

Article

Maritime Inventory Routing for LNG Distribution from an FSRU Under Carbon Tax: A Column Generation Approach with a Diving Heuristic

Kanietova Aiymbubu ¹, Kang Chen ^{1,*} , Xiaoya An ¹, Xu Xin ^{2,*} , Jichun Li ³, Haiyan Dai ⁴ and Chunliang Liu ⁴

¹ School of Maritime Economics and Management, Dalian Maritime University, Dalian 116026, China; kanietovaaiym@gmail.com (K.A.); w2803317311@dlmu.edu.cn (X.A.)

² Department of Logistics and Maritime Studies, Faculty of Business, The Hong Kong Polytechnic University, Hung Hom, Hong Kong, China

³ China Waterborne Transport Research Institute, Beijing 100088, China; lj217@wti.ac.cn

⁴ DaLian CIMC Special Logistics Equipment Co., Ltd., Dalian 116000, China; haiyan.dai@cimc.com (H.D.); chunliang.liu@cimc.com (C.L.)

* Correspondence: chenkan@dlmu.edu.cn (K.C.); xu-david.xin@connect.polyu.hk (X.X.)

Highlights

Please indicate how your work links to systems science via your contributions to systems practice, theory, and or methodology.

- It models LNG distribution from an FSRU as an integrated maritime inventory routing system.
- It coordinates routing, inventory, berth use, speed, and backup supply through column generation.

What are the main findings and/or the implications of the main findings?

- Intermediate carbon taxes alter vessel deployment and reduce direct operational emissions.
- Carbon tax transition points depend on fleet, infrastructure, and time discretization.

Abstract

A floating storage and regasification unit (FSRU) provides flexible infrastructure for liquefied natural gas (LNG) import, storage, regasification, and regional distribution. Its operation requires coordinated decisions on vessel deployment, sailing speed, loading and discharge timing, terminal inventory, berth use, and backup pipeline supply, while carbon taxation changes the relative cost of alternative vessel and operating choices. This paper formulates this operational planning problem as a maritime inventory routing problem (MIRP) for an LNG distribution network centered on an FSRU. The MIRP formulation integrates FSRU and terminal inventories, heterogeneous vessel capacities, berth constraints, discrete speed choices, pipeline backup, and direct operational carbon costs. To solve the resulting large model, we use a column generation framework with a labeling algorithm for the pricing subproblem, dual stabilization, and a diving heuristic for integer recovery. Each column represents a complete vessel schedule, and the restricted master problem coordinates schedule selection, task coverage, vessel use, backup supply, and aggregate berth capacity. Computational experiments are conducted on calibrated synthetic instances. A complete enumeration benchmark for a small instance provides a diagnostic check, while larger experiments examine computation time, feasible incumbent values, time discretization, instance size, and sensitivity to carbon taxes. The results show nonlinear responses in cost and emissions: low tax rates produce limited operational changes, intermediate rates induce clearer shifts toward vessels with lower emissions at moderate cost increases, and very high stress test rates yield diminishing marginal emission reductions. These findings



Academic Editor: Wen-Chyuan Chiang

Received: 15 June 2026

Revised: 21 July 2026

Accepted: 24 July 2026

Published: 1 August 2026

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show how integrated scheduling can support coordinated FSRU planning under carbon costs, while the observed transition points remain specific to the tested instances.

Keywords: floating storage and regasification unit (FSRU); liquefied natural gas (LNG) distribution; carbon taxation; maritime inventory routing problem (MIRP); column generation; diving heuristic

1. Introduction

As the global energy system transitions toward energy sources with lower carbon emissions, liquefied natural gas (LNG) continues to serve as a transition fuel in many regional markets. Its role depends not only on production and procurement but also on reliable maritime distribution, in which dedicated vessels connect large supply sources with geographically dispersed demand centers [1]. Against this background, a floating storage and regasification unit (FSRU) has become a practical infrastructure option because it can generally be deployed more rapidly than an onshore terminal, requires lower initial investment, and provides flexible offshore storage and regasification capacity [2]. Operationally, an FSRU receives and stores LNG, regasifies part of the inventory for pipeline delivery, and may also serve as a hub for downstream maritime distribution by regional LNG vessels. This operating structure links inventory management, vessel routing and scheduling, berth utilization, and environmental regulation.

In many regional markets, the FSRU operations management department (hereafter referred to as the “decision maker”) is responsible for both FSRU operations and downstream LNG distribution to receiving terminals that differ in location, infrastructure, demand, and handling conditions [3,4]. The decision maker manages the FSRU as the central storage and loading facility and can deploy a set of LNG vessels that may be owned or chartered and differ in capacity, propulsion or fuel type, availability, charter cost, fuel consumption, and direct emissions. For each delivery requirement, the decision maker selects the vessel, sequences its delivery tasks, determines the loading and discharge times, and chooses discrete sailing speeds for the outbound and return legs. In each maritime delivery, a vessel loads LNG at the FSRU, transports it to the designated terminal, discharges the LNG to replenish terminal inventory, and returns to the FSRU for its next assignment. The operating plan must coordinate FSRU and terminal inventories, vessel availability and capacity, loading and discharge windows, and finite berth capacity. When maritime service is infeasible or uneconomic, backup pipeline supply can cover the corresponding demand at an explicit cost. The 2023 greenhouse gas (GHG) strategy of the International Maritime Organization (IMO) targets net zero GHG emissions from international shipping by or around 2050 [5]; carbon taxation can therefore change the preferred vessel and sailing speed through their effects on fuel use and direct emissions. Figure 1 illustrates this operating process and the associated planning decisions.

This problem belongs to the maritime inventory routing problem (MIRP) family, which integrates vessel routing and scheduling with inventory management over a planning horizon [6,7]. However, the MIRP for LNG distribution from an FSRU (FSRU-MIRP) considered here combines several features that are often treated separately: inventory at the offshore hub and receiving terminals, loading at the FSRU and discharge at the terminals, heterogeneous vessel capacities and costs, discrete speed choices, backup pipeline supply, and carbon tax payments. These decisions are strongly interdependent. A vessel schedule that is attractive from a routing perspective may be infeasible because of inventory or berth constraints, while a carbon tax may change both the preferred vessel type and the timing

of operations. The emissions calculation adopts a tank-to-wake (TTW) CO₂ accounting boundary; upstream fuel production, methane slip, and other well-to-wake (WTW) effects are outside the numerical model and are discussed as limitations.

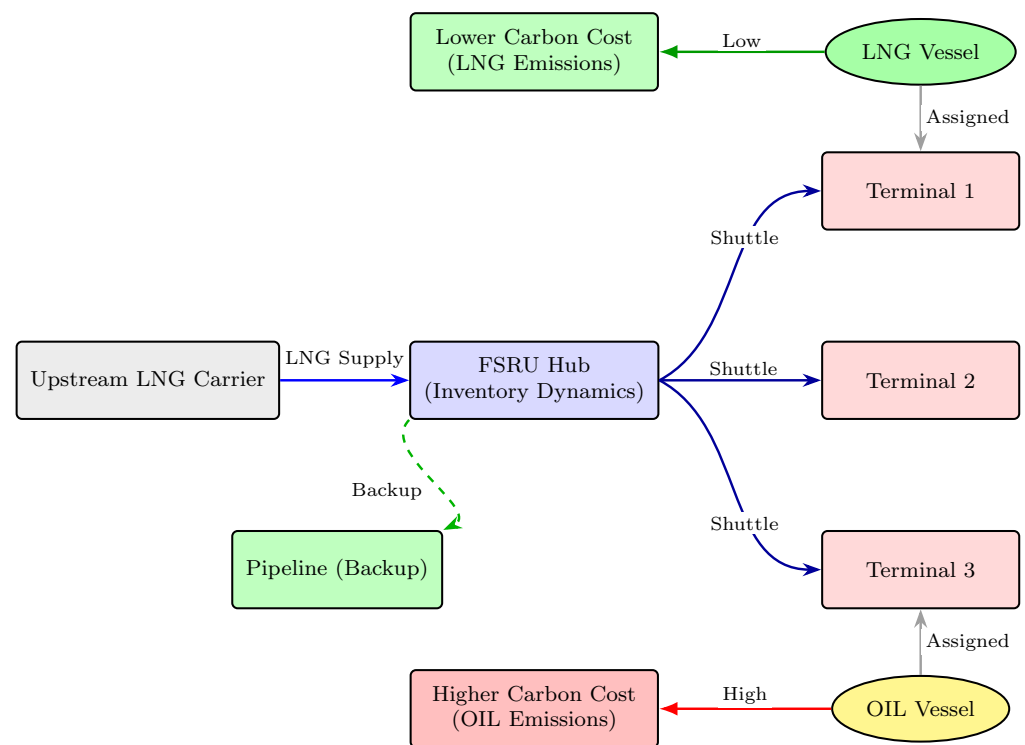


Figure 1. LNG distribution from an FSRU under carbon taxes.

To address this planning problem, we formulate the FSRU-MIRP with schedule columns. Each column represents a complete feasible schedule for one vessel, including its assigned tasks, loading and discharge times, speed choices, berth occupancy profile, operating cost, and direct carbon cost. The restricted master problem (RMP) coordinates schedule selection, vessel use, task coverage, backup pipeline supply, and aggregate berth capacity. The linear programming (LP) relaxation of the RMP is solved by column generation (CG) with a labeling algorithm for the pricing subproblem and dual stabilization, after which a diving heuristic is used to obtain a feasible integer schedule. Our computational analysis combines a complete enumeration benchmark for a small diagnostic instance with experiments on time discretization, instance size, and carbon tax levels.

Overall, the contributions of our paper are threefold. First, we formulate an FSRU-MIRP that jointly represents the principal operational decisions of regional LNG distribution. We model the FSRU as a storage and loading hub, track inventory at the FSRU and receiving terminals, limit berth occupancy at loading and discharge locations, accommodate heterogeneous vessel capacities and discrete sailing speeds, allow backup pipeline supply, and price direct TTW CO₂ emissions in the objective function. The formulation therefore evaluates operational feasibility, distribution cost, and direct carbon cost within the same planning model.

