

## Article

# Chinese Bureaucratic System Algorithm: A Rank-Weighted Institutional Search Framework for Continuous and Routing Decision Systems

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## Highlights

**Please indicate how your work links to systems science via your contributions to systems practice, theory, and or methodology.**

- The Chinese Bureaucratic System Algorithm (CBSA) separates rank aggregation from representation-specific transition and feasibility mechanisms in continuous and routing decision systems.
- The framework leverages population rank, memory, and recovery mechanisms as a structured search-policy architecture applicable to both continuous and discrete decision systems.

**What are the main findings and/or the implications of the main findings?**

- The continuous variant of CBSA (CBSA-C) obtains the best average rank among five methods on the 2014 continuous benchmark suite and is strongest on multimodal and hybrid functions.
- The discrete variant of CBSA (CBSA-D) is competitive on capacitated routing and clustered time-window instances, while mature local search remains preferable when reusable adjacency is weak.

## Abstract

Many decision systems require search over continuous parameters or discrete route structures. This study examines whether a rank-weighted population architecture can support both representations while retaining an interpretable decision mechanism. We propose the Chinese Bureaucratic System Algorithm (CBSA), which transforms objective-based population ranks into either continuous movement or edge-based routing probabilities. We expect rank aggregation to be most useful when higher-ranked solutions contain stable information that can be reused in later searches. The framework is instantiated as a continuous variant (CBSA-C) and a discrete routing variant (CBSA-D), and its computational complexity and limiting search properties are analyzed. On the 2014 Congress on Evolutionary Computation (CEC2014) benchmark suite, CBSA-C obtains the best average rank among five algorithms and is strongest on multimodal and hybrid functions. On 50 capacitated vehicle routing instances, CBSA-D ranks second behind iterated local search while outperforming four population- or edge-based baselines. Across all 56 Solomon time-window routing instances, it is most competitive on clustered classes and weaker on random and mixed classes. These results support a conditional interpretation: rank-weighted aggregation is useful when objective ranks are informative and the representation



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contains reusable structure, while stronger local-search methods remain preferable when such structure is absent.

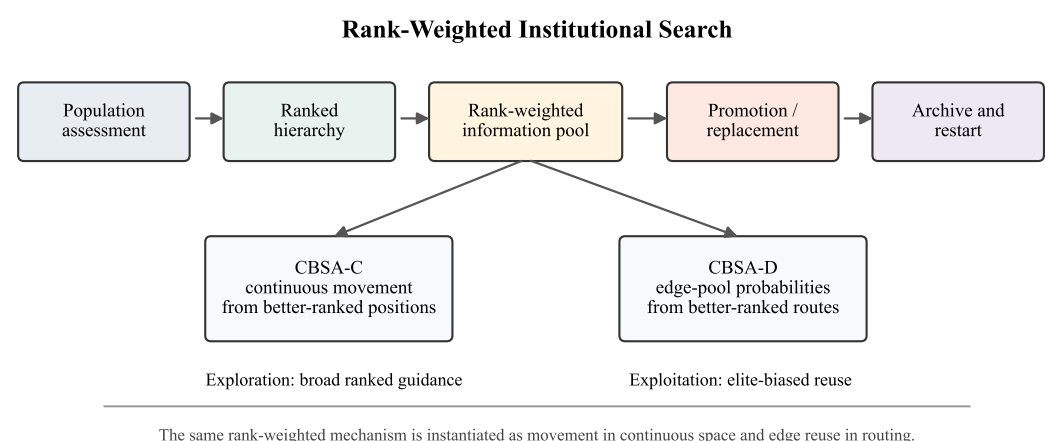
**Keywords:** evolutionary algorithm; rank-based population search; vehicle routing problem; black-box optimization; edge-pool model; metaheuristics

## 1. Introduction

Many practical systems require search over continuous or discrete decision variables. Continuous problems involve tuning parameters, balancing trade-offs, or calibrating control variables. Discrete problems involve assigning routes, sequences, or resources under hard feasibility constraints. In both settings, optimization requires not only a good solution but also a search process that can learn from the relative quality of multiple candidate states.

Three related issues motivate this study. Leader-centered updates can accelerate exploitation, but may overcommit the population when early leaders are unreliable [1,2]. Continuous attraction operators also do not transfer directly to capacity- and time-constrained permutations. Meanwhile, edge-aware routing methods can learn useful local structure, but their information models are usually tied to a particular construction process rather than derived from an objective-ranked contributor set [3,4]. We therefore ask whether objective-based ranks can provide a common information architecture whose transition rule changes with the representation: continuous movement for real-valued states and edge-guided construction for routes. The institutional metaphor determines this architecture; the scientific claim concerns rank-weighted information transfer.

This paper introduces the Chinese Bureaucratic System Algorithm (CBSA) as a rank-weighted institutional search framework. The historical analogy supplies a vocabulary for a two-level search mechanism: CBSA-C applies rank-weighted gravitation in continuous space, whereas CBSA-D transfers the ordering principle to an edge-pool probability model for routing. The framework uses a small set of core operators so that its mechanisms can be traced across problem classes. Figure 1 maps the institutional vocabulary to the rank, memory, recovery, and representation-specific operators developed below.



**Figure 1.** Conceptual mapping from the institutional hierarchy to the technical CBSA operators.

We examine two classes of decision systems. The 2014 Congress on Evolutionary Computation (CEC2014) continuous benchmark suite provides a controlled test of exploration and exploitation across unimodal, multimodal, hybrid, and composition landscapes [5]. The capacitated vehicle routing problem (CVRP) and the vehicle routing problem with time windows (VRPTW) test whether the same framework can exploit reusable adjacency

structure in logistics and distribution planning [6–8]. Although their representations differ, both require a search rule that turns ranked candidate states into new decisions.

The paper makes four contributions. First, it formulates CBSA as a population-based framework with rank weighting, archive memory, stagnation handling, local refinement, and diversity recovery. Second, it develops CBSA-C for continuous decision systems and CBSA-D for routing decision systems. Third, it provides complexity and limited convergence analysis to clarify the computational cost and search behavior of the framework. Fourth, it evaluates the method on CEC2014, CVRP, and VRPTW to identify when rank-weighted aggregation is useful and when stronger local-search structure remains necessary. The experiments identify representation- and structure-dependent performance boundaries rather than uniform dominance.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature and positions the framework. Section 3 presents the CBSA design and its continuous and routing instantiations. Section 4 describes the experiments. Section 5 reports the results. Section 6 discusses limitations and implications for system design; and Section 7 concludes.

## 2. Background and Positioning

### 2.1. Population-Based Metaheuristics

Population-based metaheuristics maintain multiple candidate solutions and update them through selection, recombination, attraction, or collective learning. The genetic algorithm (GA) introduced selection, crossover, and mutation as a general evolutionary template [9]. Particle swarm optimization (PSO) models the motion of particles under individual and social memory [1]. Differential evolution (DE) generates trial vectors through scaled differences between population members and remains a strong continuous optimizer [10]. The grey wolf optimizer (GWO) uses a simple hierarchy of leading solutions to guide the population [2]. Surveys and monographs emphasize that these methods are best understood as families of search principles rather than single fixed algorithms [11,12]. Recent studies also emphasize component-level descriptions, reusable implementations, and common experimental protocols for metaheuristic research [13,14]. These methods are flexible, but their continuous-space operators often require careful adaptation before they can exploit the edge structure of routing problems.

Ant colony optimization (ACO) is especially relevant to routing because pheromone values encode edge desirability and are updated from constructed tours [3]. Early routing variants showed that edge-level memory is useful for vehicle routing problem (VRP) search [15]. CBSA-D shares the idea that edges are useful units of information, but it differs in how the edge model is estimated: CBSA-D uses a rank-weighted aggregation over the current population instead of pheromone deposition by ants.

The most relevant distinction for this paper is the difference between leader following and rank-weighted aggregation. In leader-following methods, only a small subset of elite individuals determines most of the population movement. This can accelerate convergence when the current leaders are reliable, but it can also amplify early sampling errors. CBSA-C aggregates every better-ranked individual, whereas CBSA-D applies the same rank weighting to a variant-specific contributor set. In both cases, the weight function prioritizes stronger contributors without reducing the update to a single leader. CBSA is therefore closer to a weighted population-estimation mechanism than to pure leader following.

### 2.2. Probability-Model Search and Routing Heuristics

Estimation of distribution algorithms (EDAs) replace crossover and mutation with an explicit probability model learned from selected solutions [4]. The cross-entropy method

(CEM) similarly updates a sampling distribution from elite samples [16]. Recent theoretical and methodological reviews emphasize that the representation, probability model, parameter control, and use of problem structure jointly determine the behavior of an EDA, particularly in discrete spaces [17,18]. CBSA-D can be viewed as a rank-weighted permutation EDA: it estimates an edge-pool probability matrix from a ranked contributor set, with higher ranks receiving larger weights.

For routing, specialized local-search and hybrid methods remain highly competitive. Tabu search and probabilistic diversification were historically important for VRP local search [19,20]. Evolutionary VRP solvers include early genetic search for VRPTW [21]. Adaptive large neighborhood search and related destroy–repair frameworks are strong for rich VRP variants [22,23]. A recent operator-level review confirms the importance of representation-aware removal and insertion choices [24]. Hybrid genetic search (HGS) combines population diversity management with powerful local search and has become a leading approach for CVRP and related variants [25,26]. Recent open-source solvers such as PyVRP make high-quality HGS-style methods easier to reproduce [27]. Learning-based combinatorial optimization now includes neural improvement and hybrid search policies [28–32]. These methods are not direct baselines here because they require separate training and computational protocols; recent guidance also cautions that routing comparisons should align instance distributions, budgets, baselines, and hardware [33]. The experimental design therefore includes an iterated local search (ILS) baseline [34], so that CBSA-D is not compared only against weak discrete adaptations of continuous algorithms.

A routing method that outperforms only naive GA, PSO, or GWO discretizations may still underperform relative to local-search-centered baselines. We therefore include ILS as a strong local-search baseline so that CBSA-D is evaluated against a representative local-search method rather than only against discrete adaptations of continuous algorithms.

### 2.3. Metaphor-Based and Institutional Algorithms

Many recent metaheuristics draw inspiration from natural or social systems. Examples include teaching–learning-based optimization and imperialist competitive algorithms [35,36]. This practice is useful only when the metaphor leads to a clear computational mechanism. Sörensen argued that metaphor alone should not be mistaken for methodological contribution [37]; subsequent critiques similarly call for explicit operators, identifiable novelty, and rigorous validation rather than metaphor-centered claims [14,38]. CBSA is positioned accordingly: the imperial bureaucracy metaphor organizes the design vocabulary, whereas the technical claims concern rank-weighted information aggregation and its representation-specific realization in continuous and routing spaces.

The term institutional is used here in a limited technical sense. It refers to a system with ranks, selection, assessment, memory, promotion, and corrective mechanisms. CBSA borrows this vocabulary only as a design scaffold: civil-service examinations motivate selection, official rank motivates weighted influence, performance assessment motivates stagnation handling, the Hanlin Academy motivates elite memory, honorary inheritance motivates controlled elite propagation, and exile motivates diversity recovery. These references are only used to name the operator roles, as summarized in Table 1. The validity of the algorithm does not depend on historical authenticity.

### 2.4. Systems Perspective on Decision Structure

We treat CBSA as a decision policy rather than as a metaphorical optimizer. In a decision system, the current state, transition rule, and feasibility constraints matter as much as the objective value. CBSA maps ordered candidate information into either a movement vector or an adjacency model, and is most useful when state transitions are

regular enough for rank information to remain informative across iterations. The central question is whether this policy matches the structure of the controlled system.

**Table 1.** Institutional metaphors and their technical roles in CBSA.

| Institutional Metaphor                 | Technical Operator             | Interpretation                                                         |
|----------------------------------------|--------------------------------|------------------------------------------------------------------------|
| Imperial examination (Keju)            | Rank sorting and selection     | Selects higher-quality candidates from the population.                 |
| Official rank and titles               | Rank-weighted gravitation      | Gives stronger influence to higher-ranked candidates.                  |
| Performance assessment (Kaoji law)     | Stagnation detection and reset | Triggers replacement or reinitialization when progress stalls.         |
| Hanlin Academy                         | Elite archive                  | Stores historically strong solutions for later reuse.                  |
| Honorary inheritance/patronage (Enyin) | Elite cloning and mutation     | Preserves strong structures while allowing controlled variation.       |
| Exile/demotion                         | Diversity restart              | Reintroduces exploration when the population becomes too concentrated. |
| Inner cabinet                          | Local refinement around elites | Performs extra improvement on promising solutions.                     |

### 3. Framework and Methods

Consider a minimization problem  $\min_{\mathbf{x} \in \Omega} f(\mathbf{x})$ . At iteration  $t$ , the population is

$$P^{(t)} = \{\mathbf{x}_1^{(t)}, \dots, \mathbf{x}_N^{(t)}\},$$

where individuals are sorted in nondecreasing objective value. Thus, rank 1 is the best individual. CBSA assigns each rank a weight:

$$w_j(t) = \frac{1}{j^{p(t)}}, \quad p(t) = p_0 + (p_1 - p_0) \left( \frac{t}{T} \right)^\gamma, \quad (1)$$

Here,  $t \in \{0, \dots, T-1\}$ ,  $T \in \mathbb{N}_+$  is the iteration budget,  $j \in \{1, \dots, N\}$ ,  $p_0, p_1 \geq 0$ , and  $\gamma > 0$ . Let  $p_{\min} = \min(p_0, p_1)$  and  $p_{\max} = \max(p_0, p_1)$ . Then,  $p(t) \in [p_{\min}, p_{\max}]$  and  $w_j(t) \in [N^{-p_{\max}}, 1]$ . For candidate  $i > 1$ , the eligible contributor set is  $\mathcal{B}_i = \{1, \dots, i-1\}$  and the normalized influence is

$$q_{ij}(t) = \frac{w_j(t)}{\sum_{k \in \mathcal{B}_i} w_k(t)}, \quad q_{ij}(t) > 0, \quad \sum_{j \in \mathcal{B}_i} q_{ij}(t) = 1. \quad (2)$$

These are deterministic objective-rank weights, rather than learned attribute weights. Preference-disaggregation methods provide a complementary direction by inferring feature weights from observed decision outcomes [39]; such data-driven weight-determination approaches could inform future adaptive rank-weight schedules that adjust contributor influence based on historical search performance. In the present work, CBSA uses the observed ordering of current candidate solutions to allocate search influence.

For a non-best individual  $i$ , the rank-weighted gravitation direction is

$$\mathbf{G}_i^{(t)} = \frac{\sum_{j=1}^{i-1} w_j(t) (\mathbf{x}_j^{(t)} - \mathbf{x}_i^{(t)})}{\sum_{j=1}^{i-1} w_j(t)}. \quad (3)$$

In Equation (3),  $\mathbf{x}_j^{(t)} - \mathbf{x}_i^{(t)} \in \mathbb{R}^D$  is the displacement from follower  $i$  to better-ranked contributor  $j$ ,  $w_j(t)$  is given by Equation (1), and the denominator normalizes their aggregate.

Equivalently,  $\mathbf{G}_i^{(t)} = \sum_{j \in \mathcal{B}_i} q_{ij}(t)(\mathbf{x}_j^{(t)} - \mathbf{x}_i^{(t)})$ . The direction therefore uses every better-ranked candidate, while the power exponent controls how concentrated that influence is.

The exponent schedule in Equation (1) controls the balance between exploration and exploitation. Early in a run,  $p(t)$  is close to  $p_0$ , so differences between adjacent ranks are moderate and the update reflects a broad population consensus. Later,  $p(t)$  approaches  $p_1$  and the best-ranked candidates dominate the weighted average. The mechanism gradually shifts from broad aggregation to elite-centered exploitation without a hard switch between two algorithms.

Two limiting cases clarify the role of the weight function. If  $p(t) = 0$ , all better-ranked individuals have equal influence and Equation (3) points toward the centroid of the better-ranked subset. If  $p(t) \rightarrow \infty$ , only the best-ranked individual matters and the update approaches a one-leader attraction rule. Between these limits, several high-ranked candidates retain influence while inferior candidates move toward promising regions.

### 3.1. Search-Bias Comparison

Table 2 identifies the primary information channel used by each method. GA uses parent recombination and mutation; PSO uses personal and global memories; GWO uses a small leader set; ACO accumulates pheromone on components; and EDA and CEM estimate a probability distribution from selected samples. CBSA uses rank-weighted information from the current population and changes the representation of that information according to the search space.

**Table 2.** Mechanistic comparison between CBSA and representative metaheuristics.

| Method  | Primary Information Source                | Update Object                        | Main Risk                                     |
|---------|-------------------------------------------|--------------------------------------|-----------------------------------------------|
| GA      | Selected parents                          | Genotype through crossover/mutation  | Premature convergence or disruptive crossover |
| PSO     | Personal and global bests                 | Continuous velocity/position         | Weak handling of structured permutations      |
| GWO     | Three leading individuals                 | Continuous attraction toward leaders | Over-reliance on early leaders                |
| ACO     | Pheromone and heuristic edge desirability | Edge probabilities                   | Parameter-sensitive pheromone stagnation      |
| EDA/CEM | Elite selected samples                    | Explicit probability model           | Information loss from hard elite truncation   |
| CBSA    | Rank-weighted population ordering         | Direction vector or edge-pool model  | Dependence on useful rank-induced structure   |

The common framework ends at the ranked information architecture and the shared memory, stagnation, and diversity controls. For real-valued box-constrained states, CBSA-C converts weighted contributor positions into a projected continuous update. For routing states, CBSA-D converts weighted route adjacencies into edge scores and then relies on a task-specific constructor and decoder to preserve capacity or time-window requirements. Figure 1 depicts this shared layer and the two representation-specific transitions. Thus, CBSA-C and CBSA-D are related instantiations rather than identical low-level algorithms; their computational costs also differ, as quantified after both variants in Section 3.4.

