

## Article

# Ultra-Wideband Antenna Design for 5G NR Using the Bezier Search Differential Evolution Algorithm

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**Abstract:** As the energy crisis is leading to energy shortages and constant increases in prices, green energy and renewable energy sources are trending as a viable solution to this problem. One of the most rapidly expanding green energy methods is RF (RadioFrequency) energy harvesting, as RF energy and its corresponding technologies are constantly progressing, due to the introduction of 5G and high-speed telecommunications. The usual system for RF energy harvesting is called a rectenna, and one of its main components is an antenna, responsible for collecting ambient RF energy. In this paper, the optimization process of an ultra-wideband antenna for RF energy harvesting applications was studied, with the main goal of broadening the antenna's operational bandwidth to include 5G New Radio. For this purpose, the Bezier Search Differential Evolution Algorithm (BeSD) was used along with a novel CST-Matlab API, to manipulate the degrees of freedom of the antenna, while searching for the optimal result, which would satisfy all the necessary dependencies to make it capable of harvesting RF energy in the target frequency band. The BeSD algorithm was first tested with benchmark functions and compared to other widely used algorithms, which it successfully outperformed, and hence, it was selected as the optimizer for this research. All in all, the optimization process was successful by producing an ultra-wideband optimal antenna operating from 1.4 GHz to 3.9 GHz, which includes all vastly used telecommunication technologies, like GSM (1.8 GHz), UMTS (2.1 GHz), Wi-Fi (2.4 GHz), LTE (2.6 GHz), and 5G NR (3.5 GHz). Its ultra-wideband properties and the rest of the characteristics that make this design suitable for RF energy harvesting are proven by its  $S_{11}$  response graph, its impedance response graph, its efficiency on the targeted technologies, and its omnidirectionality across its band of operation.

**Keywords:** ultra-wideband antenna; optimization methods; Bezier Search Differential Evolution Algorithm; RF energy harvesting; 5G new radio



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## 1. Introduction

A significant aspect of emerging Internet of Things (IoT) technologies is low-powered sensor networks and the effort to achieve self-sustainability in both rural and urban envi-

ronments. With the introduction of 5G-and-Beyond technologies, in cases such as smart cities, the availability of ambient RF (RadioFrequency) energy is vastly increased, making it a perfect source for green energy applications. To collect this type of energy, dedicated compact devices, called rectennas, are used. In the meantime, great effort is made in the domain of machine learning, regarding optimization methods and, specifically, evolutionary algorithms, as they provide an efficient way of finding the optimal solution in complex but well-parameterized problems.

Lately, a huge scientific effort has been put into proposing new rectenna designs with a variety of applications, characteristics, and frequency bands that they collect energy from, as well as optimizing one of its components, i.e., the antenna. In [1], a cutting-edge rectenna with ultra-wideband features is proposed that efficiently harvests energy from frequencies ranging between 1.7 and 2.7 GHz. This covers bands used by technologies such as that of GSM at 1.8 GHz, UMTS at 2.1 GHz, Wi-Fi at 2.4 GHz, and LTE at 2.6 GHz. In [2] the antenna component of this rectenna is optimized in order to refine the design by reducing its degrees of freedom while preserving its wideband performance. In the present research effort, the goal is to broaden the bandwidth of the initial antenna to include 5G New Radio (frequency range 3.3–3.7 GHz), and if possible, to further simplify the design by reducing its degrees of freedom, making it a suitable solution for most low-power self-sustainable sensor networks, both in rural and urban scenarios.

Thus, the research problem can be defined as designing a multi-band rectenna that operates in several popular frequency bands like that of GSM at 1.8 GHz, UMTS at 2.1 GHz, Wi-Fi at 2.4 GHz, and LTE at 2.6 GHz and the 5G NR FR1 band at 3.5 GHz. We used the CST Studio Suite for antenna simulations, which is a high-performance full-wave commercial software.

In order to perform the optimization process, a CST-Matlab Wrapper was designed by the authors, which contained all the functions to manipulate a CST antenna design, using a corresponding Matlab script. Furthermore, a variety of optimization methods were studied, including evolutionary algorithms, which were tested on several benchmarks, resulting in the Bezier Search Differential Evolution Algorithm (BeSD) [3] as a suitable algorithm for increase in antenna bandwidth.

The goal of the optimization process was achieved by providing an antenna that fulfills all the aforementioned goals, both increasing the operation bandwidth and simplifying the design as much as possible, while presenting all the other important characteristics for the harvesting of RF energy. Specifically, the operation bandwidth of the antenna is extended to include 5G NR at 3.5 GHz, starting from 1.4 GHz to 3.9 GHz, also enabling technologies around 1.5 GHz that are suitable for mid-range high-speed communications. Meanwhile, it is still capable of harvesting RF energy from all technologies supported by the initial design, namely that of GSM at 1.8 GHz, UMTS at 2.1 GHz, Wi-Fi at 2.4 GHz, and LTE at 2.6 GHz. Last but not least, the optimization process leads to reduction in design complexity by discarding all structural parameters or modifications that are unnecessary for the goal to be achieved, resulting in a simple, compact, and robust design of a circular antenna with a stub extension on it that operates in all the bands of the vastly used technologies. All in all, the aforementioned characteristics of the optimal antenna, in addition to its efficiency, impedance response (around  $50 + 0j$  Ohm),  $S_{11}$  response (below  $-10$  dB), and omnidirectionality, across the frequency band, make it a sufficient design for a variety of applications, including most rectenna scenarios, as it is capable of meeting all the conditions to efficiently harvest energy from all the vastly used telecommunication technologies.

The contribution of this work can be summarized as follows:

- We design a novel ultra-wide band antenna for RF energy harvesting applications with more than a 2 GHz bandwidth.

- We introduce a recent optimization algorithm (BeSD) for antenna design. To the best of our knowledge, this is the first time BeSD has been applied to an optimization problem in electromagnetics.

In the remaining sections, this work is detailed as follows. Section 2 describes the optimization process, focusing on the BeSD algorithm approach, as well as on the implementation of the CST-Matlab API. In Section 3 the initial and optimal antenna designs are compared, in terms of factors like the  $S_{11}$  response, the Z response, efficiency, complexity, omnidirectionality, etc. Lastly, Section 4 concludes the research by summarizing the results and making closing arguments about future efforts to optimize antenna structures.

## 2. Materials and Methods

A common harvesting system is a rectenna, which consists of an antenna to collect ambient energy, a matching network, and a rectifier to transform the signal from AC to DC (Figure 1). In this paper, only the antenna component of a rectenna is studied and optimized, in regard to its ultra-wideband properties, specifically broadening its bandwidth to include 5G NR, as well as maintaining all other characteristics that make an antenna suitable for rectenna applications, like omnidirectionality, etc.

