

Article

Physics-Guided Neural Network for Predicting the Thermo-Hydraulic Performance of Concentric Tube Heat Exchangers: Toward Improved Prediction Accuracy

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Abstract

Accurate prediction of coupled heat-transfer and fluid-flow phenomena is essential for the thermal design, performance assessment, and optimization of heat exchangers (HXs). Among key HXs, concentric tube heat exchangers (CTHXs) are widely used in thermal energy systems, where their performance is governed by the coupled interaction of fluid flow and heat transfer. Although computational fluid dynamics (CFD) provides detailed insights into these transport phenomena, its high computational cost limits its applicability in design optimization and real-time monitoring applications. To overcome this limitation, the present study proposes a physics-guided neural network (PGNN) for the accurate and efficient prediction of CTHX thermo-hydraulic performance, including the overall heat-transfer coefficient (U) and the pressure drops of the cold (ΔP_c) and hot (ΔP_h) streams. The PGNN introduces correlation-based physical guidance through established Nusselt number, overall thermal-resistance, and Darcy–Weisbach pressure-drop relations. Accordingly, the proposed framework is a correlation-guided PGNN rather than a residual-based physics-informed model, because the local conservation-equation residuals are not explicitly enforced during training. For comparison, a standard artificial neural network (ANN) with the same architecture and input parameters was also developed. Both models were trained on a dataset generated from 1575 CFD simulations covering a wide range of operating and geometric conditions, including the Reynolds and Prandtl numbers of both fluids, inner and outer tube diameters, and inlet temperatures. A comprehensive error analysis demonstrates the superior predictive capability of the PGNN over the ANN under various flow and geometric conditions. On the unseen test dataset, the PGNN achieved mean absolute percentage errors of 2.03%, 1.09%, and 1.11% for predicting U , ΔP_c , and ΔP_h , respectively. The proposed PGNN therefore provides a reliable, high-fidelity, and computationally efficient alternative to CFD, supporting the analysis, optimization, and operation of thermal energy systems.



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1. Introduction

Heat exchangers (HXs) are critical devices designed to transfer thermal energy efficiently between two or more media, enabling key processes such as heating, cooling, and drying. They are widely employed across a range of industries, including thermal energy storage [1,2], sustainable heating [3], and evaporative cooling [4], where their performance directly affects the overall system's energy efficiency. Among the various configurations, concentric tube heat exchangers (CTHXs) are widely used due to their compact geometry, straightforward construction, and high thermal effectiveness [5]. Their versatility in facilitating efficient heat transfer between different fluids, such as air and water, makes them particularly suitable for numerous industrial and engineering applications.

Accurate and high-fidelity evaluation of HX performance is crucial for enhancing operational efficiency and for optimizing design strategies [6–12]. Both thermal and hydraulic characteristics strongly influence the overall performance of HXs, and improving thermal performance often increases hydraulic losses [10]. Recent advances in computational methods have enabled detailed performance prediction through Computational Fluid Dynamics (CFD). CFD techniques solve the governing Navier–Stokes and energy equations to simulate conjugate heat transfer within heat exchangers, providing detailed insights into their thermal and hydraulic behavior. Nevertheless, achieving such high-resolution results typically requires significant computational time and resources.

Over recent years, data-driven surrogate models have gained prominence as an effective approach for simulating heat transfer and fluid flow in heat exchangers [13–16]. These models leverage machine learning techniques to approximate complex, nonlinear relationships between input parameters such as fluid properties, flow rates, and geometric features and output responses, such as heat-transfer coefficients, without relying on explicit analytical expressions. Artificial neural networks (ANNs) have been particularly successful in this context. For instance, Çolak et al. [17] employed an ANN to predict critical heat-transfer parameters in a shell-and-helically-coiled-tube heat exchanger, achieving an R^2 coefficient of 0.998. Other machine learning methods, including random forest ensembles [11], have also been applied to estimate convective heat transfer and pressure drop, offering rapid predictions once the models are trained. When trained on high-fidelity simulation or experimental data, these surrogate models provide a computationally efficient alternative to conventional CFD simulations, enabling fast yet reliable performance evaluations [18].

Despite their advantages, purely data-driven approaches face significant limitations. First, they typically require large, high-quality datasets to achieve accurate predictions, making them “data-intensive.” Second, such models do not inherently incorporate the underlying physical laws, which can lead to unrealistic or non-physical results when applied outside the range of their training data [19]. In the case of HXs, generating sufficient training data is often challenging: experimental measurements are expensive and time-consuming, while high-fidelity CFD simulations demand substantial computational resources. Moreover, black-box models may violate fundamental principles such as energy conservation or flow continuity, when extrapolated beyond the regime covered by the training dataset. These challenges have motivated the development of hybrid modeling strategies that integrate data-driven flexibility with the enforcement of physical constraints, combining the strengths of both approaches.

Physics-guided neural networks (PGNNs) have recently emerged as a robust framework for surrogate modeling in heat-transfer and fluid-flow applications [20,21]. These models extend conventional deep learning approaches by embedding physical knowledge commonly represented by partial differential equations or empirical correlations into the training procedure as constraints [22]. This integration enables the model to respect fundamental physical principles, thereby reducing its reliance on extensive labeled datasets and enhancing its generalization. For instance, Abdulkarim et al. [20] developed two PGNNs to predict the Nusselt number and performance evaluation criterion (PEC) in shell-and-helical coil heat exchangers by incorporating empirical heat-transfer correlations into the loss function, demonstrating superior accuracy and robustness compared to conventional correlations and various machine learning models, achieving a high coefficient of determination R^2 of 0.997. Bokil et al. [21] developed a geometry-adaptive, physics-informed convolutional neural network surrogate model to predict velocity, pressure, and temperature fields in HX tube bundles by embedding RANS equations and boundary conditions into the training process. Their approach achieved approximately 97% prediction accuracy on unseen geometries and outperformed purely data-driven models, demonstrating improved reliability while reducing the need for extensive CFD simulations. Liang et al. [23] developed a physics-informed deep residual network to predict the performance of finned-tube evaporators by integrating prior physical knowledge into the model architecture. Their model achieved high predictive accuracy, with mean absolute errors of 0.14% for heat-transfer rate, 0.36% for refrigerant pressure drop, and 0.49% for air pressure drop, outperforming conventional machine learning approaches in capturing detailed physical behavior. Moreover, Cai et al. [24] applied physics-informed neural networks to solve a range of heat-transfer problems, including convection and phase-change phenomena, using sparse and noisy data while enforcing governing physical laws. Their results demonstrated that PGNNs can accurately reconstruct temperature and velocity fields and address ill-posed problems beyond the capabilities of traditional computational methods. Majumdar et al. [25] introduced HxPINN, a hypernetwork-based physics-informed neural network for real-time monitoring of industrial heat exchangers under varying operating conditions. Their model achieved temperature prediction errors below 5 °C while accelerating inference by approximately 35–40 times compared to conventional physics-based approaches, demonstrating strong potential for digital twin applications. Shan et al. [26] proposed a physics-guided collaborative modeling (PGCM) framework for air source heat pump systems, integrating a heat exchanger physics-informed neural network (HX-PINN) with a system-level solver to capture nonlinear heat transfer and thermodynamic interactions. Their approach achieved a high predictive accuracy of 99.3% for pressure drop and 98.5% for enthalpy change while reducing computational time by up to 95.2% and maintaining a mean absolute COP (coefficient of performance) error of 1.5%. Collectively, these studies highlight the capability of PGNNs to act as accurate, physics-guided surrogate models for thermofluid systems, addressing the data scarcity and interpretability limitations commonly associated with purely empirical or black-box approaches.

The present work develops a physics-guided neural network (PGNN) framework for predicting the thermo-hydraulic performance of concentric tube heat exchangers (CTHXs) through the simultaneous estimation of the overall heat-transfer coefficient (U) and the pressure drops of the cold (ΔP_c) and hot (ΔP_h) streams. Although PGNNs have been successfully applied in various engineering disciplines, their application to the thermo-hydraulic modeling of CTHXs has received limited attention. The scientific contributions of this work are summarized as follows:

- Development of a PGNN specifically designed for the thermo-hydraulic modeling of CTHXs, enabling the simultaneous prediction of the overall heat-transfer coefficient and the pressure drops of both hot and cold streams.
- Integration of validated thermo-hydraulic empirical correlations into the training process as physics-based constraints, thereby enforcing physically consistent predictions while improving the model's generalization capability.
- Systematic evaluation of the proposed PGNN against a conventional artificial neural network (ANN), demonstrating the advantages of physics-guided learning in terms of prediction accuracy and physical consistency.
- Demonstration that the proposed PGNN achieves high predictive accuracy on unseen test data, with mean absolute percentage errors of 2.03%, 1.09%, and 1.11% for U , ΔP_c , and ΔP_h , respectively, confirming its effectiveness for accurate and reliable thermo-hydraulic performance prediction of CTHXs.

