

Article

Optimization of Two-Stage Military Product Revenue-Sharing Game Model Based on Particle Swarm Algorithm

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Highlights

Contributions to systems-science (theory and methodology)

- We establish a bi-level Stackelberg game framework for the two-stage profit-sharing problem separating military R&D and production, and prove the uniqueness of the Nash equilibrium grounded on the Brouwer fixed-point theorem, which enriches the system-based hierarchical game theory for complex military–industrial cooperation.
- A Bi-PSO algorithm with nested two-layer optimization is designed to match the sequential decision-making logic of the Stackelberg game. Compared with conventional intelligent algorithms, it eliminates logically invalid solutions and provides a reliable methodological tool for solving non-convex multi-constraint system-optimization problems.

Main findings and practical implications

- The overall profit maximization of the cooperative system depends on the bi-level game structure, while fair income distribution can only be realized by introducing additional fairness-weighted objectives. The stable range of profit-sharing ratios for prototype and mass-production stages is quantified to guide practical contract formulation.
- Improvements in participants' technical efficiency combined with positive reward–penalty policies generate synergy gains for the whole system. The derived parameter-interaction rules help the military department control procurement costs and balance benefits across the whole military-industry system.

Abstract

To address three major industry pain points—the lack of quantified profit-sharing standards in the two-stage pricing under the separation model of military research and production, the absence of stable Nash equilibrium in single-layer synchronous optimization, and insufficient incentives for full-cycle process optimization in design units—this paper constructs a two-level Stackelberg leader–follower game model with the general contracting unit as the leader and design and general contracting units as followers. This aligns with current prototype incentives and phased pricing policies for production rewards and penalties. At the theoretical level, it improves the complete proof system for the two-stage concave profit two-level Stackelberg Nash equilibrium, distinguishes the mathematical differences in equilibrium existence between sequential decision-making and synchronous optimization, and extracts general rules for phased differentiated profit sharing: high-innovation segments should be allocated more profit weight; simply maximizing

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total alliance profit may cause imbalanced interests, while introducing a minimum net profit-weighted objective can achieve Pareto improvements without profit loss. This conclusion can be applied to multi-stage general contracting scenarios across industries, such as EPC and military–civil collaborative innovation, enriching the basic theory of profit sharing and hierarchical games. Theoretically, the existence of the lower-level Nash equilibrium is proven using Brouwer’s fixed-point theorem, and combining it with the strictly monotonically decreasing feature of the best response function, uniqueness of the equilibrium is derived. Multiple sets of differentiated initial values are simulated to rule out multi-equilibrium bifurcation risk. The model incorporates the military’s reward and penalty policies as rigid exogenous constraints, sets dual individual rationality constraints of ‘cooperative profit greater than baseline profit with no allocation, and both parties’ net profit non-negative,’ and introduces differentiated cost-reduction efficiency and quadratic increasing effort costs to characterize the heterogeneous input of the two types of development entities. For models with piecewise nonlinearity and multi-constraint nonconvex structures, this paper modifies the standard PSO into a Bi-PSO solving framework through hierarchical temporal adaptation. It does not innovate the underlying particle update mechanism and is only used to match the sequential decision order of the leader–follower game. By comparing five algorithms—IPM, GA, SA, DE, and adaptive PSO—through 20 repeated simulations: gradient-based interior point methods easily get stuck in locally invalid solutions that violate cooperation thresholds; differential evolution has the best numerical global search performance, but all general evolutionary algorithms optimize allocation and effort variables simultaneously, disrupting the Stackelberg hierarchical timing. Only Bi-PSO maintains consistent game logic. Using a pricing case for a certain type of equipment and jointly calibrating all parameters with policy documents, three simulation scenarios were set up: no allocation, equal 50/50 split, and single-layer profit maximization. Under the no-allocation mode, R&D investment from the design unit drops to zero and alliance benefits plummet; a blanket equal split ignores differences in technical contributions across two stages, leading to clear efficiency losses; single-layer optimization only pursues total profit maximization, causing a severe imbalance in profit distribution. The two-layer basic framework can achieve the upper limit of alliance benefits, and by adding a weighted optimization goal that considers both total profit and cooperation fairness, it can achieve equal net profits for both parties without reducing overall profit. Through single-parameter sweeps and two-factor heatmap simulations, the study further revealed the coupled effects of main party efficiency and mass production rewards and penalties on equilibrium input and optimal sharing ranges. A robust check was performed by replacing the logarithmic concave output function, producing a standardized allocation range resilient to parameter perturbations: optimal split for the prototype stage is 0.4–0.6 for the design unit, and for mass production stage 0.7–0.9. The findings suggest that high-contribution stages in multi-phase collaboration contracts should receive more benefits, and a weighted fairness objective can achieve Pareto improvements. These conclusions can extend to multi-stage collaboration scenarios such as EPC and military–civilian cooperation. Theoretically, this research further completes the equilibrium proof system for two-party concave payoff two-layer games, providing a new reference for the theory of phased differentiated benefit-sharing contracts in the military sector. Methodologically, it proposes a two-layer intelligent solving tool adapted to leader–follower sequential decisions, effectively mitigating issues where single-layer model equilibria fail or analytical algorithms struggle with multi-constraint nonconvex games. The results can provide quantitative support for the military, general contracting unit, and design unit in drafting equipment incentive pricing contracts and managing full-cycle cost collaboration.

Keywords: military product design; two-stage pricing; revenue sharing; two-level Stackelberg game; particle swarm optimization; parameter sensitivity

1. Introduction

Deepening the market-oriented reform of the national defense science and technology industry and establishing a phased equipment incentive pricing system are currently the key tools for managing the lifecycle cost of military equipment. At present, the development of equipment generally separates research from production: design units are responsible for the overall equipment plan and core technology development, while the general contracting unit handles prototype testing and mass production. The current military product pricing rules only require both parties to negotiate the allocation of incentive profits and do not provide phased, quantifiable standards for profit sharing. This leads to three major engineering pain points: first, design units cannot get matching economic returns for their full-cycle technical contributions such as early R&D, mass production support, and process iteration, which weakens R&D motivation and directly drives up overall equipment procurement costs; second, using a single-layer synchronous optimization framework to solve for both profit sharing and R&D effort results only in internal profit transfers within the alliance, without reducing overall costs, and the model has no unique stable Nash equilibrium, making contract calculations unreliable; third, without differentiated incentives, design units just deliver blueprints and refuse to participate in mass production process optimization, causing full-cycle collaboration to fail. Against this backdrop, building a profit-sharing game model that fits prototype incentives and two-stage pricing rules with rewards and penalties for mass production has important theoretical value and practical significance.

The coordination of benefits in military product development essentially falls under the category of multi-agent collaborative benefit contract design, and it shares common theoretical logic with civil engineering supply chain benefit coordination, such as EPC (Engineering, Procurement, and Construction) and IPD (Integrated Project Delivery). Existing game studies in the civil sector can provide a basic paradigm for building military models. Currently, mainstream modeling tools for benefit distribution include Nash bargaining, Shapley value, fuzzy cooperative games, and multi-objective optimization. Scholars mostly focus on key variables like participant effort levels, risk sharing, and fairness preferences to build alliance profit coordination models. Wang and Zhang optimized EPC project profit distribution rules from the perspective of cooperation satisfaction [1,2]; Li and others constructed game models for contractor supply chain benefit sharing and cost sharing, quantifying the impact of participant effort on alliance benefits [3–5]; Guo's team introduced fuzzy variables to depict the uncertainty in resource input and analyzed distribution equilibrium characteristics in fuzzy information scenarios [6–9]; Liu, Teng, and others combined the Shapley value with dynamic alliance theory to clarify how R&D effort and risk preferences affect cooperative benefits [10–14]. These distribution paradigms have gradually expanded to multi-party cooperation scenarios like TOT projects, manufacturing logistics, and incomplete information alliances, providing a mature analytical framework for civil multi-stage collaborative pricing [15–18].

Focusing on the niche area of incentive pricing for military equipment, existing research can be divided into two separate lines, but both show clear adaptation flaws. The first type is single-stage military procurement incentive models: Suhonen, Sun, and others built static pricing games aimed at a single equipment procurement cycle, which only fit one-off equipment procurement scenarios. They cannot match the current two-stage differentiated pricing system of 'prototype incentive followed by batch production rewards

and penalties,' nor distinguish the differences in technological contributions between the R&D and mass production stages [19,20]. The second type is military–civilian collaborative intelligent optimization games: Liang, Duan, and others combined differential games and particle swarm algorithms to solve the military–civilian innovation benefit optimization problem. However, these models do not incorporate the military's exogenous reward and penalty policy constraints and solve all decision variables simultaneously, which goes against the sequential decision logic of Stackelberg leader–follower games, leading to theoretical distortions [21–23]. By comparing with the latest work from Tu, Guo Shuwen, Wang Lunyuan, Wang Chaoqian, and others [24–28], current research still has three areas needing improvement: first, the model structure lacks adaptability. Most military game models are single-stage and static, not distinguishing the differentiated benefit rules between prototype and mass production stages, making it impossible to align with current staged pricing policies; second, the solution methods are limited. Existing analytic derivation and Hessian matrix approaches only apply to unconstrained simple games, while this study's model includes piecewise policy boundaries, two-layer decisions, multiple individual rationality hard constraints, and a multi-constrained non-convex objective function. Analytic methods tend to converge to local optima, making large-scale parameter sensitivity simulations difficult; third, there are prominent game identification flaws. Single-layer simultaneous optimization treats benefit sharing and both parties' efforts as parallel variables. Allocation parameters only internally transfer profit without changing overall cost-reduction benefits, so mathematically there is no unique equilibrium, and the model loses contract evaluation capability. Existing military intelligent optimization literature generally optimizes allocation and effort variables simultaneously, breaking the Stackelberg leader–follower temporal relationship, and the equilibrium only satisfies numerical optimality, not Nash optimal response conditions. This study uses a hierarchical Bi-PSO to forcibly separate the upper and lower iterative processes, correcting common modeling flaws in multi-layer intelligent game solving, and provides a theoretically self-consistent standard solution paradigm for staged military equipment pricing.

To address the dual gaps in theory and engineering, this paper is based on the current reality where military research and production are separated, aligns with the existing two-stage equipment pricing policy, and builds a two-level Stackelberg leader–follower game model. In this model, the top-level general contracting unit decides on phased profit sharing, while the lower-level design units and the general contracting unit solve for equilibrium effort simultaneously.

On the theoretical side, based on Brouwer's fixed-point theorem, we fully prove the existence of lower-level Nash equilibria, and through the strict monotonicity of the best response functions, we rigorously derive the uniqueness of the equilibrium. By pairing this with numerical simulations using multiple initial values, we eliminate multiple equilibrium branches and complete a thorough proof system for two-party concave payoff bi-level games. We differentiate technological contributions between prototype and mass-production stages, introduce exogenous military reward and penalty constraints, and dual individual rationality participation thresholds, building a game paradigm tailored to military-industry-specific payoff contracts and expanding the application boundary of hierarchical game theory in defense equipment pricing. Meanwhile, this study further enriches multi-stage collaborative contract theory: stages with high innovation should get higher reward allocation weights, and simply maximizing total alliance payoff can easily cause imbalances in stakeholders' interests. Introducing a fair-weighted optimization objective can achieve a Pareto improvement in game outcomes, and the related findings can be extended to multi-stage cooperation scenarios like engineering procurement and construction (EPC) projects and military–civil integration.

At the method level, targeting the model's segmented nonlinearity and multi-constraint nonconvex features, the standard particle swarm algorithm was adapted in a hierarchical, time-sequenced way to build the Bi-PSO double-layer solving framework. There is no innovation in the underlying particle update algorithm; it simply restores the Stackelberg sequential logic of 'general contracting unit first sets the revenue contract, design unit adjusts R&D investment afterwards' through stepwise iteration between inner and outer layers, making up for the timing distortion issues of general evolutionary algorithms in game scenarios. Using a unified simulation benchmark, Bi-PSO was compared multidimensionally with IPM, GA, SA, DE, and adaptive PSO, screening the best solving tool in terms of compliance, stability, and game-theory consistency, providing a standardized simulation scheme for similar double-layer military-industry games.

This article uses equipment project pricing examples and official industry pricing standards to jointly calibrate all simulation parameters. It sets up three comparison schemes—no profit sharing, equal sharing, and single-layer Nash optimization—to verify the overall advantages of the model. Using single-parameter sweeps and two-factor coupled heatmap simulations, it clarifies how participant efficiency and reward/punishment mechanisms in the mass production stage interact with balanced strategies, and it also replaces the logarithmic concave output function for robustness testing. At the same time, combining with practical bulk equipment procurement, it conducts empirical comparative analysis and provides standardized profit-sharing ranges that can adapt to parameter fluctuations, offering a quantitative analysis tool for buyers, general contracting units, and design units to formulate equipment incentive contracts and implement full-cycle cost coordination.

This article is organized as follows: Section 2 defines the game players and the two-stage pricing rules; Section 3 goes through all the model assumptions and the theoretical support for the functions; Section 4 derives the mathematical expressions for two-stage pricing, revenue, and costs; Section 5 builds a two-level Stackelberg optimization model and explains the Bi-PSO solution logic, while also conducting comparison experiments with multiple algorithms; Section 6 completes numerical simulations for benchmark equilibrium, allocation scheme comparison, and multidimensional parameter sensitivity and robustness; Section 7 summarizes research limitations and directions for future work; and Section 8 distills the core research conclusions.

2. Problem Description and Definition of Game Participants

2.1. Two-Stage Pricing System Rules for the Military Industry

Under the model of separating research and development from production, the design unit is responsible for planning and technical design, while the general contractor undertakes prototype development and mass production. Both parties collaboratively manage cost and quality under the constraints of the target price for military product orders. Military product revenue is divided into two stages: during the initial batch stage, reward profit is accrued according to the policy-defined incentive coefficient based on the difference between the order target price and the actual expected price; during the mass production stage, stage earnings are determined based on the comprehensive evaluation results of performance, quality, and cost according to the reward and penalty ratio. The design unit not only undertakes technical design but also fully participates in production guidance, process optimization, and cost source control, and should reasonably receive design returns that match its contribution. Taking the design unit and the general contractor as two competing entities and aiming to maximize the overall revenue of the consortium, a two-stage revenue-sharing optimization model is constructed.

2.2. Roles of the Three Parties

1. Military procurement department (exogenous constraint entity, not a participant in the game): Implements a two-stage pricing policy, limiting the incentive coefficient $\alpha \in [0.5, 0.7]$ and production bonus/penalty $\beta \in [-0.05, 0.05]$ as an externally fixed constraint outside the model; not involved in the profit-sharing game;
2. Lower-level follower 1: Design unit: invests in R&D and follow-on production efforts to maximize their own net returns after deducting effort costs;
3. Lower-level follower 2: General contracting unit: invests in production management efforts, anticipates the design unit's equilibrium behavior, and decides on the two-stage revenue shares λ_1, λ_2 in the upper-level decision to maximize the total joint benefits.

2.3. Explanation of the Two-Level Game Logic

If a single-layer synchronous optimization is used, the revenue sharing only transfers profits within the alliance and cannot change the total revenue, so mathematically there is no unique optimal λ . This paper restructures a two-layer nested framework:

1. Lower-level subgame: Given any input λ_1, λ_2 , both parties independently solve for their optimal efforts $\varphi_1^*(\lambda_1, \lambda_2), \varphi_2^*(\lambda_1, \lambda_2)$ to maximize their own net benefits R_1, R_2 ;
2. Upper-level main optimization decisions: The general contractor predicts the lower-level equilibrium effort and optimizes accordingly λ_1, λ_2 ; by changing both parties' equilibrium inputs and adjusting the overall cost-reduction benefits, it achieves the optimal allocation of the sharing ratio, which is uniquely solvable, thus eliminating the identification defect of the single-level model.

