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Review

The Warm Inflation Story

Arjun Berera

School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3FD, UK; ab@ph.ed.ac.uk

Abstract: Warm inflation has normalized two ideas in cosmology, that in the early universe the initial primordial density perturbations generally could be of classical rather than quantum origin and that during inflation, particle production from interactions amongst quantum field, and its backreaction effects, can occur concurrent with inflationary expansion. When we first introduced these ideas, both were met with resistance, but today they are widely accepted as possibilities with many models and applications based on them, which is an indication of the widespread influence of warm inflation. Open quantum field theory, which has been utilized in studies of warm inflation, is by now a relevant subject in cosmology, in part due to this early work. In this review I first discuss the basic warm inflation dynamics. I then outline how to compute warm inflation dynamics from first-principles quantum field theory (QFT) and in particular how a dissipative term arises. Warm inflation models can have an inflaton mass bigger than the Hubble scale and the inflaton field excursion can remain sub-Planckian, thus overcoming the most prohibitive problems of inflation model building. I discuss the early period of my work in developing warm inflation that helped me arrive at these important features of its dynamics. Inflationary cosmology today is immersed in hypothetical models, which by now are acting as a diversion from reaching any endgame in this field. I discuss better ways to approach model selection and give necessary requirements for a well constrained and predictive inflation model. A few warm inflation models are pointed out that could be developed to this extent. I discuss how, at this stage, more progress would be made in this subject by taking a broader view on the possible early universe solutions that include not just inflation but the diverse range of options.

Keywords: early universe cosmology; warm inflation; quantum field theory; model building

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1. Introduction

Warm inflation was introduced 28 years ago. At that time the standard inflation scenario, hereafter called cold inflation, was overwhelming accepted as the valid description of the early phases of the universe, with much anticipation of its confirmation from the planned cosmic microwave background (CMB) experiments within the coming decades. In that time warm inflation has gone from being considered by many in cosmology as a distraction to one of the most promising solutions. The idea stems from an elementary observation. The central theme of inflationary dynamics has been the evolution of a scalar field, which during inflation carries most of the energy of the universe and which interacts with other fields. On the one hand, in the standard inflation picture the tacit assumption made is that these interactions have no effect apart from modifying the scalar field effective potential through quantum corrections. On the other hand, in the warm inflation picture interactions not only do that but also lead to fluctuation and dissipation effects. In condensed matter systems, interactions certainly lead in general to all three of these effects (some examples are given in [1]). Moreover, from a statistical mechanics perspective, the scalar field would want to dissipate its energy to other fields, and the system as a whole would try to equally distribute the available energy. Ultimately a thorough dynamical calculation is needed to address the question.

In cosmology, there is one important way this scalar field dynamics differs from condensed matter systems, which is that all processes for the former occur in an expanding universe. Expansion acts to constantly alter the state of the cosmological system. For example, due to expansion, radiation energy in the universe is continually being diluted. Similarly, the configuration of any cosmological scale process is being altered over time. Thus if the quantum mechanical processes that lead to dissipation operate at a time scale much slower than the expansion rate of the universe, then these processes would be totally shut down due to expansion, even if in a nonexpanding system, such as a condensed matter system, the same processes operate efficiently. This is the important question that must be understood. In the early years of inflation, there was a viewpoint that inflation had to be in a supercooled phase, since expansion would be too fast for any such microphysical processes to occur that lead to dissipation. However our work in warm inflation changed this point of view. Today this possibility is accepted without much question. Thus, one indicator of the wide influence and success of warm inflation.

The other major influence warm inflation has had is in normalizing the possibility of the initial primordial fluctuations being classical, not quantum. Again due to our timescale analysis, we showed there is considerable dynamical range in the early universe for multiparticle processes, such as those leading to thermalization or other statistical states. When Li-Zhi Fang and I first were working on this idea [2], neither of us had a full scenario in mind. We simply wanted to demonstrate that the prevailing idea of the times that primordial fluctuations, whether during inflation or otherwise, had to be quantum could be questioned. When I first discussed these ideas back in the mid-90s, researchers were surprised. I think one reason the initial paper by Fang and I on thermally induced primordial fluctuations generated interest when it came out was it was very novel for its time, yet we presented the idea in a way that made it look familiar. On its own this paper would probably have been forgotten had it not been that a few years afterwards with Marcelo Gleiser and Rudnei Ramos, we showed within a quantum field theory model that relevant microphysical timescales were possible to allow for such classical fluctuations [3,4]. Over the years since then, there has been plenty of scrutiny as to whether a full warm inflation scenario is viable, but the idea that primordial fluctuations could be classical rather than quantum had permanently taken root. This changed the prevailing thinking about primordial fluctuations dating back well before inflation, that they are of quantum origin. By now various ideas about classical primordial fluctuations have been suggested, and one of the successes of warm inflation has been that the plausibility of such ideas is routinely accepted.

In this review I will discuss the warm inflation scenario and the history of its development. In the next two Sections 2 and 3, I will discuss the basic scenario and how to realize this picture from first-principles quantum field theory (QFT). Then in Section 4, I will turn to the background of the idea, discussing my own steps in realizing and developing warm inflation. In Section 5, I will discuss density perturbations in warm inflation and the crucial difference between the weak and strong dissipative regimes of warm inflation. In Section 6, I will discuss some of the first-principles models of warm inflation that have been constructed. In Section 7, I will then talk about the advantages the warm inflation scenario has over cold inflation. Intrinsic features about its dynamics allows warm inflation to occur with the inflaton mass bigger then the Hubble scale $m_\phi > H$ and the inflaton field excursion less then the Planck scale $\phi < m_p$. In Section 8, a critique is given about difficulties warm inflation had faced in the field for several years arising more from attitude than due to any scientific shortcoming. The particle physics consequences from warm versus cold inflation are very different. This makes it important for CMB data to be viewed with a broad perspective, with any lingering attitudes now best set aside. I discuss warm inflation within the wider context of early universe cosmology and the general direction in which this field it is headed. In Section 9, I suggest ways to better select the optimum inflation models. There is a proliferation of inflation models in the literature and some of this is taking the focus away from addressing the key quantum-field-theory-based problems to model building. I suggest it would be useful to recognize the degree of speculation in any given inflation model as one means to separate out relevant models. In the final Discussion

Section 10, I point out the varied types of ideas and directions of research related to warm inflation, which I was unable to cover in detail. I also give some closing comments.

2. The Basic Dynamics

Inflation in the most general terms is a phase in which the scale factor grows at an accelerating rate, $\ddot{a} > 0$. To derive inflation, one utilizes the cosmological Einstein equations, for example the scale factor equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \,. \tag{1}$$

It can be seen from this equation that obtaining an accelerating scale factor $\ddot{a} > 0$, requires $p < -\rho/3$, so a substance with negative pressure. This means a universe where the dominant form of matter produces a repulsive form of gravity. Vacuum energy has an equation of state $p_v = -\rho_v$, which when dominant leads to inflation. If the vacuum energy is the only energy in the universe and it is constant, it leads to an exponential scale factor growth, with this de Sitter space being the most common behavior associated with inflation. In simplest terms, a universe undergoing accelerated expansion grows much bigger in the same amount of time as a universe undergoing decelerated expansion.

As it turns out, the equation of state of a scalar fields contain terms with negative pressure. The energy and pressure densities of a scalar field are

$$\begin{aligned} \rho &= \frac{\dot{\Phi}^2}{2} + V(\Phi) + \frac{(\nabla\Phi)^2}{2a^2} \\ p &= \frac{\dot{\Phi}^2}{2} - V(\Phi) - \frac{(\nabla\Phi)^2}{6a^2}, \end{aligned} \tag{2}$$

so that a potential energy, $V(\Phi)$, dominated state of a scalar field has a negative pressure. The idea generally adopted for realizing inflation from particle physics is to obtain the potential energy of some scalar field to dominate the energy density of the universe for some short period of time in the early universe, thereby generating the requisite amount of inflation needed to solve the cosmological puzzles. After sufficient inflation has then occurred, we must somehow place the universe back into a radiation-dominated hot Big Bang regime. The scalar field that performs the task in driving inflation is called the inflaton. The inflaton must perform two essential roles. These are to supply an appropriate energy density to be conducive for inflation and that the inflaton fluctuations should have the appropriate features to seed primordial density fluctuations in the universe.

In both the warm and cold inflation pictures, to realize inflation the scalar inflaton field must be dominated by potential energy. The difference is that cold inflation is synonymous with supercooling of the universe during inflation [5–8] (a good early review is [9]), whereas in warm inflation the inflaton is not assumed to be an isolated, noninteracting field during the inflation period. This means, rather than the universe supercooling, it instead maintains some radiation during inflation, to the extent to noticeably alter inflaton dynamics. In particular, the dividing point between warm and cold inflation is roughly at $\rho_r^{1/4} \approx H$, where ρ_r is the radiation energy density present during inflation and H is the Hubble parameter, which during the potential energy dominated inflation phase is $H^2 = 8\pi V/(3m_p^2)$, where m_p is the Planck mass. Here the warm inflation regime is $\rho_r^{1/4} > H$ and the cold inflation regime is $\rho_r^{1/4} \lesssim H$. These criteria are independent of thermalization, but if such were to occur, one sees that the warm inflation regime basically corresponds to when $g^{*1/4}T > H$, where g^* are the number of particle species in the universe. The relevance of this separation into these two regimes is since the typical inflaton mass during inflation is $m_\phi \approx H$, so that when $T \gtrsim H$, thermal or thermally induced fluctuations of the inflaton field will become important.

The interaction of the inflaton with other fields in general implies its effective evolution equation has terms representing dissipation or emission of energy going out of the inflaton system into other particles. Once the conceptual realization is made that there could be the significant level of particle production during inflation discussed in the above paragraph, the whole host of nonequilibrium QFT dynamics is open to consideration. The nomenclature of warm inflation is meant to capture the kinematic property of the presence of radiation without specific choice of production mechanism, with this broad meaning discussed in the early papers [10,11]. Mainly warm near-thermal equilibrium inflation has been examined, since quite evidently this is a difficult problem and that is the simplest case to first study, although the requirement the inflaton move slowly suggests this may still be the best regime. That will be the main focus in this review and for brevity we will refer to it simply as warm inflation. Note that the coupled set of Green's function equations for a general nonequilibrium treatment of warm inflation has been developed in early work by us and others [12–16], including for expanding spacetime [17,18], as well as suggestions for an alternative oscillation particle production based mechanism of warm inflation [19] and adjusted thermalization [20]. For warm near thermal equilibrium inflation, the dynamics can be expressed as a simple phenomenological Langevin type equation:

$$\ddot{\phi} + [3H + \Upsilon]\dot{\phi} - \frac{1}{a^2(t)}\nabla^2\phi + \frac{\partial V}{\partial \phi} = \zeta. \tag{3}$$

In this equation, ζ is a fluctuating random force and $\Upsilon\dot{\phi}$ is a dissipative term. Both these are effective terms that arise due to the interaction of the inflaton with other fields. A fluctuation–dissipation relation in general will relate these two terms, with details depending on the microscopic dynamics and the statistical state of the system.

For warm inflation to occur, the potential energy ρ_v must be larger than both the radiation energy density ρ_r and the inflaton's kinetic energy. A major difference to cold inflation is the evolution of the energy densities. In warm inflation, vacuum energy is continuously being dissipated at the rate $\dot{\rho}_v = -\Upsilon\dot{\phi}^2$ and so causing the radiation energy not to vanish. The General Relativity (GR) cosmological energy conservation equation,

$$\dot{\rho} = -3H(\rho + p)\ , \tag{4}$$

for this system of vacuum and radiation becomes

$$\dot{\rho}_r = -4H\rho_r + \Upsilon\dot{\phi}^2. \tag{5}$$

The first term on the right-hand side is a sink term that is depleting radiation energy, whereas the second term is sourcing this energy. These equations are meant to demonstrate the basic idea, so the notation is kept simple, but to be clear in the above the field ϕ is just the background mode, whereas in Equation (3) it represents both the background and fluctuating modes. Noting from the sink term that the rate of depletion is proportional to the amount of radiation present, in general it implies there will be a nonzero approximate steady state point for ρ_r controlled by the source term. This holds when ϕ, H, and Υ are slowly varying, which is a good approximation during inflation, As an example, if the source term is just a constant, which is a good approximation during the slow roll evolution of the inflaton, then $\Upsilon\dot{\phi}^2 = const. \equiv c_0$. In that case the solution to Equation (5) will be $\rho_r \approx c_0/(4H) + (\rho_{r0} - c_0/(4H))\exp(-4Ht)$. The second term on the RHS of this solution decays away any initial radiation, but at large time radiation does not entirely vanish because of the first term on the RHS. Thus at large time, the radiation in the universe depends only on the rate at which the source is producing it and so becomes independent of initial conditions.