### 3.2. Continuous Variant: CBSA-C

CBSA-C realizes the rank-weighted principle through contributor positions. Let  $\mathcal{B}_i = \{1, \dots, i-1\}$  for follower  $i > 1$ ; let  $K \in \{1, \dots, N-1\}$  denote the number of annealed leading contributors; and define

$$\alpha(t) = 2 - 2\left(\frac{t}{T}\right)^K, \quad A_{ij}(t) = 2\alpha(t)r_{1,ij} - \alpha(t), \quad C_{ij} = 2r_{2,ij}, \quad (4)$$

where  $r_{1,ij}, r_{2,ij} \stackrel{\text{iid}}{\sim} \mathcal{U}(0, 1)$  are resampled for each follower–contributor pair and iteration. Consequently,  $\alpha(t) \in (0, 2]$ ,  $A_{ij}(t) \in [-\alpha(t), \alpha(t)]$ , and  $C_{ij} \in [0, 2]$  over the implemented range  $t = 0, \dots, T - 1$ . The position proposed by contributor  $j$  is

$$\mathbf{y}_{ij}^{(t)} = \begin{cases} \mathbf{x}_j^{(t)} - A_{ij}(t) \left| C_{ij} \mathbf{x}_j^{(t)} - \mathbf{x}_i^{(t)} \right|, & j \leq K, \\ \mathbf{x}_j^{(t)}, & j > K, \end{cases} \quad (5)$$

where the absolute value is component-wise. To retain the gravitation notation of the general framework, define the implemented transformed direction and follower update as

$$\begin{aligned} \hat{\mathbf{G}}_i^{(t)} &= \sum_{j \in \mathcal{B}_i} q_{ij}(t) (\mathbf{y}_{ij}^{(t)} - \mathbf{x}_i^{(t)}), \\ \mathbf{x}_i^{(t+1)} &= \Pi_{\Omega} [\mathbf{x}_i^{(t)} + \hat{\mathbf{G}}_i^{(t)} + \boldsymbol{\epsilon}_i^{(t)} + \mathbf{b}_i^{(t)}], \quad i = 2, \dots, N. \end{aligned} \quad (6)$$

while  $\mathbf{x}_1^{(t+1)} = \mathbf{x}_1^{(t)}$  before the subsequent recovery operators. Here,  $\hat{\mathbf{G}}_i^{(t)}$  retains the directional role of  $\mathbf{G}_i^{(t)}$  in Equation (3); the hat indicates that transformed contributor proposals are used. Since  $\sum_{j \in \mathcal{B}_i} q_{ij}(t) = 1$ ,  $\mathbf{x}_i^{(t)} + \hat{\mathbf{G}}_i^{(t)} = \sum_{j \in \mathcal{B}_i} q_{ij}(t) \mathbf{y}_{ij}^{(t)}$ , so Equation (6) is exactly the normalized aggregate executed by the code. Further,  $\mathbf{y}_{ij}^{(t)} \in \mathbb{R}^D$  is an unprojected contributor proposal,  $\Pi_{\Omega} : \mathbb{R}^D \rightarrow \Omega$  projects each coordinate component-wise to the box domain,  $\boldsymbol{\epsilon}_i^{(t)} \sim \mathcal{N}(\mathbf{0}, \eta^2(t) \mathbf{I}_D)$ , with  $\eta(t) = \eta_0(1 - t/T)L$  and domain width  $L > 0$ , and  $\mathbf{b}_i^{(t)}$  is an archive pull applied stochastically according to the archive-pull probability recorded in Section 4.2. The verified experimental values are reported in Section 4.2. The exponent  $\kappa$  controls the annealing curvature in Equation (4): larger  $\kappa$  delays elite concentration, while smaller  $\kappa$  accelerates the transition to exploitation.

Equations (5) and (6) preserve the two framework roles. Rank weights determine how strongly all the better candidates contribute, whereas  $\alpha(t)$  controls the declining displacement around the top  $K$  contributors. The remaining better candidates contribute their positions without the annealed transformation. This distinction is an implementation choice within the broader rank-weighted framework, not a requirement that every future CBSA-C instantiation use the same perturbation.

The Gaussian perturbation prevents premature collapse and vanishes linearly over the run. If a candidate remains stagnant beyond the patience threshold, a better-half candidate is reset around the global best and a worse-half candidate is reinitialized. Periodic local perturbation, occasional elite cloning, and a diversity-triggered partial refresh are then applied as recovery operators. Elitist retention and the archive protect strong historical candidates during these operations. Table 3 summarizes the resulting implementation sequence.

**Table 3.** CBSA-C pseudocode.

- 
- |   |                                                                                                                                                 |
|---|-------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 | Initialize $N$ individuals in $\Omega$ and evaluate $f(\mathbf{x})$ .                                                                           |
| 2 | For $t = 0, \dots, T - 1$ : sort the population by nondecreasing objective value.                                                               |
| 3 | Update the elite archive with the best candidates.                                                                                              |
| 4 | Compute $p(t)$ , $w_j(t)$ , $q_{ij}(t)$ , and $\alpha(t)$ from Equations (1), (2) and (4).                                                      |
| 5 | Retain the best individual; for each $i = 2, \dots, N$ , construct $\mathbf{y}_{ij}^{(t)}$ from every $j \in \mathcal{B}_i$ using Equation (5). |
| 6 | Compute $\hat{\mathbf{G}}_i^{(t)}$ and apply the continuous update, Gaussian perturbation, archive pull, and box projection in Equation (6).    |
| 7 | Apply stagnation recovery, periodic local refinement, elite cloning, and diversity refresh when triggered.                                      |
| 8 | Return the best solution found.                                                                                                                 |
-

The current implementation uses basic operators and does not rely on covariance adaptation, surrogate modeling, or problem-specific gradient information. Extensions incorporating such techniques may further improve performance.

### 3.3. Discrete Variant for Routing: CBSA-D

Routing solutions are permutations or sets of routes, so Equation (6) cannot be applied directly. CBSA-D instead transfers the common rank-weighting principle to a symmetric edge-score model. Let  $n_c$  denote the number of customers and let  $\mathcal{V} = \{0, 1, \dots, n_c\}$  contain the depot 0 and those  $n_c$  customers; let  $\mathcal{I}_t \subseteq \{1, \dots, N\}$  be the variant-specific set of contributing population ranks, and let  $\pi_i$  denote the permutation or decoded-route representation of contributor  $i$ . Retaining the indicator notation of the submitted manuscript,  $\mathbb{I}[\{u, v\} \in \pi_i]$  equals one when the unordered pair  $\{u, v\}$  is an adjacency represented by  $\pi_i$  and zero otherwise. Define

$$\tilde{E}_{uv}^{(t)} = \sum_{i \in \mathcal{I}_t} w_i(t) \mathbb{I}[\{u, v\} \in \pi_i], \quad E_{uv}^{(t)} = \frac{\tilde{E}_{uv}^{(t)}}{\sum_{a \in \mathcal{V}} \sum_{b \in \mathcal{V}} \tilde{E}_{ab}^{(t)}}. \quad (7)$$

Thus,  $E_{uv}^{(t)} = E_{vu}^{(t)} \in [0, 1]$  and  $\sum_{u,v} E_{uv}^{(t)} = 1$  whenever at least one route edge is present. The matrix is a globally normalized population-support score, not a row-stochastic successor matrix. Hereafter, “edge pool” denotes only this iteration-specific score matrix; it is neither an additional persistent archive nor an entropy-based component. If a construction branch is currently at node  $u$  and admits the finite nonempty candidate set  $C_t(u)$ , it samples the next node from the categorical distribution

$$\Pr(v \mid u, C_t(u)) = \frac{\exp(E_{uv}^{(t)} / \tau(t))}{\sum_{z \in C_t(u)} \exp(E_{uz}^{(t)} / \tau(t))}, \quad v \in C_t(u), \quad (8)$$

where  $\tau(t) > 0$  is the branch-specific temperature at iteration  $t$ . For every  $v \in C_t(u)$ , the resulting probability lies in  $(0, 1)$  and the probabilities sum to one over  $C_t(u)$ . Equation (8) normalizes only over the candidates admitted at that construction state. The only randomness described by this equation is the next-node categorical draw conditional on the current population, state, and eligible set. CBSA-D neither minimizes a cross-entropy objective nor estimates an  $f$ -divergence. The divergence-based probabilistic sorting framework developed in [40] offers a principled alternative for decision problems where explicit divergence objectives are appropriate; such methods could guide future extensions of CBSA-D toward more rigorous probabilistic modeling of the edge-pool matrix. In the present implementation, rank-weighted adjacency support is used solely to bias stochastic construction.

**CVRP.** CVRP uses a size-dependent subset of the highest-ranked contributors. Adjacencies are extracted from capacity-feasible decoded routes, including depot edges. The edge-guided walk constructs a customer permutation, after which greedy decoding enforces vehicle capacity. A size-adjusted temperature schedule and periodic diversity burst regulate exploration.

**VRPTW.** VRPTW uses all population ranks and combines three complementary branches: feasibility-aware route construction, edge-guided permutation construction, and route crossover. Only the route-level branch filters successor candidates incrementally against capacity and time-window feasibility; the other branches rely on the subsequent decoder and evaluator. This mixed policy preserves feasible route-building information without making a single constructor mandatory for the framework.

The two instantiations share a decision architecture, but not an identical low-level operator. Rank order supplies the common information hierarchy, Equation (7) translates

that hierarchy into adjacency support, and Equation (8) defines the stochastic decision rule. The contributor set, candidate filter, decoder, and temperature policy remain task-specific interfaces. CBSA-D thus specifies how ranked population information enters structured discrete decisions while allowing the feasibility mechanism to match the application. Table 4 summarizes the shared procedure and task-specific interfaces; Section 4.2 reports the exact settings of the two routing instantiations.

**Table 4.** Shared CBSA-D procedure and task-specific routing interfaces.

|   |                                                                                                                               |
|---|-------------------------------------------------------------------------------------------------------------------------------|
| 1 | Initialize permutations and evaluate their decoded routes.                                                                    |
| 2 | For $t = 0, \dots, T - 1$ : sort solutions and preserve the best solution.                                                    |
| 3 | Select $\mathcal{I}_t$ for the routing variant and build the symmetric edge-score matrix $E^{(t)}$ using Equation (7).        |
| 4 | Select the CVRP constructor or one of the three VRPTW branches and define its eligible set $C_t(u)$ .                         |
| 5 | When the selected branch is edge-guided, sample successive customers using Equation (8) and the variant-specific temperature. |
| 6 | Decode and evaluate the resulting permutation under the applicable capacity and time-window rules.                            |
| 7 | Apply mutation, archive inheritance, stagnation recovery, diversity refresh, and scheduled local refinement when triggered.   |
| 8 | Return the best feasible routing solution found.                                                                              |

### 3.4. Computational Complexity

For CBSA-C, the main cost per iteration comes from objective evaluation, sorting, and contributor-specific position calculation. Let  $C_f(D)$  be the cost of one objective evaluation in  $D$  dimensions. Sorting costs  $O(N \log N)$ . A direct implementation of the contributor aggregation costs  $O(N^2 D)$  for dimension  $D$ , because each individual aggregates all better-ranked individuals. The reported per-iteration cost is therefore

$$O(NC_f(D) + N \log N + N^2 D),$$

before the periodically triggered local and recovery evaluations. Population and archive storage require  $O(ND)$  memory for fixed archive capacity. Cumulative weighted sums could reduce an untransformed centroid calculation, but they do not directly eliminate the contributor-specific random transformation in Equation (5); the reported implementation uses the explicit  $O(N^2 D)$  calculation.

For CBSA-D, sorting costs  $O(N \log N)$  and edge-pool construction costs  $O(|\mathcal{I}_t|n_c)$  because every contributing permutation or decoded route contains  $O(n_c)$  adjacencies. A direct softmax constructor examines up to all remaining customers at each of  $n_c$  positions, giving  $O(n_c^2)$  per generated solution and  $O(Nn_c^2)$  per iteration. For CVRP, the capacity decoder is linear after construction, so the non-local-search part of an iteration is  $O(Nn_c^2 + N \log N)$ . For VRPTW, the reported cheapest-insertion decoder can test  $O(n_c)$  insertion positions for each of  $n_c$  customers, and each direct route-feasibility check can cost  $O(n_c)$ . Its worst-case decoding cost is therefore  $O(n_c^3)$  per solution, giving  $O(Nn_c^3 + N \log N)$  per iteration for the direct implementation. Periodic route-level local search and the final improvement phases add separate instance-dependent costs and are excluded from these per-iteration bounds. Both variants require  $O(Nn_c + n_c^2)$  memory for the population and dense edge matrix. These bounds identify the direct VRPTW decoder, rather than edge-pool construction itself, as the principal scaling bottleneck.

### 3.5. Convergence Properties

We provide limited convergence results: boundedness for the continuous variant under projection, positive reachability for the discrete variant under nonzero temperature,

and elitist convergence in probability under infinite-time idealization. These results formalize the search dynamics under idealized assumptions and should not be interpreted as finite-time optimality guarantees.

**Proposition 1** (Boundedness of CBSA-C). *Assume that the feasible domain  $\Omega \subset \mathbb{R}^D$  is compact and that every CBSA-C update is followed by projection onto  $\Omega$ . Then, all iterates generated by CBSA-C remain in  $\Omega$ , and the elite archive contains at least one solution whose objective value is no worse than the best objective value previously observed.*

**Proof.** Projection maps every updated point back into  $\Omega$ , so all iterates remain feasible. The archive update keeps the best candidates among the union of the previous archive and the current population. Therefore, the best archived objective value is monotonically nonincreasing for minimization.  $\square$

**Proposition 2** (Positive reachability of CBSA-D). *Consider a finite routing instance with  $n_c$  customers. At each construction step, suppose that the eligible set  $C_t(u)$  is finite and nonempty, every score  $E_{uv}^{(t)}$  for  $v \in C_t(u)$  is finite, and the temperature satisfies  $\tau(t) > 0$ . Then, every finite successor sequence admitted by the implemented constructor has positive probability under Equation (8). Consequently, every route representation generated by such an admitted sequence and its associated decoder has positive sampling probability.*

**Proof.** For every  $v \in C_t(u)$ , finiteness of  $E_{uv}^{(t)}$  and positivity of  $\tau(t)$  imply  $\exp(E_{uv}^{(t)}/\tau(t)) > 0$ . The denominator in Equation (8) is a finite sum of positive terms, so each admitted successor has strictly positive conditional probability. The probability of any finite admitted successor sequence is the product of finitely many positive conditional probabilities and is therefore positive.  $\square$

**Theorem 1** (Elitist convergence in probability under idealized infinite sampling). *Assume that the routing search space is finite and that the archive always preserves the best solution observed so far. Conditional on no global optimum having been sampled before iteration  $t$ , suppose that the probability of sampling a global optimum at iteration  $t$  is at least  $\rho_t > 0$ . If  $\sum_{t=1}^{\infty} \rho_t = \infty$ , then the probability that the archive contains a globally optimal solution converges to one as  $t \rightarrow \infty$ .*

**Proof.** Let  $A_t$  be the event that no global optimum has been sampled up to iteration  $t$ . By the conditional lower-bound assumption,

$$\Pr(A_t) \leq \prod_{s=1}^t (1 - \rho_s) \leq \exp\left(-\sum_{s=1}^t \rho_s\right).$$

Because  $\sum_{s=1}^{\infty} \rho_s = \infty$ , the right-hand side converges to zero. Once a global optimum is sampled, elitist archiving preserves it. Thus, the probability that the archive contains a global optimum converges to one.  $\square$

The theorem is intentionally stated in terms of sampling probability rather than a specific cooling law. Classical simulated annealing convergence theory uses cooling assumptions to preserve global reachability [11,41]. For the implemented softmax constructor, a positive temperature and finite scores preserve positive probability for every currently admitted successor. Reachability may nevertheless be restricted by the eligible-set rules, decoder, or other construction branches, and the theorem does not establish finite-time optimality. This theoretical observation is consistent with the empirical need for diversity restart and stagnation handling.

## 4. Experimental Design

### 4.1. Benchmarks and Baselines

The continuous experiments use all 30 CEC2014 functions at dimension  $D = 30$  [5]. The standard fixed shift vectors, rotation matrices, and shuffle data distributed with the benchmark are used. CBSA-C is compared with GA, PSO, GWO, and DE. Each algorithm is run 30 independent times. The primary comparison uses  $T = 200$  iterations; a separate long-run convergence diagnostic uses  $T = 500$ .

