Communication

On the Possibility of Observing Negative Shapiro-like Delay Using Michelson–Morley-Type Experiments

Arunava Bhadra [1,†], Ramil N. Izmailov [2,*,†] and Kamal K. Nandi [2,†]

1   High Energy and Cosmic Ray Research Center, University of North Bengal, Siliguri 734013, India;
     aru_bhadra@yahoo.com
2   Zel'dovich International Center for Astrophysics, Bashkir State Pedagogical University, 3A,
     October Revolution Street, Ufa 450008, RB, Russia; kamalnandi1952@rediffmail.com
*   Correspondence: izmailov.ramil@gmail.com
†   These authors contributed equally to this work.

Abstract: The possibility of observing *negative* Shapiro-like gravitational time delay (or time advancement) due to the Earth's gravity employing interferometric experiments on the Earth's surface is discussed. It is suggested that such a measurement may be realized in the near future with the help of modern versions of Michelson–Morley-type experiments.

Keywords: gravitational time advancement; interferometric experiments; Earth's gravity

1. Introduction

The pivotal role of the Michelson–Morley (MM) experiment [1–3] in the empirical foundations of the special theory of relativity is well known. The null result of this famous experiment in measuring the motion of ether relative to the Earth finally led to the notion of Lorentz symmetry that underlies the relativity theory (both special theory and general theory of relativity) as well as the standard model of particle physics.

In recent years, there has been a resurgence of interest in sensitive tests [4–11] of Lorentz symmetry, which essentially states that the experimental results are independent of the orientation or the boost velocity of the laboratory. This is largely due to the theoretical suggestions according to which Lorentz invariance may not be an exact symmetry of nature at all energies; a minuscule violation of local Lorentz invariance (LLI) has been considered in several efforts to unify known forces and in quantum gravity models such as the string theory, loop quantum gravity, etc.

Among the different experiments for testing LLI, the modern versions of the classic MM experiments are particularly sensitive to such measurements. Essentially, while examining the LLI, the present-generation MM experiments look for any directional anisotropy in the speed of light (c) by comparing the resonant frequency of a microwave or optical rotating cavity with respect to a fixed frequency of reference or to the frequency of a second rotating cavity with an axis of rotation at the right angles with respect to the first one.

The angular frequencies ω of standing electromagnetic waves in resonant cavities of length L are

$$\omega = 2\pi \frac{mc}{2nL} \qquad (1)$$

where $m (\equiv 1, 2, 3, .)$ denotes the (constant) mode number and n is the index of medium if a medium is present. A violation of Lorentz symmetry may affect the isotropy (or orientation independence) in c [12,13] or even in L [14], n [15] (which are properties of macroscopic matter and are sensitive to Lorentz violation), and consequently lead to an orientation dependence of the resonant frequency. To distinguish the influences of L and n from c on resonant frequencies, dissimilar cavities have been used [11]. To date,

no orientation dependence in the resonant frequency has been found from the cavity experiments, implying that up to an accuracy of $\Delta c_\theta / c \sim 10^{-17}$, there is isotropy [11,16].

The purpose of this paper is to suggest the possibility of measuring the *negative* Shapiro-like time delay [17], which we call time advancement [18], in the gravitational field of the Earth using modern MM experiments. As is well known, the Shapiro time delay (also known as gravitational time delay of signals) is a classic test of general relativity (GR) performed by Shapiro et al. [19]. The effect comes from the space–time metric coefficients. Today, this classic effect has been employed to obtain information on the distribution of matter, including dark matter, in the Universe [20–22]. In particular, the effect is of importance in detecting dark matter in our own galaxy [21,23]. It should be noted that gravitational time delay measurement using a Cassini spacecraft have verified GR with a remarkable accuracy of about 2.3 parts in 10^5 [24] in the Sun's gravitational field. The testing of GR through the time delay measurement in the Earth's gravity field should thus be an attractive idea in itself (only Gravity Probe B seems to currently test GR in the gravitational field of the Earth). More importantly, it may be the first detection of gravitational time advancement (negative time delay) effect, as in this case the measurement will be made at a higher curvature region in the whole light trajectory [18]. To date, in all efforts to detect gravitational time delay, the observers were assumed to be located in the weak field of the central gravitating object, and as a result the time delay was found to be positive in these cases [19,25]. If gravitational time advancement is confirmed experimentally, it may be employed, at least in principle, as a tool to examine different gravitational phenomena such as to probe dark matter and dark energy [23,26] or to distinguish the Gravity Rainbow (photons of different energies experiencing different levels of gravity) from pure GR effects [27]. The time advancement effect has been recently studied in the context of Lorentz violating Bumblebee gravity [28].

