

Article

BiONet-TSE: A Bidirectional Operator-Guided Neural Network for Traffic State Estimation

Zhihao Li ^{1,2}, Baozhen Jiang ^{1,2}, Ruofei Wang ^{1,2} and Ye Li ^{1,2,*}

- ¹ The Key Laboratory of Road and Traffic Engineering, Ministry of Education, Tongji University, Shanghai 201804, China; 2410791@tongji.edu.cn (Z.L.); jbz@tongji.edu.cn (B.J.); 2410870@tongji.edu.cn (R.W.)
² College of Transportation Engineering, Tongji University, Shanghai 201804, China
* Correspondence: jamesli@tongji.edu.cn

Highlights

How this work links to systems science via your contributions to systems practice, theory, and/or methodology?

- This study addresses the reconstruction of hidden states in a dynamic transportation system, where speed and density evolve jointly but are only partially observed.
- BiONet-TSE offers a data-driven systems modeling approach that learns bidirectional relations among state variables and adaptively controls their influence through reliability gates.

What are the main findings and/or the implications of the main findings?

- Experiments on the NGSIM US101 dataset demonstrate that BiONet-TSE improves the reconstruction of both speed and density fields compared with NN and PINN baselines.
- The results indicate that learned operator relations can serve as effective auxiliary system constraints for traffic state estimation under sparse sensing conditions.

Abstract

Accurate traffic state estimation (TSE) is critical for the deployment of intelligent transportation systems. Physics-informed neural networks represent a significant approach for achieving TSE; however, their applicability is constrained by the availability and accuracy of physical information. To address this limitation, this study proposes a multi-task TSE method guided by operator learning (BiONet). The core architecture of BiONet is a multi-task network. Prior to network training, a bidirectional DeepONet is employed to pre-learn the mapping operators among various physical variables, thereby enabling a generalized representation of physical laws. Building upon this foundation, BiONet incorporates a reliability-gated mechanism to modulate the influence of the operator guidance, effectively preventing performance degradation caused by operator failure. Experimental results on real-world data demonstrate that BiONet can accurately reconstruct traffic states and exhibits enhanced robustness across varying levels of data penetration.



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Keywords: traffic state estimation; neural operator; physic-informed neural network; reliability-gated mechanism

1. Introduction

Intelligent transportation systems constitute a critical technological foundation for enhancing the efficiency of road operations, mitigating traffic congestion, ensuring traffic safety, and facilitating proactive traffic management. Advances in sensing technologies

have enabled more frequent monitoring of road traffic dynamics. Nevertheless, practical transportation systems often encounter significant sparsity and incompleteness in traffic observations due to factors such as sensor deployment costs, communication quality limitations, equipment malfunctions, and privacy concerns. Consequently, reconstructing a comprehensive traffic state field from sparse observational data has emerged as a fundamental challenge in traffic monitoring, congestion detection, state prediction, and active control within intelligent transportation systems [1].

Traffic state estimation (TSE) seeks to reconstruct continuous traffic state variables—such as speed, density, and flow—across a road network or segment based on limited observational data [2], as shown in Figure 1. However, traffic flow exhibits pronounced nonlinear, non-stationary, and multiphase characteristics, particularly during phenomena such as congestion formation, shock wave propagation, capacity reduction, and phase transitions. Traditional physical models often prove inadequate in accurately capturing these complex traffic dynamics, especially when the assumed traffic flow model deviates from the actual traffic regime [3]. In recent years, deep learning approaches have gained prominence in TSE and reconstruction tasks due to their strong capacity for modeling nonlinear relationships [4]. Nonetheless, purely data-driven models typically require extensive high-quality labeled datasets and lack explicit incorporation of traffic flow conservation laws and inter-variable coupling constraints. Under conditions of sparse observations, these models are susceptible to physical inconsistencies and limited generalizability.

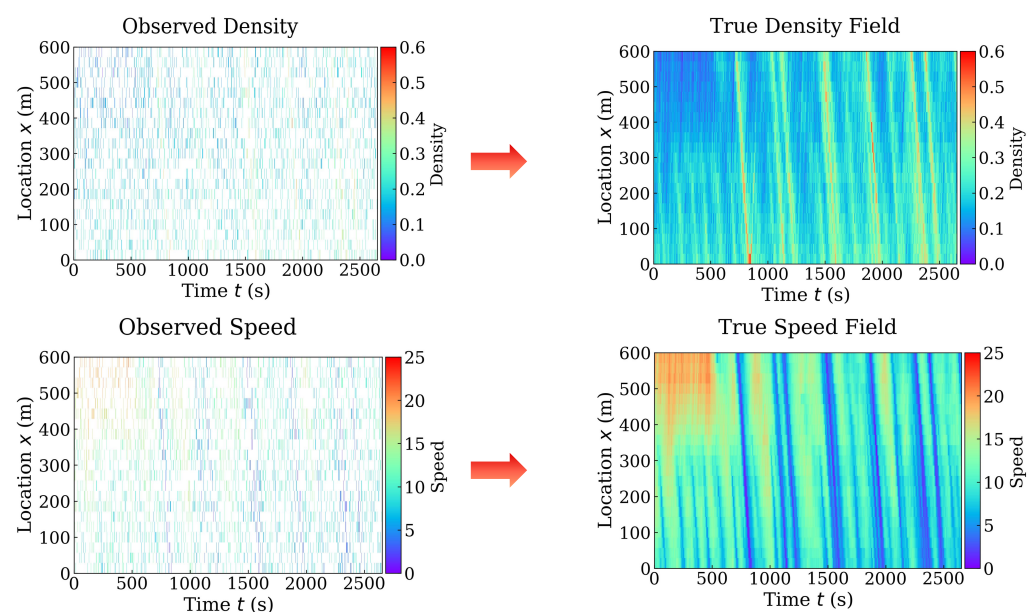


Figure 1. Schematic diagram of TSE.

In this context, physics-informed deep learning offers a promising avenue for TSE by integrating data fitting with mechanistic constraints [5,6]. Compared to conventional numerical methods, physics-informed neural networks (PINNs) can directly utilize spatiotemporal coordinates as inputs and traffic state variables as outputs, thereby naturally accommodating the estimation of continuous traffic state fields. Prior research has demonstrated that embedding classical traffic flow models—such as the Lighthill–Whitham–Richards (LWR) model, second-order traffic flow models, or fundamental diagram relationships—within neural network architectures can effectively mitigate the underdetermination arising from sparse observations and enhance the physical consistency and interpretability of estimation outcomes [7]. However, the performance of such methods still depends on the availability, validity, and calibration accuracy of the selected physical model. When the predefined traffic flow equation or fundamental diagram cannot fully represent the

true speed–density relationship under complex traffic conditions, the resulting physical regularization may become biased or even restrictive.

Neural operators represent a novel methodological framework for addressing this problem. Unlike traditional neural networks that learn mappings between finite-dimensional vectors, neural operators aim to learn mappings between function spaces, thereby directly characterizing operator relationships between input and output functions [8,9]. Techniques such as Deep Operator Networks (DeepONet) and Fourier Neural Operators have been extensively applied in parameterized partial differential equations, complex physical system modeling, and scientific machine learning [10,11]. In the domain of TSE, speed, density, and flow fields are inherently functions defined over spatiotemporal domains, with intrinsic coupling relationships that bear significant traffic flow implications. Recent studies have also attempted to reformulate TSE as an operator learning problem by using DeepONet to map sparse inputs to full spatiotemporal traffic state fields [12]. Accordingly, mappings from speed to density fields and vice versa can be conceptualized as neural operators between traffic state variables, which can be leveraged to guide the estimation process within coordinate-based neural networks.