Second, we adapt an established decomposition framework to the operational structure of the FSRU-MIRP. CG, labeling, dual stabilization, and diving are standard algorithmic components, while the design specific to this problem concerns the vessel schedule columns, pricing resources, resource extension and dominance rules, and integer recovery procedure used to represent loading at the FSRU, terminal discharge, deliveries with feasible inventory, berth occupancy, heterogeneous vessel characteristics, and carbon costs that depend on

sailing speed. We present the method as a tailored implementation for the FSRU-MIRP, with the contribution centered on the schedule columns, pricing resources, resource extension and dominance rules, and integer recovery design.

Third, we conduct a computational analysis with clearly defined evidence and claim boundaries. We compare the small diagnostic instance with a complete enumeration benchmark and report computation times and feasible incumbent values for larger instances under different time discretizations and instance sizes. We then examine how carbon tax levels affect vessel deployment, total cost, and direct operational emissions. The results reveal nonlinear tradeoffs between cost and emissions. The larger-instance results are reported as feasible incumbents, and the observed transitions under carbon taxation are specific to the tested instances.

The remainder of the paper is organized as follows: Section 2 reviews maritime inventory routing, environmental considerations in maritime transportation, and decomposition methods, and identifies the research gaps addressed by our paper. Section 3 describes the LNG distribution system centered on an FSRU and the associated planning problem. Section 4 presents the assumptions, notation, and formulation, including the RMP and the conditions for column feasibility. Section 5 describes the CG procedure, labeling algorithm for the pricing subproblem, dual stabilization, diving heuristic, and search acceleration. Section 6 reports the diagnostic check for a small instance and the sensitivity analyses of time discretization, instance size, and carbon taxation. Section 7 discusses the findings, managerial implications, and limitations. Section 8 concludes the paper.

2. Literature Review

This section reviews the literature most closely related to our problem setting and solution approach. Section 2.1 discusses MIRPs, with particular attention to LNG logistics, heterogeneous vessel fleets, and time representation. Section 2.2 reviews environmental considerations in maritime transportation, Section 2.3 summarizes CG and B&P methods, and Section 2.4 identifies the research gaps addressed by our paper.

2.1. MIRPs

MIRPs integrate vessel routing and scheduling with inventory management at loading and discharge locations over a finite planning horizon. The main decisions typically include vessel routes, port call sequences and times, and loading and discharge quantities, while inventory levels at production and consumption ports must remain within prescribed bounds. Unlike standard vehicle routing problems, delivery quantities and visit times are endogenous and are coordinated with time varying production or consumption. Fagerholt et al. [8] placed MIRPs within the broader vendor-managed inventory (VMI) setting and emphasized that vessel heterogeneity, arising from differences in capacity, speed, cost, port accessibility, initial position, or availability, can materially affect the operating plan. In LNG logistics, these decisions are further complicated by cryogenic handling, boil-off gas (BOG), and specialized loading and discharge operations [8].

Several LNG inventory routing papers provide the methodological foundation for our paper. Grønhaug et al. [9] developed a B&P algorithm for an LNG inventory routing problem with a heterogeneous fleet, Rakke et al. [10] introduced formulations based on patterns, and Andersson et al. [11] proposed a decomposition algorithm for LNG inventory routing. Li et al. [12] considered a large LNG project using rolling horizon heuristics. These papers demonstrated the value of coordinating inventory, routing, and fleet decisions, but comparatively few papers have represented an FSRU simultaneously as a storage facility, regasification unit, and loading hub for downstream maritime distribution.

The choice of time representation is also central to MIRP modeling. Discrete time formulations facilitate the representation of time varying inventory and berth use, whereas continuous time formulations can provide greater temporal precision with fewer time indices [8]. Eide et al. [13] showed that integrating speed optimization and operations that depend on vessel load can generate substantial cost savings in the application studied. In VMI settings, Hameri et al. [14] reported that coordinated inventory policies can reduce inventory while maintaining service levels. Ghiami et al. [15] examined heterogeneous fleets and deteriorating inventory in inland LNG distribution, which have close operational analogues in maritime LNG transport. Uncertain sailing and port service times were addressed through stochastic programming [16], while dos Santos Diz et al. [17] applied robust optimization to crude oil transportation with a heterogeneous fleet. Foss et al. [18] considered nonmixable cargoes, and Cifuentes-Lobos et al. [19] proposed an enhanced mixed-integer programming (MIP) approach for an MIRP with multiple vessels in Antarctic logistics. Because realistically sized MIRPs are often difficult to solve directly with commercial MIP solvers, the literature also contains numerous customized heuristics, decomposition methods, and matheuristics for obtaining high quality feasible solutions [20].

2.2. Environmental Considerations in Maritime Transportation

Environmental regulation has made the tradeoff between operating cost and emissions increasingly important in maritime planning. Three research streams are particularly relevant to our paper: speed optimization, carbon pricing, and heterogeneous fleet deployment. The 2023 IMO GHG Strategy sets an ambition of achieving net-zero GHG emissions from international shipping by or around 2050 and identifies both fuel standards and an economic emissions pricing mechanism as candidate midterm measures [5]. The International Monetary Fund also examined global carbon pricing for aviation and shipping, including a price path aligned with net-zero targets that rises to approximately \$170/tCO₂ by 2035 [21]. Using computable general equilibrium models, Pereda et al. [22] estimated that a carbon tax of up to \$100/tCO₂ could reduce shipping emissions by approximately 3.6%, while deeper reductions may require higher prices or complementary measures.

At the operational level, sailing speed is a direct decision lever because fuel consumption is commonly approximated as a cubic function of speed. Eide et al. [13] showed that speed optimization can materially reduce operating costs and emissions, while Cariou [23] estimated that slow steaming contributed to an approximately 11% reduction in CO₂ emissions from container shipping after 2008. Other papers linked carbon regulation to routing, inventory, and supply chain coordination. Caballero-Morales and Martínez-Flores [24] showed that integrated inventory management under carbon constraints can reduce both operating costs and emissions. Liu et al. [25] examined emission reduction technologies in shipping supply chains under carbon taxes, and Tian et al. [26] proposed a carbon allocation policy for liner shipping. Xu et al. [27] analyzed the spatiotemporal evolution of carbon emissions in European Union coastal countries, while Wu and Li [28] studied the effects of carbon taxes on freight forwarding services and consumer rebates. Carbon costs were also incorporated into routing models [29,30], although these papers did not address the operating structure of LNG distribution from an FSRU. Li et al. [31] examined port investments that support operations with lower carbon emissions, complementing optimization at the vessel level.

Fleet heterogeneity adds another dimension to the policy question. Under the TTW accounting boundary adopted in many operational models, vessels fueled by LNG may have lower direct CO₂ emission factors than vessels fueled by conventional heavy fuel oil. However, methane slip and upstream fuel production remain important in a full WTW

assessment. Zhao et al. [32] and El Shar et al. [33] argued that fleet composition, speed, and fuel choice should be optimized jointly. Kwon et al. [34] showed that carbon trading mechanisms can reduce emissions in heterogeneous vehicle routing problems without prohibitive cost increases. Few papers, however, have embedded these decisions in an MIRP centered on an FSRU, where vessel deployment, sailing speed, inventory feasibility, berth availability, and carbon taxation interact within the same planning horizon.

2.3. CG and B&P Methods

The combination of routing decisions, inventory constraints, berth capacities, and environmental costs makes large MIRPs difficult to solve by direct enumeration or with a standard MIP solver. Decomposition methods are therefore widely used when complete vessel schedules can be represented as columns in a master problem [20]. CG, labeling algorithms for pricing problems, dual stabilization, and diving are established techniques. Our methodological contribution lies in implementing these components for a problem in which each column must represent loading at the FSRU, terminal discharge, sailing speed choices, inventory feasibility, berth occupancy, and carbon tax payments.

In CG, an RMP is solved over a limited column pool, and a pricing subproblem for each vessel generates additional schedules with negative reduced cost [35]. This structure was applied to LNG inventory routing problems by Grønhaug et al. [9], Rakke et al. [10], and Andersson et al. [11]. Related extensions addressed shipping with multiple products [36] and continuous time formulations [37]. Hybrid approaches were also used: El Shar et al. [33] integrated reinforcement learning into inventory decisions, and Heidari and Hemmati [38] developed an adaptive large neighborhood search matheuristic. These papers showed how decomposition and heuristic components could be combined. Our paper extends this line of work by jointly representing backup pipeline supply, berth coordination, heterogeneous vessel speeds, and direct carbon tax payments within vessel schedules for LNG distribution from an FSRU.

2.4. Research Gaps

The literature review reveals three gaps. First, few papers have jointly represented an FSRU as an inventory and loading hub while coordinating downstream terminal inventories, berth use, heterogeneous vessel deployment, sailing speed, backup pipeline supply, and direct carbon tax payments. Second, the closest paper, Chen et al. [3], emphasized uncertainty protection and developed a robust B&P method for a heterogeneous fleet. This leaves room for deterministic operational scenario analysis of time discretization, backup supply, and vessel and speed responses to carbon taxation. Third, although CG, labeling, stabilization, and diving are established techniques, their implementation requires columns and resource feasibility checks specific to the FSRU-MIRP. Table 1 summarizes these gaps and the scope of our paper. The solution approach therefore combines CG with a diving heuristic and is evaluated as a matheuristic.

These research gaps also define the scope of our paper. We integrate FSRU inventory, terminal handling, backup pipeline supply, vessel selection, sailing speed decisions, and direct carbon tax payments in one MIRP. The computational analysis examines how these decisions change with the carbon tax rate under a direct TTW accounting boundary, using feasible incumbent solutions for the larger instances and instance-specific transition ranges.

Table 1. Research gaps relative to the related literature.

Research Stream or Representative Paper	Main Planning Elements	Connection to Our Paper
LNG inventory routing with decomposition [9–11]	Vessel routing and scheduling, inventory coordination, heterogeneous fleets, and CG or solution methods based on patterns	Provides the main decomposition logic; our paper extends the operational setting to an FSRU hub with backup pipeline supply and carbon tax sensitivity.
Inventory routing through an FSRU hub [3]	Single FSRU hub, heterogeneous fleet, carbon pricing, uncertainty, and robust B&P	Closest application setting. Our paper focuses on deterministic scenario analysis, backup pipeline supply, time discretization, and feasible incumbent solutions obtained by CG and diving.
Environmental maritime routing [26,29,30]	Carbon regulation, routing costs, speed choice, and emissions accounting	Motivates the inclusion of carbon tax payments; our model links these costs with FSRU inventory, berth use, and downstream LNG delivery tasks.
MIRP and shipping matheuristics [20,38]	Hybrid mathematical programming and heuristic methods for large routing and inventory problems	Supports the use of decomposition and heuristics for integer recovery; our paper develops the operational representation for the FSRU distribution setting.

