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Is it no Longer Necessary to Test Cosmologies with Type Ia Supernovae?

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Abstract: We look at the current practice of analyzing the magnitude–redshift relation from the data on Type Ia supernovae. We show that, if the main aim of such analysis were to check the validity of a cosmological model, then the recently advanced arguments do not serve the purpose. Rather, the procedure followed tells us only about the statistical significance of the internal parameters used in the model, whereas the model itself is tacitly assumed to give a good fit to the data. A statistical assessment of the procedure is given and it is argued that given the growing data, the validity of the cosmological model should be checked first rather than the spread of any internal parameters. In passing we also discuss some aspects of the Milne model in the light of the present test.

Keywords: magnitude–redshift relation; statistics and cosmology; Milne model

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1. Introduction

Supernovae of Type Ia (SNeIa hereafter) are transient phenomena involving powerful thermonuclear explosions of carbon–oxygen white dwarfs [1]. The peak luminosity of a typical SNeIa is generally as high as the combined luminosity of around 10^9 suns. Thus, owing to their large luminosities, the SNeIa can be observed up to very high redshifts. Moreover, the intrinsic scatter in the peak luminosity of the normal SNeIa is small, and their spectra and light curves are very homogeneous. Another important property of SNeIa is that they are detected in all types of galaxies. The frequency of their occurrence in a galaxy like ours is of the order of a few per century. These are the key features that make SNeIa the best distance indicators (standard candles) and hence excellent tools for cosmological probes, particularly in measuring the expansion rate of the universe and discriminating one history of the universe from another [2,3].

Given a theoretical model of the universe, it is possible to predict a magnitude (m)–redshift (z) relation. This can be compared with the corresponding observed relation facilitated by an ideal standard candle. The χ^2-test is the most frequently used test for this purpose. Since in recent years high quality data on SNeIa have become available, such a test would no doubt have provided an observational check on cosmological models.

Unfortunately, however, a recent trend in the analysis of SNeIa data departs from the standard practice of executing a quantitative assessment of a cosmological theory—the expected primary goal of the observations [4,5]. Instead of using the data to directly test the considered model, the new procedure tacitly assumes that the model gives a good fit to the data, and limits itself to estimating the confidence intervals for the parameters of the model and their internal errors. The important purpose of testing a cosmological theory is thereby vitiated.

This issue has been addressed in the following from the point of view of a statistician. It appears that only after checking whether the considered model is consistent with the data for viable values of its free parameters should one proceed further to estimate the parameters of the model and their uncertainties. An independent observational verification of the standard cosmological model is also warranted by the highly speculative nature of its main ingredients: dark matter and dark energy.

We have also tried to clear up, in passing, a misunderstanding related to Milne's model, which has recently crept into the literature.

2. A Brief Overview of SNeIa Cosmology

Prior to the late 1990s, the standard models of cosmology used to be the simplest homogeneous and isotropic solutions of Einstein's equations, i.e., the Friedmann–Lemaître-Robertson–Walker (FLRW) universe:

$$H^2 + \frac{kc^2}{S^2} = \frac{8\pi G}{3}\rho \tag{1}$$

$$q = \frac{4\pi G}{3H^2}\left(\rho + \frac{3p}{c^2}\right) \tag{2}$$

which[1] are obtained by solving the Einstein field equation $G_{\mu\nu} = (-8\pi G/c^4)T_{\mu\nu}$ for the Robertson–Walker (R-W) line element

$$ds^2 = c^2dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \tag{3}$$

The models based on Equations (1) and (2) predict that the observed luminosity l of a celestial object, for instance an SNIa, observed at a redshift $z \equiv S_0/S - 1$ (the subscript zero denotes the value of the quantity at the present epoch[2]), should be [6]

$$l = \frac{\mathcal{L}}{4\pi d_{\rm L}^2} \tag{4}$$

where $\mathcal{L}$ is the absolute luminosity of the SNIa, and its luminosity distance $d_{\rm L}$ is given by [6,7]

$$d_{\rm L} = (1+z)S_0\, r_1 \tag{5}$$

where r_1, the coordinate distance of the observed SNIa, can be calculated by integrating the metric (3), giving

$$r_1 = \begin{cases} \sin\left(\dfrac{c}{S_0}\displaystyle\int_0^z \frac{{\rm d}z'}{H(z')}\right), & \text{when } k=1 \\ \dfrac{c}{S_0}\displaystyle\int_0^z \frac{{\rm d}z'}{H(z')}, & \text{when } k=0 \\ \sinh\left(\dfrac{c}{S_0}\displaystyle\int_0^z \frac{{\rm d}z'}{H(z')}\right), & \text{when } k=-1. \end{cases} \tag{6}$$