Building upon these insights, this study proposes a neural operator-guided multi-task TSE framework. The framework employs a PINN-based coordinate network as its backbone, taking spatiotemporal coordinates as inputs and producing two traffic state variables—speed and density—as outputs. Distinct from conventional PINNs that rely on a predetermined traffic flow equation as the sole physical regularization term, this approach first utilizes DeepONet to learn bidirectional mapping operators between speed and density fields. During the training of the state estimation network, these pretrained operators are incorporated to formulate a speed–density consistency regularization term. This term ensures that the network outputs not only conform to observed data constraints but also maintain consistency with the learned coupling relationships between traffic variables. Moreover, to prevent undue influence from potentially unreliable operators, the framework introduces a reliability gating mechanism based on operator validation errors. This mechanism adaptively weights the regularization terms associated with each operator direction, enabling dynamic adjustment of operator guidance in accordance with the operators' respective credibility. The principal contributions of this work are summarized as follows:

- We develop a multi-task TSE framework guided by neural operators, which is capable of accurately predicting traffic states solely based on spatiotemporal coordinates.
- We employ neural operators to separately learn the speed–density and density–speed mappings, incorporating these learned operators as regularization terms within the neural network to enable adaptive learning of physical laws. Additionally, we introduce a reliability gating mechanism to prevent network degradation in cases where the operators fail.
- Experimental results on real-world datasets demonstrate that, compared to approaches lacking physical information or relying on deterministic physical information, the proposed framework more effectively guides the regularization of neural networks and exhibits superior adaptability across diverse scenarios.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 illustrates the framework of BiONet. Section 4 presents the experimental results and analysis. Section 5 concludes the paper and discusses future research directions.

2. Related Work

2.1. Traffic State Estimation

Early studies predominantly employed model-driven frameworks that integrated macroscopic traffic flow models with filtering algorithms to correct system states in real time. For instance, Wang and Papageorgiou proposed an extended Kalman filter approach that provides a general framework for real-time highway TSE, enabling online updates of density and speed conditioned on traffic flow models and detector observations [1]. However, the accuracy of model-driven methods depends not only on the model structure and parameters but also heavily on the sensor deployment configuration. Ahmed et al. highlighted that sensor placement significantly affects the precision and stability of real-time TSE [13]. Consequently, subsequent research expanded from fixed roadside detectors to cooperative sensing and probabilistic modeling. De Heij et al. treated infrastructure sensors and connected vehicles as collaborative nodes within a V2X network, employing distributed Kalman filtering, consensus mechanisms, and physical projection to reconstruct traffic states, demonstrating that distributed observations can alleviate observability issues caused by sparse fixed detectors [14]. Chen et al. embedded the LWR and ARZ traffic flow models into Gaussian process kernels via linearized differential operators, simultaneously providing state estimation and uncertainty calibration under low probe vehicle penetration rates, thereby illustrating the advantages of integrating physical models with probabilistic machine learning [15].

With the advancement of probe vehicles, mobile crowd sensing, video, trajectory, and multi-source detection data, data-driven approaches have increasingly become a pivotal direction in TSE. Xu et al. introduced GE-GAN, which employs generative adversarial learning to recover missing or sparse traffic states, indicating that generative models can learn traffic state distributions and complete unobserved regions [16]. Abdelraouf et al. constructed a Seq2Seq recurrent graph convolutional network (GCN) using sparse connected probe vehicle data to simultaneously estimate and predict network-level traffic volume and speed, demonstrating that graph convolution and sequential modeling can extract spatiotemporal correlations from low-penetration mobile observations [17]. Addressing more complex real-world observation environments, Zhang et al. developed a data-driven optimization framework by fusing heterogeneous sensor data; Xue et al. employed spatiotemporal Transformer graph neural networks to enhance low-cost estimation under sparse mobile crowd sensing; and Wu et al. utilized anisotropic Gaussian processes to interpolate traffic states from vehicle trajectories and quantify uncertainty [18–20]. Collectively, these studies indicate that the core challenge of data-driven TSE has shifted from pointwise prediction to reconstructing complete spatiotemporal state fields under sparse observations. To this end, Zhou et al. proposed the Dirichlet Graph Auto-Encoder, which enhances high-speed traffic estimation and cross-city transferability through feature propagation mechanisms tailored for directed traffic networks [21]; Liu et al. combined mask-aware diffusion models with physics-guided projection to reconstruct speed fields from extremely low observability data [22]. Concurrently, the expansion of observation modalities has broadened TSE applications to diverse traffic monitoring scenarios: Yajima et al. extended data assimilation techniques to distributed fiber optic sensing (DFOS) in low-speed congestion contexts; Gao et al. proposed a Kalman-based neural network for estimating queue lengths at signalized intersections; Yao et al. investigated the synergy between UAVs and ground sensors to reduce uncertainty in signalized network observations; and Inoue et al. selected additional measurement points based on traffic pattern coverage diversity to improve urban total traffic volume estimation [23–26]. Nevertheless, purely data-driven models often rely heavily on the training data distribution and may

exhibit physical inconsistencies when confronted with traffic wave propagation, boundary discontinuities, or cross-scenario generalization.

To bridge the gap between the strong assumptions of model-driven methods and the weak physical constraints of data-driven approaches, physics-informed TSE explicitly incorporates traffic flow conservation laws, fundamental diagrams, and nonlocal interactions into the learning process. Shi et al. introduced physics-informed deep learning combined with a fundamental diagram learner to jointly perform TSE, parameter identification, and fundamental diagram estimation, ensuring that the model adheres to traffic flow physical relationships even under limited observations [3]. Subsequently, a physics-informed spatiotemporal learning framework for urban TSE further validated the robustness enhancement provided by physical constraints under sparse observations [27]. Huang et al. integrated nonlocal traffic flow models into physics-informed neural networks (PINNs) to capture broader vehicle interactions during congestion propagation; Zhang et al. unified traffic flow models, fundamental diagram parameters, and state reconstruction within a differentiable optimization framework via computational graph methods [28]. Moving closer to realistic traffic flow stochasticity, Wang et al. proposed stochastic physics-informed deep learning by introducing stochastic fundamental diagrams into a knowledge-data fusion framework to characterize traffic flow scatter effects, thereby improving estimation stability under noise and sparse observations [29]. Overall, TSE is evolving from local state correction reliant on fixed detectors and deterministic models toward an integrated modeling paradigm that fuses multi-source observations, physical constraints, uncertainty quantification, and spatiotemporal field reconstruction.

2.2. Applications of Neural Operators in Traffic

Neural operators aim to learn mappings between function spaces, enabling direct prediction of continuous spatiotemporal solution functions from input functions. This characteristic makes them inherently suitable for addressing traffic flow dynamics governed by partial differential equations (PDEs). Representative architectures include DeepONet, which employs branch and trunk networks to encode input functions and output locations respectively, achieving universal approximation of nonlinear operators [8], and the Fourier Neural Operator (FNO), which parameterizes integral kernels in the frequency domain to efficiently learn input-to-solution mappings of parameterized PDEs [10].

In the traffic domain, neural operators initially focus on forward and inverse problems of macroscopic traffic flow PDEs and state estimation. Thodi et al. applied physics-informed FNO to learn solution operators of macroscopic traffic flow models, regularizing density field recovery and parameter inversion with LWR conservation laws, demonstrating that operator learning can accelerate PDE solving while preserving physical constraints [30]. Gao and Di compared DeepONet, FNO, and other neural operators on simulated and real traffic data for TSE, finding that neural operators adapt well to varying initial and boundary conditions and hold promise for real-time estimation [31]. Building on this, Li et al. formulated TSE as an operator learning problem mapping sparse observations to complete spatiotemporal state fields, embedding traffic flow conservation equations and fundamental diagram constraints during training in their PI-DeepONet framework [12]. Yu et al. further extended PI-DeepONet for real-time highway TSE by incorporating two-dimensional inputs, nonlinear extension layers, attention mechanisms, multi-input multi-output (MIMO) outputs, and parameter identification networks to accommodate larger-scale and more complex real-world road networks [32]. These studies illustrate that neural operators elevate TSE from discrete grid-based function approximation to learning mappings between continuous state fields, thereby naturally handling estimation challenges under varying sensor deployments, boundary conditions, and spatial resolutions.