3. Problem Description

We consider an FSRU that serves a set of receiving terminals L . The FSRU and the receiving terminals together form the set of inventory nodes N , and the planning horizon is divided into a set of discrete time points T . At each node $n \in N$, the inventory level is monitored at every $t \in T$ and must remain between the lower and upper limits $\underline{q}_n$ and $\bar{q}_n$. The FSRU receives LNG from upstream supply, stores the product offshore, regasifies part of the inventory for pipeline delivery when required, and loads regional LNG vessels for downstream maritime distribution. In this way, the FSRU acts as both an inventory buffer and the loading hub for deliveries to geographically dispersed terminals.

Regional deliveries are represented by a set of tasks I , including the dummy initial task 0 and dummy final task e used to construct vessel schedules. Each actual task $i \in I$ has cargo volume d_i and is associated with one receiving terminal $l \in L$, as indicated by the binary parameter δ_{il} . Its admissible loading start window at the FSRU is $[\tilde{t}_i^s, \tilde{t}_i^e]$, and its admissible discharge start window is $[\tilde{t}_i^{ds}, \tilde{t}_i^{de}]$. Loading and discharge require t_i^{h1} and t_i^{h2} time units, respectively. Terminal l has n_l^b discharge berths, while the FSRU has n_0^b loading berths. If a task cannot be completed by maritime service within its operating windows or at an acceptable cost, it may be covered by backup pipeline supply at cost c_i^{pipe} . This option represents an external supply channel.

The set of available LNG vessels is denoted by V . Each vessel $v \in V$ has capacity w_v , earliest availability time $\tilde{t}_v^0$, a propulsion or fuel type, and charter cost coefficient c_v^{char} . Sailing speed is selected from the discrete set K . For task i , vessel v , and speed option $k \in K$, the outbound and return sailing times are $t_{vik}^{\text{or}1}$ and $t_{vik}^{\text{or}2}$, and the corresponding sailing costs, including operating and carbon tax payments, are $c_{vik}^{\text{r}1}$ and $c_{vik}^{\text{r}2}$. Vessel assignment and speed choice therefore determine sailing time, cost, and direct emissions and must be coordinated with loading at the FSRU, terminal discharge, inventory availability, and berth capacity.

For each vessel, a feasible schedule $p \in \Omega$ specifies an ordered sequence of delivery tasks, loading and discharge times, and outbound and return speed choices. The coefficient a_{ip} equals one if task i is included in schedule p , while s_{vp} equals one if schedule p is assigned to vessel v . The berth occupancy coefficients b_{ipt}^l and b_{ipt}^d indicate whether task i in schedule p occupies an FSRU loading berth or a terminal discharge berth at time t . The inventory

trajectory induced by schedule p is denoted by $q_{nt}(p)$, and its total operating cost is c_p^* . A schedule enters Ω only if it respects vessel availability and capacity, task time windows, sailing time relationships, inventory bounds, berth requirements, and continuity of vessel operations over the planning horizon.

At the planning level, the binary decision variable x_p selects schedule $p \in \Omega$, while the binary decision variable γ_i equals one when task i is covered by backup pipeline supply. Each actual delivery task must be covered exactly once by one of these alternatives. The selected schedules may use each vessel at most once and must jointly respect the finite berth capacities of the FSRU and receiving terminals. We minimize the total operating cost of the selected schedules and backup supply. Because c_p^* already includes charter cost, sailing costs at the selected speeds, port handling, and carbon tax payments, the formulation evaluates vessel assignment, task sequencing, loading and discharge timing, sailing speed, and carbon tax payments jointly during schedule construction. Inventory holding costs are excluded because storage arrangements are assumed to be governed by separate contracts, but all inventory trajectories must remain within their prescribed bounds. Table 2 summarizes how the main operating features enter the formulation.

Table 2. Operational features and their representation in the model.

Operating Feature	Representation in the Formulation
FSRU inventory and loading	The FSRU is the loading hub for vessel schedules. Loading intervals determine the FSRU berth occupancy coefficients b_{ipt}^l , and each generated schedule must satisfy the relevant inventory and loading time conditions.
Terminal inventory and discharge	Each delivery task specifies a cargo quantity, receiving terminal, discharge time window, and terminal berth occupancy coefficients b_{ipt}^d . Inventory and handling requirements are incorporated into schedule feasibility.
Heterogeneous vessel fleet	Each schedule is assigned to one vessel through s_{vp} . Vessel capacity w_v , availability, charter cost, operating costs at each sailing speed, and emission factors determine whether a task sequence is feasible and economical.
Sailing speed and emissions	The decision variables for discrete speed choices o_{vikp}^{r1} and o_{vikp}^{r2} determine sailing times, fuel consumption, direct CO ₂ emissions, and the carbon tax component of c_p^* .
Backup pipeline supply	The decision variable γ_i covers delivery task i through backup pipeline supply when it is not served by a selected vessel schedule. The corresponding cost c_i^{pipe} is included explicitly in the objective.

4. Model Construction

This section constructs the mathematical model. Section 4.1 presents the modeling assumptions and notation, while Section 4.2 introduces the schedule column formulation, including the RMP in Section 4.2.1 and the conditions for column feasibility in Section 4.2.2.

4.1. Notation and Assumptions

For modeling convenience, we adopt the following assumptions:

- (1) Demand, sailing times, port service times, and task time windows are deterministic over the planning horizon.
- (2) Each vessel performs at most one operation at a time, and each voyage uses one fuel mode.
- (3) The FSRU and receiving terminals have finite berth capacities.
- (4) Inventory holding costs are excluded, but inventory levels remain within prescribed bounds.
- (5) Backup pipeline supply covers a task externally and is not represented as a sailing leg.

- (6) Vessels select speeds from a discrete set, and fuel consumption the standard cubic relationship with speed [39].
- (7) The emissions accounting covers direct tank-to-wake CO₂ emissions and excludes upstream fuel production, methane slip, and other well-to-wake effects.

Table 3 summarizes the sets, parameters, and decision variables used in the formulation. Within each category, symbols follow their order of introduction in Section 3; decision variables used only in pricing and the large constant for temporal linking are introduced when the conditions for column feasibility are presented.

Table 3. Sets, parameters, and decision variables used in the model.

Notation	Description
Sets	
L	Set of receiving terminals (discharge ports)
N	Set of inventory nodes, including the FSRU and receiving terminals
T	Set of discrete time points in the planning horizon
I	Set of delivery tasks, including the dummy initial task 0 and dummy final task e
V	Set of available vessels
K	Set of discrete sailing speed options
Ω	Set of feasible schedules for one vessel (columns)
Parameters	
$q_n, \bar{q}_n$	Minimum and maximum inventory levels at node n
d_i	Cargo volume associated with delivery task i
δ_{il}	Binary parameter equal to 1 if task i is discharged at terminal l
$\tilde{t}_i^s, \tilde{t}_i^{le}$	Earliest and latest feasible loading start times for task i
$\tilde{t}_i^{ds}, \tilde{t}_i^{de}$	Earliest and latest feasible discharge start times for task i
t_i^{h1}, t_i^{h2}	Loading and discharge durations for task i , excluding waiting time
n_l^b, n_0^b	Numbers of berths at receiving terminal l and the FSRU, respectively
c_i^{pipe}	Cost of covering task i through backup pipeline supply
w_v	Capacity of vessel v
$\tilde{t}_v^0$	Earliest available time of vessel v
c_v^{char}	Charter cost coefficient of vessel v
$t_{vik}^{or1}, t_{vik}^{or2}$	Outbound and return sailing times of vessel v for task i at speed k
$c_{vik}^{r1}, c_{vik}^{r2}$	Outbound and return sailing costs for task i by vessel v at speed k , including operating and carbon tax payments
a_{ip}	Column coefficient equal to 1 if task i is included in schedule p
s_{vp}	Column coefficient equal to 1 if schedule p is assigned to vessel v
b_{ipt}^l, b_{ipt}^d	Column coefficients equal to 1 if task i occupies the FSRU loading berth or a terminal discharge berth at time t in schedule p
$q_{nt}(p)$	Inventory level at node n and time t induced by schedule p and the deterministic consumption profile
c_p^*	Total operational cost of schedule p , including charter, sailing at the selected speeds, port handling, and carbon tax payments
M	Sufficiently large constant used in the temporal linking constraints
Decision Variables	
x_p	Binary master decision variable equal to 1 if vessel schedule p is selected
γ_i	Binary master decision variable equal to 1 if task i is covered by backup pipeline supply
$\tilde{x}_{ijp}$	Binary decision variable in the pricing problem equal to 1 if task j immediately follows task i in schedule p
$\tilde{t}_{ip}^s$	Loading start time of task i in schedule p during pricing
t_{ip}^{w1}, t_{ip}^{w2}	Waiting times before loading and discharge for task i in schedule p
t_{ip}^{r1}, t_{ip}^{r2}	Outbound and return sailing durations for task i in schedule p
$\tilde{a}_{ivp}$	Auxiliary binary pricing variable linking task inclusion and vessel assignment
$o_{vikp}^{r1}, o_{vikp}^{r2}$	Binary decision variables in the pricing problem, equal to 1 if vessel v uses speed k on the outbound or return leg of task i in schedule p

4.2. Schedule Column Formulation

This section presents the schedule column formulation used in the computational analysis. In the RMP, each column p represents a feasible schedule for one vessel. Accordingly, a_{ip} , s_{vp} , b_{ipt}^l , b_{ipt}^d , and c_p^* are fixed column attributes, and the master problem decides which schedules to select. During pricing, these attributes are generated from the routing, timing, capacity, inventory, and speed decisions. This separation preserves a linear master formulation.

