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Star and Black Hole Formation at High Redshift

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Abstract: Evidence for dark matter (DM) was originally discovered in 1933 by Zwicky (Zwicky 1933, 1937), and has defied all explanations since then. The original discovery was based on the motions of galaxies in clusters of galaxies. The Micro Wave Back Ground (MWBG) observations by the Planck mission and other satellites give definitive numbers. Galaxy correlations give results down to small galaxies, which match theoretical expectations. Here we focus on a few interesting aspects, that may allow to determine the nature of dark matter: (1) Ultra Faint Dwarf (UFD) galaxies, that represent the oldest galaxies known. UFDs are almost devoid of baryonic matter. (2) Calculations show that there can be super-sonic flow of baryonic matter. It follows that there are ubiquitous shockwaves; commonly oblique they generate vorticity. (3) Early virialized clumps, mini-halos, have a density that is consistent with the density implied by Super Massive Black Holes (SMBHs) today, if we assume that SMBHs grow by merging, akin to the Press & Schechter (1974) picture for galaxies. This implies that the oldest SMBHs observed today give powerful constraints on the very early phases.

Keywords: dark matter; ultra faint dwarf galaxies; early shock waves; super-massive black holes

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1. Introduction

Dark matter (DM) has been with us since Zwicky noted its presence in clusters of galaxies [1,2]. Nowadays due to extensive work using X-rays to probe the potential well of galaxies, groups and clusters (e.g., [3–7]), as well as the work on rotation curves of disk galaxies [8–13], we know that DM is ubiquitous, and far more abundant than baryonic matter. Its density ρ_{DM} in galaxies follows to a fair approximation the isothermal gas sphere law by Emden [14] of $\rho_{DM} \sim r^{-2}$ for large distances, where r is the radial distance. This then gives immediately the flat rotation curves observed, showing that DM reaches out far further than the baryonic matter, that is traced by hot and cold gas, stars, while both kinds of matter are shown by gravitational lensing. Today the strongest constraints on DM derive from Micro Wave Back Ground (MWBG) observations and of its angular structure and polarization on the sky [15]; all data are consistent with a Cold Dark Matter (CDM) approach, but do not exclude a Warm Dark Matter (WDM) model with a judicious choice of parameters. In addition, galaxy correlations add further support for the Planck results and their interpretation. However, we still do not know what DM actually is. There are a number of coincidences that speak for a massive neutral particle, or an extremely light particle, and others, that suggest a neutral Fermi particle: in one hypothesis this is a right-handed neutrino of a few keV, commonly referred to as Warm Dark Matter (WDM). The WDM argument has used galaxy structure arguments showing correlations between galaxies and their inner DM halo, which compete with the complexity of baryonic matter cooling (see, e.g., [16]), which might yield the same observed properties (e.g., [17–19]). To quote Nadler et al. [20]: "Non-CDM physics suppresses the linear matter power spectrum on small scales (their Figure 1, left panel), which manifests as an underabundance of subhalos (their Figure 1, middle panel) and faint MW satellite galaxies (their Figure 1, right panel) relative to CDM predictions". For any proposed WDM model it needs to be shown that it can reproduce both the MWBG fluctuations, as well as the smaller scale observations available, as was done by the Planck-collaboration [15]. Such a model also has to show how it can allow to interpret the Gaia data. In the following we assume that the WDM models used all have passed such a series of tests.

A full review of what the motivation is behind Warm Dark Matter (WDM) can be found in the 2017 White Paper [21], with useful work [22–24]. An updated review has been given by Paduroiu [25]. An extensive discussion of WDM galaxy models with and without central SMBHs has been given by de Vega & Sanchez [26,27]. A comparison with data was made in [28].

There are a number of astronomical observations that may allow to verify its nature, or falsify the various proposals.