Recent applications of neural operators in traffic have expanded from open-loop state reconstruction to online estimation, uncertainty quantification, and traffic control. Harting et al. proposed a closed-loop FNO traffic density observer that incorporates real-time sensor feedback to correct neural operator predictions, enhancing robustness under sparse detection and noisy conditions [33]. Rap and Das developed ON-Traffic, which utilizes Lagrangian mobile sensors for online traffic flow estimation and explicitly outputs uncertainty, suitable for handling sparse, irregular, and time-shifted observations [34]. In the control domain, Zhang et al. applied operator learning to the ARZ traffic model for mitigating stop-and-go congestion, accelerating backstepping kernel computations while maintaining closed-loop stability [35]. Nonetheless, traffic flow phenomena such as shock waves, congestion boundaries, and sharp density gradients impose stringent demands on neural operators.

2.3. Summary

Existing operator-based TSE methods predominantly focus on directly learning the mapping from input conditions or sparse observations to the complete traffic state field. Some approaches further incorporate explicit physical residuals or online feedback mechanisms. In contrast, BiONet employs a bidirectionally learned speed-density operator as a regularization module to guide a coordinate-based multitask state estimation network. Additionally, it adaptively modulates the influence of operators in different directions through a reliability gating mechanism.

3. Materials and Methods

3.1. Problem Formulation

TSE aims to reconstruct continuous traffic state fields from sparse and partially observable measurements. Let $\Omega = X \times T$ denote the spatiotemporal domain, where X and T represent the spatial and temporal domains, respectively. For each spatiotemporal coordinate $(x, t) \in \Omega$, the traffic state is characterized by the speed $v(x, t)$ and density $\rho(x, t)$. In practice, only a limited number of observations are available due to the sparse deployment of sensors and incomplete data acquisition. The observed dataset can be written as

$$\mathcal{D}_{obs} = \{(x_i, t_i, v_i, \rho_i)\}_{i=1}^{N_{obs}} \quad (1)$$

where N_{obs} is much smaller than the total number of spatiotemporal grid points. The objective is to learn a continuous estimator

$$f_{\theta} : (x, t) \mapsto [\hat{v}(x, t), \hat{\rho}(x, t)] \quad (2)$$

Based on this foundation, a comprehensive speed field and density field can be obtained by querying the entire spatiotemporal grid.

$$\begin{aligned} \hat{\mathbf{V}} &= \{\hat{v}(x_i, t_j)\}_{i=1, j=1}^{N_x, N_t} \\ \hat{\mathbf{P}} &= \{\hat{\rho}(x_i, t_j)\}_{i=1, j=1}^{N_x, N_t} \end{aligned} \quad (3)$$

3.2. Overall Framework

Figure 2 illustrates the overall architecture of the proposed BiONet-TSE. Specifically, BiONet-TSE comprises three primary components: a bidirectional DeepONet neural operator learning module, a multitask coordinate-based state estimation network, and a reliability-gated operator regularization module. Initially, DeepONet is employed to learn the bidirectional mapping operators between speed fields and density fields, capturing the transformation from speed to density and vice versa. This approach enables the model to

adaptively learn the intrinsic coupling relationships among traffic variables directly from data, rather than relying on a fixed underlying graph structure. Subsequently, a multitask coordinate neural network, termed StateNet, is constructed to serve as the principal traffic state estimator. StateNet takes normalized spatiotemporal coordinates as input and simultaneously outputs speed and density estimates. Finally, the two pretrained DeepONet operators are frozen and utilized to impose constraints on StateNet's outputs. Concretely, the speed output from StateNet is fed into the speed-to-density operator to produce the operator-induced density $\rho_{op}(x, t)$, while the density output is input into the density-to-speed operator to yield the operator-induced speed $v_{op}(x, t)$. Consistency constraints are then enforced between StateNet's own outputs and these operator-induced results. Recognizing that the neural operators in the two directions may exhibit varying degrees of reliability, a reliability gating mechanism is introduced. This mechanism dynamically adjusts the weighting of the operator loss terms based on reconstruction errors observed on a validation set. Operators with higher reliability impose stronger constraints, whereas those with lower reliability are automatically down-weighted, thereby mitigating the risk of unreliable operators adversely affecting the training of the main network. A detailed exposition of this framework follows.

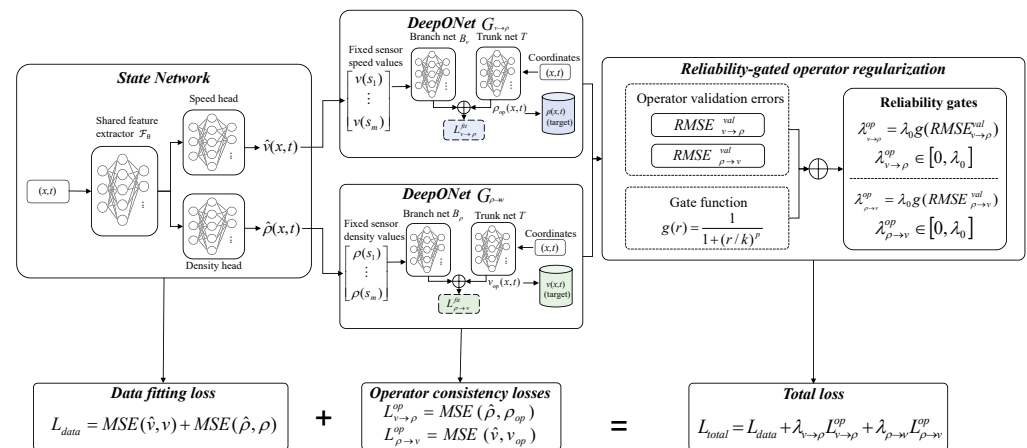


Figure 2. The overall architecture of the proposed BiONet-TSE.

3.3. Bidirectional DeepONet Neural Operator Learning

3.3.1. DeepONet Structure

DeepONet is utilized to approximate functional mappings. Each DeepONet architecture comprises two components: a branch network and a trunk network. The branch network encodes the values of the input function sampled at a predetermined set of sensor locations, while the trunk network encodes the query points corresponding to the output function.

Let the set of fixed sensor locations be defined as

$$\mathcal{S} = \{s_j = (x_j, t_j)\}_{j=1}^m, \quad (4)$$

Let m represent the total number of sensors. In the context of the speed-to-density operator, the input to the branch network consists of the speed field values measured at these predetermined sensor positions:

$$\mathbf{b}_v = [v(s_1), v(s_2), \dots, v(s_m)]. \quad (5)$$

The input to the trunk network is the spatiotemporal coordinate at which the output is queried:

$$\mathbf{y} = (x, t). \quad (6)$$

Accordingly, the output of the DeepONet for the speed-to-density mapping can be expressed as

$$\rho_{\text{op}}(x, t) = \mathcal{G}_{v \rightarrow \rho}(\mathbf{b}_v)(x, t). \quad (7)$$

In practice, the branch and trunk networks produce latent feature vectors of the same dimension, and the final output is obtained through their inner product:

$$\mathcal{G}_{v \rightarrow \rho}(\mathbf{b}_v)(x, t) = \sum_{k=1}^p B_k^v(\mathbf{b}_v) T_k(x, t) + b. \quad (8)$$

In this context, $B_k^v(\cdot)$ represents the k -th element of the feature vector generated by the speed branch network, while $T_k(\cdot)$ corresponds to the k -th element of the feature vector produced by the trunk network. Here, p denotes the dimensionality of the latent feature space, and b signifies a trainable bias parameter.