The initial antenna design, presented in [1], represents an ultra-wideband patch antenna with slots, stubs, and partial grounding, capable of harvesting RF energy signals from 1.7 to 2.7 GHz, while in [2] an optimized version of this antenna is proposed that operates in the same bandwidth, but with many fewer degrees of freedom than the initial structure. This research constitutes an extension of the initial effort in [2] to optimize the structure presented in [1].

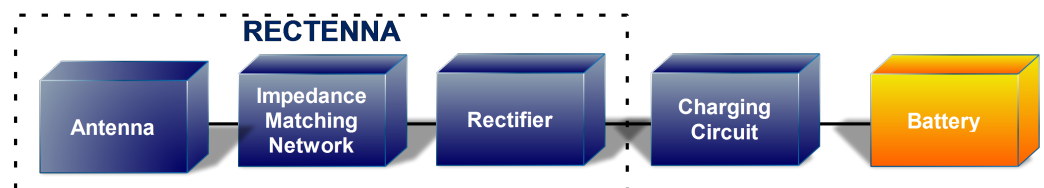


**Figure 1.** Rectenna components.

In the literature, a variety of antennas for RF energy harvesting applications are proposed, including almost all possible types of antenna. In [4], a square, electromagnetically coupled, two-layer antenna, capable of harvesting energy from three bands, namely at 2.1 GHz, 2.4–2.48 GHz, and 3.3–3.8 GHz, is proposed. The authors in [5] suggest a slotted fractal patch with partial grounding, operating from 2.15 to 2.9 GHz. Then, in [6], a cross-dipole slotted antenna is presented, harvesting from 1.8 to 2.5 GHz. The broad-band  $1 \times 4$  quasi-Yagi-Uda antenna array in [7] harvests from 1.8 to 2.2 GHz, while, the 16-port wideband dual-polarized patch antenna presented in [8] operates from 1.74 to 2.57 GHz. The authors in [9] propose a compact broad-band slotted patch antenna operating from 2.1 to 3.5 GHz, for harvesting energy from the LTE band. In Table 1, a comparative study of the optimal antenna proposed and broad-band antennas from the literature in the same frequency band is displayed.

It is obvious that the proposed antenna outperforms the previously suggested antennas for RF energy harvesting for this frequency band, as it has a greater operational bandwidth and is the only one capable of harvesting from all the vastly used wireless telecommunications technologies, i.e., GSM (1.8 GHz), UMTS (2.1 GHz), Wi-Fi (2.4 GHz), LTE (2.6 GHz), and 5G NR (3.5 GHz). This makes the proposed design one of the most suitable antenna components for rectennas that aim to collect as much energy as possible in modern telecommunications networks, as its huge frequency band of operation can cover the majority of RF energy harvesting scenarios.

**Table 1.** Comparative table of antennas operating in target frequency band

| Reference              | Design Type                                                                             | Frequency Bands                        |
|------------------------|-----------------------------------------------------------------------------------------|----------------------------------------|
| [4]                    | square, two layer, coupled antenna                                                      | 2.1 GHz, 2.4–2.48 GHz, and 3.3–3.8 GHz |
| [5]                    | slotted fractal patch antenna with partial grounding                                    | 2.15–2.9 GHz                           |
| [6]                    | cross-dipole slotted antenna                                                            | 1.8–2.5 GHz                            |
| [7]                    | 1 × 4 quasi-Yagi-Uda antenna array                                                      | 1.8–2.2 GHz                            |
| [8]                    | 16-port dual-polarized patch antenna                                                    | 1.74–2.57 GHz                          |
| [9]                    | compact slotted patch antenna                                                           | 2.1–3.5 GHz                            |
| [1]                    | wideband patch antenna with slots, stubs, and partial grounding                         | 1.7–2.7 GHz                            |
| [2]                    | wideband patch antenna with stubs and partial grounding, optimized with BES [10]        | 1.7–2.7 GHz                            |
| proposed patch antenna | ultra-wideband patch antenna with a stub and partial grounding, optimized with BeSD [3] | 1.4–3.9 GHz                            |

### 2.1. Optimization Process

The optimization of the antenna structure to achieve the desired goal (broadening the antenna bandwidth to include 5G NR) uses novel metaheuristic algorithms, known for their efficiency in solving a wide range of well-defined and parametrized problems, as highlighted in the existing literature. To make the algorithms applicable to the structure, the authors first parameterized the antenna (at least the degrees of freedom that would be used for the optimization), and then a fully functional API between the electromagnetic simulation software CST 2023 and the Matlab programming environment was designed by the authors to automatically implement the changes in the structural parameters and obtain the behavioral responses (i.e.,  $S_{11}$ ).

These changes are defined by the BeSD algorithm [3], during the iteration process, in which each agent corresponds to a possible solution (antenna design with algorithm-defined parameters) and is evaluated by its user-defined cost function, which is calculated based on the  $S_{11}$  graph. The design with the lowest cost function (minimization problem) is deemed optimal, and the algorithm ultimately converges at it, successfully achieving the posed goal. Our objective is to develop an antenna that includes a wide operating bandwidth. Therefore, the objective function is defined as

$$F(\bar{x}) = S_{11}^{f_c} + \eta \times \max(0, \gamma(\bar{x}) - U_{dB})$$

$$\gamma(\bar{x}) = \max\{S_{11}^{f_k}(\bar{x}) : k = 1, \dots, K\} \quad (1)$$

where  $\bar{x}$  is the vector of the geometric parameters,  $\eta$  denotes a very large number,  $U_{dB}$  is the limit in dB,  $K$  is the number of frequency points taken, and  $f_c = 3.55$  GHz the center frequency of the n78 5G NR band. We set  $U_{dB} = -10$  dB while the frequency points are taken from 1.4 to 3.9 GHz. The optimization process was carried out using the CST Matlab API in conjunction with the BeSD algorithm code.

### 2.2. Matlab—CST API

A novel Matlab-CST wrapper was designed by the authors, allowing the manipulation of CST antenna designs, using the BeSD Matlab script code. This API enables the optimization process to take place, in which the varying values of the degrees of freedom are provided as input by the algorithm, in an effort to reach the optimal design, and the evaluation of the corresponding antenna is obtained by its cost function. In this minimization problem, the cost function is defined by the  $S_{11}$  response graph of the antenna and then returned to the algorithm. Each simulated antenna design corresponds to an agent of a

given iteration, with algorithm-defined parameters, in an effort to find the optimal result (minimum cost function).