2.4. Proof of the Existence and Uniqueness of Lower-Level Nash Equilibrium

Given the two-stage revenue-sharing ratios λ_1, λ_2 set by the upper level, the total contractor's equilibrium effort φ_1 and the design unit's equilibrium effort φ_2 form a lower-level two-person non-cooperative Nash subgame. In this section, we strictly prove the existence of equilibrium based on Brouwer's fixed-point theorem, and then complete a full derivation of equilibrium uniqueness using the concavity/convexity of the profit function and the monotonicity of the best response functions.

2.4.1. Basic Prep: Brouwer's Fixed Point Theorem

Theorem: Let set $D \subset R^n$ be non-empty, compact, and convex. If mapping $F: D \rightarrow D$ is a continuous self-mapping, then there must exist at least one fixed point $x^* \in D$ that satisfies $F(x^*) = x^*$.

Among them:

1. Compact set: a bounded closed set, where the variable values have upper and lower bounds and the boundaries are included in the set;
2. Convex set: the line segment connecting any two points in the set lies entirely within the set;
3. Continuous self-mapping: a mapping where both input and output belong to set D , and the mapping function is continuous everywhere.

2.4.2. The Feasible Region D Meets All the Prerequisites of the Theorem

Define $D = \{(\varphi_1, \varphi_2) | 0 \leq \varphi_1 \leq 100, 0 \leq \varphi_2 \leq 100\}$ as the feasible effort domain for both sides, and verify the set properties one by one:

1. Non-empty: There are infinitely many valid effort values in the interval $[0, 100]$, so the set is not empty;
2. Compactness: Both φ_1 and φ_2 have clear upper and lower bounds of 0 and 100, forming a bounded closed rectangle in two dimensions, which meets the definition of compactness;
3. Convexity: For any two points $(\varphi_1^a, \varphi_2^a), (\varphi_1^b, \varphi_2^b) \in D$, take any $\theta \in [0, 1]$, and with the convex combination: $(\theta\varphi_1^a + (1-\theta)\varphi_1^b, \theta\varphi_2^a + (1-\theta)\varphi_2^b)$, the x and y coordinates still fall within $[0, 100]$, so the convex combination belongs to D , meaning the set is convex.

In summary, the effort feasible domain D fully satisfies the set prerequisites for Brouwer's fixed point theorem.

2.4.3. Construct Continuous Best-Response Self-Mapping

1. Definition of Single-Agent Best Response Function

Given the other party's fixed effort level, the main subject aims to maximize its own net profit to find the optimal input, defining the optimal response function:

Given the effort φ_2 of the design unit, the total contractor's optimal effort is: $\varphi_1^* = BR_1(\varphi_2) = \arg \max_{\varphi_1 \in [0, 100]} R_1(\varphi_1, \varphi_2, \lambda_1, \lambda_2)$. Given the total contractor's effort φ_1 , the design unit's optimal effort is: $\varphi_2^* = BR_2(\varphi_1) = \arg \max_{\varphi_2 \in [0, 100]} R_2(\varphi_1, \varphi_2, \lambda_1, \lambda_2)$. In the formula, R_1 and R_2 are the net profit functions of the total contractor and the design unit after deducting effort costs, derived from the two-stage pricing and revenue formulas in Section 4.

2. Proof of Mapping Continuity

Composition of the payoff function: The joint cost-reduction output uses an exponential concave function $f(\varphi_1, \varphi_2) = e^{\mu\varphi_1 + \nu\varphi_2}$, the effort cost is a quadratic polynomial $\frac{1}{2}h_1\varphi_1^2, \frac{1}{2}h_2\varphi_2^2$, and the two-stage reward profit is a piecewise linear function; after combining, $R_1(\cdot)$ and $R_2(\cdot)$ are infinitely differentiable, belonging to smooth continuous functions;

The objective function is strictly concave: As shown later in Section 2.4.4, R_1 is strictly concave in φ_1 , R_2 is strictly concave in φ_2 , and a single-variable concave function has a unique maximum point on a closed interval.

3. Continuity of argmax output

The optimal solution of a strictly concave function changes smoothly with exogenous variables, so $BR_1(\varphi_2)$ and $BR_2(\varphi_1)$ are both continuous functions of one variable.

Explanation of project management: This continuity feature reflects the actual operating patterns of military projects. Investments in R&D and process optimization do not have sudden jumps in returns. Companies can gradually adjust manpower and work hours through small iterations to get closer to the optimal plan. There will not be extreme situations where tiny changes in investment cause huge swings in project profit. The model's balance is practical and adjustable, and the mathematical conclusions can directly

guide companies in planning stage-by-stage investments, rather than being purely theoretical math.

4. 2D Self-Mapping Construction

Combine the two sets of optimal response functions to construct a two-dimensional self-mapping $F : D \rightarrow D$: $F(\varphi_1, \varphi_2) = (BR_1(\varphi_2), BR_2(\varphi_1))$, and with input $(\varphi_1, \varphi_2) \in D$, the output optimal response combination still satisfies $0 \leq BR_1(\bullet) \leq 100$ and $0 \leq BR_2(\bullet) \leq 100$, that is $F(\varphi_1, \varphi_2) \in D$; at the same time, $BR_1(\varphi_2)$ and $BR_2(\varphi_1)$ are continuous, so F is a continuous self-mapping defined on D .

2.4.4. Conclusion on the Existence of Nash Equilibrium

The feasible region D is non-empty, compact, and convex, and the mapping F is a continuous self-map on D , fully meeting all the conditions of Brouwer's fixed-point theorem. Therefore, there must exist a fixed point in $(\varphi_1^*, \varphi_2^*) \in D$, denoted as

$$\begin{cases} \varphi_1^* = BR_1(\varphi_2^*) \\ \varphi_2^* = BR_2(\varphi_1^*) \end{cases} .$$

This fixed point is defined as the pure strategy Nash equilibrium of the

lower-level two-player game, meaning a lower-level Nash equilibrium must exist.

Engineering management explained: Mathematically, we can rigorously prove that there is always a balance between R&D and production efforts at the lower levels. In real-world engineering practice, this means that no matter what kind of two-stage profit-sharing contract the general contracting unit comes up with, both the design team and the general contracting unit can find stable, workable schedules and R&D investment plans, avoiding extreme situations where efforts are wasted or the whole process stalls, basically making sure the incentive contracts actually work.

2.4.5. A Complete Guide to Deriving the Uniqueness of Nash Equilibrium

Just proving the existence of an equilibrium does not guarantee a unique solution or rule out multiple solution branches. In this section, we strictly prove the uniqueness of the equilibrium through the second-order concavity/convexity and the monotonicity of the best response function.

Step 1: Prove that the net profit function is strictly concave with respect to its own effort.

Take the second-order partial derivative of the general contracting unit's net profit

R_1 with respect to their own effort φ_1 : $\frac{\partial^2 R_1}{\partial \varphi_1^2}$ is the second-order term of revenue.

Since the combined cost-reduction output is a concave function, the second-order revenue term is always less than 0; with effort cost coefficient $h_1 > 0$, this makes $\frac{\partial^2 R_1}{\partial \varphi_1^2} < 0$,

meaning $R_1(\varphi_1, \varphi_2)$ is a strictly concave function of φ_1 . Similarly, the second-order partial derivative of the design unit's net profit R_2 with respect to its own effort φ_2 is

$\frac{\partial^2 R_2}{\partial \varphi_2^2} < 0$, so $R_2(\varphi_1, \varphi_2)$ is a strictly concave function of φ_2 . A strictly concave function

has only one maximum point on a closed interval, so for any given effort from the other party, the optimal responses $BR_1(\varphi_2)$ and $BR_2(\varphi_1)$ have a unique solution.

Step 2: Prove that the optimal response function is strictly decreasing.

Take the total contractor's optimal response $BR_1(\varphi_2)$ as an example and do implicit differentiation on the first-order optimal condition. The first-order optimal condition is: $\frac{\partial R_1(BR_1(\varphi_2), \varphi_2)}{\partial \varphi_1} = 0$. Differentiating both sides with respect to φ_2 gives:

$$\frac{\partial^2 R_1}{\partial \varphi_1^2} \times \frac{dBR_1}{d\varphi_2} + \frac{\partial^2 R_1}{\partial \varphi_1 \partial \varphi_2} = 0. \text{ Rearranging gives the slope of the response function:}$$

$$\frac{dBR_1}{d\varphi_2} = - \frac{\frac{\partial^2 R_1}{\partial \varphi_1 \partial \varphi_2}}{\frac{\partial^2 R_1}{\partial \varphi_1^2}}. \text{ Because of the cross second-order derivative } \frac{\partial^2 R_1}{\partial \varphi_1 \partial \varphi_2} < 0, \text{ the efforts}$$

of the two parties are strategic substitutes. When the designer increases their effort, the marginal gain for the total contractor to invest more is diluted, so the total contractor will reduce their own effort; the second-order derivative with respect to itself is $\frac{\partial^2 R_1}{\partial \varphi_1^2} < 0$.

Plugging in gives: $\frac{dBR_1}{d\varphi_2} < 0$ and $BR_1(\varphi_2)$, which are strictly decreasing functions.

Similarly, $\frac{dBR_2}{d\varphi_1} < 0$ and $BR_2(\varphi_1)$ are also strictly decreasing.

Step 3: There is only one intersection point on the $2D$ plane.

In the $2D$ feasible region D , there are two curves: $L_1: \varphi_1 = BR_1(\varphi_2)$ and $L_2: \varphi_2 = BR_2(\varphi_1)$. Both are strictly decreasing and continuous, and within a bounded rectangular area, they can meet at most once. According to 2.4.4, an equilibrium must exist, so the two curves meet at exactly one point. That single point is the lower-level Nash equilibrium, and the lower-level pure strategy Nash equilibrium is mathematically unique.

Explanation of engineering management: If there are multiple Nash equilibria, both parties may disagree on their optimal investment, making it easy for mass production and process optimization to stall. The unique equilibrium property derived in this paper ensures that after both parties sign a fixed, phased revenue-sharing contract, they have a unified and stable expectation of their optimal working hours and R&D investment. This removes cooperation game conflicts from the root of the contract, and the mathematical results can directly help design risk management for military contracts.

2.4.6. Explanation of Theory and Numerical Connection

On the theoretical side, relying on Brouwer's fixed-point theorem, the existence of equilibrium was proven. By using concavity/convexity and implicit function differentiation, the monotonicity of optimal responses was fully derived, strictly proving the uniqueness of equilibrium and eliminating the multi-equilibrium splitting problem in single-layer games. For numerical verification, Section 6.2.2 sets up five sets of completely different initial effort combinations (10, 10), (30, 50), (60, 20), (80, 40), (20, 90) to run Bi-PSO iterative simulations. All initial values eventually converge to the same set of equilibrium efforts ($\varphi_1^* = 29.79$, $\varphi_2^* = 85.79$), with all parties' net profits exactly matching and no multiple solution splits. Combining theory and simulation, the analytical derivation mathematically guarantees a unique equilibrium, while the multiple initial value numerical experiments confirm equilibrium stability from the simulation standpoint, forming a complete dual evidence system for existence and uniqueness.

From a logical standpoint, the analytical derivation guarantees the uniqueness of the equilibrium mathematically, while multi-initial-value numerical simulations verify the stability of the equilibrium from the perspective of algorithm implementation. Together, they form a complete dual-proof system for the existence and uniqueness of the equilibrium. The theoretical proof answers whether the model has a stable cooperative strategy, and the multi-initial-value simulations check if multiple feasible equilibrium solutions could appear in actual calculations. Both support each other, ensuring that the staged profit-sharing ratios and equilibrium effort levels obtained from this model can be directly applied to the quantitative assessment of military equipment contracts. The related mathematical derivations have practical engineering value and are not just purely theoretical exercises.

To unify symbol definitions throughout the paper and facilitate formula replication and reading, all decision variables, policy parameters, endogenous functions, and algorithm parameters are summarized in the complete comparison table below in Table 1.

Table 1. Definition of model variables and parameters.

Symbol	Variable/Parameter Meaning	Dimension/Unit
φ_1	Effort balance of R&D and production by the general contracting unit	Dimensionless
φ_2	Effort balance of design unit in planning and follow-up production	Dimensionless
λ_1	Profit-sharing ratio for the design unit in the first phase	Dimensionless
λ_2	Profit-sharing ratio for the second stage reward and punishment design unit	Dimensionless
C_t	Target order price determined at the military product project approval stage	Ten thousand yuan
C_a	Final expected price for the first phase of military products	Ten thousand yuan
α	Incentive constraint coefficient for the first phase of military products	Dimensionless
β	Comprehensive evaluation reward ratio for the second phase of military products	Dimensionless
Q_1	Production quantity in the first phase	Set
Q_2	Order quantity in the second phase	Set
μ	Cost-reduction efficiency coefficient of the general contracting unit	Dimensionless
ν	Technical cost-reduction efficiency coefficient of the design unit	Dimensionless
h_1	Effort cost coefficient of the general contracting unit	Dimensionless
h_2	Effort cost coefficient of the design unit	Dimensionless
$f(\varphi_1, \varphi_2)$	Cost reduction brought by joint efforts (concave function)	Dimensionless
P_1	Procurement price per unit in the first phase	Ten thousand yuan
P_2	Procurement price per unit in the second phase	Ten thousand yuan
R_1	Net profit of the general contracting unit after deducting effort costs	Ten thousand yuan
R_2	Net profit of the design unit after deducting effort costs	Ten thousand yuan
R_0	Total profit of the joint entity of general contracting unit and design unit	Ten thousand yuan
<i>WeightedFit</i>	Upper-level weighted optimization objective: R_0 and $0.1 \cdot \min(R_1, R_2)$	Ten thousand yuan

N	PSO population size	Piece
$MaxIter$	PSO maximum iterations	Time
ω	PSO inertia weight	Dimensionless
c_1, c_2	P_1, P_2 learning factors	Dimensionless
$seed$	Random seed (fixed 12,345, to ensure reproducibility)	Dimensionless
R_1^0	General contractor unit benchmark with no allocated profit	Ten thousand yuan
R_2^0	Design unit benchmark without allocated profits	Ten thousand yuan

3. Model Assumptions and Function Theory Support

3.1. Basic Assumptions About the Behavior of Game Players

Hypothesis 1. Both the general contracting unit and the design unit are completely rational and risk-neutral collaborative entities. They form an equipment development interest consortium and each strives to maximize its own net profit. The general contracting unit, as the project leader, simultaneously coordinates the overall alliance benefits and the fairness of long-term cooperation [24].

Hypothesis 2. The game information is completely symmetric. The general contracting units and design units know each other's efficiency coefficients, effort cost coefficients, and all parameters of the two-stage pricing policy, with no hidden information or differences in information advantage.

Hypothesis 3. The military only acts as an external policy-making body and does not get involved in profit distribution games. Fixed incentive coefficients $\alpha \in [0.5, 0.7]$ and mass production rewards and penalties $\beta \in [-0.05, 0.05]$ are rigid constraints outside the model and do not change with decisions.

3.2. Effort and Cost Function Assumptions

Hypothesis 4. The effort levels of the general contracting unit and the design unit φ_1, φ_2 are continuous non-negative measurable variables, with effort costs being a quadratic concave function

$$C_i = \frac{1}{2} h_i \varphi_i^2 \text{ and marginal input costs increasing [29,30].}$$

3.3. Assumptions of the Joint Cost-Reduction Output Function

Hypothesis 5. Joint efforts by both parties can reduce the expected procurement price of the equipment, with the price reduction $f(\varphi_1, \varphi_2)$ following a continuously increasing concave function, satisfying first $f' > 0$ and second $f'' < 0$ derivative conditions, meaning that the marginal cost-saving benefit of effort keeps decreasing. Select the exponential form $f = I - e^{-\mu\varphi_1 - \nu\varphi_2}$ as the base function for simulation, and use the logarithmic function for robustness testing. This type of function is widely used in R&D output modeling [12].

