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Capacity Bounds for Fluid-Antenna-Assisted MIMO in Nakagami- m Channels

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Abstract

Conventional multiple-input multiple-output (MIMO) systems rely on static antenna placement. To exploit additional spatial degrees of freedom, the fluid antenna (FA) concept has emerged as a promising solution for improving data rates and diversity performance. Most existing FA studies assume Rayleigh fading, whereas analytical characterization under Nakagami- m fading is more challenging. This article investigates the ergodic capacity of FA-assisted MIMO systems over Nakagami- m fading channels. By applying majorization theory, upper and lower bounds on the ergodic capacity are derived. High signal-to-noise ratio (SNR) approximations are then obtained to clarify the role of the fading parameter and the number of propagation paths. The large-system behavior is also studied, and Monte Carlo simulations are used to assess the tightness of the proposed bounds. The results show that the upper bound closely tracks the simulated capacity, while the lower bound remains useful mainly in the low-SNR regime.

Keywords: fluid-antenna systems; multiple-input multiple-output; MIMO; Nakagami- m fading; ergodic capacity; majorization theory

1. Introduction

Conventional multiple-input multiple-output (MIMO) systems generally employ fixed-position antennas. To enhance both diversity and multiplexing gains, fluid antenna (FA) technology has recently emerged as a promising alternative [1]. In FA-assisted architectures, antennas are allowed to move freely within a predefined spatial region rather than being confined to fixed locations. By exploiting the extra spatial degrees of freedom, the integration of FA with MIMO can significantly boost spatial diversity performance [2,3].

Numerous studies have explored the performance of FA-enabled systems. Fluid-antenna systems (FASs) were first put forward in [1], where their operation and performance were investigated for single-antenna settings. Subsequent results in [1] showed that a single-antenna FAS confined within a compact region could emulate the maximum-ratio-combining (MRC) gains usually associated with multi-antenna receivers. Building on these foundations, ref. [4] provided closed-form formulas for the average level-crossing rate of FASs. Furthermore, ref. [5] proposed a movable antenna-based MIMO architecture, which was essentially a MIMO implementation of FASs, and investigated capacity optimization using instantaneous channel state information (CSI). Moreover, ref. [6] proposed a channel model and evaluation framework tailored to MIMO systems aided by movable antennas.

Recent advances have further broadened the scope of FASs and movable-antenna research. In particular, movable-antenna-assisted MIMO systems have been shown to provide additional spatial degrees of freedom by jointly exploiting antenna-position optimization



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and conventional MIMO processing [5]. Moreover, recent studies have investigated outage probability, diversity gain, spatial correlation modeling, channel estimation, and multiple-access operation in FAS-enabled networks [7–9]. Beyond conventional FAS architectures, recent works have also explored the integration of fluid antennas with emerging programmable wireless technologies, such as stacked intelligent metasurfaces [10]. Survey and overview articles have further highlighted the role of FAS and movable antennas as candidate technologies for reconfigurable 6G wireless systems, including their potential integration with artificial intelligence, integrated sensing and communication, non-terrestrial networks, and reconfigurable intelligent surfaces [11,12].

In parallel, Nakagami- m fading, though mathematically more involved than Rayleigh or Rician models, has attracted attention due to its versatility in capturing a wide range of propagation conditions within a unified framework. It includes Rayleigh fading as a special case ($m = 1$) and approximates Rician behavior for $m > 1$, enabling modeling of both moderate and severe fading environments. This flexibility is particularly relevant for movable antenna systems, whose dynamic positions may experience diverse propagation conditions. Despite recent developments in FAS and movable-antenna systems, most available analytical results rely either on Rayleigh fading models or on instantaneous-CSI-based optimization. In contrast, Nakagami- m fading provides a more flexible statistical model that can capture a wider range of propagation conditions. Closed-form approximations for the outage probability of single-port fluid-antenna systems under correlated Nakagami- m fading were derived in [13], and copula-based outage expressions were obtained in [7]. However, the ergodic capacity characterization of FA-assisted MIMO systems under Nakagami- m fading remains largely unexplored. This motivated the present work, which derives tractable capacity bounds for finite-dimensional FA-assisted MIMO channels in Nakagami- m fading environments.

The main novelty of this work is the development of a tractable capacity-bound framework for finite-dimensional FA-assisted MIMO systems under Nakagami- m fading. This distinguishes the present study from conventional fixed-position MIMO capacity analyses, where the channel matrix is usually modeled statistically without antenna-position dependence, and from most existing FAS studies, which mainly focus on outage probability, diversity gain, level-crossing rate, average fade duration, multiple-access operation, or antenna-position optimization under instantaneous CSI. In the considered FA-assisted MIMO setting, the effective channel depends not only on the random path-gain matrix but also on the transmit and receive field-response matrices induced by the fluid-antenna positions. Hence, the channel behavior is shaped by the antenna locations, angles of arrival (AoAs), angles of departure (AoDs), number of propagation paths, fading severity, and effective channel rank.

The specific contributions of this paper are summarized as follows. First, a geometry-based FA-assisted MIMO channel model under Nakagami- m fading is considered, where the transmit and receive field-response matrices capture the impact of antenna positions and angular propagation parameters. Second, capacity bounds on the ergodic capacity are derived without requiring the exact eigenvalue distribution of the effective MIMO channel. Third, high-SNR approximations are obtained to provide analytical insight into the role of fading severity, number of paths, geometry-dependent spectral factors, and effective channel rank. Fourth, Monte Carlo simulations are used to validate the analytical bounds, examine their tightness, compare FA-assisted MIMO with conventional fixed-position MIMO, and quantify the practical capacity gain enabled by fluid antennas.

2. System Model

We consider a MIMO system in which the transmitter and receiver are equipped with M and N FAs, respectively, as shown in Figure 1. Each FA is connected to its corresponding RF chain via flexible interconnects, which allow the antenna element to move freely within a prescribed spatial region. The admissible displacement regions at the transmitter and receiver are denoted by $\mathcal{S}_t$ and $\mathcal{S}_r$, respectively. A two-dimensional Cartesian coordinate system is adopted to describe the FA locations. The position of the m th transmit FA, for $m = 1, \dots, M$, is given by $\mathbf{t}_m = [x_m^{(t)}, y_m^{(t)}]^\top \in \mathcal{S}_t$, while the position of the n th receive FA, for $n = 1, \dots, N$, is expressed as $\mathbf{r}_n = [x_n^{(r)}, y_n^{(r)}]^\top \in \mathcal{S}_r$. Both deployment regions, $\mathcal{S}_t$ and $\mathcal{S}_r$, are modeled as square areas of side length A (i.e., area $A \times A$). Specifically, we have $\mathcal{S}_t = \mathcal{S}_r = [-A/2, A/2] \times [-A/2, A/2]$.

The displacement regions $\mathcal{S}_t$ and $\mathcal{S}_r$ are assumed to be electrically compact compared with the transmitter–receiver separation. Hence, changing the FA positions affects the relative path phases and the field-response matrices but does not change the large-scale path loss or invalidate the far-field approximation. The FA positions are assumed to be configured according to the channel geometry and then kept fixed during the corresponding channel-use interval.

The complete sets of positions for the M transmit and N receive antenna elements are denoted by

$$\mathbf{t} = [\mathbf{t}_1, \dots, \mathbf{t}_M] \in \mathbb{R}^{2 \times M} \quad \text{and} \quad \mathbf{r} = [\mathbf{r}_1, \dots, \mathbf{r}_N] \in \mathbb{R}^{2 \times N},$$

respectively.