The MM experiments or their improved versions are conducted on the surface of the Earth, and since gravity cannot be shielded off, the motion of light in an MM apparatus should be influenced by the Earth's gravity. However, since the Earth is in free-fall relative to the Sun, there is no effect of the Sun's gravity on any Earth-bound experiment. Here, we would like to suggest that, as a consequence of the Earth's gravity, the resonant frequency of a cavity (Fabry–Perot) resonator will be different depending on whether the axis of rotation is parallel or perpendicular to the Earth's surface, and consequently the detection of the effect should be feasible in the near future, exploiting the modern versions of MM experiments, provided these experiments attain a slightly better accuracy ($\Delta c_\theta / c \sim 5 \times 10^{-19}$) than the level achieved so far ($\Delta c_\theta / c \sim 10^{-17}$) [16].

The structure of this paper is as follows. In the next section, we shall briefly describe gravitational time advancement. In Section 3, we estimate gravitational time advancement for a small distance travel, along horizontal and radial directions, from the surface of the Earth. The possibility of detecting time advancement from an Earth-bound experiment using two orthogonal Fabry–Perot interferometers is discussed in Section 4. Finally, we conclude this paper in Section 5.

2. Gravitational Time Advancement

The gravitational time delay is simply the extra time required by a photon or particle to travel along a trajectory in a massive object's gravitational field over the required time when the massive object is not present. This time delay is often evaluated in terms of the (Schwarzschild) coordinate time difference, which is not a measurable quantity. It must be converted into the proper time difference. Such a conversion leads to an opposite type of effect, namely gravitational time advancement (GTA), which is essentially a negative time delay, which occurs when the observer is in a stronger gravitational field relative to the gravitational field that the photon encounters during its journey. The GTA effect results from the fact that clocks run differently in gravitational fields depending on their curvature. Let us suppose an electromagnetic/gravitational wave is moving between two points in the gravitational field due to a spherically symmetrical matter distribution. Restricting to the

test a particle motion in the equatorial plan, in Schwarzschild geometry the time required by a particle to travel a distance from an arbitrary position r to the observer position r_o, which follows from the geodesic equations, is given by

$$\Delta t\,(r, r_o) = \frac{1}{c} \int_{r_o}^{r} \sqrt{P(r)}\, dr\,, \tag{2}$$

where,

$$P(r) = \frac{\left(1 - \frac{2\mu}{r}\right)^{-2}}{\left[1 - \frac{r_o^2(1 - 2/mu/r)}{r^2(1 - 2/mu/r_o)}\right]} \tag{3}$$

where $\mu = GM/c^2$, G is the gravitational constant and c is the speed of light. The proper time difference for the journey up to the first order accuracy of μ is given by

$$\begin{aligned}
\Delta \tau_m^{Sch} &= \sqrt{1 - 2\mu/r_o}\,\Delta t(r, r_o) \simeq \frac{1}{c}\left[\left(\sqrt{r^2 - r_o^2}\right)\left(1 - \frac{\mu}{r_o}\right) + \right. \\
&\quad \left. +2\mu \ln \frac{r + \sqrt{r^2 - r_o^2}}{r_o} + \mu \sqrt{\frac{r - r_o}{r + r_o}}\,\right].
\end{aligned} \tag{4}$$

When $r \gg r_o$, the first term in the right hand side gives a negative gravitational contribution $\left(-\frac{\mu r}{c r_o}\right)$, which dominates over the rest of the gravitational terms, which leads to gravitational time advancement.