### 2.3. Bezier Search Differential Evolution Algorithm Description

In the literature, many metaheuristic algorithms have been applied as optimizers in the domain of electromagnetics [2,11–16]. For example, these include the most popular Genetic Algorithm [17,18] and Particle Swarm Optimization [19,20] models, as well as the Gray Wolf Optimizer [21,22], the Differential Evolution (DE) algorithm [23], the CMA-ES algorithm [24], Biogeography Based Optimization (BBO) [25,26], and the Bald Eagle Search Algorithm [10].

The evolutionary algorithm used in this research paper to achieve the optimal antenna goal is the Bezier Search Differential Evolution Algorithm or BeSD algorithm [3]. BeSD is a stochastic evolutionary search algorithm based on matrix patterns and is a modified version of the Differential Evolution (DE) algorithm [23], which is a fast and reliable algorithm. It is designed as a global optimizer for real-valued numerical problems and was selected among a variety of candidate algorithms as it outperforms all of them when tested in benchmark functions, as it produces more accurate optimal results faster than the rest. As a general note, the ultimate advantage of the BeSD algorithm is its rapid convergence to the optimal solution, as already tested in research [27].

More specifically, the main advantages of the BeSD algorithm [3], which make it a better choice than other conventional optimization algorithms, are the following:

- Its unique (Benzier polynomials) mutation algorithm is multi-component, including a weighted-elitist component that strengthens exploitation. It differs from the mutation operators of conventional algorithms, providing an enhanced exploration aspect to BeSD.
- In contrast to most conventional algorithms, its crossover process has no control parameters and is randomized, favoring both exploration and exploitation.
- BeSD's characteristic structure showcases simplicity and efficiency, due to its limited computational complexity, which leads to accurate optimal solutions with a relatively quick convergence, compared to conventional algorithms.
- Moreover, BeSD can be applied to parallel computing scenarios, as its patterns evolve separately, leading to a non-recursive algorithm.

Briefly describing the optimization technique of the BeSD algorithm, the following definitions are considered. A random solution to the problem, consisting of as many variables as the size of the problem, is described as a pattern. A matrix of patterns is the total number of possible solutions used in the algorithm, which is reconstructed at the beginning of each iteration by randomly selecting  $N$  patterns from the pre-pattern matrix. Hence, the size of the pre-pattern matrix can be expressed as  $L$  times the size of the pattern matrix,  $M$  (Equation (2)).

$$G_{p,j} \sim U(\text{MinVar}_j, \text{MaxVar}_j) \quad \text{where} \quad p = [1 : L \cdot N], j = [1 : D], 1 \leq L \leq 5 \quad (2)$$

where  $\text{MinVar}$  and  $\text{MaxVar}$  are the lower and upper search boundaries of the  $j$ th variable of patterns, accordingly, and  $L$  is the size control value of the pre-pattern matrix  $G$ . The values of the objective function,  $\mathcal{OF}(\cdot)$ , for the matrix  $G$  are expressed by Equation (3).

$$\text{fit}G_p = \mathcal{OF}(G_p) \quad \text{where} \quad p = [1 : L \cdot N] \quad (3)$$

The initial value of the global solution,  $\text{best}_{\text{global}}$ , and its objective function,  $\text{min}_{\text{global}}$ , are given by Equation (4).

$$[best_{global}, min_{global}] = [G_{\delta}, fitG_{\delta}] \mid fitG_{\delta} = \min(fitG), \delta \in \{1 : L \cdot N\} \quad (4)$$

Then, the iterative process described in Algorithm 1 takes place, until the predefined number of iterations is met,  $It = MaxIt$ . A variety of index vectors are used in this algorithm, to balance both its exploration and exploitation properties. That is,  $v_0, v_1$ , and  $v_2$  are used to generate the initial pattern matrix  $M$  and  $fitM$  and the b-subjective vectors  $dw_1$ . The row order of patterns of  $dw_1$ , which are bijective patterns, is different from the row order of  $M$ 's patterns, assuring that the latter will always evolve towards a different pattern. The vectors  $v_3$  and  $v_4$  are used to compute the Bezier mutation vectors  $dw_2$ , in which  $B_{0,3}, B_{1,3}, B_{2,3}$ , and  $B_{3,3}$  are the real values given by the Bernstein polynomials.  $dw_2$  patterns define Bezier locations on the hyperline segments, which vary according to the row order of the Y pattern matrix. As  $Permutation(1 : N)$  replaces the row numbers of the Y patterns,  $dw_2 - M$  change direction frequently and efficiently, enhancing the exploitation attributes of the algorithm in a search for the local optima.

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**Algorithm 1** Pseudo-code for the Bezier Search Differential Evolution Algorithm.
 

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Define the Objective Function:  $OF$ , Search Space Limits:  $(MinVar, MaxVar)$ , Size of the Pattern Matrix  $M$ :  $N$ , Dimensions of the problem:  $D$ , Maximum Number of Iterations:  $MaxIt$ , and size control value of pre-pattern Matrix  $G$ :  $L$

Initialize pre-pattern matrix  $G$ , objective function  $OF$ , and the  $best_{global}$  solution along with its value  $min_{global}$  using Equations (2)–(4), accordingly

**for**  $It = 1$  **to**  $MaxIt$  **do**

    Generate vectors  $v_0, v_1, v_2$

    count = 0

**while** true **do**

$v_1 = Permutation(1 : L \cdot N), v_2 = Permutation(1 : L \cdot N)$

$v_1 := v_1(1 : N), v_2 := v_1(1 : N)$

$\mid \text{ ' := ' update operator}$

**if**  $((v_1 \neq [1 : N]) \wedge (v_2 \neq [1 : N]) \wedge (v_1 \neq v_2)) \vee (\text{count} > (N \cdot D))$  **then**  
             false

**end if**

        count = count + 1

**end while**

$v_0 = v_1$

    Compute pattern matrix  $M$ , and  $fitM$

**for**  $i = 1$  **to**  $N$  **do**

$[M_i, fit(M_i)] = [G_{v_1}(i), fit(G_{v_1}(i))]$

**end for**

    Generate Bijective vectors  $dw_1$

$dw_1 = G_{v_2}$

    Select top  $N$  best pattern vectors  $N_{best}$

**for**  $i = 1$  **to**  $N$  **do**

$N_{best} = H_{[\kappa^3 \cdot K \cdot N]} \mid H = G_{\beta}$

$\beta \leftarrow \text{sort}(fit(G), \text{ascend}), \beta \in \{1 : L \cdot N\}$

**end for**

    Compute vectors  $v_3$  and  $v_4$

    count = 0

**while** true **do**

$v_3 = Permutation(1 : N), v_4 = Permutation(1 : N)$

**if**  $((v_3 \neq [1 : N]) \wedge (v_4 \neq [1 : N]) \wedge (v_3 \neq v_4)) \vee (\text{count} > N \cdot D)$  **then**  
             false

