predicted values almost matched the original data. However, in subsequent years, although the overall trend aligned, there were slight differences. Compared with the FGRM, the modeling results of the FGM were also similar to the original data from 2015 to 2017 but showed relatively larger differences in subsequent years. The modeling results of the CFGM were similar to those of the FGRM, but in some years (such as 2019 and 2021), the differences from the original data were slightly greater than those of the FGRM. Among all models, the CFGOM had the closest modeling results to the original data from 2015 to 2020, especially from 2018 to 2020, where the predicted values almost matched the original data. However, in 2021 and 2022, the

predicted values were higher than the original data. Compared with other models, the ARIMA's modeling results showed significant differences from the original data in most years. Particularly from 2019 to 2022, the predicted values of this model were significantly lower or higher than the original data.

Table 1. Modeling results and errors in case 1.

Year	Raw Data (EJ)	FGRM	FGM	CFGM	CFGOM	ARIMA
2015	0.480	0.480	0.480	0.480	0.461	0.480
2016	0.490	0.490	0.490	0.490	0.503	0.520
2017	0.540	0.548	0.549	0.548	0.549	0.530
2018	0.590	0.599	0.603	0.599	0.599	0.580
2019	0.670	0.647	0.651	0.647	0.653	0.630
2020	0.710	0.694	0.693	0.694	0.712	0.710
2021	0.720	0.741	0.730	0.740	0.777	0.750
2022	0.810	0.788	0.762	0.787	0.847	0.760
r		0.864	0.504	0.812	1	/
MRSPE (%)		1.668	1.743	1.908	2.096	2.843
MRFPE (%)		2.746	5.886	2.869	4.628	6.173
CMRPE (%)		1.803	2.261	2.028	2.412	3.259

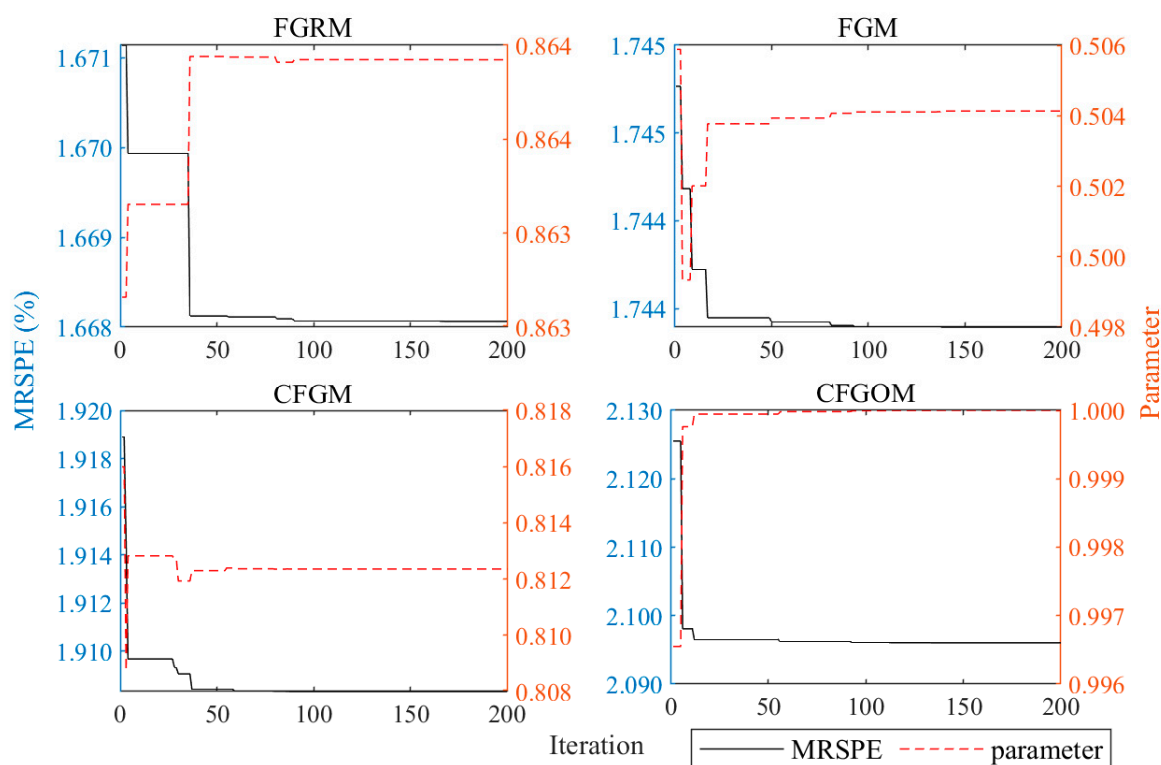


Figure 2. PSO algorithm iteration situation in case 1.

In terms of error results, among all models, the FGRM had the smallest MRSPE of only 1.668%, indicating the smallest error during the simulation process. The ARIMA model had the largest MRSPE of 2.843%, indicating the largest error during the simulation process. The MRFPE of the FGRM was relatively small, at only 2.746%, while the ARIMA model had the largest MRFPE of 6.173%, indicating that the FGRM had smaller error when predicting future values compared with the ARIMA model. The CMRPE of the FGRM was also the smallest, at only 1.803%, while the ARIMA model had the largest CMRPE of 3.259%. In summary, among all models, the FGRM exhibited the smallest simulation error, prediction error, and combined error, demonstrating its high accuracy and stability in processing such data. In contrast, the ARIMA model had the largest errors and was less suitable for modeling and predicting this type of data.

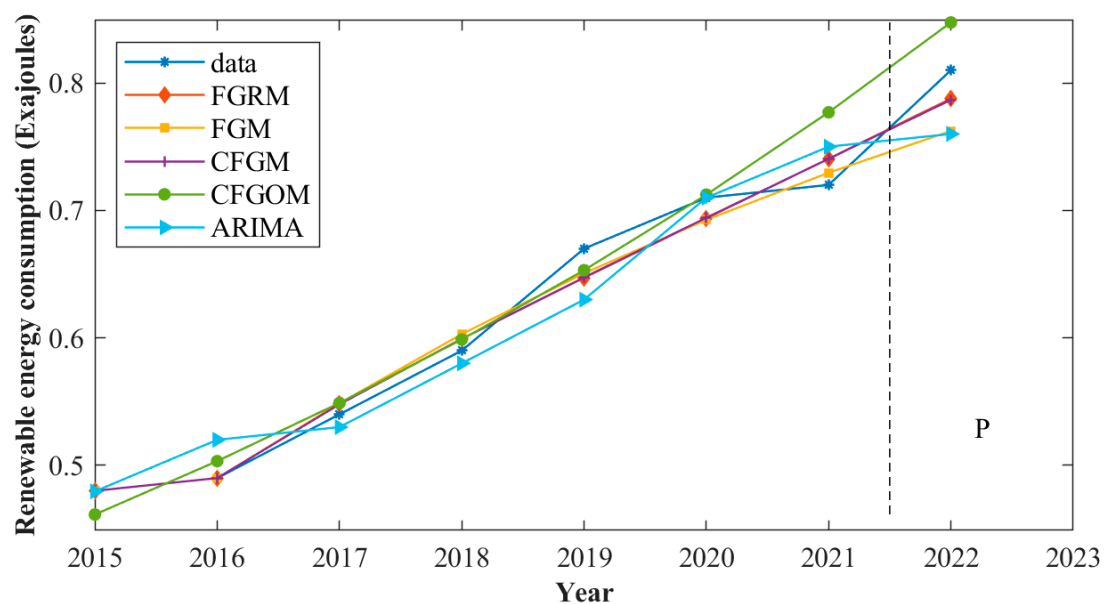


Figure 3. Fitting and predicting curve in case 1.