3. Gravitational Negative Time Delay for Small Distance Travel from the Earth's Surface

Our aim is to obtain a model independent magnitude of the Shapiro-like gravitational time delay observed on Earth. To that end, we start with the general static (ignoring the Earth's axial rotation, the effect being comparatively small [29]) and spherically symmetric space–time in isotropic coordinates (t, ρ, θ, ϕ) given by

$$ds^2 = -B(\rho)c^2 dt^2 + A(\rho)\left(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2\right), \tag{5}$$

where c is the isotropic speed of light in vacuum. Using the above generic metric, which describes the effects in a weak gravitational field, the standard procedure is to apply the post–Newtonian (PN) expansion up to certain orders [30]. This PN formalism allows one to compare predictions of GR with those from alternative metric theories of gravity. It should also be noted that, in order to discuss the propagation of light to any desired order, knowledge of the space–time metric to that order is required [31]. Expanding the metric coefficients up to second-PN order, we obtain

$$B(\rho) = 1 - 2\frac{\mu}{\rho} + 2\beta\frac{\mu^2}{\rho^2} \tag{6}$$

and

$$A(\rho) = 1 + 2\gamma\frac{\mu}{\rho} + \frac{3}{2}\delta\frac{\mu^2}{\rho^2} \tag{7}$$

where β, γ, δ are the PN parameters (these parameters in fact distinguish different theories since, in GR, all of them are equal to 1) and $\mu = GM/c^2$ as before, M being the mass of the central gravitating object.

At this point, we take into account the fact that light propagation can take place both in the transverse direction (horizontal direction to the Earth's surface (Figure 1a)) as well in the radial direction (vertical to the Earth's surface (Figure 1b)). Accordingly, let us first assume that a light signal is sent horizontally from the surface of the Earth (say, from a

point Q as in Figure 1a) to a nearby point, say P, from where it is reflected back to the same point Q along the same trajectory. The horizontal distance (ΔX) between the points P and Q is assumed to be very small in comparison to the Earth's radius R. The trajectory of the to-and-fro light signal traveling between the points P and Q will be along a null curve satisfying $ds^2 = 0$.
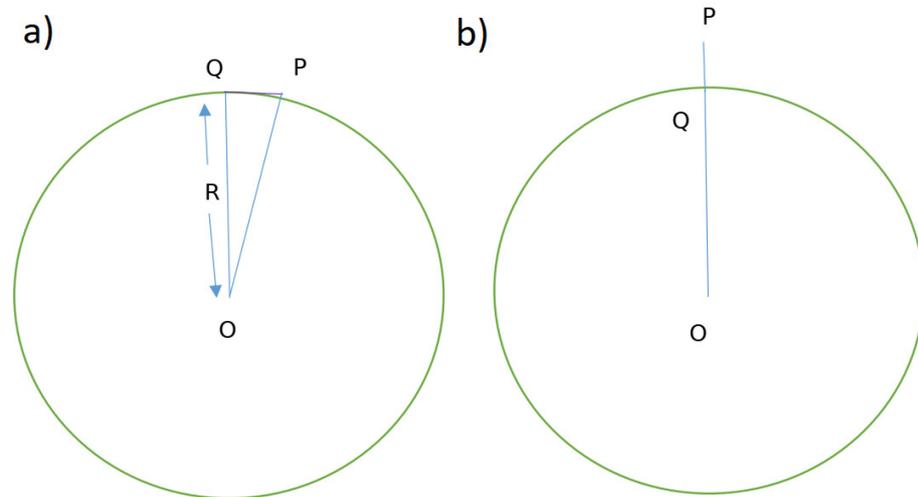

Figure 1. Schematic view of gravitational time delay/advancement: (**a**)—Light motion in the horizontal direction; (**b**)—Light motion in the vertical direction.

In order to derive time delay to the order of μ^2, it is necessary to know how much the photon path deviates from the Euclidean trajectory during its motion Q to P. The geodesic equations reveal that, within the accuracy of order μ^2, light trajectory does not depend on the azimuthal angle θ and follows a straight Euclidean path between Q and P as long as ΔX is small [31]. Hence, to the order μ^2, the coordinate time Δt_t taken by light in traveling from Q to P along the horizontal direction is

$$
\begin{aligned}
\Delta t_t &= \int_R^{r_P} \left(\frac{A(r)/B(r)}{1 - \frac{R^2 B(r)}{r^2 B(R)}} \right)^{1/2} dr \\
&= \Delta L_h \left[1 + \frac{\mu}{R} + \left(\frac{3}{2} - \beta \right) \frac{\mu^2}{R^2} \right],
\end{aligned}
\tag{8}
$$

where the subscript t denotes transverse direction, ΔL_h is the proper distance between Q and P when the motion is along the horizontal direction, and R is the distance of closest approach ($r_Q = R$). Therefore, the proper time interval measured by the observer at B between the two-way motion of the signal is

$$
\Delta \tau_t = 2B^{-1/2}(R)\Delta t_t = 2\Delta L_h.
\tag{9}
$$

This suggests that, for the motion, there is no gravitational time delay (or advancement) effect in the transverse direction at least up to the second-PN order, when the distance (ΔX) is small. With regard to the effect of the Earth's rotation on gravitational time delay for the null trajectory in the horizontal direction, it has been found in [29] that the rotational contribution to delay is much smaller than the second-order PN effect.