**end if**

        count = count + 1

**end while**

---

**Algorithm 1** *Cont.*


---

```

    Compute Bezier mutation vectors  $dw_2$ 
    for  $i = 1$  to  $N$  do
         $Y_i = [M_i, M_{v_3,i}, M_{v_4,i}, N_{best_i}]^T$ 
         $Y := Y_{Permutation(1:N)}$ 
         $dw_{2i} = [B_{0,3}(q) B_{1,3}(q) B_{2,3}(q) B_{3,3}(q)] \cdot Y \mid q \sim U(0,1)$ 
    end for
    Compute evolutionary step size  $F$ 
    for  $i = 1$  to  $N$  do
         $F_i = a \cdot b^e, \mid a \sim \mathcal{N}(0,1), b = 1 + \kappa$ 
         $e = a^c \mid c \sim U\{1,2,\dots,7\}$ 
    end for
    Compute crossover control matrix  $C$ 
     $C_1(1:N, 1:D) = C_2(1:N, 1:D) = 0$ 
    for  $i = 1$  to  $N$  do
         $C_1(i, h(u)) = 1 \mid h = \text{Permuting}(1:D), u = 1: \lceil \kappa^c \cdot D \rceil$ 
         $C_2(i, h(u)) = 1 \mid h = \text{Permuting}(1:D), u = 1: \lceil 1 - \kappa^c \cdot D \rceil$ 
        if  $\kappa < \kappa$  then
             $C = C_1$ 
        else
             $C = C_2$ 
        end if
    end for
    Compute trial pattern vectors  $T$ 
    for  $i = 1$  to  $N$  do
         $z_1 = \kappa, z_2 = \kappa$ 
         $T = M + C \circ F \circ (z_1 \circ (dw_1 - M) + z_2 \circ (dw_2 - M))$ 
    end for
    Check that  $T_i$  values conform to the search space boundaries, and if not, recalculate  $T_i$ 
    and  $fitT_i$ 
    for  $i = 1$  to  $N$  do
        for  $j = 1$  to  $D$  do
            if  $T_{i,j} < MinVar_j$  or  $T_{i,j} > MaxVar_j$  then
                 $T_{i,j} = MinVar_j + \kappa \cdot (MaxVar_j - MinVar_j), \kappa \in [0,1]$ 
            end if
        end for
    end for
    Update pattern matrix  $M$  and  $fitM$ 
    for  $i = 1$  to  $N$  do
         $fitT_i = \mathcal{OF}(T_i)$ 
        if  $fitT_i < fitM_i$  then
             $[M_i, fitM_i] = [T_i, fitT_i]$ 
        end if
    end for
    Update the best solution  $best_{global}$  and its value  $min_{global}$ 
     $[M_{best}, Val_{best}] = [M(\lambda), fitM(\lambda)] \mid fitM(\lambda) = \min(fitM) \mid \lambda \in \{1:N\}$ 
    if  $BestVal < min_{global}$  then
         $best_{global} = M_{best}, min_{global} = Val_{best}$ 
    end if
    Update the pattern matrix  $G$  and its value  $fitG$ 
     $[G_{v0}, fitG_{v0}] = [M, fitM]$ 
    Report the best solution  $best_{global}$  and the corresponding global minimum value  $min_{global}$ 
end for

```

---

Using the bijective and Bezier mutation vectors, as well as the crossover control matrix  $C$  and the scaling factor  $F$ , the trial pattern vectors  $T$  are computed, and they are recalculated in cases where they do not conform to the search boundaries ( $MinVar$ ,  $MaxVar$ ). The scaling factor  $F$  is simple, fast, and easy to generate, thus making it efficient in finding the best global solution, while the crossover and mutation operators produce trial patterns in an efficient manner, substantially speeding up the search process for the global solution. The crossover operator injects information from the parental to the mutation patterns, preventing, in this way, the excessive degeneration of the latter. The crossover operator is controlled by the binary-valued map matrix, in which, for parental patterns with  $C = 0$ , they are preserved and unchanged in the mutation patterns.

Then,  $T$  vectors update the pattern matrix  $M$  and  $fitM$  and track the best pattern (minimum value),  $M_{best}$ , to compare its value,  $Val_{best}$ , with  $min_{global}$  so far, and  $best_{global}$  is replaced with  $M_{best}$  if the value of the latter is smaller. This process is repeated in each iterative step, until the user-defined number of iterations is reached, when the last  $best_{global}$  pattern is given as the optimal solution to the minimization problem, with a value of  $min_{global}$ .

The time complexity of the BeSD [3] is  $\mathcal{O}(N \cdot D \cdot MaxIt)$ . Moreover, the space complexity of the algorithm is similar to that of the original differential evolution and equal to  $\mathcal{O}(N \cdot D)$ .

It should be noted that BeSD outperformed other competitive algorithms in [3]. More specifically, in [3], the authors compared BeSD with the Covariance Matrix Learning and Searching Preference (CMLSP) [28], Mean Variance Optimization (MVO) [29], Without Approximation (WA)-Based Optimization [30], Success History-Based Adaptive Differential Evolution (SHADE) [31], and Linear Population Size Reduction Adaptive Differential Evolution (LSHADE) [32] algorithms. The above algorithms competed with other algorithms and were reported as the top methods according to the CEC2014 criteria.

The computational complexity of BeSD is generally superior to that of CRMLS, MVO, WA, SHADE, and LSHADE according to [3]. Furthermore, according to the numerical results presented in ([3], Table 5), the complexity of the algorithm  $C_{time}$  in [3] of BeSD was better than RMLSP and MVO for problems with {30, 50, 100} dimensions. Additionally, BeSD obtained better  $C_{time}$  than WA for {100} dimensions. BeSD also obtained better results for  $C_{time}$  than LSHADE for all dimensions.

In terms of computational time, BeSD has some advantages over other algorithms. In contrast to SHADE and LSHADE, BeSD does not use sophisticated statistical distributions to calculate the value of the mutation parameter,  $F$ . The BeSD method for producing  $F$  is quick and effective. Unlike SHADE and LSHADE, BeSD does not use an archiving mechanism. As a result, BeSD requires comparatively little memory.

#### 2.4. Performance Evaluation

In this optimization process, the BeSD [3] algorithm is proposed, after being evaluated based on its performance, compared to other algorithms, namely the Differential Evolution (DE) algorithm [23], the CMA-ES algorithm [24], the Genetic Algorithm (GA) [17,18], and Biogeography-Based Optimization (BBO) [25,26]. To successfully assess those algorithms, a comparative test was performed with highly used benchmark functions [33], in which BeSD outperformed all the aforementioned algorithms, as depicted in Table 2. The benchmark functions were Rotated Hyper-ellipsoid,  $f1$ , Rastrigin,  $f2$ , Ackley,  $f3$ , Sphere,  $f4$ , Powell,  $f5$ , Three-Hump Camel,  $f6$ , Beale,  $f7$ , Sum Squares,  $f8$ , Bohachevsky No. 3,  $f9$ , and Sum of Different Powers,  $f10$ , and they are given in detail in Appendix A. For the benchmark test to be valid, all algorithms had to be tested for the same parameters across the aforementioned functions. Hence, for all test cases, the following were applied:

- Population: 100.
- Iterations: 1000.
- Problem Variables: 30.
- Variable Boundaries:  $[-10, 10]$ .
- Independent Trials: 100.