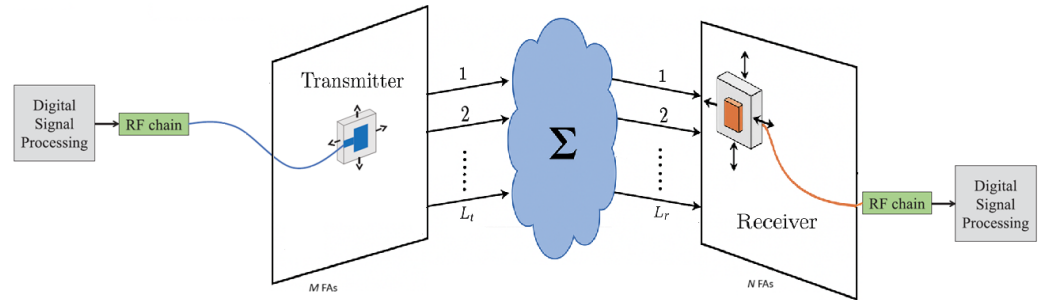


Figure 1. An FA-assisted MIMO system with M transmit FAs and N receive FAs.

The considered FA architecture follows the principle that each radiating element, or equivalently its effective radiation point, can be repositioned within a compact predefined region. In practice, this can be implemented through flexible feeding structures, movable conductive elements, switchable ports, or electronically reconfigurable fluidic/radiating media. Each FA is connected to one RF chain, and the antenna position determines the phase response observed along the different propagation paths. Therefore, the main role of the FA mechanism in this work is to provide additional spatial sampling capability within the available aperture, rather than to change the number of RF chains.

The MIMO input–output relationship is given by

$$\mathbf{y}(\mathbf{t}, \mathbf{r}) = \mathbf{H}(\mathbf{t}, \mathbf{r})\mathbf{x} + \mathbf{w}, \quad (1)$$

where $\mathbf{H}(\mathbf{t}, \mathbf{r}) \in \mathbb{C}^{N \times M}$ denotes the channel matrix between a transmitter with M antennas at positions $\mathbf{t}$ and a receiver with N antennas at positions $\mathbf{r}$. The vector $\mathbf{x}$ is the transmitted signal with covariance $\mathbf{Q}$, and the noise is modeled as circularly symmetric complex Gaussian, $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$. (It is assumed that the region within which the antenna elements are allowed to move is negligible compared to the link distance between the transmitter and receiver, thereby validating the far-field approximation).

The number of propagation paths at the transmitter and receiver are denoted by L_t and L_r , respectively. At the transmitter side, each of the L_t paths is characterized by an elevation angle $\theta_t^p \in [0, \pi]$ and an azimuth angle $\phi_t^p \in [0, \pi]$, where $1 \leq p \leq L_t$. For the p th transmit path, the relative propagation distance offset of the m th transmit element with respect to a reference point at the origin $\mathbf{t}_0 = (0, 0)^T$ is expressed as [5]

$$\rho_t^p(\mathbf{t}_m) = x_m \sin \theta_t^p \cos \phi_t^p + y_m \cos \theta_t^p. \quad (2)$$

Accordingly, the corresponding phase difference introduced by the p th path is $\frac{2\pi}{\lambda} \rho_t^p(\mathbf{t}_m)$, where λ denotes the wavelength of the transmit signal. Based on this, the transmit field-response vector is expressed as

$$\mathbf{g}(\mathbf{t}) \triangleq [e^{j\frac{2\pi}{\lambda} \rho_t^1(\mathbf{t})}, \dots, e^{j\frac{2\pi}{\lambda} \rho_t^{L_t}(\mathbf{t})}]^T \in \mathbb{C}^{L_t \times 1}. \quad (3)$$

The overall transmit field-response matrix for all M transmit elements is then defined as

$$\mathbf{G}(\mathbf{t}) \triangleq [\mathbf{g}(\mathbf{t}_1), \mathbf{g}(\mathbf{t}_2), \dots, \mathbf{g}(\mathbf{t}_M)] \in \mathbb{C}^{L_t \times M}. \quad (4)$$

On the receiver side, the AoAs of the q th path ($1 \leq q \leq L_r$) are parameterized by elevation $\theta_r^q \in [0, \pi]$ and azimuth $\phi_r^q \in [0, \pi]$. The associated excess path length, measured between the n th receive element at $\mathbf{r}_n = (x_n, y_n)^T$ and the reference $\mathbf{r}_0 = (0, 0)^T$, is

$$\rho_r^q(\mathbf{r}_n) = x_n \sin \theta_r^q \cos \phi_r^q + y_n \cos \theta_r^q. \quad (5)$$

The resulting phase shift is $\frac{2\pi}{\lambda} \rho_r^q(\mathbf{r}_n)$, and the corresponding receive field-response vector becomes

$$\mathbf{f}(\mathbf{r}_n) \triangleq [e^{j\frac{2\pi}{\lambda} \rho_r^1(\mathbf{r}_n)}, \dots, e^{j\frac{2\pi}{\lambda} \rho_r^{L_r}(\mathbf{r}_n)}]^T \in \mathbb{C}^{L_r \times 1}. \quad (6)$$

The complete receive field-response matrix for all N receive elements is then expressed as

$$\mathbf{F}(\mathbf{r}) \triangleq [\mathbf{f}(\mathbf{r}_1), \dots, \mathbf{f}(\mathbf{r}_N)] \in \mathbb{C}^{L_r \times N}. \quad (7)$$

The channel interaction between the reference points $\mathbf{t}_0$ and $\mathbf{r}_0$ is captured by the path-response matrix $\mathbf{\Sigma} \in \mathbb{C}^{L_r \times L_t}$, where each element $\Sigma_{q,p}$ represents the complex gain associated with the p th transmit path and the q th receive path. It is assumed that these entries are independent and identically distributed (i.i.d.) and have Nakagami- m -distributed magnitude and uniformly distributed phase. Therefore, the overall MIMO channel matrix between the antenna arrays located at $\mathbf{t}$ and $\mathbf{r}$ is given by [5] (We adopt a geometry-based stochastic channel model that captures the spatial response of fluid antennas and the multipath propagation environment. Specifically, the channel formulation in (8) follows a field-response representation, where the transmit and receive fluid antennas generate position-dependent responses determined by their locations within the aperture regions. The adopted decomposition separates the spatial field responses from the stochastic path gains, which allows tractable statistical analysis while preserving the physical interpretation of fluid-antenna positioning and multipath propagation).

$$\mathbf{H}(\mathbf{t}, \mathbf{r}) = \mathbf{F}^H(\mathbf{r}) \mathbf{\Sigma} \mathbf{G}(\mathbf{t}), \quad (8)$$

where the PDF of $x = |\Sigma_{q,p}|$ is

$$p(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} e^{-(m/\Omega)x^2}, \quad (9)$$

where $m = \mathbb{E}[x^2]/\text{Var}[x^2]$ and $\Omega = \mathbb{E}[x^2]$, with $\Gamma(\cdot)$ denoting the gamma function.

In this model, the matrices $\mathbf{G}(\mathbf{t})$ and $\mathbf{F}(\mathbf{r})$ capture the deterministic spatial responses induced by the FA positions and by the AoDs/AoAs, respectively. The random matrix $\mathbf{\Sigma}$ contains the small-scale path gains between the transmit-side and receive-side propagation paths. Thus, the FA positions affect the effective MIMO channel through the geometry-dependent field-response matrices, while the Nakagami- m fading statistics are assigned to the path gains in $\mathbf{\Sigma}$.