As already noted, the presence of radiation during inflation is fully consistent with the equations of General Relativity, since the single requirement by the scale factor equation to realize an inflationary scale factor growth is that the vacuum energy density is the dominant component of energy in the universe. This means supercooling is only one special limiting regime of this general case implied by these equations. Thus inflation would still happen if there was, say, a 10% or 1% etc., admixture of radiation, in addition to the vacuum energy. This is an important point. To appreciate it, note that there are at least five scales in inflation, the vacuum energy $E_v \equiv \rho_v^{1/4}$, the radiation energy $E_r \equiv \rho_r^{1/4}$, the Hubble scale H, the inflaton mass $m_\phi^2 \equiv V''(\phi)$, and the dissipative coefficient Υ. In the cold inflation picture, these five energy scales are related as (i) $E_v \gg E_r$, (ii) $H > m_\phi$, (iii) $m_\phi > E_r$, and (iv) $H \gg \Upsilon$. Condition (i) is simply a minimal General Relativity requirement to have inflation. Condition (ii) is necessary for the slow roll regime. Condition (iii) implies the universe is in a low-temperature regime where radiation has an insignificant effect on inflaton fluctuations. Finally condition (iv) implies dissipation has an insignficant effect on inflaton evolution.

For warm inflation, there are two regimes that must be addressed: weak and strong dissipative warm inflation. In both these regimes the following energy scales are the same (i) $E_v > E_r$, (ii) max $(\Upsilon, H) > m_\phi$, and (iii) $E_r > m_\phi$. Condition (i) is required again by General Relativity to realize inflation. Condition (ii) is the warm inflation equivalent to the slow roll regime. Condition (iii) implies the inflaton fluctuations are no longer in a zero temperature state, so that radiation will have nontrivial effects on inflaton dynamics and fluctuations. Finally the last condition, and the one that leads to two regimes of warm inflation, are (iv) $\Upsilon > 3H$, strong dissipative warm inflation and (iv) $\Upsilon \leq 3H$, weak dissipative warm inflation. The notation here is almost self-explanatory, with the strong dissipative regime, where the dissipative coefficient Υ controls the damped evolution of the inflaton field and the weak dissipative regime, where the Hubble damping still is the dominant term.

Even if the presence of radiation does not hinder inflationary growth, it can still influence inflaton dynamics. For example consider inflation at the Grand Unified Theory (GUT) scale, so $V^{1/4} \equiv E_v \sim 10^{15}$ GeV, which means the Hubble parameter, is $H \sim V^{1/2}/m_p \sim 10^{11}$ GeV. For cold inflation and weak dissipative warm inflation, since the Hubble damping term $3H\dot{\phi}$ must be adequate to produce slow roll inflaton evolution, it requires that the inflaton mass $m_\phi \sim 10^{9-10}$ GeV $\lesssim 3H$. The key point to appreciate is that there are five orders of magnitude difference here between the vacuum energy scale and the scale of the inflaton mass. In other words there is a huge difference in scales between the energy scale m_ϕ governing inflaton dynamics and the energy scale E_v driving inflation. This implies, for example, that in order to excite the inflaton fluctuations above their ground state, it only requires a minuscule fraction of vacuum energy dissipated at a level as low as 0.001%. This is good indication that dissipative effects during inflation can play a noticeable role. This is only an energetic assessment, but it is suggestive of interesting physics. It leaves then a question for a full dynamical calculation to answer. In particular, the universe is expanding rapidly during inflation at a rate characterized by the Hubble parameter H. The question then is whether the fundamental dynamics responsible for dissipation occurs at a rate faster than Hubble expansion.

The other difference between warm and cold inflation is how dissipation affects the parameters of the underlying first-principles model, which becomes most evident in the strong dissipative regime, $\Upsilon > 3H$. To understand this point, recall that in cold inflation the inflaton motion is damped by only the $3H\dot{\phi}$ term. Thus slow roll evolution requires the inflaton mass, $\sim\sqrt{V''}$, to be less than $\sim H$. However in typical quantum field theory models of inflation, it is very difficult to maintain such a tiny inflaton mass, a point which is further addressed in Section 6. In one form this is called the "η-problem" [21,22]. To contrast, slow roll motion in warm inflation only requires $V'' < (3H + \Upsilon)^2$, so for $\Upsilon > 3H$ it means the inflaton mass can be bigger than in the cold inflation case, and in particular bigger than the Hubble scale. This relaxation of the inflaton mass constraint permits much greater freedom in building realistic inflaton models, since this "η-problem", infra-red, and/or swampland problem is comfortably eliminated.

Another model-building feature that differentiates warm and cold inflation is where inflation occurs in regards the region of the scalar field background mode amplitude, which is the zero-mode of the field $\phi \equiv \langle \Phi \rangle$. For cold inflation with the simplest types of potentials, which also are the most commonly used, $V = \lambda\Phi^4/4!$ and $V = m_\phi^2\Phi^2/2$, calculations show that the initial inflaton amplitude has to be above the quantum gravity Planck scale $\phi_i > m_p$. This is because in these models, H in the Hubble damping term, $3H\dot{\phi}$, increases with larger field amplitude, so in order to achieve an adequately long slow roll period to yield the desired 50 or so e-folds of inflation, this large field amplitude is required. However from the perspective of the ultimate goal of building a realistic particle physics inflation model, this condition poses a problem, which forces more complications into the model building. This will be discussed later in this review. On the other hand in warm inflation, both dissipation and radiation work to reduce the inflaton field amplitude. Since thermal fluctuations will always be larger than quantum, in order to constrain the scalar perturbation amplitude to be the desired $\sim 10^{-5}$, it requires fixing the other parameters such as the inflaton coupling and field amplitude to be smaller, which in turn lowers the tensor to scalar ratio. Moreover, when $\Upsilon > 3H$, this larger dissipation implies the the inflaton traverses a much smaller region of the field amplitude in the slow roll phase, so allowing its field amplitude to be smaller. The basic point detailed calculations show is that for these simple monomial potentials, in warm inflation the inflaton field amplitude can be below the Planck scale $\phi < m_p$, thus avoiding the quantum gravity scale.

3. First Principles Dynamics

As already pointed out in the previous Section, the conversion of even a little vacuum energy into radiation can have significant effects during inflation. Furthermore, Equation (3) has been presented as an effective evolution equation for the inflaton field once interactions with other fields are integrated out. The questions that remain to be answered from quantum field theory are whether both these effects actually occur and, if so, then in what models. In order to address these questions, in this Section our task is to understand how to derive the effective equation of motion for the scalar inflaton field starting from a fundamental Lagrangian.

The basic Lagrangian quite generally for any inflaton model has the form $\mathcal{L} = \mathcal{L}_S + \mathcal{L}_R + \mathcal{L}_I$. Here $\mathcal{L}_S$ is the inflaton system Lagrangian, which has the general form,

$$\mathcal{L}_S = \frac{1}{2}\dot{\Phi}^2 - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi). \tag{6}$$

The inflaton in any model must interact with other fields, since channels must exist from which the vacuum energy contained in the inflaton field ultimately can be released into radiation energy, so that inflation ends and the universe is put into a Hot Big Bang evolution. These interactions are contained in the $\mathcal{L}_I$ part of the above Lagrangian. The question is whether this conversion process occurs exclusively at the end of inflation, as pictured in cold inflation, or does it occur concurrently with inflation, as pictured in warm inflation. Some common types of interactions are the inflaton coupled to bosonic fields such as $0.5g^2\Phi^2\chi^2$ or fermion fields as $h\Phi\bar{\psi}\psi$, i.e., $-\mathcal{L}_I = 0.5g^2\Phi^2\chi^2 + h\Phi\bar{\psi}\psi$. Finally $\mathcal{L}_R$ contains all other terms associated with all fields aside from the inflaton that form the radiation bath or reservoir, such as the χ and ψ fields in this example.

For this Lagrangian, the quantum operator equations of motion can be immediately written down, one for each field. These equations are generally coupled to each other due to nonlinear interactions. We are interested in the evolution equation of the fields, and in particular the expectation value of the evolution equation of the inflaton field, given the state of the system at some initial time t_i. Thus we wish to obtain the effective equation of motion for the scalar inflaton field configuration $\phi \equiv \langle\Phi\rangle$, after integrating out the quantum fluctuations in Φ, and the effects of all other fields with which ϕ interacts, such as the χ and ψ fields in $\mathcal{L}_I$. This is a typical "system-reservoir" decomposition of the problem, as familiar in statistical mechanics [1]. In our case the system is ϕ and the reservoir is all the other dynamical degrees of freedom.

The system-reservoir approach has applications to many problems in physics. It is instructive to state a few examples here. One of the most common examples is Brownian motion, where the evolution of one singled out particle is of interest, when it is immersed in a fluid and interacts with particles in that fluid. One seeks the evolution equation for this Brownian particle, once the effect of all the other particles are integrated out and represented in this equation through effective terms. In our problem the background field ϕ is the analog of the Brownian particle and the reservoir bath in this case contains the Φ quantum modes, the scalar χ, and the spinor ψ. In condensed matter physics, the system-reservoir, or open quantum system, approach is widely used. Some examples are the tunnelling of a trapped flux in a SQUID, interaction in a metal of electrons with polarons, and in Josephson junction arrays [1,23].

In order to obtain the ϕ effective equation of motion, the procedure is first to replace the field Φ in the Lagrangian by $\Phi = \phi + \kappa$, where $\langle\Phi\rangle \equiv \phi$ and κ are the quantum fluctuations of the Φ field. Taking for example the potential $V = m_\phi^2 \Phi^2/2$, the equation of motion for ϕ, then becomes

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi - \frac{1}{a^2(t)}\nabla^2\phi + g^2\phi\langle\chi^2\rangle + g^2\langle\kappa\chi^2\rangle + h\langle\bar{\psi}\psi\rangle = 0\,. \tag{7}$$

One now wishes to solve the quantum operator equations of motion for all the other fields, i.e., κ, χ and ψ, as a function of ϕ, substitute these above in Equation (7), and then take the specified expectation values. What would emerge from this is the sought-after effective evolution equation for ϕ. This is possible in principle, but in practice it cannot be done exactly, so various perturbative and resummation methods are used. In this review we will not explore these approximation methods, but the interested reader can examine [3,12–18,24]. Here only a few general features of the effective ϕ equation of motion are highlighted. First, since ϕ is singled out, it becomes an open system, so it is expected that the ϕ effective equation of motion will be nonconservative. Second, the fields χ, ψ etc., at a given time t_0 in general will be functions of ϕ at all earlier times $t < t_0$. Thus the expectation values $\langle\chi^2\rangle$ etc., in Equation (7) will be nonlocal in time with respect to ϕ, so consistent with the first general fact, as this will lead to a nonconservative equation. These time nonlocal terms are then expressed in a derivative expansion of ϕ with respect to time, and for adequately slow evolution, only the leading term is retained to give the dissipative $\Upsilon\dot{\phi}$ term in Equation (3).

For the scalar field background mode, the emerging evolution equation is simply $\ddot{\phi} + [3H + \Upsilon]\dot{\phi} + dV/d\phi = 0$, which, after multiplying through by a factor of $\dot{\phi}$, can also be written in terms of the scalar field Hamilitonian as $dH_S/dt = -[3H + \Upsilon]\dot{\phi}^2$ [25]. This equation conveys that the loss in energy in the scalar inflaton field sector is from the two terms on the right hand side, one due to cosmological expansion and the other due to dissipation, with the dissipative term then sourcing that energy to radiation, as shown in Equation (5). The derivative expansion mentioned above means that the leading time nonlocal term associated with dissipation goes as $\dot{\phi}$, with Υ, which has dimensions of rate (energy dimension one), controlling how fast the kinetic motion of the scalar field decays its energy into particles due to its coupling to other fields. The QFT formalism for computing Υ from the microphysical dynamics that folds into producing this macroscopic dissipation term can be found in [3,13,24,26].