The article is structured as follows: Section 2 presents the detailed methodology of CFD simulations, Section 3 describes the dataset preprocessing, Section 4 introduces the machine learning methodology including the ANN model and the proposed PGNN model, Section 5 provides a detailed error analysis to compare the predictive performance of the ANN and PGNN models, and Section 6 concludes with key findings and future perspectives.

2. Computational Fluid Dynamics (CFD)

2.1. Computational Domain

Figure 1 presents the layout of the counter-flow concentric tube heat-recovery exchanger (CTHX) considered in this study. The warm greywater stream flows through the central tube, while the cold-water stream passes through the annular space between the inner and outer walls. To reduce computational cost, the exchanger is modeled in a two-dimensional meridional plane under the axisymmetric assumption. This approach allows the selected section to reproduce the behavior of the complete three-dimensional device. The geometry is characterized mainly by the inner-tube diameter D_i , the outer diameter D_o , and the exchanger length L .

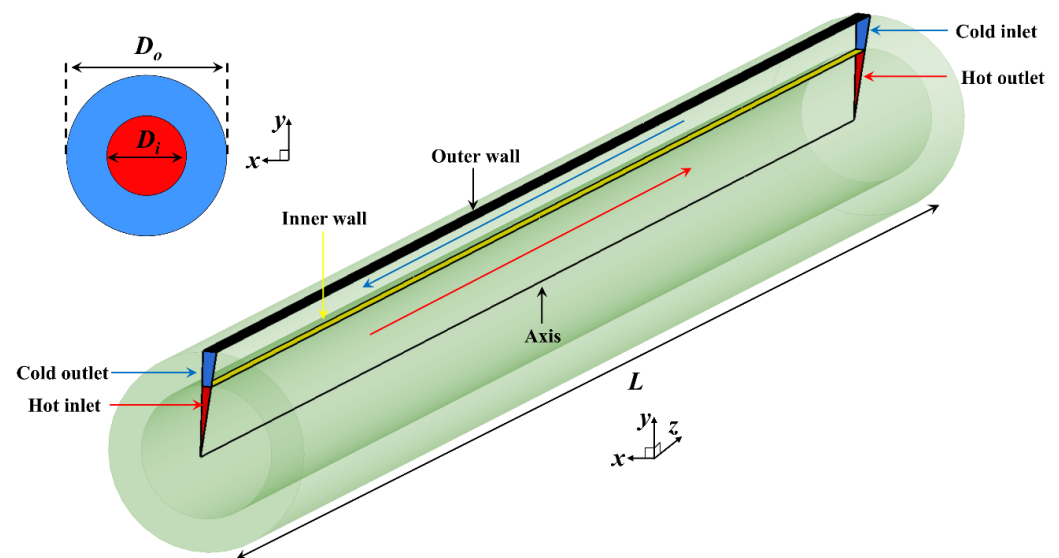


Figure 1. Computational domain of the CTHX.

A wide range of flow conditions is examined by considering inlet Reynolds numbers of 1000, 2000, 4000, 10,000, and 20,000. The cases at Reynolds numbers of 1000 and 2000 are classified as laminar, whereas those at Reynolds numbers of 4000, 10,000, and 20,000

are classified as turbulent. No cases are considered within the transitional range between the Reynolds numbers of 2300 and 4000. The temperature dependence of water properties is accounted for using a piecewise-linear interpolation based on the data reported by Bergman et al. [27]. The geometric parametric study includes three values of the inner diameter, namely 0.01, 0.02, and 0.05 m, combined with three diameter ratios (D_o/D_i) equal to 1.25, 1.5, and 3. Four exchanger lengths are also investigated: 0.4, 0.6, 2, and 4 m.

To represent practical operating conditions, three inlet temperatures are imposed for each stream. The hot greywater enters at 365.15 K (92 °C), 338.15 K (65 °C), or 313.15 K (40 °C), whereas the cold-water inlet temperature is set to 298.15 K (25 °C), 288.15 K (15 °C), or 280.15 K (7 °C). The selected geometric and operating conditions were chosen to ensure that the investigated cases are representative of practical liquid-to-liquid CTHX applications. In particular, the Reynolds number ranges were selected to cover both laminar and turbulent-flow regimes, enabling the model to learn the thermo-hydraulic behavior over the operating conditions commonly encountered in industrial practice. Moreover, the selected hot- and cold-water temperature ranges are representative of typical liquid-to-liquid heat-transfer applications, thereby enhancing the practical relevance and applicability of the proposed PGNN framework.

The CFD dataset was generated using a deterministic reduced-factorial design rather than random sampling or Latin hypercube sampling. Although nine variables were subsequently provided as inputs to the machine learning models, they were not all sampled independently. The independently selected parameters were the exchanger length, inner-tube diameter, diameter ratio, hot- and cold-stream inlet Reynolds numbers, and hot- and cold-fluid inlet temperatures. The outer diameter was calculated from the selected inner diameter and diameter ratio, whereas the inlet Prandtl numbers were determined from the temperature-dependent thermophysical properties of water.

The reduced-factorial design consisted of two complementary blocks. The first block was developed to investigate the effects of exchanger geometry, length, and flow conditions. It included three exchanger lengths, $L = 0.4, 0.6$, and 4 m; three inner-tube diameters, $D_i = 0.01, 0.02$, and 0.05 m; three diameter ratios, $D_o/D_i = 1.25, 1.5$, and 3; and five inlet Reynolds number levels for each stream. The inlet temperatures were fixed at $T_{h,i} = 365$ K and $T_{c,i} = 280$ K. The number of simulations in this block was therefore:

$$N_1 = 3 \times 3 \times 3 \times 5 \times 5 = 675 \quad (1)$$

The second block focused primarily on the influence of the hot- and cold-fluid inlet temperatures. The exchanger length was fixed at $L = 2$ m, and two inner-tube diameters, $D_i = 0.01$ and 0.05 m, were considered. The same three diameter ratios and five inlet Reynolds number levels for each stream were retained. Three hot-fluid inlet temperatures, $T_{h,i} = 313, 338$, and 365 K, and two cold-fluid inlet temperatures, $T_{c,i} = 288$ and 298 K, were investigated. The number of simulations in this block was:

$$N_2 = 1 \times 2 \times 3 \times 5 \times 5 \times 3 \times 2 = 900 \quad (2)$$

The complete CFD dataset therefore contained:

$$N_{\text{total}} = N_1 + N_2 = 675 + 900 = 1575 \quad (3)$$

For each selected case, the inlet velocities were recalculated using the temperature-dependent water density and viscosity to maintain the prescribed Reynolds numbers, while the corresponding Prandtl numbers were obtained directly from the water properties at the respective inlet temperatures.

2.2. Governing Equations and Numerical Settings

The fluid motion is described using the incompressible Reynolds-averaged Navier–Stokes (RANS) equations. Conservation of mass is enforced through the continuity equation, given in Equation (1), whereas the averaged momentum equations are written in Equation (2):

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (4)$$

$$\rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \rho \overline{u_i' u_j'} \right], \quad (5)$$

In these expressions, the overbar denotes Reynolds-averaged variables, ρ is the fluid density, and μ is the dynamic viscosity. The Reynolds stresses are closed using the Boussinesq approximation, through which turbulence effects are represented by an eddy viscosity μ_t . The flow formulation is selected independently for each fluid passage according to its inlet Reynolds number. For cases with a Reynolds number below 2300, the laminar formulation is employed, for which the turbulent viscosity and Reynolds stress contributions are set to zero. For cases with a Reynolds number equal to or greater than 4000, the Reynolds-averaged equations are closed using the k - ω SST turbulence model. Consequently, in configurations where the two streams belong to different flow regimes, the laminar formulation is applied to the low-Reynolds number passage and the SST formulation is applied to the turbulent passage.

Heat transfer within the exchanger is governed by the energy equation, written in Equation (3):

$$\rho C_p \bar{u}_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} \left(k_{\text{eff}} \frac{\partial T}{\partial x_i} \right), \quad (6)$$

where C_p denotes the specific heat capacity at constant pressure. The effective thermal conductivity is defined as

$$k_{\text{eff}} = k + k_t, \quad k_t = C_p \mu_t / \text{Pr}_t. \quad (7)$$

so that both molecular and turbulent contributions to heat transport are included.

For turbulent cases, turbulence is modeled using the k - ω shear-stress-transport formulation introduced by Menter [28]. This model combines the near-wall capabilities of the k - ω formulation with the stable free-stream behavior of the k - ϵ formulation. The SST model is not applied to the laminar cases. No transitional turbulence model is employed because Reynolds numbers within the transitional interval are excluded from the investigated dataset.