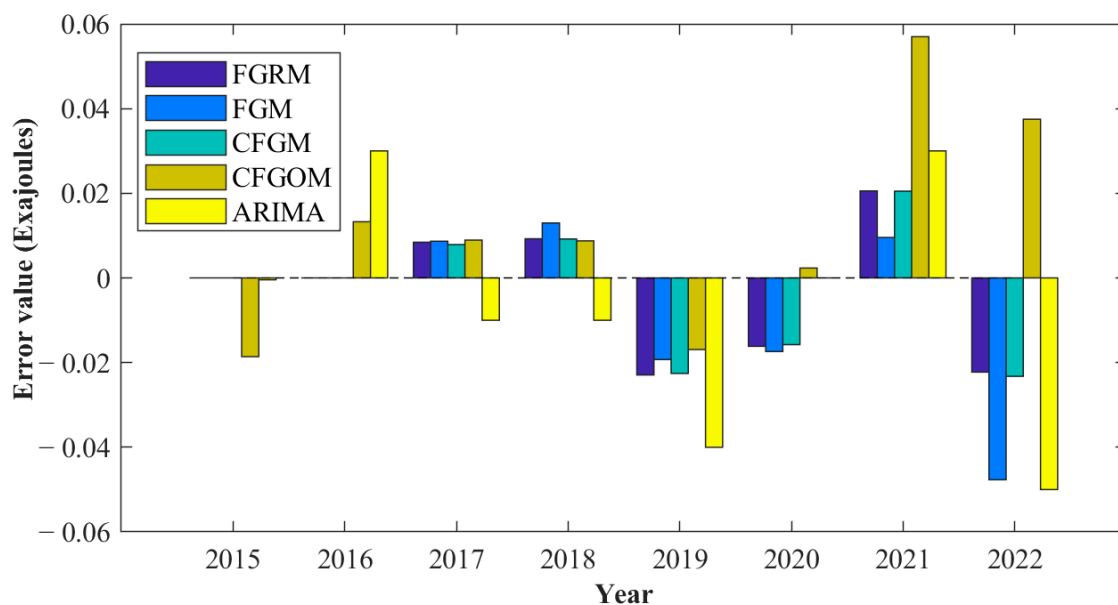


Figure 4. Discrepancy between the modeling results and raw data in case 1.

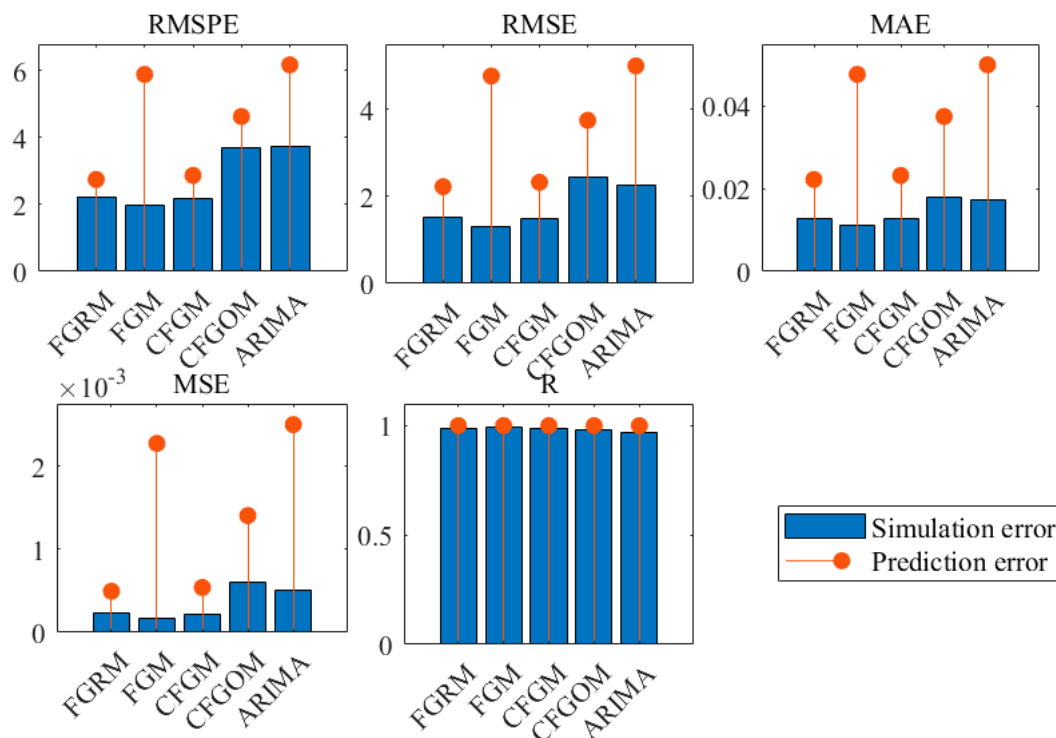


Figure 5. Modeling errors under other error indexes in case 1.

3.2. Case 2: REC in Spain

In May 2021, Spain approved its first Energy Transition Law, which set goals such as reducing greenhouse gas emissions, increasing the penetration of renewable energy in final energy consumption, and achieving a power system fully based on renewable energy generation. According to the Renewable Energy Country Attractiveness Index (RECAI), Spain ranks ninth globally and third within the European Union in terms of attractiveness for renewable energy investment and deployment opportunities. The proportion of renewable energy in Spain's electricity mix has been increasing annually, reaching 50.8% in 2023, higher than the 42.2% in 2022. The total generation of renewable energy has also continued to grow, exceeding 135,000 GWh in 2023 compared with 116,695 GWh in 2022. Therefore, we validated the effectiveness of the FGRM using Spain's REC. Models were built using data from 2015 to 2021, and predictions were made for 2022. Table 2 presents the modeling results for each model. The optimal accumulation orders for the four grey models were 1.338, 0.017, 1, and 1 for the FGRM, FGM, CFGM, and CFGOM, respectively. Figure 6 shows the PSO iterations for the accumulation order, Figure 7 illustrates the fitting and prediction curves for each model, Figure 8 displays the deviations of each model's modeling results from the original system at various timepoints, and Figure 9 shows the simulation and prediction errors under other error indexes.

It can be observed that the modeling results of the FGRM aligned well with the overall trend of the original data, although there were slight differences in some years. Particularly in 2021 and 2022, the predicted values of the FGRM were higher than the original data, but the differences were not significant. The modeling results of the FGM were relatively close to the original data from 2015 to 2018. However, after 2019, the predicted values of this model began to deviate significantly from the raw data, especially in 2022, where the predictions were much higher than the raw data. The CFGM's modeling results were relatively close to the original data from 2015 to 2017. However, from 2018 to 2022, the predicted values of this model were higher or lower than the original data in some years, with relatively large differences. The modeling results of the CFGOM were in good agreement with the original

data from 2015 to 2018. However, after 2019, the predicted values of this model started to deviate from the original data, especially in 2021 and 2022, where the predictions were significantly lower than the original data. The ARIMA's modeling results were relatively close to the original data from 2015 to 2018. However, from 2019 to 2022, the predicted values of this model showed significant differences from the original data in some years, without a clear pattern.

Table 2. Modeling results and errors in case 2.