For readers familiar with the effective potential in Lagrangian quantum field theory, there is a heuristic way to understand the origin of the ϕ effective equation of motion. The effective potential calculation applies when ϕ is a static background. The interaction of ϕ with the other quantum fields leads to the creation of quantum fluctuations, which are emitted off ϕ, propagate in space and time, and then are reabsorbed by ϕ. These processes typically are known as loop corrections, which modify the classical potential and lead to the effective potential. Now suppose ϕ is not in a static situation, but that it is changing in time. In this case the same loop corrections mentioned above would occur. However at the time of emission and absorption, the state of ϕ has changed. Thus these loops no longer simply modify the potential of ϕ, but also introduce terms which mix products of

ϕ at different times, therefore introducing temporally nonlocal terms into the ϕ evolution equation.

Thus the key question is, given a particle interaction structure in the Lagrangian, what types of dissipative effects does this lead to during inflation? In Section 6 we will review various models. Just as one example, here is a two stage mechanism, involving the inflaton field coupled to a heavy scalar field χ which in turn is coupled to light fermion fields ψ as $g^2\Phi^2\chi^2 + h\chi\bar{\psi}\psi$ [27,28]. In this case the background inflaton field ϕ acts as a time-dependent mass to the χ field. As ϕ changes over time, the χ mass changes, thus altering the χ vacuum. This leads to virtual χ production, which then decay into real ψ particles. There are also direct interactions of ϕ and χ particles. This type of interaction structure is very common in particle physics models, thus conducive to warm inflationary dynamics.

4. Background

My own background in physics was not in cosmology. My PhD was on string theory and during that time I also did considerable work in statistical physics and condensed matter physics. After finishing my PhD and during my first postdoc in Tucson, Arizona, I started working on perturbative QCD. During that time I was the seminar organizer. One academic who was in our department there was Fang. I frequently had conversations with him and found he had broad interests, with thought given to many things. So I asked him to give us a seminar and he said he would on inflationary cosmology. This was not a subject I had studied before. During his talk as he discussed the scalar inflaton field and how it can drive inflation, I raised my hand and asked him where the dissipative term was in the inflaton evolution equation. Coming from a background that included condensed matter physics, I found it unusual that an interacting system did not have a term accounting for dissipation. Fang paused his talk and said I should come by afterwards for a discussion. So I did and I explained to him how I would expect the inflaton evolution equation to have a standard dissipative term and possibly even be governed by a Langevin type evolution. This seemed to be a direction Fang had given thought to before. He showed me a paper he had written in 1980 which had basically suggested inflation [29]. It also had radiation production during inflation, but neither of us felt the dynamics of that paper was the direction to develop further.

The basic idea of an exponential expansion phase in the early Universe was first suggested in the highly insightful work by Gliner starting in the mid-1960s [30,31]. Then during the 70s, Kirzhnits and Linde developed the foundations for application of particle physics to phase transitions in cosmology, including explaining how they could be important in explaining the cosmological puzzles [32–36]. By the late 70s and early 80s there were several papers using these seminal ideas including Fang's paper. His was one of the set of papers prior to Guth's [5] that had suggested the inflation idea as a solution to the horizon problem [37–43], but without the catchy name that Guth finally gave the scenario (Guth's paper also pointed out that inflation could solve the flatness problem). All this work aside from Fang's was developing cold inflation dynamics.

Fang and I started developing our ideas and wrote the paper [2]. This addressed dissipation and noise and came up with an expression for inflaton fluctuations that were thermally induced. At that point my postdoc there was coming to an end, and I was moving to Penn State. During that period I started thinking about formulating a full scenario and showing one could realize inflation and have dissipation concurrently with these thermally induced fluctuations. I did eventually arrive at a model and I sent my results to Fang. I had naturally considered him a collaborator on this work. Fang found the results very interesting and encouraged me to write it up and publish it [44]. However he said he had not thought about our idea to this extent and so I should write this paper on my own. He and I even had a discussion about what to name this new scenario, which led to calling it warm inflation.

At the time I considered this work as just a side project to my main interests developing in perturbative QCD. However I kept thinking about it and wrote a couple more papers in the next couple of years. In one paper I studied warm inflation trajectories computed from the Friedmann equations for a system with both a decaying vacuum energy and radiation, and showing how such evolution could smoothly go from a warm inflationary phase to the radiation-dominated regime, thus offering a graceful exit [10]. I also started becoming interested in how an inflaton Langevin-type equation could be derived from first principles. After all my initial comment in Fang's talk had been that there should be a dissipation and noise term in such equations. In my first attempt to understand such dynamics [25], resting on my condensed matter background, I studied the Caldeira–Leggett model [23]. The original was a quantum mechanical model of a single coordinate, the system, coupled linearly to many other coordinates, all being harmonic oscillator Hamiltonians. My paper made a quantum field theory extension of this model to study inflationary expansion concurrent with dissipation, which set some foundations for a QFT derivation of warm inflation dynamics.

Around this point I was looking for a new postdoc position. By now my research interests had turned heavily toward developing warm inflation and not many people were interested in the idea, so I had difficulty getting hired. However Robert Brandenberger responded to my efforts by reaching out in an email to me saying he found my ideas interesting. He was one of the first researchers in cosmology to support my work. I am not sure that Robert believed warm inflation was necessarily THE idea of cosmology. I think his attitude, like mine, is that any reasonable idea in cosmology needs to be fully examined. There is no way that any idea in cosmology can become the single adopted picture until all reasonable ideas have been fully vetted. In that context I think he felt warm inflation deserves its time to be developed and considered. Through his help I was able to secure a postdoc position at Vanderbilt in the group of Tom Kephart and Tom Weiler. It was here that my work in warm inflation developed considerably. They had a very open minded attitude toward developing new ideas in theoretical physics and were not too bothered about just following the mainstream. This provided a conducive environment.

After writing these initial papers on warm inflation, I was contacted by Gleiser and Ramos. They had written one of the pivotal papers in deriving dissipation in a scalar quantum field theory [45] (other papers on scalar field dissipation around or previous to this are [46–51]). They saw the connection between their work and what I was trying to achieve. We started working together and wrote a paper of a scalar field ϕ, meant to be the inflaton, coupled to N other scalar fields which are integrated out to arrive at an effective equation of motion for ϕ [3]. This effective equation of motion would contain then a dissipative term. This model contained all the basic features for realizing warm inflation dynamics. Because all masses were much bigger than the Hubble scale and all microphysical dynamics was fast compared to the expansion scale, this model could be calculated still within a flat spacetime framework. This is a noteworthy feature of warm inflation that on a relative scale for a nonequilibrium open QFT problem, the underlying dynamics are fairly simple to calculate. These conditions on the field theory were imposed as consistency conditions. In particular we introduced adiabatic conditions that the dynamic time scale of evolution of the scalar field be much larger than typical collision and decay time scales Γ^{-1},

$$\phi/\dot{\phi} \gg \Gamma^{-1} \, . \tag{8}$$

We also imposed that these microphysical collision and decay time scales be much shorter than the Hubble time,

$$\Gamma \gg H \, . \tag{9}$$

As I mentioned earlier, a key initial barrier to why no one before me had suggested the warm inflation scenario seems to have been due to a lack of understanding of dynamical time scales relevant to inflation and in particular most researchers in this field held to a belief that inflation happens too quickly for particle production to occur. In this paper with Gleiser and Ramos my initial thoughts about such time scales was examined within quantum field theory, and we recognized that indeed timescales can allow for particle production, although it would not be easy to realize. This was one major accomplishment of this paper.

Several months after we put this paper on the arXiv, Junichi Yokoyama and Andrei Linde (YL) wrote a paper titled 'Is warm inflation possible?' and seemed to have answered their question within a one sentence abstract that said it was 'extremely difficult and perhaps even impossible' [52]. The analysis in their paper was not very different from ours. They did compute the fermionic channel of dissipation whereas our paper had considered the bosonic, so that added a useful new result. However, their basic analysis of warm inflation followed ours, as did the consistency conditions. Only the conclusions differed. Where they saw impossible we simply saw a set of constraints that would help guide us towards building a first-principles model of warm inflation.

After our paper but before Yokoyama and Linde's, Ramos and I had presented our work at PASCOS98 in Boston. Linde was in the audience for our talks and told us that he did not think our warm inflation work was correct, although subsequently still during the conference he told us there was some merit to our dissipation calculations. Nevertheless some months later he wrote the above mentioned paper. Yokoyama and Linde did share the results of their upcoming paper with us with this claim about the impossibility of warm inflation. Thus, we got to work on building a quantum field theory model that demonstrated warm inflation is possible. Within a few weeks after they arXived their paper, we put out the first quantum field theory model, which we called the distributed mass model (DMM, explained below in Section 6), demonstrating that the 'impossible' was really not quite so [4]. The following year Tom Kephart and I built a string theory motivated realization of this model [53], further solidifying that not only is warm inflation possible but it has attractive model-building prospects.

Around this time my interest turned to taking a deeper examination of warm inflation dynamics. The regime of warm inflation that to me seemed most interesting, and still does, is the strong dissipative regime of warm inflation,

$$\Upsilon > 3H \,, \tag{10}$$

so that the damping of the inflaton motion was dominated by the thermal damping and Hubble damping did not play a major role. In the first paper with Fang, we had computed the density perturbations only in the weak dissipative regime of warm inflation,

$$\Upsilon \leq 3H \,. \tag{11}$$

In [11] I determined the expression for the density perturbations in the strong dissipative regime. In this paper I also looked more closely at the quantum field theory dynamics using the distributed mass model we had introduced. One of the key observations I made in that paper was that in the strong dissipative regime $\Upsilon > 3H$, the mass of the inflaton could be larger than the Hubble scale. This allowed for warm inflation models unlike anything that could be made for cold inflation, where slow roll under Hubble friction required that $m_\phi < 3H$. A mass less than H implies a Compton wavelength bigger than the horizon. For such a case, in the rest frame the particle is not localizable, so the field associated with it has no particle interpretation within the conventional sense. Thus a matter field with $m_\phi < H$ was unlike any kind of quantum field we had any empirical knowledge about from collider experiments (note that photons being massless have no rest frame and so are not localizable, but this is due to their Abelian gauge symmetry which makes them very different from material particles such as from a scalar field). It opened up the possibility for infra-red problems. So one of the conditions I imposed for what I considered the ideal inflation model was that $m_\phi > H$ and I called it the infra-red condition [11]. I had also understood that dissipation could lower the background inflaton field amplitude and in particular for monomial potentials could allow $\phi < m_p$ [54]. Both these conditions I recognized by the early 2000s from simple reasoning as being important for an ideal inflation model, well before they emerged in the swampland conditions [55,56]. These conditions set the goal for what to look for in deriving a warm inflation model from first-principles quantum field theory. There has been some success in this direction, which I will discuss in Section 6. However it has proven very difficult to find models in this regime. Nevertheless in principle it is possible, which is very different from cold inflation where such a regime is very difficult to achieve and in particular having an inflaton mass larger than the Hubble scale is out of the question. It remains an open model building question for warm inflation to find such models.

At this point Rudnei and I set out to build more first-principles warm inflation models. Initially we explored supersymmetry (SUSY), which was one of the most well utilized symmetries in inflation model building to realize the ultraflat inflaton potential that was needed. In warm inflation there were the dual requirements for having this very flat inflaton potential but at the same time creating a large enough dissipative term. This effort resulted in the SUSY two-stage dissipation model [28] (further explained in Section 6). We computed the dissipative coefficient, the radiative corrections, and consistency conditions for this model and obtained a warm inflation regime, although like DMM once again this model required a large number of fields going upward of thousands. At around this time, I also started working with Mar Bastero-Gil and we further examined this and other SUSY models [57]. Around this time Ian Moss also became interested in warm inflation, with Hall, Moss, and Berera [58] doing a more detailed examination of density perturbations. He continued to study this even further, with Graham and Moss [59] finding a certain growing mode for the fluctuations if the dissipative coefficient was temperature dependent. This result added yet more numerical difficulties in computing warm inflation from a model. Bastero-Gil and Ramos developed a set of codes that could carry out the needed calculations. The warm inflation power spectrum would have contribution from both quantum and thermal noise. Ramos and da Silva [60] conducted a careful analysis of these contributions starting with my basic expressions for the primordial density perturbations and came up with a total power spectrum.