All simulations are performed using ANSYS Fluent 2023 [29]. The pressure–velocity coupling is handled with the Coupled algorithm. Convective fluxes are discretized using a second-order upwind scheme, whereas diffusive terms are evaluated by central differencing. A solution is considered converged once the residuals of continuity, momentum, k , and ω fall below 10^{-5} , and the energy residual decreases below 10^{-8} .

2.3. Boundary Conditions

The following boundary conditions are applied in the numerical model:

- **Hot and cold inlets:** Uniform inlet velocity and temperature are prescribed. The hot-stream inlet temperature is fixed at 365.15, 338.15, or 313.15 K. In contrast, the cold-stream inlet temperature is set to 298, 288, or 280 K. For each case, the inlet velocity is adjusted so that the desired Reynolds number is achieved in the corresponding flow passage.
- **Outlets:** Both outlets are treated with a zero-gauge-pressure condition.

- **Outer wall:** The external wall is assumed adiabatic; therefore, no heat flux is allowed through this boundary.
- **Inner wall (interface between tube and annulus):** A coupled thermal condition is imposed to ensure continuity of both temperature and heat flux across the separating wall.
- **Axis of symmetry:** An axisymmetric boundary condition is specified along the exchanger centerline.

Together, these conditions aim to reproduce the CTHX's real behavior while maintaining a computationally efficient numerical setup.

2.4. Mesh-Independence Assessment

A mesh-sensitivity study was conducted to ensure that the numerical predictions are not significantly influenced by grid resolution. The analysis follows the **Grid Convergence Index (GCI)** procedure proposed by Celik et al. [30]. Three fully structured meshes of increasing resolution were generated for this purpose. Special care was devoted to the near-wall regions on both the inner and outer surfaces: the first cell height was selected to maintain $y^+ \approx 1$, and 15 inflation layers were introduced to resolve the steep gradients within the viscous sublayer.

The mesh refinement strategy was selected in accordance with Celik's recommendation to use a refinement factor $r \geq 1.3$. The resulting refinement ratios were 1.68 from Mesh 1 to Mesh 2 and 1.42 from Mesh 2 to Mesh 3. The comparison was performed under a demanding operating condition corresponding to $Re = 20,000$ for both streams, a hot-side inlet temperature of 333 K, a cold-side inlet temperature of 280 K, and a geometry defined by $D_i = 0.02$ m, $D_o/D_i = 1.5$, and $L = 2$ m. The finest grid, Mesh 3, contains 1,893,428 control volumes and is used as the reference mesh.

The selected quantity for the GCI analysis is the heat-recovery rate $\dot{Q}_{rec}$, determined from the inlet–outlet energy balance as expressed in Equation (5). Since the outer wall is assumed perfectly insulated, the thermal energy lost by the hot stream is equal to that gained by the cold stream.

$$\dot{Q}_{rec} = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o}) = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) \quad (8)$$

Here, $\dot{Q}_{rec}$ is the heat-recovery rate; $\dot{m}_h$ and $\dot{m}_c$ are the hot- and cold-stream mass flow rates, respectively; $c_{p,h}$ and $c_{p,c}$ are their corresponding specific heat capacities; and $T_{h,i}$, $T_{h,o}$, $T_{c,i}$, and $T_{c,o}$ denote the hot-stream inlet, hot-stream outlet, cold-stream inlet, and cold-stream outlet temperatures, respectively. The subscripts h , c , i , and o denote the hot stream, cold stream, inlet, and outlet, respectively.

Table 1 lists the main features of the three meshes, and Table 2 summarizes the GCI results. The apparent order of convergence was found to be $p_c = 2.17$, while the estimated discretization uncertainty on the finest mesh was 1.23%. In addition, the difference in $\dot{Q}_{rec}$ between Mesh 2 and Mesh 3 was only 1.13%. Based on these observations, Mesh 3 was selected for the remainder of the study.

Table 1. Characteristics of the meshes used for the grid-sensitivity analysis.

	Mesh 1	Mesh 2	Mesh 3
Cell count, N	332,000	936,000	1,893,428
Characteristic grid size, s (m)	9.51×10^{-4}	5.66×10^{-4}	3.98×10^{-4}
Refinement factor, r	–	$s_1/s_2 = 1.68$	$s_2/s_3 = 1.42$

Table 2. GCI results based on the heat-recovery rate.

	ϕ_1	ϕ_2	ϕ_3	ϕ_{ext}	e_a^{23} (%)	p_c	GCI (%)
$\dot{Q}$ (W)	15,730.7	16,239.7	16,425.4	16,587.3	1.13	2.17	1.23

2.5. Numerical Verification and Consistency Assessment

In the absence of experimental measurements or a directly comparable benchmark case covering the same geometry, boundary conditions, Reynolds numbers, and thermal operating conditions, the present assessment is limited to a numerical verification and consistency comparison. The CFD-predicted heat-recovery rate is compared with independent estimates obtained using conventional empirical correlations for turbulent forced convection. This comparison is not intended to constitute a formal validation of the CFD model, but rather to evaluate whether its predictions remain within the range expected from established engineering correlations. The validation is performed for a high-flow case in which both the hot and cold streams operate at $Re = 20,000$. The geometry used in this comparison has an inner diameter $D_i = 0.05$ m and a diameter ratio $D_o/D_i = 3$, with hot- and cold-side inlet temperatures of 333 K and 280 K, respectively.

The analytical reference is based on the effectiveness-NTU method for counter-flow heat exchangers. The exchanger effectiveness ε_{HX} is defined as the ratio of the actual heat-transfer rate to the maximum possible heat-transfer rate:

$$\varepsilon_{HX} = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}. \quad (9)$$

The number of transfer units is calculated as:

$$NTU = \frac{UA}{C_{\min}} \quad (10)$$

where U is the overall heat-transfer coefficient, A is the heat-transfer area, and $C_{\min}$ is the smaller heat-capacity rate between the two streams. The heat-capacity-rate ratio is given by $C_r = C_{\min}/C_{\max}$.

In the present model, the inner-tube wall thickness is assumed to be negligible. Consequently, the inner and outer surface areas of the separating tube are considered identical, and the heat-transfer area in Equation (7) is evaluated as

$$A = \pi D_i L, \quad (11)$$

where D_i is the inner-tube diameter and L is the exchanger length.

The overall heat-transfer coefficient is obtained from the hot- and cold-side convective coefficients according to:

$$U = \left(\frac{1}{h_h} + \frac{1}{h_c} \right)^{-1} \quad (12)$$

where h_h and h_c are evaluated using standard Nusselt number correlations applicable to turbulent internal flow. Because the tube wall is assumed highly conductive, conduction resistance through the wall is neglected.

Four widely used turbulent-flow correlations are employed to estimate the convective coefficients: Dittus–Boelter [31], Petukhov–Kirillov [32], Gnielinski [33], and Chilton–Colburn [34]. Their expressions are reported in Table 3. It should be noted that these correlations were originally derived for fully developed turbulent flow in simple circular tubes under idealized thermal boundary conditions, and therefore do not exactly reproduce the thermal and geometric complexity of the present concentric tube exchanger.

Table 3. Empirical Nusselt number correlations used for turbulent internal flow.

Author(s)	Correlation Expression
Dittus and Boelter [31]	$Nu = 0.023Re^{0.8}Pr^n$, where $n = 0.3$ for cooling, 0.4 for heating
Pethukov and Kirillov [32]	$Nu = \frac{(f/8)Re \cdot Pr}{1.07 + 900/Re - 0.36/(1 + 10Pr) + 12.7\sqrt{f/8}(Pr^{2/3} - 1)}$
Gnielinski [33]	$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)}$
Chilton and Colburn [34]	$Nu = 0.125fRePr^{1/3}$

Once U is determined for each correlation, the associated values of NTU and ε_{HX} are calculated, and the heat-recovery rate is obtained from:

$$\dot{Q} = \varepsilon_{HX} \dot{Q}_{\max}. \quad (13)$$

Table 4 compares the CFD-predicted heat-recovery rate with the estimates obtained from the four empirical correlations. The CFD model predicts a heat-recovery rate of 16,425 W. The Petukhov–Kirillov and Gnielinski correlations provide the closest estimates, with relative deviations of approximately 9.39% and 10.01%, respectively. In contrast, the Dittus–Boelter and Chilton–Colburn correlations show larger deviations of approximately 20.22% and 22.27%, respectively.

Table 4. Comparison of the heat-recovery rate predicted by CFD with estimates obtained from empirical correlations.

