

# Double Hawking Temperature: From Black Hole to de Sitter

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**Abstract:** The double Hawking temperature  $T = 2T_H$  appears in some approaches to the Hawking radiation when the radiation is considered in terms of the quantum tunneling. We consider the origin of such unusual temperature for the black hole horizon and also for the cosmological horizon in de Sitter spacetime. In the case of the black hole horizon, there are two contributions to the tunneling process of radiation, each being governed by the temperature  $T = 2T_H$ . These processes are coherently combined to produce the radiation with the Hawking temperature  $T_H$ . This can be traditionally interpreted as the pair creation of two entangled particles, of which one goes towards the center of the black hole, while the other one escapes from the black hole. In the case of the cosmological horizon, the temperature  $T = 2T_H$  is physical. While the creation of the entangled pair is described by the Hawking temperature, the de Sitter spacetime allows for another process, in which only a single (non-entangled) particle inside the cosmological horizon is created. This process is characterized by the local temperature  $T = 2T_H$ . The local single-particle process also takes place outside the black hole horizon, but it is exponentially suppressed.

**Keywords:** black hole; white hole; de Sitter spacetime; Hawking radiation; Painleve–Gullstrand coordinates; quantum tunneling



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## 1. Introduction

The problem of the doubling of the Hawking temperature of black holes was already discussed in 1984 [1] (see recent continuation of this discussion in Ref. [2]). The double Hawking temperature  $2T_H$  also appears in some approaches to the Hawking radiation from the black hole and cosmological horizons, where the process of Hawking radiation is considered as quantum tunneling. In the case of quantum tunneling from the black hole horizon, the  $2T_H$  can be attributed to the improper choice of the reference frame. The static Schwarzschild coordinates have the nonphysical coordinate singularity at the horizon, which influences the result. The choice of the Painleve–Gullstrand coordinate system, which is valid both inside and outside the horizon and has no singularity at the horizon, gives the Hawking temperature  $T_H$  [3,4]. In the de Sitter Universe, the situation is different; the temperature  $T = 2T_H$  is physical. It is the local temperature experienced by matter well inside the cosmological horizon [5]. We discuss the possible connections between these two manifestations of the double Hawking temperature.

## 2. $2T_H$ Problem for Black Holes

The quantum tunneling approach allows us to study different processes without considering the details of the microscopic physics or quantum field theory. The Hawking radiation from the black hole [6] provides an example when the complicated process is highly simplified by the method of quantum tunneling [3–5], which does not require the consideration based on quantum field theories. The Hawking temperature is obtained when the tunneling rate is compared to the Boltzmann factor.

In this approach, the Painleve–Gullstrand (PG) coordinate system [7,8] is used with the metric:

$$ds^2 = -dt^2(1 - \mathbf{v}^2) - 2dt \mathbf{dr} \cdot \mathbf{v} + d\mathbf{r}^2, \quad (1)$$

where  $v^2(r) = R/r$ ;  $R = 2M$  is the position of the black hole horizon ( $G = c = \hbar = 1$ ); the contracting and expanding velocities  $\mathbf{v}$  correspond to the black hole and white hole correspondingly (for extension of the PG metric to the black and white holes with several horizons, see Ref. [9]). This PG metric is the stationary (but not static) metric, which does not have singularity at the horizon. This allows for analytic considerations both inside and outside the horizon and also allows for the extension to the complex plane. The latter is important for calculations of the tunneling exponent by the analytic continuation of the action to the complex plane.

For the black hole realization, this procedure gives the tunneling exponent corresponding to the Hawking temperature  $T_H = 1/8\pi M$ . The process of Hawking radiation can be interpreted as the pair production of two entangled particles, of which one goes towards the center of the black hole, while the other one escapes from the horizon. Let us stress that Hawking radiation perceived by observers generally depends on the trajectory of the observer, see, e.g., [10]. In this section, we consider the Hawking temperature measured by the static observer at  $r \rightarrow \infty$ .