Here we confine ourselves to three topics, since all three may lead to serious constraints on the property and nature of dark matter: (1) Ultra Faint Dwarf (UFD) galaxies: UFD galaxies are now recognized as the oldest stellar systems known. UFDs are almost devoid of baryonic matter, but are usually detected by the presence of a few stars moving together. (2) Simulations of early times after recombination, as well as simple quantitative estimates, both show supersonic flow of baryonic matter [29]. This implies shocks, with density jumps of up to a factor 4 or possibly even more. Post-shock densities of baryonic matter locally can be about the same density as DM, and so can decouple and form stellar systems devoid of DM. Is this connected to globular clusters? Most shocks are oblique, and oblique shocks generate vorticity, possibly related to rotation. (3) Simulations let us expect an abundance of early virialized clumps, also called mini-halos, around redshift 20, of mass a few $10^5\,M_\odot$ to about $10^7\,M_\odot$, all dominated by dark matter. They have a mass spectrum of approximately M^{-2} [30]: Their predicted density is consistent with the mass range and original density of SMBHs if they grow mostly by merging. That suggests that mini-halos could be the seed environment of many perhaps all SMBHs observed today. In our discussion of WDM the only property which we actually use is the possible degeneracy in its 6D phase space distribution.

Before we delve into these observations and their interpretation, we need to strongly emphasize one warning: No gravitational system is ever in thermodynamic equilibrium [31,32]. In a spherical stellar system for instance the density contrast slowly increases with time, until it hits a threshold, when a gravothermal catastrophe takes over, collapsing the central region, and ejecting the rest [33,34]. In observed globular stellar clusters this clearly has not happened yet, and in larger stellar systems like elliptical galaxies the time-scale of this process is orders of magnitude larger than the Hubble-Lemaitre time. In large disk galaxies interactions between molecular clouds and stars drive the very slow evolution [35], also on time scales of orders of magnitude beyond the Hubble-Lemaitre time.

We do observe old and young stars and stellar systems, out to a relatively high redshift today, so clearly in the early universe many of the processes driving stellar systems, with or without dark matter (DM), were faster than the Hubble-Lemaitre time. De Propris et al. [7] noted that even DM may not be in equilibrium yet around clusters of galaxies.

2. What Galaxies and Their Central SMBHs Tell Us

There is ample evidence how galaxies and their central SMBHs change with time. The concept of continuous merging, especially of early Hubble type galaxies, argued about very early on, was quantified by Press & Schechter [36,37]. Now we also know that the galaxy types are more complex, with ultra-diffuse galaxies [38] and ultra faint dwarf galaxies [39]. SMBHs correlate well with the bulges of their galaxy hosts (e.g., [40–46]). We observe the merger of these SMBHs via the spin-flip of the two participating SMBH through the swiveling and precession of the associated powerful jets, as shown by, e.g., the radio galaxies M87 [47] and Cen A [48,49]. If the spin-flip observed via powerful relativistic jets swerving around is through a large angle it implies [50] that the two SMBHs are of similar mass. First of all this implies that SMBHs can grow quite fast, as can their host galaxies. Radio data [51] show that every radio galaxy may have undergone a SMBH merger in cosmologically recent times, with an associated galaxy merger; this suggests that they underwent repeated mergers throughout their evolution (see, e.g., [44]). Andrade-

Santos et al. [52] showed that in some cases one can discern a direct sequence of mergers. All this suggests that galaxies grow with their central SMBHs, but it does not say how this relation got started. It is consistent with the notion, that as soon as the host galaxy baryonic stellar bulge and perhaps the DM halo pass a certain mass threshold an initial SMBH forms, and then grows with the galaxy. If this is true, then the extreme mass SMBH in galaxies such as in M87 formed the earliest, and SMBHs such as in our own Milky Way later. This suggests a top-down structure formation initially (see, e.g., [22–25]), and a bottom-up phase more recently (e.g., [30]), as already proposed by Zeldovich [53].

Through the detailed Gaia results the formation of our own Galaxy can now be studied star by star in very large numbers [54]. These data show how our Galaxy formed out of many dwarf galaxies [55–57].

3. Ultra Faint Dwarf (UFD) Galaxies

The oldest galaxies now known are the Ultra Faint Dwarf Galaxies (UFDs) [39]. They are almost devoid of baryonic matter, showing their existence only through a few old stars, all moving together, often with an extremely low metal abundance.

Under the hypothesis that the WDM particle is a keV Fermion, there is a minimum mass of clumps in the clump spectrum (see, e.g., [30]). In the observed clump size or galaxy mass distribution there should be a scarcity at very low masses, allowing to differentiate between CDM and WDM. This distribution has now been observed and used to determine a lower limit to the WDM particle mass, and the result was 6.5 keV [20]. However, there are quite a few selection effects in determining this distribution, since often very few stars in each such galaxy are well measured, and the discovery rate has not plateaued yet.