Method	Predicted Heat-Recovery Rate (W)	Error Relative to CFD (%)
CFD (present study)	16,425.41	–
Dittus and Boelter [31]	13,662.11	20.22
Pethukov and Kirillov [32]	15,015.89	9.39
Gnielinski [33]	14,930.31	10.01
Chilton and Colburn [34]	13,433.77	22.27

These differences should be interpreted in view of the applicability limits of the empirical correlations. The selected correlations were mainly developed for fully developed turbulent flow in simple circular tubes under idealized thermal boundary conditions. They do not explicitly account for the annular flow passage, diameter-ratio effects, inlet-region development, conjugate heat transfer, or temperature-dependent thermophysical properties considered in the CFD model. Consequently, the observed deviations cannot be attributed exclusively to inaccuracies in the numerical solution.

The correlation-based comparison therefore provides only a partial verification of the order of magnitude and physical consistency of the CFD predictions. When considered together with the mesh-independence assessment, the low discretization uncertainty, and the agreement between the heat lost by the hot stream and that gained by the cold stream, the results support the internal numerical consistency of the CFD model. Nevertheless, they do not replace validation against experimental measurements or a directly comparable benchmark case.

3. Dataset Preprocessing

The preprocessing of the dataset generated by 1575 CFD simulations constitutes an essential step to ensure an effective training of machine learning (ML) models. The principal stages of the preprocessing workflow are summarized as follows:

- Identification of the input features and output variables:** The heat-transfer and fluid-flow physical phenomena in the concentric tube heat exchangers (CTHXs) are governed by several interacting physical parameters. Based on their relevance to the governing transport phenomena, nine key parameters were selected as input features for the ML models. These parameters include the length of the CTHX (L), the inlet Reynolds number of the hot flow (Re_h), the inlet Reynolds number of the cold flow (Re_c), the Prandtl number of the hot fluid (Pr_h), the Prandtl number of the cold fluid (Pr_c), the inner-tube diameter (D_i), the outer tube diameter (D_o), and the inlet temperatures of the cold ($T_{c,i}$) and hot ($T_{h,i}$) fluids. The ML model aims to predict the thermo-hydraulic performance of CTHXs. In this context, the thermal performance is quantified using the overall heat-transfer coefficient (U). In contrast, the hydraulic performance is assessed based on the pressure drops of the cold (ΔP_c) and hot (ΔP_h) flows. It is noteworthy that the input features were carefully selected based on an in-depth analysis of their physical relevance to the output variables, which characterize the governing physical phenomena of CTHXs. This procedure follows a feature selection strategy similar to that reported in [13]. Moreover, the distributions of all dataset variables, including the nine previously identified input features and the three output variables, are shown in Figure 2, as red and blue histograms, respectively.

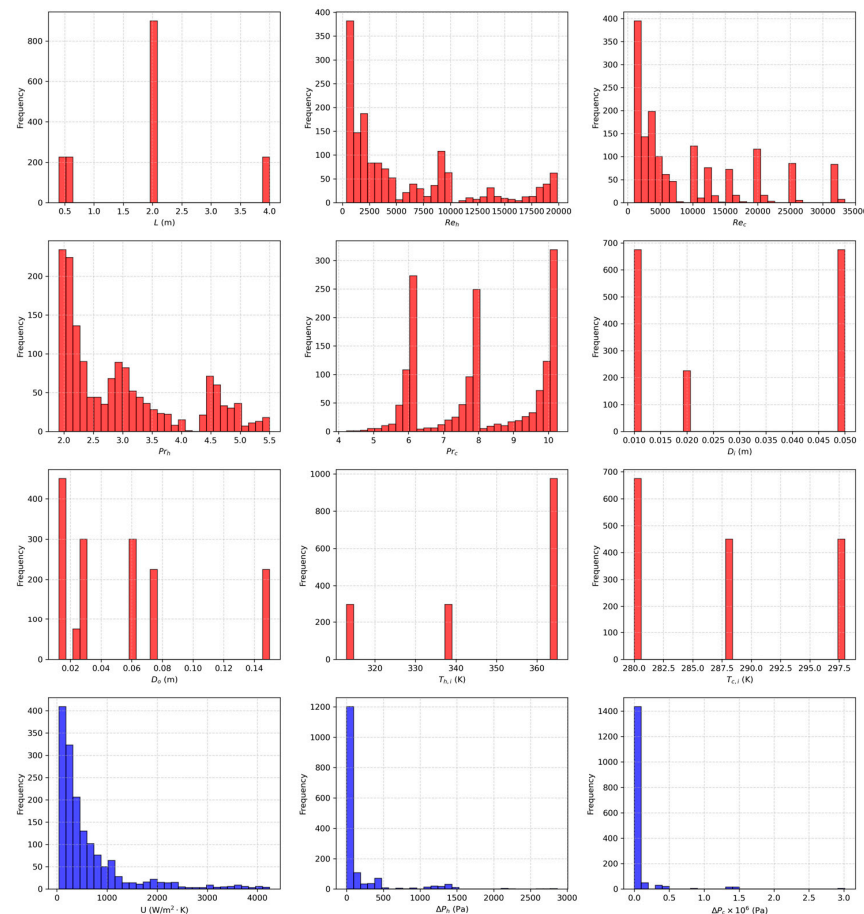


Figure 2. Distributions of the nine input features (red histograms) and the three output variables (blue histograms) in the CFD-generated dataset.

- Feature standardization:** The nine selected features were standardized using the *StandardScaler* available in the *scikit-learn* Python 3.7 library. This normalization of the input features ensures that all features contribute equally during ML model training by preventing variables with larger numerical ranges from dominating the learning

process. On the other side, the output variables of the ML models, namely the overall heat-transfer coefficient (U), the cold flow pressure drop (ΔP_c) and the hot flow pressure drop (ΔP_h), were not scaled to maintain the physical interpretability.

- **Data splitting:** The dataset comprises 1575 samples. The *train_test_split* function from the *scikit-learn* Python library was used to divide the dataset into a training set (80%, 1260 samples) and a testing set (20%, 315 samples). The training set was exclusively used for training the ML models, whereas the test set was retained as unseen data for model evaluation and comparative analysis.

4. Machine Learning Methodology

This section presents the detailed methodologies of the ANN and the PGNN.

4.1. Artificial Neural Network (ANN)

ANNs are among the most advanced machine learning (ML) techniques and are widely used across various fields due to their strong ability to capture complex patterns in data. The training process of ANNs generally involves two main stages: forward propagation and backward propagation. Initially, the weights and biases of each neuron in the network are randomly initialized. Subsequently, the input information propagates forward through the hidden layers to the output layer, where the network generates its prediction. The forward propagation process in each neuron of the ANN can be described as follows:

$$\sigma(\mathbf{Z}) = \sigma(\mathbf{W} \cdot \mathbf{X} + \mathbf{B}) \quad (14)$$

where $\mathbf{B}$ is the bias vector, $\mathbf{W}$ is the weight matrix and σ is the activation function. In the backward propagation process, the loss function is computed using the predicted output from the forward propagation and the actual (CFD) value. Then it is propagated backward from the output layer through the hidden layers until reaching the neurons of the input layer. The forward and backward propagation stages are iteratively repeated until the loss function converges to a minimum. The ANN's loss function used in the present study is defined as follows:

$$L_{ANN} = \frac{1}{m} \sum_{j=1}^3 \sum_{i=1}^m \left| \frac{y_{i,j} - \hat{y}_{i,j}}{y_{i,j}} \right| + \frac{\beta}{2m} \sum_{i=1}^m \|\mathbf{W}\| \quad (15)$$

where $y_{i,j}$ represent the CFD-derived (true) values of the output variables, $\hat{y}_{i,j}$ denote the corresponding predicted values, j is the index of the three output variables (U , ΔP_{cold} and ΔP_{hot}), m is the number of samples in the mini-batch and β is the L2 regularization coefficient. The ANN model was constructed using Keras with the Tensorflow Python library [35].

4.2. Physics-Guided Neural Network (PGNN)

The physics-guided neural network (PGNN) is an enhanced version of ANN that aims to incorporate the underlying physical phenomena of CTHXs into the learning process of the neural network through the loss function [36]. Empirical correlations are incorporated to constrain and guide the learning process, thereby improving the network's ability to capture the heat-transfer and fluid-flow behavior within CTHXs and to predict their thermo-hydraulic performance (U , ΔP_c and ΔP_h) with high accuracy. More specifically, the physical guidance introduced in the present model operates at the level of the global thermo-hydraulic output quantities. For the thermal prediction, the Baehr–Stephan and Dittus–Boelter correlations relate the Nusselt number to the Reynolds number, Prandtl number, hydraulic diameter, and exchanger length, as presented in Equations (17) and (18). The resulting convective heat-transfer coefficients are subsequently combined through the

overall thermal-resistance relation in Equation (20) to provide a correlation-based estimate of the overall heat-transfer coefficient. For the hydraulic predictions, the Darcy–Weisbach relation in Equation (21), together with regime-dependent friction-factor expressions, introduces the expected dependence of pressure drop on the length-to-diameter ratio, fluid density, velocity, and Reynolds number. The neural network does not replace these baseline relations; instead, it predicts non-negative multiplicative correction factors that account for deviations between the simplified correlations and the CFD-generated data.