Alongside these developments of the theory, we carried out model building and made predictions for the CMB from the two-stage mechanism model. In 2009 the paper by Bastero-Gil and me [61] demonstrated that in warm inflation the tensor-to-scalar ratio, r, would be suppressed compared to the comparable cold inflation model for the monomial potentials Φ^2 and Φ^4. At the time this was a result that went against the growing tide of expectation of finding a high scale tensor mode at the Grand Unified Theory scale. In 2014 our paper [19] carried out a more detailed study to show the suppression of the tensor mode for the warm Φ^4 model and also consistency for n_s with the Planck 2013 results. This paper also showed that as the dissipative coefficient increased, r would decrease, thus demonstrating again the parametric suppression of the tensor mode with increasing dissipation. This was a significant finding. Although by this time there was a trend in CMB data that the upper bound on r was decreasing, it was edging on ruling out the cold inflation Φ^4 model, there was anticipation that a tensor mode would be found. Our results went contrary to such expectations and indicated that the tensor mode would be suppressed.

Although the two-stage model was very successful in developing a working warm inflation model, the fact that it required a huge number of fields was an issue we were not too happy about. We wanted to find a simple warm inflation model that contained only a small number of fields. Our efforts in doing that with SUSY we felt had been exhausted, so we started exploring other symmetries. One that we had been thinking about for some time was the pseudoscalar symmetry. This led us to construct the warm little inflaton model [62]. This model will be discussed in more detail below in Section 6, but in short it obtained warm inflation with just a few fields, thus was a major step forward in constructing successful particle physics models of warm inflation.

There had been work previous to my warm inflation paper in 1995 and that by Fang and I earlier that year, which had discussed dissipation during inflation. To start with was Fang's 1980 paper [29] that I already mentioned. He examined a source of dissipation associated with bulk viscosity specific to a phase transition and was not the direction that seemed could be developed in any detail. Subsequently in the mid-1980s, Moss [63] and Yokoyama and Maeda [64] suggested the idea of dissipation in the inflaton evolution equation similar to warm inflation, though we were not aware of these two papers when initially developing warm inflation. The success of warm inflation gave these interesting early works a new lease on life. However none of this early work appreciated the importance of time scales, so that the dissipation could only be present if the microphysical dynamics producing it operated faster than the macroscopic time scale of expansion. This is really the key question to answer as to whether warm inflation is a viable idea. Nor did these early works pick up on the underlying fluctuation–dissipation dynamics of warm inflation, and that property is more general than just the thermal limit. Finally an expression for density perturbation was obtained in Moss's paper, but it was only for the weak dissipative regime, with his paper not understanding the distinction between dissipative regimes. In fact Fang and I made a similar oversight in deriving the same expression and thinking it was generally valid, when actually it was only for the weak regime [2] unaware of the Moss paper [63]. Yokoyama and Linde [52] had briefly commented that the presence of the dissipative coefficient in the inflaton evolution equation may affect the expression for the density perturbation but gave no details and just used the expression of Fang and me. It was only after a few years of studying warm inflation did I realize there are two very distinct regimes of warm inflation, strong $\Upsilon > 3H$ and weak $\Upsilon \leq 3H$. I then understood that Fang's and my original expression for the density perturbation was only valid in the weak regime, and I then obtained the expression for the density perturbation in the strong regime [11]. It is the strong regime that has the most interesting features of warm inflation, since with $\Upsilon > 3H$ it allows $m_\phi > H$, thus cleanly solving the η-problem [54]. Moreover, the strong regime can allow $\phi < m_p$, thus allowing all scales in the model to be below the quantum gravity scale. In more recent terms this is the best regime to overcome [65–68] all the swampland difficulties [55,56,69].

5. Density Perturbations

In the most common realization of warm inflation, density perturbations are induced from a thermal bath. They are classical on creation and thus the scenario has no quantum-to-classical transition problem as is the case for cold inflation. In cold inflation the inflaton density perturbations are dictated by the Hubble scale where modes freeze out to give $\delta\phi \sim H$. In contrast in warm inflation there are three scales: the Hubble scale, the dissipation scale, and the temperature during inflation. The dissipative term, Υ, in warm inflation can be much larger than the Hubble damping term H during inflation. Due to the Υ term, this freeze-out momentum scale can be much larger than that in cold inflation, which is $\sim H$. At the freeze-out time t_F, when the physical wavenumber $k_F = k/a(t_F)$, the mode amplitude $\overline{\delta\phi}$ can be estimated using a purely thermal spectrum,

$$\overline{\delta\phi}^2(k_F) \approx \int_{k<k_F} \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} (e^{\beta\omega_k} - 1)^{-1} \overset{T\to\infty}{\approx} \frac{k_F T}{2\pi^2} \,. \tag{12}$$

To estimate k_F, one must determine when the damping rate of Equation (3) falls below the expansion rate H, which occurs at $k_F^2 \approx (3H + \Upsilon)H$. Thus, in the strong dissipative regime $Q \equiv \Upsilon/(3H) \gg 1$, which implies $k_F \sim \sqrt{H\Upsilon}$. Substituting for k_F in Equation (12), one finds the expression for the inflaton fluctuation amplitude at freeze-out,

$$\overline{\delta\phi}^2 \sim \frac{\sqrt{H\Upsilon}T}{2\pi^2} \,. \tag{13}$$

This expression was first derived by me in [11]. In the weak dissipative regime $Q \ll 1$, the freeze-out wavenumber $k_F \sim H$, which is consistent with cold inflation, thus giving the inflaton fluctuation amplitude at freeze-out,

$$\overline{\delta\phi}^2 \sim \frac{HT}{2\pi^2}. \tag{14}$$

This expression was found by Moss [63] and then independently rediscovered by Berera and Fang [2]. In both cases the regime of its validity was wrongly understood, and in [11] the appropriate regime in which it was valid, the weak dissipative regime, was clarified.

The fact that the density perturbations in warm inflation are classical and of thermal origin some regard as an unappealing picture for the early universe, but there is no concrete argument behind this attitude. The idea that the initial primordial perturbations are quantum in origin has over the years become encased in the lore of early universe cosmology, but the factual basis for having such a beginning is lacking. There was initially some ideas about the universe being created as a quantum fluctuation. Moreover the chaotic inflation model [8] did provide some kind of dynamical picture motivating the origin of quantum fluctuations. However this model required inflation at a very high energy at the GUT scale, which has been ruled out by CMB Planck data [70] and further constrained by more recent BICEP data [71].

There has been furious effort over the decades to develop ever more cold inflation models with their own unique signatures from the density perturbations. However the data themselves have been far from revealing and the fact of the matter is what is seen there can equally be explained by the quantum fluctuations of cold inflation or the classical ones of warm inflation. So far observational data shows absolutely no preference. Moreover, of the myriad of possible interesting effects that might emerge from density perturbations, it is a model-building game and one can concoct various such features both from warm and cold inflation models. More recently there has been work to look for intrinsic features that signal a quantum origin or in its absence a classical origin [72–75]. However such tests appear to be extremely difficult and, if the CMB data remain without significant features, maybe impossible to decisively measure. The real lesson that can be taken away from these papers is just how hard it is to discriminate between quantum versus classical primordial perturbations during inflation. This makes it all the more perplexing how some adhere to the belief that the primordial fluctuations must have been quantum. In actuality there is equal reason to believe both classical and quantum processes play roles in phenomena in the early universe. This is the correct unbiased initial assumption that should be taken for a robust examination of early universe cosmology.

This adherence to the primordial fluctuations being quantum is more a statement about present attitudes in theoretical physics. This has historical parallels. Somewhat more than a century back, the established thinking was that the world was governed by deterministic classical physics. By now attitudes in theoretical physics tend to almost the other extreme and identifying quantum phenomenon where possible is all the rage. We are more fortuitous compared to the state of physics a century back in that we have an extensive understanding of both classical and quantum physics. The rational attitude is to accept both possibilities for the origin of density perturbations and let the science decide. The early universe was large enough to allow for classical behavior. In fact, for anything bigger than the quantum gravity scale, there is no argument to favor quantum fluctuations over classical ones. From what we know, inflation had to occur below the Planck scale, below the string scale and even below the lower end of the GUT scale. As the bounds on the tensor mode decreases, if interpreted in terms of inflationary dynamics, meaning the energy scale of inflation decreases, so moves even further below the quantum gravity scale, the arguments for a quantum origin of perturbations become less compelling. The universe may still have initially emerged from some type of quantum gravity scale fluctuation, but after that the ensuing dynamics need not all then be quantum. It is well possible that particle production occurred and density fluctuations then had a classical characteristic to them whether thermally induced, thermal, or any other statistical state. There is also an intermediate possibility, namely, that these primordial fluctuations have mixed quantum and classical properties.

The idea of quantum fluctuations seeding the initial density perturbations was suggested early on in the 1950s by Wheeler [76] and later considered by Harrison [77]. In these early works there was no mechanism suggested for producing such fluctuations, but it was simply asserted that their presence could explain what was known at the time about large scale structure. A noteworthy point about these early papers is they assumed these fluctuations would have been created at the quantum gravity scale m_p, with their simple argument being that at that scale classical physics fails. Linde's chaotic inflation scenario offered a mechanism that linked quantum gravity scale physics down to the GUT scale, where he postulated quantum fluctuations. His scenario did not require at that stage quantum fluctuations. There were known dynamical QFT models at that scale, so some dynamical processes such as thermalization, etc., could still be conceivable. At the moment the most stringent bound on the tensor-to-scalar ratio is from BICEP, placing the current upper bound of $r \sim 0.03$ [71]. This would correspond to an energy scale during inflation less than $\sim 10^{15}$ GeV. This implies a Hubble scale during inflation $H \sim \sqrt{V}/m_p \approx 10^{11}$ GeV, which is seven orders of magnitude above the Large Hadron Collider energy scale. From our developed theoretical knowledge about the early universe, this does not come across as a particularly fast timescale. Moreover, at this conceivable energy scale $\sim 10^{15}$ GeV for inflation, there are plenty of particle physics models that have been constructed, GUT etc., that could provide degrees of freedom operating fast enough to create a thermal or some type of multiparticle statistical state leading to classical fluctuations. Thus there is no reason to expect at this conceivable energy scale of inflation or lower, which is much below the quantum gravity scale, that primordial fluctuations must be uniquely quantum. Should a tensor mode eventually be found, based on the present bounds, we know the corresponding energy scale will be below the GUT scale. In such a case, the arguments are very compelling that such an outcome is favoring warm, not cold, inflation.

The gamble taken by cold inflationary cosmology was that the tensor mode signatures for inflation would be found at the GUT scale, based on a chaotic inflation explanation involving the simple monomial potentials. Nevertheless, there always were ample grounds to be cautious about these models, since their predictions came from a questionable regime of QFT with a sub-Hubble inflaton mass and super-Planckian field excursion, and eventually the data ruled them out. With the trends in the data not supporting the simple monomial cold inflation models, it is best now to be more open-minded. Meaningful progress in these theoretical questions about the early universe will only happen by taking a broad view. If data do eventually confirm a low tensor-to-scalar ratio, there are strong arguments that it is confirming warm rather than cold inflation.

In order to decide which is the more compelling origin of density perturbations thus the more compelling inflation picture, the data alone will not be sufficient, since there is only a limited amount of information we can measure about such an early time period of the Universe. Equally it needs to be seen which scenario is most compelling from a theoretical perspective, a point that will be discussed in greater detail in Section 9. A minimal requirement has to be that the scenario can be cleanly derived from quantum field theory. The success of quantum field theory in collider physics implies this is the best and only tool we have to explore the high energy regime. Moreover, for energy regimes yet far beyond measurement, the best we can do is rely on the rules of quantum field theory that we have learned at these lower energy scales, and if some consistent picture based on those rules emerges for higher energies, then that is the best possible prediction we can make. This of course means building a model beyond the Standard Model (SM), however using types of fields and if possible even symmetries that are known in the Standard Model. In particular such a model should not rely on gravity, since we know nothing definitive about its quantum nature and even have limited knowledge about its classical nature. In this respect the cold inflation scenario, though simple in appearance, hides many problems.