However, some calculations of the Hawking radiation, including the quantum tunneling approach, lead to the double Hawking problem [1,2,11–13], where the obtained radiation is thermal but with temperature  $T = 2T_H$ . We consider this problem in the semiclassical tunneling approach using the Klein–Gordon equation for a massive field in a curved background [11], which leads to the relativistic Hamilton–Jacobi equation for the classical action:

$$g^{\mu\nu} \partial_\mu S \partial_\nu S + m^2 = 0. \tag{2}$$

Let us start with the PG metric in Equation (1) since it does not have coordinate singularity at the horizon. One has for the fixed energy  $E$  [11]:

$$-E^2 + (1 - v^2) \left( \frac{dS}{dr} \right)^2 + 2vE \frac{dS}{dr} + m^2 = 0. \tag{3}$$

For the classical action on two different trajectories of the propagating massive particle, this gives:

$$S = - \int dr \frac{Ev}{1 - v^2} \pm \int \frac{dr}{1 - v^2} \sqrt{E^2 - m^2(1 - v^2)}. \tag{4}$$

For the plus sign in the second term, the imaginary parts of the two terms cancel each other. This corresponds to the incoming trajectory of the particle, which enters the black hole and moves toward the black hole singularity at  $r = 0$ . For the minus sign in the second term, one obtains the tunneling exponent describing the radiation of a particle from the region inside the black hole to the region outside the black hole. The combination of two terms in Equation (4) describes the Hawking radiation with the Hawking temperature  $T_H$ :

$$\exp(-2 \text{Im} S) = \exp\left(-\frac{E}{2T_H}\right) \exp\left(-\frac{E}{2T_H}\right) = \exp\left(-\frac{E}{T_H}\right). \tag{5}$$

Note that each of the two terms in Equation (5) corresponds to the effective temperature  $2T_H$ . That is why if one of the two terms is lost in calculations, the temperature  $2T_H$  erroneously emerges. A similar product of each of the terms with  $2T_H$  has also been obtained for the black holes with several horizons [14].

In Ref. [11], the Schwarzschild coordinates were also used (see also Ref. [15]):

$$ds^2 = -\left(1 - \frac{R}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{R}{r}} + r^2 d\Omega^2. \tag{6}$$

This gives only the second term in Equation (4), and as a result, the  $2T_H$  temperature is obtained. In Refs. [12,15], it was argued that the Hawking temperature can be restored by considering the balance between the emission  $p_{\text{em}} \propto \exp(-E/2T_H)$ , which comes from

the minus sign, and the absorption  $p_{\text{abs}} \propto \exp(E/2T_H)$ , which comes from the plus sign. Then, the ratio  $p_{\text{em}}/p_{\text{abs}}$  gives the Hawking temperature.

However, the consideration of the exponentially large absorption is somewhat unnatural. The first term in Equation (4) can be naturally restored if one takes into account the coordinate singularity of the Schwarzschild metric at the horizon. To avoid this singularity, one should make the transformation from the Schwarzschild coordinates to the nonsingular PG coordinates:

$$dt \rightarrow d\tilde{t} + dr \frac{v}{1-v^2}. \tag{7}$$

Now, we must take into account that the action also contains the  $\int Edt$  term:

$$S = - \int Edt \pm \int dr \frac{1}{1-v^2} \sqrt{E^2 - m^2(1-v^2)}, \tag{8}$$

and the transformation of this term gives the missing tunneling exponent:

$$\begin{aligned} \exp\left(-2\text{Im} \int Edt\right) &= \exp\left(-2\text{Im} \int \left(Ed\tilde{t} + dr \frac{Ev}{1-v^2}\right)\right) \\ &= \exp\left(-2\text{Im} \int dr \frac{Ev}{1-v^2}\right) = \exp(-2\pi ER) = \exp\left(-\frac{E}{2T_H}\right). \end{aligned} \tag{9}$$

Thus, the total tunneling exponent again contains the product of two exponents:

$$\exp(-2\text{Im} S) = \exp\left(-\frac{E}{2T_H}\right) \exp\left(\pm \frac{E}{2T_H}\right). \tag{10}$$

The minus sign corresponds to the radiation from the PG black hole considered in the Schwarzschild coordinates, and it gives the Hawking radiation with  $T = T_H$ .

In this approach, Equation (10) also contains two contributions, each corresponding to the effective temperature  $2T_H$ . However, the absorption is now  $p_{\text{abs}} \propto \exp(-E/2T_H) \exp(E/2T_H)$ , and the emission is now  $p_{\text{abs}} \propto \exp(-E/2T_H) \exp(-E/2T_H)$ . Their ratio naturally reflects the detailed balance principle for the black hole.