These relations constitute the exact physical knowledge used to guide the present PGNN. However, the model does not explicitly evaluate the residuals of the continuity, momentum, or energy equations, nor does it enforce the local boundary and interface conditions used in the CFD simulations. Consequently, the model cannot guarantee local conservation of mass, momentum, or energy. The proposed framework should therefore be interpreted as a correlation-guided PGNN, rather than as a governing-equation-residual-based PINN or a fully physics-constrained neural network.

Therefore, the loss function of the PGNN relates the actual (CFD) values of the output variables with the values obtained from physical correlations by a non-negative correction factor Φ . The loss function of the PGNN can be written as follows:

$$L_{PGNN} = \frac{1}{m} \sum_{j=1}^3 \sum_{i=1}^m \left| \frac{y_{i,j} - \Phi_{i,j} \tilde{y}_{i,j}}{y_{i,j}} \right| + \frac{\beta}{2m} \sum_{i=1}^m \|\mathbf{W}\| \quad (16)$$

where $\tilde{y}_{i,j}$ are the values of the output variables (U , ΔP_c and ΔP_h) obtained using physical correlations. It should be noted that the PGNN does not directly predict the thermo-hydraulic performance parameters (U , ΔP_c and ΔP_h). Instead, the network predicts a non-negative correction factor (Φ), which is applied to the empirical correlations to obtain the corrected predictions of the output variables. For computing the correlated overall heat-transfer coefficient ($\tilde{U}$), Baehr and Stephan's correlation [37] is first employed to calculate the Nusselt number (Nu) for the laminar flow regime, which can be expressed as follows:

$$Nu = \frac{\frac{3.66}{\tanh(2.264 Gz^{-1/3} + 1.7 Gz^{-2/3})} + 0.0499 Gz \tanh(Gz^{-1})}{\tanh(2.432 Pr^{1/6} Gz^{-1/6})} \quad (17)$$

where Gz is the Graetz number expressed as $Gz = (D_h / L) Re Pr$, D_h is the hydraulic diameter and L is the length of the CTHX. For the turbulent-flow regime, the Nu correlation developed by Dittus and Boelter [31] is used, and can be expressed as follows:

$$Nu = 0.023 Re^{0.8} Pr^n \quad (18)$$

where n is 0.3 for the hot fluid (cooling process) and 0.4 for the cold fluid (heating process). After evaluating the Nusselt number in both the laminar and turbulent-flow regimes using the aforementioned correlations, the convective heat-transfer coefficients of the cold and hot fluids (h_c and h_h) can be directly calculated as follows:

$$h_{c,h} = \frac{Nu k}{D_h} \quad (19)$$

where k is the thermal conductivity of the cold or hot fluid ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$). Subsequently, the correlated overall heat-transfer coefficient ($\tilde{U}$) can be determined directly, assuming no wall and fouling resistances, as expressed in the following equation:

$$\tilde{U} = \frac{1}{\frac{1}{h_c} + \frac{1}{h_h}} \quad (20)$$

Regarding the hydraulic performance of CTHXs, the correlated pressure drops for the cold and hot fluids ($\Delta\tilde{P}_c$ and $\Delta\tilde{P}_h$) are evaluated using the Darcy–Weisbach equation [38]:

$$\Delta\tilde{P}_{c,h} = \frac{\rho V_{in}^2 f L}{2D_h} \quad (21)$$

where ρ is the density of the cold or hot fluid, V_{in} denotes the inlet velocity of the cold or hot fluid and f is the friction coefficient of the cold or hot fluid. In the laminar flow regime, the friction coefficient f is computed as $f = 64/Re$ [38], whereas in the turbulent-flow regime, it is calculated using $f = 0.316 Re^{-0.25}$ [38]. Although the Reynolds number for the annular flow is evaluated using the hydraulic diameter, the classical friction-factor and Nusselt number correlations are employed only as baseline physical constraints within the PGNN model. To account for deviations associated with annular geometry, diameter ratio, and thermal boundary conditions, non-negative correction factors are incorporated into the PGNN loss function as presented in Equation (16). These correction factors enable the model to compensate for discrepancies between the classical correlations and the actual annular flow behavior while preserving the underlying physical trends.

4.3. Architecture and Input Parameters of the Neural Networks

Starting with the neural network architecture, both the ANN and the PGNN share the same architecture. The network consists of an input layer with 9 neurons (corresponding to the number of input features), followed by 5 hidden layers containing 128, 256, 512, 256 and 128 neurons, respectively. The output layer comprises 3 neurons associated with the three output variables (U , ΔP_c and ΔP_h). The architecture of the ANN and PGNN models is illustrated in the schematic shown in Figure 3. For both the ANN and PGNN models, the Rectified Linear Unit (ReLU) function is employed as the activation function for the neurons in the hidden layers, and can be expressed as follows [39]:

$$\sigma_{ReLU} = \max(0, Z) \quad (22)$$

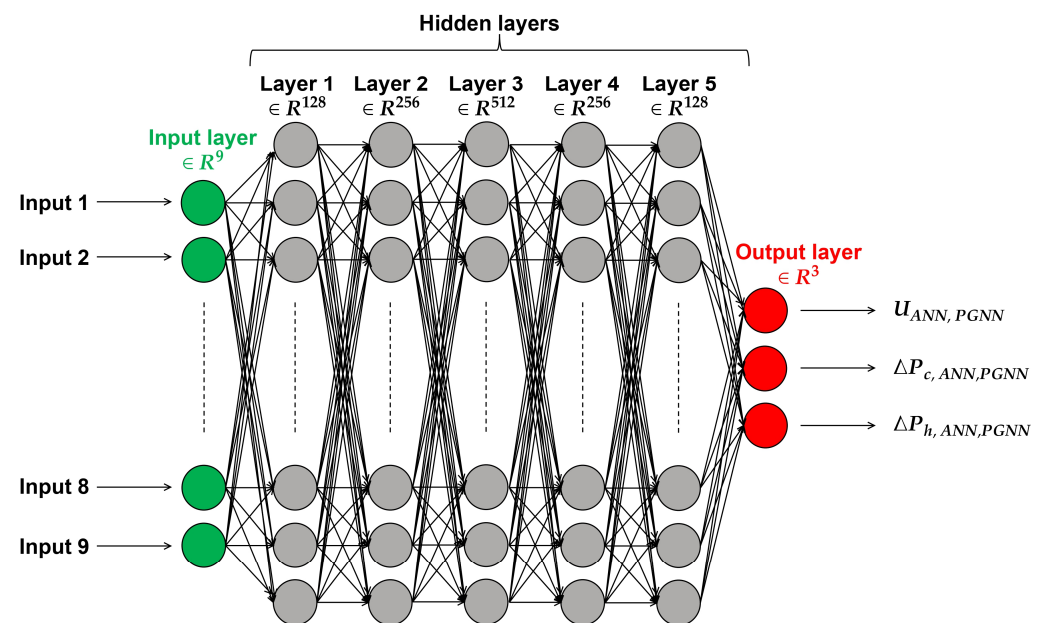


Figure 3. Architecture of the ANN and PGNN.

The ReLU function is commonly adopted for neural networks due to its ability to mitigate the vanishing gradient problems that may be encountered during the training

process. As for the output layer, a linear function is used for the ANN, while a softplus activation function is employed for PGNN to ensure a non-negative correction factor Φ , as defined by the following equation:

$$\sigma_{softplus} = \ln(1 + e^Z) \quad (23)$$

Gradient optimization is performed in both the ANN and PGNN models using the Adam optimizer [40] with an initial learning rate of 0.001. The reason for choosing the Adam optimizer is its ability to adapt the learning rate, leading to an accelerated optimization process. The Adam optimizer adjusts the weights and biases of the network's neurons to reduce the loss function. L2 regularization is used to avoid overfitting with a regularization coefficient β of 0.001. Moreover, a seed value of 42 is used for both the ANN and PGNN to ensure reproducibility and avoid randomness. Furthermore, the early stopping method was employed with a patience of 200 epochs, meaning that if the loss function did not improve in 200 epochs the training process will be stopped and the best weights that lead to the lowest Loss function will be restored. Lastly, the maximum number of epochs was set to 5000; however, due to early stopping, the training process typically converged in fewer epochs. The input parameters of the ANN and PGNN models are summarized in Table 5.

Table 5. Summary of input parameters and training settings for ANN and PGNN models.