Analogously, for the density-to-speed operator, the input to the branch network comprises the values of the density field measured at predetermined sensor locations:

$$\mathbf{b}_\rho = [\rho(s_1), \rho(s_2), \dots, \rho(s_m)]. \quad (9)$$

The corresponding operator output is

$$v_{\text{op}}(x, t) = \mathcal{G}_{\rho \rightarrow v}(\mathbf{b}_\rho)(x, t). \quad (10)$$

3.3.2. Training of Speed to Density Operator

The speed-to-density operator is trained utilizing a dataset specifically designed for operator fitting. For any given query point (x_i, t_i) , the input to the branch network consists of the fixed speed-sensor vector $\mathbf{b}_v$, whereas the associated supervisory target corresponds to the density value ρ_i at that particular query point. The training loss is formulated as follows:

$$\mathcal{L}_{v \rightarrow \rho}^{\text{fit}} = \frac{1}{N_{\text{op}}} \sum_{i=1}^{N_{\text{op}}} |\mathcal{G}_{v \rightarrow \rho}(\mathbf{b}_v)(x_i, t_i) - \rho_i|^2. \quad (11)$$

Following the training phase, the reconstruction performance of the operator is assessed using an independent operator-validation dataset. The associated validation error is defined as follows:

$$e_{v \rightarrow \rho}^{\text{val}} = \text{RMSE}(\mathcal{G}_{v \rightarrow \rho}(\mathbf{b}_v)(x, t), \rho(x, t)). \quad (12)$$

This validation error is subsequently used to compute the reliability weight of the speed-to-density operator.

3.3.3. Training of Density to Speed Operator

The density-to-speed operator is trained in a manner analogous to that of the speed-to-density operator. Its branch-network input is the density-sensor vector $\mathbf{b}_\rho$, and the supervision target is the speed value v_i at the corresponding query point. The training loss is defined as

$$\mathcal{L}_{\rho \rightarrow v}^{\text{fit}} = \frac{1}{N_{\text{op}}} \sum_{i=1}^{N_{\text{op}}} |\mathcal{G}_{\rho \rightarrow v}(\mathbf{b}_\rho)(x_i, t_i) - v_i|^2. \quad (13)$$

The corresponding validation error is defined as

$$e_{\rho \rightarrow v}^{\text{val}} = \text{RMSE}(\mathcal{G}_{\rho \rightarrow v}(\mathbf{b}_{\rho})(x, t), v(x, t)). \quad (14)$$

Upon completion of the training process for the two DeepONet operators, their network parameters are fixed. These trained operators are then integrated as regularization components within the subsequent training phase of the StateNet model.

3.4. Multi Task Coordinate State Estimation Network

StateNet serves as the primary state estimation network in this study. Its input consists of the normalized spatiotemporal coordinates (x, t) , and its outputs are the estimated speed and density:

$$[\hat{v}(x, t), \hat{\rho}(x, t)] = f_{\theta}(x, t). \quad (15)$$

The network consists of a shared feature extraction module and two task-specific output heads. The shared feature extractor first maps the input coordinates into a latent representation:

$$\mathbf{h}(x, t) = F_{\theta_s}(x, t), \quad (16)$$

where θ_s denotes the parameters of the shared feature extraction module. Subsequently, the speed and density output heads generate the corresponding state variables:

$$\hat{v}(x, t) = H_{\theta_v}^v(\mathbf{h}(x, t)), \quad (17)$$

$$\hat{\rho}(x, t) = H_{\theta_{\rho}}^{\rho}(\mathbf{h}(x, t)). \quad (18)$$

This multi-task architecture facilitates the sharing of a unified spatiotemporal latent representation between speed and density, while preserving the distinct output features specific to each task. In contrast to training two separate networks independently, this design is more effective in learning a cohesive representation of the interrelated state fields. For the observation points extracted from the state-network fitting dataset, the data-fitting loss is formulated as follows:

$$\mathcal{L}_{\text{data}} = \mathcal{L}_v + \mathcal{L}_{\rho}. \quad (19)$$

The speed loss is given by

$$\mathcal{L}_v = \frac{1}{N_s} \sum_{i=1}^{N_s} |\hat{v}(x_i, t_i) - v_i|^2, \quad (20)$$

whereas the density loss is defined as

$$\mathcal{L}_{\rho} = \frac{1}{N_s} \sum_{i=1}^{N_s} |\hat{\rho}(x_i, t_i) - \rho_i|^2. \quad (21)$$

Here, N_s denotes the number of state-training points sampled in each mini-batch.

3.5. Operator Guided Consistency Regularization

Following the pretraining of the two DeepONet operators, these models are utilized to ensure functional consistency between the speed and density outputs produced by StateNet. In contrast to the operator pretraining phase, during the training of StateNet, the inputs to the DeepONet branch networks are not the actual speed or density measurements obtained from the sensor locations. Rather, they consist of the predictions generated by StateNet at these fixed sensor positions.

For the fixed sensor set $\mathcal{S}$, StateNet first predicts the speed and density values at these sensor locations:

$$\hat{\mathbf{b}}_v = [\hat{v}(s_1), \hat{v}(s_2), \dots, \hat{v}(s_m)]. \quad (22)$$

$$\hat{\mathbf{b}}_\rho = [\hat{\rho}(s_1), \hat{\rho}(s_2), \dots, \hat{\rho}(s_m)]. \quad (23)$$

The predicted speed sensor vector is then fed into the pretrained speed-to-density operator to obtain the operator-induced density:

$$\rho_{\text{op}}(x, t) = \mathcal{G}_{v \rightarrow \rho}(\hat{\mathbf{b}}_v)(x, t). \quad (24)$$

Similarly, the predicted density sensor vector is fed into the pretrained density-to-speed operator to obtain the operator-induced speed:

$$v_{\text{op}}(x, t) = \mathcal{G}_{\rho \rightarrow v}(\hat{\mathbf{b}}_\rho)(x, t). \quad (25)$$

Accordingly, two directional operator-consistency losses are constructed:

$$\mathcal{L}_{v \rightarrow \rho}^{\text{op}} = \frac{1}{N_c} \sum_{i=1}^{N_c} |\hat{\rho}(x_i, t_i) - \rho_{\text{op}}(x_i, t_i)|^2. \quad (26)$$

$$\mathcal{L}_{\rho \rightarrow v}^{\text{op}} = \frac{1}{N_c} \sum_{i=1}^{N_c} |\hat{v}(x_i, t_i) - v_{\text{op}}(x_i, t_i)|^2. \quad (27)$$

In this context, N_c represents the quantity of spatiotemporal points employed to assess the operator-consistency loss functions.

This regularization approach diverges from traditional physics-informed neural networks, which typically derive physics residuals directly from a predefined traffic-flow equation. Instead of compelling the model to adhere to a specific analytical traffic-flow formulation, the proposed method imposes constraints on the speed and density outputs of StateNet to ensure consistency with the bidirectional mappings inferred from empirical data. As a result, this regularization technique offers greater flexibility in capturing the underlying coupling relationships among traffic state variables present within the target dataset.

3.6. Adaptive Weighting for Reliable Gating

Given that the two neural operators may demonstrate varying levels of reconstruction performance, applying equal weights to their respective operator-consistency loss terms could result in an unreliable operator disproportionately impacting the training of StateNet. To mitigate this concern, a reliability-gating mechanism is proposed. This mechanism independently calculates the reliability scores of the two operators based on their validation error metrics.

The reliability scores of the speed-to-density and density-to-speed operators are defined as

$$r_{v \rightarrow \rho} = g(e_{v \rightarrow \rho}^{\text{val}}). \quad (28)$$

$$r_{\rho \rightarrow v} = g(e_{\rho \rightarrow v}^{\text{val}}). \quad (29)$$

Here, $g(\cdot)$ is a monotonically decreasing gating function. In this study, the following bounded gating function is adopted:

$$g(e) = \frac{1}{1 + \left(\frac{e}{\tau}\right)^\gamma}. \quad (30)$$

In this context, τ represents the reliability threshold, while γ serves as a hyperparameter that modulates the sensitivity of the gating function. A low operator validation error corresponds to a high reliability score; conversely, a high validation error results in a reduced reliability score.