### 3. $2T_H$ Problem for White Holes

According to Refs. [16–18], the hole object can be in three different states: the PG black hole, the PG white hole and the intermediate state—the neutral, fully static object described by Schwarzschild coordinates. These three states correspond to different quantum vacua, which are determined by different global coordinate systems. These states can be obtained from each other by singular coordinate transformations between the global coordinate systems. For example, the transformation from the black hole to the white hole has the following form:

$$dt \rightarrow d\tilde{t} + 2dr \frac{v}{1-v^2}. \tag{11}$$

Earlier, we used the coordinate transformation in Equation (7) and applied it for the transformation of the particle action in Equation (9). Now, we can apply such singular transformations to macroscopic objects: the black, white and neutral holes. This can be used for calculating the macroscopic tunneling transitions between these objects and for calculating the entropy of each macro-object [16–18].

For example, the probability of the quantum tunneling from the black hole to the white hole with the same mass  $M$  is given by the imaginary part in the black hole action on the following path:

$$\begin{aligned}
 P_{\text{BH} \rightarrow \text{WH}} &= \exp\left(-2\text{Im} \int M dt\right) = \exp\left(-2\text{Im} \int \left(M d\tilde{t} + 2dr \frac{Mv}{1-v^2}\right)\right) \\
 &= \exp\left(-4\text{Im} \int dr \frac{Mv}{1-v^2}\right) = \exp(-4\pi MR) = \exp(-8\pi M^2 G) \tag{12}
 \end{aligned}$$

$$P_{\text{BH} \rightarrow \text{WH}} = \exp(-2S_{\text{BH}}) = \exp(-(S_{\text{BH}} - S_{\text{WH}})). \tag{13}$$

Here,  $S_{\text{BH}} = 4\pi M^2 G$  is the entropy of the black hole, and  $S_{\text{WH}} = -4\pi M^2 G$  is the entropy of the white hole with the same mass. The latter is obtained from Equation (13). Since the tunneling transition can be considered as quantum fluctuation, the exponent in the tunneling process can be expressed as the difference between the entropies of the initial state (black hole) and the final state (white hole) [19].

The white hole, which is obtained by the coordinate transformation from the black hole, has negative entropy and thus a negative temperature. The latter can be also obtained using the methods discussed in Section 2. Since, in the white hole, the shift velocity is opposite to that in the black hole, the first term in Equation (4) has the positive sign in the exponent. As a result, one obtains  $p_{\text{em}} \propto \exp(E/2T_H) \exp(E/2T_H)$  for emission and  $p_{\text{abs}} \propto \exp(E/2T_H) \exp(-E/2T_H)$  for absorption. Only the ratio of the two processes is physical. From this ratio  $p_{\text{em}}/p_{\text{abs}} \propto \exp(E/T_H)$  and from the detailed balance principle, one obtains that the temperature of the white hole is  $T_{\text{WH}} = -T_H$ . The temperature of the white hole is the temperature of the black hole with the same mass but with a minus sign, which is similar to the relation between their entropies.

Here is the main difference between the tunneling approach and the approach in Refs. [1,2]. While in the tunneling approach the black and white holes are considered as independent quantum states, in Refs. [1,2], they are combined in a single object, where the black and white parts represent two opposite coordinate patches of the Penrose diagram. In the tunneling approach, such an object can be compared with the fully static Schwarzschild solution in Equation (6), which represents the neutral object—the intermediate state between the black and white holes. This fully static object has zero entropy, and distinct from the black hole considered either in the Painleve–Gullstrand coordinates or in the static Schwarzschild coordinates, the neutral object does not radiate. If the analogy between the combined object and the neutral object is correct, then instead of  $T = 2T_H$ , one would have  $T = 0$ . However, this requires more detailed consideration.

#### 4. $2T_H$ Problem in the de Sitter Spacetime

Let us now go to the problem of the double-Hawking temperature in de Sitter spacetime. In the dS spacetime, one has  $v^2 = r^2 H^2$ , where  $H$  is the Hubble parameter, and the cosmological horizon is at  $R = 1/H$ :

$$ds^2 = -dt^2 + (d\mathbf{x} - H\mathbf{x} dt)^2. \tag{14}$$

The same procedure as in Section 2 for the PG black hole gives the Hawking radiation in Equation (5) with the Hawking temperature  $T_H = H/2\pi$ .

However, there are several arguments that there are additional processes characterized by the local temperature, which is twice the Hawking temperature,  $T_{\text{loc}} = H/\pi = 2T_H$  [5,18]. Such a temperature is experienced by matter well inside the horizon. This is supported in particular by calculations of the tunneling rate of the ionization of atoms in the de Sitter spacetime [5,20].