Parameter	Value/Setting
Input Layer	9 neurons
Hidden Layers	5 layers {128, 256, 512, 256, 128 neurons}
Output Layer	3 neurons
Hidden Layer Activation	ReLU
Output Layer Activation	ANN: Linear/PGNN: Softplus
Optimizer (Initial Learning Rate)	Adam (0.001)
Regularization (Coefficient)	L2 ($\beta = 0.001$)
Random Seed	42
Patience Early Stopping	200 epochs
Training/Test	80%/20%
Maximum Epochs	5000 epochs

5. Results and Discussions

In this section, the mean absolute percentage error (MAPE) is used as a performance metric to evaluate the predictive accuracy of the ANN and PGNN models. It provides a clear and straightforward measure of the average percentage deviation between the CFD reference values and the model predictions, and can be computed as follows:

$$MAPE = \frac{1}{N} \sum_1^N \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100 \quad (24)$$

where N is the number of samples in the training or test dataset.

5.1. Effect of Training Set Size

Figure 4 illustrates the variation in the MAPE, evaluated on the unseen test set, as a function of the sample percentage, defined as the ratio of training samples to the total number of samples. It is important to mention that the maximum sample percentage is limited to 80%, in accordance with the data-splitting strategy described in Section 3, where 80% of the dataset is allocated for training and the remaining 20% is reserved for testing and model performance comparison. Across a wide range of training set sizes,

corresponding to percentages of samples between 8% and 80%, the test MAPE values of the three output variables U , ΔP_c and ΔP_h obtained with the PGNN model (solid lines) are considerably lower than those of the ANN model (dashed lines). This demonstrates the superior predictive performance of the PGNN model compared to the ANN model in accurately capturing the complex physical phenomena of CTHXs, with minimal deviation from the CFD results (black dashed line). This improvement can be attributed to the integration of physics-based correlations into the learning process, which constrains the model and enhances its generalization capability.

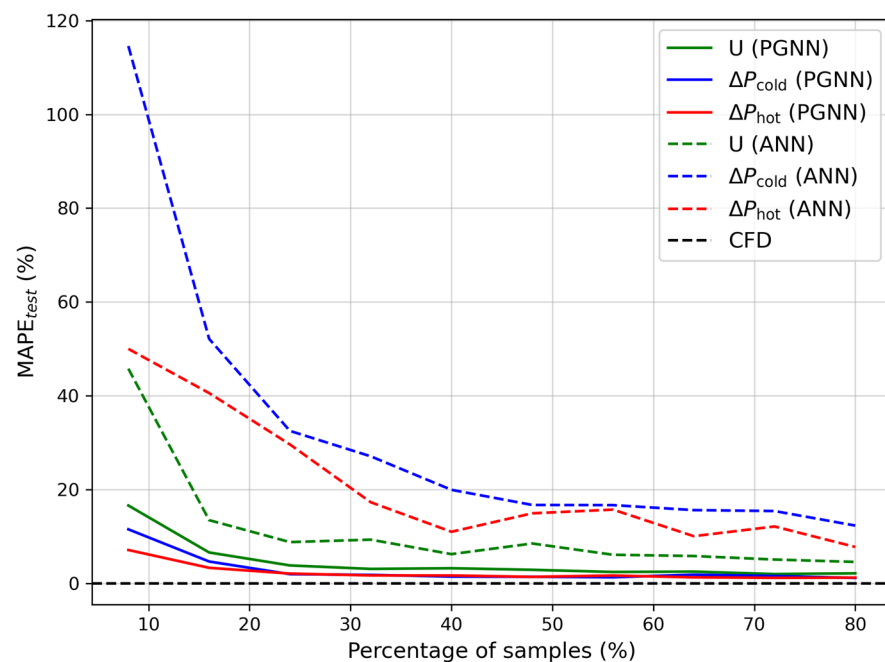


Figure 4. Variation in the MAPE evaluated on the unseen testing dataset function of the percentage of samples for the ANN and PGNN models.

Furthermore, even with a small training dataset of 315 samples (20% of the full set), the PGNN model exhibits negligible variation in MAPE compared to results obtained with the full training set (80%, i.e., 1260 samples). In contrast, for the ANN model, using 315 training samples results in a significant increase in MAPE compared to performance with the full training dataset. This reflects the robustness and high-fidelity of the PGNN model in predicting the thermo-hydraulic performance of CTHXs even when trained on small or moderate datasets. This also provides additional evidence of the PGNN model's low sensitivity to training set size, especially within the range of 20% to 80% of the total samples.

5.2. Predictive Performance Comparison of the Neural Networks

To provide a clear comparison of the predictive performance of the ANN and PGNN models on the training and unseen test datasets, histograms in Figure 5 show the MAPEs of the output variables evaluated on the training and test datasets using the full training set (1260 samples) and the test set (315 samples). The error bars shown in Figure 5 represent the standard deviation of the prediction errors, indicating the dispersion of the model predictions around the mean error values. For the PGNN model (blue bars), the MAPEs evaluated on the training set of the U , ΔP_c and ΔP_h are 1.25%, 1.09% and 0.86%, respectively, whereas the MAPE calculated on the unseen test set are 2.03%, 1.09% and 1.11%, respectively. On the other side, the MAPEs for the ANN model (red bars) computed on the training data of the U , ΔP_c and ΔP_h are 3.74%, 9.86% and 6.77%, while the corresponding test MAPEs are

4.71%, 12.75% and 7.97%, respectively. The prediction errors of the PGNN model on both training and test data are much smaller compared to those of the ANN model, particularly for predicting the hydraulic performance (ΔP_c and ΔP_h). This improved accuracy can be attributed to the incorporation of physical constraints into the PGNN's loss function, which guides the neural network to account for the underlying physical phenomena of CTHXs during training. Furthermore, the results demonstrate the superior predictive performance of the PGNN model over the ANN model in assessing the thermo-hydraulic performance of CTHXs, with consistently small errors. In addition, the relatively small gaps between the training and test errors of the PGNN model indicate strong generalization performance and a reduced possibility of overfitting, as well as lower susceptibility to high-variance or high-bias scenarios.

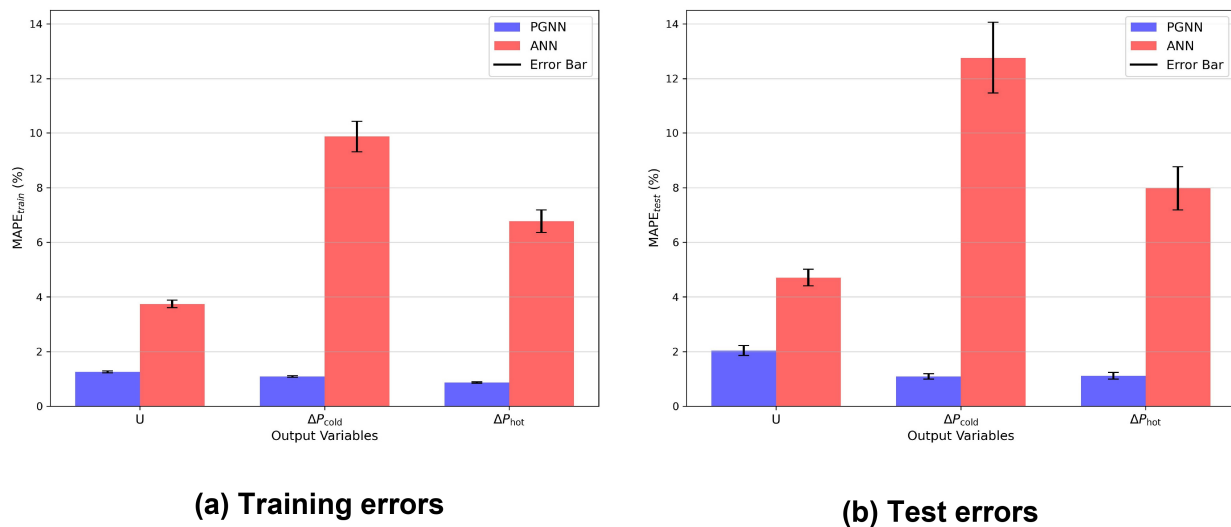


Figure 5. MAPEs of the output variables (U , ΔP_c and ΔP_h) for the PGNN and ANN models.

Figure 6a–c illustrate the variation in the U , ΔP_c and ΔP_h predicted by the ANN and PGNN models over the unseen test dataset (315 samples) as a function of the corresponding values acquired using CFD simulations. For the PGNN model, the majority of the data points (blue) are closely clustered near the ideal black dashed line ($y_{CFD} = y_{ML}$) for the three output variables U , ΔP_c and ΔP_h with small deviations, elucidating the high fidelity of the PGNN model in accurately predicting the thermo-hydraulic performance of CTHXs. This tight clustering of the blue sample points near the perfect line demonstrates that the PGNN model neither underestimates nor overestimates the output variables, providing stable predictions across both training and test data. In contrast, the red data points of the ANN model deviate from the ideal theoretical line, particularly at higher values of the U , ΔP_c and ΔP_h , reflecting the lower capability of the ANN model to generalize across the dataset and to deliver consistent predictions under a wide range of operating conditions of the CTHXs. At higher values of the output variables, the majority of ANN red points fall below the ideal line, indicating that ANN underestimates the values of the output variables in these regions. Furthermore, these results also serve as additional evidence to confirm the superior predictive accuracy of the PGNN model over the ANN model in estimating the thermal and hydraulic performances of CTHXs.