Year	Raw Data (EJ)	FGRM	FGM	CFGM	CFGOM	ARIMA
2015	0.720	0.720	0.720	0.720	0.692	0.720
2016	0.710	0.710	0.717	0.676	0.717	0.711
2017	0.730	0.708	0.727	0.722	0.743	0.700
2018	0.750	0.742	0.749	0.770	0.770	0.750
2019	0.780	0.797	0.789	0.822	0.799	0.770
2020	0.860	0.871	0.859	0.878	0.828	0.810
2021	0.980	0.961	0.979	0.937	0.858	0.940
2022	1.040	1.069	1.183	1.000	0.889	1.100
r		1.338	0.017	1	1	/
MRSPE (%)		1.36	0.488	3.417	2.6	2.21
MRFPE (%)		2.784	13.704	3.832	14.477	5.769
CMRPE (%)		1.538	2.14	3.469	4.085	2.655

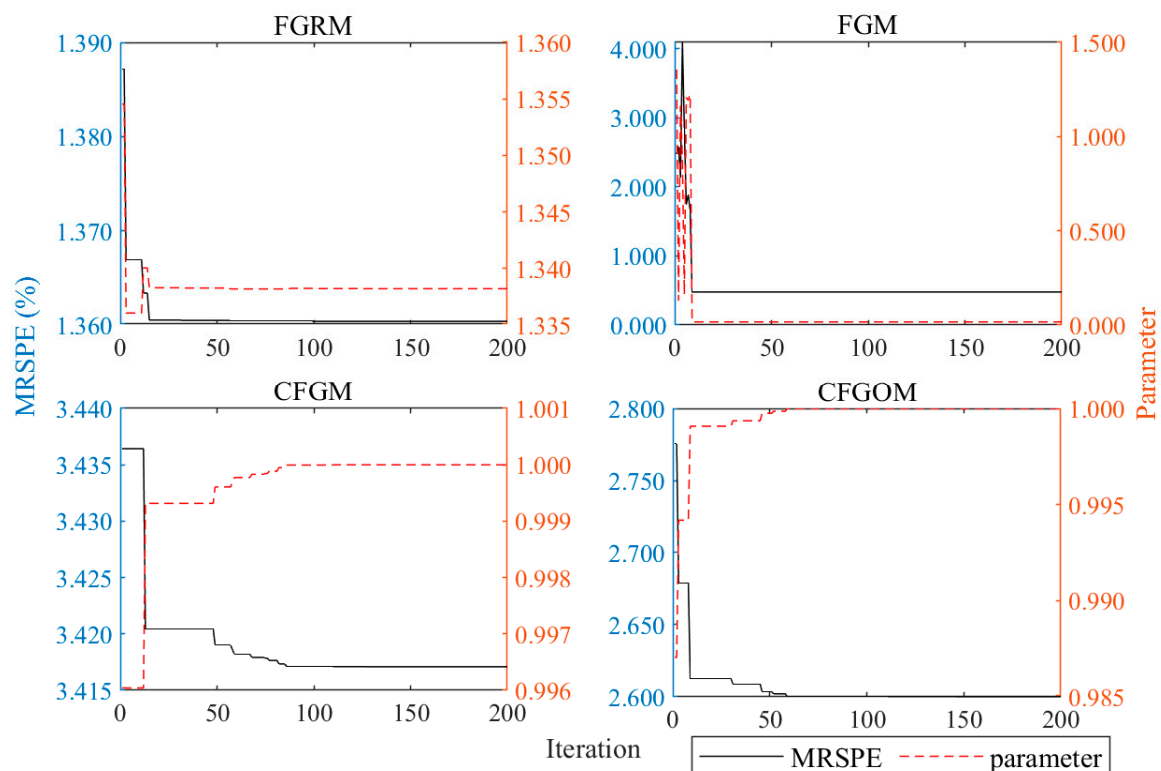


Figure 6. PSO algorithm iteration situation in case 2.

In terms of error results, the FGRM had the smallest MRSPE, indicating the smallest error during the simulation process. In contrast, the CFGM and CFGOM had larger MRSPEs, suggesting relatively larger errors during the simulation. The FGM had the largest MRFPE, reaching 13.704%, indicating a very large error in predicting future values. The MRFPE of the FGRM was relatively small, indicating smaller prediction errors. The CFGOM had the largest CMRPE, while the FGRM had the smallest CMRPE, indicating the best overall performance. In summary, among all models, the FGRM exhibited smaller errors in terms of simulation error, prediction error, and comprehensive error, demonstrating high accuracy and stability in processing such data.

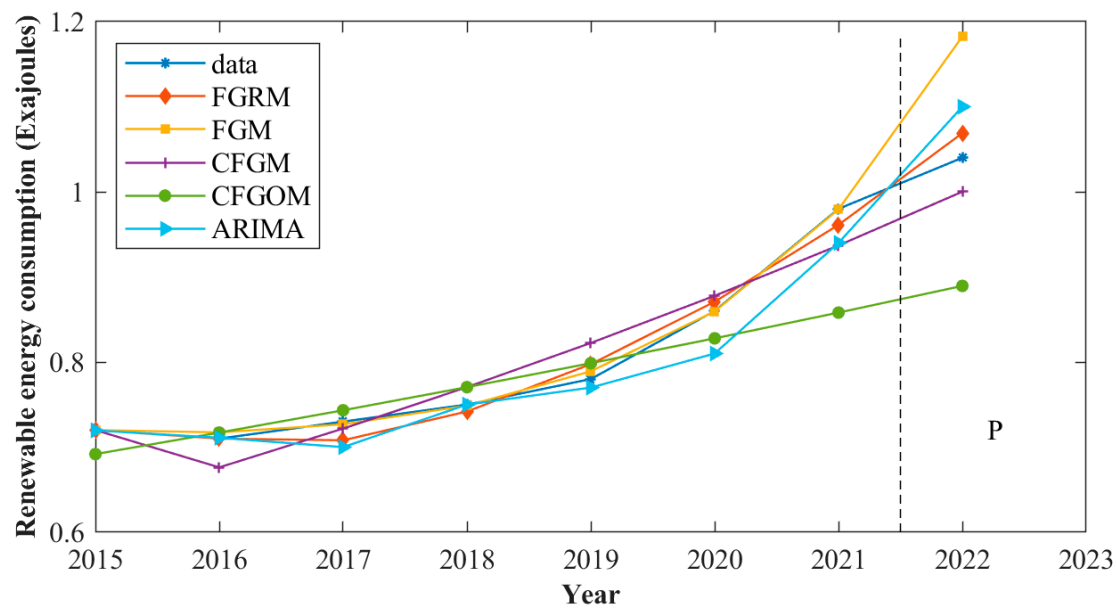


Figure 7. Fitting and predicting curve in case 2.

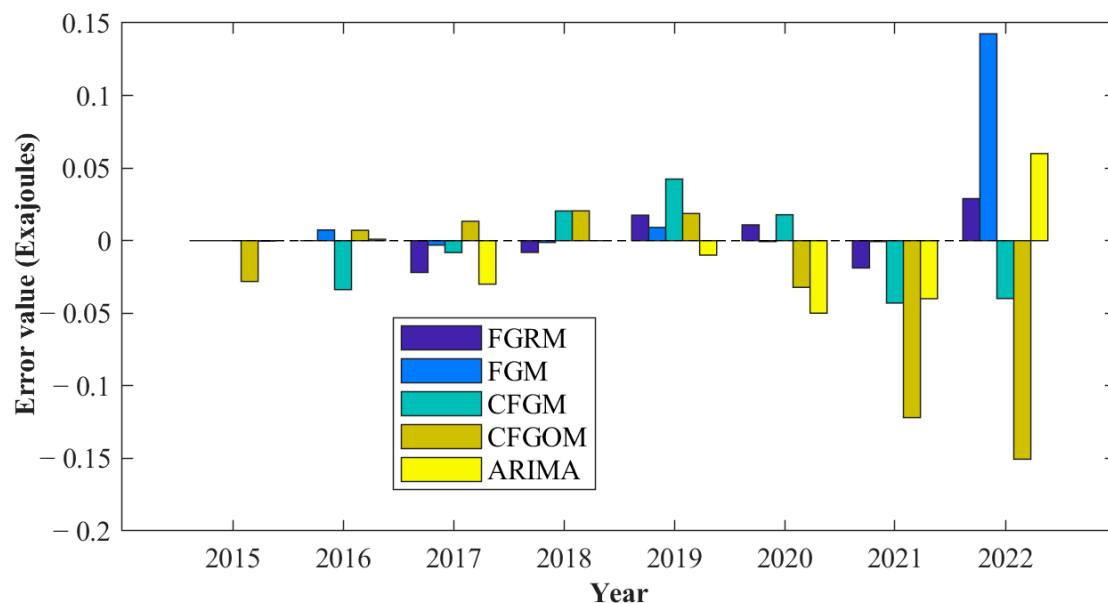


Figure 8. Discrepancy between the modeling results and raw data in case 2.