Let us consider an atom at the origin,  $r = 0$ , in the de Sitter spacetime. The atom is playing the role of the detector (or the role of the static observer) in this spacetime. The electron bounded to an atom absorbs the energy from the gravitational field of the de Sitter background and escapes from the electric potential barrier. If the ionization potential

is much smaller than the electron mass,  $H \ll \epsilon_0 \ll m$ , one can use the nonrelativistic quantum mechanics to estimate the tunneling rate through the barrier. The corresponding radial trajectory  $p_r(r)$  is obtained from the classical equation  $E(p_r, r) = -\epsilon_0$ , which is determined by the Doppler shift  $p_r v(r)$ , with  $v(r) = Hr$ :

$$-\epsilon_0 = \frac{p_r^2(r)}{2m} + p_r v(r), \quad v(r) = Hr, \tag{15}$$

$$p_r(r) = -mv(r) + \sqrt{m^2 v^2(r) - 2m\epsilon_0}. \tag{16}$$

The integral over the classically forbidden region  $0 < r < r_0 = \sqrt{2\epsilon_0/mH^2}$  gives the probability of ionization, which looks thermal with the double Hawking temperature:

$$\begin{aligned} \exp(-2 \operatorname{Im} S) &= \exp\left(-2mH \int_0^{r_0} \sqrt{r_0^2 - r^2}\right) \\ &= \exp\left(-\frac{\pi\epsilon_0}{H}\right) = \exp\left(-\frac{\epsilon_0}{2T_H}\right). \end{aligned} \tag{17}$$

The same local temperature describes the process of the splitting of the composite particle with mass  $m$  into two components with  $m_1 + m_2 > m$ , which is also not allowed in the Minkowski vacuum [5,21,22]. The probability of this process (for  $m \gg T_H$ ):

$$\Gamma(m \rightarrow m_1 + m_2) \sim \exp\left(-\frac{m_1 + m_2 - m}{2T_H}\right). \tag{18}$$

In particular, for  $m_1 = m_2 = m$ , the decay rate of a massive field in the de Sitter spacetime is obtained

$$\Gamma \sim \exp\left(-\frac{m}{2T_H}\right), \tag{19}$$

which is in agreement with Ref. [23].

In both cases, the tunneling processes are not related to the cosmological horizon; they are local processes. For example, in the case of the ionization of an atom, the electron trajectory is well inside the cosmological horizon:

$$r < r_0 = \frac{1}{H} \sqrt{\frac{2\epsilon_0}{m}} \ll \frac{1}{H}. \tag{20}$$

Another important feature of these two local processes is that they both violate the de Sitter symmetry since the atom, which is ionized, or the particle, which is split, are external objects. They do not belong to the de Sitter vacuum. The violation of symmetry can be explicitly seen from the symmetry of the de Sitter PG metric in Equation (14). This metric is invariant (symmetric) under the following shift in the coordinates:

$$\mathbf{r} \rightarrow \mathbf{r} + \mathbf{r}_c + H\mathbf{r}_c t. \tag{21}$$

However, the existence of the static observer (atom or detector) at  $\mathbf{r} = 0$  violates this symmetry because their/its position is not invariant under this transformation. This transformation shifts the position of the observer.

Note that the global process with  $T = T_H$  and the local process with  $T_{\text{loc}} = 2T_H$  take place in the same vacuum state: the state as seen by the observer at  $\mathbf{r} = 0$ . However, these are two different processes in this vacuum state. The global process, which corresponds to Hawking radiation with Hawking temperature  $T_H$ , does not depend on whether the observer at  $\mathbf{r} = 0$  exists or not. This Hawking radiation is the property of the symmetry

of the pure de Sitter vacuum. The local processes, which are characterized by the double Hawking temperature  $2T_H$ , only take place in the presence of the observer who violates the de Sitter symmetry.

The same phenomenon of local temperature  $T_{loc} = 2T_H$  can be obtained by considering the action (Equation (4)). Till now, we considered this action for the calculations of the imaginary part for particles with energy  $E > m$  on the trajectory in the complex plane, which connects the trajectory inside the horizon and the trajectory outside the horizon. This corresponds to the creation of two particles: one inside the horizon and another outside the horizon.