Additionally, Figure 7a–c show the absolute percentage error (APE) distribution of the U , ΔP_c and ΔP_h , respectively, evaluated on the unseen test dataset. The majority of samples exhibit low APE values as indicated by the tallest histogram bar being concentrated in the low-APE regions. Notably, the PGNN model has a substantially higher number of samples with low APE values than the ANN model. For instance, for APE < 3%, the

number of samples for U is 265 for PGNN and 161 for ANN (Figure 7a), for ΔP_c it is 293 for PGNN versus 117 for ANN (Figure 7b), and for ΔP_h it is 305 for PGNN versus 98 for ANN (Figure 7c). Moreover, the ANN model shows outliers with APE values exceeding 35% for ΔP_c ΔP_h , whereas the PGNN model exhibits none across all output variables. This further highlights the PGNN model's enhanced predictive accuracy over the ANN model, demonstrating its ability to precisely capture the physical phenomena of CTHXs by effectively mapping the complex relationships between input features and output variables.

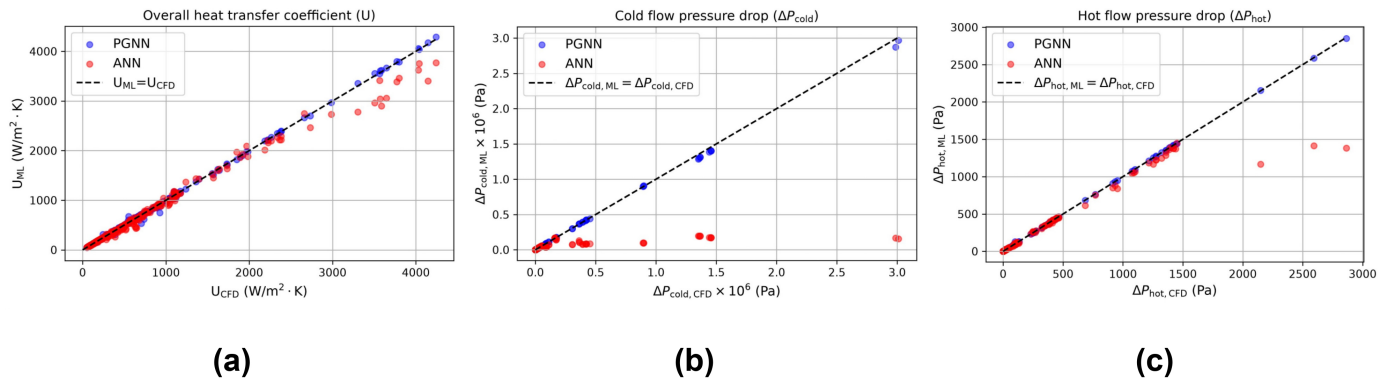


Figure 6. Variation of (a) U , (b) ΔP_c and (c) ΔP_h values predicted by the ANN (red points) and PGNN (blue points) models with respect to the corresponding values obtained using CFD simulations over the unseen test dataset.

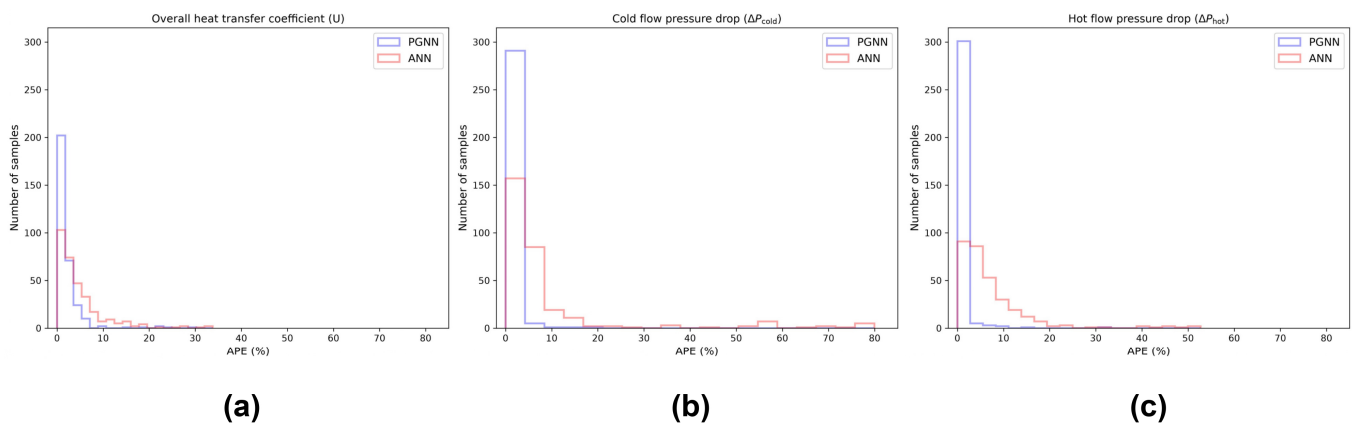


Figure 7. Absolute percentage error (APE) distribution of the (a) U , (b) ΔP_c and (c) ΔP_h over the unseen test dataset.

For further comparison of the predictive performance of the ANN and PGNN models, box plots of the APE are presented in Figure 8a–c for U , ΔP_c and ΔP_h , respectively. It is noteworthy to mention that the box plots encompass the APE values for the unseen test dataset (315 samples). The APE box plots provide a comprehensive view of the error distribution across the entire dataset, emphasizing the key statistical percentiles: the first quartile (Q1, 25th percentile), the median (50th percentile), and the third quartile (Q3, 75th percentile). In each figure, the blue and red boxes correspond to the interquartile ranges of the PGNN and ANN models, respectively, bounded by Q1 and Q3, with the median indicated by a horizontal black line within each box. The whiskers extend beyond the interquartile range to represent the spread of APE values outside the central 50% of the data, reflecting moderate deviations. Data points lying beyond the upper whisker are identified as outliers. The APE values for the Q1, median and Q3 of the box plots for both ANN and PGNN models are summarized in Table 6. As shown in Figure 8, the APE boxes for the PGNN are all smaller and narrower than those of the ANN model for the three

output variables U , ΔP_c and ΔP_h , indicating lower APE values for Q1, median, and Q3, as presented in Table 6 compared to the ANN model. Moreover, across all box plots, the lower and upper whiskers of the PGNN model are noticeably shorter than those of the ANN model, indicating lower variability in APE values. This reflects a more consistent error distribution and consequently greater stability in the PGNN model's predictions compared to the ANN model. Regarding outliers, the PGNN model exhibits no instances with APE values exceeding 35%. In contrast, the ANN model presents several outliers above this threshold, mainly in the predictions of ΔP_c and ΔP_h .

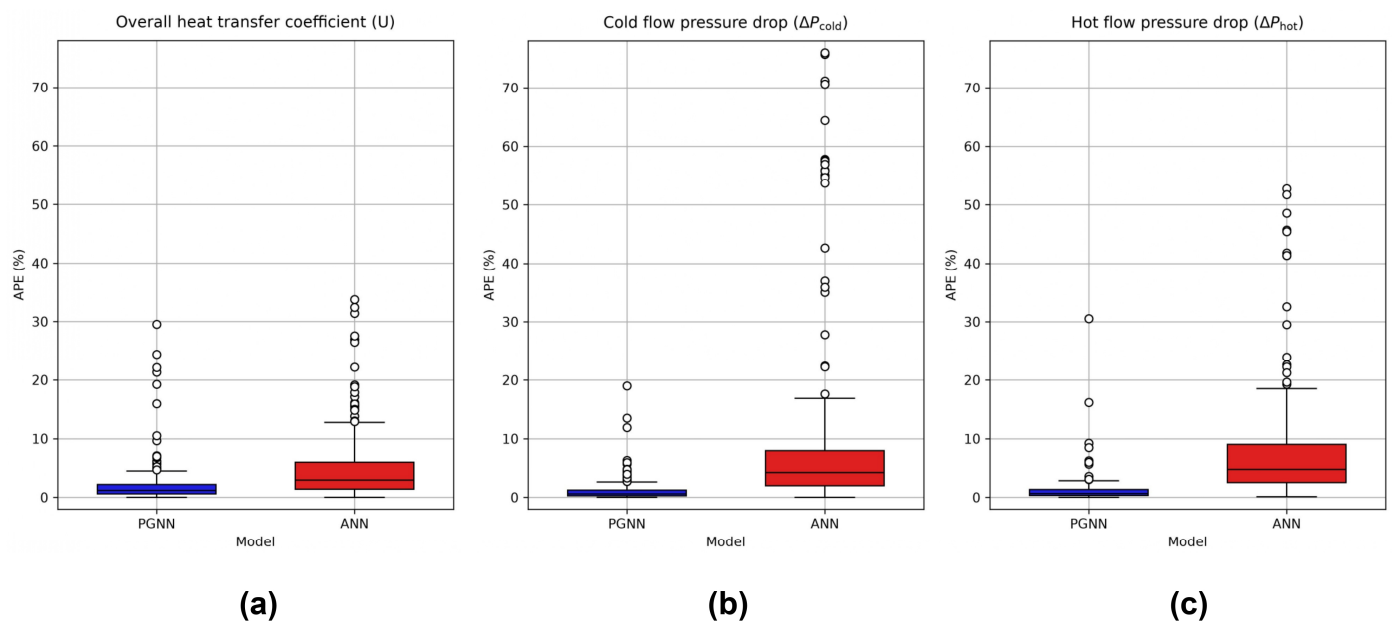


Figure 8. Box plots of the (a) U , (b) ΔP_c and (c) ΔP_h for the PGNN (blue) and ANN (red) models for the unseen test dataset.