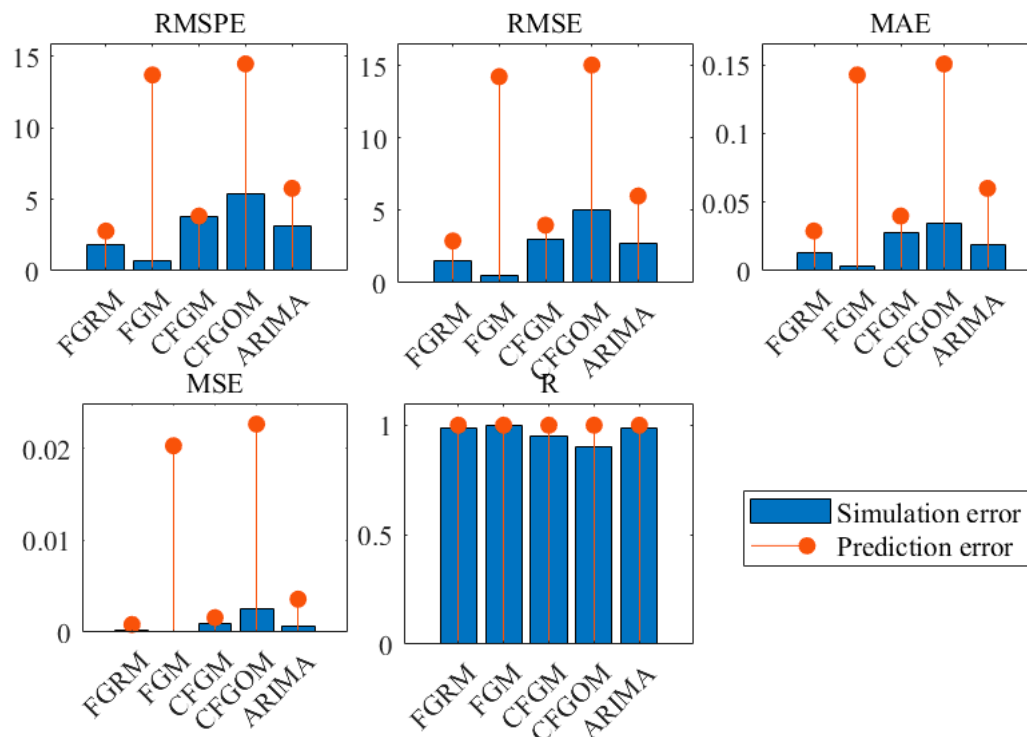


Figure 9. Modeling errors under other error indexes in case 2.

3.3. Case 3: REC in United Kingdom

The UK government has been committed to promoting energy transition and achieving the goals of a low-carbon economy and sustainable development. To this end, the government has introduced a series of policies and measures to support the development of renewable energy, including financial subsidies, tax incentives, and research and development support. Recently, renewable energy technologies have witnessed rapid development in the UK. Particularly in the fields of wind and solar energy, technological innovations have continuously driven cost reductions and efficiency improvements, providing strong support for the large-scale application of renewable energy. According to data, wind power has surpassed fossil fuels for two consecutive quarters, becoming the largest single source of electricity in the UK. This reflects the increasing share of renewable energy in the UK's electricity mix. Therefore, we validated the effectiveness of the FGRM using the REC in the UK. The models were established based on data from 2015 to 2021, and predictions were made for 2022. Table 3 presents the modeling results for each model. The optimal accumulation orders for the four grey models are 0.621, 0.505, 0.735, and 0.999 for the FGRM, FGM, CFGM, and CFGOM, respectively. Figure 10 shows the PSO iterations for the accumulation order, Figure 11 illustrates the fitting and prediction curves for each model, Figure 12 displays the deviations of each model's modeling results from the original system at various timepoints, and Figure 13 shows the simulation and prediction errors under other error indexes.

It can be observed that the modeling results of the FGRM aligned well with the general trend of the raw data, accurately reflecting the growth trend of the original data in most years, with only minor discrepancies in certain years (such as 2017 and 2019). The FGM closely matched the original data from 2015 to 2016 but started to deviate significantly from 2017 onwards. Especially for 2020 to 2022, the simulated and predicted values of the FGM were notably higher than the original data. The CFGM's results were relatively close to the original data in most years but began to diverge from 2019 onwards. For instance, the predicted values for 2021–2022 were higher than the original data. The CFGOM generally

reflected the growth trend of the original data well across most years but showed significant discrepancies in certain years (such as 2019 and 2021), particularly with the 2022 predicted value being notably lower than the original data. The ARIMA model's results closely aligned with the original data from 2015 to 2017 but started to diverge significantly from 2018 onwards.

Table 3. Modeling results and errors in case 3.

Year	Raw Data (EJ)	FGRM	FGM	CFGM	CFGOM	ARIMA
2015	0.840	0.840	0.840	0.840	0.804	0.839
2016	0.840	0.840	0.894	0.876	0.892	0.840
2017	0.990	1.008	1.006	0.994	0.990	0.840
2018	1.120	1.122	1.101	1.094	1.098	0.990
2019	1.220	1.205	1.179	1.183	1.219	1.120
2020	1.350	1.268	1.244	1.265	1.352	1.220
2021	1.240	1.316	1.298	1.343	1.500	1.350
2022	1.360	1.353	1.343	1.418	1.664	1.240
r		0.621	0.505	0.735	0.999	/
MRSPE (%)		2.201	4.276	4.131	2.117	7.651
MRFPE (%)		0.529	1.256	4.294	22.367	8.824
CMRPE (%)		1.992	3.898	4.151	4.648	7.797