The CVRP experiments use 50 standard instances from the Augerat A/B sets in the Capacitated Vehicle Routing Problem Library (CVRPLIB) [42]. Benchmark-library practice in routing has long relied on standardized instance formats and best-known reference values [43,44]. The VRPTW experiments use all 56 Solomon instances [6], grouped into clustered (C), random (R), and random-clustered (RC) customer distributions, with type 1 and type 2 time-window patterns. CBSA-D is compared with GA, discrete GWO (GWO-D), discrete PSO (PSO-D), ACO, and ILS. All routing algorithms are run 10 independent times per instance.

The routing baselines represent three search designs. GA, PSO-D, and GWO-D test CBSA-D against direct discrete population search. ACO provides an edge-aware comparison that is closer to CBSA-D in representation. ILS supplies a stronger local-search-centered comparison and prevents the evaluation from relying only on relatively weak discrete adaptations. The experiment measures both the improvement over ordinary population methods and the remaining gap to stronger local search.

Ranks are computed from mean objective values across independent runs; smaller objective values are better. For aggregate CVRP and VRPTW tables, ranks are computed within the six algorithms available for every instance. For subgroup tables that display only a subset of methods, ranks are recomputed within the displayed subset, so a three-method table has ranks between 1 and 3. Gaps are reported relative to known best or reference objective values in the benchmark data. The reporting follows standard recommendations for algorithm comparison across data sets [45].

### 4.2. Parameter Settings

The settings in Table 5 are the verified values used by the result-generating implementations. All three instantiations use  $N = 30$ ,  $p_1 = 2.0$ , and the rank-schedule exponent  $\gamma = 0.8$ , but their initial exponent and representation-specific controls are not identical. In particular,  $K = 3$  is a CBSA-C contributor setting, not a shared CBSA-D parameter. The main CEC2014 comparison uses  $T = 200$  and 30 independent runs; its separately stored convergence diagnostic uses  $T = 500$ . The CVRP and VRPTW studies use  $T = 100$  and 10 independent runs per instance. These settings describe the experimental instantiations rather than immutable constants of the broader framework.

For reproducibility, the reported CVRP instantiation defines  $s_n = \max(1, \sqrt{n/30})$  and uses the  $m(n) = \max(3, \lfloor N \min(1, 2/s_n) \rfloor)$  top-ranked solutions as edge contributors. The first two newly generated solutions use  $\tau_C(t) = 1.5 - 1.5(t/T)^{0.5/s_n}$  and the others use  $\tau(t) = 1$ . At positive iterations divisible by  $\max(1, \lfloor 20/s_n \rfloor)$ , the diversity burst sets these temperatures to 3 and 2, respectively.

The reported VRPTW instantiation uses all  $N$  ranks. Solomon's I1 insertion heuristic supplies the first  $\lfloor 0.4N \rfloor$  initial permutations when construction succeeds, and the remainder are random. The probabilities assigned to route construction, permutation construction, and route crossover are  $(0.4, 0.3, 0.3)$  for clustered instances and  $(0.6, 0.2, 0.2)$  for random and random-clustered instances. The first two newly generated solutions use  $\tau_V(t) = 2 - 2(t/T)^K$ , whereas the remaining solutions use  $\tau(t) = 1$ . These values

document the tested implementations and are not requirements of the general CBSA-D framework.

**Table 5.** Default CBSA settings used in experiments.

| Parameter                        | CBSA-C                                            | CVRP-CBSA-D                                                    | VRPTW-CBSA-D                                                   |
|----------------------------------|---------------------------------------------------|----------------------------------------------------------------|----------------------------------------------------------------|
| Population size, $N$             | 30                                                | 30                                                             | 30                                                             |
| Dimension/size                   | $D = 30$                                          | Instance customer count, $n_c$                                 | Instance customer count, $n_c$                                 |
| Main iteration budget, $T$       | 200                                               | 100                                                            | 100                                                            |
| Independent runs (seeds)         | 30 (42–71)                                        | 10 (42–51)                                                     | 10 (42–51)                                                     |
| Initial rank exponent, $p_0$     | 0.3                                               | 0.5                                                            | 0.5                                                            |
| Final rank exponent, $p_1$       | 2.0                                               | 2.0                                                            | 2.0                                                            |
| Rank-schedule exponent, $\gamma$ | 0.8                                               | 0.8                                                            | 0.8                                                            |
| Annealed scope                   | Top $K = 3$ contributors                          | First two new solutions                                        | First two new solutions                                        |
| Annealing curvature, $\kappa$    | 0.7                                               | 0.7                                                            | 0.7                                                            |
| Annealing/temperature            | $\alpha(t) = 2 - 2(t/T)^\kappa$                   | Size-adjusted schedule; periodic burst                         | $\tau_V(t) = 2 - 2(t/T)^\kappa$                                |
| Perturbation/mutation            | $\eta(t) = 0.03(1 - t/T)L$                        | Base rate: $0.05(1 - t/T)$ ; swap probability: $0.10(1 - t/T)$ | Base rate: $0.05(1 - t/T)$ ; swap probability: $0.10(1 - t/T)$ |
| Archive/stagnation               | Capacity: 12; pull probability: 0.15; patience: 8 | Capacity: 12; inheritance probability: 0.15; patience: 5       | Capacity: 12; inheritance probability: 0.15; patience: 5       |

#### 4.3. Sensitivity and Data-Noise Protocol

The revision study evaluates CBSA only and uses five matched seeds (42–46) for every condition. The one-factor-at-a-time design uses the CEC2014 functions F1, F8, F18, and F23 and routing instances A-n32-k5, B-n50-k8, C101, R101, and RC201. For CBSA-C, the tested levels are  $p_0 \in \{0.1, 0.3, 0.5\}$ ,  $p_1 \in \{1.5, 2.0, 2.5\}$ ,  $\gamma \in \{0.5, 0.8, 1.1\}$ ,  $\eta_0 \in \{0.015, 0.030, 0.045\}$ ,  $N \in \{20, 30, 40\}$ , and  $T \in \{100, 200, 300\}$ . The annealing exponent  $\kappa$  was fixed at 0.7 throughout all experiments; its sensitivity is left as future work. For CVRP and VRPTW, the corresponding levels are  $p_0 \in \{0.3, 0.5, 0.7\}$ ,  $p_1 \in \{1.5, 2.0, 2.5\}$ ,  $\gamma \in \{0.5, 0.8, 1.1\}$ ,  $N \in \{20, 30, 40\}$ , and  $T \in \{50, 100, 150\}$ . The middle value is the verified default in every case. The  $N$  and  $T$  conditions are treated as computational-budget tests and are interpreted jointly with runtime.

For CBSA-C noise tests, selection uses  $\tilde{f}(\mathbf{x}) = f(\mathbf{x}) + \varepsilon$ , where  $\varepsilon \sim \mathcal{N}(0, (\lambda s_f)^2)$ ,  $\lambda \in \{0.01, 0.05\}$ , and  $s_f$  is estimated from 1000 fixed-seed uniform samples over the function domain; the returned solution is reevaluated with the noise-free objective. Routing coordinates are perturbed by independent zero-mean Gaussian noise with a standard deviation of 0.5% or 1.0% of the coordinate span. Three fixed coordinate perturbations are combined with the five algorithm seeds, and every resulting route is reevaluated on the original instance. The complete design contains 725 runs: 300 for CBSA-C, 170 for CVRP, and 255 for VRPTW. Because each parameter condition has only five runs, the analysis is descriptive: it reports mean and best errors relative to known optima for CBSA-C, runtime for budget conditions, and original-instance feasibility for VRPTW, without adding low-powered significance tests.

#### 4.4. Evaluation Metrics

For each independent run, the best objective value found by the algorithm is recorded. For each benchmark instance, the reported mean, standard deviation, best, and worst values are computed over repeated runs. The mean objective value is used for ranking because it reflects expected algorithm performance under repeated use. The best value is also reported in Appendix A because it captures the algorithm's ability to occasionally discover high-quality solutions.

For routing instances, the mean gap is

$$\text{Gap}(\%) = 100 \times \frac{\bar{f} - f_{\text{ref}}}{f_{\text{ref}}},$$

where  $\bar{f}$  is the mean objective value over repeated runs and  $f_{\text{ref}}$  is the reference best-known value. A win is counted when a method achieves the best mean objective value among the compared algorithms on a given instance. Ranks and wins are determined from unrounded mean objective values; tied values, if present, receive the average of the ranks they jointly occupy.

## 5. Results

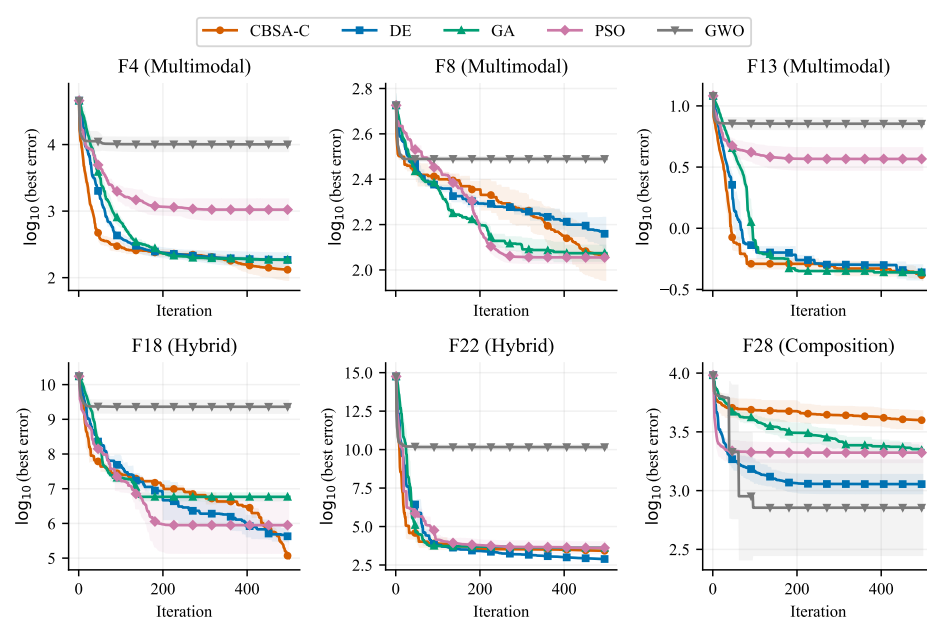
### 5.1. Continuous Optimization on CEC2014

Table 6 summarizes the main CEC2014 ranking at  $T = 200$ . CBSA-C obtains the best average rank (1.67, 19 wins) across the 30 functions, with its strongest results on multimodal and hybrid functions. CBSA-C and DE tie for the best unimodal average rank, while DE has the best average rank on composition functions. On unimodal functions (F1–F3), CBSA-C and DE both obtain an average rank of 1.67, with two and one wins, respectively. On multimodal and hybrid functions, CBSA-C obtains the best average rank (1.08 and 1.50, respectively). DE has the best average rank on F23–F30. DE obtains an average rank of 2.12 with three wins; CBSA-C obtains one win, while GWO obtains four wins but a weaker average rank of 2.88. The best-value and variance tables in Appendix A provide complementary run-level evidence. Numbers in bold denote that the corresponding algorithm achieves superior performance compared with other algorithms.

**Table 6.** Average ranks on CEC2014 functions ( $T = 200$ ,  $D = 30$ ,  $N = 30$ , 30 runs).

| Function Group       | CBSA-C           | DE              | GA       | PSO      | GWO      |
|----------------------|------------------|-----------------|----------|----------|----------|
| All 30 functions     | <b>1.67</b> (19) | 2.47 (7)        | 2.97 (0) | 3.53 (0) | 4.37 (4) |
| Unimodal: F1–F3      | <b>1.67</b> (2)  | <b>1.67</b> (1) | 2.67 (0) | 4.00 (0) | 5.00 (0) |
| Multimodal: F4–F16   | <b>1.08</b> (12) | 3.23 (1)        | 2.92 (0) | 2.92 (0) | 4.85 (0) |
| Hybrid: F17–F22      | <b>1.50</b> (4)  | 1.67 (2)        | 2.83 (0) | 4.00 (0) | 5.00 (0) |
| Composition: F23–F30 | 2.75 (1)         | <b>2.12</b> (3) | 3.25 (0) | 4.00 (0) | 2.88 (4) |

Figure 2 complements these aggregate results with the  $T = 500$  convergence trajectories generated on the same standard benchmark data.



**Figure 2.** Convergence behavior on six representative CEC2014 functions (F4, F8, F13: multimodal; F18, F22: hybrid; F28: composition).

## 5.2. CVRP Results

Table 7 shows the aggregate CVRP comparison. ILS is the strongest method overall, with the best rank, smallest mean gap, and 44 wins across 50 instances. CBSA-D ranks second and outperforms ACO, GA, GWO-D, and PSO-D under the same ranking protocol. Runtime provides additional context. ILS obtains better solutions but takes about 52.17 s on average, whereas CBSA-D takes about 13.70 s. ACO is slightly faster than CBSA-D and has a slightly larger mean gap. The current CBSA-D implementation therefore occupies an intermediate point in the quality–time trade-off: it is more accurate than the ordinary population baselines, close to ACO in runtime, and faster but less accurate than ILS.

The Appendix A give a similar picture for best values and variance. CBSA-D improves its best-value profile relative to its mean profile, but ILS still leads both tables. CBSA-D can produce strong individual runs, although it does not match the reliability of the local-search baseline and its mean gap remains larger than that of ILS.

**Table 7.** Aggregate CVRP results on 50 instances. Ranks are computed within the six algorithms available for every instance.

| Method | Average Rank | Mean Gap (%) | Wins  | Mean Time (s) |
|--------|--------------|--------------|-------|---------------|
| ILS    | 1.14         | 4.18         | 44/50 | 52.17         |
| CBSA-D | 2.20         | 8.00         | 5/50  | 13.70         |
| ACO    | 2.66         | 9.85         | 1/50  | 11.85         |
| GA     | 4.02         | 27.84        | 0/50  | 0.48          |
| GWO-D  | 4.98         | 55.30        | 0/50  | 0.92          |
| PSO-D  | 6.00         | 102.22       | 0/50  | 0.29          |

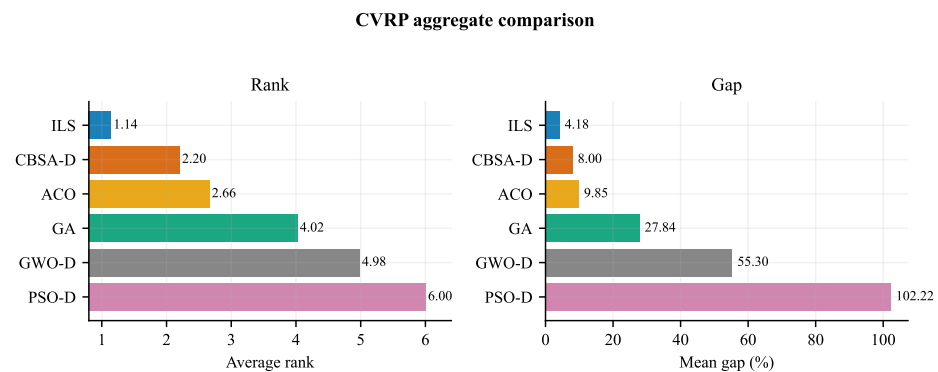
The subgroup results in Table 8 indicate where CBSA-D is most competitive. It is closest to ILS on small instances and on the B set, but the gap widens for larger and many-vehicle instances. This pattern is consistent with the edge-pool interpretation: when the routing structure is moderate in size, rank-weighted edge reuse provides useful guidance; as the number of possible route interactions grows, the local-search machinery of ILS becomes more decisive.

**Table 8.** CVRP subgroup comparison between CBSA-D and ILS. Ranks are recomputed within the two displayed algorithms, so values lie between 1 and 2.

| Subgroup                   | Instances | CBSA-D Rank | ILS Rank | CBSA-D Wins |
|----------------------------|-----------|-------------|----------|-------------|
| A set                      | 27        | 1.96        | 1.04     | 1           |
| B set                      | 23        | 1.83        | 1.17     | 4           |
| Small (<60)                | 35        | 1.86        | 1.14     | 5           |
| Large ( $\geq 60$ )        | 15        | 2.00        | 1.00     | 0           |
| Few vehicles (<9)          | 34        | 1.85        | 1.15     | 5           |
| Many vehicles ( $\geq 9$ ) | 16        | 2.00        | 1.00     | 0           |

The large-instance behavior identifies an important limitation of the current CBSA-D design. The edge pool stores pairwise adjacency frequencies, but it does not explicitly model higher-order route structure, such as the interaction between route boundaries, vehicle count, and customer clusters. In large or many-vehicle CVRP instances, these higher-order interactions become more important. A local-search method that repeatedly modifies complete route segments can exploit them more directly. This explains why the gap between CBSA-D and ILS increases for  $N \geq 60$  and for instances with at least nine vehicles.

Figure 3 visualizes the aggregate rank ordering reported in Table 7.



**Figure 3.** CVRP average-rank comparison across algorithms on 50 benchmark instances. Lower ranks indicate better mean solution quality across repeated runs.

### 5.3. VRPTW Results

The VRPTW results show that CBSA-D performance depends strongly on instance structure. Table 9 reports the aggregate comparison. Overall, CBSA-D ranks behind ILS and GA, but it remains better than ACO, GWO-D, and PSO-D. More importantly, Table 10 shows that CBSA-D is strongest on clustered C-type instances and weakest on random or mixed R/RC instances. CBSA-D's performance varies substantially by class: it ranks best on clustered C-type instances and worst on random and mixed instances.