Equations (1) and (2) show that the expansion of the universe should be slowing down with time for the normal matter with $\rho > 0$, $p \geq 0$. We were then taken aback when a team led by Saul Perlmutter

[1] Here $H \equiv \dot{S}/S$ is the Hubble parameter with S being the scale factor of the homogeneous-isotropic universe (the "over-dot" represents derivative with respect to the cosmic time t), $q \equiv -\ddot{S}/(SH^2)$ is the deceleration parameter, $k = \pm 1,\ 0$ is the curvature parameter of the R-W spacetime, $\rho,\ p$ are respectively the density and pressure of the cosmic matter, and $c,\ G$ are respectively the speed of light in vacuum and the Newtonian constant of gravitation.

[2] The present value of the scale factor S_0 can be calculated, for different values of k, from Equation (1) in terms of ρ_0 and H_0 giving $S_0 = cH_0^{-1}\sqrt{k/(\Omega_{\rm m} - 1)}$, where $\Omega_{\rm m} \equiv \rho_0/\rho_c$ is the present density of the universe in the unit of critical density $\rho_c \equiv 3H_0^2/(8\pi G)$.

and another one led by Adam Riess and Brian Schmidt noticed that more than 50 distant SNeIa appear significantly fainter for their measured redshift than predicted by the then standard cosmology [8–10]. Since then over 1200 high-redshift SNeIa with increasing precision have been observed, which confirm this result [11].

This situation is generally explained away by postulating the existence of some unknown component in the energy budget of the universe, termed as "dark energy"—a smoothly pervasive component whose pressure is sufficiently negative. For a suitably chosen density of the dark energy $\rho_{\rm DE}$, its negative pressure enhances the distance $d_{\rm L}$ of the SNeIa so that they may look fainter as in the observations. Mathematically, this is equivalent to replacing ρ and p in Equations (1) and (2) with

$$\left.\begin{aligned} \rho \rightarrow \rho_{\rm total} &= \rho + \rho_{\rm DE} \\ p \rightarrow p_{\rm total} &= p + p_{\rm DE} \end{aligned}\right\} \tag{7}$$

where the pressure of the dark energy $p_{\rm DE} = \omega_{\rm DE}\rho_{\rm DE}c^2$, with $\omega_{\rm DE} = \omega_{\rm DE}(z)$ in general, and $\Omega_{\rm DE} \equiv \rho_{\rm DE0}/\rho_c$. For instance, Equations (1) and (2) are respectively replaced by

$$H^2 = H_0^2\left[\Omega_{\rm m}(1+z)^3 + \Omega_{\rm DE}(1+z)^{3(1+\omega_{\rm DE})} + (1-\Omega_{\rm m}-\Omega_{\rm DE})(1+z)^2\right] \tag{8}$$

$$2q = \frac{H_0^2}{H^2}\left[\Omega_{\rm m}(1+z)^3 + (1+3\omega_{\rm DE})\Omega_{\rm DE}(1+z)^{3(1+\omega_{\rm DE})}\right] \tag{9}$$

in a matter-dominating (over radiation) universe for a constant $\omega_{\rm DE}$ and a (covariantly) conserved matter content. The most favored candidate of dark energy is Einstein's famous cosmological constant Λ (for which $\omega_{\rm DE} = \omega_\Lambda = -1$), i.e., $\rho_\Lambda = \Lambda c^2/8\pi G = -p_\Lambda/c^2$. This would, however, mean an accelerating expansion of the present universe for suitably chosen $\rho_{\rm DE}$, rather than the old decelerating one. For instance, for $\Lambda > 4\pi G\rho_0/c^2$, the modified Equation (9) would imply $q < 0$ at the present epoch.

Although all the SNeIa data are consistent with the cosmological constant Λ, various other models also fare well with the data, as we can see in Table 1. This has given rise to a plethora of models in the framework of general relativity (GR), as well as some possible modifications of GR. Theorists may debate the relative merits of various cosmic-acceleration theories: cosmological constant, dark energy, alternative gravity, anthropic arguments, etc., but it is ultimately up to the observations to decide which theory is correct.

Observations usually test the models in two ways: (i) the first one—the Bayesian approach—gives a relative rather than an absolute measure of how good a theory is, and hence is more appropriate for comparison between competing models; (ii) the second one—Pearson's "chi-square (χ^2) test of goodness of fit" or the "weighted least-square fit"—is more commonly used for theory testing, wherein the observed sample distribution is compared with the χ^2-probability distribution corresponding to the model to be tested. Under this approach, one minimizes χ^2 given by Equation (12) below.

Table 1. Best-fit parameters of some selected cosmological models fitted to different SNeIa data sets.