Subsequently, the weights assigned to the two directional operator-consistency loss terms are formulated as follows:

$$\lambda_{v \rightarrow \rho} = \lambda_0 r_{v \rightarrow \rho}. \quad (31)$$

$$\lambda_{\rho \rightarrow v} = \lambda_0 r_{\rho \rightarrow v}. \quad (32)$$

Here, λ_0 denotes the base regularization weight, and λ_0 has been initialized as 1.0. Finally, the total loss function of StateNet is defined as

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{data}} + \lambda_{v \rightarrow \rho} \mathcal{L}_{v \rightarrow \rho}^{\text{op}} + \lambda_{\rho \rightarrow v} \mathcal{L}_{\rho \rightarrow v}^{\text{op}}. \quad (33)$$

When both operator weights are set to zero, namely,

$$\lambda_{v \rightarrow \rho} = \lambda_{\rho \rightarrow v} = 0, \quad (34)$$

The model simplifies to a purely data-driven, multi-task, coordinate-based network in the absence of operator regularization. Consequently, comparing it with this baseline model enables a direct assessment of the impact of neural-operator regularization on the performance of traffic-state estimation.

4. Experiment and Analysis

4.1. Data Description

This study employs the real-world NGSIM dataset to empirically validate the proposed BiONet-TSE methodology. The NGSIM dataset is obtained through fixed camera systems that capture vehicle trajectory positions and associated motion status information within the monitored area, thereby providing a comprehensive representation of actual traffic flow dynamics. Specifically, this research utilizes data from the US101 highway segment, encompassing a monitoring range of approximately 680 m and an observation period of roughly 2770 s. Due to its high spatiotemporal resolution and strong empirical validity, the NGSIM dataset has been extensively applied in studies related to traffic flow modeling, TSE, and traffic system engineering.

In preparing the experimental data, the original vehicle trajectory records were initially preprocessed to exclude time intervals devoid of vehicles or containing anomalous data, ensuring the integrity and continuity of the traffic state field. Subsequently, the global spatiotemporal domain corresponding to the study area was discretized into a uniform grid with a spatial resolution of 30 m and a temporal resolution of 1.5 s, as shown in Figure 3. Utilizing this grid partitioning approach, speed and density state values were computed within each spatiotemporal unit, thereby facilitating the construction of training, validation, and testing datasets for the BiONet-TSE model.

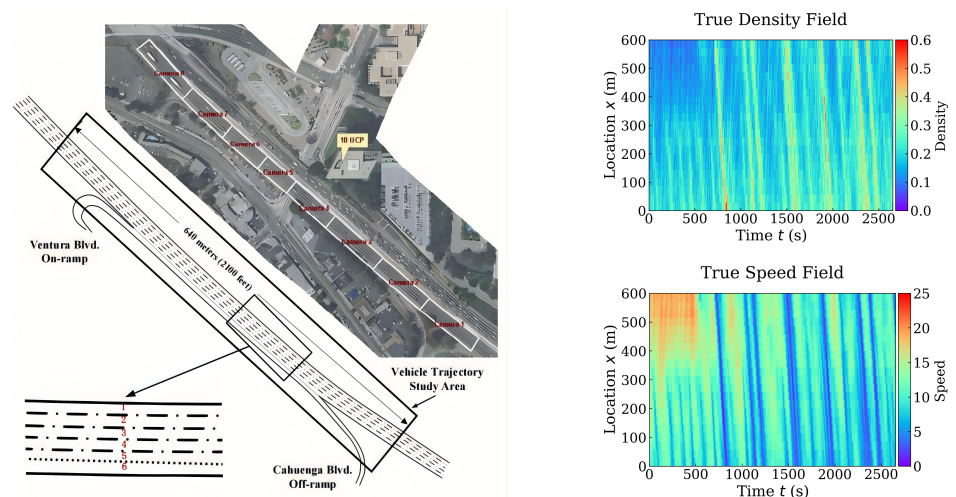


Figure 3. Research data. (left) Study area. (right) Processed research data [3].

4.2. Experimental Setting

4.2.1. Implementation Details

During the model training phase, the Adam optimizer was employed with a learning rate fixed at 0.001. Both the DeepONet operator network and StateNet implemented an early stopping mechanism, utilizing the validation set loss as the criterion for model selection. The number of sensor points for the DeepONet branch input was consistently maintained at 128. The operator network was trained for a maximum of 10,000 epochs, whereas StateNet was allowed up to 50,000 epochs. Each training batch comprised 2048 samples. To prevent the premature impact of operator constraints on the primary network's fitting during the initial training stages, a warm-up and ramp-up strategy was applied to the operator loss. Specifically, only the data fitting loss was considered during the early training phase, with the operator consistency loss being progressively incorporated thereafter. The detailed experimental configuration is presented in Table 1.

Table 1. Experimental hyperparameter settings of BiONet-TSE.

Category	Hyperparameter	Value
DeepONet operator	Number of branch sensors	128
	Hidden dimension	128
	Latent dimension	128
	Maximum training epochs	10,000
	Batch size	2048
	Learning rate	0.001
	Optimizer	Adam
	Validation ratio	0.10
	Early stopping patience	300
StateNet	Hidden dimension	64
	Maximum training epochs	50,000
	Batch size	2048
	Learning rate	0.001
	Optimizer	Adam
	Validation ratio	0.10
	Early stopping patience	30

Table 1. Cont.

Category	Hyperparameter	Value
Operator regularization	Warm-up epochs	1000
	Ramp epochs	1000
	Reliability threshold τ	0.25
	Reliability power γ	2.0
	Directional weight range	[0, 1]

4.2.2. Metrics and Baseline

To rigorously assess the model's performance and reliability, two evaluation metrics were employed: Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). The computational procedures for these metrics are detailed as follows:

$$MAE = \frac{1}{D} \sum_{i=1}^D |\hat{Y}_i - Y_i| \quad (35)$$

$$RMSE = \sqrt{\frac{1}{D} \sum_{i=1}^D (\hat{Y}_i - Y_i)^2} \quad (36)$$

where D is the number of tested samples. In order to rigorously evaluate the superiority of the proposed BiONet-TSE framework in TSE, this study conducted a comparative analysis, and the selected baseline model is detailed as follows:

NN: A neural network framework employed as a reference model for speed estimation through the learning of data characteristics [36].

PINN: A neural network incorporating traffic flow physics information as regularization [7].

Although existing DeepONet-based TSE methods are closely related to the present study, they usually formulate TSE as a direct operator-learning problem that maps sparse observations, initial conditions, or boundary conditions to the complete spatiotemporal traffic state field. The objective of BiONet-TSE is different: the pretrained bidirectional DeepONet operators are used as regularization modules to guide a coordinate-based multi-task estimator, rather than serving as the primary sparse-input-to-field predictor. Therefore, NN and PINN are selected as the main baselines to isolate the contribution of learned operator-guided regularization compared with purely data-driven learning and fixed equation-based physical regularization.

4.3. Result Analysis

4.3.1. Overall Accuracy Index

Table 2 summarizes the speed and density estimation outcomes obtained using NN, PINN, and BiONet across varying sampling rates (α). The definition of α is

$$\alpha = \frac{N_{\text{obs}}}{N_x N_t} \quad (37)$$

where N_{obs} is the number of training observation points obtained by sampling, and $N_x N_t$ is the number of complete spatiotemporal grid points. After preprocessing and removing invalid time intervals, the final speed and density fields used in the experiments contain ($N_x = 21$) spatial grid locations and ($N_t = 1770$) temporal steps. Thus, the complete spatiotemporal state field consists of ($N_x \times N_t = 37,170$) grid points. It should be noted that the DeepONet branch sensors and the sampling ratio α correspond to two independent sampling settings. The 128 branch sensors are fixed spatiotemporal locations used to

construct the input function vector for DeepONet operator learning, and they are kept unchanged under all sampling ratios to maintain a consistent operator input dimension. In contrast, α denotes the fraction of observed spatiotemporal grid points used for StateNet training. Therefore, changing α alters the sparsity level of the state estimation task, but does not change the number or locations of the DeepONet branch sensors. We chose a sampling range from 0.01 to 0.5 because this setting includes a medium to low observation ratio and is also a key issue that needs to be addressed in practical scenarios. Generally, as the sampling rate increases from 0.10 to 0.50, both the MAE and RMSE for all three methods in estimating speed and density exhibit a decreasing trend. This pattern suggests that a greater number of observed samples furnish more comprehensive information for reconstructing the traffic state field, thereby effectively reducing estimation errors. Notably, within the range of $\alpha = 0.10$ to $\alpha = 0.30$, there is a marked reduction in error metrics, highlighting that under conditions of sparse observation, increasing the sample size substantially enhances model performance.