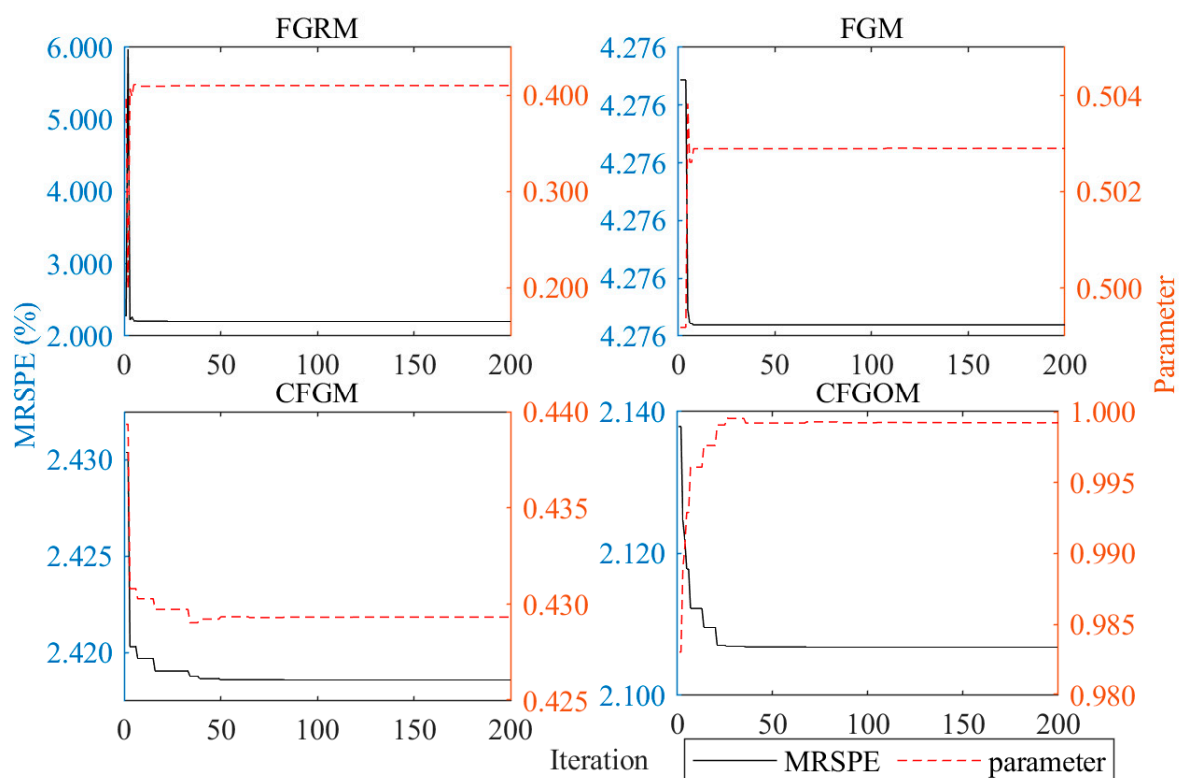


Figure 10. PSO algorithm iteration situation in case 3.

In terms of error results, the FGRM had the smallest MRSPE, indicating the least error during the simulation process. Conversely, the ARIMA model had the largest MRSPE, suggesting the greatest error during simulation. The CFGOM had the highest MRFPE, reaching 22.367%, while the FGRM's MRFPE was relatively low, at just 0.529%. Similarly, the CFGOM had the highest CMRPE, indicating the greatest overall performance error. The FGRM had the lowest CMRPE, demonstrating the best overall performance. In summary, the FGRM exhibited smaller errors in terms of simulation error, prediction error, and comprehensive error, indicating its high accuracy and stability in processing such data.

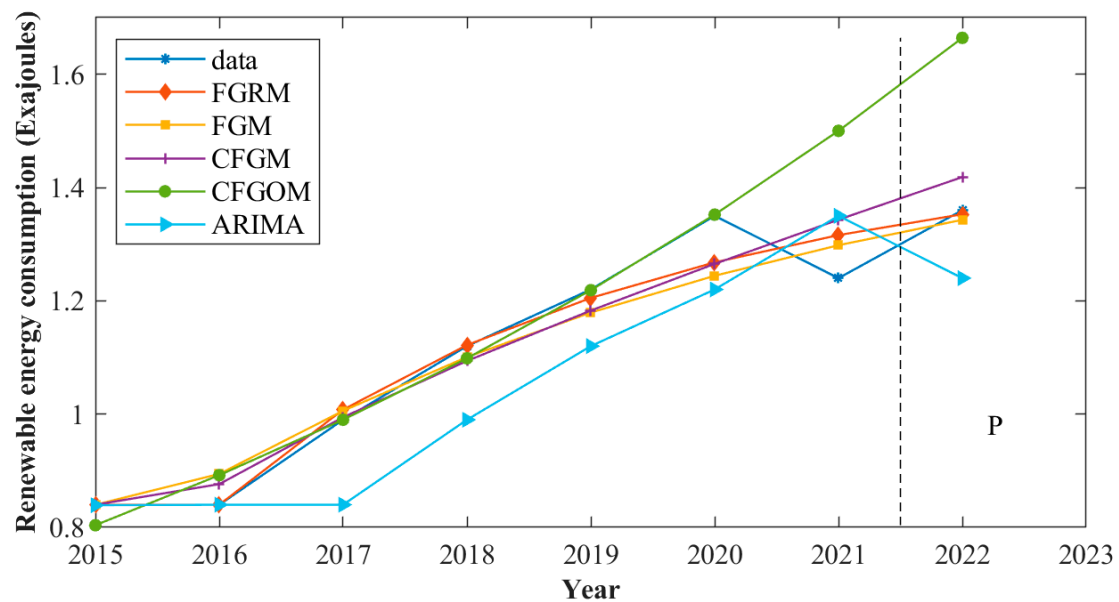


Figure 11. Fitting and predicting curve in case 3.

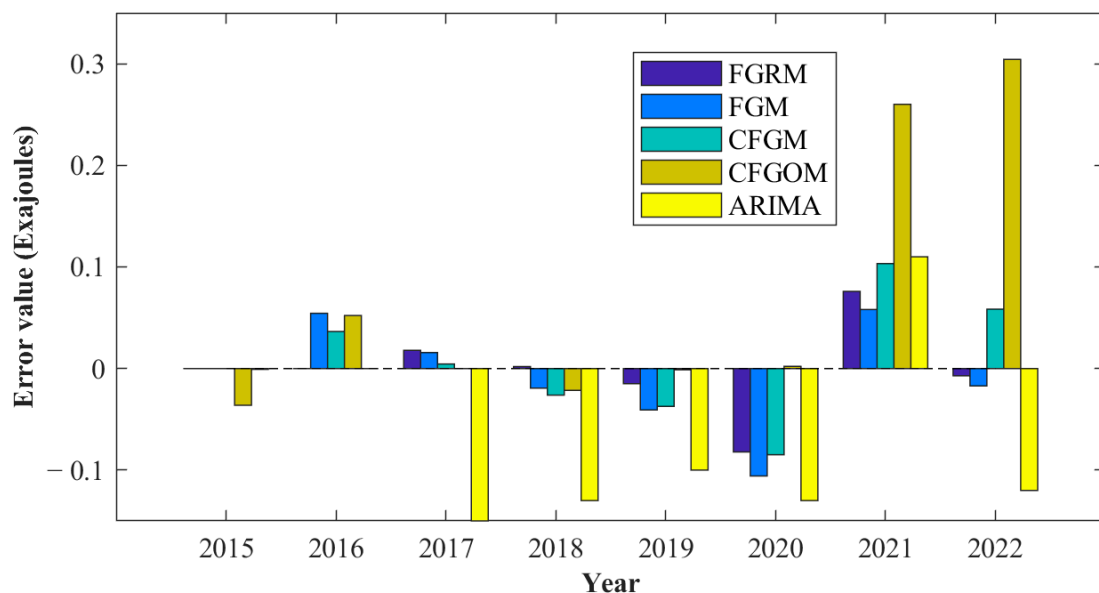


Figure 12. Discrepancy between the modeling results and raw data in case 3.