Modeling galaxies using as WDM a keV Fermion particle, it is found that the center can be Fermi degenerate; so its structure is referred to as a core, never a cusp with a divergent density distribution. If a WDM structure is degenerate, then an immersed stellar mass black hole (BH) will grow in a rapid cascade [58,59] to the scale of the degenerate configuration. That uses the full occupation number of WDM at the lowest momenta. If the stellar mass BH moves at a speed above the Fermi level of the Fermion sea, then little or nor accretion can be expected. If the momentum phase space of the background WDM Fermi sea is not full, then we also expect little or no accretion. This can define a lower limit to the WDM Fermion mass. That implies that in case, where we observe evidence of past formation of massive stars, so clearly the formation of stellar mass BHs (e.g., [60,61]), either of these two conditions must hold, or give that mass limit consistently on the basis of different galaxies. This leads to the series of models of de Vega & Sanchez [27], that show that for small galaxy masses dark-matter dominated galaxy models are possible without a central black hole, and for larger galaxy masses a central black hole is required; this is qualitatively just what is observed. Since the SMBH in our Galactic center is at the low end of the mass distribution of SMBHs, it follows, that our Galaxy must be close to the boundary between galaxies without a central SMBH and those with a central SMBH.

If the WDM is in addition a sterile neutrino [62] then runaway cooling from molecular Hydrogen formation is possible in WDM clumps, allowing star formation as early as redshift 100. However, this presupposes such a clump to have formed already, at some mass scale. There are not many simulations beyond redshift 50 to assess such possibilities. This can be tested with X-ray observations, and possibly with observing molecular Hydrogen H_2 lines as well as HD lines.

Therefore we have the following distinction (see, e.g., [27]): In CDM models the central density of merged galaxies—i.e., all of them—is predicted to diverge as $1/r$ [63,64], where r is the distance to the center. In WDM models the density never diverges on small radial scales, unless there is a central black hole. Therefore, the observation is difficult but straightforward, as a first step: If every time a divergent density is observed, a central black hole is present, and also every time a black hole is present, the density diverges, then it is inconsistent with the NFW models, but fully consistent with the WDM models. The second step is to determine in all galaxies without a central black hole, what the central density actually is. If is consistent with a degenerate configuration with the same Fermion mass every time (e.g., [59]), then we have evidence for WDM. The disk galaxy M33 could be a great test; it is a large disk galaxy without a known central SMBH.

The data listed in Simon's review of the faintest galaxies [39] allow to go one step further. In the supplementary material of this review there is a table with all Ultra Faint Dwarf (UFD) galaxies plus other dwarf galaxies well measured at the time, e.g., [65]. Many of them have scarce data, and so any averaged quantity has large error bars:

From this table we now select all dwarf galaxies simultaneously fulfilling four conditions: (i) Error in the half-light radius $R_{1/2}$ smaller than 1/5 the quantity, so $R_{1/2}/(\Delta R_{1/2}) > 5$. (ii) Similarly the error in the velocity dispersion σ should be less than 1/5 of quantity itself, so $\sigma/(\Delta\sigma) > 5$; alternatively we allow a well determined upper limit. (iii) The iron abundance should be low, below 10^{-2} of the Sun, or $[Fe/H] < -2$. (iv) The galaxies should be fainter than −7.7 in absolute visual magnitude M_V, so as to be considered as UFD galaxies [39]. As a result we obtain a subsample of eight dwarf galaxies, with up to seven references for the data for each galaxy given in Simon's [39] listing. Allowing brighter dwarf galaxies, but keeping the first three conditions expands the sample to ten dwarf galaxies. Plotting then $R_{1/2}$ versus σ reveals that all ten galaxies are consistent with a single profile, with $R_{1/2} = const.\sigma^2$, with the same value of the *const.* for all ten; as shown in [59] this is consistent with two properties: (1) All these dwarf galaxies have merged already, and so are consistent with an inner NFW profile [64] on the radial scale observed. It is important to note that for the Draco dwarf galaxy, the second brightest of the ten dwarf galaxies, this profile could be determined, and it is in fact the cuspy NFW profile [66]. Massari et al. [67] also found a NFW profile in the Sculptor dwarf galaxy, which is not in the sample of ten, because it is above the $[Fe/H] = -2$ cutoff limit used. But in $R_{1/2}$ and in σ it is in fact similar to the other two bright dwarf galaxies included here, Draco and Ursa Minor. Ursa Minor is the second bright dwarf galaxy in the sample of ten. Both, the Draco dwarf galaxy and the Ursa Minor dwarf galaxy have an Fe abundance close to the upper limit of the range considered, $[Fe/H] < -2$. These two dwarf galaxies also have the highest velocity dispersion of the sample of ten, 9.1 and 9.5 km/s, with the third highest velocity dispersion 7 km/s; Sculptor has 9.2 km/s. (2) All these dwarf galaxies are very similar and have a structure, that is just sampled very differently by the stars. This means that their central density obeys the condition, that the product of density times distance from the center—a constant with radius, a NFW-profile property—has the same value or a very similar value for all dwarf galaxies in the sample. This product is a measure of the column density of mass through the central part of the galaxy; these results imply that this measure of mass column density is the same for all these dwarf galaxies. So a challenge is for any galaxy formation process with plenty of dark matter, using CDM, WDM or any other approach: how to explain this extreme similarity of all these UFD galaxies, with a common specific column density that needs understanding.