3.4. Policy and Equilibrium Constraint Assumptions

Hypothesis 6. The profit-sharing ratio for the two stages satisfies the interval constraint: $0 \leq \lambda_1 \leq 1$, $0 \leq \lambda_2 \leq 1$. λ_1 represents the design unit's profit share in the first stage (prototype development stage), and λ_2 represents the design unit's profit share in the second stage (mass

production stage). The upper-level optimization searches within this feasible domain throughout the process.

Hypothesis 7. complete individual rationality constraints: after implementing profit sharing, the net profits for both parties are $R_1(\lambda_1, \lambda_2, \varphi_1^*, \varphi_2^*)$ and $R_2(\lambda_1, \lambda_2, \varphi_1^*, \varphi_2^*)$. To ensure cooperation is feasible, the double individual rationality constraints must be met: the net profit after cooperation must be greater than the baseline profit without sharing, and the net profit must be non-negative.

$$\begin{cases} R_1(\lambda_1, \lambda_2, \varphi_1^*, \varphi_2^*) \geq R_1^0 \\ R_2(\lambda_1, \lambda_2, \varphi_1^*, \varphi_2^*) \geq R_2^0 \\ R_1 \geq 0, R_2 \geq 0 \end{cases} \quad (1)$$

In the formula, R_1^0 and R_2^0 are the baseline net profits of both parties under a no-profit-distribution mechanism, and all simulations in the text simultaneously satisfy the above double individual rationality constraints.

3.5. Assumptions of Game Timing

Hypothesis 8. Game follows a two-level Stackelberg sequential decision timing: As the leading player in the upper-level game, the general contracting unit first determines the two-stage revenue shares λ_1 and λ_2 ; given any set of sharing ratios, the general contractor and the design unit simultaneously act as the two followers in the lower-level Nash game, independently finding their own optimal equilibrium efforts $\varphi_1^*(\lambda_1, \lambda_2)$, $\varphi_2^*(\lambda_2, \lambda_1)$.

3.6. Limitations of Idealized Model Assumptions and Ideas for Expansion

Sections 3.1–3.5 of this article propose basic assumptions like complete information, risk neutrality, binary players, and deterministic parameters. While these simplify mathematical derivations and ensure equilibrium can be solved, they deviate from real military supply chains. The specific limitations and possible improvements are as follows:

1. Limitation of the complete information assumption: In actual development, design units' process costs and cost-reduction efficiencies are private information, so the general contracting unit cannot fully observe the other party's real input, leading to adverse selection issues. Later studies could introduce incomplete information Bayesian Stackelberg games and design information-screening contracts.
2. Limitation of the risk-neutral assumption: Military projects face risks like technical failures, fluctuations in production pass rates, and cost overruns. Both design units and general contracting units generally prefer to avoid risk. Future work could incorporate CVaR and mean-variance risk utility functions to build robust profit-sharing models under risk constraints.
3. Limitation of binary players: The real military supply chain includes supporting suppliers and third-party testing institutions, creating multi-level interest games among multiple parties. Later research could expand to multi-agent evolutionary games and build a three-level game framework (military–general contracting unit–design unit–supplier).
4. Limitation of deterministic parameters: Raw material prices, order quantities, and policy incentives are subject to random disturbances, which the current static model does not capture. Fuzzy stochastic parameters could be introduced to construct a stochastic bi-level optimization model.

4. Technical Background

Military products implement a two-stage pricing system consisting of initial batch pricing and batch production rewards and penalties. Under the research-production separation model, the revenues of both the design units and the main contracting units depend on the two-stage price formation mechanism of military products. Based on the current military product pricing policy, this paper derives the pricing formulas for the initial batch and batch production stages, clarifies the composition and distribution logic of the two-stage reward profits, and introduces efficiency coefficients and effort cost coefficients to characterize the behavioral features of the entities, providing a theoretical foundation for subsequent game modeling, profit function construction, and Particle Swarm Optimization.

4.1. First Batch Price of Military Supplies

According to military product pricing rules, the price of the first batch of purchases is composed of the benchmark cost and the incentive profit:

$$P_1 = C_a + (C_t - C_a) \times \alpha \quad (2)$$

In the formula, P_1 is the procurement price for a single set of military goods in the first batch; C_t is the target order price determined at the project approval stage of the military goods; C_a is the final expected price for the first batch of the military goods; α is the incentive constraint coefficient for the first batch of military goods, generally taken as 0.5; and $(C_t - C_a) \times \alpha$ is the reward profit for the first batch.

If the general contracting unit wants to obtain more reward profits, on one hand, it strengthens project management itself, for example, by striving to optimize production technology, improve personnel skills and quality, and perfect internal control systems, trying to control the cost of military products while ensuring high quality; on the other hand, it provides incentives to the design units, allocating an appropriate portion of the reward profits to encourage design units to carry out 'six-characteristics' design and the research and use of low-cost technologies during the design phase. For design units, to obtain more design returns, as long as the proportion of reward profit allocation provided by the general contracting unit is appropriate, they will focus on cost control from the source during the design of military products, ensuring the economic efficiency of military product manufacturing.

Suppose both parties work together to reduce the expected price of military products, with the effort level of the general contracting unit being φ_1 , and the effort level of the design unit being φ_2 . The proportion by which the expected price ultimately decreases relative to the order target price through effort is $f(\varphi_1, \varphi_2)$, which is a continuously increasing concave function of φ_1 and φ_2 , i.e., $f'(\varphi_i) > 0$, $f''(\varphi_i) < 0$, assuming $f(\varphi_1, \varphi_2) = 1 - e^{-\mu\varphi_1 - \nu\varphi_2}$, where μ is the efficiency coefficient of the general contracting unit, and ν is the efficiency coefficient of the design unit. Then, the final expected price is $C_a = [1 - f(\varphi_1, \varphi_2)]C_t$. Substituting into Equation (1) gives:

$$P_1 = [1 - f(\varphi_1, \varphi_2) + f(\varphi_1, \varphi_2)\alpha]C_t \quad (3)$$

In the formula, P_1 , C_t , and α are the same as in Formula (1), $f(\varphi_1, \varphi_2)$ representing the proportion by which the final expected price falls below the target order price.

When designing military products, if the production quantity in the first phase of military products is assumed to be Q_1 , then the reward profit for the first phase is $(C_t - C_a) \times \alpha \times Q_1 = f(\varphi_1, \varphi_2)C_t\alpha Q_1$. If the general contracting unit and the design unit

agree on the sharing ratio of the first phase reward profit as λ_1 , then the reward profit the design unit receives in the first phase is $\lambda_1 f(\varphi_1, \varphi_2) C_1 \alpha Q_1$, and the reward profit the general contracting unit receives in the first phase is $(1 - \lambda_1) f(\varphi_1, \varphi_2) C_1 \alpha Q_1$.

4.2. Price During Mass Production Stage of Military Products

During the mass production phase, price adjustments for rewards and penalties are implemented based on the first batch price and comprehensive evaluation results:

$$P_2 = P_1 \times (1 + \beta) \quad (4)$$

In the formula, P_2 is the purchase price of a single unit of military equipment in the second phase, P_1 is the purchase price of a single unit in the first phase, and β is the reward rate determined after the comprehensive evaluation in the second phase, assumed to range between -5% and 5%.

When designing military products, if we assume the production quantity for the second phase is Q_2 , then the bonus profit for the second phase is $P_1 \beta Q_2$. By substituting Equation (3) into the calculation, the bonus profit for the second phase is $[1 - f(\varphi_1, \varphi_2) + \alpha f(\varphi_1, \varphi_2)] C_1 \beta Q_2$. If the total contractor and the design unit agree on the sharing ratio of the second phase bonus profit as λ_2 , then $\lambda_2 [1 - f(\varphi_1, \varphi_2) + \alpha f(\varphi_1, \varphi_2)] C_1 \beta Q_2$ is the bonus profit the design unit gets in the second phase, and $(1 - \lambda_2) [1 - f(\varphi_1, \varphi_2) + \alpha f(\varphi_1, \varphi_2)] C_1 \beta Q_2$ is the bonus profit the total contractor gets in the second phase.

Based on the two stages above, the unit price of this military product is

$$P_x = (P_1 \times Q_1 + P_2 \times Q_2) / (Q_1 + Q_2) \quad (5)$$

4.3. Revenue Model

Assuming that the various efforts made by the general contractor and the design unit can be equated to monetary costs, that is, investment, as the level of effort increases, the corresponding growth rate of monetary costs will accelerate. Suppose the investment effort cost functions of the general contractor and the design unit are represented as:

$$C_i = \frac{1}{2} h_i \varphi_i^2 \quad (i=1,2) \quad (6)$$

In the formula, h_1 and h_2 are the effort cost coefficients of the general contracting unit and the design unit, respectively.

Under the military product design return mechanism, the design unit and the general contracting unit can be regarded as a unified consortium, working together to maximize their joint benefits. Assuming R_0 is the total profit of the two parties combined, then

$$R_0 = f(\varphi_1, \varphi_2) C_1 \alpha Q_1 + [1 - f(\varphi_1, \varphi_2) + \alpha f(\varphi_1, \varphi_2)] C_1 \beta Q_2 - \frac{1}{2} h_1 \varphi_1^2 - \frac{1}{2} h_2 \varphi_2^2 \quad (7)$$

The total profit of the general contracting unit is

$$R_1 = (1 - \lambda_1) f(\varphi_1, \varphi_2) C_1 \alpha Q_1 + (1 - \lambda_2) [1 - f(\varphi_1, \varphi_2) + \alpha f(\varphi_1, \varphi_2)] C_1 \beta Q_2 - \frac{1}{2} h_1 \varphi_1^2 \quad (8)$$

The total profit of the design company is

$$R_2 = \lambda_1 f(\varphi_1, \varphi_2) C_1 \alpha Q_1 + \lambda_2 [1 - f(\varphi_1, \varphi_2) + \alpha f(\varphi_1, \varphi_2)] C_1 \beta Q_2 - \frac{1}{2} h_2 \varphi_2^2 \quad (9)$$

All in all, the existing constraints are

$$\begin{cases} 0.5 \leq \alpha \leq 0.7, -0.05 \leq \beta \leq 0.05 \\ 0 \leq \lambda_1 \leq 1, 0 \leq \lambda_2 \leq 1 \\ 0 \leq \varphi_1 \leq 100, 0 \leq \varphi_2 \leq 100 \\ R_1 \geq 0, R_2 \geq 0 \end{cases} \quad (10)$$

5. Two-Level Stackelberg Game and PSO Nested Optimization Method

5.1. Two-Level Optimization Math Model

5.1.1. Lower-Level Subproblem (Given λ_1, λ_2 , Optimize φ_1, φ_2)

$$\begin{cases} \max_{\varphi_1} R_1(\varphi_1 | \lambda_1, \lambda_2) \\ \max_{\varphi_2} R_2(\varphi_2 | \lambda_1, \lambda_2) \\ s.t. 0 \leq \varphi_1, \varphi_2 \leq 100, R_1 \geq R_1^0, R_2 \geq R_2^0, R_1 \geq 0, R_2 \geq 0 \end{cases} \quad (11)$$

In the formula, R_1 is the total profit of the main contracting unit, as shown in Formula (8), R_2 is the total profit of the design unit, as shown in Formula (9), R_1^0 and R_2^0 are the baseline net profits of both parties without any profit-sharing mechanism, as shown in Equation (1); φ_1 is the effort level of the main contracting unit, φ_2 is the effort level of the design unit, λ_1 is the design unit's first-stage profit-sharing ratio, and λ_2 is the design unit's second-stage profit-sharing ratio.

5.1.2. Upper-Level Subproblems (Anticipate Lower-Level Equilibrium $\varphi_1^*(\bullet), \varphi_2^*(\bullet)$, Optimize the Revenue-Sharing Ratio)

Define two types of upper-level optimization goals:

1. Theoretical benchmark goal

$$\begin{cases} \max_{\lambda_1, \lambda_2} R_0(\varphi_1^*(\lambda_1, \lambda_2), \varphi_2^*(\lambda_1, \lambda_2), \lambda_1, \lambda_2) \\ s.t. 0 \leq \lambda_1, \lambda_2 \leq 1, 0.5 \leq \alpha \leq 0.7, -0.05 \leq \beta \leq 0.05 \end{cases} \quad (12)$$

In the formula, R_0 represents the total profit of the union, as shown in Formula (7), $\varphi_1, \varphi_2, \lambda_1, \lambda_2$ as shown in Formula (11), α is the incentive constraint coefficient for the first stage of military products, and β is the reward ratio determined after the comprehensive evaluation in the second stage.

2. Engineering Weighted Goals

$$\begin{cases} \max_{\lambda_1, \lambda_2} \text{WeightedFit} = R_0 + 0.1 \min(R_1, R_2) \\ s.t. 0 \leq \lambda_1, \lambda_2 \leq 1, 0.5 \leq \alpha \leq 0.7, -0.05 \leq \beta \leq 0.05 \end{cases} \quad (13)$$

In the formula, all the symbols are the same as those in Equation (12). Later in Section 6.2.1, an additional control group was set up to only maximize R_0 without any fairness term. The simulation results show that the two-level framework can only ensure the total profit of the coalition is maximized; the fact that both sides have equal profits is an added effect of the weighted fairness constraint, and cannot be entirely attributed to the two-level game structure.

5.2. Basic Principles and Parameter Settings of Particle Swarm Optimization

5.2.1. Basic Principles and Parameter Settings of Particle Swarm Optimization

Particle Swarm Optimization (PSO) simulates the cooperative foraging behavior of birds, mapping each particle to a set of decision variables. Particles iteratively update their velocity and position within the feasible domain by tracking the individual best $pbest$ and the global best $gbest$, achieving global optimization of complex nonlinear functions. The algorithm has a fast convergence speed and strong robustness, making it suitable for the multi-variable, multi-constraint revenue allocation optimization problem in this paper.

Particle velocity and position update formulas:

$$\begin{cases} v_{i,d}^{k+1} = \omega v_{i,d}^k + c_1 r_1 (pbest_{i,d} - x_{i,d}^k) + c_2 r_2 (gbest_d - x_{i,d}^k) \\ x_{i,d}^{k+1} = x_{i,d}^k + v_{i,d}^{k+1} \end{cases} \quad (14)$$

In the formula, ω is the inertia weight; c_1 and c_2 are learning factors; r_1 and r_2 are uniformly random numbers in the interval $[0, 1]$; $x_{i,d}$ is the particle position, corresponding to the decision variable $\varphi_1, \varphi_2, \lambda_1, \lambda_2$; and $v_{i,d}$ is the particle iteration velocity.

Parameter Settings:

1. Computing environment: MATLAB R2025a, Intel i9 processor, Windows 11;
2. PSO hyperparameters: population size $N = 100$, maximum iterations $MaxIter = 200$, inertia weight = 0.6, learning factor = 1.6;
3. Dual stopping criteria: either 200 iterations reached or total profit change $< 10^{-4}$ for 20 consecutive iterations;
4. Fixed random seed: $seed = 12,345$, out-of-bound variables are reset by boundary truncation;
5. Constraint handling: individuals violating constraints get a heavy penalty on fitness and are removed.

5.2.2. Double-Layer Nested PSO Pseudocode

This article does not make any innovative improvements to the underlying formulas for PSO particle velocity or position updates. It just designs a nested inner–outer iterative process (called Bi-PSO) based on the business timing of Stackelberg—where the leader allocates first and the follower finds the Nash equilibrium later. It is an application-oriented adaptation for phased benefit allocation problems in the military industry, without any theoretical innovation in intelligent algorithms. Its main value lies in restoring the sequential decision logic of leader–follower games.