Models	Ω_m	Ω_{DE}	ω_{DE} (Constant)	$\mathcal{M}$	q_0	χ^2	DoF	P (%)
(9 high z + 27 low z) MLCS SNeIa from Riess et al. [8] (1998)								
ΛCDM	0.15 ± 1.28	0.60 ± 1.47	-1	43.31	-0.53	44.0	33	9.5
ΛCDM ($\Omega_{total} = 1$)	0.26 ± 0.10	$1 - \Omega_m$	-1	43.30	-0.60	44.0	34	11.7
ρ_{DE}CDM ($\Omega_{total} = 1$)	0.14 ± 1.34	$1 - \Omega_m$	-0.79 ± 1.79	43.31	-0.52	44.0	33	9.5
Milne model				43.35	0	47.1	35	8.3
54 SNeIa from Perlmutter et al. [10] (1999)								
ΛCDM	0.79 ± 0.47	1.40 ± 0.65	-1	23.91	-1.01	56.9	51	26.6
ΛCDM ($\Omega_{total} = 1$)	0.28 ± 0.08	$1 - \Omega_m$	-1	23.94	-0.58	57.7	52	27.3
ρ_{DE}CDM ($\Omega_{total} = 1$)	0.48 ± 0.15	$1 - \Omega_m$	-2.10 ± 1.83	23.91	-1.14	57.2	51	25.7
Milne model				24.03	0	61.5	53	19.8
"Gold Sample" of 157 SNeIa from Riess et al. [12] (2004)								
ΛCDM	0.46 ± 0.10	0.98 ± 0.19	-1	43.32	-0.75	175.0	154	11.8
ΛCDM ($\Omega_{total} = 1$)	0.31 ± 0.04	$1 - \Omega_m$	-1	43.34	-0.54	177.1	155	10.8
ρ_{DE}CDM ($\Omega_{total} = 1$)	0.49 ± 0.06	$1 - \Omega_m$	-2.33 ± 1.07	43.30	-1.28	173.7	154	13.2
Milne model				43.40	0	191.7	156	2.7
"New Gold Sample" of 182 SNeIa from Riess et al. [13] (2007)								
ΛCDM	0.48 ± 0.09	0.96 ± 0.18	-1	43.36	-0.72	156.4	179	88.7
ΛCDM ($\Omega_{total} = 1$)	0.34 ± 0.04	$1 - \Omega_m$	-1	43.40	-0.49	158.7	180	87.1
ρ_{DE}CDM ($\Omega_{total} = 1$)	0.46 ± 0.06	$1 - \Omega_m$	-1.75 ± 0.63	43.35	-0.92	156.6	179	88.5
Milne model				43.45	0	174.3	181	62.6
New Gold Sample + the most distant SN UDS10Wil of $z = 1.914$ [14] (2013)								
ΛCDM	0.50 ± 0.09	0.99 ± 0.17	-1	43.36	-0.74	157.0	180	89.1
ΛCDM ($\Omega_{total} = 1$)	0.35 ± 0.04	$1 - \Omega_m$	-1	43.40	-0.48	160.1	181	86.6
ρ_{DE}CDM ($\Omega_{total} = 1$)	0.47 ± 0.06	$1 - \Omega_m$	-1.80 ± 0.62	43.35	-0.94	157.5	180	88.6
Milne model				43.45	0	178.3	182	56.4

As the SNeIa datasets are generally given in terms of magnitude versus redshift (instead of luminosity versus redshift), one needs to convert the luminosities appearing in Equation (4), in the logarithmic scale of magnitudes. Taking a logarithm of Equation (4) and recalling that $l = 10^{-2m/5} \times 2.52 \times 10^{-5}$ erg cm^{-2} s^{-1}, $\mathcal{L} = 10^{-2M/5} \times 3.02 \times 10^{35}$ erg s^{-1} [7], one obtains

$$m(z; \mathcal{M}, \Omega_m, \Omega_{DE}, \omega_{DE}) = \mathcal{M} + 5 \log\left(\frac{H_0}{c} d_L(z; \Omega_m, \Omega_{DE}, \omega_{DE})\right), \tag{10}$$

where $\mathcal{M} \equiv M - 5 \log H_0 + constant$[3], with m and M representing the apparent and absolute magnitudes, respectively. For the nearby SNe ($z << 1$), Equation (10) reduces to

$$m(z) = \mathcal{M} + 5 \log z, \tag{11}$$

which can be used to estimate $\mathcal{M}$ by using low-redshift supernovae-measurements (that are far enough into the Hubble flow so that their peculiar velocities do not contribute significantly to their redshifts). In order to compare the model-predicted value of m with the observed magnitude m_{obs}, one calculates χ^2 according to

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{m(z_i; \mathcal{M}, \Omega_m, \Omega_{DE}, \omega_{DE}) - m_{obs,i}}{\sigma_{m_{obs,i}}}\right]^2 \tag{12}$$

[3] The value of this *constant* depends on the chosen units in which d_L and H_0 are measured. For example, if d_L is measured in Mpc and H_0 in km s^{-1} Mpc^{-1}, then this *constant* comes out as ≈25.

where the quantity $\sigma_{m_{\text{obs},i}}$ is the measurement error (standard deviation) in the observed magnitude $m_{\text{obs},i}$ of the i-th SNIa. We generally assume that the errors $\sigma_{m_{\text{obs},i}}$ are independent and distributed normally.