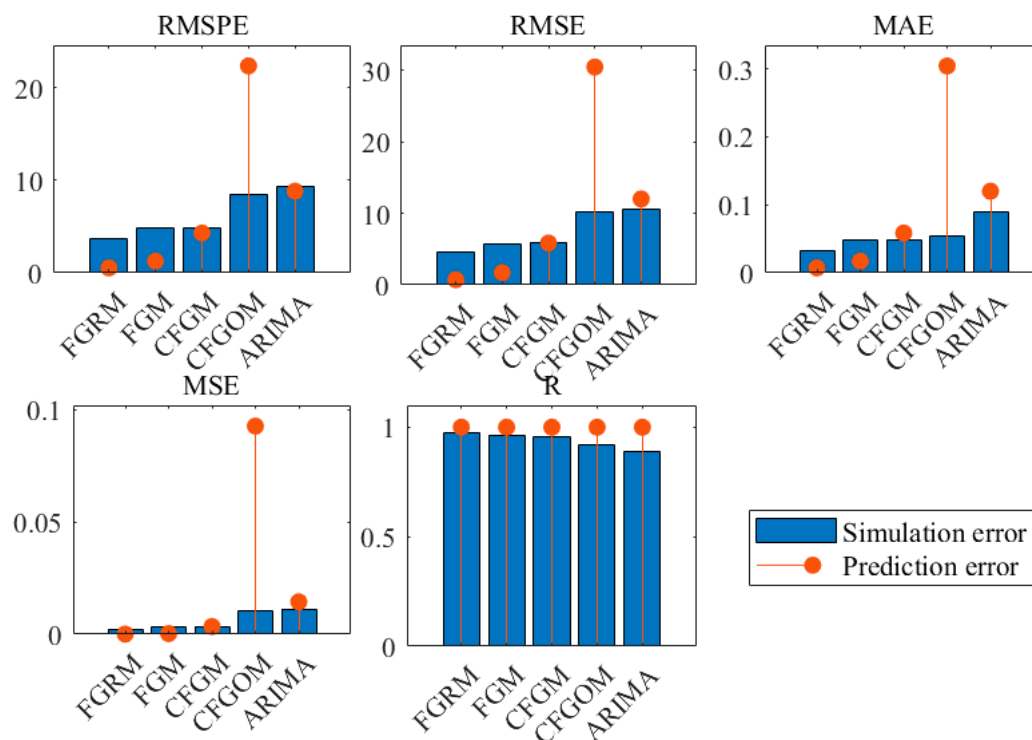


Figure 13. Modeling errors under other error indexes in case 3.

3.4. Case 4: REC in Total Europe

For a long time, Europe has had a high dependence on external energy supplies, particularly natural gas and oil. To reduce this dependence and enhance energy security, many European countries have begun to vigorously develop renewable energy. According to the “2024 Energy Union State of the Union Report”, half of the EU’s electricity generation came from renewable energy sources in the first half of 2024. This demonstrates significant progress in renewable energy generation in Europe. The EU has set a target for renewable energy to account for more than 40% of final energy consumption by 2030, and some countries have even proposed higher targets. Therefore, we validated the effectiveness of the FGRM using REC from Europe. The models were established based on data from 2015 to 2021, and predictions were made for 2022. Table 4 presents the modeling results for each model. The optimal accumulation orders for the four grey models were 0.886, 0.82, 0.842, and 1 for the FGRM, FGM, CFGM, and CFGOM, respectively. Figure 14 shows the PSO iterations for the accumulation order, Figure 15 illustrates the fitting and prediction curves for each model, Figure 16 displays the deviations of each model’s modeling results from the original system at various timepoints, and Figure 17 shows the simulation and prediction errors under other error indexes.

It can be observed that in most years, the prediction results of the FGRM were very close to the original data, especially during the period from 2015 to 2019, when its predicted values almost aligned with the original data. The prediction results of the FGM were also relatively stable, but compared with the FGRM, there was a slightly larger gap with the original data. The modeling results of the CFGM showed more significant differences from the original data in some years (such as 2018 and 2020), but overall, its simulation performance was still relatively good. The modeling results of the CFGOM exhibited greater variability in some years, particularly after 2020, when the gap between its predicted values and the original data gradually widened. The ARIMA’s modeling results were relatively poor, especially after 2016, when the gap between its predicted values and the original data significantly increased.

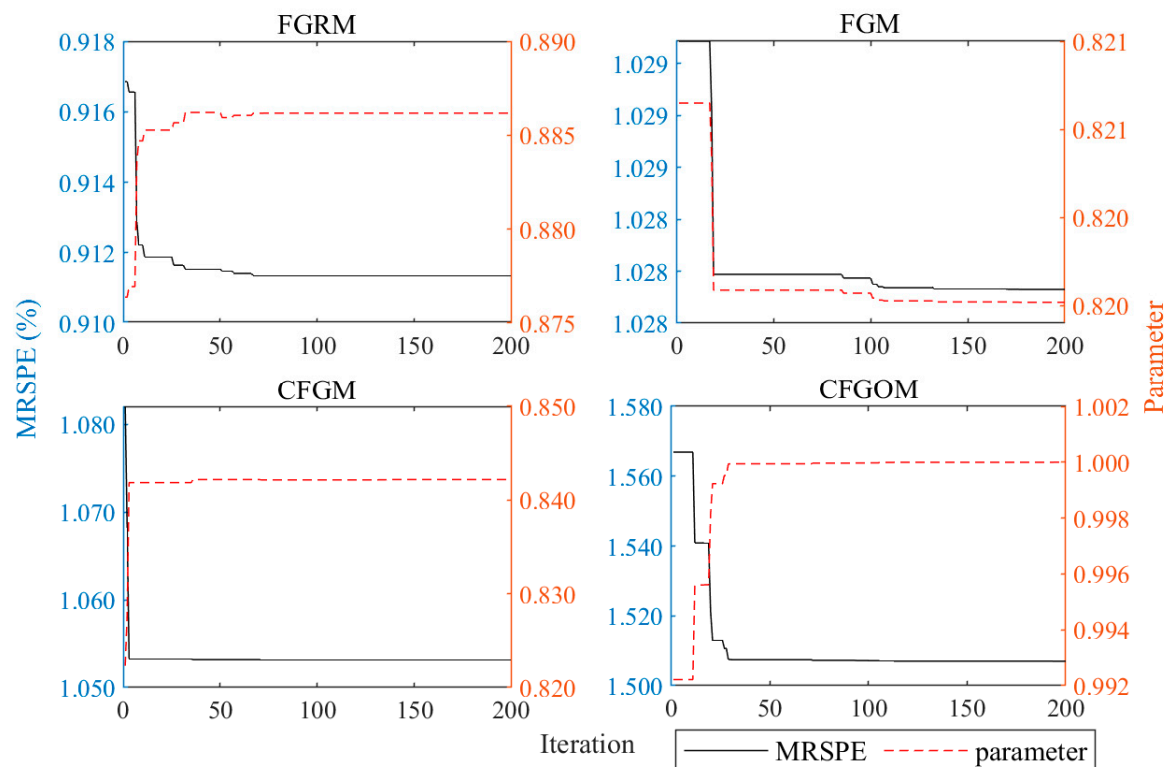


Figure 14. PSO algorithm iteration situation in case 4.

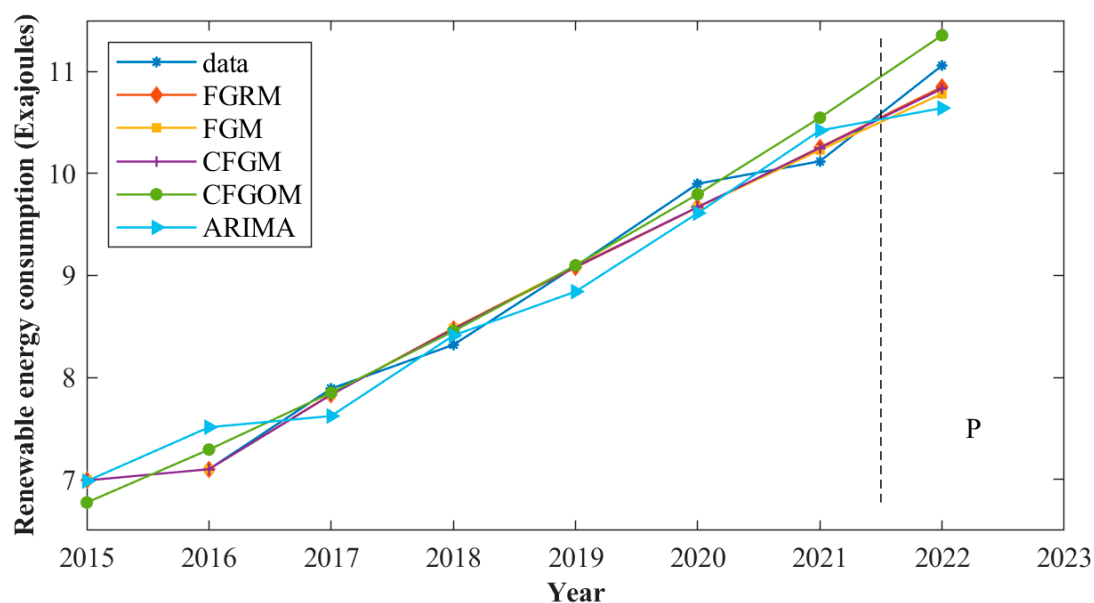


Figure 15. Fitting and predicting curve in case 4.