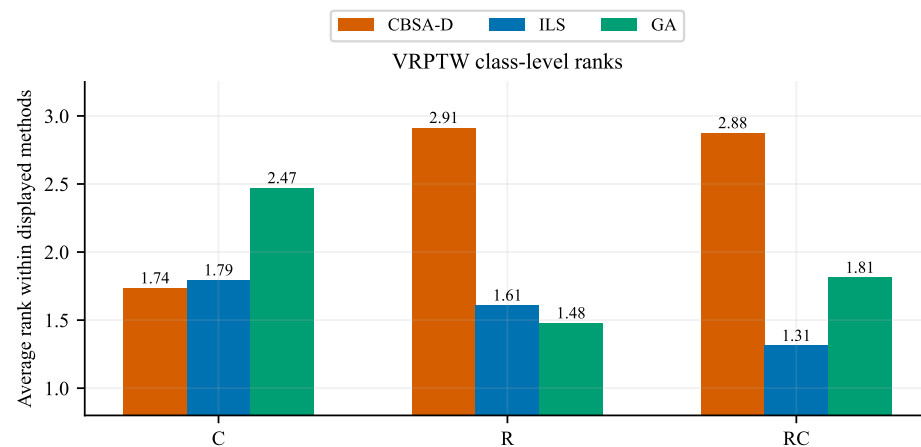
**Table 9.** Aggregate VRPTW results on 56 Solomon instances. Ranks are computed within the six algorithms available for every instance.

| Method | Average Rank | Mean Gap (%) | Wins  | Mean Time (s) |
|--------|--------------|--------------|-------|---------------|
| ILS    | 1.79         | 14.71        | 27/56 | 861.96        |
| GA     | 2.12         | 19.47        | 18/56 | 425.58        |
| CBSA-D | 2.86         | 24.95        | 5/56  | 594.08        |
| ACO    | 4.02         | 39.28        | 6/56  | 142.88        |
| GWO-D  | 4.54         | 58.99        | 0/56  | 353.00        |
| PSO-D  | 5.68         | 71.60        | 0/56  | 348.42        |

**Table 10.** VRPTW class-level ranks for the three strongest displayed methods. Ranks are recomputed within CBSA-D, ILS, and GA, so values lie between 1 and 3.

| Class | Instances | CBSA-D Rank | ILS Rank | GA Rank |
|-------|-----------|-------------|----------|---------|
| C     | 17        | 1.74        | 1.79     | 2.47    |
| C1    | 9         | 1.61        | 1.61     | 2.78    |
| C2    | 8         | 1.88        | 2.00     | 2.12    |
| R     | 23        | 2.91        | 1.61     | 1.48    |
| R1    | 12        | 2.83        | 1.58     | 1.58    |
| R2    | 11        | 3.00        | 1.64     | 1.36    |
| RC    | 16        | 2.88        | 1.31     | 1.81    |
| RC1   | 8         | 2.88        | 1.50     | 1.62    |
| RC2   | 8         | 2.88        | 1.12     | 2.00    |

Figure 4 visualizes this class-level rank pattern and places the clustered, random, and mixed groups on the same scale.



**Figure 4.** VRPTW class-level ranking pattern. CBSA-D is most competitive on clustered C-type instances and less competitive on random and mixed classes.

#### 5.4. Sensitivity and Data-Noise Results

CBSA shows local stability on the tested representative cases around the verified default configuration. For each mechanism parameter, a bracketed cell gives the range of median percentage changes in mean performance across the two non-default tested levels relative to the verified default. In the four budget rows, an entry written as  $x$  ( $y$ ) means a median performance change of  $x\%$  and a median runtime change of  $y\%$ , both relative to the verified default. Negative performance values indicate improvement, whereas negative runtime values indicate reduced computation time. For every case and condition, each run contributes its final best-found objective, and the condition-level performance is the mean of these values over the five matched seeds. CBSA-C entries express this mean as error relative to the known CEC2014 optimum (F1, F8, F18, F23); routing entries use the corresponding mean final best-found objective (CVRP: A-n32-k5, B-n50-k8; VRPTW: C101, R101, RC201). After rerunning the continuous sensitivity study with the standard fixed CEC2014 shift, rotation, and shuffle data, median mean-error changes across the four functions remain within  $[-5.1\%, +13.9\%]$  for the tested mechanism-parameter levels. The corresponding routing-objective ranges are  $[-0.3\%, +0.4\%]$  for CVRP and  $[+0.6\%, +1.5\%]$  for VRPTW. Budget adjustments ( $N$  and  $T$ ) show a quality–time trade-off for CBSA-C, whereas routing budget effects remain within  $\pm 2\%$ , without a consistent direction. Table 11 summarizes these ranges.

**Table 11.** Parameter and budget sensitivity of CBSA.

| Parameter (Tested Levels)                       | CBSA-C          | CVRP           | VRPTW          |
|-------------------------------------------------|-----------------|----------------|----------------|
| $p_0 \in \{0.1, 0.3, 0.5\} / \{0.3, 0.5, 0.7\}$ | $[-5.1, +3.9]$  | $[-0.3, +0.0]$ | $[+1.0, +1.4]$ |
| $p_1 \in \{1.5, 2.0, 2.5\}$                     | $[-4.3, +6.7]$  | $[+0.2, +0.4]$ | $[+0.6, +1.5]$ |
| $\gamma \in \{0.5, 0.8, 1.1\}$                  | $[-3.1, +1.6]$  | $[-0.3, +0.0]$ | $[+1.0, +1.5]$ |
| $\eta_0 \in \{0.015, 0.030, 0.045\}$            | $[+0.7, +13.9]$ | –              | –              |
| $N = 20$                                        | $+9.7$ (–36.7)  | $-0.9$ (–10.3) | $+1.1$ (–28.0) |
| $N = 40$                                        | $-16.0$ (+45.5) | $-1.0$ (+45.7) | $+0.0$ (+23.8) |
| $T$ low (100, 50)                               | $+47.2$ (–49.4) | $-2.0$ (–16.6) | $+0.0$ (–33.9) |
| $T$ high (300, 150)                             | $-11.6$ (+47.2) | $-1.0$ (+33.4) | $+0.0$ (+37.4) |

The objective-noise and coordinate-perturbation results appear in Table 12. Each numerical cell is written as mean/best, and lower values indicate better performance. For CBSA-C, the reported quantity is the error relative to the known optimum under additive Gaussian noise with standard deviation  $\lambda s_f$ , where  $s_f$  is estimated from 1000

fixed uniform samples of the standard function and  $\lambda \in \{0.01, 0.05\}$ . For routing, the reported quantity is the objective value of the returned solution after reevaluation on the original instance under coordinate perturbation with  $\sigma \in \{0.5\%, 1.0\%\}$ . The routing defaults are reevaluated using this same solution-level convention and therefore differ from the best-found search objectives in the main-study tables. Each routing noise condition comprises 15 runs, combining three fixed coordinate perturbations with five algorithm seeds; the “Feas.” column gives the minimum feasibility rate across the two noise levels. All VRPTW runs remain feasible, and VRPTW mean-objective increases stay below 4%. CVRP performance under coordinate noise is instance-dependent. For CBSA-C, the effect is landscape-dependent: relative to the default mean error, the two tested noise levels increase the mean error by approximately 7.5–22 times on F1 and 840–2250 times on F18, but by only 13–24% on F8 and 10–35% on F23.

**Table 12.** Sensitivity to objective noise and coordinate perturbation.

| Variant | Case     | Default                               | $\lambda = 0.01 / \sigma = 0.5\%$     | $\lambda = 0.05 / \sigma = 1.0\%$     | Feas. |
|---------|----------|---------------------------------------|---------------------------------------|---------------------------------------|-------|
| CBSA-C  | F1       | $2.63 \times 10^7 / 1.08 \times 10^7$ | $1.97 \times 10^8 / 1.41 \times 10^8$ | $5.82 \times 10^8 / 3.19 \times 10^8$ | –     |
| CBSA-C  | F8       | 119/78                                | 134/120                               | 148/132                               | –     |
| CBSA-C  | F18      | $1.25 \times 10^5 / 1.07 \times 10^5$ | $1.05 \times 10^8 / 4.09 \times 10^7$ | $2.81 \times 10^8 / 1.61 \times 10^8$ | –     |
| CBSA-C  | F23      | 243/242                               | 267/259                               | 328/296                               | –     |
| CVRP    | A-n32-k5 | 1175/1098                             | 1039/832                              | 979/832                               | –     |
| CVRP    | B-n50-k8 | 1581/1495                             | 1605/1430                             | 1544/1398                             | –     |
| VRPTW   | C101     | 828.9/828.9                           | 850.8/828.9                           | 860.8/828.9                           | 100%  |
| VRPTW   | R101     | 1861/1786                             | 1898/1708                             | 1919/1787                             | 100%  |
| VRPTW   | RC201    | 1954/1848                             | 1985/1778                             | 1965/1833                             | 100%  |

## 6. Discussion, Limitations, and Future Work

CBSA has three technical preconditions. Candidate evaluations must be comparable and aligned with the intended optimization outcome; the induced ranks must be sufficiently informative rather than dominated by noise; and the chosen representation must permit feasible candidates to be generated, decoded, or repaired. If these conditions fail, rank weighting can concentrate influence around unreliable early candidates, an archive can preserve stale structures, and an edge pool can amplify adjacencies that do not repeat across high-quality solutions. Excessive concentration can therefore reduce diversity and cause premature convergence, while a misspecified or manipulated objective can make the algorithm reinforce the wrong proxy. Practical use should monitor rank stability and population diversity, validate objectives and constraints, and retain restart, archive-aging, or hybrid-solver fallbacks. CBSA is a computational search framework designed for optimization. Consequential applications require independent objective validation, constraint auditing, and domain review.

The evidence has four main limitations. CBSA-D ranks second on CVRP but does not outperform ILS overall; its VRPTW performance is strongly class-dependent and is strongest on clustered instances; its pairwise edge pool does not represent higher-order route or segment interactions; and several parameters are selected empirically. The experiments also exclude OR-Tools and PyVRP, so no comparison with those solvers is claimed [27,46].

### 6.1. Cross-Case Interpretation and Boundary Conditions

The CEC2014 results are consistent with rank-weighted gravitation being most useful when several promising basins coexist. CBSA-C leads the multimodal and hybrid groups, ties DE on average rank in the unimodal group, and DE leads the composition group. The

evidence therefore supports a complementary search bias rather than uniform superiority over leader-based or differential search.

The routing results show a parallel dependence on reusable structure. CBSA-D is closest to ILS on small and B-set CVRP instances, but the gap increases for  $n_c \geq 60$  and for instances with at least nine vehicles, where route-level interactions become more important. On VRPTW, clustered instances provide more stable spatial and temporal adjacency than random or mixed instances, matching the observed advantage on C-type classes and the weaker R/RC results.

The three metrics describe different aspects of performance: the mean characterizes a typical run, the best value records whether the search can reach a better region, and the variance measures repeatability. The mean and best-value rankings favor CBSA-C on CEC2014, while the variance results indicate comparatively consistent outcomes on this benchmark suite. The CBSA-D profile is more mixed: it ranks second on CVRP by mean objective value, its best-value profile is weaker, and its VRPTW results are less stable. Table 13 condenses these cross-case observations into a structure-to-policy guide.

**Table 13.** Which system structures are well matched to CBSA and when stronger local-search solvers remain preferable.

| System Structure                               | Observed Signal                                          | CBSA Behavior                                          | Practical Implication                                   |
|------------------------------------------------|----------------------------------------------------------|--------------------------------------------------------|---------------------------------------------------------|
| Smooth or weakly rugged continuous state space | Few basins; direct descent signals                       | Competitive, but often matched by leader-based methods | Consider CBSA-C when moderate search bias is sufficient |
| Multimodal or hybrid continuous landscape      | Multiple basins; delayed exploitation                    | Rank-weighted consensus helps avoid early commitment   | Consider CBSA-C when several good basins coexist        |
| Clustered routing structure                    | Stable adjacency; reusable local patterns                | Edge-pool model captures useful successor relations    | Use CBSA-D when route reuse is strong                   |
| Random or mixed routing structure              | Weak adjacency stability; frequent feasibility conflicts | Edge statistics are less persistent across runs        | Prefer stronger local-search or hybrid routing solvers  |
| Large or highly constrained routing systems    | Route-level interactions dominate pairwise edges         | Pairwise edge pool is too local                        | Hybridize CBSA-D with route-segment neighborhoods       |

## 6.2. Design Implications and Future Work

CBSA is better treated as a structure-aware search component than as a replacement for domain-specific solvers. CBSA-C can identify promising continuous regions before local refinement, while CBSA-D can generate or reshape routes before repair, insertion, or exchange operators. Random, mixed, or strongly constrained routing instances should instead favor local-search-centered solvers or hybrid designs.

Future work should adapt the rank schedule and temperature to population diversity or score-matrix concentration, extend CBSA-D from pairwise edges to route-segment neighborhoods, and evaluate validated OR-Tools, PyVRP, and HGS-style baselines under comparable model construction, stopping criteria, and computational budgets. These steps directly address the empirical and representational limitations identified above.

## 7. Conclusions

This paper proposed CBSA, a rank-weighted institutional search framework inspired by the selection and hierarchy mechanisms of the ancient Chinese bureaucracy. Its shared technical principle is to aggregate information from higher-ranked contributors using decaying rank weights. CBSA-C realizes this principle through continuous position updates, whereas CBSA-D transfers it to routing through an edge-pool probability model. CBSA-C performs strongly on CEC2014, especially on multimodal and hybrid functions. CBSA-D is competitive on CVRP and comparatively more competitive on the tested clustered VRPTW instances, where the edge pool can capture reusable adjacency. Strong local-search and

hybrid methods nevertheless remain better overall in several routing settings. The results support rank-weighted aggregation as a mechanism for transforming population order into continuous movement or discrete edge probabilities, subject to the structure of the problem.

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## Abbreviations

The following abbreviations are used in this manuscript:

|         |                                             |
|---------|---------------------------------------------|
| ACO     | Ant colony optimization                     |
| CBSA    | Chinese Bureaucratic System Algorithm       |
| CEC     | Congress on Evolutionary Computation        |
| CEM     | Cross-entropy method                        |
| CVRP    | Capacitated vehicle routing problem         |
| CVRPLIB | Capacitated Vehicle Routing Problem Library |
| DE      | Differential evolution                      |
| EDA     | Estimation of distribution algorithm        |
| GA      | Genetic algorithm                           |
| GWO     | Grey wolf optimizer                         |
| HGS     | Hybrid genetic search                       |
| ILS     | Iterated local search                       |
| PSO     | Particle swarm optimization                 |
| VRP     | Vehicle routing problem                     |
| VRPTW   | Vehicle routing problem with time windows   |

## Appendix A. Supplementary Experimental Tables

Table A1 reports the per-function CEC2014 mean errors and winner labels from the rerun using the standard fixed benchmark data. Tables A2–A4 report the CEC2014 mean, best, and variance results. Tables A5–A7 give the corresponding CVRP results, while Tables A8–A10 give the VRPTW results. In the mean and best-run tables, boldface identifies the smallest underlying unrounded value in each row; exact ties receive joint boldface, while rounding is used only for compact display. The variance tables are not bolded, because lower variance does not necessarily indicate superior optimization quality—it may also reflect premature convergence—and bolding it could be misinterpreted. Table A11 links each benchmark family to the structural property diagnosed in the cross-case analysis.

Numbers in bold denote that the corresponding algorithm achieves superior performance compared with other algorithms.