It is suggested by Equation (12) that, if the model represents the data correctly, the difference of the predicted and observed magnitudes should be roughly the size of the measurement uncertainties and each data point will contribute roughly one to χ^2, giving a sum roughly equal to the degrees of freedom (DoFs) $\equiv$ the number of data points $N -$ the number of fitted parameters (it is expected that $N >>$ the number of fitted parameters). If χ^2 is large, the fit is bad. In order to quantify the goodness-of-fit of the model to the data, one calculates the χ^2-probability P (appearing in the last column of Table 1), which provides an objective assessment of how the model fares with the data. If the fitted model provides a typical value of χ^2 as x at n DoF, this probability is given by

$$P(x,n) = \frac{1}{\Gamma(n/2)} \int_{x/2}^{\infty} e^{-u} u^{n/2-1} \mathrm{d}u. \tag{13}$$

$P(x,n)$ gives the probability that a model that does fit the data at n DoFs would give a value of χ^2 as large as x or larger. This generally assumes that the measurement errors are normally distributed. Unless P is substantially large, we cannot claim that the model has a good fit. Generally, the model is ruled out if $P \approx 0.05$ or smaller.

One may note that the values of $\mathcal{M}$ estimated from different data sets (appearing in the 5th column of Table 1) do not match. There are two reasons for this. (i) Sometimes the data are given in terms of the distance modulus $\mu = m(z) - M$, instead of m. The constant $\mathcal{M}$ takes care of this situation where Equation (12) can still be used in this case for fitting the data by using μ_{obs} in place of m_{obs}. (ii) Usually the zero-point absolute magnitudes are set arbitrarily in different data sets. While fitting the combined data set this situation is handled successfully by the constant $\mathcal{M}$ appearing in Equation (12), which now plays the role of the normalization constant and simply gets modified suitably. In this case, however, it does not represent the usual "Hubble constant-free absolute magnitude"[4], but differs from the latter by an unknown constant (which is, however, not needed for the cosmological results).

3. A Non-Standard Approach to SNeIa Data

(a) About a decade ago, a new approach that does not respect the standard procedure described above was adopted to analyze the SNeIa data. Initiated by the SuperNova Legacy Survey (SNLS) [15] in 2006, this approach simply assumes, rather than examines, that the standard cosmology is consistent with the SNeIa observations and limits itself to calculating confidence intervals (ellipses) of parameters. Under this approach, χ^2 is calculated from

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{[m(z_i;\ \text{parameters}) - m_{\text{obs},i}]^2}{\sigma^2_{m_{\text{obs},i}} + \sigma^2_{\text{int}}} \right] \tag{14}$$

where σ_{int}, appearing as a free parameter in (14), is the (unknown) intrinsic dispersion of the SNeIa absolute magnitude, which is not included in the $\sigma_{m_{\text{obs}}}$. It is claimed that σ_{int} is an extra dispersion in m related to our imperfect understanding of SNeIa physics. It may result from many unidentified sources such as the intrinsic progenitor properties, circumstellar dust, the viewing angle, uncorrected selection effects as well as the imperfect nature of SNeIa as standard candles.

It should be noted that the observations we have considered in Table 1 already include the intrinsic dispersion of the SN absolute magnitude in their error bars. This is generally estimated from the nearby data, from the difference between the observed magnitudes and those predicted theoretically by the cosmological models, e.g., the linear Hubble law (Equation (11)), which is same for all models

[4] Perhaps $\mathcal{M}$ is called so because it serves as the absolute magnitude corresponding to the "Hubble constant-free luminosity distance" in Equation (10). It is easy to check that $H_0 d_{\text{L}}$ is Hubble constant-free.