Table 2. Comparison results of different methods under various sample ratios. Results are reported as mean $\pm$ standard deviation over three random seeds.

Result	Model	$\alpha = 0.10$		$\alpha = 0.20$		$\alpha = 0.30$		$\alpha = 0.40$		$\alpha = 0.50$	
		MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
Speed	NN	1.0527	1.4796	0.8566	1.2341	0.8040	1.1758	0.7507	1.0852	0.7757	1.1115
		± 0.0351	± 0.0487	± 0.0283	± 0.0415	± 0.0246	± 0.0378	± 0.0219	± 0.0326	± 0.0208	± 0.0304
	PINN	0.9497	1.3532	0.7929	1.1521	0.7750	1.0965	0.7189	0.9695	0.6733	0.9017
		± 0.0272	± 0.0398	± 0.0225	± 0.0340	± 0.0201	± 0.0315	± 0.0184	± 0.0277	± 0.0169	± 0.0259
	BiONet	0.9416	1.3447	0.7707	1.0418	0.6783	0.9072	0.6633	0.8764	0.6565	0.8733
		± 0.0198	± 0.0316	± 0.0174	± 0.0271	± 0.0152	± 0.0235	± 0.0148	± 0.0222	± 0.0135	± 0.0214
Density	NN	0.0334	0.0440	0.0307	0.0403	0.0294	0.0386	0.0286	0.0372	0.0291	0.0380
		± 0.0009	± 0.0012	± 0.0008	± 0.0011	± 0.0007	± 0.0010	± 0.0007	± 0.0009	± 0.0007	± 0.0009
	PINN	0.0330	0.0435	0.0293	0.0385	0.0295	0.0383	0.0276	0.0351	0.0270	0.0343
		± 0.0008	± 0.0011	± 0.0007	± 0.0009	± 0.0007	± 0.0009	± 0.0006	± 0.0008	± 0.0005	± 0.0007
	BiONet	0.0329	0.0433	0.0290	0.0372	0.0272	0.0344	0.0269	0.0342	0.0273	0.0349
		± 0.0006	± 0.0008	± 0.0005	± 0.0007	± 0.0005	± 0.0006	± 0.0004	± 0.0006	± 0.0005	± 0.0007

The best mean performance under each sample ratio and metric is highlighted in bold. Standard deviations are shown below the mean values.

Figure 4 illustrates the performance variation of the three models across different sampling proportions. Overall, BiONet consistently achieves the lowest errors for most sample ratios, with its superiority being particularly evident in speed estimation tasks. For instance, considering Speed RMSE and Speed MAE, the BiONet's performance curve (depicted in red) generally lies below those of NN and PINN, indicating that the incorporation of bidirectional neural operator regularization facilitates a more accurate reconstruction of the speed field. Compared to conventional NN approaches, BiONet not only fits coordinates directly to state variables but also enforces consistency between these variables by learning operator constraints linking speed and density. In contrast to traditional PINNs, BiONet eschews reliance on fixed-form physical equations, instead adopting a data-driven approach to learn variable mappings, thereby enhancing adaptability to complex, real-world traffic scenarios.

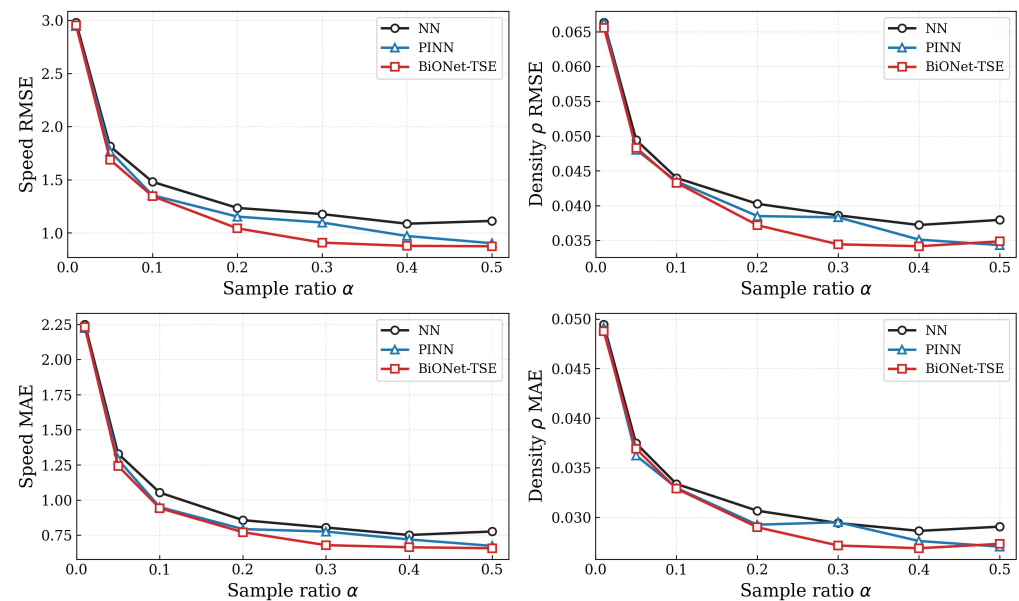


Figure 4. Performance variation curves of different models with sampling rate.

The density estimation results exhibit a similar pattern. For the metrics Density ρ RMSE and Density ρ MAE, BiONet generally outperforms NN and PINN, especially within the $\alpha = 0.20$ to 0.40 range. This finding indicates that neural operator guidance not only improves speed field estimation but also contributes to higher-quality density field reconstruction. Given the intrinsic coupling between speed and density, the bidirectional mappings learned by DeepONet provide additional consistency constraints for the primary network, mitigating the underdetermined nature of the problem under sparse observation conditions.

It is important to note that at extremely low sampling ratios, such as $\alpha = 0.01$, all three methods exhibit relatively high errors with minimal performance differences. This observation suggests that under conditions of extreme data sparsity, fully recovering the complete traffic state field from a limited number of sampling points remains challenging. Although operator regularization offers some improvement, it cannot fully substitute for sufficient observational data. As the sample size increases, the advantages of BiONet become increasingly pronounced, indicating its suitability for scenarios characterized by limited, yet not exceedingly sparse, observational data.

4.3.2. Estimation Results of Global Heatmap

Figure 5 presents the thermal maps of the reconstructed traffic state fields generated by BiONet. Each column corresponds to reconstruction outcomes at varying sampling rates, denoted by α , with the final column depicting the actual traffic state field. The first row illustrates the speed field, while the second row represents the density field. The horizontal axis indicates temporal progression, the vertical axis denotes spatial position, and the color gradients reflect the magnitude of the respective traffic state variables.

Analysis of the speed field reconstructions reveals that BiONet effectively captures the principal spatiotemporal evolution patterns observed in the true speed field across different sampling rates. Notably, multiple distinct low-speed bands are evident in the real speed data, exhibiting continuous temporal distributions and forming propagation structures along the spatial dimension. These low-speed bands typically correspond to congestion waves, traffic disturbances, or localized speed reductions. From $\alpha = 0.10$ to $\alpha = 0.50$, the model consistently identifies the locations and developmental trends of these key low-speed features, demonstrating that the proposed approach transcends

simple interpolation near sparse observation points by learning a more continuous and comprehensive representation of the traffic state field.