4.2.1. RMP

The RMP selects a set of feasible vessel schedules and uses a backup supply variable for any delivery task covered by pipeline. Equation (1) minimizes the cost of the selected schedules and backup pipeline supply:

$$\min \sum_{p \in \Omega} c_p^* x_p + \sum_{i \in I} c_i^{\text{pipe}} \gamma_i. \quad (1)$$

Constraint (2) requires each delivery task to be covered exactly once, either by one selected vessel schedule or by backup pipeline supply:

$$\sum_{p \in \Omega} a_{ip} x_p + \gamma_i = 1 \quad \forall i \in I. \quad (2)$$

Constraints (3)–(5) limit each vessel to at most one selected schedule and impose berth capacity in each period at the receiving terminals and the FSRU:

$$\sum_{p \in \Omega} s_{vp} x_p \leq 1 \quad \forall v \in V \quad (3)$$

$$\sum_{i \in I} \sum_{p \in \Omega} \delta_{il} b_{ipt}^d x_p \leq n_l^b \quad \forall l \in L, t \in T \quad (4)$$

$$\sum_{i \in I} \sum_{p \in \Omega} b_{ipt}^l x_p \leq n_0^b \quad \forall t \in T. \quad (5)$$

The binary restrictions $x_p \in \{0, 1\}$ and $\gamma_i \in \{0, 1\}$ complete the integer master problem. In the LP relaxation solved during CG, these binary restrictions are relaxed in the usual way. Equation (1) accounts for the selected vessel schedules and backup pipeline supply. Constraint (2) enforces task coverage, Constraint (3) limits each vessel to one selected schedule, and Constraints (4)–(5) enforce aggregate berth capacities at the receiving terminals and the FSRU. The detailed sequencing, timing, inventory, capacity, and speed conditions are enforced during CG.

4.2.2. Column Feasibility Conditions

A column is feasible only if the corresponding schedule can be operated by a single vessel within the planning horizon. Let $\tilde{x}_{ijp}$ equal one if task j immediately follows task i in schedule p . Constraints (6)–(9) construct a task sequence that starts at the dummy initial task 0, ends at the dummy final task e , satisfies flow conservation, and links task inclusion to the selected arcs:

$$\sum_{j \in I} \tilde{x}_{0jp} = 1 \quad \forall p \in \Omega \quad (6)$$

$$\sum_{i \in I} \tilde{x}_{iep} = 1 \quad \forall p \in \Omega \quad (7)$$

$$\sum_{j \in I} \tilde{x}_{ijp} = \sum_{j \in I} \tilde{x}_{jip} \quad \forall i \in I, p \in \Omega \quad (8)$$

$$a_{ip} = \sum_{j \in I} \tilde{x}_{ijp} \quad \forall i \in I, p \in \Omega. \quad (9)$$

Constraints (6) and (7) define the start and end of a vessel schedule, Constraint (8) maintains flow at each delivery task, and Constraint (9) defines whether task i belongs to schedule p .

Delivery tasks within a schedule must follow a feasible temporal sequence. Let $\tilde{t}_{ip}^s$ denote the scheduled loading start time of task i in schedule p , and let $\mathcal{M}$ be a sufficiently

large constant. If task j follows task i , Constraint (10) requires its start time to account for waiting before loading, loading, outbound sailing, waiting before discharge, discharge, and return sailing:

$$\tilde{t}_{jp}^s \geq \tilde{t}_{ip}^s + t_{ip}^{w1} + t_i^{h1} + t_{ip}^{r1} + t_{ip}^{w2} + t_i^{h2} + t_{ip}^{r2} - \mathcal{M}(1 - \tilde{x}_{ijp}). \quad (10)$$

Constraints (11)–(14) enforce vessel availability and the loading and discharge time windows. A selected task must start within its loading window, and the implied discharge start time must lie within the corresponding admissible window:

$$\tilde{t}_{ip}^s \geq \tilde{t}_i^s - \mathcal{M}(1 - a_{ip}) \quad \forall i \in I, p \in \Omega \quad (11)$$

$$\tilde{t}_{ip}^s \leq \tilde{t}_i^{le} + \mathcal{M}(1 - a_{ip}) \quad \forall i \in I, p \in \Omega \quad (12)$$

$$\tilde{t}_i^{ds} - \mathcal{M}(1 - a_{ip}) \leq \tilde{t}_{ip}^s + t_{ip}^{w1} + t_i^{h1} + t_{ip}^{r1} \quad \forall i \in I, p \in \Omega \quad (13)$$

$$\tilde{t}_{ip}^s + t_{ip}^{w1} + t_i^{h1} + t_{ip}^{r1} \leq \tilde{t}_i^{de} + \mathcal{M}(1 - a_{ip}) \quad \forall i \in I, p \in \Omega. \quad (14)$$

Constraints (11) and (12) bound the loading start time, whereas Constraints (13) and (14) bound the implied discharge start time. Inventory feasibility is checked before a generated schedule is admitted to the master problem. Let $q_{nt}(p)$ denote the inventory trajectory induced at node n by schedule p , accounting for scheduled vessel deliveries, deterministic consumption, and the backup supply convention. A schedule is retained only if Constraint (15) is satisfied:

$$\underline{q}_n \leq q_{nt}(p) \leq \bar{q}_n \quad \forall n \in N, t \in T. \quad (15)$$

This condition is imposed as a column admission rule because the RMP selects from schedules that are already feasible with respect to the modeled inventory bounds. Equation (16) defines the outbound and return sailing times as functions of the selected vessel and speed:

$$t_{ip}^{r1} = \sum_{v \in V} \sum_{k \in K} o_{vikp}^{r1} t_{vik}^{or1}, \quad t_{ip}^{r2} = \sum_{v \in V} \sum_{k \in K} o_{vikp}^{r2} t_{vik}^{or2} \quad \forall i \in I, p \in \Omega. \quad (16)$$

Constraints (17)–(20) link the selected speed to the vessel assigned to the generated schedule through $\tilde{a}_{ivp}$:

$$\sum_{v \in V} s_{vp} = 1 \quad \forall p \in \Omega \quad (17)$$

$$\sum_{k \in K} o_{vikp}^{r1} = \tilde{a}_{ivp} \quad \forall i \in I, v \in V, p \in \Omega \quad (18)$$

$$\sum_{k \in K} o_{vikp}^{r2} = \tilde{a}_{ivp} \quad \forall i \in I, v \in V, p \in \Omega \quad (19)$$

$$\tilde{a}_{ivp} \leq a_{ip}, \quad \tilde{a}_{ivp} \leq s_{vp}, \quad \tilde{a}_{ivp} \geq a_{ip} + s_{vp} - 1 \quad \forall i \in I, v \in V, p \in \Omega. \quad (20)$$

Constraint (17) assigns one vessel to each generated schedule, Constraints (18) and (19) select the outbound and return speeds, and Constraint (20) links vessel assignment and task inclusion.

Vessel capacity is enforced by Constraint (21) for each task included in a generated schedule:

$$\sum_{v \in V} w_v \tilde{a}_{ivp} \geq d_i a_{ip} \quad \forall i \in I, p \in \Omega. \quad (21)$$

The berth occupancy indicators b_{ipt}^l and b_{ipt}^d are derived from the loading and discharge intervals. Once a feasible schedule has been generated, its task incidence, vessel assignment, timing, speed, berth use, and cost attributes become fixed coefficients in the RMP. Because

the berth occupancy indicators are fixed column attributes, they are multiplied directly by the master decision variable x_p , and no additional bilinear linearization variables are required in the RMP.

5. Solution Methodology

The model is difficult to solve directly for practically sized instances because the number of feasible vessel schedules grows rapidly with the number of delivery tasks, time windows, vessels, and speed options. We therefore use a matheuristic based on CG in which each column represents a complete schedule for one vessel. A column records the selected delivery tasks, FSRU loading times, terminal discharge times, outbound and return sailing speeds, vessel capacity use, inventory feasibility, berth occupancy profile, operating cost, and direct CO₂ emissions.

The solution procedure has three phases. First, the LP relaxation of the RMP is solved over an initial column pool. Second, a pricing subproblem for each vessel generates additional feasible schedules with negative reduced cost. Third, when the final RMP solution is fractional, a diving heuristic fixes one fractional variable at a time and follows a single search path to obtain a feasible integer solution. We therefore classify the procedure as a CG-based matheuristic with diving. A global optimality certificate would require CG to be embedded in a complete branch-and-bound tree with global bounds and systematic node exploration.

The standard algorithmic components are CG, a labeling algorithm for the pricing subproblem, dual smoothing, and diving. The design specific to this problem lies in the column definition, resource extension functions, feasibility checks, and reduced cost calculations required by the FSRU-MIRP. Individual schedules incorporate FSRU and terminal inventory conditions, loading and discharge timing, vessel capacity, sailing speed choices, berth occupancy coefficients, and carbon tax payments. The RMP then coordinates the selected schedules, backup pipeline supply, and aggregate berth capacities. The algorithmic contribution is the integrated design tailored to the FSRU-MIRP.

Section 5.1 presents the CG procedure, Section 5.2 describes the labeling algorithm for the pricing subproblem, Section 5.3 explains integer recovery by diving, and Section 5.4 introduces stabilization, search acceleration, and the overall algorithm.

5.1. CG Procedure

CG starts from an initial pool of feasible schedules and solves the LP relaxation of the RMP. Let π_i , λ_v , μ_{lt} , and ν_t denote the dual variables associated with the task coverage Constraint (2), vessel use Constraint (3), terminal berth capacity Constraint (4), and FSRU berth capacity Constraint (5), respectively. For each vessel $v \in V$, the pricing subproblem searches for a schedule with negative reduced cost. The reduced cost balances the direct cost of a candidate schedule against the dual values of the tasks and shared resources it uses. Under the sign convention adopted for the RMP relaxation, the objective of the pricing subproblem for vessel v is

$$\min \left\{ \text{ScheduleCost} - \sum_{i \in I} \pi_i a_{ip} - \lambda_v - \sum_{i \in I} \sum_{t \in T} \sum_{l \in L} \mu_{lt} \delta_{il} b_{ipt}^d - \sum_{i \in I} \sum_{t \in T} \nu_t b_{ipt}^l \right\},$$

where

$$\text{ScheduleCost} = \sum_{i \in I} \left(\sum_{k \in K} c_{vik}^{r1} o_{vikp}^{r1} + \sum_{k \in K} c_{vik}^{r2} o_{vikp}^{r2} \right) + c_v^{\text{char}} (\tilde{t}_{ep}^s - \tilde{t}_{0p}^s).$$

The pricing problem is subject to the schedule feasibility Constraints (6)–(21) and the inventory admission Constraint (15). It is formulated as a shortest path problem with resource constraints on an expanded network whose nodes represent delivery tasks. A

labeling algorithm based on resource extension functions and dominance rules is used to solve the pricing problem. Each label records a partial task sequence, reduced cost, current time, vessel load, visited tasks, selected speeds, and the timing information needed to construct the FSRU loading and terminal discharge intervals. These resources are required because a sequence with a low sailing cost may still be infeasible owing to time window, capacity, inventory, or service time restrictions. The resulting berth occupancy profile is retained as a column attribute so that aggregate berth capacities can be enforced in the RMP.