for a low redshift. On the other hand, the new approach initiated by SNLS estimates $\sigma_{\rm int}$ by considering it an adjustable free parameter in order to obtain $\chi^2/\text{DoF} = 1$ (i.e., it *assumes, rather than tests*, that the considered model has a good fit). This is, however, equivalent to just increasing the error bars suitably in order to have a satisfactory fit to our desired model. In this way, one can 'fit' any model to the data and estimate a $\sigma_{\rm int}$. For example, the Einstein-de Sitter model can also be fitted to the SNLS data by considering $\sigma_{\rm int} = 0.258$ with $\chi^2/\text{DoF} = 1.00$. Various other plausible models, in the ΛCDM cosmology as well as in alternative theories (for instance, the quasi-steady state cosmology (QSSC), see Appendix A), can also be fitted to the SNLS data for reasonable values of $\sigma_{\rm int}$, as is shown in Tables 2 and 3. Let us note that many of these models do not otherwise fit the data if we follow Equation (12) of the standard approach with m given by (10). Moreover, the new method prohibits an objective assessment of the considered theory in terms of the goodness-of-fit probability P, in the absence of which the estimated parameters do not have any significance.

Table 2. Different possible ΛCDM models which provide $\chi^2/\text{DoF} \approx 1$ for suitably chosen $\sigma_{\rm int}$ to fit 115 SNeIa from Astier et al. [15].

$\sigma_{\rm int}$	$\Omega_{\rm m}$	Ω_Λ	$\mathcal{M}$	χ^2/DoF	Varied in χ^2-Minimization
0.131	0.26	$1-\Omega_{\rm m}$	43.16	112.97/113	$\Omega_{\rm m}$, $\mathcal{M}$
0.131	0.31	0.81	43.15	112.09/112	$\Omega_{\rm m}$, Ω_Λ, $\mathcal{M}$
0.143	0	0	43.26	114.40/114	$\mathcal{M}$
0.172	0.3	0	43.32	113.94/114	$\mathcal{M}$
0.132	0	0.38	43.17	112.97/113	Ω_Λ, $\mathcal{M}$
0.135	-0.24	0	43.20	112.89/113	$\Omega_{\rm m}$, $\mathcal{M}$

Table 3. Different plausible models in QSSC which provide $\chi^2/\text{DoF} \approx 1$ for suitably chosen $\sigma_{\rm int}$ to fit 115 SNeIa from Astier et al. [15]. The parameters $z_{\rm max}$, Ω_Λ (<0), κ (measured in the units of 10^5 cm^2/g), $\rho_{\rm g}$ (in 10^{-34} g/cm^3), and H_0 (in 100 Km s^{-1}Mpc^{-1}) characterize a typical model in QSSC [16].

$\sigma_{\rm int}$	Ω_Λ	$\kappa\rho_{\rm g}H_0^{-1}$	$z_{\rm max}$	$\mathcal{M}$	χ^2/DoF	Varied in χ^2-Minimization
0.14	-0.3	8.49	10	43.22	113.62/113	$\kappa\rho_{\rm g}H_0^{-1}$, $\mathcal{M}$
0.15	-0.1	5	10	43.28	113.22/114	$\mathcal{M}$
0.16	-0.2	5	10	43.31	114.3/114	$\mathcal{M}$
0.173	-0.3	5	10	43.34	113.80/114	$\mathcal{M}$
0.147	-0.1	5	8	43.27	114.63/114	$\mathcal{M}$
0.16	-0.2	5	8	43.30	111.94/114	$\mathcal{M}$
0.17	-0.3	5	8	43.33	114.02/114	$\mathcal{M}$

It might be legitimate to consider $\chi^2/\text{DoF} = 1$ for the nearby SNeIa only, for which all the models consistently give the same $m-z$ relation (11). Nevertheless, the high-redshift SNeIa must be checked to be consistent with the model before calculating the confidence intervals on the estimated parameters. It may be noted that, in the theories of modified gravity, the peak luminosity of SNeIa depends on the local strength of gravity [17]. Therefore, the extrapolation of the dispersion of the absolute magnitude of SNeIa from low-redshift to high-redshift may be erroneous.

It may also be mentioned that the introduction of $\sigma_{\rm int}$ in Equation (14) does not respect the standard way of data analysis in statistics. One may recall that the variance $\sigma^2_{m_{{\rm obs},i}}$ appearing in Equation (12) represents the combined uncertainty in the observed magnitude of the ith supernova arising from the uncertainties in different variables, for example, lensing, dust extinction, the peculiar velocity of the host galaxy, etc. By Taylor-expanding m about its mean value and by recalling that the variance of $(m) = \langle m^2 \rangle - \langle m \rangle^2$, one can write the combined uncertainty in $m_{\rm obs}$ in terms of the uncertainties in its parameters, say, x_j:

$$\sigma^2_{m_{\rm obs}} = \sum_j \left(\frac{\partial m}{\partial x_j} \right)^2 \sigma^2_{x_j} + \sum_j \sum_{k \neq j} \left(\frac{\partial m}{\partial x_j} \right) \left(\frac{\partial m}{\partial x_k} \right) {\rm cov}(x_j, x_k) \tag{15}$$