**Table A1.** Per-function mean errors on the standard CEC2014 suite over 30 independent runs.

| Function | Group       | CBSA-C                | DE                    | GA                    | PSO                   | GWO                   | Winner |
|----------|-------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|--------|
| F1       | Unimodal    | $2.44 \times 10^7$    | $2.73 \times 10^7$    | $6.75 \times 10^7$    | $2.50 \times 10^8$    | $1.05 \times 10^9$    | CBSA-C |
| F2       | Unimodal    | $1.01 \times 10^7$    | $2.81 \times 10^8$    | $1.14 \times 10^9$    | $1.85 \times 10^{10}$ | $6.02 \times 10^{10}$ | CBSA-C |
| F3       | Unimodal    | $6.38 \times 10^4$    | $4.03 \times 10^3$    | $2.55 \times 10^4$    | $1.10 \times 10^5$    | $1.14 \times 10^5$    | DE     |
| F4       | Multimodal  | $1.53 \times 10^2$    | $1.83 \times 10^2$    | $2.46 \times 10^2$    | $2.44 \times 10^3$    | $9.99 \times 10^3$    | CBSA-C |
| F5       | Multimodal  | $2.08 \times 10^1$    | $2.11 \times 10^1$    | $2.10 \times 10^1$    | $2.10 \times 10^1$    | $2.12 \times 10^1$    | CBSA-C |
| F6       | Multimodal  | $2.28 \times 10^1$    | $2.08 \times 10^1$    | $2.36 \times 10^1$    | $2.44 \times 10^1$    | $4.17 \times 10^1$    | DE     |
| F7       | Multimodal  | $1.10 \times 10^0$    | $4.17 \times 10^0$    | $1.13 \times 10^1$    | $2.04 \times 10^2$    | $4.68 \times 10^2$    | CBSA-C |
| F8       | Multimodal  | $1.16 \times 10^2$    | $2.10 \times 10^2$    | $1.55 \times 10^2$    | $1.32 \times 10^2$    | $3.01 \times 10^2$    | CBSA-C |
| F9       | Multimodal  | $1.37 \times 10^2$    | $2.31 \times 10^2$    | $2.43 \times 10^2$    | $1.75 \times 10^2$    | $3.34 \times 10^2$    | CBSA-C |
| F10      | Multimodal  | $2.99 \times 10^3$    | $6.57 \times 10^3$    | $4.11 \times 10^3$    | $3.74 \times 10^3$    | $7.87 \times 10^3$    | CBSA-C |
| F11      | Multimodal  | $3.57 \times 10^3$    | $7.99 \times 10^3$    | $5.64 \times 10^3$    | $4.57 \times 10^3$    | $7.78 \times 10^3$    | CBSA-C |
| F12      | Multimodal  | $1.30 \times 10^0$    | $3.65 \times 10^0$    | $1.96 \times 10^0$    | $1.95 \times 10^0$    | $4.10 \times 10^0$    | CBSA-C |
| F13      | Multimodal  | $5.38 \times 10^{-1}$ | $5.92 \times 10^{-1}$ | $5.47 \times 10^{-1}$ | $3.32 \times 10^0$    | $6.80 \times 10^0$    | CBSA-C |
| F14      | Multimodal  | $3.28 \times 10^{-1}$ | $1.13 \times 10^0$    | $5.30 \times 10^{-1}$ | $5.10 \times 10^1$    | $1.93 \times 10^2$    | CBSA-C |
| F15      | Multimodal  | $1.64 \times 10^1$    | $3.58 \times 10^1$    | $4.25 \times 10^1$    | $1.31 \times 10^5$    | $1.39 \times 10^5$    | CBSA-C |
| F16      | Multimodal  | $1.25 \times 10^1$    | $1.35 \times 10^1$    | $1.30 \times 10^1$    | $1.28 \times 10^1$    | $1.32 \times 10^1$    | CBSA-C |
| F17      | Hybrid      | $2.90 \times 10^6$    | $6.03 \times 10^5$    | $2.83 \times 10^6$    | $6.70 \times 10^6$    | $1.77 \times 10^8$    | DE     |
| F18      | Hybrid      | $1.09 \times 10^5$    | $7.90 \times 10^6$    | $9.48 \times 10^6$    | $2.38 \times 10^7$    | $1.73 \times 10^9$    | CBSA-C |
| F19      | Hybrid      | $5.57 \times 10^2$    | $1.58 \times 10^4$    | $3.60 \times 10^4$    | $2.16 \times 10^5$    | $2.40 \times 10^8$    | CBSA-C |
| F20      | Hybrid      | $1.00 \times 10^5$    | $1.59 \times 10^5$    | $7.44 \times 10^5$    | $7.26 \times 10^{10}$ | $6.35 \times 10^{13}$ | CBSA-C |
| F21      | Hybrid      | $7.70 \times 10^5$    | $4.04 \times 10^5$    | $1.17 \times 10^6$    | $8.29 \times 10^7$    | $5.55 \times 10^8$    | DE     |
| F22      | Hybrid      | $2.84 \times 10^3$    | $2.88 \times 10^3$    | $4.71 \times 10^3$    | $7.13 \times 10^5$    | $4.75 \times 10^{12}$ | CBSA-C |
| F23      | Composition | $2.42 \times 10^2$    | $2.40 \times 10^2$    | $2.49 \times 10^2$    | $3.49 \times 10^2$    | $2.34 \times 10^2$    | GWO    |
| F24      | Composition | $2.13 \times 10^2$    | $2.44 \times 10^2$    | $2.52 \times 10^2$    | $2.96 \times 10^2$    | $2.00 \times 10^2$    | GWO    |
| F25      | Composition | $2.01 \times 10^2$    | $2.14 \times 10^2$    | $2.23 \times 10^2$    | $2.33 \times 10^2$    | $2.00 \times 10^2$    | GWO    |
| F26      | Composition | $1.94 \times 10^2$    | $1.11 \times 10^2$    | $1.70 \times 10^2$    | $1.63 \times 10^2$    | $1.91 \times 10^2$    | DE     |
| F27      | Composition | $9.83 \times 10^2$    | $7.59 \times 10^2$    | $9.51 \times 10^2$    | $1.05 \times 10^3$    | $1.47 \times 10^3$    | DE     |
| F28      | Composition | $3.81 \times 10^3$    | $1.13 \times 10^3$    | $2.67 \times 10^3$    | $2.21 \times 10^3$    | $3.96 \times 10^3$    | DE     |
| F29      | Composition | $1.46 \times 10^6$    | $1.05 \times 10^7$    | $7.58 \times 10^6$    | $5.64 \times 10^7$    | $2.00 \times 10^2$    | GWO    |
| F30      | Composition | $3.40 \times 10^7$    | $4.01 \times 10^7$    | $4.53 \times 10^7$    | $1.29 \times 10^8$    | $4.54 \times 10^{11}$ | CBSA-C |

**Table A2.** Per-function mean objective values on the standard CEC2014 suite over 30 independent runs.

| Function | Group      | CBSA-C                               | DE                                   | GA                                   | PSO                                  | GWO                                  |
|----------|------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| F1       | Unimodal   | <b><math>2.44 \times 10^7</math></b> | $2.73 \times 10^7$                   | $6.75 \times 10^7$                   | $2.50 \times 10^8$                   | $1.05 \times 10^9$                   |
| F2       | Unimodal   | <b><math>1.01 \times 10^7</math></b> | $2.81 \times 10^8$                   | $1.14 \times 10^9$                   | $1.85 \times 10^{10}$                | $6.02 \times 10^{10}$                |
| F3       | Unimodal   | $6.41 \times 10^4$                   | <b><math>4.33 \times 10^3</math></b> | $2.58 \times 10^4$                   | $1.10 \times 10^5$                   | $1.15 \times 10^5$                   |
| F4       | Multimodal | <b><math>5.53 \times 10^2</math></b> | $5.83 \times 10^2$                   | $6.46 \times 10^2$                   | $2.84 \times 10^3$                   | $1.04 \times 10^4$                   |
| F5       | Multimodal | <b><math>5.21 \times 10^2</math></b> | <b><math>5.21 \times 10^2</math></b> | <b><math>5.21 \times 10^2</math></b> | <b><math>5.21 \times 10^2</math></b> | <b><math>5.21 \times 10^2</math></b> |
| F6       | Multimodal | $6.23 \times 10^2$                   | <b><math>6.21 \times 10^2</math></b> | $6.24 \times 10^2$                   | $6.24 \times 10^2$                   | $6.42 \times 10^2$                   |
| F7       | Multimodal | <b><math>7.01 \times 10^2</math></b> | $7.04 \times 10^2$                   | $7.11 \times 10^2$                   | $9.04 \times 10^2$                   | $1.17 \times 10^3$                   |
| F8       | Multimodal | <b><math>9.16 \times 10^2</math></b> | $1.01 \times 10^3$                   | $9.55 \times 10^2$                   | $9.32 \times 10^2$                   | $1.10 \times 10^3$                   |
| F9       | Multimodal | <b><math>1.04 \times 10^3</math></b> | $1.13 \times 10^3$                   | $1.14 \times 10^3$                   | $1.08 \times 10^3$                   | $1.23 \times 10^3$                   |
| F10      | Multimodal | <b><math>3.99 \times 10^3</math></b> | $7.57 \times 10^3$                   | $5.11 \times 10^3$                   | $4.74 \times 10^3$                   | $8.87 \times 10^3$                   |
| F11      | Multimodal | <b><math>4.67 \times 10^3</math></b> | $9.09 \times 10^3$                   | $6.74 \times 10^3$                   | $5.67 \times 10^3$                   | $8.88 \times 10^3$                   |
| F12      | Multimodal | <b><math>1.20 \times 10^3</math></b> | <b><math>1.20 \times 10^3</math></b> | <b><math>1.20 \times 10^3</math></b> | <b><math>1.20 \times 10^3</math></b> | <b><math>1.20 \times 10^3</math></b> |
| F13      | Multimodal | <b><math>1.30 \times 10^3</math></b> | <b><math>1.30 \times 10^3</math></b> | <b><math>1.30 \times 10^3</math></b> | <b><math>1.30 \times 10^3</math></b> | $1.31 \times 10^3$                   |
| F14      | Multimodal | <b><math>1.40 \times 10^3</math></b> | <b><math>1.40 \times 10^3</math></b> | <b><math>1.40 \times 10^3</math></b> | $1.45 \times 10^3$                   | $1.59 \times 10^3$                   |

Table A2. Cont.

| Function | Group       | CBSA-C             | DE                 | GA                 | PSO                   | GWO                   |
|----------|-------------|--------------------|--------------------|--------------------|-----------------------|-----------------------|
| F15      | Multimodal  | $1.52 \times 10^3$ | $1.54 \times 10^3$ | $1.54 \times 10^3$ | $1.32 \times 10^5$    | $1.41 \times 10^5$    |
| F16      | Multimodal  | $1.61 \times 10^3$ | $1.61 \times 10^3$ | $1.61 \times 10^3$ | $1.61 \times 10^3$    | $1.61 \times 10^3$    |
| F17      | Hybrid      | $2.90 \times 10^6$ | $6.05 \times 10^5$ | $2.83 \times 10^6$ | $6.71 \times 10^6$    | $1.77 \times 10^8$    |
| F18      | Hybrid      | $1.10 \times 10^5$ | $7.90 \times 10^6$ | $9.48 \times 10^6$ | $2.38 \times 10^7$    | $1.73 \times 10^9$    |
| F19      | Hybrid      | $2.46 \times 10^3$ | $1.77 \times 10^4$ | $3.79 \times 10^4$ | $2.18 \times 10^5$    | $2.40 \times 10^8$    |
| F20      | Hybrid      | $1.02 \times 10^5$ | $1.61 \times 10^5$ | $7.46 \times 10^5$ | $7.26 \times 10^{10}$ | $6.35 \times 10^{13}$ |
| F21      | Hybrid      | $7.72 \times 10^5$ | $4.06 \times 10^5$ | $1.17 \times 10^6$ | $8.29 \times 10^7$    | $5.55 \times 10^8$    |
| F22      | Hybrid      | $5.04 \times 10^3$ | $5.08 \times 10^3$ | $6.91 \times 10^3$ | $7.16 \times 10^5$    | $4.75 \times 10^{12}$ |
| F23      | Composition | $2.54 \times 10^3$ | $2.54 \times 10^3$ | $2.55 \times 10^3$ | $2.65 \times 10^3$    | $2.53 \times 10^3$    |
| F24      | Composition | $2.61 \times 10^3$ | $2.64 \times 10^3$ | $2.65 \times 10^3$ | $2.70 \times 10^3$    | $2.60 \times 10^3$    |
| F25      | Composition | $2.70 \times 10^3$ | $2.71 \times 10^3$ | $2.72 \times 10^3$ | $2.73 \times 10^3$    | $2.70 \times 10^3$    |
| F26      | Composition | $2.79 \times 10^3$ | $2.71 \times 10^3$ | $2.77 \times 10^3$ | $2.76 \times 10^3$    | $2.79 \times 10^3$    |
| F27      | Composition | $3.68 \times 10^3$ | $3.46 \times 10^3$ | $3.65 \times 10^3$ | $3.75 \times 10^3$    | $4.17 \times 10^3$    |
| F28      | Composition | $6.61 \times 10^3$ | $3.93 \times 10^3$ | $5.47 \times 10^3$ | $5.01 \times 10^3$    | $6.76 \times 10^3$    |
| F29      | Composition | $1.46 \times 10^6$ | $1.05 \times 10^7$ | $7.58 \times 10^6$ | $5.64 \times 10^7$    | $3.10 \times 10^3$    |
| F30      | Composition | $3.40 \times 10^7$ | $4.01 \times 10^7$ | $4.53 \times 10^7$ | $1.29 \times 10^8$    | $4.54 \times 10^{11}$ |

Table A3. Per-function best objective values on the standard CEC2014 suite over 30 independent runs.

| Function | Group       | CBSA-C             | DE                 | GA                 | PSO                | GWO                   |
|----------|-------------|--------------------|--------------------|--------------------|--------------------|-----------------------|
| F1       | Unimodal    | $1.08 \times 10^7$ | $7.60 \times 10^6$ | $2.66 \times 10^7$ | $2.79 \times 10^7$ | $3.19 \times 10^8$    |
| F2       | Unimodal    | $4.66 \times 10^6$ | $1.89 \times 10^7$ | $7.36 \times 10^8$ | $3.91 \times 10^9$ | $3.48 \times 10^{10}$ |
| F3       | Unimodal    | $4.11 \times 10^4$ | $8.21 \times 10^2$ | $4.71 \times 10^3$ | $1.21 \times 10^4$ | $7.77 \times 10^4$    |
| F4       | Multimodal  | $4.83 \times 10^2$ | $5.08 \times 10^2$ | $5.85 \times 10^2$ | $5.76 \times 10^2$ | $2.81 \times 10^3$    |
| F5       | Multimodal  | $5.21 \times 10^2$ | $5.21 \times 10^2$ | $5.21 \times 10^2$ | $5.21 \times 10^2$ | $5.21 \times 10^2$    |
| F6       | Multimodal  | $6.17 \times 10^2$ | $6.12 \times 10^2$ | $6.20 \times 10^2$ | $6.19 \times 10^2$ | $6.36 \times 10^2$    |
| F7       | Multimodal  | $7.01 \times 10^2$ | $7.01 \times 10^2$ | $7.07 \times 10^2$ | $7.77 \times 10^2$ | $9.72 \times 10^2$    |
| F8       | Multimodal  | $8.78 \times 10^2$ | $9.76 \times 10^2$ | $9.03 \times 10^2$ | $8.83 \times 10^2$ | $1.04 \times 10^3$    |
| F9       | Multimodal  | $9.83 \times 10^2$ | $1.10 \times 10^3$ | $1.10 \times 10^3$ | $1.01 \times 10^3$ | $1.17 \times 10^3$    |
| F10      | Multimodal  | $2.79 \times 10^3$ | $6.29 \times 10^3$ | $3.83 \times 10^3$ | $2.80 \times 10^3$ | $8.13 \times 10^3$    |
| F11      | Multimodal  | $3.67 \times 10^3$ | $7.55 \times 10^3$ | $5.56 \times 10^3$ | $4.57 \times 10^3$ | $7.67 \times 10^3$    |
| F12      | Multimodal  | $1.20 \times 10^3$ | $1.20 \times 10^3$ | $1.20 \times 10^3$ | $1.20 \times 10^3$ | $1.20 \times 10^3$    |
| F13      | Multimodal  | $1.30 \times 10^3$ | $1.30 \times 10^3$ | $1.30 \times 10^3$ | $1.30 \times 10^3$ | $1.31 \times 10^3$    |
| F14      | Multimodal  | $1.40 \times 10^3$ | $1.40 \times 10^3$ | $1.40 \times 10^3$ | $1.41 \times 10^3$ | $1.52 \times 10^3$    |
| F15      | Multimodal  | $1.51 \times 10^3$ | $1.52 \times 10^3$ | $1.53 \times 10^3$ | $1.76 \times 10^3$ | $1.87 \times 10^4$    |
| F16      | Multimodal  | $1.61 \times 10^3$ | $1.61 \times 10^3$ | $1.61 \times 10^3$ | $1.61 \times 10^3$ | $1.61 \times 10^3$    |
| F17      | Hybrid      | $4.14 \times 10^5$ | $1.58 \times 10^5$ | $5.13 \times 10^5$ | $1.58 \times 10^5$ | $2.89 \times 10^7$    |
| F18      | Hybrid      | $6.84 \times 10^4$ | $7.23 \times 10^5$ | $3.95 \times 10^6$ | $1.27 \times 10^5$ | $2.21 \times 10^8$    |
| F19      | Hybrid      | $1.93 \times 10^3$ | $3.19 \times 10^3$ | $7.03 \times 10^3$ | $1.92 \times 10^3$ | $3.11 \times 10^6$    |
| F20      | Hybrid      | $5.77 \times 10^4$ | $7.85 \times 10^4$ | $1.32 \times 10^5$ | $1.33 \times 10^5$ | $1.41 \times 10^{11}$ |
| F21      | Hybrid      | $1.33 \times 10^5$ | $5.04 \times 10^4$ | $3.04 \times 10^5$ | $6.41 \times 10^4$ | $8.64 \times 10^7$    |
| F22      | Hybrid      | $4.23 \times 10^3$ | $3.97 \times 10^3$ | $4.77 \times 10^3$ | $4.78 \times 10^3$ | $3.63 \times 10^5$    |
| F23      | Composition | $2.54 \times 10^3$ | $2.52 \times 10^3$ | $2.54 \times 10^3$ | $2.56 \times 10^3$ | $2.50 \times 10^3$    |
| F24      | Composition | $2.61 \times 10^3$ | $2.63 \times 10^3$ | $2.64 \times 10^3$ | $2.64 \times 10^3$ | $2.60 \times 10^3$    |
| F25      | Composition | $2.70 \times 10^3$ | $2.71 \times 10^3$ | $2.72 \times 10^3$ | $2.72 \times 10^3$ | $2.70 \times 10^3$    |
| F26      | Composition | $2.70 \times 10^3$ | $2.70 \times 10^3$ | $2.70 \times 10^3$ | $2.70 \times 10^3$ | $2.71 \times 10^3$    |
| F27      | Composition | $3.14 \times 10^3$ | $3.14 \times 10^3$ | $3.15 \times 10^3$ | $3.14 \times 10^3$ | $3.94 \times 10^3$    |
| F28      | Composition | $5.09 \times 10^3$ | $3.69 \times 10^3$ | $4.11 \times 10^3$ | $4.11 \times 10^3$ | $3.84 \times 10^3$    |
| F29      | Composition | $3.53 \times 10^4$ | $5.72 \times 10^4$ | $1.16 \times 10^6$ | $1.84 \times 10^6$ | $3.10 \times 10^3$    |
| F30      | Composition | $2.04 \times 10^6$ | $1.77 \times 10^6$ | $1.01 \times 10^7$ | $3.37 \times 10^6$ | $5.52 \times 10^9$    |