Another hint at a correlation appears, when we plot absolute visual magnitude M_V versus metal abundance $[Fe/H]$: Up to the cutoff for UFD galaxies the metal abundance appears uncorrelated with absolute visual magnitude, and above a correlation appears consistent with the change in visual luminosity proportional to the metal abundance; this has a relatively large scatter. If these dwarf galaxies grow by merging, then every merger will generate a brief starburst. Star formation is enhanced with enhanced cooling by dust, depending on heavy element abundance. Enhanced metal abundance also allows more winds to be driven by massive stars, making recycling of material more important. These effects could produce such a correlation. If this is the explanation, the UFDs below the threshold might be in their original form, before any merger.

Since we can sample these dwarf galaxies only where there are stars, we have no information, how large these galaxies are; Simon (2019) noted that they could easily be of kpc scale, as some in fact are known to be. If the dark matter in these galaxies were in fact a Fermi particle of a few keV, then the Heisenberg condition (see [59]) allows to estimate, at which point the configuration becomes degenerate; that is far below a radius of order parsec, a radial range that has not been well observed. The mass of this degenerate configuration, at $10^{3.6}\,M_\odot$ (using the expression in [59]) exceeds that of observed very massive stellar mass black holes (see [68–71]). Once these UFD galaxies merge any further, their velocity dispersion will increase, and this degenerate core would increase correspondingly. In fact, considering those dwarf galaxies with higher metal abundance their typical velocity dispersion distribution reaches out to larger values. If these UFD galaxies correspond to the earliest virialized clumps, then they should have an overall differential mass spectrum of approximately M_{cl}^{-2}. Using the case of UFD galaxies in the center of the diagram, as reference, that is a half light radius $R_{1/2} = 100\,pc$ and a velocity dispersion $\sigma = 4\,\mathrm{km/s}$ their mass is $M_{UFD} = 10^{5.6}\,M_\odot\,(R_{1/2}/\{100\ pc\})^2$, but of course only up to this radius. This mass varies among the ten observed UFD galaxies in the sample selected by a factor of 2.5^2 down, and a factor of 4^2 up from this reference number; the masses just of the UFD galaxies over the radial range detected span a factor of 100.

The observed SMBH mass spectrum runs as M_{BH}^{-2} just as the early virialized clumps, and ranges from a bit below $10^7\,M_\odot$ to a bit above $10^8\,M_\odot$ [72], with a steeper spectrum beyond reaching near to $10^{10}\,M_\odot$. Therefore if the degenerate cores of the UFD galaxies as observed were to turn into SMBHs, with some more merging they might reproduce both the lower range of masses of the SMBHs observed today, up to about $10^8\,M_\odot$, with some accretion or yet more merging. Merging of these initial SMBHs could result in the observed spectrum, across all masses (see, e.g., [73]). To obtain more precise and specific numbers requires a full-scale modeling similar to what King [74] did for globular clusters, but then allowing for the possibility of degeneracy, including both stars and DM particles separately. One should allow for a non-isotropic momentum phase space distribution (e.g., [66,67,75]). As these galaxies are thought to be the observed galaxies closest to the first structures, it is noteworthy, that these ten are consistent with being sampled from the earliest predicted virialized clumps. That is plausible in a picture, in which there is a minimum clump size scale, and therefore a peak in the distribution (see also [30]). These galaxies of very low metal abundance may represent that peak. Confining ourselves to the metal poorest galaxies among the UFD galaxies we may have identified a sample of really old dwarf galaxies, possibly before any merger.