**Table 2.** Tested algorithms' performance with benchmark functions based on the optimal result. The bold font shows lower values.

|          | BeSD                                     | GA                                       | BBO                     | DE                                       | CMA-ES                  |
|----------|------------------------------------------|------------------------------------------|-------------------------|------------------------------------------|-------------------------|
| $f_1$    | <b><math>0.000 \times 10^{00}</math></b> | $1.170 \times 10^{-33}$                  | $1.606 \times 10^{-02}$ | $4.216 \times 10^{-13}$                  | $5.819 \times 10^{-25}$ |
| $f_2$    | <b><math>0.000 \times 10^{00}</math></b> | $13.867 \times 10^{00}$                  | $3.349 \times 10^{00}$  | $5.980 \times 10^{01}$                   | $1.593 \times 10^{02}$  |
| $f_3$    | <b><math>0.000 \times 10^{00}</math></b> | $1.563 \times 10^{-12}$                  | $0.491 \times 10^{-01}$ | $7.846 \times 10^{-09}$                  | $9.731 \times 10^{-10}$ |
| $f_4$    | <b><math>0.000 \times 10^{00}</math></b> | $9.356 \times 10^{-33}$                  | $2.149 \times 10^{-02}$ | $7.462 \times 10^{-16}$                  | $9.874 \times 10^{-30}$ |
| $f_5$    | <b><math>0.000 \times 10^{00}</math></b> | $6.950 \times 10^{-02}$                  | $4.350 \times 10^{-02}$ | $1.429 \times 10^{02}$                   | $9.343 \times 10^{03}$  |
| $f_6$    | <b><math>0.000 \times 10^{00}</math></b> | <b><math>0.000 \times 10^{00}</math></b> | $5.343 \times 10^{-60}$ | $2.625 \times 10^{-73}$                  | $3.118 \times 10^{-08}$ |
| $f_7$    | $5.580 \times 10^{-08}$                  | <b><math>0.000 \times 10^{00}</math></b> | $3.592 \times 10^{-15}$ | $1.987 \times 10^{-21}$                  | $8.680 \times 10^{-07}$ |
| $f_8$    | <b><math>0.000 \times 10^{00}</math></b> | $2.478 \times 10^{-34}$                  | $2.027 \times 10^{-02}$ | $3.771 \times 10^{-13}$                  | $1.800 \times 10^{-26}$ |
| $f_9$    | <b><math>0.000 \times 10^{00}</math></b> | <b><math>0.000 \times 10^{00}</math></b> | $1.665 \times 10^{-15}$ | <b><math>0.000 \times 10^{00}</math></b> | $1.296 \times 10^{-06}$ |
| $f_{10}$ | <b><math>0.000 \times 10^{00}</math></b> | $4.005 \times 10^{-18}$                  | $1.131 \times 10^{-15}$ | $1.976 \times 10^{-38}$                  | $4.778 \times 10^{-12}$ |

The data in Table 2 depict the optimum result that each algorithm could track for a given benchmark function, in all of which the optimum result was a global minimum. It should be noted that BeSD outperformed all other algorithms, successfully finding the global minimum in all functions except one,  $f_7$ , for which the global solution was 0, as shown in Appendix A, and was achieved by the GA instead. This is in accordance with the “no free lunch” (NFL) theorem by David Wolpert and William Macready [34]. Thus, no optimization algorithm can perform better than others in all optimization problems. In more detail, as depicted in the Friedman test (Table 3), BeSD ranked first among all algorithms tested and therefore was selected as the optimizer in the antenna bandwidth broadening problem. In second place was the GA with a very good performance as well; meanwhile, the rest of the algorithms performed exceptionally too, but in some cases, their search process was trapped in local minima, not successfully reaching the global best solution. The results, shown in Tables 2 and 3, confirm the authors' claim [3] that BeSD is a fast and efficient algorithm, with a perfect balance between exploration and exploitation, making it suitable for minimization problems, such as antenna design problems.

**Table 3.** Friedman non-parametric statistical testing of algorithms' performance

|                    | BeSD | GA  | BBO | DE  | CMA-ES |
|--------------------|------|-----|-----|-----|--------|
| Friedman           | 1.3  | 2.1 | 3.6 | 3.0 | 3.7    |
| Normalized Ranking | 1    | 2   | 4   | 3   | 5      |

The time to run the algorithms for each iteration of the Rastrigin function ( $f_2$ ) was about 3.2 ms (BeSD), 3.3 ms (GA), 3.4 ms (BBO), 3 ms (DE), and 3.2 ms (CMA-ES).

### 3. Optimization Results and Discussion

The optimization process was utterly successful by producing a result that included 5G NR in its operational bandwidth. This Figure 2 shows the original antenna from [1]. The resulting antenna, as depicted in Figure 3, was much more simplified compared to the initial one, shown in Figure 2. During the optimization process, after a number of iterations, some of the variables were set to such values by the algorithm, which deemed them non-essential for the optimal result, i.e., the width of the initial slots is equal to 0. This

was either due to their inefficiency in changing the response of the antenna design or due to their non-essentiality, as the result could be achieved with fewer degrees of freedom. After discarding those variables from the design, a simplified version of the antenna was optimized from scratch using the BeSD algorithm, and the optimal result consisted of the main circular patch with partial grounding and a stub on it. That is, the substrate was Isola FR408 ( $\epsilon_r = 3.75$  and  $\tan\delta = 0.012$ ), with a thickness equal to 1.6 mm. This stub was the main reason for the widening of the bandwidth to include 5G NR, as well as including Wi-Fi in the harvested technologies, as proven by comparing the response graph of the antenna  $S_{11}$  in Figure 4 with (black line) and without (red dashed line) the stub. The width and length of the stub, along with the design scaling factor, were the three optimization variables of the minimization problem that produced this particular result. Specifically, those parameters are depicted in Figure 3 and are given in detail in Table 4 below.