5.2. Labeling Algorithm for the Pricing Subproblem

The pricing subproblem is an elementary shortest path problem with resource constraints (ESPPRC). It is elementary because each delivery task can be visited at most once in a vessel schedule, and it is resource-constrained because every path extension must satisfy vessel availability, loading and discharge time windows, vessel capacity, sailing and service times, inventory admissibility, and the discrete speed choices. A labeling algorithm is therefore suitable for the ESPPRC because it constructs feasible partial paths while carrying the resource states needed to evaluate subsequent extensions, and its dominance rules discard partial paths that cannot lead to a better complete schedule.

The labeling algorithm extends a partial schedule from the source node, which represents the vessel's initial state, to feasible successor tasks. A task extension is retained only if vessel capacity, FSRU loading time, terminal discharge time, outbound and return sailing times, and the associated cost and emissions remain feasible. A label ℓ stores the attributes required for feasibility and dominance tests: accumulated reduced cost (ℓ .cost), current time (ℓ .time), visited tasks (ℓ .visits), and the speed choices for the constructed sailing legs (ℓ .speed).

For two labels ℓ_1 and ℓ_2 at the same node, ℓ_1 dominates ℓ_2 if it is no worse in the relevant resource dimensions and strictly better in at least one dimension. The dominance rule is given by Equation (22):

$$\ell_1 \prec \ell_2 \quad \text{if} \quad \begin{cases} \ell_1.\text{cost} \leq \ell_2.\text{cost}, \\ \ell_1.\text{time} \leq \ell_2.\text{time}, \\ \ell_1.\text{visits} \subseteq \ell_2.\text{visits}, \end{cases} \quad (22)$$

with at least one strict inequality or proper inclusion. A dominated label can be discarded because it cannot lead to a better feasible completion than the label that dominates it.

5.3. Diving Heuristic for Integer Recovery

CG solves the LP relaxation of the RMP, and the schedule selection variables x_p may therefore be fractional. A diving heuristic is used to obtain a feasible integer solution. At each iteration, the heuristic selects a fractional variable x_p and fixes it to 1, prioritizing the variable whose value is closest to 1 and breaking ties by reduced cost or total schedule cost. The RMP is then updated to reflect the fixed task coverage, vessel use, and berth occupancy, and its LP relaxation is solved again. The procedure continues until a feasible integer solution is obtained or a stopping criterion is reached. The single sequence of variable fixings produces a feasible incumbent. Global optimality certification would require systematic exploration of alternative branches.

5.4. Stabilization and Search Acceleration

CG can suffer from unstable dual prices, especially when many columns have similar reduced costs. We therefore use a dual smoothing mechanism:

$$\bar{\pi}_i^{(k+1)} = \alpha \pi_i^{(k)} + (1 - \alpha) \pi_i^{(k+1)},$$

where $\bar{\pi}_i^{(k+1)}$ is the stabilized dual price for constraint i in iteration $k + 1$, $\pi_i^{(k)}$ is the dual price from the previous iteration, and $\pi_i^{(k+1)}$ is the raw dual price obtained from the current RMP solution. The parameter $\alpha \in [0, 1]$ controls the weight placed on historical dual information. We begin with a relatively high α (e.g., 0.8) and gradually decrease it as the dual sequence stabilizes. If the pricing subproblem fails to identify a new column with negative reduced cost, α is reduced to allow more responsive updates. Dual bounds are also imposed to limit oscillations. Dual smoothing serves as an integrated mechanism for improving convergence stability.

The implementation also permits compatible delivery tasks to be grouped to accelerate pricing. Tasks serving nearby terminals, with compatible time windows and short connecting voyages, may form promising partial sequences. Candidate groups guide label extensions in the pricing network, but a grouped sequence is retained only when the relevant capacity, timing, inventory, and cost conditions remain feasible. Tasks that cannot be combined remain available for individual extension. This mechanism reduces unpromising sequence evaluations without changing the feasibility requirements imposed on generated schedules. The computational experiments evaluate the integrated implementation; a separate ablation analysis of task grouping remains a direction for future research.

Together, these components form the overall solution procedure. A greedy initialization first constructs a feasible column pool while respecting the modeled inventory, vessel, timing, and berth conditions. The algorithm then alternates between solving the RMP and the pricing subproblems for individual vessels. After each RMP solution, the dual values are smoothed as described above, and each pricing subproblem is solved using the updated dual information. When no additional columns with negative reduced cost are found and the RMP solution remains fractional, the diving heuristic in Section 5.3 is applied. Algorithm 1 summarizes the procedure.

Algorithm 1 CG and diving heuristic

Require: Instance data (delivery tasks, vessel characteristics, port constraints, and carbon tax rate)

Ensure: Feasible integer solution satisfying the modeled operational constraints

```

1: Generate an initial feasible column pool
2: Initialize RMP with initial columns
3: Initialize dual bounds and smoothing parameter  $\alpha$ 
4: loop ▷ Diving loop
5:   repeat ▷ CG phase
6:     Solve RMP to obtain primal solution and dual values
7:     Apply dual stabilization using current  $\alpha$ 
8:     Update the dual bounds
9:     for each vessel  $v \in V$  do
10:      Solve the pricing subproblem using the labeling algorithm
11:      if columns with negative reduced cost are found then
12:        Filter dominated columns
13:        Add the remaining columns to the RMP
14:      end if
15:    end for
16:    Update  $\alpha$  based on convergence behavior
17:  until no columns with negative reduced cost or the iteration limit is reached
18:  if the solution is integer then
19:    break ▷ Integer solution found
20:  else
21:    Identify the set of fractional  $x_p$  variables
22:    Select the fractional  $x_p$  closest to 1
23:    Fix  $x_p = 1$  and update the RMP
24:  end if
25: end loop
  
```

6. Computational Experiments

This section evaluates the FSRU-MIRP formulation and the matheuristic based on CG. The computational analysis addresses three questions: whether the formulation produces schedules that satisfy the interacting inventory, vessel, time window, berth, and speed constraints; how time discretization and instance size affect computation time and the best feasible solution value; and how carbon tax scenarios affect vessel deployment and sailing speed decisions. A small instance is first used for detailed feasibility checks and comparison with a complete enumeration benchmark. The remaining experiments report feasible incumbent solutions for larger synthetic instances. The larger-instance results are feasible incumbent diagnostics, and global optimality remains uncertified. Specifically, Section 6.1 describes the data, test instances, and computational configuration; Section 6.2 examines schedule feasibility and solution quality using a small benchmark instance; and Section 6.3 analyzes the effects of time discretization, instance size, and carbon taxation.

6.1. Experimental Setup

6.1.1. Data Sources and Test Instances

The experiments use calibrated synthetic instances designed for transparent reporting and controlled sensitivity analysis. The instances are designed to preserve the scale and principal operational relationships of regional LNG distribution from an FSRU while allowing the data and results to be reported transparently. The network follows the distribution structure described in Section 3, with the FSRU as the loading hub and geographically dispersed receiving terminals as discharge locations. Delivery tasks, cargo volumes, and service windows are generated from calibrated patterns informed by industrial production data and regional LNG distribution practice. Cargo volumes and vessel capacities are aligned with representative regional LNG carrier sizes reported in industry sources [1]. The loading and discharge windows are calibrated to create nontrivial interactions among vessel availability, sailing time, berth use, and task sequencing.

The test fleet is heterogeneous. It includes vessels fueled by conventional heavy fuel oil, labeled OIL, and vessels fueled by LNG, labeled LNG. Under the TTW accounting boundary, the vessels fueled by LNG have lower calibrated direct CO₂ emission rates but higher charter costs. In the calibrated data, their charter costs are approximately 70–80% higher than those of comparable vessels using conventional fuel. Vessel capacities range from approximately 50,000 to 120,000 m³, and the vessels have different availability times, charter costs, operating costs that depend on sailing speed, and emission rates at the three discrete speed levels (fast, medium, and slow). The parameters reflect the standard cubic relationship between sailing speed and fuel consumption. In the calibrated dataset, an OIL vessel sailing at the highest speed emits approximately 60–70% more CO₂ per time period than a comparable vessel fueled by LNG. These values create internally consistent tradeoffs between cost and emissions and are specific to the calibrated vessel classes and regional setting. The main test instances differ in the number of delivery tasks. In an instance label such as *P30S6M8*, *P* denotes the number of time periods, *S* the number of available vessels, and *M* the number of delivery tasks.

6.1.2. Experimental Parameters and Configuration

The planning horizon is 10 days (240 h) and is represented using three time discretizations: $P = 30$ with 8 h periods, $P = 60$ with 4 h periods, and $P = 120$ with 2 h periods. The larger sensitivity instances use a fleet of six vessels and contain $M = 4, 8, 12, 16, 20, 24, 28$, or 32 delivery tasks. The separate small instance experiment in Section 6.2 uses three vessels and four tasks.

The scaled carbon tax parameter ranges from $\text{carbon} = 0.0$ to $\text{carbon} = 1.1$ in increments of 0.1, where one unit corresponds to 1000 USD/tCO₂. Thus, $\text{carbon} = 0.1$ corresponds to 100 USD/tCO₂, and $\text{carbon} = 0.2$ corresponds to 200 USD/tCO₂. The lower portion of the grid is comparable in magnitude to values discussed in international policy analyses; for example, one proposal for aviation and shipping considers a path rising to approximately 170 USD/tCO₂ by 2035 [21]. Higher values are included as stress test scenarios to reveal vessel deployment and speed responses under strong carbon cost signals. All computation time comparisons use the same implementation and stopping rules and describe the relative computational burden of the tested instances on the specified computing platform.