We have already mentioned the problems that emerge due to the inflaton mass being less than the Hubble scale. Whether they are infra-red, η, or swampland problems, this small mass scale seem something unwanted by quantum field theory. Likewise, a scalar field amplitude above the Planck scale introduces unknown quantum gravity concerns. Then there are quantum-to-classical transition issues.

One note here on terminology. Nowadays many researchers refer to the type of quantum field theory the Standard Model is built on as an effective field theory, suggesting it is subservient to some higher theory. Nevertheless to date there is no such established higher theory. This is all still a matter of research and speculation. This type of nomenclature is fine as a matter of convenience for those working on higher theories. However when talking about predictions and comparing to experiment, it can be misleading. It can suggest the quantum field theory we know and understand is somehow less predictive than the higher theory. Until there is an established higher theory, the quantum field theory we know, and the rules it embodies, is the most predictive tool we have. As such in this review I will refer to the quantum field theory of the Standard Model as simply quantum field theory, first-principles quantum field theory, conventional quantum field theory, or the quantum field theory we understand etc., I will include in this terminology effective field theories that have cutoff scales below the Planck scale, such as sigma models and such models involving pseudo Nambu–Goldstone bosons or other models built on symmetries found in the Standard Model.

6. Model Building

The ultimate goal of warm inflation model building is to find models computed from first-principles quantum field theory. This requires that the model produces dissipation and the microphysical dynamics operates faster than the macroscopic dynamics, so faster than the evolution of the inflaton field and the expansion rate, H, of the universe. These requirements emerge as consistency conditions in a warm inflation calculation. Finally, once a working model has been developed, one then needs to check whether its predictions are consistent with observation. Achieving all this is a very difficult task and so far only a few such warm inflation models have been developed.

Alongside this first-principles QFT model building, there has also been phenomenological warm inflation model building. In this approach one simply puts in by hand a dissipative coefficient in the inflaton evolution and then computes the resulting warm inflation. This approach is useful for exploring types of dissipative behavior that can lead to observationally consistent warm inflation models. Given how hard the first-principles approach is, this approach provides an intermediate step to studying the types of warm inflation models that could be relevant.

Here I will discuss some of the first-principles quantum field theory warm inflation models that have been developed. The first such model is what we called the distributed mass model (DMM) [4]. In these models there are a set of bosonic fields χ_i which interact with the inflaton field through shifted couplings. The interaction term in the Lagrangian which realizes such shifted couplings has the form,

$$\frac{g^2}{2}(\Phi - M_i)^2\chi_i^2, \tag{15}$$

so that when $\langle\Phi\rangle = \phi \sim M_i$, the χ_i field mass becomes small. In particular when the mass of a χ_i field decreases below the temperature scale in the Universe, it becomes thermally excited. Once thermally excited, as the background inflaton field evolves, it is able to dissipate energy into these fields. This creates a dissipative term in the inflaton evolution equation [3]. As an aside, the idea of these shifted couplings of the inflaton in our DMM model has subsequently been used to develop other types of warm inflation models including trapped inflation [78–80].

For the DMM, if these mass scales M_i are now distributed over some range that ϕ will traverse, then during evolution of ϕ, some subset of these χ fields will be light and generate a dissipative term. In order to control the radiative corrections, this needs to be extended to a Supersymmetric model. A simple superpotential that realises this model is [81],

$$W = 4m_\phi S^2 + \lambda S^3 + \sum_{i=1}^{N_M} \left[2gM_i X_i^2 + fX_i^3 - 2gSX_i^2 \right] . \tag{16}$$

Here the bosonic part of the chiral superfield $S = \Phi + \theta\psi + \theta^2 F$, with $\theta\psi \equiv \theta^\alpha \psi_\alpha$ and $\theta^2 \equiv \theta^\alpha \theta_\alpha$, is the inflaton field Φ, with $\langle \Phi \rangle = \phi$, and it interacts with both the Bose and Fermi fields of the chiral superfields $X_i = \chi_i + \theta\psi_{\chi_i} + \theta^2 F_{\chi_i}$. The potential terms of the Lagrangian are obtained from Equation (16) by standard procedures; the potential is $L_V = \int d^4x d^2\theta W(S, \{X_i\}) + h.c.$, and the auxiliary fields F and F_χ are eliminated through the "field equations", $\partial W/\partial F = \partial W/\partial F_{\chi_i} = 0$, which results in the Lagrangian only being in terms of the Bose and Fermi fields. For the above superpotential Equation (16), this leads to a Φ^4 inflaton potential with interactions to the χ_i fields similar to Equation (15) and in addition corresponding interaction terms to the Fermi fields ψ_{χ_i}. The distribution of the mass scales M_i are along the interval which ϕ traverses during the inflationary period. The Φ^4 self-coupling must remain small for successful inflation. In this SUSY theory it occurs because the renormalization group equations for the quartic coupling are proportional to the coupling itself, which means, even if there is another large coupling, this will not lead to a problem. This model can generate warm inflation with adequate e-foldings to solve the horizon and flatness problems [4], as well as produce observationally consistent primordial fluctuations [11]. It should be noted that the most general superpotential would include a term in Equation (16) linear in the X_i fields, $S^2 X_i$. It has been eliminated by hand. This term induces a ϕ-dependent mass term to all the X_i fields and so it must be very small for the success of this model. The stability of the SUSY theory under radiative corrections allows this term to be eliminated by hand. A more elegant way to prohibit the linear term in the superpotential would be by imposing a charge under some, for example GUT, symmetry so that these X_i fields are not singlets.

More recently this model was studied in [82] for various types of mass distributions. It was found the model can realize warm inflation for a a wide parameter range and in good agreement with Planck legacy data. We also found parameter ranges for this model entering into the strong dissipative regime, with the inflation mass m_ϕ just over the Hubble scale. This is not a clean solution to the swampland criteria but it comes very close.

In [53,81] it was shown that the DM model can arise from a fine structure splitting of a *single* highly degenerate mass level. Let $M \approx g|M_{i+1} - M_i|$ denote the characteristic splitting scale between adjacent levels. For typical cases studied in [4,11], it was shown in [53] that for significant expansion e-folding, $N_e > 50$, warm inflation occurred in the interval $10^3 M \lesssim \phi \lesssim 3 \times 10^3 M$ and, of note, at temperature $M \lesssim T$ and not T at the much higher scale of the mass levels $\sim 10^3 M$. The shifted mass coupling is precisely what makes these massive states light. In the string picture, this arrangement corresponds to a fine structure splitting of a highly degenerate state of very large mass, around the string scale $\sim M_{\rm string}$, with the fine structure splitting scale several orders of magnitude less than the mass of the state, say $M \lesssim M_{GUT} \sim 10^{-3} M_{\rm string}$.

The following string scenario was suggested for this model in [53]. Initially in the high temperature region, some highly degenerate and very massive level assumes a shifted mass coupling to ϕ. All the states in this level are degenerate, so at this point they all couple identically as $g^2 \sum_i (\phi - M)^2 \chi_i^2$. The string then undergoes a series of symmetry breaking that split the degeneracy and arrange the states into a DM model $\sum_i (\phi - M_i)^2 \chi_i^2$ with $0 < (M_i - M_{i+1})/M_i \ll 1$.

In many models, thermal loop corrections are difficult to control adequately to maintain the required flatness of the potential and tiny inflaton mass. The DM model could adequately control loop corrections, but the interest was to find more models. This led to developing the two stage dissipative mechanism of warm inflation based on supersymmetry [28], in which the inflaton Φ is coupled to a set of heavy fields χ and ψ_χ, which in turn are coupled to light fields y and ψ_y. The key point is that the heavy fields are not thermally excited, which means the loop corrections to the inflaton potential are only from vacuum fluctuations, and these SUSY can control. A generic superpotential that realises the two stage mechanism is

$$W_I = \sum_{i=1}^{N_\chi} \sum_{j=1}^{N_{\rm decay}} \left[gSX_i^2 + 4mX_i^2 + hX_iY_j^2 \right] , \tag{17}$$

where $S = \Phi + \psi\theta + \theta^2 F$, $X = \chi + \theta\psi_\chi + \theta^2 F_\chi$, and $Y = y + \theta\psi_y + \theta^2 F_y$ are chiral superfields. The field Φ is identified as the inflaton in this model with $\Phi = \phi + \kappa$ and $\langle\Phi\rangle = \phi$. In the context of the two stage mechanism, X are the heavy fields to which the inflaton is directly coupled and these fields in turn are coupled to light Y fields. A specific inflaton potential has to be chosen in order to assess the effect of this interaction structure on radiative corrections. Consider the case of a monomial inflaton potential with the additional superpotential term $W_\phi = \sqrt{\lambda}S^3/3$ so that

$$W = W_\phi + W_I \, . \tag{18}$$

At tree-level the inflaton potential from this is

$$V_0(\phi) = \frac{\lambda}{4}\Phi^4 \, . \tag{19}$$

When $\langle\Phi\rangle = \phi \neq 0$, observe that the vacuum energy is nonzero, which means SUSY is broken. This manifests in the splitting of masses between the χ and ψ_χ SUSY partners as,

$$\begin{aligned} m_{\psi_\chi}^2 &= \left[2g^2\phi^2 + 16\sqrt{2}mg\phi + 64m^2 \right] , \\ m_{\chi 1}^2 &= \left[\frac{1}{8}(g^2 + \frac{1}{2}\sqrt{\lambda}g)\phi^2 + \sqrt{2}mg\phi + 4m^2 \right] = m_{\psi_\chi}^2 + \sqrt{\lambda}g\phi^2 , \\ m_{\chi 2}^2 &= \left[\frac{1}{8}(g^2 - \frac{1}{2}\sqrt{\lambda}g)\phi^2 + \sqrt{2}mg\phi + 4m^2 \right] = m_{\psi_\chi}^2 - \sqrt{\lambda}g\phi^2 . \end{aligned} \tag{20}$$

This implies the one loop zero temperature effective potential correction

$$V_1(\phi) \approx \frac{9}{128\pi^2}\lambda g^2 \phi^4 \left(\ln \frac{m_{\psi_\chi}^2}{m^2} - 2 \right) \ll V_0(\phi) = \frac{\lambda}{4}\phi^4 \, . \tag{21}$$

This is more suppressed than the tree level potential Equation (19) and so will not alter the flatness of the inflaton potential. In [19] the two-stage mechanism was applied to the Φ^4 potential and we showed that dissipation suppresses the tensor-to-scalar ratio. The same behavior was observed a few years earlier in [61], well before the Planck data indicating the suppression of the tensor mode. At around this time there was anticipation based on the cold inflation chaotic Φ^4 model that a high tensor-to-scalar ratio would be found placing inflation at the GUT energy scale. Warm inflation demonstrated that this need not be the case for the Φ^4 model and that the presence of radiation during inflation could suppress the tensor mode.

The above early models of warm inflation demonstrated that this type of dynamics can be realized within quantum field theory. However these models required a large number of fields and so were quite complicated. On the one hand such large number of fields can be accommodated within string-theory-based models, as also has been demonstrated for the above models. Nevertheless a significant step forward we felt would be to find warm inflation dynamics within a much simpler model. One of the things we began to realize from developing the above models is supersymmetry, though it can control the inflaton potential, is clumsy to work with. We started exploring other symmetries that might also maintain the ultraflat potential required for inflation. One idea we had been thinking about was the inflaton as a Nambu–Goldstone boson of a broken gauge symmetry. This eventually led in 2016 to the warm little inflaton model [62]. In this model the inflaton field corresponds to the relative phase between two complex Higgs scalars that collectively break a local U(1) symmetry. Fermions couple to these complex scalars through Yukawa interactions and both set of fields satisfy a discrete interchange symmetry, essentially leading to an effective theory below the symmetry breaking scale $M \ll m_p$ involving the inflaton field and two Dirac fermions with a Lagrangian density,

$$-\mathcal{L} = gM\cos(\Phi/M)\bar{\psi}_1\psi_1 + gM\sin(\Phi/M)\bar{\psi}_2\psi_2\,, \tag{22}$$

where g is a dimensionless coupling and $\langle\Phi\rangle = \phi$. The original Lagrangian is actually written in terms of two complex scalar fields Φ_1 and Φ_2 and then these fields are represented in terms of modulus and phase. Thus there is no nonrenormalizable operators in this Lagrangian. It is just a matter of field representation, which is convenient when the two complex scalars develop nonzero vacuum expectations values, $\langle\Phi_1\rangle = \langle\Phi_2\rangle = M/\sqrt{2}$. For this Lagrangian the fermion masses are bounded from above, such that large inflaton field values do not lead to heavy fermions, and in addition there is a cancellation of the leading thermal contributions of the fermion fields to the inflaton's mass.