**Table 4.** The geometric values of the best optimized antenna parameters.

| Parameter               | Value (mm) |
|-------------------------|------------|
| Stub width              | 1.02       |
| Stub length             | 41.34      |
| Circular patch diameter | 51.66      |
| Substrate side          | 93.67      |
| Ground small side       | 20.34      |
| Transmission line width | 3.44       |

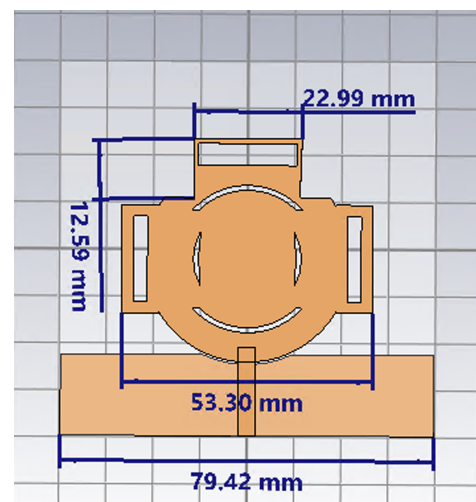


**Figure 2.** Initial antenna design [1].

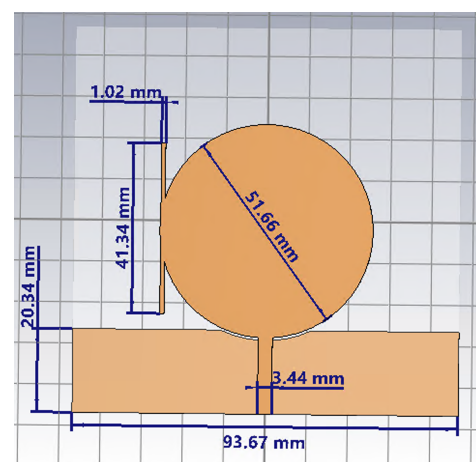


**Figure 3.** Optimal antenna using the BeSD algorithm.

The cost function is a value that is returned to the algorithm and is how the design is evaluated (Equation (1)). In this minimization problem, the cost function of each agent was defined by two factors, firstly by whether the antenna operated in the whole target bandwidth (from 1.4 GHz to 3.9 GHz), or in a fraction of it, so that the condition  $S_{11} < -10$  dB was met for all frequency points, and by the response  $S_{11}$  of the center frequency of the simulation process,  $f_c = 3.55$  GHz. More specifically, the input arguments of BeSD, as depicted in Algorithm 1, were set as follows:

- Independent Trials: 5.
- Size Control Value of Pre-Pattern Matrix, L: 2.5, to assure balance between exploration and exploitation mechanisms.
- Number of Iterations: 100.
- Size of Pattern Matrix, N (Size of Population): 40.
- Problem Dimensions, D (Variables): 3.
- Variable Boundaries:
  - Structure Scaling Factor: [0.5, 1.5].
  - Stub Width: [1, 10] (mm).
  - Stub Length: [10, 35] (mm).

The calculated number of antenna design simulations and cost functions, which corresponded to the stopping criteria of the optimization process, was 4000, and an acceptable solution was any design with the reflection coefficient  $S_{11} \leq -10$  dB across the target operation band. As the main frequency, that of the center of the n78 band was selected, at  $f_c = 3.55$  GHz, for the optimizer algorithm to focus on minimizing the  $S_{11}$  value in this frequency band, to include 5G NR in the operational bandwidth.

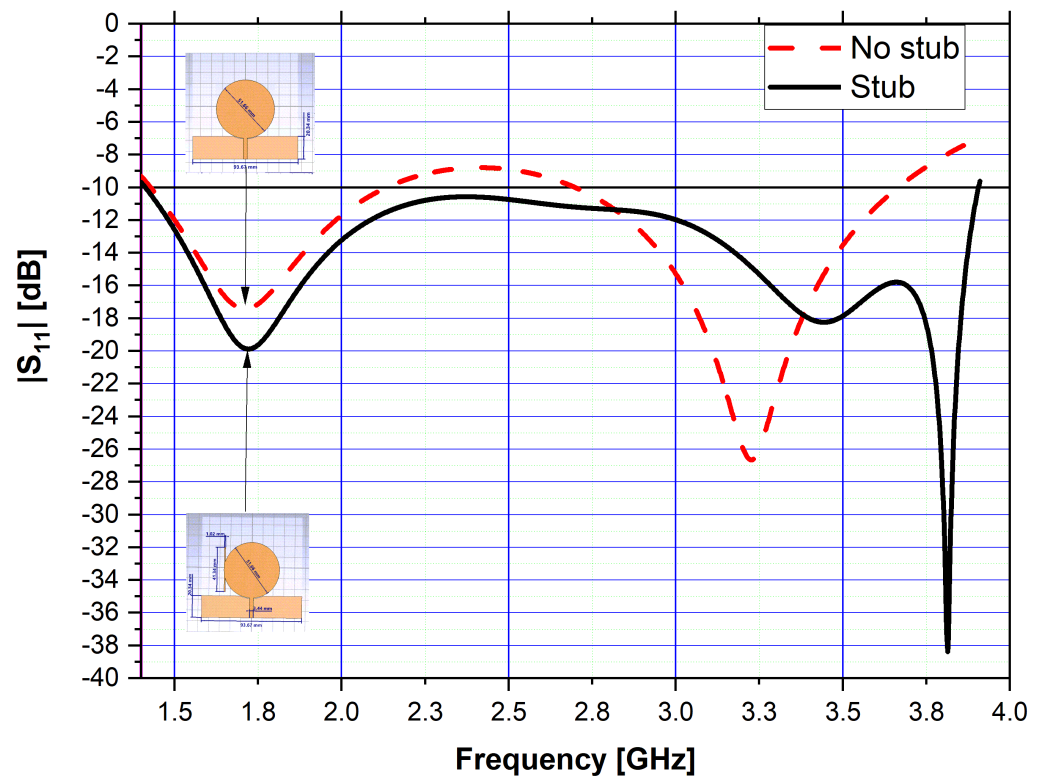


**Figure 4.** The frequency response of the best antenna found: black solid line (with the stub) and red dashed line (without the stub).

Then, as depicted in detail in the graph ( $S_{11}$ , Figure 4), the optimal antenna result operates efficiently from 1.4 GHz to 3.9 GHz, as  $S_{11} < -10$  dB is satisfied across this band,

while two minima are also demonstrated in the graph, which represent resonances in the frequencies  $f_1 = 1.72$  GHz with  $S_{11} = -20$  dB and  $f_2 = 3.8$  GHz with  $S_{11} = -38$  dB. This frequency band includes all the technologies that the original design was capable of harvesting from, namely GSM at 1.8 GHz, UMTS at 2.1 GHz, Wi-Fi at 2.4 GHz, and LTE at 2.6 GHz, but also includes 5G NR technologies at 3.5 GHz. As a result of the bandwidth broadening in the optimal antenna, not only 5G NR is included in it, but other technologies that were not the initial target of the optimization also are, as the bandwidth of the original antenna was 1 GHz (from 1.7 to 2.7 GHz), and that of the optimal antenna is 2.5 GHz (from 1.4 to 3.9 GHz).