6.2. Small Instance Feasibility and Solution Quality Check

We first consider a small instance for which schedule feasibility can be inspected in detail and complete enumeration is computationally tractable. The instance covers a 10-day (240 h) planning horizon divided into 30 periods of 8 h. The network contains one FSRU loading berth and two receiving terminals, P1 and P2, each with one discharge berth, while a heterogeneous fleet of three vessels (Oil1, Oil2, and Oil3) serves four delivery tasks (m1–m4). The instance includes asymmetric sailing times, heterogeneous vessel capacities, distinct vessel availability times, and overlapping service windows. Loading at the FSRU requires two or three periods, whereas discharge at a receiving terminal requires one period.

For task m1, the outbound and return sailing times at the highest speed are five and four periods, respectively, representing voyage asymmetry caused, for example, by routing restrictions or prevailing currents. Oil1 has a capacity of 90,000 m³, while Oil2 and Oil3 have capacities of 50,000 and 60,000 m³, respectively. Tasks m1 and m2 share an early loading window, and tasks m3 and m4 share a later loading window, so the formulation must coordinate vessel assignment, task sequencing, sailing speed, and berth occupancy. Tables 4 and 5 report the task and vessel data. This instance with three vessels is separate from the P30S6M4 sensitivity instance, which uses the test fleet of six vessels; their objective values are therefore not directly comparable. The carbon tax rate is set to zero for this small instance check.

The resulting schedule assigns the four delivery tasks to the three vessels labeled OIL. Oil1 serves m1 at P1, loading during periods 5–8, sailing outbound during periods 8–15 at the lowest speed, discharging during periods 15–16, and returning during periods 16–21 at the medium speed. Oil2 first serves m2 at P1 and then m4 at P2. For m2, it loads during periods 3–5, sails outbound during periods 5–8 at the highest speed, discharges during periods 8–9, and returns during periods 9–12. For m4, it loads during periods 12–14, sails outbound during periods 14–19 at the medium speed, discharges during periods 19–20, and returns during periods 20–25. Oil3 serves m3 at P2 at the medium speed, loading during periods 14–16, sailing outbound during periods 16–21, discharging during periods 21–22, and returning during periods 22–27. Its charter period begins in period 14, which avoids unnecessary charter cost before the second loading window.

The loading and discharge operations satisfy the service windows and berth capacity constraints. The total costs are 167.22, 127.12, and 73.83 kUSD for Oil1, Oil2, and Oil3, respectively, yielding a total schedule cost of 368.17 kUSD, and the corresponding direct emissions are 398.25, 370.50, and 202.50 tCO₂. We also solved the small instance by complete enumeration under the discrete time assumptions and the data in Tables 4 and 5. The best enumerated solution has a total cost of 367.99 kUSD, so the solution obtained by the CG and diving procedure is 0.05% above this benchmark. This comparison provides a

direct solution quality check for the small instance; its benchmark scope is limited to that diagnostic case. Figures 2 and 3 show the vessel schedules and berth occupancy.

Table 4. Delivery task data for the small instance.

Task ID	Vol. (10 ³ m ³)	Load Window	Disch. Window	Load Time	Outbound Sailing Time			Return Sailing Time			Disch. Time	Disch. Port
					F	M	S	F	M	S		
m1	80	1–10	11–20	3 *	5	6	7	4	5	6	1	P1
m2	45	1–10	11–20	2 *	3 **	4 **	5 **	3 **	4 **	5 **	1	P1
m3	40	11–20	21–30	2 *	4 **	5 **	6 **	4 **	5 **	6 **	1	P2
m4	50	11–20	21–30	2 *	4 **	5 **	6 **	4 **	5 **	6 **	1	P2

Note: (1) Time windows, handling times, and sailing times are measured in 8 h periods. (2) F/M/S denotes fast/medium/slow speed. (3) *: Loading time is set within the range of 2–3 periods used in the instance design. (4) **: Sailing times for m2–m4 follow the symmetry assumption used in this instance.

Table 5. Vessel data for the small instance.

Vessel ID	Capacity (10 ³ m ³)	Available From (Period)	Charter Cost (kUSD/Period)	Operating Cost by Speed		
				(Fast)	(Medium)	(Slow)
Oil1	90	3	5.71	11.00	7.57	5.43
Oil2	50	2	2.57	5.40	3.80	2.86
Oil3	60	0	2.71	5.71	3.86	2.90

Note: (1) Capacity is measured in 10³ m³; availability is measured in 8 h periods. (2) Charter and operating costs are measured in kUSD per 8 h period. (3) Cost parameters are calibrated for the small instance.

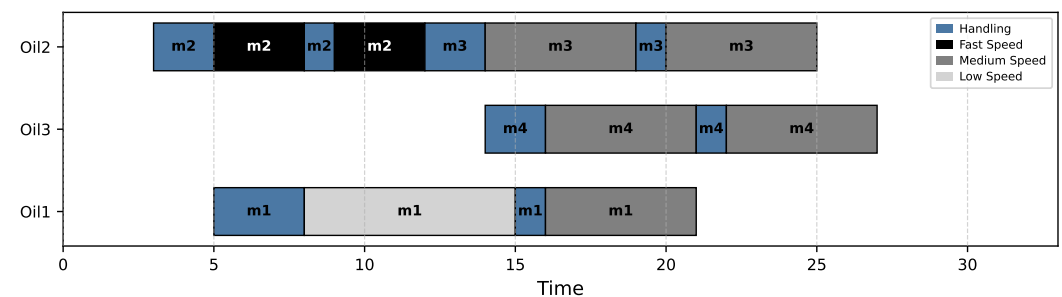


Figure 2. Vessel schedules for the small instance under zero carbon tax.

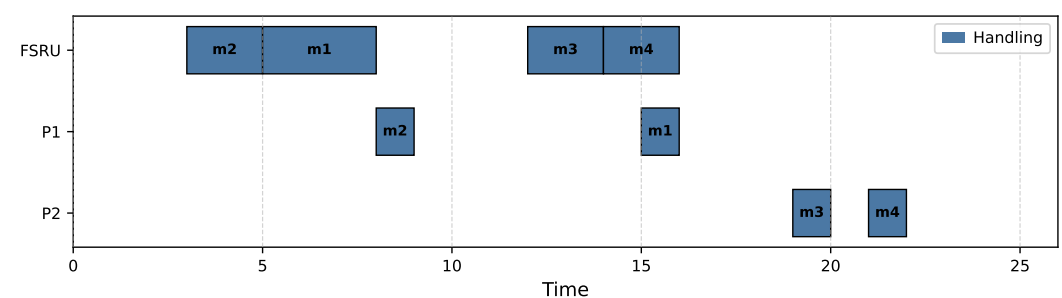


Figure 3. FSRU loading and terminal discharge operations for the small instance under zero carbon tax.

6.3. Sensitivity Analysis

The sensitivity analysis examines two sets of factors. Section 6.3.1 varies the time discretization and number of delivery tasks, and Section 6.3.2 varies the carbon tax rate for four instance sizes.

6.3.1. Impact of Time Discretization and Instance Size

We consider $P = 30, 60$, and 120 periods over the 10-day horizon, corresponding to time steps of 8, 4, and 2 h, and $M = 4, 8, 12, 16, 20, 24, 28$, and 32 delivery tasks. Table 6

reports computation times, and Table 7 reports the best feasible solution values obtained under zero carbon tax. A 6 h elapsed limit is imposed. Runs that reach this limit without producing a reported result are denoted by N/A; this notation records only the stopping outcome.

Table 6. Computation time (hh:mm:ss) by time discretization (P) and instance size (M).

Time Discretization (Periods, P)	Number of Delivery Tasks, M							
	$M = 4$	$M = 8$	$M = 12$	$M = 16$	$M = 20$	$M = 24$	$M = 28$	$M = 32$
$P = 30$ (8 h periods)	0:00:08	0:01:09	0:01:25	0:04:36	0:10:26	0:12:29	0:16:33	0:33:25
$P = 60$ (4 h periods)	0:01:07	0:06:41	0:18:20	0:32:28	1:05:44	1:29:32	2:51:45	4:39:04
$P = 120$ (2 h periods)	0:10:56	0:54:53	2:20:57	5:54:01	N/A	N/A	N/A	N/A

Note: N/A indicates runs exceeding a 6 h computation time limit.

Table 7. Best feasible solution cost (kUSD) by time discretization (P) and instance size (M) under zero carbon tax.

Time Discretization (Periods, P)	Number of Delivery Tasks, M							
	$M = 4$	$M = 8$	$M = 12$	$M = 16$	$M = 20$	$M = 24$	$M = 28$	$M = 32$
$P = 30$ (8 h periods)	329.82	676.53	1029.40	1399.54	1766.63	2141.64	2514.15	2898.52
$P = 60$ (4 h periods)	303.29	619.56	934.5	1259.96	1596.59	1937.94	2272.38	2634.69
$P = 120$ (2 h periods)	297.29	606.78	916.31	1242.08	N/A	N/A	N/A	N/A

Note: (1) Costs are reported in kUSD and correspond to final feasible integer solutions. (2) N/A denotes runs that reached the 6 h limit without a reported result.

Tables 6 and 7 show a clear tradeoff between temporal detail and computational effort. For a fixed M , the finer discretizations generally yield lower best feasible costs. For example, the reported value for $M = 8$ decreases from 676.53 kUSD at $P = 30$ to 606.78 kUSD at $P = 120$. A finer grid can represent waiting, berth occupancy, and task sequencing more precisely and may admit schedules that are unavailable under a coarser discretization. Because each value is obtained from a different discretized model, the differences describe discretization effects across distinct formulations; a common-formulation optimality gap is unavailable.

The improved temporal detail substantially increases computation time. For $M = 8$, the computation time rises from 0:01:09 at $P = 30$ to 0:54:53 at $P = 120$. For $M = 16$, it increases from 0:04:36 to 5:54:01. The effect of the time grid becomes more pronounced as the number of delivery tasks increases because both the RMP and the pricing networks become larger. At $P = 120$, no result is reported within the 6 h limit for $M \geq 20$. These results indicate that the time step length must be selected by balancing temporal detail against the available computational budget.

The values in Table 7 are final feasible incumbents obtained by the integrated implementation for the larger instances. Complete enumeration is used for the small instance in Section 6.2. Tables 6 and 7 provide descriptive evidence on computation time and best feasible solution values for the tested instances, while controlled comparisons with a commercial exact solver and alternative published heuristics remain future work.