**Table A4.** Per-function objective-value variances on the standard CEC2014 suite over 30 independent runs.

| Function | Group       | CBSA-C                | DE                    | GA                    | PSO                   | GWO                    |
|----------|-------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|
| F1       | Unimodal    | $6.49 \times 10^{13}$ | $1.42 \times 10^{14}$ | $1.20 \times 10^{15}$ | $7.61 \times 10^{16}$ | $1.21 \times 10^{17}$  |
| F2       | Unimodal    | $1.67 \times 10^{13}$ | $1.70 \times 10^{17}$ | $3.77 \times 10^{16}$ | $7.64 \times 10^{19}$ | $9.67 \times 10^{19}$  |
| F3       | Unimodal    | $1.54 \times 10^8$    | $1.82 \times 10^7$    | $2.56 \times 10^8$    | $3.96 \times 10^9$    | $7.49 \times 10^8$     |
| F4       | Multimodal  | $1.63 \times 10^3$    | $2.67 \times 10^3$    | $2.22 \times 10^3$    | $4.45 \times 10^6$    | $1.09 \times 10^7$     |
| F5       | Multimodal  | $1.47 \times 10^{-2}$ | $3.66 \times 10^{-3}$ | $4.83 \times 10^{-3}$ | $1.16 \times 10^{-2}$ | $3.87 \times 10^{-3}$  |
| F6       | Multimodal  | $1.16 \times 10^1$    | $3.89 \times 10^1$    | $7.74 \times 10^0$    | $1.11 \times 10^1$    | $1.17 \times 10^1$     |
| F7       | Multimodal  | $1.89 \times 10^{-3}$ | $1.53 \times 10^1$    | $5.08 \times 10^0$    | $8.56 \times 10^3$    | $8.90 \times 10^3$     |
| F8       | Multimodal  | $2.71 \times 10^2$    | $3.01 \times 10^2$    | $4.70 \times 10^2$    | $8.50 \times 10^2$    | $7.65 \times 10^2$     |
| F9       | Multimodal  | $7.69 \times 10^2$    | $2.31 \times 10^2$    | $5.98 \times 10^2$    | $1.60 \times 10^3$    | $9.52 \times 10^2$     |
| F10      | Multimodal  | $4.34 \times 10^5$    | $2.74 \times 10^5$    | $2.23 \times 10^5$    | $5.82 \times 10^5$    | $1.59 \times 10^5$     |
| F11      | Multimodal  | $4.25 \times 10^5$    | $1.64 \times 10^5$    | $1.78 \times 10^5$    | $6.35 \times 10^5$    | $3.70 \times 10^5$     |
| F12      | Multimodal  | $3.50 \times 10^{-1}$ | $2.20 \times 10^{-1}$ | $1.08 \times 10^{-1}$ | $8.76 \times 10^{-1}$ | $5.31 \times 10^{-1}$  |
| F13      | Multimodal  | $6.36 \times 10^{-3}$ | $6.46 \times 10^{-3}$ | $1.02 \times 10^{-2}$ | $1.83 \times 10^0$    | $1.09 \times 10^0$     |
| F14      | Multimodal  | $2.56 \times 10^{-3}$ | $1.20 \times 10^1$    | $6.91 \times 10^{-2}$ | $7.16 \times 10^2$    | $1.04 \times 10^3$     |
| F15      | Multimodal  | $1.12 \times 10^1$    | $1.16 \times 10^3$    | $2.40 \times 10^2$    | $5.71 \times 10^{10}$ | $6.39 \times 10^9$     |
| F16      | Multimodal  | $3.86 \times 10^{-1}$ | $5.20 \times 10^{-2}$ | $1.49 \times 10^{-1}$ | $1.89 \times 10^{-1}$ | $1.80 \times 10^{-1}$  |
| F17      | Hybrid      | $3.98 \times 10^{12}$ | $1.03 \times 10^{11}$ | $1.03 \times 10^{13}$ | $7.27 \times 10^{13}$ | $6.32 \times 10^{15}$  |
| F18      | Hybrid      | $8.22 \times 10^8$    | $3.54 \times 10^{13}$ | $1.50 \times 10^{13}$ | $5.00 \times 10^{15}$ | $2.89 \times 10^{18}$  |
| F19      | Hybrid      | $1.38 \times 10^6$    | $2.05 \times 10^8$    | $5.97 \times 10^9$    | $2.20 \times 10^{11}$ | $1.10 \times 10^{17}$  |
| F20      | Hybrid      | $6.25 \times 10^8$    | $3.00 \times 10^9$    | $1.21 \times 10^{12}$ | $1.21 \times 10^{23}$ | $1.78 \times 10^{28}$  |
| F21      | Hybrid      | $3.69 \times 10^{11}$ | $9.60 \times 10^{10}$ | $6.16 \times 10^{11}$ | $2.51 \times 10^{16}$ | $1.15 \times 10^{17}$  |
| F22      | Hybrid      | $2.27 \times 10^5$    | $7.18 \times 10^5$    | $1.52 \times 10^6$    | $1.50 \times 10^{13}$ | $3.33 \times 10^{26}$  |
| F23      | Composition | $7.03 \times 10^{-1}$ | $3.96 \times 10^1$    | $3.07 \times 10^1$    | $3.14 \times 10^3$    | $2.70 \times 10^3$     |
| F24      | Composition | $2.29 \times 10^1$    | $4.92 \times 10^1$    | $2.45 \times 10^1$    | $7.13 \times 10^2$    | $3.23 \times 10^{-4}$  |
| F25      | Composition | $3.21 \times 10^0$    | $1.25 \times 10^1$    | $2.52 \times 10^1$    | $2.27 \times 10^2$    | $3.92 \times 10^{-25}$ |
| F26      | Composition | $6.39 \times 10^2$    | $9.44 \times 10^2$    | $2.40 \times 10^3$    | $4.11 \times 10^3$    | $6.83 \times 10^2$     |
| F27      | Composition | $6.17 \times 10^4$    | $1.52 \times 10^4$    | $3.46 \times 10^4$    | $3.86 \times 10^4$    | $1.15 \times 10^4$     |
| F28      | Composition | $5.40 \times 10^5$    | $4.94 \times 10^4$    | $7.44 \times 10^5$    | $3.09 \times 10^5$    | $2.84 \times 10^6$     |
| F29      | Composition | $1.26 \times 10^{13}$ | $1.54 \times 10^{14}$ | $5.25 \times 10^{13}$ | $2.25 \times 10^{15}$ | $4.92 \times 10^{-12}$ |
| F30      | Composition | $1.98 \times 10^{15}$ | $2.96 \times 10^{15}$ | $9.18 \times 10^{14}$ | $6.40 \times 10^{16}$ | $1.14 \times 10^{24}$  |

**Table A5.** CVRP per-instance results: mean objective values over repeated runs.

| Instance  | CBSA-D        | ILS            | ACO            | GA      | GWO-D   | PSO-D   |
|-----------|---------------|----------------|----------------|---------|---------|---------|
| A-n32-k5  | <b>830.20</b> | 830.50         | 881.80         | 885.40  | 1094.10 | 1330.10 |
| A-n33-k5  | 676.90        | <b>673.30</b>  | 713.90         | 730.30  | 893.50  | 1109.90 |
| A-n33-k6  | 767.00        | <b>750.50</b>  | 787.00         | 814.20  | 942.10  | 1138.60 |
| A-n34-k5  | 810.30        | <b>790.20</b>  | 817.40         | 875.30  | 1009.40 | 1234.20 |
| A-n36-k5  | 859.10        | <b>824.70</b>  | 873.40         | 925.30  | 1102.10 | 1305.00 |
| A-n37-k5  | 700.00        | <b>687.10</b>  | 757.50         | 786.70  | 1022.80 | 1282.00 |
| A-n37-k6  | 1040.60       | <b>975.10</b>  | 1061.40        | 1106.80 | 1296.40 | 1485.80 |
| A-n38-k5  | 769.80        | <b>742.60</b>  | 804.90         | 904.40  | 1120.20 | 1380.20 |
| A-n39-k5  | 905.70        | <b>849.90</b>  | 911.40         | 1018.50 | 1236.90 | 1430.80 |
| A-n39-k6  | 884.10        | <b>856.90</b>  | 945.00         | 985.70  | 1234.20 | 1486.20 |
| A-n44-k6  | 1033.90       | <b>977.90</b>  | 1014.70        | 1157.40 | 1440.90 | 1709.60 |
| A-n45-k6  | 1008.80       | <b>990.60</b>  | 1048.80        | 1258.80 | 1536.10 | 1952.50 |
| A-n45-k7  | 1284.00       | 1226.50        | <b>1221.80</b> | 1372.30 | 1658.70 | 1894.00 |
| A-n46-k7  | 985.60        | <b>943.40</b>  | 1026.80        | 1136.40 | 1466.00 | 1792.80 |
| A-n48-k7  | 1179.40       | <b>1136.90</b> | 1232.00        | 1300.90 | 1709.40 | 2013.50 |
| A-n53-k7  | 1111.50       | <b>1050.80</b> | 1139.70        | 1369.90 | 1761.80 | 2236.30 |
| A-n54-k7  | 1327.60       | <b>1225.10</b> | 1292.40        | 1486.50 | 1965.80 | 2376.50 |
| A-n55-k9  | 1185.10       | <b>1115.70</b> | 1171.80        | 1368.60 | 1702.90 | 2312.80 |
| A-n60-k9  | 1548.00       | <b>1462.50</b> | 1495.10        | 1743.00 | 2248.10 | 2833.50 |
| A-n61-k9  | 1122.00       | <b>1104.60</b> | 1151.50        | 1457.20 | 1799.30 | 2316.40 |
| A-n62-k8  | 1462.90       | <b>1398.10</b> | 1440.40        | 1786.10 | 2336.50 | 2958.90 |
| A-n63-k10 | 1496.40       | <b>1395.20</b> | 1536.20        | 1696.50 | 2233.20 | 2814.80 |

Table A5. Cont.

| Instance  | CBSA-D         | ILS            | ACO     | GA      | GWO-D   | PSO-D   |
|-----------|----------------|----------------|---------|---------|---------|---------|
| A-n63-k9  | 1884.60        | <b>1744.70</b> | 1791.80 | 2088.20 | 2653.50 | 3292.00 |
| A-n64-k9  | 1583.70        | <b>1496.90</b> | 1580.40 | 1766.40 | 2333.50 | 2952.70 |
| A-n65-k9  | 1306.10        | <b>1203.00</b> | 1358.20 | 1833.60 | 2253.80 | 2942.50 |
| A-n69-k9  | 1282.40        | <b>1218.60</b> | 1323.40 | 1707.00 | 2188.50 | 3032.10 |
| A-n80-k10 | 1994.80        | <b>1919.50</b> | 2018.30 | 2439.90 | 3283.00 | 4032.70 |
| B-n31-k5  | <b>682.80</b>  | 695.20         | 693.70  | 702.70  | 742.80  | 892.20  |
| B-n34-k5  | 795.30         | <b>794.60</b>  | 818.40  | 857.70  | 919.70  | 1121.60 |
| B-n35-k5  | 965.40         | <b>965.10</b>  | 1012.00 | 1024.40 | 1172.60 | 1493.70 |
| B-n38-k6  | <b>819.40</b>  | 822.00         | 855.40  | 888.10  | 1006.00 | 1308.30 |
| B-n39-k5  | 577.80         | <b>568.70</b>  | 599.30  | 649.40  | 800.70  | 1236.50 |
| B-n41-k6  | 890.50         | <b>849.10</b>  | 863.10  | 1010.80 | 1129.40 | 1499.50 |
| B-n43-k6  | 770.60         | <b>758.50</b>  | 806.40  | 884.40  | 1095.10 | 1358.00 |
| B-n44-k7  | 984.00         | <b>946.00</b>  | 994.10  | 1087.00 | 1252.00 | 1650.20 |
| B-n45-k5  | 782.70         | <b>755.40</b>  | 801.80  | 1033.70 | 1154.30 | 1631.20 |
| B-n45-k6  | 735.30         | <b>697.80</b>  | 729.10  | 887.90  | 970.40  | 1233.00 |
| B-n50-k7  | <b>750.90</b>  | 764.70         | 796.90  | 949.50  | 1160.10 | 1845.10 |
| B-n50-k8  | 1435.00        | <b>1346.80</b> | 1367.30 | 1533.80 | 1781.40 | 2077.30 |
| B-n51-k7  | <b>1026.90</b> | 1035.30        | 1065.30 | 1558.60 | 1543.30 | 2371.50 |
| B-n52-k7  | 782.60         | <b>760.70</b>  | 850.30  | 1024.60 | 1244.30 | 1924.60 |
| B-n56-k7  | 767.90         | <b>733.90</b>  | 784.50  | 1024.70 | 1169.30 | 1899.30 |
| B-n57-k7  | 1197.70        | <b>1196.10</b> | 1319.60 | 1663.50 | 1876.60 | 2605.00 |
| B-n57-k9  | 1740.20        | <b>1681.40</b> | 1724.20 | 1930.20 | 2199.90 | 2729.90 |
| B-n63-k10 | 1669.60        | <b>1591.50</b> | 1651.20 | 1956.30 | 2513.70 | 3114.30 |
| B-n64-k9  | 929.10         | <b>915.90</b>  | 937.50  | 1429.30 | 1535.80 | 2334.40 |
| B-n66-k9  | 1464.80        | <b>1356.20</b> | 1394.50 | 1685.20 | 2260.70 | 2887.80 |
| B-n67-k10 | 1140.80        | <b>1117.50</b> | 1179.30 | 1541.00 | 1790.30 | 2638.10 |
| B-n68-k9  | 1404.90        | <b>1345.40</b> | 1405.50 | 1782.60 | 2281.80 | 2995.70 |
| B-n78-k10 | 1380.20        | <b>1321.00</b> | 1345.10 | 1788.50 | 2411.20 | 3447.70 |

Table A6. CVRP per-instance results: best objective values over repeated runs.

| Instance  | CBSA-D         | ILS            | ACO            | GA            | GWO-D   | PSO-D   |
|-----------|----------------|----------------|----------------|---------------|---------|---------|
| A-n32-k5  | <b>799.00</b>  | 819.00         | 819.00         | 832.00        | 1033.00 | 1183.00 |
| A-n33-k5  | <b>664.00</b>  | 667.00         | 678.00         | 669.00        | 850.00  | 1029.00 |
| A-n33-k6  | <b>743.00</b>  | 744.00         | 754.00         | <b>743.00</b> | 888.00  | 1074.00 |
| A-n34-k5  | 790.00         | <b>778.00</b>  | 809.00         | 827.00        | 917.00  | 1123.00 |
| A-n36-k5  | 840.00         | <b>807.00</b>  | 810.00         | 858.00        | 1023.00 | 1196.00 |
| A-n37-k5  | <b>669.00</b>  | 670.00         | 712.00         | 725.00        | 973.00  | 1128.00 |
| A-n37-k6  | 1007.00        | <b>951.00</b>  | 1012.00        | 1034.00       | 1218.00 | 1349.00 |
| A-n38-k5  | 741.00         | <b>731.00</b>  | 755.00         | 837.00        | 1059.00 | 1296.00 |
| A-n39-k5  | 869.00         | <b>834.00</b>  | 839.00         | 943.00        | 1167.00 | 1350.00 |
| A-n39-k6  | 863.00         | <b>837.00</b>  | 892.00         | 906.00        | 1164.00 | 1362.00 |
| A-n44-k6  | 964.00         | <b>947.00</b>  | 962.00         | 1072.00       | 1364.00 | 1641.00 |
| A-n45-k6  | 983.00         | <b>979.00</b>  | <b>960.00</b>  | 1009.00       | 1458.00 | 1838.00 |
| A-n45-k7  | 1251.00        | 1195.00        | <b>1192.00</b> | 1279.00       | 1547.00 | 1788.00 |
| A-n46-k7  | 968.00         | <b>924.00</b>  | 940.00         | 1011.00       | 1406.00 | 1633.00 |
| A-n48-k7  | <b>1110.00</b> | 1117.00        | 1131.00        | 1140.00       | 1599.00 | 1913.00 |
| A-n53-k7  | 1044.00        | <b>1029.00</b> | 1067.00        | 1182.00       | 1646.00 | 2107.00 |
| A-n54-k7  | 1259.00        | <b>1204.00</b> | 1232.00        | 1342.00       | 1869.00 | 2278.00 |
| A-n55-k9  | 1115.00        | <b>1085.00</b> | 1109.00        | 1208.00       | 1572.00 | 2236.00 |
| A-n60-k9  | 1485.00        | 1424.00        | <b>1412.00</b> | 1562.00       | 2060.00 | 2630.00 |
| A-n61-k9  | <b>1086.00</b> | 1092.00        | 1097.00        | 1321.00       | 1608.00 | 2191.00 |
| A-n62-k8  | 1414.00        | <b>1353.00</b> | 1418.00        | 1652.00       | 2144.00 | 2823.00 |
| A-n63-k10 | 1432.00        | <b>1366.00</b> | 1410.00        | 1535.00       | 2116.00 | 2662.00 |
| A-n63-k9  | 1835.00        | <b>1711.00</b> | 1729.00        | 1988.00       | 2553.00 | 3157.00 |
| A-n64-k9  | 1520.00        | <b>1469.00</b> | 1506.00        | 1683.00       | 2054.00 | 2840.00 |
| A-n65-k9  | 1272.00        | <b>1190.00</b> | 1269.00        | 1687.00       | 2158.00 | 2879.00 |
| A-n69-k9  | 1219.00        | <b>1204.00</b> | 1237.00        | 1505.00       | 2098.00 | 2909.00 |
| A-n80-k10 | 1929.00        | <b>1887.00</b> | 1931.00        | 2219.00       | 3058.00 | 3883.00 |