We showed that this model can realize warm inflation, just requiring in addition to the inflaton field, two fermonic fields and another scalar field, and for the Φ^2 inflaton potential leads to predictions for n_s and r consistent with Planck observational data. Moreover, increasing the dissipation would decrease the tensor-to-scalar ratio. This model established that warm inflation can be realized in a simple model and showed the scenario has significant relevance to observational data. We also obtained the strong dissipative regime for this model in [83], thus allowing an inflaton mass $m_\phi > H$, so cleanly overcoming any infra-red or swampland problems that a light inflaton mass can lead to.

More recently a model was suggested by Berghaus et al. [84], where the inflaton Φ has an axion-like coupling to a pure Yang–Mills gauge group,

$$\mathcal{L}_{\rm int} = \frac{\alpha}{16\pi}\frac{\Phi}{f}\tilde{G}_a^{\mu\nu}G^a_{\mu\nu}\,. \tag{23}$$

Here $G^a_{\mu\nu}$ is the field strength of an arbitrary Yang–Mills group with $\alpha = g^2_{YM}/(4\pi)$, where g_{YM} is the gauge coupling. This model was named minimal warm inflation. They showed that for a modest coupling this led to a thermal friction and a thermal bath during inflation. They also showed this model could achieve the strong dissipative regime. It would be of interest to develop more first-principles QFT warm inflation models, including exploring nonequilibrium dynamics beyond the warm near thermal equilibrium models.

7. Advantages

Before the inflation idea, Harrison [77] and Zeldovich [85] had already recognized that primordial fluctuations could be seeds for large scale structure and noted they needed to be scale invariant and even came up with an approximate value for the amplitude to be around 10^{-4}. Inflation then built on these ideas to provide a mechanism for producing the primordial fluctuations. The goal was to realize this from a consistent quantum field theory model and not just symbolic scalar field potentials, of which one can concoct many, as seen in the literature. This main goal of inflation so far has not been realized. Until it is, inflation remains only an interesting idea that still needs theoretical validation. One of the successes of inflation is that it can realize a Harrison–Zeldovich (HZ) spectrum and improve on it by providing a dynamical means to slightly alter the scale invariant spectrum through introducing a tilt. These features all emerge from a almost flat scalar field potential that is driving inflation.

Inflation would have occurred at a high energy scale above any scale for which quantum field theory has been empirically tested. From the success of nucleosynthesis we know our understanding of cosmological evolution is correct from the MeV scale to today. We also know from collider experiments how high energy physics behaves up to the 10 TeV scale in the context of the Standard Model. We also know there is no mechanism in the Standard Model for realising inflation. Thus it is safe to say that if inflation occurred, it must have been at a scale beyond where physics has been tested. Under these circumstances if we are trying to build an inflation model, the first question one must ask is what ground rules should be followed to produce a plausible model. One argument is to build models that predict interesting, often called smoking gun, predictions. Then if data show such effects, one could claim an indirect evidence for the model. A problem with that is cosmological data show precious little evidence of such exciting effects. In fact one could say, if inflation is correct, what empirical information we are able to gather about it is quite boring. Even by the time of the COBE data, it started becoming clear that inflation as seen from data was likely to be boring. Thus a second argument is that alongside the search for better empirical data as already mentioned, concurrently we have to ask the question can we build a realization of inflation that is consistent with everything we have learned theoretically and confirmed empirically about quantum fields. In the program of warm inflation this has been one of our main goals.

In order to pursue that, certain requirements have been imposed on the inflaton field for what is considered the ideal inflation model. One of these, which in my paper in 2000 [11] I called the infra-red condition, is I required that the mass of the inflaton field should be larger than the Hubble scale $m_\phi > H$. This means that the Compton wavelength of the inflaton field would be sub-Hubble. All the quantum matter fields that we have measured from colliders have masses above the Hubble scale. A quantum matter field in which the mass is sub-Hubble scale means it does not realize particles, certainly for field modes less than the Hubble scale. Our empirical understanding of quantum field theory so far has been in terms of a field and particle duality. We have no empirical knowledge what a quantum matter field is with masses that imply super-Hubble scale Compton wavelengths, or whether that even makes sense. The second requirement I imposed was that the scale of the inflaton amplitude $\langle\Phi\rangle$ (which we will just denote as ϕ) should be below the Planck scale, $\phi < m_p$. This condition arises since we have no knowledge of dynamics at the quantum gravity scale (actually we only know for sure how QFT, as we understand it, behaves up to the LHC scale, so $\sim$10 TeV but we think the next scale where our fundamental understanding breaks down is all the way up at the quantum gravity scale. It could be that our understanding of QFT breaks down at an even much lower scale than that). Thus we should not build an inflation model that breaches that scale. Some argue that the inflaton field amplitude is not the relevant scale, but rather the inflaton mass, which for example in the Φ^4 model would be $\sim\lambda\Phi^2$. Since λ is tiny in inflation models, even if $\phi > m_p$, the mass itself is sub-Planckian. However the inflaton field could also directly couple to gravity or other fields. Thus if it is of Planckian scale, it gets into uncertain dynamics. In particular when $\phi > m_p$, from an effective field theory perspective higher dimensional non-renormalizable operators, such as dimension six $V\Phi^2/m_p^2$, become important and thus can ruin the flatness of the inflaton potential [22].

I imposed these conditions more than two decades ago based entirely on empirical reasoning. Back then of course the η-problem was known as were the effects of higher dimensional operators. There was belief that these issues could be overcome with adequate model building. However today these constraints have been realized from string theory in the context of the swampland conditions, which indicates a more fundamental problem in violating them [55,56]. The swampland conditions are an elaborate argument built for a very sophisticated model. Nevertheless its final conclusion is supported by arguments based on simple reasoning. This is to say if a model deviates from the regime in which QFT has been empirically verified, thus minimally if the inflaton mass is smaller than the Hubble scale,

$$m_\phi < H ,$$

or the inflaton field amplitude is above the Planck scale,

$$\phi > m_p \, ,$$

then we are now entering the twilight zone of quantum of field theory. The path of least resistance in building a theoretically consistent model of inflation is to use only the quantum field theory as we understand it, which minimally means to not breach these boundaries.

Whether infra-red, η, higher dimensional operators, or swampland problems, etc., the writing on the wall has been clear for decades, that having an inflaton mass less than the Hubble scale or an inflaton field amplitude above the Planck scale is fraught with problems. Rather than fight against what quantum field theory clearly has difficulty with, it is prudent to explore an alternative approach that sits very comfortably in quantum field theory by looking for inflation models where the inflaton mass is larger than the Hubble scale and where the inflaton field amplitude remains sub-Planckian. If the inflaton mass is bigger than the Hubble scale, Hubble damping will have little effect in slowing down the inflaton field. Thus a dissipation term (or some other backreaction effect on the inflaton arising from particle production) is required with dissipative coefficient $\Upsilon > m_\phi > H$. However the presence of such a dissipative term will imply radiation production during inflation. This logic guided by quantum field theory consistency leads in a natural way to warm inflation. There are some first-principles quantum field theory warm inflation models which have been shown to achieve the strong dissipative regime [82–84]. There is still much to be done in this direction, but the evidence is convincing that warm inflation avoids the major model-building hurdles that hamper cold inflation.

8. Critique

Within the field of cosmology, warm inflation seems to have developed into a rebel idea. It was never my intention for that to happen. As already mentioned, when I first proposed the basic warm inflation scenario, I had little background in cosmology and in particular inflation. In science, new ideas often are greeted with interest, which has been my experience from the different areas of physics I have worked in. The general attitude is no one idea can possibly be accepted until all reasonable ideas are given full consideration. In this respect my experience in cosmology has been somewhat unusual, as I unintentionally discovered. I found that this field had a certain large group of researchers, who advocated for cold inflation to the extent that they seemed to have already decided it is the right answer and had minimal interest in considering any alternative picture of the early universe. Somehow they knew that their idea is exactly what happened in that brief minuscule fraction of a second 14 billion years ago, and there is little need for broader thinking. The only thing this attitude has accomplished is slowing the development of the field and wasting resources in the process, which could better have served research.

From my own part, I have never tried to hype up the warm inflation idea. In fact most of my effort seems to have gone in looking for all the ways warm inflation will not work. However in the process, a few gems of ideas have emerged that do work, and there are a handful of quantum-field-theory-based models that look promising for producing a complete first-principles solution of inflation. I hope that the scrutinizing attitude I have taken with warm inflation has helped to keep other researchers working on it, focused on the central problems or to do interesting model building or comparison of models with data. Oftentimes the success of cold inflation is explained in terms of the number of citations and papers it has generated. However much of the work on cold inflation, though contains its compelling features, also perpetuates the same ignorances or builds more complicated theory or QFT machinery on top of the same core problems. Science is not a democracy nor a popularity contest. Ultimately the cold (or warm) hard truths catch up to you, and for cold inflation, despite its over four decades of existence and despite its simple picture in appearance, there remain some very difficult unanswered questions about the viability of this idea from first-principles quantum field theory. Without there being clear and unambiguous answers to those questions, there is not much there. The same holds for warm inflation, but it seems to be a bit ahead in addressing the fundamental problems. Nevertheless warm inflation still needs to be better understood in terms of the underlying first-principles QFT dynamics and separately in terms of the GR evolution of density perturbations in the presence of dissipation and radiation. Inflationary cosmology has been around now long enough, that the age of innocence for this field has long past. Enthusiasm for the basic picture can no longer be sufficient to justify its prevalence.

In this respect the fundamental problems confronting inflation provide ample justification to those who have altogether given up the inflation habit and are looking for very different solutions for early universe cosmology. After the first minute or so of amazement one has at all the cosmological problems inflation can solve in one fell swoop, here the subject is a career lifetime later and yet there is no fully consistent, viable dynamical model of inflation. In theoretical physics, inflation may be one of the great ideas of our time, which upon second consideration is maybe one dare say, not so great. Inflation certainly generates a large number of papers and citations. This apparent indicator of success of inflation is also its problem, in that the idea is so vague that for almost any feature one can imagine in the CMB data or in model building, one can invent some inflation model to explain it. However this does not feel quite like the success we are typically used to calling success in theoretical physics. In theoretical physics, success has a more rigorous foundation, where the theory makes definitive prediction based on a derivation that is widely accepted. Herein lies the key point, that if inflation model building was restricted to models that were theoretically consistent, there would be a vast reduction in possible models and possible predictions from inflation. Under such restrictions whether inflation really can produce a viable model remains to be seen. It may well be that theoretical cosmology comes full circle and the ideas considered early on by Wheeler and Harrison that the superhorizon primordial density perturbations were somehow fixed by yet unknown quantum gravity dynamics, may be the right answer. The great mystery of causality may reveal itself in solving the great mystery of gravity, and that entirely may change our perspective, especially about cosmology. If we are unsuccessful in finding a fully consistent dynamical model of inflation, then we have to keep open-minded that perhaps this physics was already fixed at the quantum gravity scale, and the whole inflation program is wrong.

I am being critical here just as much of warm inflation as cold inflation. The inflation ideas as a whole may yet prove to be the ether of our time. The ether idea was of a mysterious substance that drastically alters spacetime. That very much also describes inflation. The ether idea was motivated by the best concepts of its time, electromagnetism and stress tensor, just as inflation today is motivated by General Relativity. The idea was simple but implementing it led to many complications, just like inflation. It is ironic that General Relativity marked the final end of the ether idea and yet today is used to provide the strongest argument for inflation. Some things never change.

The mysterious substance in the case of inflation is vacuum energy. Within the GR equations, such a substance leads to an exponential expansion, which is characteristic of inflation. However there has been no direct detection of such a substance exhibiting anti-gravity. Within conventional QFT as we so far understand it, the vacuum energy up to a constant is arbitrary and fundamentally unnecessary. Thus from this perspective, conventional QFT can have an arbitrary amount or not of inflation, it is completely unconstrained.