In terms of error results, the FGRM had the smallest MRSPE, at just 0.911%, indicating the best simulation performance. The ARIMA model had the largest MRSPE, reaching 2.717%, suggesting relatively poor simulation performance. The MRFPEs of the FGRM and CFGM were relatively small, at 1.902% and 2.067%, respectively. The ARIMA model still had the largest MRFPE, at 3.782%. The FGRM had the smallest CMRPE, at 1.035%, indicating the best overall performance. The ARIMA model had the largest CMRPE, at 2.85%, suggesting relatively poor overall performance. Taken together, the FGRM demonstrated the best performance in simulating and predicting a certain data indicator

for Europe, with its prediction results closely matching the original data and exhibiting the smallest errors.

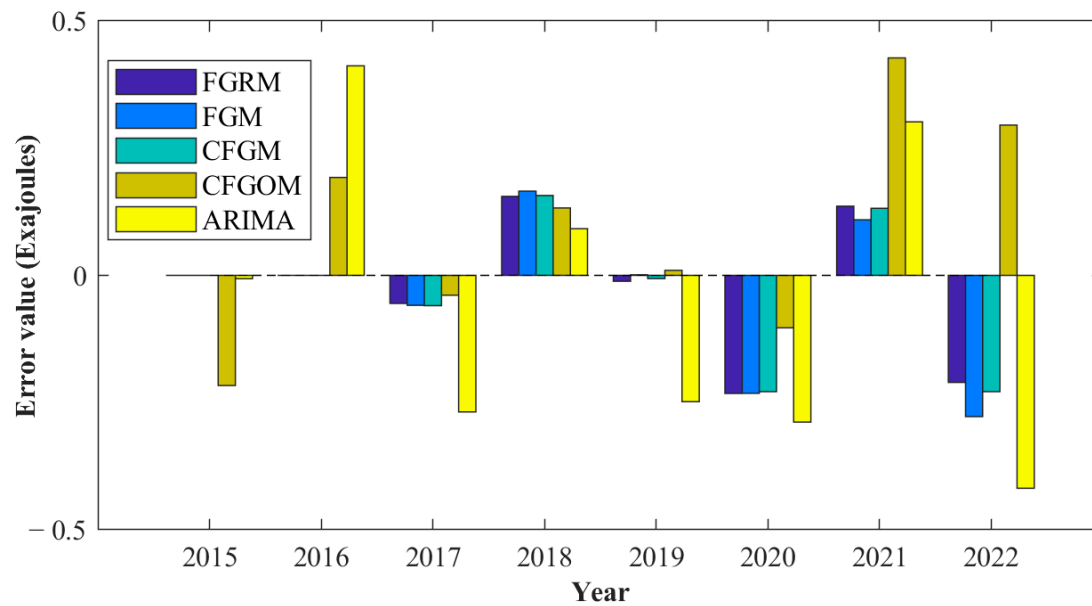


Figure 16. Discrepancy between the modeling results and raw data in case 4.

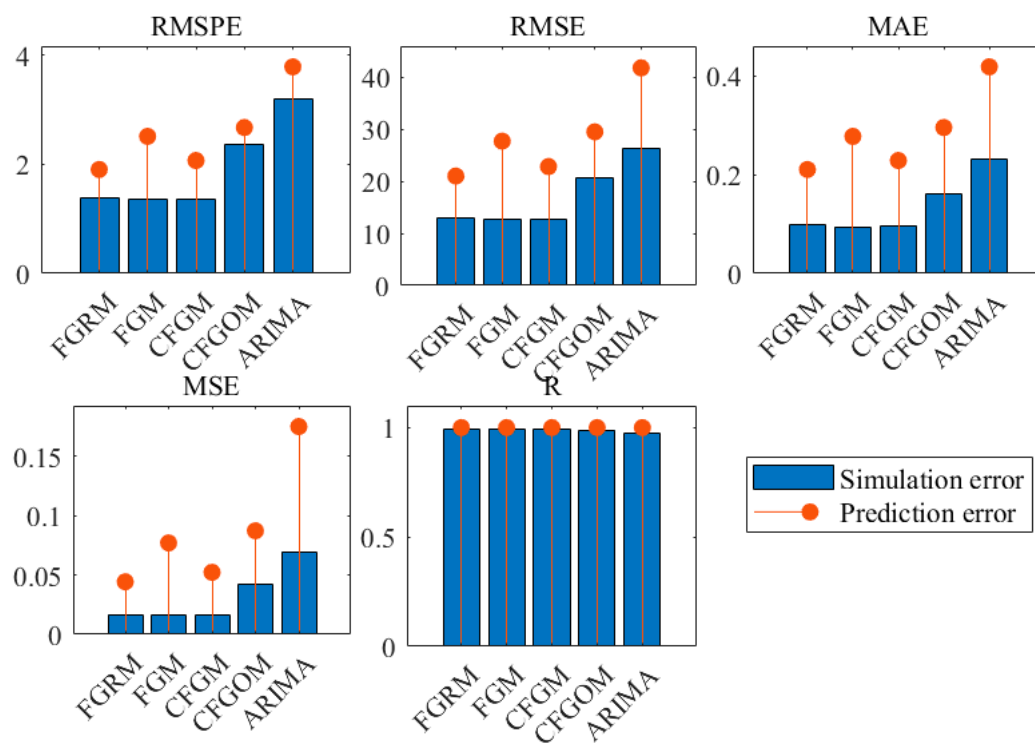


Figure 17. Modeling errors under other error indexes in case 4.

Table 4. Modeling results and errors in case 4.

Year	Raw Data (EJ)	FGRM	FGM	CFGM	CFGOM	ARIMA
2015	6.990	6.990	6.990	6.990	6.773	6.984
2016	7.100	7.100	7.100	7.100	7.292	7.512
2017	7.890	7.835	7.831	7.830	7.851	7.622
2018	8.320	8.475	8.485	8.477	8.452	8.412
2019	9.090	9.078	9.091	9.084	9.100	8.842
2020	9.900	9.668	9.668	9.672	9.797	9.612
2021	10.120	10.256	10.229	10.252	10.547	10.422
2022	11.060	10.850	10.783	10.831	11.355	10.642
<i>r</i>		0.886	0.82	0.842	1	/
MRSPE (%)		0.911	1.028	1.053	1.507	2.717
MRFPE (%)		1.902	2.509	2.067	2.671	3.782
CMRPE (%)		1.035	1.213	1.18	1.652	2.85

4. Forecasting REC

Based on a comprehensive comparison of the aforementioned models, it is evident that the FGRM exhibited the highest level of modeling accuracy, closely aligning with the original data and outperforming the other models in terms of precision and reliability. Therefore, using REC data from 2015 to 2022 as the basis, the FGRM was employed to predict the REC for France, Spain, the United Kingdom, and Europe as a whole from 2023 to 2030. The relevant forecast results are summarized in Table 5, and for better visualization of the data trends, they are also plotted in Figure 18.

Table 5. Forecast results of REC from 2023 to 2030 (EJ).