Table A6. Cont.

| Instance  | CBSA-D         | ILS            | ACO            | GA      | GWO-D   | PSO-D   |
|-----------|----------------|----------------|----------------|---------|---------|---------|
| B-n31-k5  | 676.00         | 683.00         | <b>672.00</b>  | 685.00  | 708.00  | 763.00  |
| B-n34-k5  | 791.00         | <b>788.00</b>  | 802.00         | 797.00  | 886.00  | 1061.00 |
| B-n35-k5  | <b>955.00</b>  | 957.00         | 967.00         | 986.00  | 1125.00 | 1313.00 |
| B-n38-k6  | <b>806.00</b>  | 809.00         | 810.00         | 846.00  | 939.00  | 1210.00 |
| B-n39-k5  | 554.00         | <b>553.00</b>  | 562.00         | 567.00  | 692.00  | 1072.00 |
| B-n41-k6  | 849.00         | 836.00         | <b>834.00</b>  | 932.00  | 1084.00 | 1277.00 |
| B-n43-k6  | 760.00         | <b>752.00</b>  | 770.00         | 790.00  | 1025.00 | 1276.00 |
| B-n44-k7  | 938.00         | 925.00         | <b>911.00</b>  | 997.00  | 1127.00 | 1552.00 |
| B-n45-k5  | 758.00         | <b>751.00</b>  | 771.00         | 894.00  | 1065.00 | 1472.00 |
| B-n45-k6  | 715.00         | <b>683.00</b>  | 702.00         | 793.00  | 857.00  | 1155.00 |
| B-n50-k7  | <b>741.00</b>  | <b>741.00</b>  | 759.00         | 755.00  | 999.00  | 1727.00 |
| B-n50-k8  | 1387.00        | <b>1335.00</b> | 1344.00        | 1406.00 | 1704.00 | 1985.00 |
| B-n51-k7  | <b>1018.00</b> | 1019.00        | 1044.00        | 1233.00 | 1436.00 | 2095.00 |
| B-n52-k7  | 755.00         | <b>751.00</b>  | 767.00         | 875.00  | 1057.00 | 1707.00 |
| B-n56-k7  | 722.00         | <b>716.00</b>  | 748.00         | 869.00  | 999.00  | 1790.00 |
| B-n57-k7  | <b>1145.00</b> | 1175.00        | 1223.00        | 1348.00 | 1750.00 | 2302.00 |
| B-n57-k9  | 1723.00        | <b>1649.00</b> | 1683.00        | 1806.00 | 2081.00 | 2625.00 |
| B-n63-k10 | 1632.00        | 1564.00        | <b>1558.00</b> | 1809.00 | 2333.00 | 2983.00 |
| B-n64-k9  | 908.00         | <b>897.00</b>  | 904.00         | 1199.00 | 1443.00 | 2124.00 |
| B-n66-k9  | 1422.00        | <b>1334.00</b> | 1346.00        | 1427.00 | 2019.00 | 2572.00 |
| B-n67-k10 | <b>1080.00</b> | 1092.00        | 1117.00        | 1480.00 | 1682.00 | 2450.00 |
| B-n68-k9  | 1376.00        | <b>1313.00</b> | 1330.00        | 1522.00 | 2073.00 | 2775.00 |
| B-n78-k10 | 1347.00        | 1300.00        | <b>1279.00</b> | 1449.00 | 2213.00 | 3282.00 |

Table A7. CVRP per-instance results: variances over repeated runs.

| Instance  | CBSA-D  | ILS    | ACO     | GA        | GWO-D     | PSO-D     |
|-----------|---------|--------|---------|-----------|-----------|-----------|
| A-n32-k5  | 169.51  | 23.61  | 1518.40 | 1422.27   | 1642.10   | 5856.77   |
| A-n33-k5  | 52.54   | 17.79  | 632.99  | 1221.12   | 910.94    | 2595.21   |
| A-n33-k6  | 244.00  | 29.39  | 1448.89 | 2388.62   | 1518.77   | 3653.82   |
| A-n34-k5  | 191.57  | 43.07  | 56.27   | 2088.46   | 2309.82   | 5170.18   |
| A-n36-k5  | 104.32  | 106.46 | 1280.04 | 891.79    | 3249.66   | 7524.89   |
| A-n37-k5  | 262.89  | 95.66  | 600.06  | 2716.01   | 1433.07   | 7922.67   |
| A-n37-k6  | 700.04  | 166.32 | 1645.82 | 2373.07   | 3226.04   | 6180.40   |
| A-n38-k5  | 523.73  | 51.16  | 1165.88 | 4063.60   | 1350.18   | 5313.96   |
| A-n39-k5  | 562.90  | 128.77 | 1869.60 | 5187.39   | 3221.43   | 3518.40   |
| A-n39-k6  | 338.10  | 225.43 | 851.78  | 7888.68   | 1817.29   | 4094.18   |
| A-n44-k6  | 1577.21 | 403.88 | 1434.01 | 5074.49   | 2569.21   | 2225.38   |
| A-n45-k6  | 381.51  | 182.93 | 2270.84 | 33,677.07 | 2275.43   | 4890.72   |
| A-n45-k7  | 887.56  | 332.06 | 606.40  | 2763.57   | 5110.01   | 8665.11   |
| A-n46-k7  | 232.49  | 453.16 | 2724.40 | 7562.93   | 2064.22   | 6887.07   |
| A-n48-k7  | 1167.60 | 305.21 | 3120.00 | 7103.21   | 5337.60   | 4926.06   |
| A-n53-k7  | 1857.83 | 247.73 | 2242.68 | 7367.66   | 6654.18   | 6363.12   |
| A-n54-k7  | 1427.60 | 131.88 | 794.93  | 5008.72   | 6739.96   | 3692.50   |
| A-n55-k9  | 2209.43 | 465.57 | 1951.96 | 12,546.93 | 4105.88   | 4209.29   |
| A-n60-k9  | 1677.11 | 450.06 | 3176.32 | 10,594.44 | 8449.43   | 12,530.28 |
| A-n61-k9  | 272.89  | 173.60 | 2024.28 | 7421.73   | 11,393.57 | 4648.27   |
| A-n62-k8  | 1457.21 | 888.10 | 625.82  | 10,447.88 | 15,137.61 | 7078.99   |
| A-n63-k10 | 1512.27 | 164.40 | 3559.73 | 3951.83   | 5038.18   | 6349.07   |
| A-n63-k9  | 1725.82 | 450.68 | 791.07  | 5492.84   | 4454.50   | 9179.33   |
| A-n64-k9  | 1299.34 | 224.54 | 1340.71 | 8270.04   | 13,634.28 | 8756.46   |
| A-n65-k9  | 863.88  | 150.44 | 3088.18 | 11,713.82 | 3920.84   | 3319.17   |
| A-n69-k9  | 1387.16 | 99.38  | 2719.16 | 23,067.56 | 4866.06   | 6108.54   |
| A-n80-k10 | 2122.62 | 354.28 | 3175.12 | 17,689.43 | 14,414.67 | 6539.79   |
| B-n31-k5  | 12.84   | 52.18  | 260.68  | 108.01    | 359.73    | 4306.18   |
| B-n34-k5  | 7.12    | 14.71  | 159.38  | 2758.68   | 1708.01   | 1604.04   |
| B-n35-k5  | 73.60   | 15.66  | 1088.00 | 2225.60   | 1434.49   | 9998.68   |
| B-n38-k6  | 111.38  | 82.44  | 944.71  | 2814.32   | 1608.89   | 5461.34   |
| B-n39-k5  | 369.29  | 256.01 | 836.68  | 2120.71   | 3413.79   | 9651.83   |

Table A7. Cont.

| Instance  | CBSA-D  | ILS    | ACO     | GA        | GWO-D     | PSO-D     |
|-----------|---------|--------|---------|-----------|-----------|-----------|
| B-n41-k6  | 717.61  | 75.66  | 220.10  | 2218.62   | 1171.38   | 8940.06   |
| B-n43-k6  | 59.38   | 22.72  | 521.60  | 5058.04   | 1198.99   | 2918.22   |
| B-n44-k7  | 778.44  | 230.67 | 1148.54 | 6525.33   | 5212.67   | 5737.73   |
| B-n45-k5  | 387.12  | 13.82  | 799.73  | 7731.12   | 4786.46   | 7575.07   |
| B-n45-k6  | 277.79  | 94.84  | 556.32  | 7825.88   | 3289.82   | 2271.56   |
| B-n50-k7  | 40.32   | 104.68 | 893.43  | 14,299.39 | 6624.99   | 5989.21   |
| B-n50-k8  | 924.22  | 124.84 | 711.12  | 6055.51   | 4178.93   | 7970.90   |
| B-n51-k7  | 194.32  | 102.23 | 306.46  | 35,368.49 | 4410.90   | 36,822.50 |
| B-n52-k7  | 534.49  | 54.46  | 3762.46 | 9922.04   | 9050.68   | 17,009.82 |
| B-n56-k7  | 1036.77 | 170.99 | 466.72  | 14,499.12 | 6166.90   | 7107.79   |
| B-n57-k7  | 2330.23 | 502.32 | 2780.27 | 30,372.94 | 6625.16   | 23,662.44 |
| B-n57-k9  | 226.18  | 383.16 | 1182.40 | 7000.18   | 6353.88   | 4372.32   |
| B-n63-k10 | 441.82  | 398.72 | 3052.84 | 5565.79   | 13,288.46 | 10,960.90 |
| B-n64-k9  | 348.77  | 280.54 | 1030.28 | 26,479.34 | 6052.40   | 12,435.60 |
| B-n66-k9  | 2005.29 | 172.40 | 1142.72 | 13,751.96 | 12,207.57 | 20,939.51 |
| B-n67-k10 | 1928.84 | 215.61 | 1432.46 | 2696.00   | 9604.90   | 10,364.54 |
| B-n68-k9  | 560.77  | 511.38 | 1746.94 | 13,071.60 | 18,638.18 | 12,383.12 |
| B-n78-k10 | 1291.29 | 301.78 | 3563.88 | 40,369.83 | 17,563.96 | 7952.90   |

Table A8. VRPTW per-instance results: mean objective values over repeated runs.

| Instance | CBSA-D        | ILS            | ACO           | GA             | GWO-D   | PSO-D   |
|----------|---------------|----------------|---------------|----------------|---------|---------|
| C101     | <b>828.94</b> | <b>828.94</b>  | <b>828.94</b> | 850.54         | 1185.06 | 1313.73 |
| C102     | 963.53        | <b>928.46</b>  | 1274.00       | 959.22         | 1372.57 | 1393.34 |
| C103     | 1145.04       | <b>921.28</b>  | 1548.94       | 1111.57        | 1381.17 | 1454.74 |
| C104     | 1048.79       | <b>960.71</b>  | 1457.45       | 1269.96        | 1535.43 | 1669.66 |
| C105     | <b>828.94</b> | 851.87         | <b>828.94</b> | 937.09         | 1587.53 | 1601.00 |
| C106     | <b>834.21</b> | 857.97         | 875.92        | 927.81         | 1708.38 | 1769.88 |
| C107     | <b>828.94</b> | 878.43         | 844.16        | 987.76         | 1755.21 | 1862.48 |
| C108     | 854.41        | 944.31         | <b>837.36</b> | 1043.32        | 1944.12 | 2017.44 |
| C109     | 956.69        | 1029.38        | <b>862.67</b> | 1219.03        | 2037.93 | 2193.68 |
| C201     | <b>591.56</b> | 597.48         | <b>591.56</b> | 606.26         | 684.83  | 976.94  |
| C202     | 625.27        | 647.37         | 859.26        | <b>624.18</b>  | 833.10  | 1078.00 |
| C203     | 749.78        | 706.61         | 1295.45       | <b>686.34</b>  | 960.94  | 1120.29 |
| C204     | 876.18        | <b>675.33</b>  | 1177.95       | 759.13         | 1116.95 | 1255.44 |
| C205     | 596.20        | 616.24         | <b>588.88</b> | 604.98         | 1000.04 | 1300.88 |
| C206     | 657.43        | 620.25         | <b>595.91</b> | 679.66         | 1190.84 | 1418.33 |
| C207     | 637.13        | 668.51         | <b>622.90</b> | 640.95         | 1169.95 | 1323.07 |
| C208     | 645.70        | 634.12         | <b>600.63</b> | 714.07         | 1246.88 | 1493.79 |
| R101     | 1722.39       | 1766.71        | 1954.68       | <b>1683.71</b> | 1818.99 | 1894.31 |
| R102     | 1618.75       | 1696.29        | 1897.98       | <b>1513.63</b> | 1628.02 | 1676.51 |
| R103     | 1455.09       | 1319.72        | 1751.63       | <b>1280.24</b> | 1432.25 | 1453.38 |
| R104     | 1327.80       | <b>1048.84</b> | 1441.31       | 1174.85        | 1342.62 | 1465.40 |
| R105     | 1557.76       | 1540.22        | 1662.03       | <b>1454.50</b> | 1770.02 | 1819.62 |
| R106     | 1519.34       | 1400.69        | 1677.08       | <b>1345.36</b> | 1552.08 | 1630.42 |
| R107     | 1336.39       | <b>1169.49</b> | 1517.27       | 1212.01        | 1449.60 | 1482.44 |
| R108     | 1369.05       | <b>1013.51</b> | 1210.21       | 1198.93        | 1488.07 | 1616.60 |
| R109     | 1594.08       | <b>1298.53</b> | 1473.13       | 1400.30        | 1724.46 | 1813.48 |
| R110     | 1624.40       | <b>1193.54</b> | 1470.17       | 1337.05        | 1663.23 | 1824.74 |
| R111     | 1431.11       | <b>1152.52</b> | 1458.19       | 1234.26        | 1503.36 | 1589.44 |
| R112     | 1486.02       | <b>1051.98</b> | 1222.03       | 1312.88        | 1794.25 | 1914.08 |
| R201     | 1451.52       | 1379.05        | 1586.16       | <b>1338.96</b> | 1546.95 | 1622.74 |
| R202     | 1343.93       | 1261.07        | 1517.01       | <b>1188.22</b> | 1345.21 | 1415.73 |
| R203     | 1133.14       | 1055.52        | 1403.91       | <b>1006.01</b> | 1175.66 | 1262.26 |
| R204     | 888.44        | <b>839.45</b>  | 1172.78       | 848.46         | 954.13  | 1032.67 |
| R205     | 1220.44       | <b>1137.01</b> | 1282.03       | 1151.34        | 1523.62 | 1546.27 |
| R206     | 1137.61       | 1074.96        | 1305.74       | <b>1062.30</b> | 1355.43 | 1393.11 |
| R207     | 1016.67       | 979.09         | 1248.74       | <b>941.91</b>  | 1164.48 | 1225.27 |
| R208     | 863.93        | <b>780.85</b>  | 1020.62       | 808.97         | 1014.09 | 1053.60 |

Table A8. Cont.