This implies that we could differentiate between WDM and CDM by observing the central distribution of the density of stars, and their velocities: If the stars follow a divergent power law all the way to the center, and if some stars show excessive velocities, there is a central BH. If there is a core, so a convergent density towards the center then there is no central BH, and the DM distribution could be modeled by a Fermion of order keV. A key question is in this case, whether data of all galaxies give the same Fermion mass. The data for UFDs are not clearly pointing to one or the other option yet. All those UFDs showing a cusp profile down to far below parsec scale should contain a central BH. This is a clear test, albeit very difficult.

4. Supersonic Flow and Shockwaves

In the early universe molecular Hydrogen (i.e., H_2 and HD both) is the main cooling agent ahead of dust emission which requires a sufficient abundance of heavy elements. The criterion to understand when this cooling leads to star formation is when the time-scale of cooling is shorter than the local Hubble-Lemaitre time. Stars then form, but at the time-scale of the universe evolving, since that is the free-fall time [76]. When super-sonic flow is present [77–81], we will have shock waves, and with it the enhanced cooling, as in a strong shock the density is increased by a factor of 4 (for an adiabatic index of 5/3), and the temperature is also raised. Since the cooling runs essentially as the density of molecular Hydrogen, to be compared with the heat content running also with density, the immediate cooling time-scale is not changed. However, the increased density leads to new molecular Hydrogen formation, and that leads to increased cooling. If we are near the peak of the cooling curve, the combined effect of increased density and increased temperature will drastically increase molecular Hydrogen formation (see also [62]). A consequence is that star formation will proceed on a much shorter time scale than the Hubble-Lemaitre time; star formation then can be a run-away process. The combined conditions on post-shock temperature and density puts the optimum redshift about a factor of 2.5 below 80, so about 30, to reach the peak of molecular Hydrogen formation. This is in fact consistent with other arguments (see, e.g., [30]).

This implies that post-shock the baryonic density is about the same as the DM density, possibly even more. If in these supersonic flows there is shear, as likely, then any magnetic seed field will be enhanced, and particle acceleration is possible. Similarly, if the shock is turbulent, magnetic fields can be enhanced (e.g., [82]). One consequence could be that the effective adiabatic gas index is lowered from 5/3 towards 4/3; this has as a consequence that the density jump in a shock is larger: this allows a density jump approaching 4 already for modest Mach numbers, and a density jump approaching 7 for large Mach numbers. In a shockwave the free-fall time scale is halved (for an adiabatic gas constant of 5/3; for a relativistic adiabatic gas constant of 4/3 the speeding of free fall is a factor of $\sqrt{7} \simeq 2.6$). That means it is the only mechanism to speed collapse. This is likely to entail DM free clump formation, so purely baryonic collapse very early in the universe. A solution could be that upon the decoupling of the baryon temperature from the MWBG temperature the baryon temperature is cooled down by molecular Hydrogen emission to a factor of 2 below the MWBG temperature. This would have the consequence that the shock waves are quite strong, enabling density jumps of a factor of 4, and so decoupling from the DM fluctuations.