	France	Spain	United Kingdom	Total Europe
<i>R</i>	0.890	1.282	0.624	0.905
MRSPE (%)	1.581	1.226	1.957	0.945
2023	0.856	1.158	1.386	11.645
2024	0.913	1.287	1.408	12.335
2025	0.972	1.434	1.425	13.052
2026	1.035	1.602	1.436	13.799
2027	1.100	1.794	1.444	14.578
2028	1.169	2.012	1.448	15.391
2029	1.242	2.261	1.448	16.241
2030	1.318	2.543	1.447	17.130

France's REC is projected to show a steady growth trend in the coming years. It is expected to increase from 0.856 EJ in 2023 to 1.318 EJ in 2030, representing a growth rate of 53.97%. This is slightly higher than the forecast result (1.19 EJ) reported by Şahin [49]. The possible reason lies in the fact that Şahin [49] utilized modeling data from the period 2004–2018, whereas the rapid development of renewable energy in recent years has far exceeded expectations. This also indicates that France is making good progress in the field of renewable energy and is likely to achieve its energy transition goals. This positive trend may be attributed to the French government's commitment to promoting energy transition, reducing dependence on fossil fuels, and vigorously developing renewable energy sources such as wind and solar power. Additionally, France possesses relatively well-developed energy infrastructure and advanced energy technologies, providing strong support for the development of renewable energy. Spain's REC is forecasted to experience rapid growth

in the coming years. It is anticipated to rise from 1.158 EJ in 2023 to 2.543 EJ in 2030, with a growth rate significantly higher than that of France, reaching 119.60%. This is also significantly higher than the forecast result reported by Şahin [49], which may be attributed to Spain's considerable potential for development in the renewable energy sector and its readiness to achieve a more pronounced energy transition. This potential is likely due to Spain being one of the key countries for renewable energy development in Europe, with abundant solar and wind resources. The government actively promotes renewable energy and has implemented a series of supportive policies. Furthermore, Spain's relatively open energy market has attracted substantial domestic and foreign investment, further driving the development of renewable energy. The United Kingdom's REC is also expected to increase in the coming years, albeit at a slower rate. It is projected to grow from 1.386 EJ in 2023 to 1.447 EJ in 2030, with a growth rate of only 4.40%, significantly lower than those of France and Spain. This is also largely consistent with the forecast result (1.54 EJ) reported by Şahin [49]. This slower growth may be associated with policy adjustments, market changes, and resource endowments in the renewable energy sector in the UK. Europe's overall REC is forecasted to exhibit a rapid growth trend in the coming years. It is anticipated to increase from 11.645 EJ in 2023 to 17.130 EJ in 2030, with a growth rate of 47.10%. This is largely consistent with the forecast results reported by Mehedintu et al. [53]. This also indicates that Europe has strong momentum in the field of renewable energy and is likely to achieve more significant energy transition and greenhouse gas emission reduction targets. The overall development context for renewable energy in Europe is similar to that of the aforementioned countries.

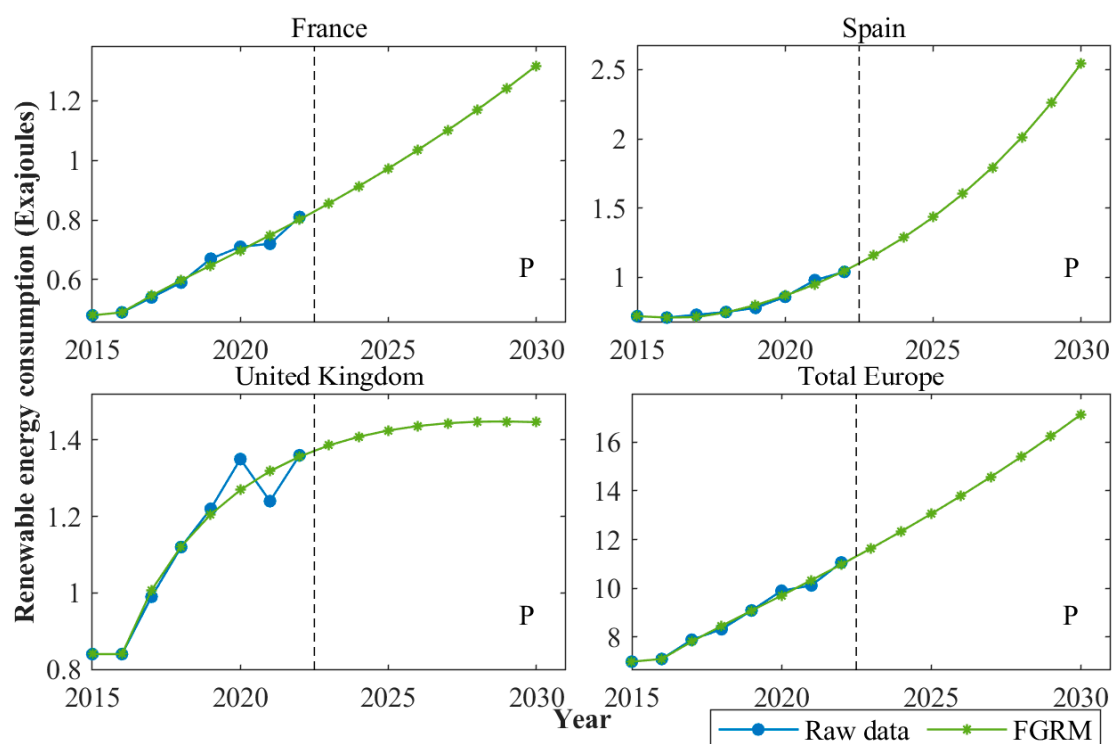


Figure 18. Fitting and forecasting curve of REC from 2023 to 2030.

5. Conclusions

Amidst the dual challenges of global climate change and energy crises, renewable energy, as a clean and sustainable energy form, has garnered widespread consensus within the international community for its development and utilization. Accurate forecasting of REC is crucial for ensuring national energy security. However, predicting re REC is a complex and challenging task, and traditional forecasting methods often struggle to

achieve satisfactory results when dealing with such intricate data. Although grey models have been widely applied in the field of energy consumption forecasting, one of their key principles—the priority of new information—has not been fully realized in practical applications. The traditional GM(1,1) employs a forward sequence accumulation method, which conflicts with the principle of new information priority. Despite the emergence of various improved inverse accumulation grey models in recent years, these models typically perform well in the fitting stage but fail to provide effective extrapolation predictions. This limitation is primarily due to the insufficient accuracy of inverse accumulation operations during extrapolation, which restricts their predictive capability in practical applications. To address this challenge, this paper innovatively proposes the FGRM(1,1) to solve the aforementioned issues. The core conclusions of this paper are summarized below:

Firstly, building upon the existing mode, this paper introduces a novel fractional reverse accumulation and derives the operations along with several of its characteristics. Subsequently, a novel grey model, designated as the FGRM(1,1), is formulated. To optimize the performance of this model, its parameters are determined through the application of the PSO algorithm.

Secondly, the validation results based on REC data from France, Spain, the United Kingdom, and Europe demonstrated that the FGRM(1,1) exhibited the smallest simulation error, prediction error, and comprehensive error in all four cases. Its prediction performance significantly outperformed that of other compared models, indicating that the novel reverse accumulation operation can effectively utilize new information.

Thirdly, the prediction results indicated that France and Spain will achieve significant growth in REC by 53.97% and 119.60%, respectively, from 2023 to 2030, reflecting the positive trends in their energy transitions. In contrast, the growth in the United Kingdom was predicted to be relatively slow, at only 4.40%. It was predicted that overall, Europe's REC would increase by 47.10%, showcasing robust development momentum and emission reduction potential.

This paper applied the new model solely to the prediction of REC. In the future, it can be extended to predict other energy consumption or production scenarios. Additionally, the reverse accumulation method can be combined with other state-of-the-art grey models to construct superior grey models for application in a wider range of fields.

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