| Instance | CBSA-D  | ILS            | ACO     | GA             | GWO-D   | PSO-D   |
|----------|---------|----------------|---------|----------------|---------|---------|
| R209     | 1199.52 | 1101.70        | 1417.33 | <b>1046.00</b> | 1304.09 | 1350.96 |
| R210     | 1157.26 | 1097.14        | 1492.19 | <b>1032.80</b> | 1272.84 | 1337.11 |
| R211     | 1072.51 | <b>932.33</b>  | 1160.37 | 986.53         | 1242.62 | 1312.64 |
| RC101    | 1800.94 | 1802.08        | 1878.42 | <b>1717.90</b> | 2035.39 | 2131.79 |
| RC102    | 1705.79 | 1620.66        | 1936.12 | <b>1598.32</b> | 1882.09 | 1973.95 |
| RC103    | 1645.70 | <b>1397.10</b> | 1825.98 | 1460.57        | 1739.36 | 1850.05 |
| RC104    | 1552.17 | <b>1227.93</b> | 1497.60 | 1409.26        | 1804.79 | 2009.24 |
| RC105    | 1808.12 | 1693.84        | 2009.88 | <b>1645.14</b> | 1922.72 | 1985.81 |
| RC106    | 1682.71 | <b>1528.22</b> | 1610.09 | 1610.15        | 2028.50 | 2151.93 |
| RC107    | 1633.71 | <b>1374.08</b> | 1593.63 | 1509.44        | 1997.95 | 2185.80 |
| RC108    | 1568.95 | <b>1235.89</b> | 1446.36 | 1472.49        | 2029.63 | 2185.26 |
| RC201    | 1548.97 | <b>1510.80</b> | 1784.47 | 1562.00        | 2022.54 | 2033.62 |
| RC202    | 1458.52 | <b>1363.21</b> | 1637.70 | 1380.10        | 1689.35 | 1754.35 |
| RC203    | 1200.80 | <b>1139.65</b> | 1535.16 | 1146.19        | 1429.71 | 1502.42 |
| RC204    | 1018.17 | <b>904.74</b>  | 1225.01 | 932.09         | 1144.53 | 1249.21 |
| RC205    | 1521.43 | <b>1396.50</b> | 1731.10 | 1402.70        | 1740.58 | 1812.89 |
| RC206    | 1314.45 | <b>1280.84</b> | 1525.09 | 1281.84        | 1795.12 | 1913.27 |
| RC207    | 1338.14 | 1290.17        | 1720.13 | <b>1231.85</b> | 1582.68 | 1713.83 |
| RC208    | 1194.84 | <b>1071.03</b> | 1361.47 | 1170.03        | 1607.42 | 1631.86 |

Table A9. VRPTW per-instance results: best objective values over repeated runs.

| Instance | CBSA-D        | ILS            | ACO           | GA             | GWO-D         | PSO-D   |
|----------|---------------|----------------|---------------|----------------|---------------|---------|
| C101     | <b>828.94</b> | <b>828.94</b>  | <b>828.94</b> | <b>828.94</b>  | 1104.10       | 1203.76 |
| C102     | <b>865.92</b> | 878.90         | 1137.29       | 869.96         | 1218.16       | 1253.22 |
| C103     | 921.84        | <b>880.20</b>  | 1451.32       | 1021.59        | 1183.26       | 1289.06 |
| C104     | 914.09        | <b>878.75</b>  | 1378.32       | 1161.65        | 1439.05       | 1509.00 |
| C105     | <b>828.94</b> | 830.46         | <b>828.94</b> | <b>828.94</b>  | 1416.04       | 1495.32 |
| C106     | <b>828.94</b> | 830.46         | 854.31        | <b>828.94</b>  | 1519.89       | 1538.97 |
| C107     | <b>828.94</b> | 833.77         | <b>828.94</b> | 867.18         | 1524.21       | 1692.74 |
| C108     | <b>828.94</b> | 876.28         | <b>828.94</b> | 901.16         | 1836.30       | 1868.96 |
| C109     | 850.07        | 921.71         | <b>833.59</b> | 1008.54        | 1820.68       | 2124.46 |
| C201     | <b>591.56</b> | <b>591.56</b>  | <b>591.56</b> | <b>591.56</b>  | <b>591.56</b> | 702.00  |
| C202     | <b>591.56</b> | 611.08         | 731.92        | <b>591.56</b>  | 717.72        | 972.20  |
| C203     | 703.61        | 672.04         | 947.88        | <b>591.17</b>  | 815.07        | 952.34  |
| C204     | 845.87        | <b>614.29</b>  | 843.25        | 614.49         | 1041.75       | 1165.64 |
| C205     | 591.04        | <b>588.88</b>  | <b>588.88</b> | <b>588.88</b>  | 864.09        | 1142.46 |
| C206     | 591.04        | <b>588.49</b>  | 588.88        | 592.21         | 1049.76       | 1136.64 |
| C207     | 590.66        | 599.59         | 588.49        | <b>588.29</b>  | 866.26        | 1150.99 |
| C208     | 591.78        | <b>588.32</b>  | 588.49        | <b>588.32</b>  | 1114.23       | 1193.73 |
| R101     | 1679.88       | 1739.94        | 1826.81       | <b>1665.82</b> | 1745.75       | 1862.92 |
| R102     | 1549.73       | 1587.48        | 1867.04       | <b>1496.35</b> | 1573.84       | 1635.88 |
| R103     | 1345.68       | 1290.57        | 1492.75       | <b>1239.32</b> | 1404.65       | 1387.02 |
| R104     | 1236.84       | <b>1023.26</b> | 1332.23       | 1083.24        | 1198.89       | 1399.47 |
| R105     | 1431.06       | 1475.80        | 1586.26       | <b>1401.45</b> | 1681.82       | 1647.73 |
| R106     | 1411.71       | 1354.52        | 1578.61       | <b>1293.75</b> | 1482.91       | 1589.14 |
| R107     | 1194.01       | <b>1134.37</b> | 1409.11       | 1152.63        | 1347.52       | 1400.41 |
| R108     | 1213.64       | <b>986.04</b>  | 1041.63       | 1109.82        | 1453.64       | 1498.37 |
| R109     | 1482.54       | <b>1236.74</b> | 1277.88       | 1321.43        | 1654.23       | 1736.28 |
| R110     | 1541.16       | <b>1156.39</b> | 1322.30       | 1248.35        | 1597.65       | 1704.78 |
| R111     | 1334.36       | <b>1109.08</b> | 1251.19       | 1177.14        | 1436.62       | 1500.96 |
| R112     | 1364.65       | <b>1026.15</b> | 1096.14       | 1204.90        | 1762.56       | 1857.61 |
| R201     | 1369.60       | 1350.31        | 1364.46       | <b>1299.69</b> | 1458.50       | 1537.60 |
| R202     | 1225.49       | 1230.05        | 1351.66       | <b>1089.38</b> | 1289.67       | 1343.10 |
| R203     | 1070.34       | 1020.23        | 1156.93       | <b>965.59</b>  | 1126.23       | 1219.64 |
| R204     | 841.14        | 793.35         | 1044.59       | <b>777.63</b>  | 877.69        | 973.89  |
| R205     | 1156.97       | 1090.93        | 1145.36       | <b>1043.55</b> | 1431.66       | 1475.18 |
| R206     | 1064.29       | <b>996.21</b>  | 1134.98       | 1004.89        | 1250.65       | 1306.77 |

Table A9. Cont.

| Instance | CBSA-D  | ILS            | ACO     | GA             | GWO-D   | PSO-D   |
|----------|---------|----------------|---------|----------------|---------|---------|
| R207     | 947.65  | 924.93         | 1040.55 | <b>915.20</b>  | 1109.24 | 1196.07 |
| R208     | 773.89  | <b>765.78</b>  | 841.41  | 786.33         | 991.42  | 991.27  |
| R209     | 1120.61 | 1047.18        | 1135.08 | <b>940.34</b>  | 1251.09 | 1237.13 |
| R210     | 1095.62 | 1049.59        | 1280.06 | <b>964.44</b>  | 1192.62 | 1243.96 |
| R211     | 954.17  | <b>888.64</b>  | 938.82  | 932.21         | 1121.82 | 1237.73 |
| RC101    | 1714.64 | 1760.39        | 1772.67 | <b>1689.61</b> | 2002.70 | 2063.11 |
| RC102    | 1535.21 | 1558.25        | 1824.39 | <b>1518.11</b> | 1770.02 | 1864.71 |
| RC103    | 1517.44 | <b>1343.12</b> | 1668.97 | 1421.33        | 1633.92 | 1684.50 |
| RC104    | 1438.96 | <b>1165.87</b> | 1368.48 | 1316.40        | 1695.85 | 1817.08 |
| RC105    | 1650.89 | 1636.91        | 1711.38 | <b>1601.32</b> | 1858.47 | 1865.26 |
| RC106    | 1615.92 | <b>1485.96</b> | 1529.64 | 1536.73        | 1930.95 | 2076.53 |
| RC107    | 1536.79 | <b>1316.77</b> | 1379.71 | 1367.55        | 1851.40 | 2044.13 |
| RC108    | 1486.93 | <b>1200.11</b> | 1312.56 | 1356.19        | 1878.90 | 2038.44 |
| RC201    | 1489.30 | <b>1437.65</b> | 1527.76 | 1483.28        | 1920.37 | 1935.72 |
| RC202    | 1355.07 | 1292.14        | 1509.24 | <b>1266.96</b> | 1599.35 | 1648.92 |
| RC203    | 1101.84 | <b>1031.88</b> | 1201.31 | 1051.43        | 1340.70 | 1432.05 |
| RC204    | 875.98  | <b>848.25</b>  | 1070.39 | 855.02         | 1054.96 | 1151.81 |
| RC205    | 1425.41 | 1372.75        | 1544.09 | <b>1327.92</b> | 1668.48 | 1697.48 |
| RC206    | 1247.90 | 1225.57        | 1267.00 | <b>1210.47</b> | 1581.00 | 1845.25 |
| RC207    | 1223.79 | 1156.22        | 1368.90 | <b>1136.00</b> | 1435.26 | 1593.02 |
| RC208    | 1117.62 | <b>983.74</b>  | 1230.04 | 1069.75        | 1542.04 | 1516.69 |

Table A10. VRPTW per-instance results: variances over repeated runs.

| Instance | CBSA-D    | ILS     | ACO       | GA        | GWO-D     | PSO-D     |
|----------|-----------|---------|-----------|-----------|-----------|-----------|
| C101     | 0.00      | 0.00    | 0.00      | 1218.98   | 3198.26   | 12,321.62 |
| C102     | 6767.59   | 1590.58 | 8395.75   | 2460.84   | 5868.00   | 8612.98   |
| C103     | 12,362.32 | 1660.98 | 5094.39   | 5508.56   | 8430.08   | 13,940.66 |
| C104     | 8428.35   | 2901.83 | 4321.35   | 6020.72   | 2844.49   | 15,030.95 |
| C105     | 0.00      | 750.50  | 0.00      | 12,552.56 | 16,444.57 | 18,727.60 |
| C106     | 277.55    | 1013.48 | 1062.23   | 4849.80   | 10,022.60 | 18,029.23 |
| C107     | 0.00      | 1945.78 | 171.73    | 6172.00   | 9646.61   | 31,946.94 |
| C108     | 1550.18   | 3072.73 | 593.98    | 7084.81   | 6780.29   | 8646.71   |
| C109     | 8541.63   | 3136.70 | 319.91    | 18,765.54 | 12,877.36 | 2293.33   |
| C201     | 0.00      | 350.75  | 0.00      | 1007.00   | 6888.40   | 24,764.90 |
| C202     | 400.29    | 2396.03 | 3879.37   | 2662.06   | 6480.60   | 7609.59   |
| C203     | 1234.70   | 1001.42 | 19,087.17 | 4737.64   | 8688.33   | 7622.44   |
| C204     | 607.39    | 2652.04 | 37,585.67 | 5134.89   | 3272.75   | 3479.79   |
| C205     | 118.62    | 1946.31 | 0.00      | 1182.79   | 15,021.85 | 13,616.91 |
| C206     | 4847.02   | 1697.94 | 444.71    | 4699.59   | 8927.07   | 18,862.36 |
| C207     | 2710.66   | 3116.39 | 4649.00   | 3223.69   | 16,003.19 | 15,478.12 |
| C208     | 4399.90   | 2493.21 | 39.66     | 6884.50   | 7400.69   | 14,367.96 |
| R101     | 2153.57   | 317.11  | 4490.06   | 211.35    | 1919.13   | 1029.61   |
| R102     | 2625.46   | 3627.33 | 323.79    | 168.57    | 673.48    | 852.73    |
| R103     | 3948.29   | 604.81  | 18,137.95 | 533.96    | 574.51    | 1089.72   |
| R104     | 2776.79   | 138.68  | 2116.30   | 4045.70   | 5075.06   | 4043.55   |
| R105     | 11,139.81 | 1295.57 | 1425.29   | 1073.39   | 2912.23   | 4286.89   |
| R106     | 3176.95   | 1241.70 | 4297.06   | 1000.90   | 1487.30   | 499.61    |
| R107     | 6924.76   | 593.88  | 2989.03   | 1921.21   | 2910.20   | 2386.92   |
| R108     | 7705.74   | 634.96  | 15,122.87 | 4722.40   | 614.66    | 4213.25   |
| R109     | 8493.82   | 1093.50 | 6707.13   | 3491.08   | 2328.68   | 2578.65   |
| R110     | 2426.96   | 696.18  | 5897.40   | 5168.56   | 1718.52   | 3974.09   |
| R111     | 1828.43   | 1156.00 | 9032.67   | 3330.69   | 1612.90   | 3800.13   |
| R112     | 4221.94   | 393.14  | 3479.11   | 4949.23   | 694.68    | 1925.30   |
| R201     | 2265.01   | 447.31  | 20,769.34 | 873.04    | 2194.55   | 2457.44   |
| R202     | 2797.71   | 553.26  | 11,598.13 | 3135.58   | 1851.43   | 1730.26   |
| R203     | 1730.17   | 1376.50 | 26,628.39 | 1150.42   | 874.56    | 730.93    |
| R204     | 876.07    | 793.94  | 4720.37   | 1749.04   | 1523.48   | 2012.53   |

Table A10. Cont.

| Instance | CBSA-D    | ILS     | ACO       | GA      | GWO-D     | PSO-D     |
|----------|-----------|---------|-----------|---------|-----------|-----------|
| R205     | 1774.27   | 504.43  | 8214.89   | 4360.31 | 1559.13   | 1914.86   |
| R206     | 2817.06   | 1640.16 | 11,309.57 | 1195.10 | 2404.32   | 2371.12   |
| R207     | 1690.35   | 896.59  | 40,162.95 | 847.34  | 1686.57   | 743.04    |
| R208     | 1631.82   | 121.43  | 11,004.84 | 233.84  | 308.12    | 1028.14   |
| R209     | 3818.17   | 1206.24 | 29,745.79 | 2957.87 | 933.86    | 3518.99   |
| R210     | 2293.91   | 1475.09 | 25,178.10 | 2745.24 | 2710.17   | 2741.48   |
| R211     | 4640.23   | 1087.42 | 33,535.46 | 2663.53 | 3938.88   | 1703.57   |
| RC101    | 3757.73   | 661.26  | 3455.45   | 785.34  | 1361.03   | 2885.81   |
| RC102    | 13,067.01 | 1522.03 | 3967.50   | 2574.27 | 3249.12   | 6204.86   |
| RC103    | 6700.13   | 1216.65 | 12,868.39 | 1538.77 | 4319.65   | 8840.38   |
| RC104    | 5269.16   | 2209.36 | 9483.36   | 3919.23 | 4935.71   | 8937.86   |
| RC105    | 5488.55   | 2110.55 | 31,767.11 | 312.98  | 2585.03   | 2986.60   |
| RC106    | 2030.77   | 863.21  | 2857.31   | 5885.50 | 3291.40   | 1964.78   |
| RC107    | 4264.52   | 1060.45 | 13,651.61 | 9031.92 | 4362.10   | 6418.73   |
| RC108    | 2998.91   | 1485.45 | 5833.79   | 3788.50 | 6717.51   | 10,358.66 |
| RC201    | 2194.99   | 1374.12 | 33,402.28 | 4982.38 | 2633.87   | 4090.97   |
| RC202    | 4770.79   | 2346.67 | 5855.94   | 7195.67 | 2785.30   | 3790.10   |
| RC203    | 1732.13   | 2446.91 | 28,705.50 | 3494.00 | 3968.04   | 1532.47   |
| RC204    | 4840.30   | 1704.59 | 12,149.05 | 2555.98 | 3508.98   | 4543.64   |
| RC205    | 3361.06   | 334.67  | 40,086.85 | 6402.45 | 979.88    | 4682.38   |
| RC206    | 5032.05   | 1685.67 | 35,135.47 | 1295.66 | 11,535.81 | 2909.57   |
| RC207    | 4521.10   | 4700.07 | 33,737.83 | 4059.95 | 5812.01   | 2635.31   |
| RC208    | 1680.63   | 1566.39 | 11,735.76 | 7565.48 | 1530.99   | 3891.77   |

Table A11. Benchmark families as system diagnostics. The appendix organizes the main test sets by the type of decision structure they expose.

| Family                     | Structural Property                       | What it Diagnoses                                                        | CBSA Signal                                        |
|----------------------------|-------------------------------------------|--------------------------------------------------------------------------|----------------------------------------------------|
| CEC2014 unimodal           | Smooth local structure                    | Whether the policy can exploit a direct improvement path                 | Rank weighting is enough but not unique            |
| CEC2014 multimodal         | Multiple competing basins                 | Whether the policy can avoid over-committing to a single basin too early | Strongest relative advantage for CBSA-C            |
| CEC2014 hybrid/composition | Mixed interactions and ruggedness         | Whether the policy remains stable under nonseparable structure           | CBSA-C remains competitive but not dominant        |
| CVRP                       | Route adjacency with capacity constraints | Whether edge reuse is informative when routes are repeatedly similar     | CBSA-D is competitive against population baselines |
| VRPTW clustered            | Adjacency plus time feasibility           | Whether local structure survives a feasibility filter                    | CBSA-D is most effective here                      |
| VRPTW random/mixed         | Weakly reusable adjacency                 | Whether pairwise edge statistics are stable enough to guide search       | Local-search-heavy methods remain preferable       |

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