6.3.2. Impact of Carbon Tax Rate

The carbon tax analysis uses four instance sizes at $P = 30$: $M = 8, 12, 16$, and 20. For each instance, the scaled tax parameter is varied in increments of 0.1, where one unit corresponds to 1000 USD/tCO₂. The reported vessel mix, total cost, and direct emissions are calculated from the final feasible schedules. The larger tax values are used as stress

test scenarios. Because results are not available for every high carbon tax run of the largest instance, Table 8 states the range included for each case.

Table 8. Available carbon tax sensitivity results at $P = 30$.

Instance	Available Carbon Range	Step	Interpretation
P30S6M8	0.0–1.1	0.1	Full range used for representative Gantt charts and aggregate curves.
P30S6M12	0.0–1.1	0.1	Full range available in the reported sensitivity batch.
P30S6M16	0.0–1.1	0.1	Full range available in the reported sensitivity batch.
P30S6M20	0.0–0.8	0.1	The aggregate uses this instance through 0.8.

Figures 4 and 5 summarize the fleet composition and responses in cost and emissions across the carbon tax scenarios. Figure 4 reports the share of vessels fueled by LNG in the selected fleet, while Figure 5 reports the average percentage changes in total cost and direct CO₂ emissions relative to the zero carbon tax baseline. At each tax level, the average includes only instances for which a result is available, so the $M = 20$ instance is excluded above carbon = 0.8.

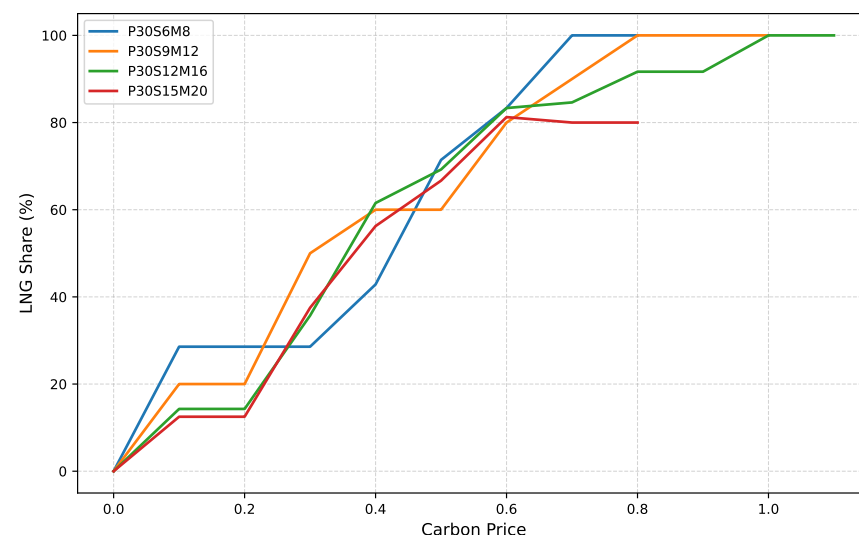


Figure 4. Share of vessels fueled by LNG in the selected fleet versus the scaled carbon tax parameter ($P = 30$, $M = 8, 12, 16, 20$).

The vessel share response is nonlinear and stepwise. Vessels fueled by LNG first enter the selected schedules at scaled tax levels of approximately 0.3–0.5, depending on the instance, and their share increases further over the 0.5–0.8 range. In the smaller instances for which the full tax grid is available, the selected fleet is dominated by vessels fueled by LNG at the highest tax levels. The curves indicate that intermediate carbon tax levels can change fleet composition and reduce direct emissions, while the highest stress test levels produce larger emissions reductions at substantially higher total cost. These transition points depend on the available vessels and scheduling constraints and are specific to the tested instances.

Figures 6–8 provide representative schedules for the P30S6M8 instance under the zero carbon tax baseline, an intermediate carbon tax scenario, and a high carbon tax stress test. The comparison focuses on changes in vessel assignment, task sequencing, and sailing

speed choice as the carbon cost signal strengthens. Together, the three schedules provide an operational view of the stepwise aggregate response reported in Figures 4 and 5.

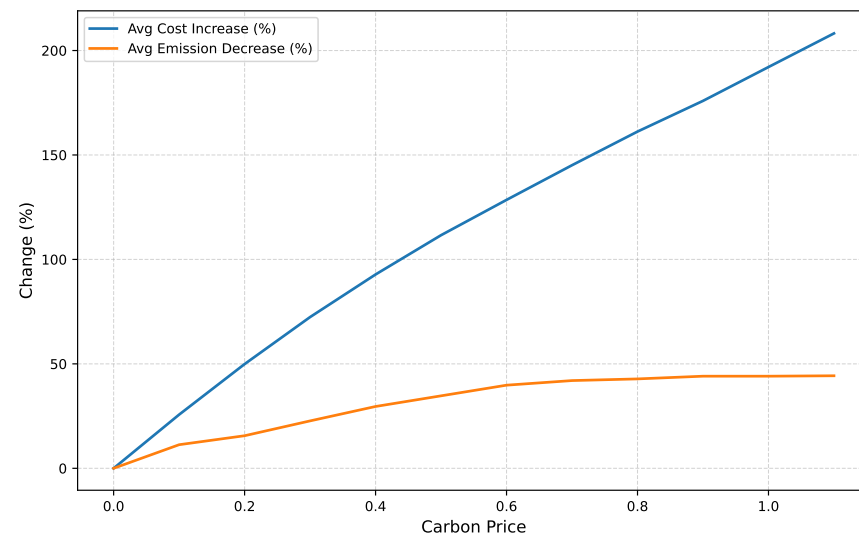


Figure 5. Average changes in total cost and CO₂ emissions relative to the zero carbon tax baseline, using the instances available at each carbon tax level.

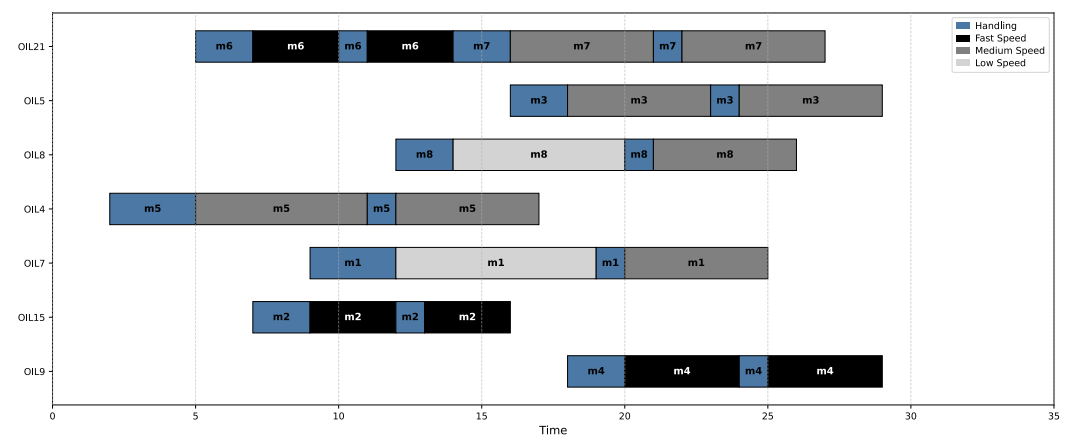


Figure 6. Representative schedule for *P30S6M8* at carbon = 0 (baseline).

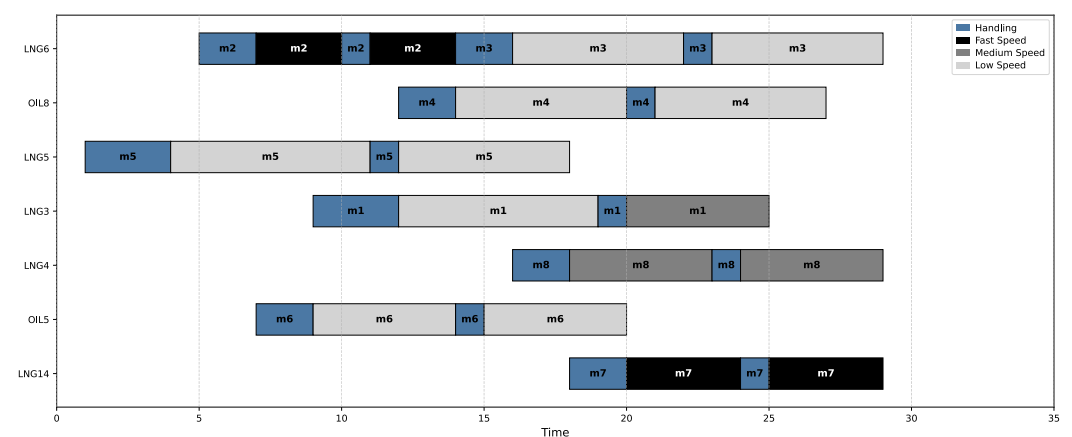


Figure 7. Representative schedule for *P30S6M8* under an intermediate carbon tax scenario (carbon = 0.5).

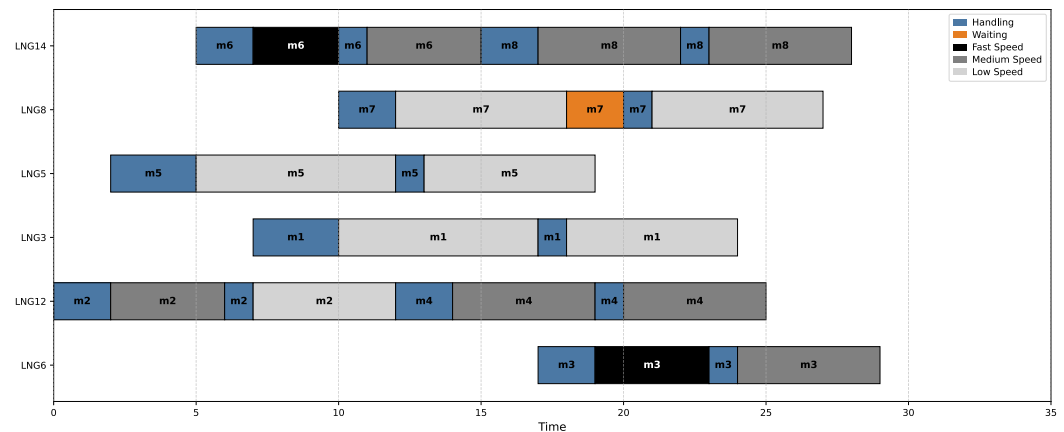


Figure 8. Representative schedule for *P30S6M8* under a high carbon tax stress test (carbon = 1.1).