3. Capacity Bounds of Fluid MIMO Systems

In this analysis, it is assumed that the CSI is perfectly known at the receiver, and that the total transmit power is evenly allocated across all transmit antennas. Under equal power allocation across the M transmit elements, the transmit covariance is $\mathbf{Q} = \frac{P}{M} \mathbf{I}_M$, where P is the total transmit power. Moreover, we exploit that the AoAs/AoDs typically evolve on a much slower time scale than the small-scale path gains in time-varying channels [14]. Operationally, the FA positions are assumed to be known at the receiver after configuration, so that the receiver can estimate the resulting effective channel $\mathbf{H}(\mathbf{t}, \mathbf{r})$. The present work focuses on the capacity analysis for given FA positions and channel geometry, rather than on the real-time mechanical or electronic control problem required to select the positions. Position optimization and hardware-control latency are therefore outside the scope of this paper. Based on these assumptions and under the condition that $M \geq N$, the ergodic capacity can be formulated as

$$C = \mathbb{E} \left[\log_2 \det \left(\mathbf{I}_N + \frac{P}{M\sigma^2} \mathbf{H}(\mathbf{t}, \mathbf{r}) \mathbf{H}(\mathbf{t}, \mathbf{r})^H \right) \right]. \quad (10)$$

When $M < N$, the capacity expression can be equivalently rewritten using the identity $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$, which holds for any appropriately dimensioned matrices $\mathbf{A}$ and $\mathbf{B}$. The subsequent analysis is grounded in the principles of majorization theory [15].

3.1. Upper Bound

This subsection is dedicated to deriving an upper bound on the ergodic capacity of general Nakagami- m fading channels.

Theorem 1. For fixed antenna positions, AoAs, and AoDs, a position- and angle-dependent upper bound on the ergodic capacity of the FA-assisted MIMO system over Nakagami- m fading is

$$C \leq C_U = \frac{N}{\ln 2} \frac{1}{\Gamma(mL_r L_t)} G_{3,2}^{1,3} \left(\begin{matrix} 1 - mL_r L_t, 1, 1 \\ 1, 0 \end{matrix} \middle| \frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})\Omega}{MN\sigma^2 m} \right), \quad (11)$$

where

$$\beta_F(\mathbf{r}) = \lambda_{\max}(\mathbf{F}(\mathbf{r})\mathbf{F}^H(\mathbf{r})), \quad \beta_G(\mathbf{t}) = \lambda_{\max}(\mathbf{G}(\mathbf{t})\mathbf{G}^H(\mathbf{t})). \quad (12)$$

Proof. Please see Appendix A. $\square$

The quantities $\beta_F(\mathbf{r})$ and $\beta_G(\mathbf{t})$ explicitly include the effect of the fluid-antenna positions and the angular propagation structure. In particular, the entries of $\mathbf{F}(\mathbf{r})$ depend on the receive antenna positions and AoAs, while the entries of $\mathbf{G}(\mathbf{t})$ depend on the transmit antenna positions and AoDs. Thus, the revised upper bound does not ignore the FA geometry; rather, it captures its effect through the spectral factors $\beta_F(\mathbf{r})$ and $\beta_G(\mathbf{t})$.

As the above upper bound admits a closed-form expression involving the Meijer G-function, it provides limited physical intuition regarding the capacity behavior. There-

fore, to gain more analytical insights, we focus on the high-SNR regime and derive a simplified expression.

Proposition 1. For fixed antenna positions, AoAs, and AoDs, the upper bound in Theorem 1 admits the following high-SNR approximation

$$C_{\text{HSNR}} = N \log_2 \left(\frac{P \beta_F(\mathbf{r}) \beta_G(\mathbf{t})}{MN \sigma^2} \right) + \frac{N}{\ln 2} \left[\psi(m L_r L_t) - \ln \left(\frac{m}{\Omega} \right) \right]. \quad (13)$$

Proof. Please see Appendix B. $\square$

The fading severity parameter m influences the high-SNR upper-bound approximation through the term

$$\psi(m L_r L_t) - \ln \left(\frac{m}{\Omega} \right). \quad (14)$$

This term is equal to $\mathbb{E}[\ln Y]$, where $Y = \|\mathbf{\Sigma}\|_F^2$ follows a Gamma distribution with shape parameter $m L_r L_t$ and scale parameter Ω/m . Therefore, m affects the capacity through the logarithmic moment of the aggregate path-gain power. A larger value of m corresponds to less severe fading and smaller path-gain fluctuations, which generally improves the ergodic capacity and reduces the probability of deep fading. In contrast, smaller values of m represent more severe fading and lead to a lower capacity bound.

It is also observed from the term $m L_r L_t$ that the impact of m is coupled with the number of propagation paths. When $L_r L_t$ is small, the system has fewer independent path-gain components, and the capacity is more sensitive to changes in m . As $L_r L_t$ increases, the aggregate gain $\|\mathbf{\Sigma}\|_F^2$ becomes less sensitive to individual fading fluctuations, and the effect of m on the capacity becomes less pronounced. This explains why richer multipath propagation can partially compensate for more severe fading conditions.

The high-SNR expression in Proposition 1 should be interpreted as an upper-bound approximation rather than as an exact multiplexing-gain characterization. The actual high-SNR slope of the FA-assisted MIMO channel is governed by the effective rank

$$r_{\text{eff}} = \text{rank}(\mathbf{F}^H(\mathbf{r}) \mathbf{\Sigma} \mathbf{G}(\mathbf{t})) \leq \min(M, N, L_t, L_r). \quad (15)$$

Therefore, although the upper bound scales with the prefactor N , the true number of independent spatial streams cannot exceed r_{eff} . The factors $\beta_F(\mathbf{r})$ and $\beta_G(\mathbf{t})$ also contribute important geometry-dependent constants, since they depend on the antenna positions and on the AoA/AoD configuration. Hence, the high-SNR expression should be used to understand the scaling of the proposed bound and the impact of fading and geometric parameters, but it should not be interpreted as guaranteeing an N -fold multiplexing gain in all channel conditions.

In the large-system regime, the revised upper bound indicates that increasing the number of propagation paths can improve the capacity bound through the Gamma-distribution term $\psi(m L_r L_t)$ and through the geometry-dependent spectral factors $\beta_F(\mathbf{r})$ and $\beta_G(\mathbf{t})$. For large $L_r L_t$, using $\psi(x) \simeq \ln(x)$ gives

$$C_{\text{HSNR}} \simeq N \log_2 \left(\frac{P \beta_F(\mathbf{r}) \beta_G(\mathbf{t}) \Omega}{MN \sigma^2} \right) + N \log_2(L_r L_t). \quad (16)$$

This expression shows that the upper bound increases logarithmically with the product $L_r L_t$. However, the actual achievable capacity is still limited by the effective channel rank $r_{\text{eff}} \leq \min(M, N, L_t, L_r)$. Consequently, the large-system result should be understood as a

scaling law for the analytical upper bound, not as an unconditional statement that the true capacity grows without rank limitations.

3.2. Complexity Discussion

We now discuss the computational complexity of the proposed capacity-bound evaluation. The closed-form upper bound in Theorem 1 requires the evaluation of a Meijer-G function and the computation of the geometry-dependent spectral factors $\beta_F(\mathbf{r})$ and $\beta_G(\mathbf{t})$. Specifically, $\beta_F(\mathbf{r})$ is obtained as the largest eigenvalue of $\mathbf{F}(\mathbf{r})\mathbf{F}^H(\mathbf{r}) \in \mathbb{C}^{L_r \times L_r}$, while $\beta_G(\mathbf{t})$ is obtained as the largest eigenvalue of $\mathbf{G}(\mathbf{t})\mathbf{G}^H(\mathbf{t}) \in \mathbb{C}^{L_t \times L_t}$. Hence, the dominant algebraic complexity is

$$\mathcal{O}(L_r^3 + L_t^3), \quad (17)$$

assuming standard dense eigenvalue decomposition. The remaining operations, including the evaluation of the Gamma, digamma, and Meijer-G functions, are scalar special-function evaluations and do not scale with the antenna-array dimensions in the same way as repeated matrix decompositions.