Consider a DM clump of mass $10^7\, M_\odot$ virialized at redshift 30: The original scale is $R = 10^{23.2}\,(1+z)^{-1}$ cm. This scale has contracted to a scale about $(200)^{1/2}$ smaller upon virialization, in a time given by $t = 10^{17.7}\,(1+z)^{-3/2}$. The graphs shown in Tegmark et al. [30] about the virialization of clumps show that a clump virialized at redshift z stars its serious contraction at redshift about $3z/2$; this means that effective contraction time is about a factor of 2 shorter than the local Hubble-Lemaitre time. That means that the effective speed is about $10^{5.8}\,(1+z)^{1/2}$ cm/s, which is at that redshift $z = 30$ then $v = 10^{6.4}$ cm/s. The speed of sound is given by $c_s^2 = \gamma\,\frac{2\,n\,k_B\,T}{n\,m_p}$. This is $c_s = 10^{4.3}\,T^{1/2}$ cm/s, which is at redshift $z = 30$ then $10^{5.3}$ cm/s. This implies a contracting super-sonic flow, which at some radius will lead to a standing shock wave. We note that clusters of galaxies have an accretion standing shockwave today, as may our Galaxy in addition to a standing shock of a Galactic wind - the latter in the polar region and the other in an equatorial belt region. There are observations suggesting that we have detected the accretion shock to a large scale structure filament [83]. That implies the Mach-number M of the contracting flow is about $M = 10$. The expression $1/M^2 \simeq 1/100$ enters the formula for the density jump, and at this Mach-number we are in the strong shock regime, when the density jump becomes independent of Mach-number. That implies that a standing shock wave would have a density jump of order 4. For clumps of another mass, virialized at a different redshift (higher mass at lower redshift), the situation may be similar.

This leads to another test: The location of birth of DM-free gravitational systems, i.e., globular clusters of stars, should all be outside much heavier gravitational systems, which form later (see [84]). That suggests that globular clusters may not be quite as old as the oldest DM-dominated gravitational systems. Since in a WDM model, in which the WDM-particle is a sterile neutrino, molecular Hydrogen formation is spurred on [62] and so early star formation, Pop III stars may be present in the oldest and smallest DM-dominated gravitational systems.

There is another effect of shock-waves: Most shock-wavs are oblique, sometimes to the point of getting super-sonic flow on both sides of the shock. Of course, then the density jump in the shock can be highly reduced, possibly to the point of being negligible, i.e., 1:1. But one key aspect of oblique shock-waves, however, even slightly oblique, is that they produce vorticity (see the original paper [85], the review [86], and also related questions on magnetic helicity [87]). That implies that any structure that started to form pre-shock, and then fully develops post-shock, rotates. That translates to parallel spin for all such structures along such a shock. That is a different source of vorticity than tidal forces. If tidal forces are the source of vorticity, then paired galaxies ought to have opposite spin. If oblique shock are the source of vorticity then neighboring galaxies ought to have parallel spin.

The mass density of dark matter is about 4 times the baryon density, so in mass density $\sim 4 \times 10^{-6.7}\, m_p\, \mathrm{g\,cm}^{-3}\, (1+z)^3$, which translates to number density $\sim\; 10^{-0.6}\, m_{3\,keV}^{-1}\, \mathrm{cm}^{-3}\, (1+z)^3$ if we assume a WDM Fermion of 3 keV. The property of being a Fermion in turn implies that the velocity must be larger the Heisenberg limit $v_{3\,keV} \;=\; 10^{2.1}\, f\, m_{3\,keV}^{-4/3}\, \mathrm{cm\,s}^{-1}\, (1+z)$, where $f \;\geq\; 1$. $f \;=\; 1$ reproduces the limit given by the Heisenberg uncertainty principle.

5. The Early Mini-Halos

The original density of mini-halos [88] is consistent with the inferred density of the original density of Super Massive Black Holes (SMBHs), if they all grow mostly by merging, see Table 1. That implies, that in any model of DM, we have to explain the numbers. In mini-halos we have the original choices to make a first generation super-massive black hole [32], (i) of a gravothermal catastrophe of the stellar distribution, with the core collapsing to a SMBH, (ii) a direct collapse of a baryonic cloud to form a SMBH, (iii) a merger runaway of the final BHs. (iv) If mini-halos are Fermi-degenerate then of course a single stellar mass BH can eat all the degenerate matter, and grow to a SMBH.

5.1. Virialized Clump Statistics

Some helpful numbers from standard simulations are in Table 1:

Table 1. Virialized clump statistics from standard modeling. The density is referred to today as comoving.

Density n (in Mpc^{-3})	**Redshift z**	**Mass M (in $M_\odot$)**
10^{0}	20	$10^{6.9}$
10^{-1}	20	$10^{7.4}$
10^{0}	30	$10^{5.2}$
10^{-1}	30	$10^{5.7}$
10^{-2}	30	$10^{6.1}$

These statistics show: (i) Clumps grow in mass between redshifts 30 and 20 by a factor of about 50 in mass. Towards today they grow much further. (ii) Their mass spectrum corresponds to density proportional to about M^{-2}. A density of today of $10^{-2}\, Mpc^{-3}$ corresponds at redshift 30 to a clump of $10^{6.1}\, M_\odot$, and at redshift 20 to a clump of about $10^{7.8}\, M_\odot$. These virialized clumps are often referred to as mini-halos.