However, there is also the trajectory that allows for the creation of a single particle fully inside the cosmological horizon. In this creation from “nothing”, the particle with mass  $m$  must have zero energy,  $E = 0$ . This is possible, as follows from the second term in Equation (4), which gives the following imaginary part of the action at  $E = 0$ :

$$\text{Im } S(E = 0) = m \int_0^{1/H} \frac{dr}{\sqrt{1 - r^2 H^2}} = \frac{\pi m}{2 H}. \tag{22}$$

The probability of radiation

$$\exp(-2 \text{Im } S) = \exp\left(-\frac{\pi m}{H}\right) = \exp\left(-\frac{m}{2T_H}\right). \tag{23}$$

again corresponds to the thermal creation of particles by the environment with a local temperature equal to the double Hawking temperature,  $T_{loc} = H/\pi = 2T_H$ . Furthermore, again, the tunneling trajectory is fully inside the horizon, and this process is possible because the de Sitter symmetry is violated by the detector.

### 5. Two Processes of the Black Hole Radiation

Two processes—related and not related to the event horizon—can be also found in the black hole physics. Let us consider the process of creating a particle with zero energy,  $E = 0$ , without creating its partner inside the black hole. The observation of such single-particle creation is possible if the detector is at a finite distance  $R_0$  from the black hole. Then, the second term in Equation (4) gives

$$\text{Im } S(E = 0) = m \int_R^{R_0} \frac{dr}{\sqrt{1 - R/r}}. \tag{24}$$

Far from the horizon, at  $R_0 \gg R$ , this process is exponentially suppressed:

$$p \sim \exp(-2mR_0), \quad R_0 \gg R. \tag{25}$$

This process is possible because the existence of the classical detector, which is not at  $r = \infty$ , disturbs the vacuum of quantum fields near the black hole.

The more general case with nonzero  $E < m$  was considered in Ref. [24]. It is the combined process, at which the Hawking radiation is measured by the observer at a finite distance  $R_0$  from the black hole. The process is described by two tunneling exponents. For  $R_0 \gg R$ , one obtains [24]:

$$p \sim \exp\left(-\frac{E}{T_H}\right), \quad E > m, \tag{26}$$

$$p \sim \exp\left(-\frac{E}{T_H}\right) \exp\left(-2R_0 \sqrt{m^2 - E^2}\right), \quad E < m. \tag{27}$$

The first exponent in Equation (27) comes from the horizon and corresponds to the conventional Hawking radiation, and the second one describes the process of tunneling the

created particle, which occurs outside the horizon. The second process, which only takes place at  $E < m$ , is exponentially suppressed for  $R_0 \gg R$  and thus does not look thermal.

This is distinct from the local processes in the de Sitter case, where the detector at  $r = 0$  still conserves the part of the symmetry of the original de Sitter space—the spherical symmetry. Because of that, the radiation looks thermal, though with a factor of two.

**6. Acceleration: Unruh Effect vs. Local Processes**

Let us now consider the Unruh effect [25], where one may also expect two independent processes. One of them corresponds to the thermodynamics of the vacuum in the accelerated frame. The second one corresponds to the radiation experienced by an accelerated external object, such as an atom. Let us consider these two processes in the Rindler spacetime. For the 1 + 1 case, one has

$$ds^2 = g_{\mu\nu}d^\mu x d^\nu x = (1 + ax)^2 dt^2 - dx^2, \tag{28}$$

$$g^{\mu\nu} p_\mu p_\nu = \frac{E^2}{(1 + ax)^2} - p^2 = m^2. \tag{29}$$

The first process describes the pair creation from the horizon in the Rindler spacetime at  $x = -1/a$ . It is characterized by the Unruh temperature  $T_U = a/2\pi$ :

$$\exp\left(-2\text{Im} \int dx p(x)\right) = \exp\left(-\frac{2\pi E}{a}\right) = \exp\left(-\frac{E}{T_U}\right). \tag{30}$$

One may expect that the ionization of an atom is also determined by the thermal bath with the Unruh temperature.

However, there is also the local process of the ionization of an atom, which takes place well inside the horizon at  $x \ll 1/a$ . The trajectory of the nonrelativistic electron with  $\epsilon_0 \ll m$

$$-\epsilon_0 = \frac{p^2(x)}{2m} + max, \tag{31}$$

gives the following rate of ionization [26]:

$$\exp\left(-\frac{4\sqrt{2}}{3} \frac{\epsilon_0}{a} \left(\frac{\epsilon_0}{m}\right)^{1/2}\right). \tag{32}$$

Since  $\epsilon_0 \ll m$ , the local process of ionization essentially exceeds the rate of the thermal ionization  $\exp\left(-\frac{2\pi\epsilon_0}{a}\right)$ .