Input: fixed parameters $C_1, \alpha, \beta, Q_1, Q_2, \mu, v, h_1, h_2$ for defense industry, and PSO hyperparameters $N, MaxIter, \omega, c_1, c_2$

Set random seed to 12,345, initialize upper-level particles $X = [\lambda_1, \lambda_2]$ and their velocities V

Upper-level iteration loop (1– $MaxIter$):

For each upper-level particle $[\lambda_1, \lambda_2]$:

Solve lower-level Nash sub-PSO to get equilibrium effort φ_1^*, φ_2^*

Initialize lower-level particles $[\varphi_1, \varphi_2]$, iterate until convergence condition is met (profit change $< 10^{-4}$ for 20 consecutive generations)

Check individual rationality constraint: apply huge penalty if violated

Output lower-level equilibrium φ_1^*, φ_2^* , calculate $R_1, R_2, R_0, WeightedFit$

Update upper-level particles' pbest, gbest, iterate velocity/position, apply boundary corrections $\lambda_1, \lambda_2 \in [0, 1]$

Stopping criterion: if WeightedFit changes <10 for 20 consecutive generations, stop early

Output: optimal λ_1^*, λ_2^* , equilibrium φ_1^*, φ_2^* , profits R_1, R_2 , and total alliance profit R_θ

5.3. Comparison and Analysis of Multiple Optimization Algorithms

To test how well the double-layer nested particle swarm (Bi-PSO) works in solving the two-layer Stackelberg military–industrial revenue game model, we selected five algorithms for comparison: gradient-based interior point method (IPM), standard genetic algorithm (GA), simulated annealing (SA), differential evolution (DE), and adaptive-weight PSO [29,30]. All algorithms shared the same double-layer nested structure: the outer layer optimizes λ_1 and λ_2 , while the inner layer solves the Nash equilibrium for both parties with fixed revenue splits. The main difference among the five algorithms lies only in how they handle the sequence of moves in the game.

1. This Bi-PSO: strictly follows the Stackelberg sequential decision-making, where the upper level first determines the profit sharing, and the lower level then simultaneously solves for the equilibrium efforts of both parties, fully matching the 'general contractor decides first, design responds later' game sequence.
2. GA, SA, DE, Adaptive PSO: Although the lower-level equilibrium solution module is nested, during optimization λ_1 , λ_2 , φ_1 and φ_2 are treated as synchronous parallel decision variables, which weakens the Stackelberg leader–follower timing constraints. At the game theory level, there is a deviation from equilibrium, making it only suitable for single-point benchmark cases and unable to support large-scale batch parameter simulations for 2D heat maps.

We set up four quantitative evaluation metrics: the alliance's maximum total profit under 20 independent optimizations, the average number of iteration steps, the profit standard deviation, and the average computation time per run. Additionally, we added the number of invalid equilibrium solutions (where the net profit of total contractors/designers is less than 0) as a supplementary measure of constraint compliance. The experimental results are shown in Figure 1 and Table 2.

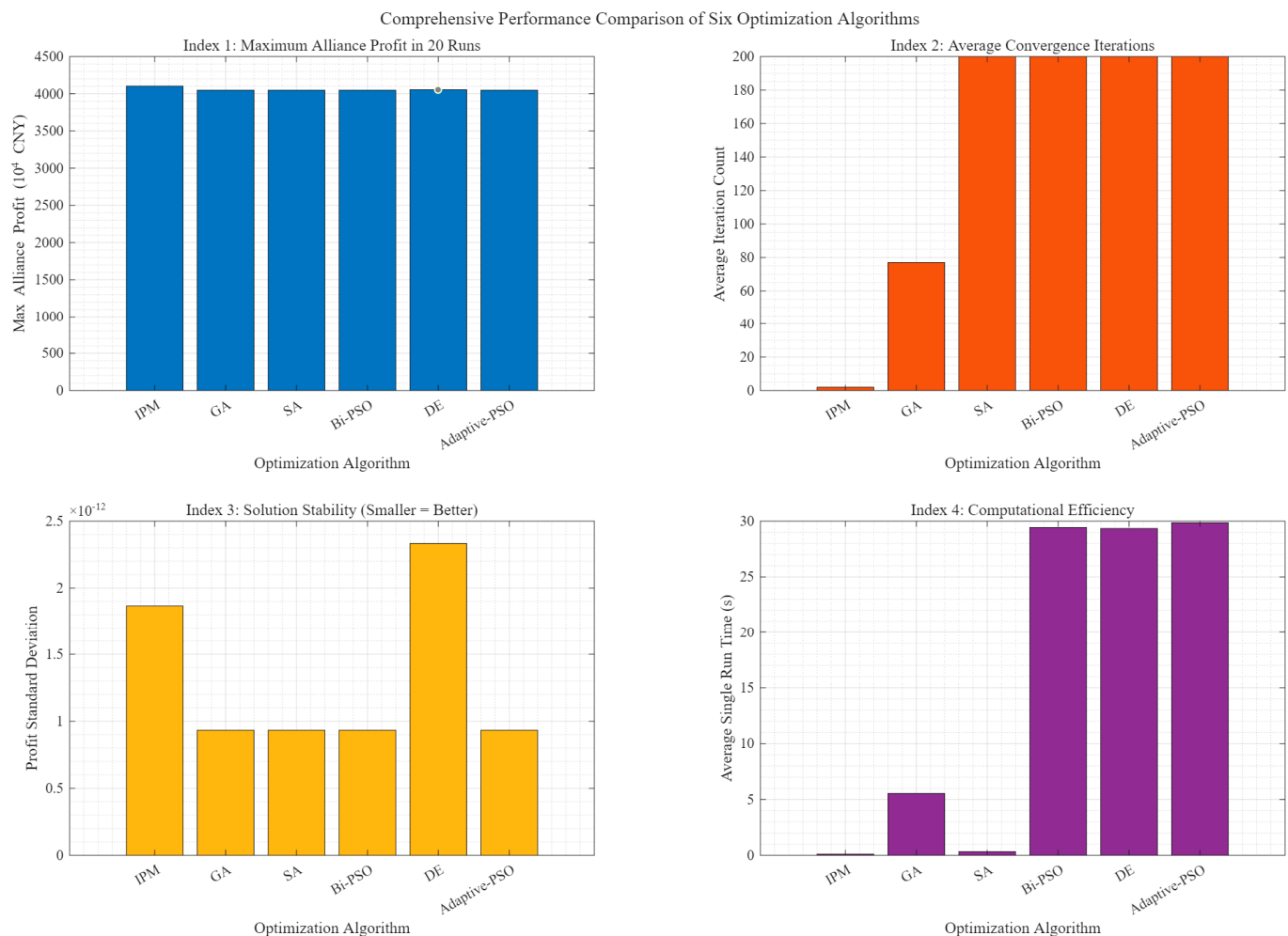


Figure 1. Comparison chart of four indicators for six types of optimization algorithms.

Table 2. Comparison of various metrics for six types of optimization algorithms.

Algorithm	Total Profit of 20-Round Optimal Consortium (10,000 Yuan)	Average Number of Iterations (Times)	Standard Deviation of 20 Independent Runs (Ten Thousand Yuan)	Average Time Per Run (Sec-onds)	Number of Times Invalid Output Occurs
IPM	4103.18	2	0.0000	0.1082	20
GA	4047.66	77	0.0000	5.5401	0
SA	4047.66	200	0.0000	0.3393	0
Bi-PSO	4047.66	200	0.0000	29.4292	0
DE	4054.48	200	0.0000	29.3426	0
Adaptive-PSO	4047.66	200	0.0000	29.8552	0

Comparing the repeated simulation results of six types of algorithms shows that gradient-based IPM requires the fewest iterations to converge. However, due to the non-convex structure of the model and multiple constraints, all 20 independent repeated tests produced invalid local optima that violated individual rationality constraints, making it less practical for engineering. GA, SA, DE, adaptive PSO, and the Bi-PSO in this paper all belong to evolutionary global optimization algorithms, and each algorithm includes a subroutine for solving lower-level Nash equilibria. From a numerical perspective, DE has the strongest global search capability, achieving a total alliance profit of 4054.48 ten thousand yuan; GA, SA, adaptive PSO, and Bi-PSO stabilize total alliance profits at 4047.66 ten thousand yuan under equilibrium conditions. With fixed simulation conditions, the standard

deviation of 20 independent runs for the five evolutionary algorithms approaches zero, indicating good iteration stability. But, looking at it from the standpoint of two-level Stackelberg game theory compliance: GA, SA, DE, and adaptive PSO treat the upper-level allocation ratios λ_1 , λ_2 and lower-level efforts ϕ_1 , ϕ_2 as parallel synchronous optimization variables during iterations, weakening the sequential relationship of “leader makes decisions first, follower responds later.” These algorithms only pursue numerical optimization of the objective function, and the resulting solutions cannot guarantee Nash equilibrium conditions, showing theoretical flaws. Only Bi-PSO in this paper uses a hierarchical iterative structure: the outer iteration optimizes the two-stage profit-sharing ratio, and each fixed allocation is passed to the inner layer to independently solve the lower-level Nash equilibrium, strictly aligning with the standard mathematical definition of a leader–follower game. Although a single Bi-PSO simulation takes up to 29.4292 s, which is more computationally expensive than GA and SA, in large-scale parameter sweeps and 2D heatmap grid simulations, the completeness of the game logic is prioritized over local numerical indicators. Considering the core standards of game theory self-consistency and constraint compliance, all subsequent numerical simulations and parameter sensitivity analyses use Bi-PSO.

The Bi-PSO constructed in this paper does not introduce any fundamental innovations in particle velocity or position update rules. The core improvement idea is: embedding double-layer leader–follower game timing constraints into the standard particle swarm iteration framework. This is an application-oriented adaptation for phased military industrial benefit distribution scenarios, rather than a completely new intelligent optimization algorithm. The differences in output results of the six types of algorithms stem from two independent mechanisms: the numerical optimization mechanism and the game timing mechanism:

- (1) Numerical optimization mechanism. Gradient-based IPM relies on local gradient information for iteration. The model in this paper includes piecewise policy constraints, rigid dual individual rationality boundaries, and non-convex payoff functions. The feasible domain contains numerous local extrema, causing gradient algorithms to easily fall into invalid solutions that violate the cooperation premise, and all 20 simulations failed. GA, SA, PSO, and DE are population-based global stochastic search algorithms, and repeated simulations can all output feasible solution combinations that satisfy the constraints. Among them, the DE differential mutation operator has a stronger ability to expand the search range, so the alliance benefit values are slightly higher than those of the other evolutionary algorithms. However, this advantage is only numerical and does not guarantee satisfaction of the game equilibrium conditions.
- (2) Game timing mechanism (core difference). GA, SA, DE, and adaptive PSO simultaneously optimize upper-level allocation parameters and lower-level effort variables within the same loop, eliminating the Stackelberg sequential decision order. Mathematically, this weakens the leader–follower hierarchy, meaning the obtained optimal combination is only the global numerical optimum and cannot guarantee satisfaction of the Nash equilibrium response logic, which is a theoretical flaw. In contrast, Bi-PSO enforces a hierarchical loop: the outer layer only optimizes the phased benefit proportions, and after an allocation scheme is given, the inner layer independently solves the Nash equilibrium and feeds back the objective function. This replicates the business process where “the general contracting unit first signs the benefit contract, and the design unit subsequently adjusts the R&D input,” and the results strictly conform to the definition of a two-layer Stackelberg game, which other evolutionary algorithms lack.

In summary, DE only has a numerical optimization advantage, but its underlying game timing logic has inherent flaws. Bi-PSO takes longer for a single computation, but in large-scale heatmaps and parameter grid simulation scenarios, theoretical self-consistency of the game is far more important than minor numerical gains. Therefore, all simulations in this paper use Bi-PSO.

6. Numerical Simulation Experiments and Robustness Analysis

6.1. Experimental Parameter Settings

This article uses a certain type of gas turbine project as a simulation example. During the project initiation stage, the target procurement price for the equipment was set at a = 30000 ten thousand yuan; in the first phase, prototype trials were carried out with a production of $Q_1 = 2$ units, and in the second phase, mass production of $Q_2 = 3$ units was carried out along with a comprehensive performance evaluation. The incentive constraint coefficient for the first phase was $\alpha = 0.5$; after completing the mass production evaluation, the reward and punishment ratio for the second phase was $\beta = 0.03$. The other basic parameters were set as $\mu = 0.0010$, $\nu = 0.0018$, $h_1 = 0.8$, and $h_2 = 0.5$.

As a point of reference: under the traditional model, no profit-sharing mechanism is set up for the design units. The design units only deliver the drawings and do not put extra effort into optimizing the mass production process, making the decision variable $\varphi_2 = 0$, while keeping the other basic parameters unchanged. The calculated benchmark profit of the total manufacturing unit in this scenario is $R_1^0 = 1695.04$ ten thousand yuan.

The benchmark simulation parameters come from desensitized data of a certain gas turbine project, and all simulation conditions are reproducible. The parameters are divided into two categories based on their sources:

- (1) Policy-driven external parameters (C_1 , Q_1 , Q_2 , α , β): taken from the gas turbine development task book, equipment procurement contracts, and industry standards, serving as the predetermined constraints of the project;
- (2) Key efficiency and cost characteristic parameters (μ , ν , h_1 , h_2): based on three consecutive years of prototype trials and full-process desensitized labor and material records from mass production, samples were obtained; using joint cost-reduction benefits as the dependent variable and R&D labor input from both parties as independent variables, a multiple nonlinear regression was conducted to fit an exponential production function to get μ , ν ; combined with R&D labor costs and equipment depreciation data, a quadratic effort cost function was regressed to calibrate and obtain h_1 , h_2 .

The benchmark scenario profit R_1^0 , R_2^0 is calculated by inputting a fixed design unit effort level $\varphi_2 = 0$ into the project's historical static cost data, which corresponds to the industry's traditional collaboration model of 'the design unit only delivers drawings and does not participate in mass production process iteration and optimization.'

6.2. Simulation Results

6.2.1. Analysis of Loss Function Convergence

This article uses the Particle Swarm Optimization (PSO) algorithm to perform global optimization on the two-stage revenue-sharing model. The algorithm parameters are set as follows: population size $N = 100$, maximum number of iterations $MaxIter = 200$,

inertia weight $\omega = 0.6$, and learning factor $c_1 = c_2 = 1.6$, and the results are made uniquely reproducible by fixing the random seed. The loss function is shown in Figure 2.