Finally, we consider the case of radial motion restricting to orbits in the equatorial plane $\theta = \pi/2$. Then, the geodesic equation for ϕ leads to

$$
\rho^2 \frac{d\phi}{dt} = \frac{B}{A} J^2
\tag{10}
$$

where the constant of integration J can, as per usual, be identified with the angular momentum of the photon motion in the weak field limit. Thus, if photon motion is restricted initially along the radial direction (that is, $J = 0$), Equation (10) suggests that ϕ may always remain unchanged (say, ϕ_0), so that the light trajectory may remain radial throughout its motion.

For the PN metric (Equations (6) and (7)), the coordinate time delay for light travel from the point $\left(R, \frac{\pi}{2}, \phi_0\right)$ to the point $\left(R + \Delta R, \frac{\pi}{2}, \phi_0\right)$ and back along the same path is given by

$$\Delta t_\rho = 2\Delta L_r \left(1 - \frac{\mu \Delta L}{2R^2}\right)\left[1 + \frac{\mu}{R} + \left(\frac{3}{2} - \beta\right)\frac{\mu^2}{R^2}\right], \tag{11}$$

where ΔL_r is the proper distance between the points R and $R + \Delta R$. Now, translating the coordinate time interval into that of proper time interval, we have

$$\Delta \tau_\rho = B^{-1/2}(\rho)\Delta t_\rho = 2\Delta L_r \left(1 - \frac{\mu \Delta L}{2R_\oplus^2}\right), \tag{12}$$

where we have identified $R = R_\oplus$ without incurring significant error because $R_\oplus \gg \frac{2GM_\oplus}{c^2}$, the Schwarzschild radius of Earth. It is evident from Equation (12) that the light signal suffers a negative time delay due to the negative sign in the round bracket, i.e., there occurs a gravitational time advancement. It should also be mentioned that the correction term also follows from dimensional arguments [32]. A curious aspect of Equation (12) is that the time advancement factor appears in the first order in μ but in the second order in $1/\rho$. To date, gravitational theories have been tested only to the first order both in μ and $1/\rho$ in the Solar System. It is evident from the above discussion that there should be an anisotropy in the measured average speed of light due to the effect given in Equations (9) and (12). The magnitude of the anisotropy in the radial direction is

$$\frac{c_R}{c} \simeq \frac{M_\oplus \Delta L_r}{2R_\oplus^2} \sim 5.4 \times 10^{-19}\left(\frac{\Delta L_r}{1\,\text{cm}}\right). \tag{13}$$

One may note here that, as ΔL_r tends to zero, there will be perfect isotropy, in accordance with the basic principle of general relativity.

4. Possibility of Experimental Detection of the Negative Time Delay

An interferometric experiment may be ideal to detect the anisotropy in the measured speed of light. In the case of cavity resonators, the eigenfrequency of the resonators with a symmetry axis perpendicular to the Earth's surface should be modulated due to the gravitational effect by

$$\Delta \nu = \nu \frac{GM_\oplus \Delta L_r}{2cR_\oplus^2}, \tag{14}$$

whereas that of the cavity resonators with a symmetry axis parallel to the Earth's surface may remain unmodulated within the accuracy of $\mu_\oplus^2/R_\oplus^2$. Hence, by comparing eigenfrequencies of a pair of resonator cavities with a symmetry axis of one of them in the horizontal direction and of the other in the vertical direction, or a pair of resonator cavities with both of their symmetry axes perpendicular to the Earth's surface but having unequal lengths, the gravitational time advancement effect can be detected, provided the experiment attains the necessary accuracy level, which is slightly higher than that achieved presently ($\Delta c_\theta/c \sim 10^{-17}$ [16]). It is worth noting that a technical difficulty for a cavity experiment with a horizontal rotation axis could be that the elastic bending of the cavities due to their own weight may be a dominating factor which would change the cavity length, and thus the resonance frequencies. Such an effect may have a different dependence on the length of the cavities as well as the materials of the cavities, and hence the effect may be isolated by using multiple cavities of different lengths/materials. More studies are needed in this regard.