For these reasons one needs to be very careful in working with inflation. It should not be viewed as a goal that whatever is the evolving data and theory, at all costs a model of inflation must exist. Rather it should be question, a matter of scientific enquiry, that can we find a sensible and theoretically consistent inflation model in line with the data, and if we cannot, then there is no validation of it. If that becomes the case, then serious thought needs to be given to alternative ideas about the early universe beyond inflation.

At least if one has a QFT model of inflation which is otherwise consistent, thus minimally with $m_\phi > H$ and $\phi < m_p$, then such a model has only one unknown fundamental quantity, this vacuum energy, and if a tensor mode is found in the CMB, indicative of a vacuum energy, it could then more uniquely be attributed to a QFT model. If on the other hand the QFT model requires other unknown fundamental assumptions such as sub-Hubble masses or super-Planckian field excursions, then there are too many unknowns for the model to be uniquely predictive and it leaves open a bigger range of interpretations about what has been found in the CMB data.

For those of us who work on warm inflation, the alternatives are acknowledged. We have approached warm inflation as just one relevant idea that needs to be fully vetted. Suppressing alternative ideas or not fully recognizing the success of alternatives is not helpful to the development of this field. In fact, everything found in the CMB data to date that is talked about in support of cold inflation, equally is a success for warm inflation, yet this point is rarely acknowledged. Along with this, today it is often stated as a general fact that the Φ^2 and fairly closely the Φ^4 models of inflation have been ruled out due in particular to the lowering of the upper bound on any possible tensor-to-scalar ratio, but in fact such models are still consistent within warm inflation [19,62,83,86]. These have been the models for three decades that the advocates of cold inflation had been pinning their hopes on. Yet when they were ruled out, rather than even a brief moment of reflection and reassessment, thus noting the continued success of these models within warm inflation, their interest immediately turned to other more exotic models. What's going on?

I am not saying cold inflation is wrong. How do I know? Everything I know is based on what I learn from the data and theory, and so far both these are inconclusive. There is no one who knows more than that. We do not have a Gandalf here showing us the way. Our only guides are the theory and experiment. Although there are pockets of success for inflation, they should not be exaggerated, since the whole picture does not quite add up neither from the side of theory nor observation. Everyone is capable of thinking for themselves in seeing the the science is inconclusive about inflation, and even more about warm versus cold. However warm inflation does score a bit higher than cold both in having anticipated a lower tensor-to-scalar ratio, which could still be found, and in addressing the main model-building problems. There is no principled argument for focusing exclusively on cold inflation.

The introduction of the Planck 2018 data was a watershed moment [87], where one would have to conclude that there is no longer a benckmark paradigm of the early universe. Cold inflation can no longer claim that mantle. Even if any researcher still wishes to remain ignorant about warm inflation or the many other interesting alternative ideas about the early universe, the quantum gravity possibility still hangs over the whole subject. Thus inflation has a very tough requirement to come up with a fully consistent dynamical model. That is why for warm inflation, as I already mentioned, I have maintained very stringent requirements in building a model that is fully consistent with the quantum field theory we know to have worked at tested collider energy scales. As such this minimally means no scale in the model, such as the inflaton field amplitude, should be larger than m_p, since that enters the quantum gravity regime, for example in generating nonrenormalizable operator corrections. In going above the m_p scale, one must already make an assumption that quantum gravity effects somehow are subdominant to the low energy model. However as soon as any assumptions are required about quantum gravity, it is not much of a leap to simply assume quantum gravity solves all the problems. Though I have not achieved the complete goal up to now of coming up with such a fully consistent warm inflation model, I have at least shown that warm inflation can avoid the most ambiguous regimes of quantum field theory by models remaining where

$$m_\phi > H, \text{ and} \tag{24}$$
$$\phi < m_p \,. \tag{25}$$

Thus I have shown warm inflation has the basic ingredients to solve the most fundamental and pressing problems of inflation model building. Moreover, with collaborators we have developed the first simple quantum field theory model that can realize these properties and give an observationally consistent result for inflaion [62,83]. These properties arise as consequences ultimately of the particle production from interactions amongst quantum fields during inflation. As an aside, note these effects differ from the coupling of matter fields to classical gravity that then leads to particle production arising from an expanding background and/or curvature, which was examined in the seminal work of Parker [88] and subsequently by others [89,90]. Some papers also discussed this in the context of inflation [91–94], although these effects showed no appreciable change from cold inflation to the large scale predictions.

There is a more general point here that my work has demonstrated. Just as radiative and thermal corrections affect the background scalar field physics by altering its effective potential, the warm inflation lesson is that when time evolution is involved, additional QFT effects will occur, such as particle production, and these effects will react back on the background scalar field and alter its dynamics, such as by possibly allowing a larger inflaton mass or affecting the extent of the background inflaton's field value. This broader lesson from our work may be useful for other early universe models, to not ignore particle production from QFT interactions and the associated effects that come with it. For inflation, accounting for this from the time evolving background scalar field could be the missing ingredient that has balanced inflation models to allow for more sensible model building.

My above critical comments about the attitudes in the early universe cosmology field arise from direct experience in trying to develop warm inflation. Although there have been many researchers who have contributed their insights and skill into developing warm inflation, it has also faced an almost stauch lack of acceptance from that larger portion of the cosmology community that advocated for cold inflation. Mainly they have just minimized acknowledging the idea at all but also argued that the idea is in someway or another inferior. At the start their arguments centered around the Yokoyama and Linde work, despite us having shown fairly soon after that warm inflation could be realized from QFT. Once such arguments clearly became dated, others followed, such as warm inflation not being as simple as cold inflation since it requires an additional parameter, the dissipation coefficient. Cold inflation also needs an additional parameter for reheating. However in that scenario, it seemed to have been decided without any empirical evidence supporting the picture, that inflation is synonymous with a supercooled phase and that reheating must be a distinct separate phase. By now, though criticism of warm inflation is much more muted, with some past critics even working on the idea, the main defence I have heard by some of the cold inflation advocates is they believe in cold inflation or like it. Belief is a fine quality in a devotee but needs to be tempered by the facts in a scientist.

The simplicity argument, despite it obvious shortcomings, carried on being used until the Planck data showed there was no tensor mode at the high energy scale predicted by the single field Φ^2 and Φ^4 chaotic cold inflation models. It became further obsolete with the swampland work that showed that many cold inflation models may not be consistent with quantum gravity, at least within the context of string theory. On the other hand, both these developments worked in favor of the warm inflation scenario. We had seen from model calculations as early as 2009 [61] and further studied in [19], that warm inflation suppresses the tensor-to-scalar ratio, including in the simple monomial models such as Φ^2 and Φ^4. Moreover the swampland conditions showed once again that an inflaton field with mass $m_\phi > H$ and $\phi < m_p$ would be the consistent regime in this case in the context of string theory.

The Planck 2018 inflation paper [87] really demonstrated the harmful degree of advocacy by the cold inflation enthusiasts. I had informed the Planck inflation team about the successes of warm inflation in demonstrating well before their data that the tensor-to-scalar ratio should be suppressed, even in the simplest monomial models. Given that the outcome of no tensor mode detected down to the GUT scale was the big result from their analysis, thus breaking decades of expectation from cold inflation of a confirmation of the simple monomial models and possibly even its consistency condition, it is a rather big deal that warm inflation all along had what was now the consistent picture with the data. Despite this information available to them, the Planck 2018 inflation paper reported amongst its *"main results"* summary that Planck 2018 *"strongly disfavors monomial models"*, which in truth only applies to cold inflation models, but the wording with no other qualifiers suggests that no science exists beyond that picture. That is misleading. In fact one significant interpretation of the Planck 2018 results is that it has found indirect evidence for radiation and dissipation during inflation, that is suppressing the tensor-to-scalar ratio. This is a very simple explanation to the Planck 2018 data. It is the sort of bread-and-butter explanation, involving simple potentials, particle production, dissipation, etc., that has a familiarity. It offers an explanation of this remote phenomenon that one ideally wants in terms of relatable analogies. This is a far cry from cold inflation, where the explanation involves string theory, modifications to gravity and other features that so far have shown no close contact with reality. Physics is an empirical science. An unknown idea used to understand another unknown idea gains very little. Neither string theory nor modifications to gravity have been shown to be theoretically consistent, and there is no experimental evidence for them. Thus it is unclear how relevant any inflation models are that are based on these ideas. On the other hand, models based on the quantum field theory we are familiar with, no doubt extending beyond the Standard Model but following the basic rules that are understood to work, have closer connection with reality. In this respect there are at least a few warm inflation models.

Nevertheless Planck 2018 gave only the vaguest mention about our successful prediction, yet the associated papers going back to 2009 that had made this prediction were not referenced. As Planck are an observation group, their responsibility is to their data and protecting its integrity from any theoretical bias. A supporting pillar of science is impartiality of experimentalists to theory. So if they insist on discussing theory, then they need to be equitable to all ideas that predicted the trends in their data. In this respect warm inflation has been spot on and with a very simple explanation, yet was given only a vague mentioned in their paper with relevant references missing. It does not benefit any field of science when objectivity is lost and advocacy reaches the point of discarding the evidence on the ground, with facts not thoroughly reported. Here the facts remain that warm inflation bucked the theoretical trends by arguing for a lower tensor-to-scalar ratio below the detection limits of Planck, and we were correct.

If a tensor mode is eventually detected at some lower energy scale, then there are very good claims to be made that this is evidence for warm inflation [19,62,83,86]. Statistically warm inflation models compare very well to Planck data. The $\Delta\chi^2$ for warm inflation with dissipation of the type found in the warm little and two-stage models are much smaller then typically found for cold inflation models. For example for the Φ^4 potential, the $\Delta\chi^2$ for warm inflation models is very small, order 1 [95], whereas for cold inflation was found to be $\sim$40 in the Planck 2015 analysis [96]. Moreover, for the warm little inflaton model in [83], for the Φ^2 potential we found that a super-Hubble scalar inflaton mass and sub-Planckian scalar field excursion throughout inflation occurs for a tensor-to-scalar ratio $r \approx 6.4 \times 10^{-6}$ and with a spectral index $n_s \approx 0.965$ so within the 68% confidence level of the Planck 2018 legacy data [87].

Imagine for a moment that tomorrow a tensor mode is found in the CMB, which corresponds to an energy scale for inflation just below the present upper bound (which itself is just below that found by the Planck 2018 results). Then if we go by the Planck 2018 inflation paper [87] *"main results"* summary, a likely best fit to that hypothetical result is D-brane inflation. So does that mean that will mark the day when string theory will have been experimentally discovered? If we go by their main results, they say *"...inflationary models such as R^2, T and E α-attractor models, D-brane inflation and those having a potential with exponential tails provide good fits..."* to their data. The first three of these involve either modifications to gravity, supergravity, conformal or superconformal field theories, and/or string theory, so in all cases would be very major discoveries. Such models are important as intellectual exercises in the overall efforts to develop more fundamental theories of physics. However they are far from established theories of the physical world. Such models have no urgency for an accurate best fit comparison by experimentalists against their data and for them to do that is misguided. The last possibility they gave of potentials with an exponential tail is rather disappointing to find in their list. This suggests that after four decades of inflation model building, they consider it a main result to give just a form of a potential, which moreover without a higher theory that might justify it, on its own is nonrenormalizable and so without merit. It only should be added that this potential is much more complicated than the simple, renormalizable, monomial potentials that work for warm inflation. All their main result possibilities are more complicated to the much simpler conclusion, that their CMB data imply an inflation with a simple, conventionally renormalizable, potential and just a bit of radiation and dissipation. It would be irresponsible and downright unscientific to favor the complicated explanation over the simpler one. This goes against all practice in science in which when given the option to take the simplest interpretation. That is the misdirection the Planck 2018 inflation paper is heading us toward, if we simply ignore the warm inflation possibility. Out of over fifty inflation models tested (this in any case seems excessive), they did not test any warm inflation models, which are amongst the only few that are conventionally renormalizable, thus by default significant. There are only two dynamical pictures of inflation, warm and cold. If a thorough scientific analysis is the goal, then models from both pictures need to be tested against data. The particle physics dynamics implied by an interpretation of the CMB data as either warm or cold inflation is very different. Thus there needs to be extreme caution in how data are interpreted, as it will become the underlying basis for future particle physics model building.