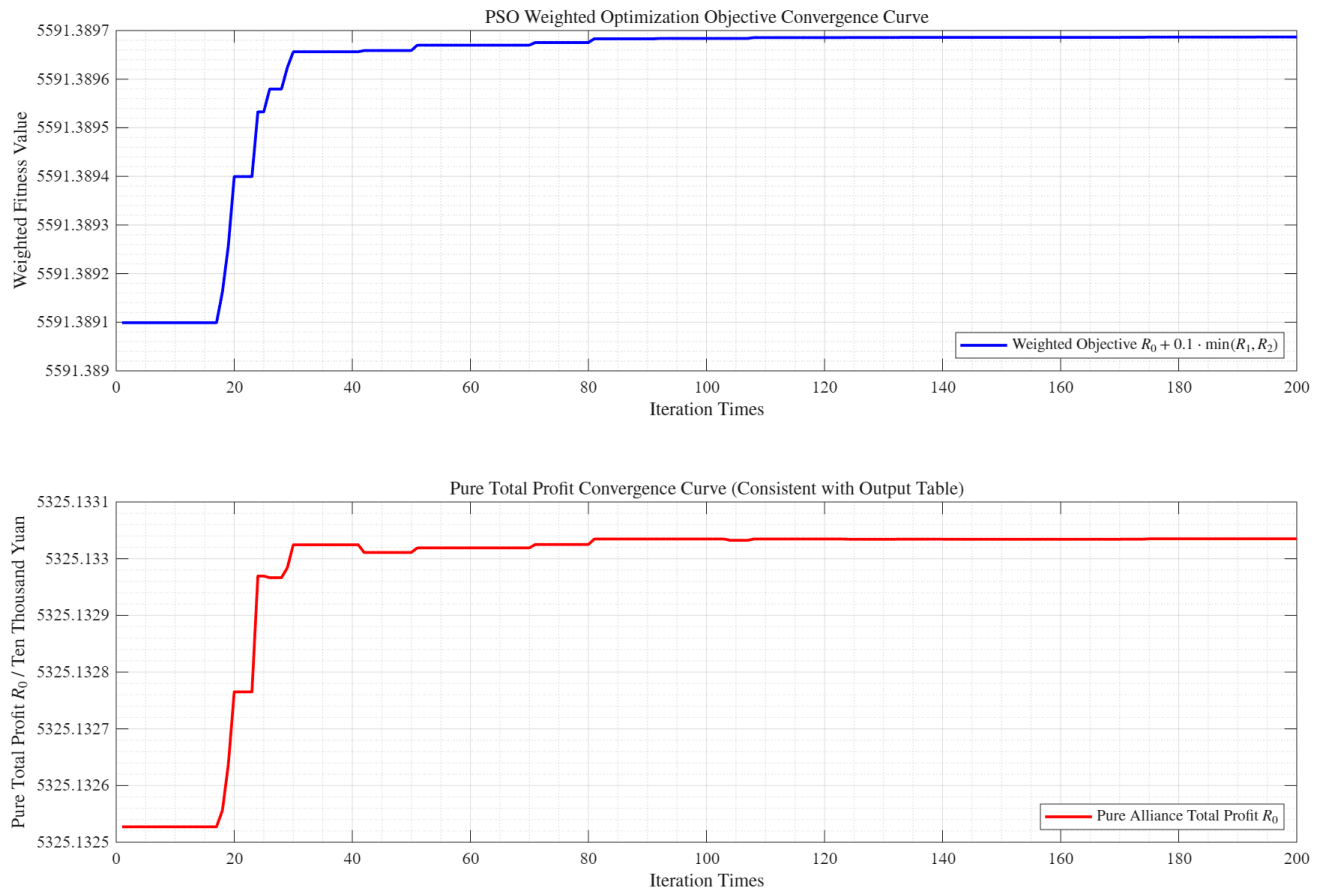


Figure 2. Double-layer nested PSO convergence curves. In the top chart, the blue curve shows the convergence of the upper-layer weighted optimization objective; in the bottom chart, the red curve shows the convergence of the original total profit R_0 of the industry chain. The lower-layer Nash equilibrium is only recalculated when the globally optimal profit-sharing ratio is updated, so the curve appears as stepped plateaus. The final stable values match the equilibrium result table exactly.

With a fixed random seed and 20 independent simulation runs, the algorithm converges to a stable and feasible equilibrium solution after iteration, as shown in Table 3.

Table 3. Global optimal solution results of the two-stage incentive model for military products. Note: All monetary values in the table are in ten thousand Chinese yuan (CNY).

Indicator	Symbol	Optimal Value
Initial Phase Revenue-Sharing Ratio	λ_1	0.49
Mass Production Phase Revenue-Sharing Ratio	λ_2	0.82
General Contracting Unit Optimal Effort Level	φ_1	29.79
Design Unit Optimal Effort Level	φ_2	85.79
General Contracting Unit Optimal Profit	R_1	2662.57
Design Unit Optimal Profit	R_2	2662.57
Consortium Optimal Total Profit	R_0	5325.13

A double-layer nested PSO is used to solve the equipment development benefit-sharing Stackelberg game model. The upper layer optimizes a weighted function $WeightedFit = R_0 + 0.1 \times \min(R_1, R_2)$, balancing the total industry chain benefits and the fairness of profits between party A and party B, while the lower layer solves the Nash equilibrium of R&D efforts for both sides. Simulations ran 200 iterations, and the convergence curve is shown in Figure 2, with the algorithm fully converging to the global optimal equilibrium after 80 generations. The optimal allocation ratio is a first-phase design unit profit share $\lambda_1 = 0.49$ and second-phase $\lambda_2 = 0.82$; at equilibrium, the optimal R&D efforts for the total contractor and design unit are 29.79 and 85.79, respectively, with both parties having equal net profits of 2662.57 ten thousand yuan, and the total pure industry chain benefit is 5325.13 ten thousand yuan. The results meet individual rational participation constraints, avoid boundary extreme distortion, and consider both overall efficiency and fair cooperative benefits.

To figure out whether the equal sharing of alliance profits is inherently effective in the two-tier Stackelberg framework or if it is just an additional effect from the weighted fairness term, we set up a control simulation: we fully keep the two-tier Stackelberg game timing structure, but in the upper level, we only optimize the pure total alliance profit R_0 and remove the $0.1 \min(R_1, R_2)$ fairness weighting term. All other constraints, algorithms, and benchmark parameters remain exactly the same. The comparison of the results from the two groups of simulations is as follows:

1. No fairness comparison group (only maximizing R_0): $\lambda_1 = 0.72$, $\lambda_2 = 0.15$, $\varphi_1 = 29.79$, $\varphi_2 = 85.80$; total contractor profit $R_1 = 3175.1$ ten thousand yuan, design unit profit $R_2 = 2150.04$ ten thousand yuan, and total alliance profit $R_0 = 5325.13$ ten thousand yuan; the alliance's revenue reaches the theoretical maximum, but there is a significant profit gap between the two sides.
2. The baseline model in this article (weighted objective $WeightedFit = R_0 + 0.1 \min(R_1, R_2)$): $\lambda_1 = 0.49$, $\lambda_2 = 0.82$, $\varphi_1 = 29.79$, $\varphi_2 = 85.79$; $R_1 = R_2 = 2662.57$ ten thousand yuan, and the total alliance profit is still 5325.13 ten thousand yuan.

Comparing the results of the two experiments, we can see that the two-layer Stackelberg sequential game structure can only maximize the overall alliance profit and cannot automatically balance the benefits between party A and party B; the equal outcome of $R_1 = R_2$ is an additional optimization effect after introducing a fairness weighting term and cannot be entirely attributed to the two-layer game framework itself.

6.2.2. Lower-Level Nash Equilibrium Uniqueness Test

To numerically show that the lower-level Nash equilibrium is unique and has no multiple solution branches, this study, under the benchmark parameters, fixes the upper-level optimal allocation ratios at $\lambda_1 = 0.49$ and $\lambda_2 = 0.82$, and sets five completely different initial effort combinations $(\varphi_1^0, \varphi_2^0)$: (10, 10), (30, 50), (60, 20), (80, 40), and (20, 90). The Bi-PSO iterations were conducted independently, and the experimental results are shown in Table 4.

Table 4. Uniqueness test of lower-level Nash equilibrium (fixed sharing ratios). Note: All monetary values are in ten thousand Chinese yuan (CNY). φ_1, φ_2 = effort levels of two participants; R_1 = General contractor profit; R_2 = Design Firm Profit.

Serial Number	$(\varphi_1^0, \varphi_2^0)$	φ_1^*	φ_2^*	R_1	R_2
1	(10, 10)	29.79	85.79	2662.57	2662.57
2	(30, 50)	29.79	85.79	2662.57	2662.57
3	(60, 20)	29.79	85.79	2662.57	2662.57
4	(80, 40)	29.79	85.79	2662.57	2662.57
5	(20, 90)	29.79	85.79	2662.57	2662.57

From the simulation results in Table 4, it can be seen that the five sets of initial values all eventually converged to the equilibrium effort levels of $\varphi_1 = 29.79$ and $\varphi_2 = 85.79$, with the net profits of all parties being exactly the same, meaning there are no multiple equilibrium solutions. This numerical experiment confirms the theoretical conclusion in Section 2.4, which based on Brouwer's fixed point theorem, showed that the equilibrium is unique, proving that the bi-level model in this paper is stable and reliable.

6.2.3. Comparison of Balanced Results of Four Types of Income Distribution Mechanisms

To verify the advantages of the two-tier Stackelberg two-stage revenue-sharing mechanism proposed in this paper, three mainstream allocation schemes are set as reference scenarios: no revenue-sharing mechanism, two-stage equal 50/50 split, and single-layer Nash synchronous optimization model. All model parameters and PSO solving hyperparameters are kept completely consistent. The equilibrium simulation results of the four mechanisms are shown in Table 5.

Table 5. Comparison table of balanced results for four types of profit distribution mechanisms. Note: All monetary values are in ten thousand Chinese yuan (CNY). λ_1, λ_2 = two-stage revenue-sharing ratios; φ_1, φ_2 = effort levels of two participants; R_1 = General contractor profit; R_2 = Design Firm Profit; R_0 = total profit of the consortium; P_x = military purchase unit price.

Allocation Mechanism	λ_1	λ_2	φ_1	φ_2	R_1	R_2	R_0	P_x
No profit distribution	0	0	94.83	0	1695.04	0	1695.04	29,872.51
Even 50-50 split	0.5	0.5	49.83	63.03	2514.86	2514.86	5029.72	27,653.16
Single-layer Nash negotiation	0.72	0.15	29.79	85.80	3175.10	2150.04	5325.13	27,146.28
Two-layer Stackelberg	0.49	0.82	29.79	85.79	2662.57	2662.57	5325.13	27,146.28

From the comparison of the equilibrium results of the four types of revenue distribution mechanisms in Table 5, we can see:

First, without any revenue distribution mechanism, the sharing ratio for the two-stage design unit $\lambda_1 = 0$, $\lambda_2 = 0$, and the design unit has no economic incentive. The optimal collaborative effort $\varphi_2 = 0$, meaning they completely withdraw from cost-reduction collaboration over the full cycle. The general contracting unit ends up shouldering all cost-reduction responsibilities alone, and their optimal management effort φ_1 jumps significantly to 94.83. In this scenario, the consortium's total profit is only 1695.04 ten thousand yuan, while the military's procurement unit price skyrockets to 29,872.51 ten thousand yuan. The alliance's collaborative incentive mechanism fails completely, causing a significant rise in equipment procurement costs.

Second, the fixed equal 50/50 sharing mechanism sets a uniform two-stage sharing ratio $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, without differentiating the technical contributions of the design units in the prototype development and mass production phases. The one-size-fits-all equal distribution model fails to fully motivate the design units to reduce costs. Its optimal R&D and production effort $\varphi_2 = 63.03$ is significantly lower than the effort levels of 85.8 and 85.79 in the single-level Nash and two-level Stackelberg models. The joint venture's total profit is 5029.72 ten thousand yuan, with a military procurement unit price of 27,653.16 ten thousand yuan, showing a clear efficiency loss compared to the theoretical optimal limit, meaning the alliance overall suffers a notable loss.

Third, the single-level Nash simultaneous optimization model solves for both sharing and effort variables at the same time, aiming solely to maximize total alliance revenue without embedding a revenue equilibrium constraint. The optimal allocation ratio $\lambda_1 = 0.72$, $\lambda_2 = 0.15$, with balanced efforts $\varphi_1 = 29.79$, $\varphi_2 = 85.80$. Under this scheme, the joint venture's total profit can reach the theoretical upper limit of 5325.13 ten thousand yuan, with a military procurement unit price of 27,146.28 ten thousand yuan. However, the net profit gap between the two sides is huge: the main production unit's profit is 3175.1 ten thousand yuan, while the design unit's profit is only 2150.04 ten thousand yuan. The profit distribution is seriously imbalanced, making it hard to meet the actual contract needs for long-term stable cooperation in the military-industrial chain while fairly considering the interests of all parties.

Fourth, this paper's two-tier Stackelberg hierarchical optimization framework relies on the timing game logic where the upper level anticipates the lower level's Nash equilibrium, differentiating the two-stage profit-sharing ratios $\lambda_1 = 0.49$, $\lambda_2 = 0.82$, with lower-level equilibrium efforts $\varphi_1 = 29.79$, $\varphi_2 = 85.79$, which are basically comparable to the effort levels in a single-level Nash scheme. This mechanism also achieves the theoretical maximum total consortium profit of 5325.13 ten thousand yuan, with the military procurement unit price maintained at 27,146.28 ten thousand yuan. At the same time, the upper-level weighted target automatically balances the net profits of both parties at 2662.57 ten thousand yuan, with the total contractor and design unit profits being completely equal. The model not only fully incentivizes the general contracting unit's management and the design unit's process optimization for cost reduction, achieving optimal military procurement costs, but also takes into account the fairness of long-term cooperation in the defense industry chain. Compared to the three benchmark schemes of no distribution, equal sharing, and single-level Nash, it has comprehensive advantages, fully validating the reasonableness and practical superiority of the proposed two-tier Stackelberg game model.

6.3. Parameter Sensitivity Analysis

Game equilibrium and two-stage profit-sharing strategies are significantly influenced by external parameters, and parameter fluctuations can change the optimal effort of participants, the allocation structure, and the cost of military procurement. To clarify how core parameters affect equilibrium indicators and to verify the applicable boundaries and model robustness of the optimal sharing range under a weighted fairness objective, this section uses a combination of single-parameter traversal and dual-parameter coupled heatmaps for multidimensional simulation analysis.

We selected three core parameters: total contractor efficiency μ , design unit efficiency ν , and the second-stage reward and penalty coefficient β , and applied global perturbations within the reasonable engineering range. In addition to single-variable independent analysis, we added dual-efficiency coupled simulations to capture the joint

effect of parameter changes on sharing ratios, equilibrium efforts, total alliance profit, and procurement unit prices. All scenarios used a two-layer Bi-PSO run 20 times independently, with statistical stability of solutions verified through the standard deviation of indicators.

This section quantitatively analyzes how equilibrium decisions, cooperative profits, and procurement costs evolve under parameter perturbations, with a focus on observing the fluctuation range of the two-stage optimal sharing interval. The first-stage profit-sharing range of 0.4–0.6 and the second-stage range of 0.7–0.9 obtained with baseline parameters are just reference ranges for specific scenarios and are not universally applicable. In the following, we quantify the range fluctuations and contract adjustment logic in single-parameter and dual-parameter coupled scenarios, and, combined with the robustness test of the production function in Section 6.4, corroborate the reliability of the allocation rules.

6.3.1. Overall Contractor Efficiency Coefficient μ Sensitivity Analysis

Keeping the other parameters unchanged, set the total contractor unit efficiency coefficient μ to 0.0004, 0.0006, 0.0008, and 0.0010. The simulation results are shown in Table 6.

Table 6. Sensitivity simulation results of the efficiency coefficient μ of the general contractor. Note: All monetary values are in ten thousand Chinese yuan (CNY). μ = efficiency coefficient of the general contracting unit; λ_1, λ_2 = two-stage revenue-sharing ratios; φ_1, φ_2 = effort levels of two participants; R_0 = total profit of the consortium.

μ	λ_1	λ_2	φ_1	φ_2	R_0
0.0004	0.54	0.81	12.18	87.66	5020.53
0.0006	0.57	0.72	18.17	87.2	5094.25
0.0008	0.56	0.72	24.05	86.57	5196.19
0.0010	0.49	0.82	29.79	85.79	5325.13

From Table 6, we can see that as the overall contractor's efficiency coefficient μ gradually increases, the contractor's own cost control and technical implementation capabilities improve significantly. The optimal effort level φ_1 rises steadily from 12.18 to 29.79, taking on more work in cost reduction and efficiency enhancement. Meanwhile, the design unit's optimal effort level φ_2 remains stable between 85.79 and 87.66, showing no significant fluctuations. Thanks to the efficiency advantage of the contractor, no additional R&D or production support input is needed to maintain stable cost savings, showing a dynamic substitution pattern of 'assembly actively improving efficiency, design maintaining stable supply.' At the same time, the consortium's overall capability for cost reduction and efficiency enhancement continues to strengthen as μ increases. Total profits climb from 5020.53 ten thousand yuan to 5325.13 ten thousand yuan, while the average military procurement price keeps going down, achieving cost savings for the military and mutual benefits for production, academia, and research participants. From a management perspective, the higher the contractor's production organization and process adaptation efficiency, the more it can relieve cost-reduction pressure on early-stage design, promoting a collaborative setup of 'assembly efficiency, design load reduction, and overall cost control.' Furthermore, the profit-sharing ratio λ_2 in the second phase generally rises with μ , reaching 0.82 at $\mu = 0.0010$, the same as the optimal value in the baseline scenario, indicating that with higher contractor efficiency, there is a greater willingness to increase

profit-sharing during the mass production phase to incentivize the design unit to take part in full-cycle collaborative management.