How our Galaxy grew by merging has now been studied with Gaia data (e.g., [55,57,84,89]), with a review by Helmi [54]. The examples of the Sculptor and Draco dwarf galaxy show how even some early galaxies grew by merging [66,67].

5.2. The First Super-Massive Black Holes (SMBHs)

Caramete & Biermann [72] determined the density of Super Massive Black Holes (SMBHs) conservatively to $10^{-2.2 \pm 0.4}\, (\{M_{BH}\}/\{10^{7}\, M_\odot\})^{-1}$, a cumulative density, consistent with other determinations. The SMBH at the Galactic Center at $10^{6.6}\, M_\odot$ [90] is consistent with the low cut-off in the distribution. There are three ways to get to today's observed distribution: (i) All SMBHs start from virialized clumps taking all their total mass, at today's mass distribution, and accretion modifies it only insignificantly. Their mass distribution is consistent with such an idea, as it is also about M_{BH}^{-2}, up to about $10^{8}\, M_\odot$, steeper for higher masses. (ii) Today's SMBHs form at the center of virialized clumps that were already much larger in mass, just as already some dwarf galaxies today exceed most of today's SMBHs in mass. (iii) They grow from the very lowest SMBH mass of all, i.e., about $10^{6.6}\, M_\odot$, mostly by merging: This yields a reference density of $10^{-1.6 \pm 0.4}\, Mpc^{-3}$ [72,91]; obviously, if some of their growth derives from accretion, then the original number could be lower, in extremis down to today's number, $10^{-2.2 \pm 0.4}\, Mpc^{-3}$. Since the determination by [72] was conservative, leaving out many SMBH candidates in early Hubble type galaxies, it follows that a large fraction of mini-halos may be required as the seeds of the first generation of SMBHs.

The question is what physical process defines this lowest mass, a mass consistent with today's mass of the Galactic Center SMBH, $10^{6.6}\ M_{\odot}$.

So under the assumption that SMBHs grow mostly via merging, their original inferred density was $10^{-1.6\pm0.4}\ Mpc^{-3}$, using $10^{7}\ M_{\odot}$ as reference. Extrapolating the numbers above this matches in density at redshift 20 clump masses of $10^{7.3\pm0.2}\ M_{\odot}$, so matches also in mass to within the uncertainties. Going to redshift 30 the density matches a mass of $10^{5.9}\ M_{\odot}$, so would require some further growth, suggesting that this phase of growth is just the natural accretion of DM and baryonic mass of any virialized clump, plus any early merging with smaller clumps. One question is whether the formation of the first SMBH stops further accretion, so that the SMBH has the possibility of staying at its mass. Today's statistics show that there are many SMBHs in the mass range close to our Galactic Center black hole, so at whatever stage they formed, they must have stopped growing right away.

The question we explore here is whether the process from virialization of a mini-halo to a dwarf galaxy or SMBH is helped along by Warm Dark Matter (WDM), because WDM can start a run-away cooling and star formation already at high redshift [62].

5.3. *The Gravothermal Catastrophe*

In any system of particles, black holes or stars, that are just governed by gravitation, the isothermal sphere is a basic configuration (Emden [14]). When such a system evolves, it moves to a higher density contrast between average density and central density [32–34,92–94]. As this contrast increases a threshold is reached, when a sudden collapse becomes imminent, the gravothermal catastrophe, with an ejection of a large fraction of particles in the system. It is not clear whether self-gravitating systems can naturally be formed close to this threshold of stability. But if that were to happen, then the time-scale of change and collapse could be quite short. This model does not naturally give the mass of the observed low mass limit of SMBHs, around the mass of our Galactic Center SMBH, $4.2 \cdot 10^{6}\ M_{\odot}$.