Moving forward, the rest of the graphs that describe the behavior of the optimal antenna should also be examined. Apart from the  $S_{11}$  graph, the impedance graph of the antenna is also significant, as its values should represent good impedance matching between the patch and the line, in order for the energy to be transferred without major losses to the following component of the system. Ideally, the value of the antenna impedance response should be as close to  $50 + 0j \Omega$  as possible across the operation bandwidth. As depicted in Figure 5, from 1.4 to 3.9 GHz, the reactance value has a mean value below  $30j \Omega$ , and the resistance is  $50 \pm 10 \Omega$  across the whole band. These values are vastly acceptable, as perfect matching cannot be realistically achieved in a bandwidth as large as 2.5 GHz, but the small reactance and resistance close to  $50 \Omega$  prove that the antenna indeed has ultra-wideband characteristics from 1.4 to 3.9 GHz, including all the aforementioned technologies (GSM, UMTS, Wi-Fi, LTE, 5G NR).

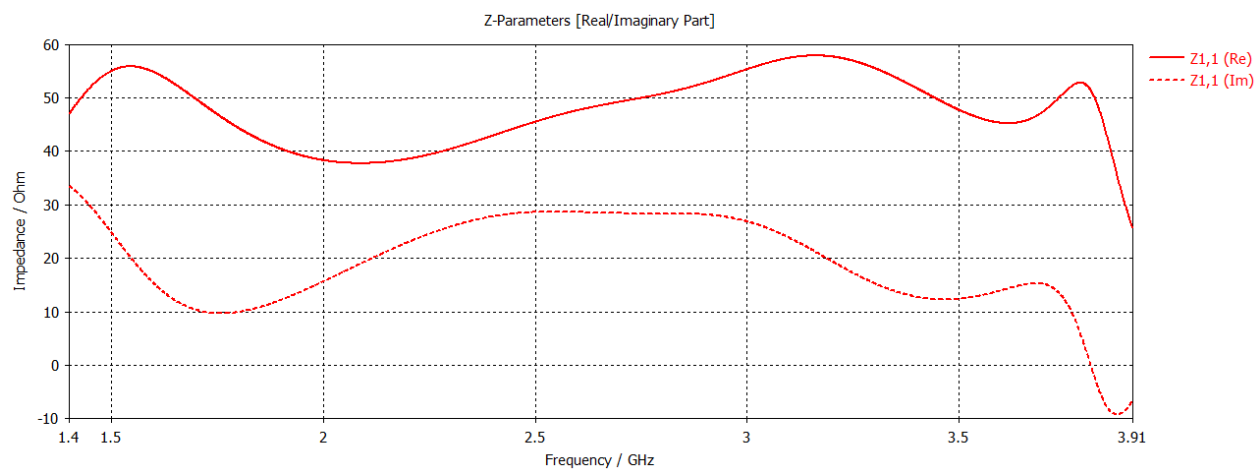


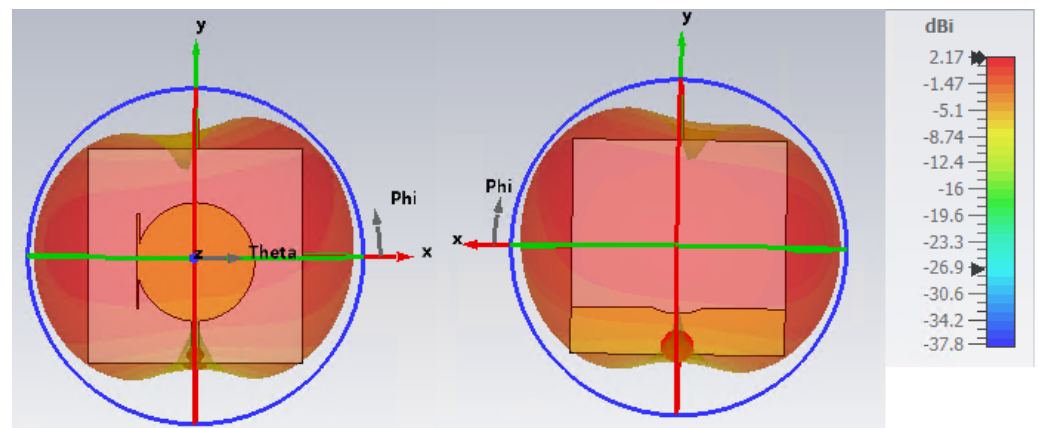
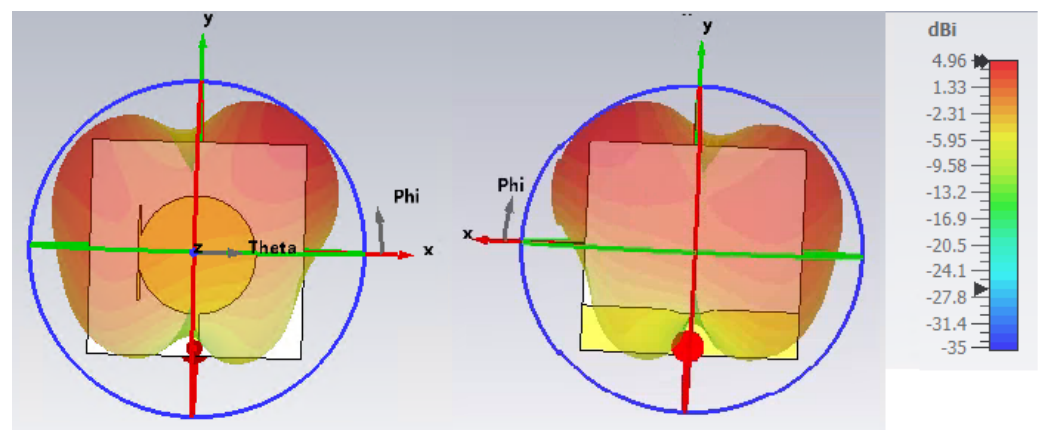
**Figure 5.** The impedance response of the optimal antenna.

In the following, since the impedance and  $S_{11}$  response graphs confirm the wideband properties of the antenna across the target frequency band, the efficiency of it is examined. These three metrics are codependent, but convenient values in one of them do not guarantee that the rest of them are as expected, i.e., in cases where the search produces structural results that contain design violations. As depicted in Table 5, the optimal antenna can successfully harvest energy from the ambient environment, with values greater than 84% for all technologies of interest, across the entire frequency band, and up to 96% for some frequency points. The efficiency in Table 5 proves that the optimal antenna is capable of harvesting energy from all target technologies, from GSM 1.8 GHz, UMTS 2.1 GHz, Wi-Fi 2.4 GHz, LTE 2.6 GHz, and 5G NR 3.5 GHz, which is also solidified by the impedance and  $S_{11}$  response graphs.