This happens because the existence of the atom violates the symmetry of the Rindler spacetime and breaks the corresponding vacuum state. Again, we have the same Rindler vacuum state in the accelerating frame, but in one case, the process is global and depends only on acceleration, while in the other case, the physical presence of the classical observer is important. The latter case is very similar to the local process of ionization in the de Sitter spacetime, where the atom violates the symmetry of the de Sitter spacetime. However, now, the violation of the symmetry is more crucial. As distinct from the de Sitter case, the ionization does not look thermal, and thus, there is no doubling of the Unruh temperature.

**7. Relation between Global and Local Processes in de Sitter Spacetime**

We considered the local processes in the de Sitter, Rindler and Painleve–Gullstrand spacetimes (the combined processes are considered in Ref. [27]). In all three cases, in addition to the processes related to horizons, there are local processes that violate the symmetries of these spacetime. The radiation in these local processes differs from the processes determined by the corresponding Gibbons–Hawking, Unruh and Hawking temperatures. Examples of local processes are the single-particle creation and the ionization

of the atom. Both do not exist in pure vacuum states. These processes are only possible due to the interaction of the quantum fields in the vacuum with external objects (the detector or an atom). It is the presence of the detector that violates the symmetry of the empty spacetime. Without the detector, the single-particle creation is impossible, and only the creation of a pair of entangled particles is allowed, which corresponds to the Hawking and Unruh radiation.

Among these three spacetimes, the de Sitter spacetime is specific since the local process looks thermal, and the corresponding local temperature is related to the Hawking temperature,  $T_{\text{loc}} = H/\pi = 2T_H$ . This suggests that the Hawking radiation can be represented as the correlated (cotunneling) creation of the entangled pair in the bath with the local temperature  $T = 2T_H$ :

$$\exp\left(-\frac{E}{2T_H}\right) \exp\left(-\frac{E}{2T_H}\right) = \exp\left(-\frac{E}{T_H}\right). \quad (33)$$

This connection between the two processes also suggests that in the presence of the detector (or atom) the symmetry of the de Sitter spacetime partially survives. In the de Sitter Universe, all the points in space are equivalent, and the position of the observer may serve as the event horizon for some distant observers [18]. The observer at a given point can see the Hawking radiation as the creation of two correlated particles according to Equation (33). On the other hand, the far-distant observer will only see a single particle, which comes from her/his horizon. For them, it will be the Hawking radiation with the Hawking temperature  $T_H$ .

This symmetry is also violated in the inflationary stage of the expansion of the Universe, which leads to a strong deviation from the thermal law except for the limit case  $M \ll H$ , where the Hawking temperature  $T_H$  is obtained [28,29].

## 8. Conclusions

There is the connection between Equations (23) and (33) for the de Sitter spacetime on one hand and the similar Equations (5) and (10) for the black hole horizon on the other hand. In both cases, the temperature  $T = 2T_H$  enters. However, the physics is different.

In the case of the PG black hole, there are two contributions to the tunneling process of radiation, each governed by the temperature  $T = 2T_H$ . They are coherently combined to produce radiation with the Hawking temperature  $T_H$ . This process can be traditionally interpreted as the creation of a pair of two entangled particles, of which one goes towards the center of the black hole, while the other one escapes from the black hole. The coherent combination of the several processes, which gives rise to the product of probabilities, is similar to the phenomenon of cotunneling in the electronic systems. In these condensed matter systems, the electron experiences the coherent sequence of tunneling events: from an initial to the virtual intermediate states and then to the final state [30,31], or two electrons tunnel simultaneously (one tunnels from the intermediate to the final state and leaves the intermediate state empty, and the other from the initial state to this empty state).

In the case of de Sitter spacetime, the temperature  $T = 2T_H$  is physical. Instead of the creation of the entangled pair, this local temperature describes the thermal creation of a single (non-entangled) particle inside the cosmological horizon. The local processes also take place outside of the black hole horizon and inside the Rindler horizon. The local process is highly suppressed in the case of the black hole—see Equations (25) and (27)—but is dominant in the case of the Rindler spacetime—see Equation (32).

How the local processes influence the thermodynamics of the de Sitter spacetime is an open question [18], as well as the problem of the radiation during the de Sitter stage of expansion [28,29,32–34].



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