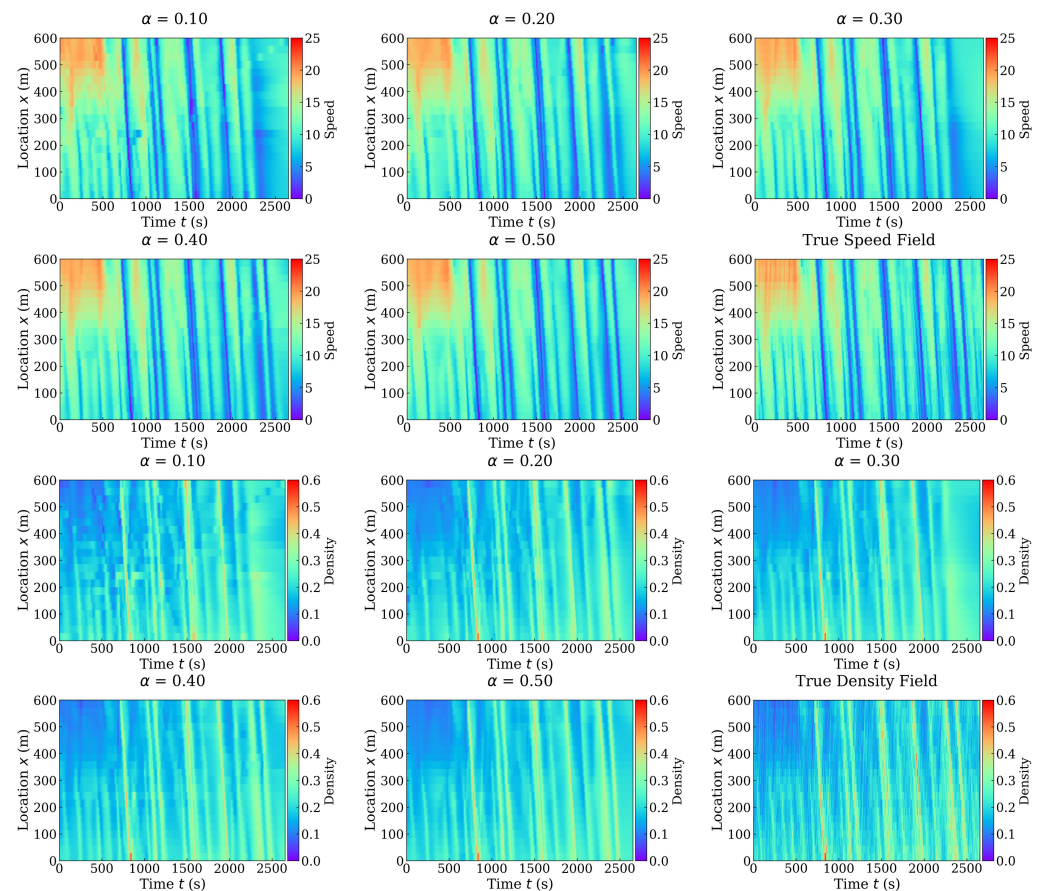


Figure 5. Global heatmap based on the proposed BiONet reconstruction field.

Regarding the density field reconstructions, BiONet similarly succeeds in restoring the overall distributional characteristics of the actual density field. Several high-density regions manifest as continuous strip-like structures along the temporal axis, which approximately align with the low-speed regions identified in the speed field. The model's ability to reconstruct these high-density bands across different sampling rates suggests it effectively captures the coupling relationship between speed and density. Specifically, the emergence of low-speed or congested propagation patterns in the speed field is generally accompanied by corresponding increases in density. This consistency is maintained through the bidirectional neural operator regularization employed by BiONet.

Nonetheless, compared to the speed field, accurately recovering local details within the density field proves more challenging. At lower sampling rates, the reconstructed density heatmaps appear smoother, and certain narrow, high-density strip structures are less distinct relative to the real field. This observation implies that the density field exhibits stronger local fluctuations and more intricate fine-grained variations, which are difficult to fully recover relying solely on sparse observations and operator regularization. As the sampling rate increases, the continuity and spatial accuracy of the high-density bands improve progressively, although some discrepancies in local intensity persist. These findings align with the density metric results presented in the accompanying table, where BiONet generally outperforms alternative methods across most sampling rates, albeit with a slightly reduced advantage in density estimation compared to speed estimation.

In summary, the thermal maps substantiate the efficacy of BiONet in reconstructing comprehensive speed and density fields at varying sampling rates, effectively preserving

critical spatiotemporal traffic features such as congestion propagation, speed reductions, and density aggregations. When considered alongside quantitative performance metrics, these results indicate that the neural operator-guided framework not only enhances numerical accuracy but also improves the overall structural coherence and data-driven coupling relationships of the reconstructed traffic state fields.

4.3.3. Estimation Results of Local Evolution

To further assess the efficacy of BiONet in localized traffic scenarios and to substantiate the contribution of the proposed reliability-gating mechanism, this study presents the local time-series reconstruction results for speed and density at representative virtual sensing locations using ($\alpha = 0.40$), as illustrated in Figure 6. Here, a virtual sensing location refers to a spatial grid location obtained after discretizing the NGSIM US101 segment, rather than a physical roadside detector. Among the 21 spatial grid locations, four representative locations, namely Locations 5, 8, 13, and 17, were selected for visualization.