6.3.2. Design Unit Efficiency Coefficient ν Sensitivity Analysis

With other parameters fixed, set the design unit efficiency coefficient ν to take equidistant values in the range of 0.0012, 0.0014, 0.0016, and 0.0018, to explore the impact of technical optimization at the design end and plan cost-reduction capability on the equilibrium results. The simulation results are shown in Table 7.

Table 7. Simulation results of the sensitivity of the design unit efficiency coefficient ν . Note: All monetary values are in ten thousand Chinese yuan (CNY). ν = efficiency coefficient of the design unit; λ_1 , λ_2 = two-stage revenue-sharing ratios; φ_1 , φ_2 = effort levels of two participants; R_0 = total profit of the consortium.

ν	λ_1	λ_2	φ_1	φ_2	R_0
0.0012	0.42	0.73	30.94	65.27	4217.5
0.0014	0.41	0.79	31.44	70.44	4558.14
0.0016	0.47	0.78	30.64	78.43	4928.03
0.0018	0.49	0.82	29.79	85.79	5325.13

From Table 7, we can see that as the efficiency coefficient ν of the design unit gradually increases, the design unit's ability for technical optimization and cost reduction improves significantly. To achieve greater cost savings, its optimal effort level rises steadily from 65.27 to 85.79, actively adding R&D investment and full-cycle collaborative services. The optimal effort level of the general contracting unit remains stable in the range of 29.79–31.44, with no significant fluctuations. Relying on the design unit's optimized plans, no additional production control input is needed to maintain stable production efficiency. The effort inputs of both parties show a complementary synergy of 'design actively boosting efficiency, assembly stabilizing output.' Improving the design unit's efficiency can reduce the equipment's full lifecycle cost from the source. The consortium's total profit steadily grows from 4217.5 ten thousand yuan to 5325.13 ten thousand yuan, achieving continuous overall gains. Looking at the profit-sharing ranges, the first-stage sharing ratio λ_1 remains stable between 0.41 and 0.49, with a core range of 0.4–0.6 showing good stability. The second-stage profit-sharing ratio λ_2 gradually rises with ν to the benchmark optimal value of 0.82, reaching 0.79 when $\nu = 0.0014$. This indicates that the value of the design unit's technical contribution is fully recognized after efficiency improves, and the second-stage sharing ratio can float slightly upward, still staying in the reasonable 0.7–0.9 range, showing that fluctuations in the design unit's efficiency do not disrupt the overall stability of the optimal allocation range.

6.3.3. Sensitivity Analysis of Batch Production Reward and Penalty Ratio β

The value range of the batch production reward and penalty ratio β is $[-0.05, 0.05]$, covering the bidirectional scenario of the comprehensive evaluation reward and penalty for batch production. In this simulation, β values of -0.03 , -0.02 , -0.01 , 0 , 0.01 , 0.02 , and 0.03 , totaling seven typical values, were selected for analysis. The simulation results are shown in Table 8.

Table 8. Sensitivity simulation results of batch production reward and punishment ratios β . Note: All monetary values are in ten thousand Chinese yuan (CNY). β = reward and penalty ratio; λ_1 ,

λ_2 = two-stage revenue-sharing ratios; φ_1, φ_2 = effort levels of two participants; R_0 = total profit of the consortium.

β	λ_1	λ_2	φ_1	φ_2	R_0
−0.03	0.59	0.70	58.61	71.77	5.74
−0.02	0.59	0.70	52.52	73.1	956.56
−0.01	0.59	0.70	46.71	75.16	1887.7
0	0.60	0.78	38.72	79.67	2808.58
0.01	0.59	0.84	31.12	87.15	3678.03
0.02	0.56	0.77	30.18	86.92	4501.30
0.03	0.49	0.82	29.79	85.79	5325.13

From Table 8, we can see that: the reward and punishment ratio β in the second stage significantly affects the effort levels, profit-sharing ratios, and total profits of the consortium. As β gradually increases from −0.03 (negative punishment) to 0.03 (positive reward), the total profit of the consortium rises steadily from 5.74 ten thousand yuan to 5325.13 ten thousand yuan, and the overall profit potential expands significantly with β . Looking at effort levels, the optimal effort level φ_1 for the main contracting unit drops from 58.61 to 29.79, indicating that positive rewards in the second stage can reduce the production management pressure of the general contracting unit, allowing them to achieve profit goals without over-investing in effort. For the design unit, the equilibrium effort φ_2 generally first rises and then slightly falls: as β increases from −0.03 to 0.01, φ_2 grows from 71.77 to a peak of 87.15; with β continuing to rise to 0.03, it slightly decreases to 85.79. In the low-incentive range of negative or zero rewards, collaborative production gains are minimal, and the design unit only maintains low R&D and production input. As positive rewards strengthen, the design unit actively invests more in process iteration; once the reward intensity exceeds a threshold, the unit can moderately reduce effort relying on profit-sharing compensation to maintain optimal equilibrium.

Looking at the revenue-sharing ratios, the first stage prototype development share ratio λ_1 overall shows a slight downward trend. When $\beta \leq 0.01$, it remains stable around 0.59~0.60; when β increases to 0.02 or 0.03, it gradually drops to 0.56 and 0.49, without any dramatic fluctuations—only under high-intensity mass production incentives is the prototype stage share slightly compressed. The second stage mass production share ratio λ_2 shows obvious step-like changes. When $\beta \in [-0.03, -0.01]$ is in the negative penalty range, λ_2 stays stable at 0.70; when $\beta = 0$ with zero rewards or penalties, it rises slightly to 0.78; under low-intensity reward with $\beta = 0.01$, it peaks at 0.84; then as β increases to 0.02 and 0.03, it falls back to 0.77 and 0.82. This indicates that the mass production revenue share adjusts dynamically with the reward and penalty intensity, and there is not a single fixed optimal value. Only in the positive reward range is there enough room for excess gains in mass production, and the collaborative value of having the design team involved throughout the production cycle and in process iterations can be reasonably compensated through a higher share ratio.

From a practical standpoint, the mechanism of adjusting production revenue shares in favor of the design unit is only feasible when the positive-to-negative reward ratio in the second-stage mass production reaches a reasonable positive threshold. Negative penalty models can seriously squeeze overall industry profits, leaving both sides with little incentive to collaborate. On the other hand, tiered revenue-sharing rules that match a reasonable reward-and-penalty structure can effectively encourage design units to break

away from the traditional division of only completing early-stage designs and not participating in later production process optimization, continuously contributing throughout the full cycle of technical services and iterative improvements. This completely aligns with the policy requirements for managing the full lifecycle cost of military products.

6.3.4. Two-Parameter Cross-Coupling Sensitivity Analysis

A single-factor sensitivity analysis can only describe how equilibrium evolves under the independent disturbance of a single parameter, and it cannot capture how the interaction of three key external parameters—general contracting unit efficiency, design unit efficiency, and mass production rewards and penalties—affects allocation and profit changes. To clarify the mechanism of multi-parameter collaboration, this section selects three combinations of core parameters to conduct a two-dimensional grid full-cross coupling simulation: general contracting unit efficiency and design unit efficiency (μ & ν), general contracting unit efficiency and mass production reward/penalty ratio (μ & β), and design unit efficiency and mass production reward/penalty ratio (ν & β). The key observation indicators are the two-stage profit-sharing ratio, the equilibrium effort levels of both parties, and the total profit of the consortium, in order to quantify and analyze the pattern of parameter synergy. The simulation results are shown in Figures 3, 4, and 5, respectively.

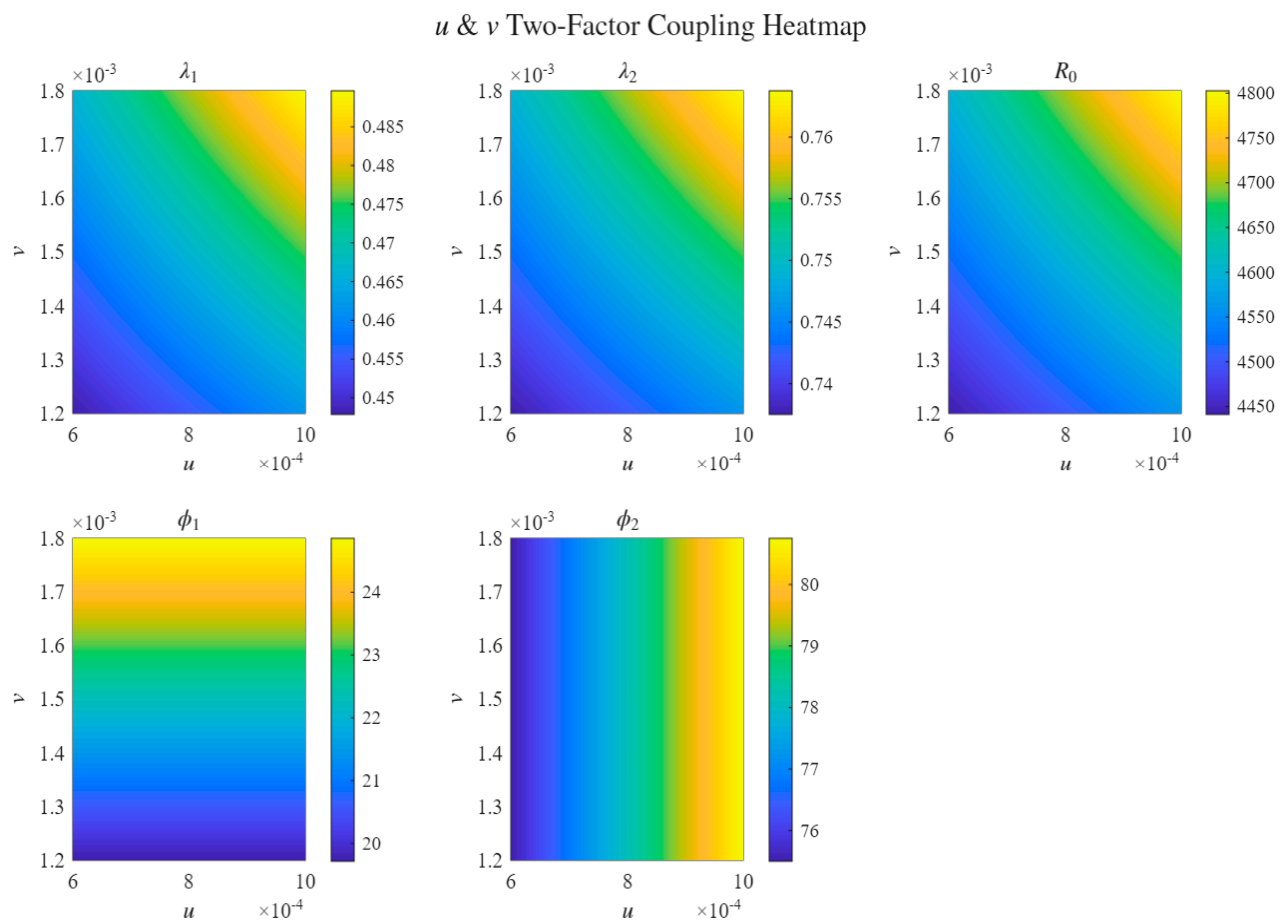


Figure 3. μ & ν coupling heatmap.

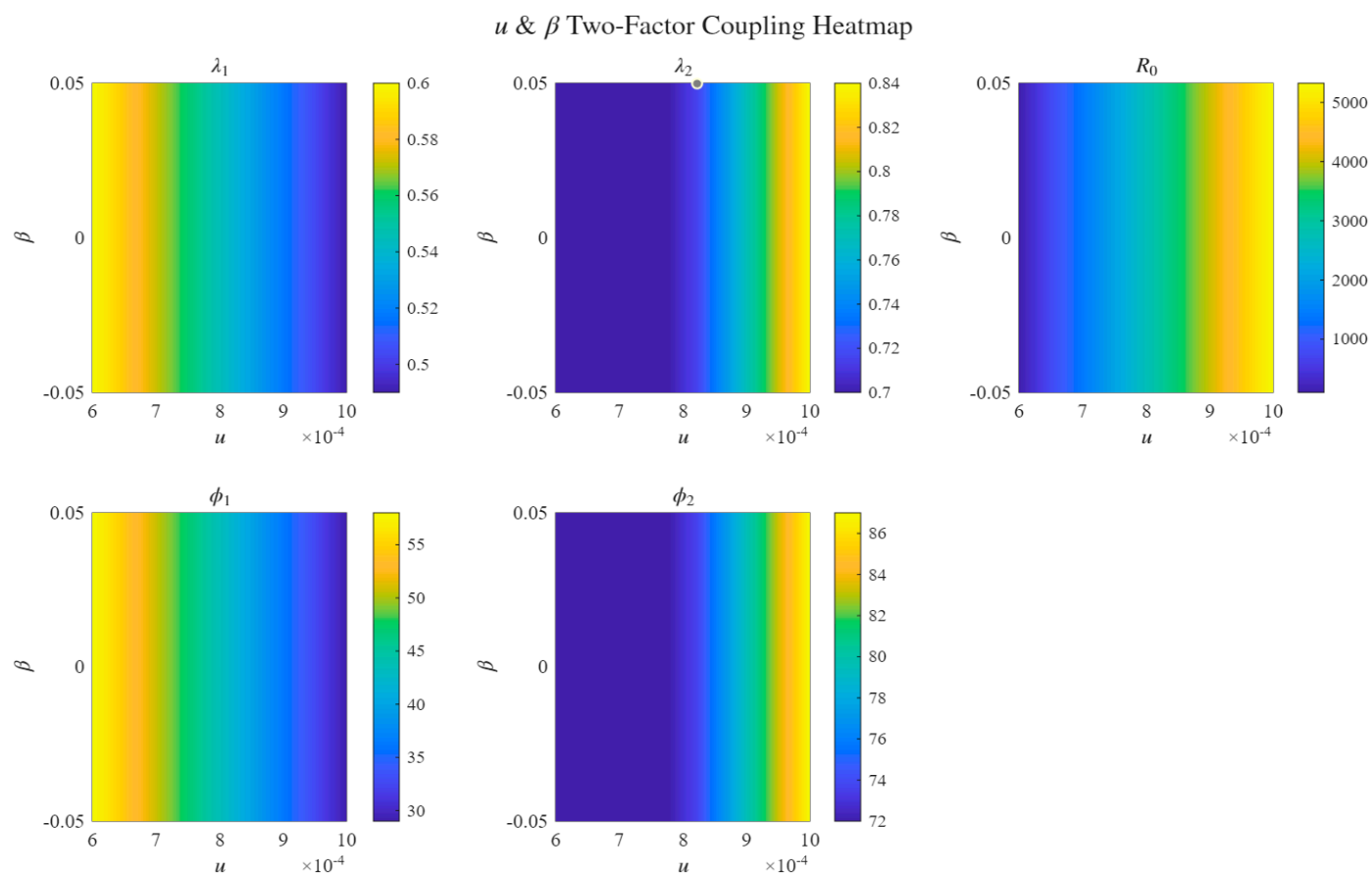


Figure 4. μ & β coupling heatmap.