By contrast, direct Monte Carlo evaluation of the ergodic capacity requires generating N_{MC} independent channel realizations. For each realization, the effective channel $\mathbf{H}(\mathbf{t}, \mathbf{r}) = \mathbf{F}^H(\mathbf{r})\mathbf{\Sigma}\mathbf{G}(\mathbf{t})$ must be formed, and a log-determinant of an $N \times N$ matrix must be computed. The corresponding complexity is approximately

$$\mathcal{O}\left(N_{MC}\left(NL_rL_t + L_rL_tM + N^3\right)\right), \quad (18)$$

where the first two terms correspond to forming the effective channel and the last term corresponds to the log-determinant computation. Therefore, the Monte Carlo approach becomes significantly more expensive as the number of realizations, antennas, or paths increases.

Alternative exact analytical approaches based on the eigenvalue distribution of $\mathbf{H}\mathbf{H}^H$ are also computationally and analytically more demanding. In FA-assisted MIMO systems, the channel matrix depends on the position-dependent field-response matrices, AoAs, AoDs, and Nakagami- m path gains. Consequently, deriving and evaluating the exact joint eigenvalue distribution is generally intractable for finite-dimensional systems with arbitrary antenna positions and propagation geometry. The proposed capacity-bound method avoids this difficulty by using deterministic geometry-dependent spectral factors and the scalar Gamma distribution of $\|\mathbf{\Sigma}\|_F^2$. This leads to a tractable evaluation procedure with substantially lower complexity than Monte Carlo simulation and without requiring exact eigenvalue statistics.

3.3. Lower Bound

Here, we aim to derive a lower bound for the ergodic capacity in the context of Nakagami- m fading.

Theorem 2. For fixed antenna positions, AoAs, and AoDs, assume that the field-response matrices have nonzero minimum spectral factors

$$\eta_F(\mathbf{r}) = \lambda_{\min}\left(\mathbf{F}(\mathbf{r})\mathbf{F}^H(\mathbf{r})\right), \quad \eta_G(\mathbf{t}) = \lambda_{\min}\left(\mathbf{G}(\mathbf{t})\mathbf{G}^H(\mathbf{t})\right). \quad (19)$$

Then, a closed-form lower bound on the ergodic capacity of the FA-assisted MIMO system over Nakagami- m fading is

$$C \geq C_L = \frac{1}{\ln 2} \frac{1}{\Gamma(mL_r L_t)} G_{3,2}^{1,3} \left(\begin{matrix} 1 - mL_r L_t, 1, 1 \\ 1, 0 \end{matrix} \middle| \frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})\Omega}{M\sigma^2 m} \right). \quad (20)$$

Proof. Please see Appendix C. $\square$

This lower bound does not remain tight for general FA-assisted MIMO channels, as becomes evident in the subsequent high-SNR analysis.

Proposition 2. For fixed antenna positions, AoAs, and AoDs, the closed-form lower bound in Theorem 2 admits the following high-SNR approximation

$$C_{L,HSNR} = \log_2 \left(\frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} \right) + \frac{1}{\ln 2} \left[\psi(mL_r L_t) - \ln \left(\frac{m}{\Omega} \right) \right]. \quad (21)$$

Proof. Please see Appendix D. $\square$

The high-SNR lower-bound approximation in (21) should be interpreted as the asymptotic form of the closed-form lower bound, not as the exact high-SNR capacity of the FA-assisted MIMO channel. Unlike the upper-bound approximation, it does not contain the prefactor N , because the lower bound is obtained by reducing the log-determinant expression to a scalar trace-based term. Therefore, it is useful as a conservative analytical benchmark, particularly at low and moderate SNR, but it does not characterize the full multiplexing behavior of the effective MIMO channel. The actual high-SNR slope remains governed by the effective rank $r_{\text{eff}} = \text{rank}(\mathbf{F}^H \mathbf{\Sigma} \mathbf{G}) \leq \min(M, N, L_t, L_r)$.

4. Numerical Results

We now present numerical results to validate the proposed analytical bounds and to examine their tightness under different system parameters. Also, we provide sufficient details for reproducibility. Monte Carlo simulations are used as a benchmark throughout this section. In particular, for each simulated point, the channel matrix was generated according to the geometric FA-assisted MIMO model in (8), where the entries of the path-response matrix followed the Nakagami- m distribution in (9). The simulated ergodic capacity was then obtained by averaging (10) over 10^3 independent channel realizations. These simulations were compared with the analytical upper and lower bounds, as well as with the corresponding high-SNR approximations, in order to assess the accuracy and tightness of the proposed results. For comparison, we also considered a conventional fixed-position MIMO benchmark. In that benchmark, the antenna elements at the transmitter and receiver were placed at fixed locations and were not allowed to move within the fluid-antenna regions. Therefore, the conventional MIMO benchmark did not exploit the additional spatial degrees of freedom offered by antenna-position adaptation. This comparison was used to highlight the performance benefit of FA-assisted MIMO and to identify the regimes in which the proposed approach was most advantageous.

Unless otherwise stated, the simulation parameters were set as follows. The transmitter and receiver were equipped with $M = N = 4$ fluid antennas. The numbers of transmit-side and receive-side propagation paths were denoted by L_t and L_r , respectively, with default values $L_t = L_r = 3$. The carrier wavelength was $\lambda = 1.5$ m, the default Nakagami fading parameter was $m = 2$, and the average path-gain power was normalized as $\Omega = 1$. The fluid-antenna displacement regions at the transmitter and receiver were modeled as square regions

$$\mathcal{S}_t = \mathcal{S}_r = [-A/2, A/2] \times [-A/2, A/2]. \quad (22)$$

In the simulations, the aperture side length was set to $A = \lambda$. Therefore, the transmit and receive displacement regions became

$$\mathcal{S}_t = \mathcal{S}_r = [-\lambda/2, \lambda/2] \times [-\lambda/2, \lambda/2]. \quad (23)$$

For the conventional fixed-position MIMO benchmark, the antenna elements were placed at the four corners of the same square aperture. Hence, for $M = N = 4$, the fixed transmit and receive antenna locations were

$$(-A/2, -A/2), \quad (A/2, -A/2), \quad (-A/2, A/2), \quad (A/2, A/2). \quad (24)$$

Thus, the conventional benchmark used the same aperture size as the FA-assisted MIMO system but without antenna-position adaptation. The AoAs and AoDs, including elevation and azimuth angles, were independently generated from a uniform distribution over $[0, \pi]$. The phases of the path gains were independently generated from a uniform distribution over $[0, 2\pi)$, while the magnitudes followed the Nakagami- m distribution in (9). For each simulation point, the ergodic capacity was obtained by averaging over 10^3 independent channel realizations. For the parameter-sensitivity figures, the SNR was fixed at $P/\sigma^2 = 15$ dB, whereas for the SNR-dependent figures, P/σ^2 was varied over the range shown in the corresponding plot.

To quantify the tightness of the proposed analytical expressions, we used the relative deviation between an analytical curve and the Monte Carlo benchmark, defined as

$$\epsilon_B(\rho, L_t) = \frac{|C_B(\rho, L_t) - C_{MC}(\rho, L_t)|}{C_{MC}(\rho, L_t)} \times 100\%, \quad (25)$$

where $C_B(\rho, L_t)$ denotes the analytical bound or the corresponding high-SNR approximation, and $C_{MC}(\rho, L_t)$ denotes the simulated ergodic capacity obtained through Monte Carlo averaging. In addition, we report the average tightness error over the considered SNR range as

$$\bar{\epsilon}_B(L_t) = \frac{1}{|\mathcal{S}_\rho|} \sum_{\rho \in \mathcal{S}_\rho} \epsilon_B(\rho, L_t), \quad (26)$$

where $\mathcal{S}_\rho$ denotes the set of simulated SNR values, as shown in Table 1. These metrics provide a quantitative measure of the agreement between the analytical expressions and the Monte Carlo benchmark.