**Table 5.** Efficiency at target frequencies.

| Frequency (GHz) | Efficiency |
|-----------------|------------|
| 1.8 (GSM)       | 95.9%      |
| 2.1 (UMTS)      | 87.9%      |
| 2.4 (Wi-Fi)     | 83.7%      |
| 2.6 (LTE)       | 84.1%      |
| 3.5 (5G NR)     | 91.8%      |

Last but not least, another key attribute of antennas for rectenna applications is their omnidirectionality. Omnidirectionality is significant in RF energy harvesting scenarios, as the direction of arrival of the incident field is usually unknown and it varies in recent technologies, i.e., beamforming in 5G NR. As depicted in Figure 6, for the first resonant frequency,  $f_1 = 1.72$  GHz, the 3D radiation pattern of the optimal antenna has omnidirectional characteristics, which are also maintained throughout the operating frequency band, making the optimal antenna a perfect fit for RF harvesting applications. Specifically, while it is higher in the frequency band, the 3D pattern slightly loses its omnidirectionality, as depicted in Figure 7, for the second resonant frequency,  $f_2 = 3.8$  GHz. However, this change is not important and the design can be considered omnidirectional across the whole band. The slight changes in the pattern are also expressed as a difference in the Gain, varying from 2.2 dBi at  $f_1 = 1.72$  GHz to 4.9 dBi at  $f_2 = 3.8$  GHz, as shown below. It must be noted that although the radiation pattern seems to have some asymmetric behavior, probably due to the asymmetric stub, the Gain values are very good for all the operating frequency bands.

**Figure 6.** 3D radiation pattern of optimal antenna at  $f_1 = 1.72$  GHz.**Figure 7.** 3D radiation pattern of optimal antenna at  $f_2 = 3.8$  GHz.

## 4. Conclusions

In this research paper, a novel ultra-wideband patch antenna with a stub and partial grounding is proposed for RF energy harvesting applications (Figure 3). This antenna is the result of an optimization process, using the Bezier Search Differential Evolution Algorithm [3] (Algorithm 1), which was tested with benchmark functions and outperformed all other candidate algorithms (Tables 2 and 3). The optimization goal was to expand the operational bandwidth of the initial antenna design proposed in [1] (Figure 2) to include the 5G FR1 band. In order to implement the optimization, a novel CST-Matlab API was designed, containing all the functions to manipulate the CST antenna structure and capable of calculating the cost function from the corresponding  $S_{11}$  graph of each agent. The optimal antenna operates from 1.4 GHz to 3.9 GHz, successfully broadening the bandwidth of the initial design which was from 1.7 GHz to 2.7 GHz and allowing harvesting from all the vastly used technologies like GSM at 1.8 GHz, UMTS at 2.1 GHz, Wi-Fi at 2.4 GHz, LTE at 2.6 GHz, and 5G NR at 3.5 GHz, which is a feature not yet present in the literature, making it one of the most useful broad-band antennas for rectenna applications (Table 1). The ultra-wideband characteristics, along with its omnidirectionality, are confirmed from the impedance response graph, the  $S_{11}$  graph, the efficiency table, and the 3D radiation pattern. More specifically, the  $S_{11}$  graph in Figure 4 has values lower than  $-10$  dB in the entire operational frequency band; the impedance graph in Figure 5 depicts appropriate matching, with a mean reactance value less than  $30 \Omega$  and resistance close to  $50 \Omega$ ; the efficiency is above 84% across the frequency band and up to 96% for specific frequency points (Table 5); and last but not least, the 3D radiation pattern shows omnidirectional behavior as expected from antennas for RF harvesting application (Figures 6 and 7). In conclusion, the optimization process was utterly successful, resulting in an optimal antenna design suitable for the majority of RF energy harvesting applications, as it presents both ultra-wideband and omnidirectional characteristics in the operation band, which spreads from 1.4 to 3.9 GHz, including the frequencies of all vastly used wireless telecommunications technologies, like GSM, UMTS, Wi-Fi, LTE, and 5G NR. Future research will focus on finding complex matching networks that will allow rectifiers to convert RF energy from more than one of these technologies to DC.

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## Appendix A. Benchmark Functions

1. Rotated Hyper-ellipsoid function:

$$f(\mathbf{x}) = \sum_{i=1}^n \left( \sum_{j=1}^i x_j^2 \right)^2$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector.

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

2. Rastrigin function:

$$f(\mathbf{x}) = An + \sum_{i=1}^n \left( x_i^2 - A \cos(2\pi x_i) \right)$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector and  $A = 10$ .

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

3. Ackley function:

$$f(\mathbf{x}) = -20 \exp \left( -0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} \right) - \exp \left( \frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) + 20 + \exp(1)$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector, and  $n \geq 1$ .

Global minimum:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

4. Sphere function:

$$f(\mathbf{x}) = \sum_{i=1}^n x_i^2$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector.

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

5. Powell function:

$$f(\mathbf{x}) = \sum_{i=1}^{n/4} \left( (x_{4i-3} + 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^4 \right)$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector, and  $n$  is divisible by 4.

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

6. Three-Hump Camel function:

$$f(\mathbf{x}) = 2x_1^2 - 1.05x_1^4 + \frac{x_1^6}{6} + x_1x_2 + x_2^2$$

where  $\mathbf{x}$  is a 2-dimensional vector  $(x_1, x_2)$ .

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, 0]$ .

7. Beale function:

$$f(\mathbf{x}) = (1.5 - x_1 + x_1x_2)^2 + \left( 2.25 - x_1 + x_1x_2^2 \right)^2 + \left( 2.625 - x_1 + x_1x_2^3 \right)^2$$

where  $\mathbf{x}$  is a 2-dimensional vector  $(x_1, x_2)$ .

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [3, 0.5]$ .

8. Sum of Squares function:

$$f(\mathbf{x}) = \sum_{i=1}^n ix_i^2$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector.

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

9. Bohachevsky No. 3 function:

$$f(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) \cos(3\pi x_2) + 0.3$$

where  $\mathbf{x}$  is a 2-dimensional vector  $(x_1, x_2)$ .

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, 0]$ .

## 10. Sum of Different Powers function:

$$f(\mathbf{x}) = \sum_{i=1}^n |x_i|^{i+1}$$

where  $\mathbf{x}$  is an  $n$ -dimensional vector and  $p$  is a positive integer.

Global solution:  $f(\mathbf{x}) = 0$  at  $\mathbf{x} = [0, \dots, 0]$ .

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