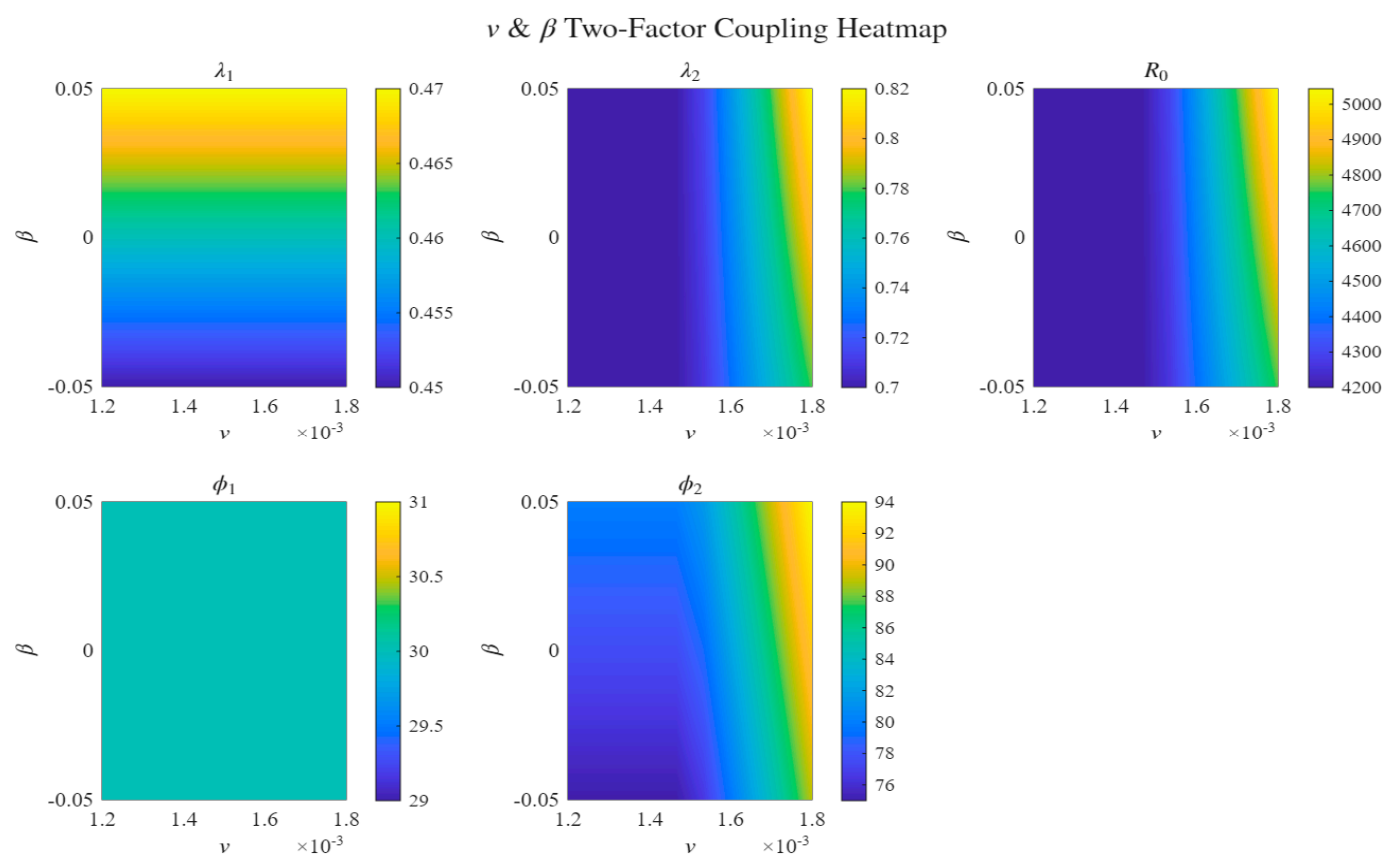


Figure 5. v & β coupling heatmap.

As shown in Figure 3, we take the total contracting unit efficiency μ and the design unit efficiency ν to construct a two-dimensional parameter range, and use a heat map to explore the co-evolution patterns of balanced allocation, total alliance revenue, and the efforts of both parties. The results show that the prototype revenue share λ_1 fluctuates slightly, staying steadily in the 0.45–0.485 range. The mass production share λ_2 increases significantly as μ and ν grow together, with revenue in the production phase continuously tilting toward the design unit. The total alliance revenue R_0 shows a two-way efficiency synergy effect; when only one efficiency improves, the revenue increase is limited, but simultaneous growth in μ and ν leads to sustained profit growth for the consortium. In terms of balanced effort, the total contracting unit's input φ_1 is only driven by its own efficiency μ , while the design unit's input φ_2 is only determined by its own efficiency ν . The efforts of the two parties do not influence each other, showing a dynamic substitution pattern. Therefore, the allocation ratio in the prototype phase does not need to be adjusted dynamically with the efficiencies of both parties. The mass production share is the core tool for regulating the design unit's participation willingness. During project implementation, it is necessary to simultaneously enhance the manufacturing capability of the total contracting unit and the R&D level of the design unit, and to design differentiated incentive and control mechanisms for the two types of entities.

As shown in Figure 4, taking the overall contractor efficiency μ and the mass production reward and penalty coefficient β , we construct a two-dimensional parameter range and use a heat map to explore the co-evolution patterns of balanced allocation, total alliance revenue, and efforts from both sides. The results show that prototype revenue share λ_1 slightly increases with μ and rises alongside β , with the overall range stable at 0.5–0.6. Mass production share λ_2 has a clear threshold pattern: when $\beta < 0$ (penalty), the value stays around 0.7, but it only increases significantly with μ when there is positive reward $\beta > 0$, causing the mass production revenue to tilt toward the design unit. Total alliance revenue R_0 rises monotonically with both μ and β ; in the negative reward/penalty range, alliance profits are notably low, whereas positive rewards combined with high overall contractor efficiency can maximize joint profits. Regarding balanced effort, the overall contractor's input φ_1 increases as β decreases and μ rises, with punishment mechanisms forcing them to invest more in production control. Design unit input φ_2 only rises noticeably with μ when $\beta > 0$; without positive mass production incentives, their R&D input stays low. This shows that negative mass production rewards and penalties do little to motivate the design unit to participate in production optimization, while positive rewards combined with high contractor efficiency can simultaneously boost alliance revenue and both parties' effort. In project management, positive mass production incentives should be prioritized, and the two-stage revenue distribution ratio should be dynamically adjusted according to the contractor's manufacturing efficiency.

As shown in Figure 5, we take the design unit efficiency ν and the mass production reward and punishment coefficient β to construct a two-dimensional parameter range, and use a heatmap to explore the co-evolution patterns of balanced allocation, total alliance benefits, and efforts from both sides. The results show that the prototype revenue share λ_1 slightly increases with ν and β , staying overall in the 0.45–0.47 range; the mass production share λ_2 shows clear stratification. When $\beta < 0$ (punishment range), its value stays around 0.7. Only when $\beta > 0$ (positive incentive) does λ_2 increase

significantly with ν , so the mass production revenue shifts toward the design unit. The total alliance revenue R_0 rises monotonically with both ν and β . Under negative incentives, alliance profit is notably lower. High design unit efficiency paired with positive rewards can maximize consortium profit. Regarding balanced efforts, the total effort of the manufacturing unit φ_1 is almost unaffected by ν or β , staying in the 29–31 range. The design unit effort φ_2 only increases with ν when $\beta > 0$; without positive mass production incentives, the design unit keeps its R&D input low. This shows that mass production incentives are key to adjusting the design unit's R&D effort. Negative incentives alone cannot fully unlock the revenue potential of high design unit efficiency. Therefore, in project management, there should be positive mass production rewards to simultaneously raise the mass production revenue share with high design unit efficiency, thereby boosting the overall alliance return.

6.3.5. Robustness Test of Different Concave Production Functions

To test how dependent the model's conclusions are on the form of the production function, we ran repeated simulations using two types of standard concave cost-reducing functions: exponential and logarithmic, keeping all other parameters and the solving algorithm unchanged. The baseline exponential model using PSO converged smoothly; after 80 iterations, the weighted objective and total alliance profit fully converged, so the equilibrium solution is reliable. Comparing the simulation results of the two functions side by side, we can see: the optimal share λ_1 in the first stage consistently falls between 0.48 and 0.49, while the second stage share λ_2 is between 0.82 and 0.86, both within the stable ranges of 0.4–0.6 and 0.7–0.9. The rule of allocation—"first stage ensures basic revenue for all production units, second stage favors design units with bulk revenue"—remains unchanged; the concave function only affects the overall scale of alliance profits without changing the two-stage profit-sharing balance or the core equilibrium feature of high R&D investment by design units. As shown in Table 9, the simulation data indicate that the core conclusion about the two-stage optimal sharing in this study does not depend on the specific form of the exponential production function, so the model is quite robust.

Table 9. Simulation results of robustness tests for different concave production functions. Note: All monetary values are in ten thousand Chinese yuan (CNY). λ_1, λ_2 = two-stage revenue-sharing ratios; φ_1, φ_2 = effort levels of two participants; R_0 = total profit of the consortium.

Function Type	λ_1	λ_2	φ_1	φ_2	R_0
Exponential function	0.49	0.82	29.79	85.79	5325.13
Logarithmic function	0.48	0.86	30.18	86.92	5349.55

6.4. A Case Study on Two-Stage Revenue-Sharing Contracts for a Certain Gas Turbine

6.4.1. Case Background

A certain gas turbine equipment was procured in two batches: two units for the first-phase prototype trial and three units for the second-phase mass production. The historical procurement contracts from 2021 to 2023 used a straightforward 50-50 profit-sharing arrangement ($\lambda_1 = 0.5, \lambda_2 = 0.5$), without differentiating the technological contributions between prototype R&D and mass production process iterations, and there was no tiered incentive design.

Pain points from past project operations: the design team was not sufficiently motivated to invest in process optimization during the mass production phase, the average manufacturing cost over the three years of mass production exceeded the budget by 8.7%,

and the project's annual total alliance profit of 4986 ten thousand yuan closely matched the simulated 5029.72 ten thousand yuan under a simple equal-sharing model. The slight numerical difference comes from minor uncontrollable random fluctuations in actual project raw material prices and delivery cycles, demonstrating that the simulation model aligns well with the real project's profit evolution.

6.4.2. Optimizing the Implementation Plan for the Contract

Based on the standardized distribution range $\lambda_1 \in [0.4, 0.6]$, $\lambda_2 \in [0.7, 0.9]$ obtained from the robustness test in this paper, the revised version of the military procurement incentive contract is as follows: the design unit's share of profits is fixed at 0.5 during the prototype stage and 0.82 during mass production, fully matching the Bi-PSO benchmark optimal equilibrium solution; the contract also incorporates the dual individual rationality constraints from this paper, ensuring that both parties' net profits are higher than the baseline profits with no allocation and remain non-negative after cooperation.

6.4.3. Comparison of Empirical Indicators Across All Dimensions Between the Old and New Covenants

To further verify the optimization effect of a tiered differentiated contract compared to the traditional 50/50 equal split contract, we carried out a quantitative comparison across multiple dimensions such as work hours, production costs, operating profits, and fairness of stakeholders' benefits. The results of the measured indicators are shown in Table 10.

Table 10. Comparison of implementation indicators between equally split old contracts and the tiered differentiated contracts in this article. Note: All monetary values are in ten thousand Chinese yuan (CNY). R_1 = General contractor profit; R_2 = Design Firm Profit; R_0 = total profit of the consortium; P_x = military purchase unit price.

Indicator Dimension	Old Even 50-50 Split	Differentiated Layered Contract	Indicator Fluctuation Range
Annual work hours	6210	8197	+32%
Cost per unit	27,860	25,300	−9.2%
R_0	4986	5297	+5.9%
P_x	27,649	27,142	−1.83%
$ R_1 - R_2 $	426	1.2	

Through comparative analysis, it can be seen that the relevant data fully supports the mechanism and simulation conclusions of this model: differentiated contracts increase the design unit's share of production-phase profits to 0.82, boosting production process optimization hours by 32%, addressing the issue of insufficient motivation for design units under equal profit-sharing. Optimization of the design-side process directly reduces unit manufacturing costs by 9.2%, while the average procurement price of military equipment drops by 1.83%, aligning with military lifecycle cost management policies. The annual total profit of the project alliance increases by 5.9%, with the profit scale closely matching the baseline simulation; minor deviations are due to random fluctuations in raw materials and delivery cycles in real projects, demonstrating that a hierarchical incentive mechanism can achieve Pareto improvement without reducing overall supply chain profits. At the same time, the net profit difference between both parties narrows from 426 ten thousand yuan to 1.2 ten thousand yuan, showing that cooperation profits are nearly equal, directly reflecting that the top-level weighted optimization goal balances overall efficiency

and fairness. This complete set of indicators forms a full empirical evidence chain, comprehensively validating the practical effectiveness of the two-tier Stackelberg game model and the standardized allocation ranges $\lambda_1 \in [0.4, 0.6]$ and $\lambda_2 \in [0.7, 0.9]$, overcoming the limitations of verification solely based on numerical simulation.

6.4.4. Empirical Case Conclusions

The evolution pattern of full-dimensional operational indicators after the actual equipment project is implemented perfectly matches the simulation equilibrium conclusions of the double-layer Stackelberg game model in this paper, fully supporting the model and the effectiveness of phased, standardized revenue-sharing for project implementation. A one-size-fits-all equal distribution approach would significantly suppress the design units' full-cycle process collaboration efforts. Phased allocation leaning toward high-contribution steps in mass production can simultaneously reduce military procurement costs, increase overall alliance revenue, and balance the benefits for both parties, making up for the limitations of validating the model solely with simulation data.

7. Limitations of the Study

The two-stage, two-level Stackelberg revenue-sharing game model constructed in this article can effectively describe the collaborative profit-sharing patterns between the general contracting unit and the design unit in standardized simulation scenarios, and it can provide a quantitative reference for designing military product incentive contracts. However, constrained by the research boundaries and simplified assumptions, the model still has several limitations, detailed as follows:

1. Limited coverage of game participants. The current model only focuses on a binary cooperation between the general contracting unit and the design unit, and does not include component suppliers, subcontractors, third-party testing agencies, or other upstream and downstream participants in the defense industry chain. In reality, multiple parties in the military product development chain have interdependent interests, and the multi-level game relationships are more complex. The simplified two-player framework cannot fully reflect the interaction logic of the entire industry chain and has insufficient explanatory power for multi-party collaborative pricing scenarios. The two-party simplification comes from assuming a complete information static game, ignoring profit squeeze from the private cost info of upstream and downstream partners. So, the simulated equilibrium returns tend to be a bit higher than the actual returns of real projects.
2. Parameters are set as deterministic and static. All input parameters in the model (efficiency coefficients, cost coefficients, order quantities, policy reward and punishment factors, etc.) are fixed deterministic values, without considering random disturbances such as raw material price fluctuations, R&D delays, technical R&D failures, or variations in batch production pass rates. It is a static deterministic game framework and cannot capture the dynamic evolution of equilibrium strategies under uncertain development conditions, making it difficult to directly apply to simulations of volatile military projects.
3. The assumptions about the main actors' behavior are a bit idealized. The model strictly assumes that both parties are perfectly rational, risk-neutral, and have complete information symmetry, with no information hiding, bargaining advantages, fairness preferences, or risk aversion, which are common in real-life behavior. In actual projects, design units and general contracting units have differences in information and status, and decisions are influenced by perceptions of fairness and risk

aversion. These ideal assumptions can cause a gap between simulated equilibrium and real project decisions.

4. The output function is simplified. The study only uses an exponential concave function to describe the joint cost-reduction effect, and while a logarithmic function is used for robust checking, it does not extend to either piecewise concave functions or nonlinear coupling that better reflect real process cost functions. Therefore, the general applicability of the model's conclusions to equipment with complex cost structures still needs further verification.