Figures 2 and 3 validate the analytical expressions against Monte Carlo simulations. In these figures, dotted lines with markers represent the analytical bounds, dashed lines represent the corresponding high-SNR approximations, and solid lines represent Monte Carlo simulations. Different marker types and colors are used to identify the different values of L_t . This plotting convention makes it possible to distinguish the analytical, asymptotic, and simulated results for each parameter setting.

The tightness is further quantified using (25) and (26). The upper bound exhibits a smaller relative deviation from the Monte Carlo capacity than the lower bound over most of the considered SNR range. The high-SNR approximation becomes more accurate as the SNR increases, as expected from the approximation $\ln(1+x) \simeq \ln(x)$. By contrast, the lower bound remains mainly useful in the low-SNR regime and becomes less accurate at moderate and high SNR because it does not capture the full multiplexing behavior of the effective MIMO channel.

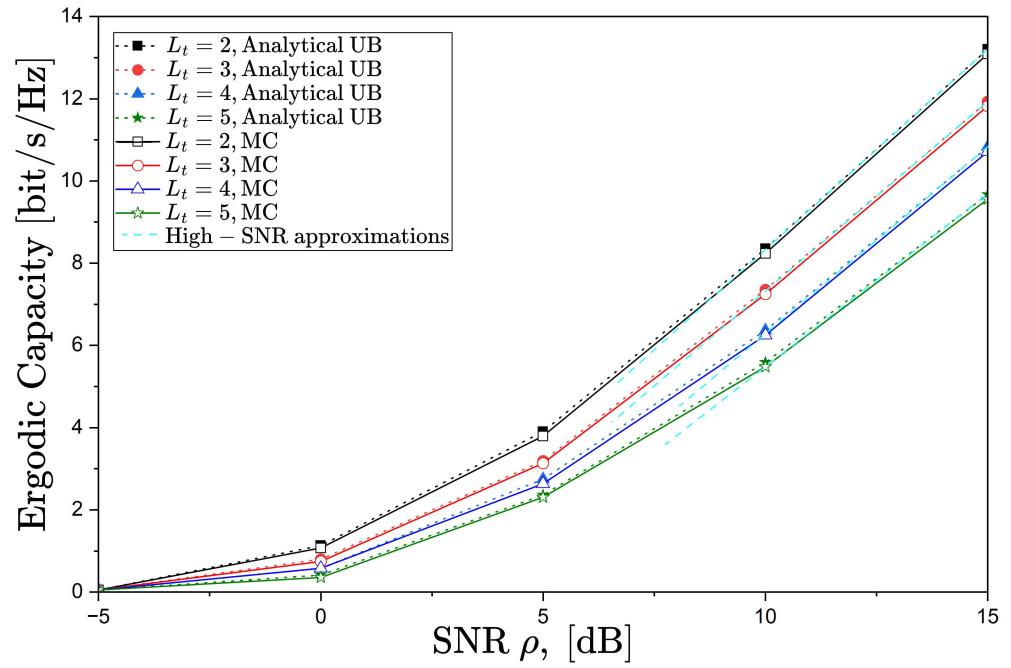


Figure 2. Ergodic capacity with respect to SNR for varying L_t (upper bound (UB), MC simulations, and high-SNR approximation).

Figure 3 evaluates the lower bound and its high-SNR approximation. The lower bound is seen to be less tight than the upper bound, particularly at moderate and high SNR values. This behavior is consistent with the analytical discussion following Proposition 2, since the lower bound does not preserve the full multiplexing-growth behavior of the MIMO channel at high SNR. Nevertheless, the lower bound remains useful in the low-SNR regime, where it provides a reasonable approximation and helps characterize the capacity trend.

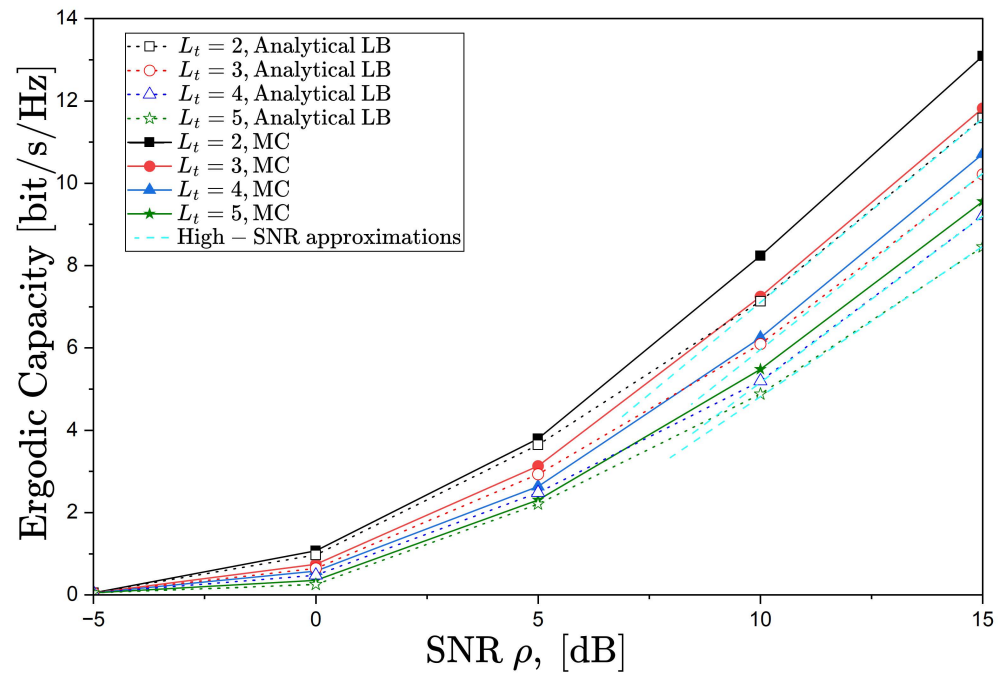
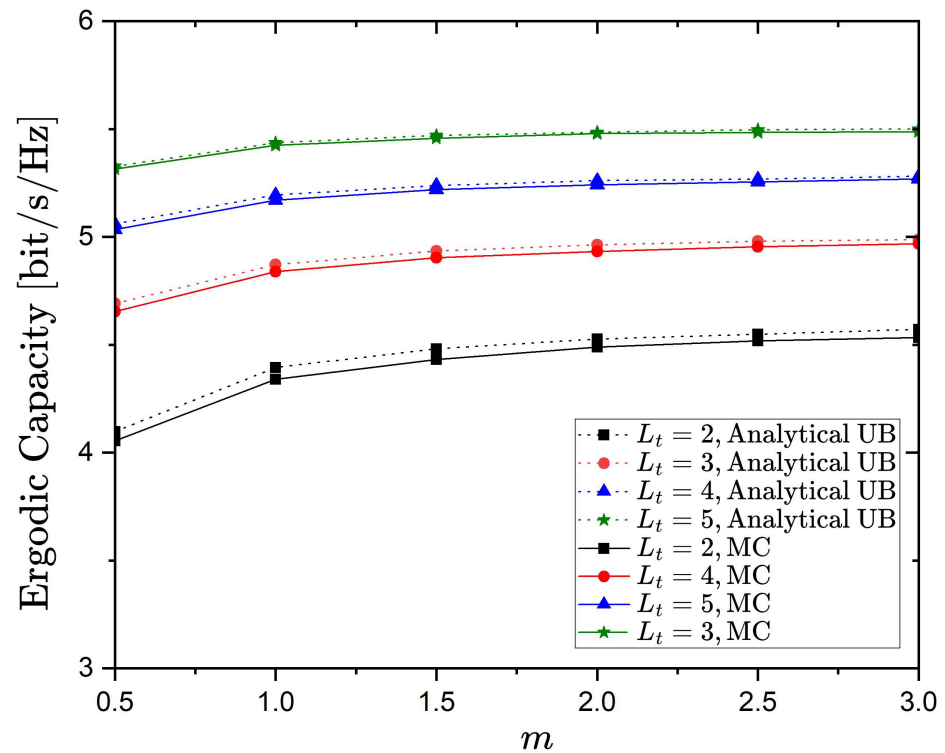


Figure 3. Ergodic capacity with respect to SNR for varying L_t (lower bound (LB), MC simulations, and high-SNR approximation).