5.4. *The Loss-Cone-Mechanism*

Hills and others [95–97] have shown that stars can get eaten by central black holes in galaxies, driven by interactions with giant molecular clouds for stars, and by stars for DM particles. This mechanism uses the fact, that particles that get lost in an accreting black hole are eliminated from momentum phase space, forming a "loss cone". This loss cone can be refilled by interactions with molecular clouds [35,98]. This mechanism leads to a radial power-law distribution of the density of stars, now observed [99] in the Galactic Center. This process is very slow. However, under proper circumstances with many massive molecular clouds it could be fast.

5.5. *Direct Collapse to a Super-Massive Star SM**

The baryonic gas just cools and collapses directly into a Super-Massive star (SM*: [32,100]). This SM* then can accrete and grow some more, but finally explodes and becomes a BH. Here that first star's evolution time scale could be the dominating factor. For a massive star that is a few 10^{6} yrs. This process gives a direct mass, of order $10^{6}\ M_{\odot}$. This is one process that clearly gives this specific mass, and this is the process used in [91].

5.5.1. Agglomeration of Stars

In this option the baryonic gas forms many massive stars, that agglomerate to a SM* [32,101]. This SM* forms a black hole, with a mass of around $10^{6}\ M_{\odot}$. This mass is given by the two factors of an instability given General Relativity, and an equation of state close to the radiation limit of an adiabatic gas constant of $4/3$. This process needs the evolution time scale of the original massive stars, then the agglomeration, which could proceed under the gravothermal catastrophe limit, so be quite fast, and then the evolutionary time scale of the SM*. This then finally gives a SMBH, just as above.

5.5.2. Baryonic Accretion

In all options above we could replace the DM accretion with baryonic accretion. In such a case the final SMBH would be mostly built from baryonic matter, and not DM. Accretion at the Eddington limit is quite slow. However, accretion might be faster than this limit [102].

5.6. Direct Growth of a Stellar Mass BH to a SMBH from Quantum Lattice

If the DM is WDM and obeys Fermi statistics, it may be in a quantum lattice, that depletes in a run-away into a stellar scale BH [58,59]. This way the BH can grow to the degenerate core of the mini-halo, and so become a SMBH. For Fermi particles this step can be quite fast. Here that first star's evolution time scale could be the dominating factor. For a massive star that is a few 10^6 yrs. This process would give a range of masses, matching the lower masses of SMBHs, and would be quite rapid.

There has now been a discovery shedding light on such an option in the Sculptor dwarf galaxy by Skúladóttir et al. [89], who could determine the occurrence of a SN-explosion of a star at almost zero metal abundance of originally 20 $M_\odot$. At such a low metal abundance such a SN-explosion probably yielded a black hole [60,61], and so a candidate to eat all the degenerate WDM, if DM has this form. If that stellar mass black hole formed, was it kicked out in the explosion, or is it still there? Is its motion low with respect to the stars? If so, it could be inside that region, that is degenerate in WDM models.

5.7. Comparison

As the list above shows, there are two ways to be quite fast, and also give the right mass scale: (i) Form a SM* and out of it a SMBH. This process gives a specific mass, which may depend on metallicity. (ii) For a stellar mass BH, that eats all of Fermi-full quantum lattice of a WDM dominated UFD galaxy, modeled as an isothermal sphere. This process gives the right mass range, and is independent of metallicity.

6. Conclusions for WDM

Ultra-faint dwarf (UFDs) galaxies are the oldest observed galaxies; we concentrate on those reasonably well observed, and with very low metal abundance: we note that those galaxies are consistent with being all very similar. The difference between these UFD galaxies is probably well described by a range of velocity dispersions and masses, so scales.

WDM helps in several ways to speed evolution up: First, in shocks it can increase the production of molecular Hydrogen, and thereby initiate star formation. Second, WDM can form a quantum lattice, that can feed an inside black hole rapidly, until the black hole has taken all available WDM. However, if the configurations of stars and black holes are generated close to the gravothermal catastrophe limit, then black hole formation can be quite fast, independent of what the nature of DM is. In this case CDM accretion is quite slow, and baryonic accretion would need an accretion disk to be reasonably fast. One of the fastest paths to a SMBH is the original degenerate WDM core of a mini-halo, made up of mostly dark matter, and all eaten up by a stellar black hole. Third, WDM with its top-down structure formation can yield the starting conditions for the later bottom-up formation stages well studied, that allows galaxies with super-massive black holes such as M87 to begin their evolution early on, and modest scale galaxies such as our Milky Way later. What Gaia observes all relates to such a later stage.

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