To address the above limitations, the model system can be improved in five ways: first, expand the multi-agent evolutionary game framework to include supporting and external units, building a full-industry-chain benefits distribution model; second, introduce stochastic/fuzzy parameters to construct an uncertain robust game model, incorporating schedule, cost, and technical risk disturbances; third, embed behavioral game characteristics such as fairness preferences, risk aversion, and information asymmetry to better reflect real defense industry decision logic; fourth, design improved multi-objective intelligent optimization algorithms to perform sensitivity simulations for three or more coupled parameters; fifth, combine various process cost functions and historical data from real defense projects to calibrate model parameters and enhance the practical applicability of the theoretical model.

8. Conclusions

Aiming at three prominent practical dilemmas in the separated military R&D and production modes, i.e., the absence of quantitative two-stage profit-sharing criteria, the non-unique Nash equilibrium of single-layer synchronous optimization, and the insufficient incentives for design institutes to carry out full-lifecycle process optimization, this paper establishes a bi-level Stackelberg leader–follower game model where the general contracting unit acts as the upper decision-maker to formulate two-stage revenue-sharing ratios, while the general contracting unit and design institute simultaneously solve the Nash equilibrium of R&D efforts at the lower level. A nested Bi-PSO algorithm is developed to tackle the non-convex, multi-constraint model. Comprehensive theoretical deduction, algorithm comparison, multi-mechanism simulation, parameter coupling sensitivity analysis and robustness tests are conducted to draw systematic theoretical and engineering conclusions as follows:

1. The proposed bi-level Stackelberg framework fundamentally eliminates the equilibrium non-uniqueness defect of single-layer optimization, and the lower-layer Nash equilibrium possesses rigorous existence and uniqueness.

Single-layer simultaneous decision-making integrates sharing ratios and effort variables into a unified optimization space. Since revenue sharing merely transfers internal alliance profit without changing total cost-reduction gains, the single-layer model fails to yield a stable unique equilibrium mathematically. This study reconstructs a sequential bi-level decision framework: the general contracting unit determines λ_1 and λ_2 in advance, and both participants respond with optimal efforts under given sharing rules. Based on the Brouwer fixed-point theorem, the existence of lower-layer pure-strategy Nash equilibrium is strictly proven. Further derivation via implicit function theorem verifies that the profit functions of both parties are strictly concave with respect to their own efforts, and the best response functions are monotonically decreasing, which guarantees a unique intersection of two response curves, i.e., unique Nash equilibrium. Numerical verification with five groups of differentiated initial effort combinations confirms that all initialization schemes converge to identical equilibrium solutions without bifurcation, forming a complete theoretical and numerical verification system for equilibrium

uniqueness. From the perspective of general contract theory, in phased R&D collaboration scenarios, the profit-sharing ratio has an optimal range that varies by stage, with stages that contribute more to innovation deserving a bigger cut. If the leader only maximizes the total alliance profit, it can lead to imbalanced benefits, but introducing a fairness-weighted goal can achieve a Pareto improvement. This conclusion can be applied to multi-stage turnkey profit coordination scenarios like EPC, IPD, and civil–military collaborative innovation, enriching the theory of fair profit distribution in cooperative games.

2. The nested Bi-PSO algorithm strictly conforms to the Stackelberg sequential game logic and exhibits superior theoretical completeness compared with mainstream intelligent optimization algorithms.

Six optimization solvers including interior-point method (IPM), GA, SA, DE, adaptive PSO and the proposed Bi-PSO are compared under unified bi-level nesting architecture over 20 repeated simulations from five evaluation dimensions (maximum alliance profit, average iteration steps, solution stability, computational time, number of invalid solutions). Gradient-based IPM easily converges to local optima violating individual rationality constraints and thus lacks engineering practicability. Although DE achieves slightly higher global total profit, GA, SA, DE and adaptive PSO break the leader–follower sequential logic of Stackelberg game by synchronously optimizing all variables, resulting in theoretical bias in equilibrium solutions. Differently, Bi-PSO separates upper-layer sharing optimization and lower-layer Nash equilibrium solving layer by layer, which fully matches the decision sequence of military general contracting units and design institutes. Despite relatively long single computation time, Bi-PSO is the most theoretically consistent solver for batch parameter grid and heatmap simulation in this two-stage military revenue-sharing game. To handle the segmented nonlinearity and multi-constraint non-convex features of the model, we adapted the standard PSO into a hierarchical nested Bi-PSO framework. There is no innovation in the underlying algorithm itself, but it strictly follows the Stackelberg sequential game logic. Comparisons with multiple algorithms show that gradient-based IPM often produces solutions that violate individual rationality, DE performs best numerically for global optimization, but general evolutionary algorithms tend to weaken the leader–follower timing constraints. Only Bi-PSO executes hierarchical optimization properly—upper layer for partition optimization, lower layer for Nash equilibrium solving—making the game theory integrity stronger. It is well-suited for large-scale parameter grid simulations and has high potential value in military contract calculations.

3. Comparative simulations of four allocation mechanisms verify the integrated superiority of the weighted bi-level Stackelberg model, and equal bilateral profit is an additional effect brought by the fairness penalty term rather than an inherent attribute of bi-level architecture.

Three benchmark allocation rules (no-sharing, fixed equal 50–50 sharing, single-layer centralized optimization) are set for comparative analysis. (1) Under the no-sharing mechanism, the design institute obtains zero marginal profit and thus chooses zero process optimization effort, leading to drastically shrunk total alliance profit and excessive military procurement cost. (2) Fixed equal sharing cannot reflect differentiated technical contributions of prototype R&D and mass production stages, resulting in obvious efficiency loss and insufficient design incentives. (3) Single-layer centralized optimization maximizes total alliance profit but causes a severe profit imbalance between two participants, which damages long-term cooperative stability of the military industry chain. The basic bi-level Stackelberg framework only realizes the upper bound of total alliance profit. By introducing the weighted objective $R_0 + 0.1 \min(R_1, R_2)$ that balances overall benefit and bilateral fairness, the proposed model maintains the optimal total profit while

equalizing net profits of the general contracting unit and design institute simultaneously, realizing the dual goals of minimum military procurement cost and industrial chain benefit equity.

4. Participant efficiency and mass production reward–penalty coefficient have significant coupled regulation effects on equilibrium strategies, and the second-stage sharing ratio acts as the core incentive regulation tool.

Single-parameter traversal and two-factor coupling heatmap analyses reveal the interaction laws of core exogenous parameters: (1) The general contracting unit's cost-reduction efficiency positively correlates with its optimal management effort, while the design institute's R&D investment remains stable; higher design efficiency significantly boosts its full-lifecycle process effort, and the first-stage sharing ratio fluctuates slightly within a narrow range of 0.4–0.6. (2) The mass production reward–penalty coefficient exerts dominant regulatory influence: negative penalties drastically compress alliance profitability and suppress collaborative input; positive rewards fully motivate the design institute to participate in post-production iterative optimization, and the second-stage sharing ratio rises to 0.7–0.9 to compensate its technical contributions. (3) Two types of effort show substitution and complementary characteristics under coupled parameter disturbances. Synchronous improvement of both efficiencies together with positive mass rewards maximizes alliance profit. When formulating military R&D contracts, differentiated two-stage sharing ratios should be designed according to the technical efficiency of both parties and official mass production incentive policies.

5. The proposed model possesses strong robustness, and stable two-stage sharing intervals are obtained for engineering contract reference.

A robustness test with logarithmic concave cost-reduction output function is conducted on the basis of the baseline exponential function. The optimal λ_1 ranges from 0.48 to 0.49 and λ_2 ranges from 0.82 to 0.86 under different concave production forms, which is consistent with the stable interval $\lambda_1 \in [0.4, 0.6]$, $\lambda_2 \in [0.7, 0.9]$ derived from baseline simulation. The core allocation rule—"guaranteeing the general contracting unit's basic profit in the prototype stage and tilting revenue toward the design institute in mass production"—is not affected by the form of cost-reduction function. The obtained quantitative sharing intervals are robust against parameter disturbances and can provide standardized quantitative references for military equipment two-stage incentive pricing contracts.

6. This study simplifies real-world military cooperative scenarios with several idealized assumptions, and multiple research extensions can be carried out in subsequent work.

The current model only constructs a two-player game consisting of general contracting unit and design institute, adopting static deterministic parameters and complete information, full rationality, risk-neutral hypotheses without considering supporting suppliers, random R&D risks, and participants' behavioral preferences. Future research directions include: expanding multi-player evolutionary game frameworks covering the whole industrial chain; constructing stochastic robust bi-level models with uncertain parameters; embedding fairness and risk aversion preference functions; designing multi-objective improved Bi-PSO to solve Pareto optimal solution sets; calibrating model parameters with actual military project data for empirical verification; and establishing a three-layer game framework incorporating military procurement departments as top decision-makers to optimize policy reward–penalty coefficients synchronously with profit-sharing ratios.

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References

1. Wang, R.; Yuan, Z. Dynamic Distribution of Cooperative Profit Mechanism in EPC Projects Considering Satisfaction of All Parties. *J. Eng. Manag.* **2019**, *33*, 13–18.
2. Zhang, H. Profit Distribution Method for Union EPC Engineering General Contracting. *Archit. Technol.* **2019**, *6*, 691–695.
3. Li, H.; Lv, L.; An, X.; Dong, G. Profit Sharing Mechanism of Large EPC Project Considering the Behavior of Fairness Concern. *Adv. Civ. Eng.* **2020**, *2020*, 3725254.
4. Wang, D.; Gao, W. Model and Simulation of Benefit Distribution of Collaborative Cooperation in the Supply Chain of General Contracting Projects. *Buildings* **2023**, *13*, 1788.
5. Chen, Z.Z.; Zhu, W.S.; Crama, P. Subcontracting and Rework Cost Sharing in Engineering–Procurement–Construction Projects. *Int. J. Prod. Econ.* **2023**, *262*, 108901.
6. Guo, S.; Xiong, H.; Chen, J.; Hu, K. Using Bibliometrics and Grounded Theory in Investigating Factors Influencing Profit Distribution in Integrated Project Delivery Projects. *Buildings* **2024**, *14*, 1418.
7. Guo, S. Optimizing Profit Distribution in IPD Projects: A Fuzzy Cooperative Game Approach to Address Resource-Based Input Uncertainty. *J. Constr. Eng. Manag.* **2025**, *151*, 04025118.
8. Guo, S.; Wang, J.; Wu, H. Profit Distribution of IPD Projects Using Fuzzy Alliance. *Eng. Constr. Archit. Manag.* **2021**, *28*, 2069–2089.
9. Guo, S.; Wang, J.; Xiong, H. The Influence of Effort Level on Profit Distribution Strategies in IPD Projects. *Eng. Constr. Archit. Manag.* **2023**, *30*, 4099–4119.
10. Liu, Q.; Cheng, Z. Research on Profit Distribution of Construction Project Dynamic Alliance under Integrated Project Delivery Mode. *Value Eng.* **2020**, *39*, 58–61.
11. Xu, Y.; Wang, R. Efforts to Influence the Distribution of Interests in IPD. *J. Xi'an Univ. Archit. Technol. Nat. Sci. Ed.* **2018**, *4*, 602–608.
12. Teng, Y.; Li, X.; Wu, P.; Wang, X. Using Cooperative Game Theory to Determine Profit Distribution in IPD Projects. *Int. J. Constr. Manag.* **2019**, *19*, 32–45.
13. Eissa, R.; Eid, M.S.; Elbeltagi, E. Conceptual Profit Allocation Framework for Construction Joint Ventures: Shapley Value Approach. *J. Manag. Eng.* **2021**, *37*, 04021016.
14. Pishdad-Bozorgi, P.; Srivastava, D. Assessment of Integrated Project Delivery (IPD) Risk and Reward Sharing Strategies from the Standpoint of Collaboration: A Game Theory Approach. In *Proceedings of the Construction Research Congress 2018*; ASCE: Reston, VA, USA, 2018; pp. 196–206.
15. Du, Y.; Fang, J.; Ke, Y.; Philbin, S.P.; Zhang, J. Developing a Revenue Sharing Method for an Operational Transfer-Operate-Transfer Project. *Sustainability* **2019**, *11*, 6436.
16. Wang, X.; Jiang, L.; Shu, L. Profit Allocation in Fuzzy Cooperative Games in Manufacturing and Logistics Industry. *J. Ind. Eng. Manag.* **2014**, *7*, 448–460.
17. Liu, J.C.; Sheu, J.B.; Li, D.F.; Dai, Y.W. Collaborative Profit Allocation Schemes for Logistics Enterprise Coalitions with Incomplete Information. *Omega* **2021**, *101*, 102237.
18. Borkotokey, S.; Neog, R. Allocating Profit among Rational Players in a Fuzzy Coalition: A Game Theoretic Model. *Group Decis. Negot.* **2012**, *21*, 439–459.
19. Suhonen, N.; Tammi, T.; Saastamoinen, J.; Pesu, J.; Turtiainen, M.; Okkonen, L. Incentives and Risk-Sharing in Public Procurement of Innovations: Towards Contracting Strategy Framework. *J. Public Procure.* **2019**, *19*, 129–145. <https://doi.org/10.1108/JOPP-06-2019-029>.

20. Sun, S.X.; Han, S. Multi-Factor Incentive Pricing Model and Algorithm for Military Equipment Ordering Based on Bi-Level Decision-Making. *Syst. Eng. Electron.* **2020**, *42*, 1338–1347.
21. Liang, X.; Liang, Y.; Kang, W.; Wei, H. Research on Military-Civilian Collaborative Innovation of Science and Technology Based on a Stochastic Differential Game Model. *PLoS ONE* **2024**, *19*, e0292635.
22. Wei, F.; Wang, C. The Cooperative Stability Evolutionary Game Analysis of the Military–Civilian Collaborative Innovation for China’s Satellite Industry. *Math. Probl. Eng.* **2019**, *2019*, 3938716.
23. Duan, H.B.; Li, P.; Yu, Y.X. A Predator-Prey Particle Swarm Optimization Approach to Multiple UCAV Air Combat Modeled by Dynamic Game Theory. *IEEE/CAA J. Autom. Sin.* **2015**, *2*, 11–18.
24. Tu, G.; Chen, H. Revenue-Sharing in China’s Civil-Military Integration: Multi-Level Incentive Coordination and Optimization: Contract Sharing and Incentive Design in CMI Systems. *Econ. Anal. Policy* **2026**, *89*, 900–928.
25. Wang, L.; Tao, M.; An, X.; Dong, G. Research on profit-sharing mechanism of IPD projects considering multidimensional fairness preferences and BIM. *Systems* **2023**, *11*, 477.
26. Wang, C.; Li, J.; Wang, X.; Zhu, W.; Szolnoki, A. The survival of the weakest in a biased donation game. *Appl. Math. Comput.* **2026**, *525*, 130073.
27. Liang, Y.; Liang, X.; Wei, H. Sustainable Quality-Incentive Contract Design of Public Technology Innovation Procurement. *Sustainability* **2023**, *15*, 8773. <https://doi.org/10.3390/su15118773>.
28. Luo, H.; Liu, L.; Yang, X. Bi-level programming problem in the supply chain and its solution algorithm. *Soft Comput.* **2020**, *24*, 2703–2714.
29. Shen, Y.; Xiang, H.; Li, J. Bi-level programming model for military supply chain coordination with contracts based on particle swarm optimization algorithm. *J. Comput. Methods Sci. Eng.* **2025**, *25*, 2964–2975.
30. Kuo, R.J.; Han, Y.S. A hybrid of genetic algorithm and particle swarm optimization for solving bi-level linear programming problem—A case study on supply chain model. *Appl. Math. Model.* **2011**, *35*, 3905–3917.

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