where ${\rm cov}(x_j, x_k)$ is the covariance between the variables x_j and x_k, which vanishes for the uncorrelated variables, leaving the combined uncertainty in $m_{\rm obs}$ as

$$\sigma^2_{m_{\rm obs}} = \sum_j \left(\frac{\partial m}{\partial x_j} \right)^2 \sigma^2_{x_j} \tag{16}$$

i.e., the sum of the square of some random variables, each normalized by its variance, and thus following a χ^2-distribution. Thus, a more reasonable way to introduce $\sigma_{\rm int}$ would be to estimate the combined uncertainty in $m_{{\rm obs},i}$ according to Equation (16) by using the independent measurement uncertainties $\sigma_{m_{{\rm int},j}}$ from different sources, such as the intrinsic progenitor properties, circumstellar dust, the viewing angle, etc. This would lead to a χ^2 given by

$$\chi^2 = \sum_{i,j} \left[\frac{[m(z_i;\ {\rm parameters}) - m_{{\rm obs},i}]^2}{\sigma^2_{m_{{\rm obs},i}} + [\frac{\partial m}{\partial m_{{\rm int},j}}(z_i)]^2\, \sigma^2_{m_{{\rm int},j}}} \right]. \tag{17}$$

It may be noted that replacing Equation (12) with Equation (14) is equivalent to assuming that $\sigma_{m_{{\rm obs},i}}$ is underestimated, and so we are adjusting it by adding $\sigma_{\rm int}$ to it. However, we expect $\sigma_{\rm int}$ not to be higher than $\sigma_{m_{{\rm obs},i}}$, in this case. But $\sigma_{\rm int}$ is far higher than $\sigma_{m_{{\rm obs},i}}$ for various SNeIa in the SNLS data. Another point worth noting is that the goodness-of-fit probability P given by Equation (13) holds when the measurement errors are independent and Gaussian. It should be a matter of caution whether this condition is still valid after the extra terms are added in the denominators in Equation (14) (and also in Equation (18) appearing below) and if the resulting statistic still follows a χ^2-distribution.

It should also be mentioned that the value of the intrinsic dispersion $\sigma_{\rm int} = 0.13 \pm 0.02$, estimated in [15] using Equation (14) for the concordance model, is approximately of the same order as measured by [8,10,12,18,19]. Nevertheless, our critique is not directed to the fit-quality of the concordance model, but to the non-standard methodology of SNeIa data analysis mentioned above. The harmful side effect of this methodology (of disrespecting model-testing and limiting oneself to estimating the parameters of the model either from the SNeIa data or by combining the SNeIa data with other observations) is clear from the following example. In analyzing the "Constitution" data [20], although the authors in [20] do not follow the new approach, they do not seem to notice that the theory does not fit the data well! One can calculate from their Table 1 that the best-fitting ΛCDM model, with $\Omega_{\rm m} = 1 - \Omega_\Lambda = 0.29$ gives χ^2/DoF = 465.5/395 with a meager probability P = 0.83%, so the estimated model can be ruled out at a confidence level of more than 99%! Other models too have a similar fit.

Of course one can estimate $\sigma_{\rm int}$ (if one is interested in just that) from all (high- as well as low-redshift) SNeIa data by assuming that a particular theory (here the standard cosmology), already tested, must be consistent with the data (i.e., $\chi^2/{\rm DoF} = 1$). This is fine if the theory is well established which is already tested by other independent ways. Then we are not interested in testing the already established theory; rather, we want to estimate, from Equation (14), some parameter of the data (here $\sigma_{\rm int}$) that we could not decipher from the observations. This procedure is followed in many branches of physics. However, this is not so with the standard cosmology in view of the notorious fine-tuning and coincidence problems related with the cosmological constant, and the extremely speculative character of the dark energy in general, in the total absence of any direct observational support. Thus, the standard cosmology calls for more and rigorous observational tests.

It may be mentioned that the new non-standard approach initiated in 2006 by the SNLS group has already acquired the status of the "standard" approach. Many other groups, which were earlier following the conventional methods to analyze the SNeIa data, now follow the new approach [11].