Table 1. Average relative tightness error $\bar{\epsilon}_B(L_t)$ with respect to Monte Carlo simulations.

L_t	Upper Bound	Upper High-SNR	Lower Bound	Lower High-SNR
2	8.7%	14.5%	18.9%	25.6%
3	7.4%	12.8%	21.3%	28.4%
4	6.8%	11.6%	23.7%	31.2%
5	6.2%	10.9%	25.1%	33.5%

Figure 4 investigates the influence of the Nakagami fading parameter m on the ergodic capacity for different numbers of propagation paths. The results show that the ergodic capacity mildly increases with m , especially when m is small. This is because larger values of m correspond to less severe fading and reduced path-gain variability.

**Figure 4.** Ergodic capacity with respect to m for varying L_t (upper bound (UB), MC simulations).

It is also observed that the sensitivity of the capacity to m becomes less pronounced as the number of propagation paths increases. This behavior is expected because additional multipath components provide averaging over independent path gains, thereby reducing the impact of fading severity. Hence, the Nakagami- m parameter has a more visible effect in relatively sparse multipath conditions, whereas its influence becomes weaker in richer multipath environments.

Figure 5 compares the ergodic capacity of the proposed FA-assisted MIMO system with that of a conventional fixed-position MIMO system. It is observed that the FA-assisted MIMO architecture achieves a higher ergodic capacity than the conventional MIMO benchmark. This gain is due to the additional spatial degrees of freedom provided by the movable antenna positions, which allow the channel response to be more effectively exploited. The improvement becomes more pronounced when the propagation environment contains a larger number of effective paths, since the fluid antennas can sample richer spatial channel variations.

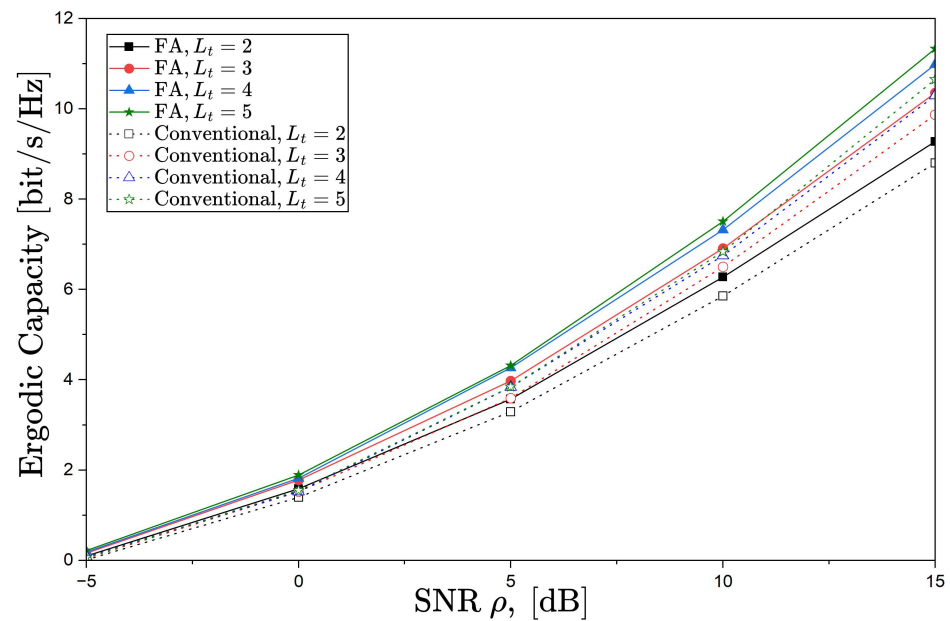


Figure 5. Ergodic capacity comparison between the proposed FA-assisted MIMO system and the conventional fixed-position MIMO benchmark versus the SNR for different numbers of transmit-side propagation paths L_t .

The comparison also reveals the limitations of FA-assisted MIMO. The capacity gain over conventional MIMO depends on the availability of sufficiently rich spatial channel variations within the antenna displacement region. When the propagation environment is poor, the number of effective paths is small, or the available movement region is highly constrained, the advantage of fluid antennas may become less significant. Therefore, FA-assisted MIMO is most beneficial in scenarios where the antenna positions can be effectively adjusted to exploit spatial diversity and multiplexing opportunities.

5. Conclusions

In this work, we utilized tools from majorization theory to derive an upper bound and a lower bound on the ergodic capacity of FA-assisted MIMO systems subject to Nakagami- m fading. A thorough analysis was performed for MIMO channels incorporating FA under Nakagami- m fading, leading to a tight upper-bound formulation involving the Meijer G-function. Also, a lower, looser, but practical lower bound was provided. Moreover, in the high-SNR regime, simplified closed-form expressions were obtained for both the upper and lower bounds, which provided analytical insight into the impact of key system parameters, including the fading figure, the number of antennas, and the number of propagation paths, on the achievable capacity. The high-SNR and large-system expressions were interpreted as scaling laws for the proposed bounds. The actual multiplexing behavior is governed by the effective channel rank, which is limited by the numbers of antennas and propagation paths.

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Conflicts of Interest: The author declares no conflicts of interest.

Appendix A. Proof of Theorem 1

We provide a detailed proof of Theorem 1 in order to clarify the main mathematical transformations used in the derivation. The proof consists of four steps. First, the log-determinant expression is upper-bounded by a trace expression using the arithmetic–

geometric mean inequality. Second, the trace is expressed as the squared Frobenius norm of the effective MIMO channel. Third, the effect of the FA field-response matrices is included through spectral-norm factors that depend on the antenna positions and AoA/AoD parameters. Fourth, the remaining random term is shown to be the Frobenius norm of the Nakagami- m path-gain matrix, whose distribution can be derived exactly.

For the sake of reference, we first recall the following majorization result, which is used to determine the correct direction of the logarithmic inequality.

Lemma A1. Let $\mathbf{A} \in \mathbb{C}^{n \times n}$ be Hermitian positive semidefinite, with diagonal vector $\mathbf{d}(\mathbf{A})$ and eigenvalue vector $\lambda(\mathbf{A})$. Then, by the Schur–Horn theorem,

$$\mathbf{d}(\mathbf{A}) \prec \lambda(\mathbf{A}). \quad (\text{A1})$$

Since

$$\Phi(\mathbf{x}) = \sum_i \log(1 + \alpha x_i), \quad \alpha > 0, \quad (\text{A2})$$

is symmetric and concave over the nonnegative orthant, it is Schur-concave. Consequently, we have

$$\sum_i \log(1 + \alpha d_i(\mathbf{A})) \geq \sum_i \log(1 + \alpha \lambda_i(\mathbf{A})). \quad (\text{A3})$$

Equivalently, we have

$$\log \det(\mathbf{I} + \alpha \mathbf{A}) = \sum_i \log(1 + \alpha \lambda_i(\mathbf{A})) \leq \sum_{i=1}^n \log(1 + \alpha A_{ii}). \quad (\text{A4})$$

The direction of the inequality in Lemma A1 follows from the Schur-concavity of the logarithmic sum. Thus, replacing the eigenvalue-dependent log-determinant expression by the corresponding diagonal-entry expression gives an upper bound.