There was a proposal for a small space mission experiment (Space–time Asymmetry Research (STAR)), which will measure anisotropy in the speed of light using two orthogonal Fabry–Perot interferometers with resolution of one-to-two orders better than that of the ground-based experiments [33]. The gravitational field of the Sun will dominate in a space-bound measurement. Such an experiment, if realized in the future, will therefore provide an opportunity to study gravitational negative delay due to solar gravity.

5. Conclusions

Our conclusion is the following. When light is sent vertically at a short distance from the Earth and reflected back along the same path to the point of origin, it may naturally suffer from the influence of the Earth's gravitational field. It is precisely this influence which gives rise to a negative gravitational time delay which could constitute a potential new test of general relativity. One plausible way of measuring this effect on the Earth's surface could be via the measurement of anisotropy in the light speed to be revealed in a high-precision MM-type experiment.

References

1. Michelson, A.A. The relative motion of the Earth and the Luminiferous ether. *Am. J. Sci.* **1881**, *22*, 120–129. [CrossRef]
2. Michelson, A.A.; Morley, E.W. On the Relative Motion of the Earth and the Luminiferous Ether. *Am. J. Sci.* **1887**, *34*, 333–345. [CrossRef]
3. Michelson, A.A.; Morley, E.W. On the relative motion of the earth and the luminiferous aether. *Philos. Mag.* **1887**, *5*, 449–463. [CrossRef]
4. Lipa, J.A.; Nissen, J.A.; Wang, S.; Stricker, D.A.; Avaloff, D. New Limit on Signals of Lorentz Violation in Electrodynamics. *Phys. Rev. Lett.* **2003**, *90*, 060403. [CrossRef]
5. Müller, H.; Herrmann, S.; Braxmaier, C.; Schiller, S.; Peters, A. Modern Michelson-Morley Experiment using Cryogenic Optical Resonators. *Phys. Rev. Lett.* **2003**, *91*, 020401. [CrossRef] [PubMed]
6. Wolf, P.; Bize, S.; Clairon, A.; Santarelli, G.; Tobar, M.E.; Luiten, A.N. Improved test of Lorentz invariance in electrodynamics. *Phys. Rev. D* **2004**, *70*, 051902. [CrossRef]
7. Stanwix, P.L.; Tobar, M.E.; Wolf, P.; Susli, M.; Locke, C.R.; Ivanov, E.N.; Winterflood, J.; van Kann, F. Test of Lorentz invariance in electrodynamics using rotating cryogenic sapphire microwave oscillators. *Phys. Rev. Lett.* **2005**, *94*, 040404. [CrossRef]
8. Stanwix, P.L.; Tobar, M.E.; Wolf, P.; Locke, C.R.; Ivanov, E.N. Improved test of Lorentz invariance in electrodynamics using rotating cryogenic sapphire oscillators. *Phys. Rev. D* **2006**, *74*, 081101. [CrossRef]
9. Herrmann, S.; Senger, A.; Kovalchuk, E.; Müller, H.; Peters, A. Test of the Isotropy of the Speed of Light Using a Continuously Rotating Optical Resonator. *Phys. Rev. Lett.* **2005**, *95*, 150401. [CrossRef]
10. Antonini, P.; Okhapkin, M.; Göklü, E.; Schiller, S. Test of constancy of speed of light with rotating cryogenic optical resonators. *Phys. Rev. A* **2005**, *71*, 050101. [CrossRef]
11. Müller, H.; Stanwix, P.L.; Tobar, M.E.; Ivanov, E.N.; Wolf, P.; Herrmann, S.; Senger, A.; Kovalchuk, E.; Peters, A. Tests of Relativity by Complementary Rotating Michelson-Morley Experiments. *Phys. Rev. Lett.* **2007**, *99*, 050401. [CrossRef] [PubMed]