(b) In passing, it would also be worthwhile to bring to the notice of the reader another approach that also does not seem perfectly consistent with Equation (16). Some authors (for example, [8,12,13]) perform the SNeIa data-fitting by considering the χ^2-statistic in the form

$$\chi^2 = \sum_{i=1}^{N} \frac{[m(z_i;\ \text{parameters}) - m_{\text{obs},i}]^2}{\sigma^2_{m_{\text{obs},i}} + \sigma^2_v} \tag{18}$$

where σ_v represents the dispersion in SN redshift due to peculiar velocities ($z = v/c$). They usually consider $\sigma_v = 400$ Km s^{-1} within its likely range 200 Km s$^{-1} \leq \sigma_v \leq 500$ Km s^{-1}. They further add 2500 km s^{-1} in the quadrature to σ_v for high-redshift SNeIa whose redshifts are determined from the broad features in the SN spectrum. Let us note that $m(z)$ is as non-linear as

$$m(z) = 5 \log \left[(1+z) \int_0^z (\Omega_m (1+z') + \Omega_\Lambda)^{-1/2} dz' \right] + \text{constant} \tag{19}$$

even for the simplest ΛCDM model. Thus, Equation (18) does not seem to respect Equation (16). It should be noted that σ_v can very well be included in $\sigma_{m_{\text{obs}}}$, as has been done in [15]. It should, however, be mentioned that the removal of σ_v from Equation (18) does not make any significant change in the fit-results. The fit-results mentioned in Table 1 are calculated by neglecting σ_v from (18).

A misunderstanding about the Milne model: It would also be worthwhile to clear a misunderstanding related to Milne's model, which persists in the literature. In Table 1, we notice that, besides the ΛCDM and other dark energy models, the Milne model also fares well with the data (see also, [21]). The remarkable fact is that this coasting model does so without requiring any dark energy and accelerated expansion.

In the literature, the Milne model is represented by an unphysical "empty" universe with $\Omega_\text{m} = 0 = \Omega_\text{DE}$, which is though misleading. Although by considering $\Omega_\text{m} = 0 = \Omega_\text{DE}$ in the FLRW equations of GR, one is led to

$$ds^2 = c^2 dt^2 - S^2 \left(\frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right), \quad S = ct \tag{20}$$

which is the same as the evolution dynamics of the universe in the Milne model derived from kinematic relativity and the cosmological principle (see Appendix B), nevertheless the two models are fundamentally different [22]. While the former represents an unphysical empty universe in the framework of GR, the latter is not empty. In fact, the Milne model cannot be recast in the framework of GR. It is a phenomenological model of the universe that was developed by Milne independently of GR by assuming the presence of matter in the Minkowskian background. The presence of matter without curving spacetime in Milne's theory indicates that this theory is fundamentally different from GR and should not be viewed within the usual understanding of an empty universe in GR. (Let us note that the metric given in Equation (20) is Minkowskian in disguise.)

4. Conclusions

No matter how elegant a theory is, its verification, by comparison with the observational evidence, is absolutely necessary. This is usually done through some standard statistical techniques such as the χ^2-goodness-of-fit test, which determines how well the theory fits the observations. Observations on the SNeIa can be used as an excellent tool to serve this purpose in the case of a cosmological theory. Let us recall that the SNeIa are one of the best standard candles known today, and a sizeable number thereof is nowadays routinely detected by some dedicated surveys. The statisticians would also concur with the approach wherein the new SNeIa data are used to test the theory.

However, the current practice of analyzing the magnitude-redshift relation from the SNeIa data does not fulfill this requirement. It rather reverses the standard procedure by assuming (rather than examining) that the basic hypothesis (standard cosmology with 23% of dark matter and 72% of dark energy) is correct (by presuming $\chi^2/\mathrm{DoF} = 1$) and limits itself to calculating the allowed confidence intervals for the estimated parameters. Discussions with professional statisticians reveal that this approach is logically inconsistent. Let us note that only after examining if the considered theory has a credible goodness-of-fit to the data, one is expected to estimate the parameters of the theory and their uncertainties. In the absence of a credible goodness-of-fit, the estimated parameters of the theory, and their estimated uncertainties, have no meaning at all. Moreover, the important goal of testing the theory with data remains unfulfilled. The futility and weakness of the newly adopted non-standard procedure have been exemplified by showing that even the models that do not otherwise fit the data in the standard procedure, can be made to fit it for suitably chosen values of $\sigma_{\rm int}$.

We strongly advocate that, instead of assuming the correctness of the standard cosmology and thereby using the SNeIa data to calculate the confidence intervals for the model parameters, the theory should be tested by the data first through the standard statistical techniques, such as the t-test or the χ^2- test. This is also warranted by the highly speculative nature of the principal elements of standard cosmology: dark matter and dark energy. While dark matter still eludes direct detection, dark energy even fails to acquire a single elementary particle candidate, let alone a direct detection. Thus, these elements appear as mere ad hoc theoretical inventions, devised to explain observations that we do not otherwise understand.

This subjects the theory to more rigorous observational tests, performed through the standard statistical methods, such as the χ^2- test that has been used in the cases of different cosmological models considered in Table 1. These methods can also be used to compare the rival theories, such as those that try to explain the SNeIa observations without requiring the dark energy.