From (8) and (10), the capacity can be written as

$$C = \mathbb{E} \left[\log_2 \det(\mathbf{I}_N + \alpha \mathbf{H} \mathbf{H}^H) \right], \quad (\text{A5})$$

where $\alpha = P/(M\sigma^2)$.

Let $\lambda_i(\mathbf{H} \mathbf{H}^H)$, $i = 1, \dots, N$, denote the eigenvalues of the positive semidefinite matrix $\mathbf{H} \mathbf{H}^H$. Then, we have

$$\log_2 \det(\mathbf{I}_N + \alpha \mathbf{H} \mathbf{H}^H) = \sum_{i=1}^N \log_2(1 + \alpha \lambda_i(\mathbf{H} \mathbf{H}^H)). \quad (\text{A6})$$

Since the geometric mean of positive numbers is not larger than their arithmetic mean, we have

$$\prod_{i=1}^N (1 + \alpha \lambda_i(\mathbf{H} \mathbf{H}^H))^{1/N} \leq \frac{1}{N} \sum_{i=1}^N (1 + \alpha \lambda_i(\mathbf{H} \mathbf{H}^H)). \quad (\text{A7})$$

Taking the logarithm of both sides gives

$$\log_2 \det(\mathbf{I}_N + \alpha \mathbf{H} \mathbf{H}^H) \leq N \log_2 \left(1 + \frac{\alpha}{N} \sum_{i=1}^N \lambda_i(\mathbf{H} \mathbf{H}^H) \right). \quad (\text{A8})$$

Finally, using $\sum_{i=1}^N \lambda_i(\mathbf{H}\mathbf{H}^H) = \text{tr}(\mathbf{H}\mathbf{H}^H)$, we obtain the trace-based upper bound

$$\log_2 \det \left(\mathbf{I}_N + \alpha \mathbf{H}\mathbf{H}^H \right) \leq N \log_2 \left(1 + \frac{\alpha}{N} \text{tr}(\mathbf{H}\mathbf{H}^H) \right). \quad (\text{A9})$$

The trace term can be written as a Frobenius norm because

$$\text{tr}(\mathbf{H}\mathbf{H}^H) = \|\mathbf{H}\|_F^2. \quad (\text{A10})$$

Using the channel decomposition $\mathbf{H} = \mathbf{F}^H \mathbf{\Sigma} \mathbf{G}$, this becomes

$$\text{tr}(\mathbf{H}\mathbf{H}^H) = \|\mathbf{F}^H \mathbf{\Sigma} \mathbf{G}\|_F^2. \quad (\text{A11})$$

The following standard inequality is then applied. For compatible matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, we have Section 5.6 in [16]

$$\|\mathbf{ABC}\|_F \leq \|\mathbf{A}\|_2 \|\mathbf{B}\|_F \|\mathbf{C}\|_2. \quad (\text{A12})$$

This gives

$$\|\mathbf{F}^H \mathbf{\Sigma} \mathbf{G}\|_F^2 \leq \|\mathbf{F}^H\|_2^2 \|\mathbf{\Sigma}\|_F^2 \|\mathbf{G}\|_2^2. \quad (\text{A13})$$

For fixed antenna positions, AoAs, and AoDs, the field-response matrices $\mathbf{F}(\mathbf{r})$ and $\mathbf{G}(\mathbf{t})$ are deterministic. Therefore, the exact trace term is a quadratic form in $\mathbf{\Sigma}$ whose coefficients depend on the antenna positions and the angular propagation parameters. Instead of assuming the distribution of this quadratic form, we use (A13) to obtain a rigorous upper bound. Define

$$\beta_F(\mathbf{r}) \triangleq \|\mathbf{F}^H\|_2^2 = \lambda_{\max}(\mathbf{F}(\mathbf{r})\mathbf{F}^H(\mathbf{r})), \quad (\text{A14})$$

and

$$\beta_G(\mathbf{t}) \triangleq \|\mathbf{G}^H\|_2^2 = \lambda_{\max}(\mathbf{G}(\mathbf{t})\mathbf{G}^H(\mathbf{t})), \quad (\text{A15})$$

where we have used that the spectral norm of a matrix $\mathbf{A}$ is defined as

$$\|\mathbf{A}\|_2 = \lambda_{\max}(\mathbf{A}), \quad (\text{A16})$$

with $\lambda_{\max}(\mathbf{A})$ denoting the largest singular value of $\mathbf{A}$.

Then, we have

$$\text{tr}(\mathbf{H}\mathbf{H}^H) \leq \beta_F(\mathbf{r}) \beta_G(\mathbf{t}) \|\mathbf{\Sigma}\|_F^2. \quad (\text{A17})$$

The factors $\beta_F(\mathbf{r})$ and $\beta_G(\mathbf{t})$ explicitly capture the effects of the receive and transmit antenna positions, as well as the AoAs and AoDs, because these quantities determine the entries of $\mathbf{F}(\mathbf{r})$ and $\mathbf{G}(\mathbf{t})$ through (6) and (3).

Substituting (A17) into the arithmetic mean–geometric mean (AM–GM) trace bound, given by (A9), results in

$$\log_2 \det \left(\mathbf{I}_N + \frac{P}{M\sigma^2} \mathbf{H}\mathbf{H}^H \right) \leq N \log_2 \left(1 + \frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})}{MN\sigma^2} \|\mathbf{\Sigma}\|_F^2 \right). \quad (\text{A18})$$

Taking the expectation over the Nakagami- m path gains yields

$$C \leq N\mathbb{E}\left[\log_2\left(1 + \frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})}{MN\sigma^2}\|\boldsymbol{\Sigma}\|_F^2\right)\right]. \quad (\text{A19})$$

The distribution required in (A19) is not the distribution of the exact trace term, but the distribution of $\|\boldsymbol{\Sigma}\|_F^2$. This quantity is

$$Y \triangleq \|\boldsymbol{\Sigma}\|_F^2 = \sum_{q=1}^{L_r} \sum_{p=1}^{L_t} |\Sigma_{q,p}|^2. \quad (\text{A20})$$

Since the entries $\Sigma_{q,p}$ are independent and have Nakagami- m distributed magnitudes, each $|\Sigma_{q,p}|^2$ follows a Gamma distribution with shape parameter m and scale parameter Ω/m . Therefore, Y is the sum of $L_r L_t$ independent Gamma random variables with the same scale parameter. Hence, we have

$$Y \sim \text{Gamma}\left(mL_r L_t, \frac{\Omega}{m}\right), \quad (\text{A21})$$

with PDF

$$p_Y(y) = \frac{1}{\Gamma(mL_r L_t)} \left(\frac{m}{\Omega}\right)^{mL_r L_t} y^{mL_r L_t - 1} e^{-(m/\Omega)y}. \quad (\text{A22})$$

Therefore, the position- and angle-dependent rigorous upper bound is

$$C_U = \frac{N}{\ln 2} \int_0^\infty \ln\left(1 + \frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})}{MN\sigma^2}y\right) p_Y(y) dy \quad (\text{A23})$$

$$= \frac{N}{\ln 2} \frac{1}{\Gamma(mL_r L_t)} G_{3,2}^{1,3}\left(1 - mL_r L_t, 1, 1 \mid \frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})\Omega}{MN\sigma^2 m}\right), \quad (\text{A24})$$

where, in (A23), the expression $\ln(1 + \alpha x)$ has been represented using the Meijer G-function, as outlined in [Equation (8.4.6.5)] in [17]. Subsequently, to carry out the integration in (A23), we apply the integral identity provided in [Equation (7.813.1)] in [18].