Three conclusions emerge for the calibrated instances. Low carbon tax levels produce limited changes in the selected schedules, intermediate levels alter vessel deployment and reduce direct emissions at a moderate increase in total cost, and the highest stress test levels shift the available solutions toward greater use of vessels fueled by LNG and larger emissions reductions but with a substantial cost increase. These results are conditional on the $P = 30$ discretization. As shown in Section 6.3.1, a finer time grid can change feasible waiting, sequencing, and sailing speed patterns, so the carbon tax transition points may shift under $P = 60$ or $P = 120$, as well as under different fleet, infrastructure, cost, or emissions assumptions. The reported values are planning results specific to the tested instances.

7. Discussion

This section interprets the computational results and clarifies their practical boundaries. Section 7.1 discusses the key findings, Section 7.2 presents managerial and policy implications, and Section 7.3 summarizes the limitations.

7.1. Interpretation of Key Findings

The results show that carbon taxation affects vessel deployment and sailing speed decisions in a nonlinear manner that depends on the instance. In Figures 4 and 5, low tax levels produce limited changes in the selected schedules, intermediate levels lead to more frequent use of vessels fueled by LNG, and the highest stress test levels further reduce direct emissions at a substantial increase in total cost. Sailing speed reductions may occur before a change in vessel type, but the lowest speed is not always selected because time windows, berth availability, and charter duration remain binding considerations. The stepwise response arises from the discrete vessel and speed choices.

The time discretization analysis identifies a separate modeling tradeoff. Finer time steps can represent waiting, service, and sequencing decisions more precisely and produced lower best feasible costs in the reported runs. They also increase the sizes of the RMP and pricing networks, leading to sharply higher computation times as the number of delivery tasks grows. The time step length should therefore be selected according to the required temporal detail and available computational budget. Because the carbon tax experiments use $P = 30$, their transition points may change under a finer discretization.

The findings are consistent with the MIRP literature in showing that routing, scheduling, and inventory decisions are strongly interdependent. They also complement environmental routing papers by showing that the response to a carbon tax depends on the available vessels and the feasibility of the resulting schedules. Chen et al. [3] examined uncertainty protection with robust B&P, and our deterministic scenario analysis adds evi-

dence on backup pipeline supply, time discretization, vessel deployment, and direct carbon tax payments. Together, the two papers cover complementary planning settings. The experiments support the internal consistency and practical use of the integrated implementation on the tested instances; broader comparisons with exact B&P, commercial MIP solvers, and alternative matheuristics remain a useful direction for future research.

7.2. Managerial Implications

For the decision maker, carbon tax payments should be incorporated directly into schedule construction because the tested scenarios show changes in both vessel deployment and sailing speed. Intermediate tax levels produce clearer shifts toward vessels with lower direct emission factors, while the highest stress test levels yield additional emission reductions at substantially higher total cost. The decision maker can therefore use carbon price scenarios to evaluate vessel assignment, chartering, task sequencing, and speed choices under different cost signals. Maintaining access to vessels with different capacities, propulsion systems, costs, and availabilities also preserves scheduling flexibility. The time discretization results provide a separate implementation guideline: finer grids can improve the represented schedule and the best feasible cost, but their computational burden grows sharply with instance size.

For receiving terminal operators, the results underline the importance of reliable discharge windows and berth availability. These constraints affect which vessel and speed combinations remain feasible and can alter the sequence of deliveries. Receiving terminal operators can support coordinated planning by sharing updated berth calendars, handling capacities, service windows, and inventory information with the decision maker. Where recurrent berth conflicts occur, flexible discharge slots and operational coordination can improve the feasibility of the integrated schedule.

For the government, the stepwise fleet response indicates that the effect of carbon pricing depends on the available vessels, infrastructure, and operational constraints. In the calibrated instances, intermediate tax levels produce the clearest changes in vessel deployment, while the highest stress test levels achieve further TTW CO₂ reductions at a considerably higher cost. The tested transition ranges are instance specific, so carbon tax design should be assessed with regional fleet and infrastructure data. Carbon pricing can also be combined with support for alternative fuels, vessel technology, bunkering facilities, and port infrastructure to expand the operational options available for emission reduction.

7.3. Limitations

The results should be interpreted within the modeling and data boundaries. Demand, sailing times, port service times, and time windows are deterministic. In practice, weather and sea conditions, port congestion, vessel delays, equipment failures, and demand fluctuations can affect schedule feasibility and cost. These uncertainties could be incorporated through stochastic programming, robust optimization, or simulation optimization. The cost model includes chartering, operating costs that depend on sailing speed, port handling, backup supply, and carbon tax payments but omits detailed maintenance costs, terminal tariffs, BOG management, and fuel price uncertainty.

The operational representation is also simplified. Detailed berth assignment within a terminal, internal FSRU regasification scheduling, and complex fuel switching operations for vessels with dual fuel engines are outside the scope of the model. The emissions calculation uses direct TTW CO₂ factors for the modeled voyages and excludes upstream fuel production, methane slip, and other WTW effects. The interpretation is therefore limited to direct voyage emissions under the TTW boundary.

The computational analysis uses calibrated synthetic instances designed to preserve the principal operating relationships of the modeled system. The exact costs, emissions, vessel transitions, and carbon tax transition points depend on the assumed fleet, infrastructure, service windows, cost parameters, and time discretization. Application to another regional system would require recalibration with local data. Complete enumeration is limited to the small instance, while the larger instance values are feasible incumbents obtained by CG and diving. Controlled comparisons with commercial exact solvers and alternative published algorithms remain future work. The current scope is finite horizon operational planning; long-term fleet sizing, network design, and FSRU deployment are possible extensions.

8. Conclusions

This section summarizes the problem and solution approach, reports the main quantitative findings, and concludes with future research directions. Liquefied natural gas (LNG) distribution from a floating storage and regasification unit (FSRU) requires coordinated decisions on inventory, vessel routing and scheduling, berth use, backup supply, and carbon costs. We formulate this operational planning problem as a maritime inventory routing problem for LNG distribution from an FSRU (FSRU-MIRP) that integrates FSRU and terminal inventories, heterogeneous vessels, loading and discharge timing, discrete sailing speeds, backup pipeline supply, and direct tank-to-wake (TTW) carbon dioxide (CO₂) costs. Each feasible vessel schedule is represented as a column. We solve the linear programming (LP) relaxation of the restricted master problem (RMP) by column generation (CG) with a labeling algorithm for the pricing problem of each vessel and dual stabilization, and then apply a diving heuristic to obtain a feasible integer schedule. The diving procedure follows one search path, so the method is classified as a matheuristic based on CG. The solutions for larger instances are reported as feasible incumbents without a global optimality certificate.

For the diagnostic instance with four tasks, the matheuristic based on CG obtains a total cost of 368.17 thousand USD, compared with 367.99 thousand USD for the best solution from complete enumeration under the same discrete time assumptions. The difference is 0.05%, which provides a limited solution quality check for the small instance. The time discretization experiments further show that finer grids can improve the best feasible solution value but substantially increase computation time. Let M denote the number of delivery tasks and P the number of time periods. For $M = 8$, the best feasible cost decreases from 676.53 thousand USD at $P = 30$ to 606.78 thousand USD at $P = 120$, while the computation time increases from 1 min 9 s to 54 min 53 s. For $M = 16$, the computation time increases from 4 min 36 s to 5 h 54 min 1 s. At $P = 120$, no result is reported within the 6 h limit for instances with $M \geq 20$. These results quantify the tradeoff between temporal detail and computational effort.

The carbon tax experiments reveal a nonlinear and stepwise operational response. Vessels fueled by LNG first enter the selected schedules at scaled tax levels of approximately 0.3–0.5, equivalent to 300–500 USD per tonne of CO₂ under the adopted scaling, and their share increases further over the 0.5–0.8 range. Low tax levels produce limited changes in vessel deployment, intermediate levels induce clearer shifts toward vessels with lower direct emission factors, and the highest stress test levels produce additional direct emission reductions at substantially higher total cost. These transition points depend on the calibrated fleet, infrastructure, cost and emission parameters, and the $P = 30$ time discretization; they are therefore planning results specific to the tested instances.

Future research could incorporate uncertainty in demand, sailing time, port service time, congestion, weather, and vessel availability through stochastic programming, robust optimization, or simulation optimization. Other extensions could consider boil-off gas management, methane slip, well-to-wake emissions, detailed berth assignment, fuel

price uncertainty, dynamic backup pipeline supply, fleet composition over the long term, retrofitting, FSRU deployment, and network design. The solution procedure could also be strengthened through restricted master integer optimization, multiple start or limited backtracking diving, parallel pricing, and controlled comparisons with exact solvers and alternative maritime inventory routing algorithms.

Author Contributions: Conceptualization, K.A., K.C. and X.X.; methodology, K.C. and X.X.; software, K.A., X.A. and J.L.; validation, X.A., J.L., H.D. and C.L.; formal analysis, K.A., H.D. and C.L.; investigation, K.A., X.A. and K.C.; resources, X.X.; data curation, K.A. and X.A.; writing—original draft preparation, K.A., K.C. and X.X.; writing—review and editing, K.A., K.C., X.A., X.X., J.L., H.D. and C.L.; visualization, X.A., H.D. and C.L.; supervision, X.X.; project administration, X.X.; funding acquisition, K.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Social Science Fund of China, grant number 23VHQ015.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this paper are included herein. Further inquiries can be directed to the corresponding author.

Acknowledgments: During the preparation of this paper, the authors used ChatGPT, version 5.6, for the purposes of language polishing. The authors have reviewed and edited the output and take full responsibility for the content of this publication.

Conflicts of Interest: Jichun Li was employed by the organization “China Waterborne Transport Research Institute.” Haiyan Dai and Chunliang Liu were employed by the company “DaLian CIMC Special Logistics Equipment Co., Ltd.” The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest.

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