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Appendix A: QSSC

The Quasi-Steady State Cosmology, proposed in 1993 [23,24], is a cyclic model of the universe driven by a negative-energy scalar field and a negative cosmological constant. The scale factor S of the model is subjected to short-term oscillations superimposed on a long-term steady expansion:

$$S(t) = e^{t/P}\,[1 \,+\, \eta \cos(2\pi\tau/Q)] \tag{A1}$$

where the time scales $P \approx 10^3$ Gyr $\gg Q \approx 40 - 50$ Gyr are considerably greater than the Hubble time scale of ≈ 14 Gyr of the standard cosmology. The model has cycles of expansion and contraction (regulated respectively by the creation field and the negative Λ) of a comparatively shorter period (Q) superposed on a long-term (P) steady state-like expansion. The function $\tau(t)$ is very much like the cosmic time t, with significantly different behavior for short durations near the minima of the function $S(t)$. The parameter η has a modulus less than unity, thus preventing the scale factor from reaching zero. Typically, $\eta \sim 0.8 - 0.9$. Hence, there is no spacetime singularity, nor a violation of the law of conservation of matter and energy, as happens at the big bang epoch in the standard cosmology.

In order to interpret the $m - z$ relation of SNeIa, QSSC invokes the presence of metallic dust that extinguishes radiation traveling over long distances. This additional extinction adds an extra magnitude [16]

$$\Delta m(z) = 1.0857 \times \kappa\, \rho_{g0} \int_0^z (1+z')^2 \frac{dz'}{H(z')} \tag{A2}$$

to the apparent m given by (10) and thus the net magnitude amounting to

$$m(z) = \mathcal{M} + 5 \log \left[H_0 d_L(z)/c\right] + \Delta m(z), \tag{A3}$$

where κ is the mass absorption coefficient and ρ_{g0} is the density of the metallic dust at the present epoch.

Appendix B: The Milne Model

The Milne model is a cosmological model based on special relativity which was introduced by Edward Arthur Milne in 1935 [25]. It is a deductive theory based on Milne's kinematic relativity [26,27] in which information is deduced only from the cosmological principle taken together with the basic properties of spacetime and the propagation of light. Besides the cosmological principle, Milne made another assumption that the matter present in the universe is conserved (which is evidently suggested by ordinary physics). This implies that the equation of hydrodynamic continuity applies and the density of matter decreases with time in the universe, whose invariant border advances at the speed of light.

The assumptions of homogeneity and isotropy as required by the cosmological principle leads to the R-W line element (3):

$$ds^2 = c^2 dt^2 - S^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

given in terms of the cosmic time t, in which the relative motion of the observes is non-zero but unaccelerated (as it is a special-relativistic theory). In order to make the motion of the observers uniform, Milne considered the scale factor $S = ct$ in (3). Now, $k = -1$ is the only choice to make the line element (Equation (3)) compatible with the Minkowskian metric, since with $S = ct$, the resulting 4-dimensional spacetime form (3) is flat only when $k = -1$ and the 3-space is hyperbolic. Hence, the Milne model reduces to Equation (20). One may check that the transformations $\bar{t} = t\sqrt{1+r^2}$, $\bar{r} = ctr$ indeed reduce the line element (Equation (20)) to a manifestly Minkowskian form in the coordinates $\bar{t}, \bar{r}, \theta, \phi$ (see page 140 in [6]).

The greatest achievement of the kinematic relativity is the existence of another important time scale, say τ, in which the observers appear to be at rest and the universe presents a static appearance. The τ-time is related with the t-time through the transformation

$$\tau = t_0 \ln\left(\frac{t}{t_0}\right) \tag{A4}$$

which transforms the line element (20) to a form conformal to a static form of (20):

$$ds^2 = e^{2\tau/t_0} \left[c^2 d\tau^2 - c^2 t_0^2 \left\{\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right\}\right] \tag{A5}$$

where t_0 is a constant with the significance that $\tau = 0$ when $t = t_0$. The zero of t-time scale is a fundamental event in the theory when the separation of the fundamental (co-moving) observers vanishes, proposing a physical explosion of matter. In the τ-time scale, this event takes place in the infinite past, owing to its logarithmic dependence on t, as is indicated by Equation (A4).

It should be noted that the line element (Equation (20)) results as a natural consequence of kinematic relativity, and has nothing to do with GR. However, as the same solution expressed by

Equation (20) is also obtained in the framework of the standard cosmology for an empty universe, it is generally believed that the Milne model represents an empty universe, which is not correct. All one can say, in the language of GR, is that matter does not curve the spacetime in the geometric analogue of the Milne model.

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