Table 6. Summary of the box plots.

	U		ΔP_c		ΔP_h	
Metric\Model	PGNN	ANN	PGNN	ANN	PGNN	ANN
Q1	0.59%	1.41%	0.28%	1.97%	0.32%	2.53%
Median	1.17%	2.93%	0.62%	4.23%	0.66%	4.76%
Q3	2.21%	5.98%	1.25%	7.96%	1.36%	9.02%

6. Conclusions and Perspectives

In the present work, a physics-guided neural network (PGNN) approach is proposed to predict the thermo-hydraulic performance of CTHXs by estimating U , ΔP_c and ΔP_h . The objective is to guide the neural network in capturing the underlying physical phenomena of the CTHXs by incorporating empirical correlations as constraints into the PGNN loss function. For comparison purposes, a standard artificial neural network (ANN) model was introduced with the same architecture and input parameters as the PGNN model. First, the effect of the training dataset size was investigated, revealing the PGNN model's low sensitivity to it. In particular, between 20% (315 training samples) and 80% (1260 training samples) of the total dataset, the PGNN model exhibited only negligible changes in the MAPE evaluated on the unseen test set. In contrast, when using 20% of the dataset (315 samples) for training, the ANN model showed a significant increase in test MAPE values for the output variables, indicating a stronger dependence on the size of the

training data. Moreover, the investigation of the training set size reveals that the PGNN model outperforms the ANN model, yielding lower MAPE values on the training and testing datasets for all output variables (U , ΔP_c and ΔP_h) across a wide range of training set sizes. Subsequently, a detailed error analysis was provided to compare the predictive performance of the ANN and PGNN models using the full training dataset (1260 samples, 80% of the total dataset). The results indicate that the MAPEs evaluated on the unseen test data for the U , ΔP_c and ΔP_h using the PGNN model are 2.03%, 1.09% and 1.11%, respectively, which are remarkably lower than those obtained with the ANN model, which are 4.71%, 12.75%, and 7.97%, respectively. This demonstrates the superior predictive accuracy of the PGNN model compared to the ANN model in predicting the thermo-hydraulic performance of CTHXs. This enhanced predictive performance can be attributed to the inclusion of physical constraints in the loss function of the PGNN model, which enforce the neural network to consider the physical phenomena of the CTHXs during the training process.

Future work could extend the present approach by applying the proposed PGNN model to predict the thermo-hydraulic performance of other types of heat exchangers, such as plate heat exchangers. Another potential direction is to validate the CFD-based results by training the neural network on experimental data, thereby further assessing the model's predictive capability under real-world conditions.

A limitation of the present study is the absence of direct experimental validation or a benchmark dataset reproducing the same concentric-tube geometry, boundary conditions, Reynolds number range, and thermal conditions. The empirical-correlation comparison and mesh-independence assessment provide numerical verification and consistency evidence, but they should not be interpreted as full validation. Experimental measurements of heat-transfer rates, outlet temperatures, and pressure drops will therefore be required to further establish the quantitative accuracy of the CFD-generated dataset and the resulting machine learning models.

A further methodological limitation concerns the level at which physics is incorporated into the proposed PGNN. The present framework uses empirical thermo-hydraulic correlations as global output-level guidance but does not include residuals of the continuity, momentum, or energy equations. Therefore, although the correlation-derived baselines introduce established heat-transfer and pressure-drop dependencies, the resulting predictions are not guaranteed to satisfy local conservation laws, boundary conditions, or conjugate-interface conditions. Moreover, because the current network maps global geometric and operating parameters directly to the global quantities U , ΔP_c , and ΔP_h , it does not predict the spatial velocity, pressure, and temperature fields required to evaluate differential governing-equation residuals. The reported results should consequently be interpreted as those of a correlation-guided surrogate model rather than a fully equation-constrained model.

Another limitation is that the greywater stream is modeled as clean water and no fouling effects are considered. Actual greywater may contain suspended particles, detergents, fats, and other contaminants that can modify its effective viscosity and promote deposition on the tube walls. Such deposits may increase surface roughness and pressure losses, reduce the available flow area, and introduce an additional thermal resistance, thereby decreasing the overall heat-transfer coefficient and heat-recovery rate. The present results should therefore be interpreted as baseline predictions under clean-surface conditions. Future work should incorporate experimentally characterized greywater properties and time-dependent fouling resistance.

Another perspective is to investigate the influence of alternative Nusselt number, friction-factor, and pressure-drop correlations under different flow regimes and quantify

how uncertainties in these correlations propagate through the correlation-guided loss function. More importantly, a governing-equation-residual-based extension of the present framework should be developed. Such an extension would require a field-based neural network architecture in which the spatial coordinates and operating conditions are provided as inputs and the local velocity, pressure, and temperature fields are predicted as outputs. The training loss could then combine CFD-data and correlation-guidance terms with residuals of the continuity, momentum, and energy equations, together with the inlet, outlet, wall, symmetry, and conjugate-interface boundary conditions. The simplified axisymmetric CTHX geometry considered in the present study provides a computationally tractable benchmark for developing and evaluating this extension. As an intermediate approach, integral energy conservation could also be introduced by jointly predicting the outlet temperatures and heat-recovery rate and penalizing any imbalance between the energy lost by the hot stream and that gained by the cold stream. These developments would permit a systematic comparison between purely data-driven, correlation-guided, integral-conservation-guided, and governing-equation-residual-based neural network formulations.

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Nomenclature

Latin letters

R^2	Coefficient of determination
L	Length of CTHX (m)
Re_h	Reynolds number of the hot fluid
Re_c	Reynolds number of the cold fluid
Pr_h	Prandtl number of the hot fluid
Pr_c	Prandtl number of the cold fluid
D_i	Inner-tube diameter of CTHX (m)
D_o	Outer tube diameter of CTHX (m)
$T_{c,i}$	Inlet temperature of the cold fluid (K)
$T_{h,i}$	Inlet temperature of the hot fluid (K)
U	Overall heat-transfer coefficient ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$)
β	Regularization coefficient
$\tilde{y}_{i,j}$	Correlated output variables values
$\mathbf{W}$	Weight matrix
$\hat{y}_{i,j}$	Predicted output variables values
$y_{i,j}$	Output variables obtained using CFD simulations
Nu	Nusselt number
Gz	Graetz number
D_h	Hydraulic diameter (m)
f	Friction coefficient
σ	Activation function
Φ	Non-negative correction factor

h_c	Convective heat-transfer coefficient of the cold fluid
h_h	Convective heat-transfer coefficient of the hot fluid
ρ	Density ($\text{kg}\cdot\text{m}^{-3}$)
V_{in}	Inlet velocity ($\text{m}\cdot\text{s}^{-1}$)
N	Number of samples
$\dot{Q}_{rec}$	Heat-recovery rate (W)
r	Refining factor
k	Turbulent kinetic energy
Greek symbols	
ΔP_c	Cold flow pressure drop (Pa)
ΔP_h	Hot flow pressure drop (Pa)
μ_t	Eddy viscosity
ε_{HX}	Effectiveness
ω	Specific rate of dissipation
ε	Rate of dissipation
Abbreviations	
PGNN	Physics-guided neural network
HX	Heat exchanger
HVAC	Heating ventilation and air conditioning
CTHXs	Concentric tube heat exchangers
CFD	Computational fluid dynamics
ML	Machine learning
ANN	Artificial neural network
ReLU	Rectified linear unit
MAPE	Mean absolute percentage error
PEC	Performance evaluation criteria
COP	Coefficient of performance
RANS	Reynolds-averaged Navier–Stokes
NTU	Number of transfer units

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