12. Kostelecky, V.A.; Mewes, M. Signals for Lorentz violation in electrodynamics. *Phys. Rev. D* **2002**, *66*, 056005. [CrossRef]
13. Kostelecky, V.A.; Mewes, M. Sensitive Polarimetric Search for Relativity Violations in Gamma-Ray Bursts. *Phys. Rev. Lett.* **2006**, *97*, 140401. [CrossRef]
14. Müller, H.; Stanwix, P.L.; Herrmann, S.; Peters, A.; Lümmerzahl, C. Electromagnetic cavities and Lorentz invariance violation. *Phys. Rev. D* **2003**, *67*, 056006. [CrossRef]
15. Müller, H. Testing Lorentz invariance by the use of vacuum and matter filled cavity resonators. *Phys. Rev. D* **2005**, *71*, 045004. [CrossRef]
16. Eisele, C.; Nevsky, A.Y.; Schiller, S. Laboratory Test of the Isotropy of Light Propagation at the 10^{-17} Level. *Phys. Rev. Lett.* **2009**, *103*, 090401. [CrossRef]
17. Shapiro, I.I. Fourth Test of General Relativity. *Phys. Rev. Lett.* **1964**, *13*, 789. [CrossRef]
18. Bhadra, A.; Nandi, K.K. Gravitational time advancement and its possible detection. *Gen. Relativ. Gravit.* **2010**, *42*, 293–302. [CrossRef]
19. Shapiro, I.I.; Pettengill, G.H.; Ash, M.E.; Stone, M.L.; Smith, W.B.; Ingalls, R.P.; Brockelman, R.A. Fourth Test of General Relativity: Preliminary Results. *Phys. Rev. Lett.* **1968**, *20*, 1265. [CrossRef]
20. Desai, S.; Kahya, E.O. The Galactic One-Way Shapiro Delay to PSR B1937+21. *Mod. Phys. Lett.* **2016**, *31*, 1650083 [CrossRef]
21. Desai, S.; Kahya, E.O.; Woodard, R.P. Reduced time delay for gravitational waves with dark matter emulators. *Phys. Rev. D* **2008**, *77* , 124041 [CrossRef]
22. Cromartie, H.T.; Fonseca, E.; Ransom, S.M.; Demorest, P.B.; Arzoumanian, Z.; Blumer, H.; Brook, P.R.; DeCesar, M.E.; Dolch, T.; Ellis, J.A.; et al. Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar. *Nat. Astron.* **2020**, *4*, 72.
23. Ghosh, S.; Bhadra, A. Influences of dark energy and dark matter on gravitational time advancement. *Eur. Phys. J. C* **2015**, *75*, 494. [CrossRef]
24. Bertotti, B.; Iess, L.; Tortora, P. A test of general relativity using radio links with the Cassini spacecraft. *Nature* **2003**, *425*, 374–376. [CrossRef]
25. Will, C.M. *Theory and Experiment in Gravitational Physics*, Revised ed.; Cambridge University Press: Cambridge, UK, 1993.
26. Ghosh, S.; Bhadra, A.; Mukhopadhyay, A. Probing dark matter and dark energy through gravitational time advancement. *Gen. Rel. Grav.* **2019**, *51*, 54. [CrossRef]
27. Deng, X.-M.; Xie, Y. Gravitational time advancement under gravity's rainbow. *Phys. Lett. B* **2017**, *772*, 152. [CrossRef]
28. Tuleganova, G.Y.; Karimov, R.K.; Izmailov, R.N.; Potapov, A.A.; Bhadra, A.; Nandi, K.K. Gravitational time advancement effect in Bumblebee gravity for Earth bound systems. *Eur. Phys. J. Plus* **2023**, *138*, 94. [CrossRef]
29. Tartaglia, A.; Ruggiero, M.L. Angular Momentum Effects in Michelson–Morley Type Experiments. *Gen. Rel. Grav.* **2002**, *34*, 1371–1382.
30. Misner, C.W.; Thorne, K.S.; Wheeler, J.A. *Gravitation*; Freeman: San Francisco, CA, USA, 1973.
31. Richter, G.W.; Matzner, R.A. Second-order contributions to relativistic time delay in the parametrized post-Newtonian formalism. *Phys. Rev. D* **1983**, *28*, 3007. [CrossRef]
32. Zhang, Y.Z. On the Theory of the Michelson-Morley Experiment in the Earth's Gravitational Field. *Europhys. Lett.* **1987**, *3*, 1251. [CrossRef]
33. Lipa, J.A.; Buchman, S.; Saraf, S.; Zhou, J.; Alfauwaz, A.; Conklin, J.; Cutler, G.D.; Byer, R.L. Prospects for an advanced Kennedy-Thorndike experiment in low Earth orbit. *arXiv* **2012**, arXiv:1203.3914.