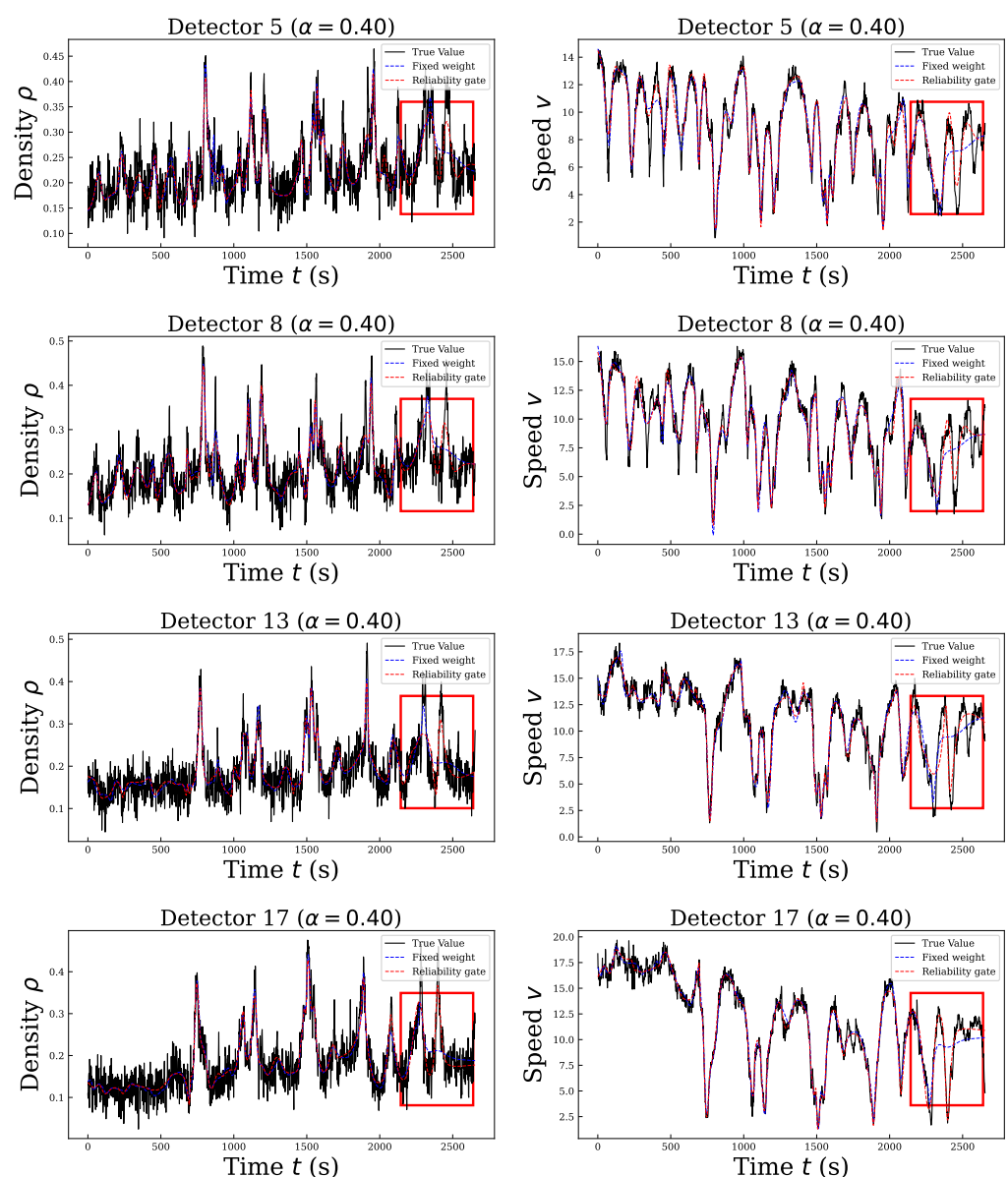


Figure 6. Comparison of reconstruction results at representative virtual sensing locations. The red boxes denote the regions exhibiting significant performance disparities between the two models, underscoring the importance of dependable gating mechanisms in accurately capturing dynamic variations in traffic conditions.

Overall, both regularization methods effectively capture the general trends of actual traffic conditions, indicating that operator regularization enhances the model's capacity to represent the temporal evolution of speed and density to some extent. However, relative to the fixed-weight approach, the reliability gating method demonstrates superior tracking performance during multiple intervals characterized by local fluctuations. Notably, within the region highlighted by the red box in the figure, the fixed-weight method tends to exhibit either excessive smoothing or insufficient responsiveness, whereas the reliability gating method more accurately captures local peaks, troughs, and rapid trend changes in the true values.

Examining the density estimation results reveals that, during the latter half of the time intervals at Detectors 5, 8, 13, and 17, the true density exhibits pronounced local fluctuations and short-term peak variations. The fixed-weight method generally recovers only relatively smooth trends in these segments and shows limited responsiveness to abrupt density spikes or drops. In contrast, the reliability gating method aligns more closely with the true density curve within the red box region, effectively characterizing local rises, falls, and peak positions. This suggests that the reliability gating mechanism mitigates excessive constraints imposed on the primary network by unreliable operators, thereby enabling the model to maintain global smoothness while enhancing adaptability to localized traffic disturbances.

Similarly, the speed estimation results underscore the advantages of the reliability gating approach. In the early phase at Detector 5 and within the red box regions during later stages at each detector, the true speed undergoes significant fluctuations, declines, and recoveries. The fixed-weight method fails to fully capture these abrupt changes, manifesting as inadequate representation of speed troughs or delayed recovery trends. Conversely, the reliability gating method more precisely reflects the processes of speed decline and rebound, particularly demonstrating improved responsiveness to local low-speed states and regions with rapid speed variations. This indicates that regularization via reliability gating operators not only improves density estimation but also enhances the model's ability to recover local dynamic changes in the speed field.

Further analysis reveals that the fixed-weight method applies a uniform constraint strength to operator losses across both directions, which can lead to excessive or inappropriate regularization when operator reliability is low or when the mapping difficulty varies between variables. This results in diminished capacity to represent local fluctuations. In contrast, the reliability gating method adaptively adjusts the weights of operator losses in different directions based on the validation errors of pre-trained operators, thereby providing strong guidance for highly reliable operators while appropriately attenuating the influence of less reliable ones. Consequently, the model achieves a more balanced trade-off between data fitting and operator constraints, preserving the overall continuity of the traffic state field while preventing local information loss associated with fixed regularization weights.

It is important to acknowledge that the true traffic state signals contain substantial high-frequency fluctuations and observational noise, and neither method fully captures all instantaneous peaks. This limitation is reasonable, as the objective of TSE is not to fit noise but to reconstruct the principal traffic evolution patterns with data-driven coupling relationships under conditions of sparse observations. Although the reliability gating method also smooths certain high-frequency disturbances, it exhibits superior recovery of critical local peaks, valleys, and trend inflection points, indicating enhanced capability to capture the principal dynamic structures inherent in real traffic conditions. In addition, more supplementary results on parameter sensitivity analysis and ablation experiments have been listed in Table A1 of Appendix A and Table A2 of Appendix B.

5. Conclusions

To address challenges related to the incorporation and availability of physical information within physical information neural networks, this study introduces a bidirectional neural operator-guided multitask TSE framework, termed BiONet. BiONet employs two bidirectional deep operator networks (DeepONets) to learn the mapping operators between traffic speed and density, thereby facilitating the guidance of state inference network training. To mitigate potential network failures arising from limitations inherent in operator learning, BiONet integrates a reliability gating mechanism to assess the trustworthiness of the learned operators. Empirical evaluations using real-world data demonstrate that BiONet effectively reconstructs traffic state fields and exhibits robust performance across varying permeability conditions. Additionally, the model successfully captures the localized evolution dynamics of traffic flow. Future investigations may focus on methodologies to extract explicit and reliable physical information from data to further enhance network training and performance. Meanwhile, as the experiment in this article is based on the NGSIM dataset, it is necessary to further validate the generalization ability of the model in other road types and larger road networks in the future.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Sensitivity Analysis

The outcomes of the sensitivity analysis concerning the reliability threshold τ and the gating sensitivity parameter γ are presented in Table A1. The findings demonstrate that the model exhibits considerable stability across a plausible range of parameter values. Specifically, setting τ to 0.25 and γ to 2.0 yields an optimal balance between operator guidance and data fitting.

Table A1. Sensitivity analysis of the reliability-gating parameters under $\alpha = 0.40$.

τ	γ	Speed MAE	Speed RMSE	Density MAE	Density RMSE
0.10	1.0	0.6815	0.9027	0.0276	0.0351
0.10	2.0	0.6758	0.8953	0.0274	0.0349
0.10	3.0	0.6796	0.8998	0.0275	0.0350
0.25	1.0	0.6689	0.8846	0.0271	0.0345
0.25	2.0	0.6633	0.8764	0.0269	0.0342
0.25	3.0	0.6667	0.8815	0.0270	0.0344
0.50	1.0	0.6724	0.8897	0.0272	0.0347
0.50	2.0	0.6708	0.8872	0.0272	0.0346
0.50	3.0	0.6741	0.8919	0.0273	0.0348

The best performance under each metric is highlighted in bold.

Appendix B. Ablation Experiment

Table A2 presents the findings of the ablation study. The results indicate that the model incorporating operator regularization consistently outperforms alternatives across all sampling rates. Furthermore, the reliability weights employed in this study demonstrate superior performance compared to fixed weights in the majority of cases, thereby validating the efficacy of this approach.

Table A2. Comparison results of different operator regularization strategies under various sample ratios.

Result	Model	$\alpha = 0.10$		$\alpha = 0.20$		$\alpha = 0.30$		$\alpha = 0.40$		$\alpha = 0.50$	
		MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
Speed	NR(NN)	1.0527	1.4796	0.8566	1.2341	0.8040	1.1758	0.7507	1.0852	0.7757	1.1115
	FW	0.9763	1.3902	0.8580	1.2200	0.7227	0.9662	0.6596	0.8665	0.8043	1.1715
	RG	0.9416	1.3447	0.7707	1.0418	0.6783	0.9072	0.6633	0.8764	0.6565	0.8733
Density	NR(NN)	0.0334	0.0440	0.0307	0.0403	0.0294	0.0386	0.0286	0.0372	0.0291	0.0380
	FW	0.0331	0.0436	0.0304	0.0400	0.0282	0.0362	0.0276	0.0348	0.0292	0.0385
	RG	0.0329	0.0433	0.0290	0.0372	0.0272	0.0344	0.0269	0.0342	0.0273	0.0349

The best performance under each sample ratio and metric is highlighted in bold. NR denotes no operator regularization, FW denotes fixed-weight operator regularization, and RG denotes reliability-gated operator regularization.

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