Appendix B. Proof of Proposition 1

The high-SNR approximation follows by applying $\ln(1 + x) \simeq \ln(x)$ when x is large. This approximation is applied to the argument of the logarithm in the rigorous upper bound, not to the exact capacity expression. Therefore, the resulting expression should be interpreted as a high-SNR approximation of the upper bound.

Hence, from (A19), we have

$$C_U \approx \frac{N}{\ln 2} \int_0^\infty \ln\left(\frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})}{MN\sigma^2}y\right) p_Y(y) dy \quad (\text{A25})$$

$$= N \log_2\left(\frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})}{MN\sigma^2}\right) + \frac{N}{\ln 2} \mathbb{E}[\ln Y], \quad (\text{A26})$$

Since $Y = \|\boldsymbol{\Sigma}\|_F^2$ follows a Gamma distribution with shape parameter $k = mL_r L_t$ and scale parameter $\theta = \Omega/m$, its logarithmic expectation is

$$\mathbb{E}[\ln Y] = \psi(k) + \ln(\theta) = \psi(mL_r L_t) - \ln\left(\frac{m}{\Omega}\right), \quad (\text{A27})$$

where $\psi(\cdot)$ is the digamma function [Equation (8.365.4)] in [18]. Substituting this expression into the high-SNR integral gives the stated result.

Substituting this expression gives

$$C_{\text{HSNR}} = N \log_2 \left(\frac{P\beta_F(\mathbf{r})\beta_G(\mathbf{t})}{MN\sigma^2} \right) + \frac{N}{\ln 2} \left[\psi(mL_r L_t) - \ln\left(\frac{m}{\Omega}\right) \right], \quad (\text{A28})$$

which completes the proof.

Appendix C. Proof of Theorem 2

Let $\lambda_i(\mathbf{H}\mathbf{H}^H)$, $i = 1, \dots, N$, denote the eigenvalues of $\mathbf{H}\mathbf{H}^H$. Since

$$\prod_{i=1}^N (1 + \alpha \lambda_i(\mathbf{H}\mathbf{H}^H)) \geq 1 + \alpha \sum_{i=1}^N \lambda_i(\mathbf{H}\mathbf{H}^H), \quad (\text{A29})$$

for $\alpha = P/(M\sigma^2) > 0$, we obtain

$$\log_2 \det(\mathbf{I}_N + \alpha \mathbf{H}\mathbf{H}^H) \geq \log_2 (1 + \alpha \text{tr}(\mathbf{H}\mathbf{H}^H)). \quad (\text{A30})$$

Using the channel decomposition $\mathbf{H} = \mathbf{F}^H \mathbf{\Sigma} \mathbf{G}$, the trace term becomes

$$\text{tr}(\mathbf{H}\mathbf{H}^H) = \left\| \mathbf{F}^H(\mathbf{r}) \mathbf{\Sigma} \mathbf{G}(\mathbf{t}) \right\|_F^2. \quad (\text{A31})$$

For fixed antenna positions, AoAs, and AoDs, the matrices $\mathbf{F}(\mathbf{r})$ and $\mathbf{G}(\mathbf{t})$ are deterministic. By the singular-value inequality for the Frobenius norm,

$$\left\| \mathbf{F}^H(\mathbf{r}) \mathbf{\Sigma} \mathbf{G}(\mathbf{t}) \right\|_F^2 \geq \eta_F(\mathbf{r}) \eta_G(\mathbf{t}) \left\| \mathbf{\Sigma} \right\|_F^2, \quad (\text{A32})$$

where

$$\eta_F(\mathbf{r}) = \lambda_{\min}(\mathbf{F}(\mathbf{r})\mathbf{F}^H(\mathbf{r})), \quad (\text{A33})$$

and

$$\eta_G(\mathbf{t}) = \lambda_{\min}(\mathbf{G}(\mathbf{t})\mathbf{G}^H(\mathbf{t})). \quad (\text{A34})$$

The factors $\eta_F(\mathbf{r})$ and $\eta_G(\mathbf{t})$ explicitly include the effects of the FA positions and the AoA/AoD configuration through the field-response matrices.

Combining (A30) and (A32) gives

$$C \geq \mathbb{E} \left[\log_2 \left(1 + \frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} \left\| \mathbf{\Sigma} \right\|_F^2 \right) \right]. \quad (\text{A35})$$

The random variable

$$Y = \left\| \mathbf{\Sigma} \right\|_F^2 = \sum_{q=1}^{L_r} \sum_{p=1}^{L_t} |\Sigma_{q,p}|^2 \quad (\text{A36})$$

is the sum of $L_r L_t$ independent Gamma random variables with shape parameter m and scale parameter Ω/m . Hence,

$$Y \sim \text{Gamma} \left(mL_r L_t, \frac{\Omega}{m} \right), \quad (\text{A37})$$

with PDF

$$p_Y(y) = \frac{1}{\Gamma(mL_r L_t)} \left(\frac{m}{\Omega}\right)^{mL_r L_t} y^{mL_r L_t - 1} e^{-(m/\Omega)y}. \quad (\text{A38})$$

Therefore,

$$C_L = \frac{1}{\ln 2} \int_0^\infty \ln \left(1 + \frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} y \right) p_Y(y) dy. \quad (\text{A39})$$

Using the Meijer-G representation of the logarithm and the standard Gamma-type integral identity, we obtain

$$C_L = \frac{1}{\ln 2} \frac{1}{\Gamma(mL_r L_t)} G_{3,2}^{1,3} \left(\begin{matrix} 1 - mL_r L_t, 1, 1 \\ 1, 0 \end{matrix} \middle| \frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})\Omega}{M\sigma^2 m} \right), \quad (\text{A40})$$

which completes the proof.

Appendix D. Proof of Proposition 2

From Theorem 2, the closed-form lower bound can be written in expectation form as

$$C_L = \mathbb{E} \left[\log_2 \left(1 + \frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} Y \right) \right], \quad (\text{A41})$$

where

$$Y = \|\mathbf{\Sigma}\|_F^2 \sim \text{Gamma} \left(mL_r L_t, \frac{\Omega}{m} \right). \quad (\text{A42})$$

At high SNR, we use the approximation $\ln(1+x) \simeq \ln(x)$ for large x . Therefore,

$$C_L \approx \frac{1}{\ln 2} \mathbb{E} \left[\ln \left(\frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} Y \right) \right]. \quad (\text{A43})$$

This gives

$$C_{L,\text{HSNR}} = \log_2 \left(\frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} \right) + \frac{1}{\ln 2} \mathbb{E}[\ln Y]. \quad (\text{A44})$$

Since Y is Gamma distributed with shape parameter $k = mL_r L_t$ and scale parameter $\theta = \Omega/m$, its logarithmic expectation is

$$\mathbb{E}[\ln Y] = \psi(k) + \ln(\theta) = \psi(mL_r L_t) - \ln\left(\frac{m}{\Omega}\right). \quad (\text{A45})$$

Substituting this expression yields

$$C_{L,\text{HSNR}} = \log_2 \left(\frac{P\eta_F(\mathbf{r})\eta_G(\mathbf{t})}{M\sigma^2} \right) + \frac{1}{\ln 2} \left[\psi(mL_r L_t) - \ln\left(\frac{m}{\Omega}\right) \right], \quad (\text{A46})$$

which completes the proof.

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