Without a rigorous model of inflation that is fully consistent with the QFT that we know and understand, it is even dangerous to interpret any hypothetical tensor mode discovery in the CMB as inflation in the way we currently understand it. There is still a possibility that the true explanation may involve quantum gravity in some as yet subtle way. For example such a tensor mode may arise from an inert vacuum energy at the measured scale, but the primordial density perturbations may still have been fixed by quantum gravity. Moreover there may be some way the inert vacuum energy is correlated with quantum gravity. It may also be that quantum gravity is creating tensor modes of a sort we do not fully comprehend at the moment and we may confuse those signals for a vacuum energy. All this may sound like contrived explanations, but we just do not know all the unknown unknowns about quantum gravity until it is fully solved. That is why it is imperative that, if we want to interpret a hypothetical tensor mode discovery in terms of inflation as we understand it, then minimally we need a rigorous QFT model that tells us how exactly do we understand inflation. Moreover, this model needs to be not half rigorous or sort of rigorous but completely rigorous. This requirement is true for both warm and cold inflation.

In fact, in the fallout of Planck 2018 it is best to not bias our thinking just toward inflation, especially by CMB experimental groups. If any experimental group insists on discussing theory, then they need to be very disciplined in giving a fair and broad overview. Experimentalists should understand what they represent as experimentalists if they make any association between their results and string theory or any of these many speculative ideas from theoretical physics. Actions such as these shift the focus away from looking for results that might be accessible to results which, for the time being, are impossibly unreachable. One expects experimentalists, especially large experimental groups, to lean on the side of restraint in associating any theoretical idea with consistency to their data until they have confidence the data are headed with high likelihood eventually to confirm that idea. They need to think very carefully whether the fragmentary information gleaned about the early universe from the culmination of all their extensive work really is enough, to start associating it with these fantastic ideas of theoretical physics. If they insist on wanting to do this, they need to ask what kind of credibility are they giving their results if they then ignore simpler, more tame, theoretical explanations, by not giving a broad assessment of theory. Very speculative directions can be left for individuals to consider, but experimental groups should act responsibly toward the whole field and also make thoughtful use of hard earned taxpayer money. Actually, there is no need for experimentalists to get into any comparison of theoretical models. Once their data are out, within a short period some theorist will analyze it and report that some string theory model or whatever is the best fit to data and so forth. However, that is different. Nobody listens to us theorists when we talk like that. However for an experimental group to make such claims is a huge deal.

The discussion in this Section has shown that at the very least there are two quite different dynamics, warm and cold inflation, with models which agree very well with CMB data. Moreover this is likely to still hold if a tensor mode is discovered in the near future. So there already is a large degeneracy of possible early universe solutions, even before considering non-inflationary models. We are far off from any conclusive early universe theory. If a tensor mode is discovered, more needs to be assessed whether to present understanding its explanation reduces just to a comparison between warm and cold inflation. Thus, further research should be conducted in searching for non-inflationary early universe models that create tensor mode signals [97], as well as more scrutiny over secondary sources of such B-mode polarization signals. Alongside this, as discussed in the next Section, attention should be given to the level of assumptions entering a model, with those requiring the least number of assumptions then being most important for comparison against data. At this point it is to our advantage to treat all viable ideas about the very early universe on an equal footing and give them all equal tests in comparison to the data. In the long run this approach is better also for cold inflation, so that it is fully vetted and scrutinized. We need to avoid the danger of talking ourselves into believing only one idea is correct. A broader perspective is needed until clear evidence favors one idea well above all others, including the possibility that none of our present ideas for now are adequately favored.

9. Model Selection

Apparently Landau once said something to the effect that cosmologists are often in error but never in doubt. I do not know if he said this but it sounds relevant. Still today this statement holds meaning for the theoretical side of cosmology. The problem with theory in cosmology is that no matter what particle physics model one builds, only a small portion will directly be tested against what intrinsically is limited empirical information about the early universe. In particular when comparing to data, all inflation models boil down to the value of a scalar field potential at one particular point of the background field and a couple of derivatives at that point (and maybe a couple more). The lower multipole spectrum can also be fit covering up to around 10 e-folds of inflation, so giving partial information about a small region of the potential about this point. There are a few more details. The potential in question must have another point at which inflation ends, and the point of interest for comparing to data is meant to be from where the inflaton evolves long enough to create around 50 e-folds of inflation. Moreover the end temperature also becomes fixed by the inflaton model and that is involved in determining exactly how many e-folds of inflation near about 50 will be needed. For warm inflation there is also the radiation density generated during inflation, which in the most common realizations is a temperature scale. A point on the scalar potential will be associated with a temperature, which itself will depend on the parameters of the potential and some set of interaction couplings in the model. So the parametric dependence from the underlying model is a bit different in warm versus cold inflation, but the basic idea is the same.

Details aside, inflation model building boils down to finding an encasing theory that can produce a scalar field potential with a point on that potential and a small region about that point that compares well against the data. If that point on that potential of that encasing theory agrees well against the data, that does not mean one has confirmation of the encasing theory. It is the other way around. One needs to show that the encasing theory which produces the potential that has that point has some claim to be a physical theory. For example, the presence of the photon does not mean string theory has been discovered. String models contain the photon, but the challenge is to show a string model also is theoretically consistent. If there was a unique encasing model that produced the point on the potential which agreed with the cosmological data, then that would at least single out that model. However, what compounds the problem is the proliferation of inflation models in the literature, which suggests not to expect such a unique association, and also not all these models ultimately could be based on underlying correct theoretical ideas. Moreover for many encasing theories, the choice of the inflaton potential has some degree of arbitrariness, thus further dissociating the potential, which is the only part of the encasing theory that cosmological data is testing, from other aspects of the encasing theory.

The cosmological data provide an upper bound on the tensor-to-scalar index r, an amplitude of the scalar perturbation, and the spectral index n_s and possibly its running. The parameters of a given inflation model need to fit as best they can to these observables alongside some constraints such as producing a sufficient duration of inflation, the final temperature after inflation, and perhaps some theoretical constraints and consistency conditions within the model. The error in the spectral index is sufficiently wide that one should not anticipate uniquely separating out a single inflation model. Cosmic variance is an ultimate limiting factor in how accurately n_s can be measured. Narrowing these errors will certainly improve predictability, but more is still needed to help determine the best model.

The requirement that the inflation model be theoretically consistent and have a claim on being part of the physical world provides a useful guide. It suggests that another helpful measure for separating inflation models would be to characterize for each such model how speculative it is. The more speculative it is, the higher the chance that once quantum field theory and particle physics model building is better understood, the prediction from the model will very likely change and in the worst case the underlying ideas the inflation model is built on are wrong or inconsistent. Alternatively, the less speculative the encasing theory is the more predictive it is. The least speculative theory would be the Standard Model, but we know that is not adequate to explain cosmology.

This gives a guiding rule, that models of the early universe should be assessed on how much more beyond the SM is required to compare well against the cosmological data. By counting the number of speculative ideas inputted to a model, it would provide some guidance over model building. At the lowest end of speculation it would still require building an extension to the Standard Model but remaining within the rules of QFT that have been understood by the Standard Model. At a somewhat higher level of speculation, effective field theory methods, though less predictive, can still be acceptable provided the cutoff scale is below the quantum gravity scale. It is fine to still make much more speculative models at a much higher end of the speculation count, but by having such a count, it makes us more aware that such models are for the most part intellectual exercises. The models with the lowest speculation count are the serious contenders for comparing to data. In that respect models that involve any assumptions about quantum gravity or rely in an essential way on some modification to gravity are not at a stage where much is gained to compare them to the very limited data that cosmological measurements can provide. That is because first our ideas about quantum gravity or even some aspects of classical gravity beyond GR could and probably are wrong at some level at the moment, so any model based on them will most likely ultimately need adjustments or simply be wrong. Second, if one is ready to make such assumptions involving quantum gravity as part of their model, then it is not a step much further to simply assume that quantum gravity might completely solve the problems of the early universe by directions already suggested in the literature or in some entirely different way, once it is much better understood. Thus for any such model, we simply must wait until quantum gravity is much better understood before we can assess its relevance. These are points I hope our research over the years in warm inflation has tried to express through our actions in attempting to find a warm inflation model that is fully consistent with the quantum field theory as we presently understand. Admittedly, we have also crossed into the higher end of the speculation count, because it has fundamental interest and is fun to do, but at the same time we have put considerable effort to find much more experimentally relevant models that were as close an extension as possible to the QFT we presently understand. Here I will try to a provide a more systematic guide as to how to do a speculation count for any cosmological model.

Quantum gravity, string theory, higher spacetime dimensions beyond four, modifications to spacetime aside from additional dimensions, loop quantum gravity, super-Planckian field excursion, higher dimensional operators, sub-Hubble masses, modified gravity, fields where any mode propagation differs from standard QFT, supergravity, supersymmetry, effective field theory with cutoff scale at m_p, using effective field theory methods, extra fields beyond those in the Standard Model which cannot be attributed to some symmetry or higher theory, symmetries not of types in the Standard Model, symmetries included in the model, model building beyond the Standard Model, etc., each add more speculation to a model. To account for these properties, the speculation count would need two separate categories. One would be for fundamental (F) attributes that alter the quantum field theory as we presently understand it. These are all attributes with unknown unknowns, for which at the moment there is no empirical evidence for whatsoever, and alongside that there is no established theory for them. Inflation models with any fundamental speculation counts are susceptible ultimately to being wrong or needing significant modifications once any of the fundamental attributes in the count are better understood in the future. The other category would be speculations of a technical (T) nature related to standard QFT model building. These are attributes with known unknowns or known knowns. The success of the Standard Model is strong evidence that standard QFT model building is immensely successful, with ample evidence of its empirical relevance. The rules for such model building were set decades ago and by now are prescriptive, leaving much of it as a technical exercise. It can still lead to novel results, but it works with symmetries and properties of the type familiar or similar to those in the Standard Model. The attributes belonging to both categories are given in the Table 1. The speculation count based on this Table 1 applies to inflation models that have a mechanism to end inflation into the radiation dominated regime and are able to protect the flatness needed of the inflaton potential from radiative/thermal corrections.

Table 1. Range of speculative attributes in cosmological models.

Fundamental [F]
Quantum gravity
Additional spacetime dimensions above four
Modifications to gravity beyond General Relativity
Sub-Hubble mass scalar fields
Supersymmetry/other new spacetime symmetries or adjustments to them
Technical [T]
Effective field theory methods with cutoff scale below m_p
Symmetries included in the model not of the type in the Standard Model and excluding new spacetime symmetries
Symmetries included in the model
Extra fields added beyond the Standard Model and not attributed to any symmetry
Model building beyond the Standard Model

The separation into two different categories is necessary because it is an apples-and-oranges-type comparison of speculation between them. Attributes in the Fundamental category involve a conceptual leap beyond our present both theoretical and empirical knowledge. On the other hand, for models with all attributes in the Technical category, they have a familiarity based on our extensive experience with the Standard Model. Models with only Technical attributes are built on types of quantum fields, all that have been tested in collider experiments, and so there is a possibility that beyond cosmological data some particle physics based collider or astrophysical tests can also be conceived to test such models, even if indirectly. To appreciate the distinction between Fundamental and Technical attributes, suppose a new particle was discovered which could be explained through standard type of model building beyond the Standard Model. That would be exciting and extremely noteworthy, but mainly because any discovery in particle physics has basic significance, happens slowly, and after a lot of work. Yet it would be a level shift fundamentally higher if, for example, the discovery directly showed that our world had an extra dimension. The Fundamental/Technical speculation count is obtained by simply counting the attributes of a model in each category.

The fundamental count includes sub-Hubble mass scalars based on my discussion earlier in the paper. Models with super-Planckian field excursions or higher dimensional operators (not from an effective field theory with cutoff below m_p), would include quantum gravity in the fundamental count. Even if quantum gravity is not explicitly used, once a model has these features, implicitly it is affected by quantum gravity and that is an uncontrolled approximation. In the technical category, any symmetry included in the inflation model is counted to assess the complexity of the model, but if that symmetry differs significantly from types found in the SM, it is counted twice to account for the higher speculation associated with it. For symmetries much different from the SM, specifically I exclude any new spacetime symmetries, such as supersymmetry or alterations to Lorentz invariance, CPT etc., as that is already counted in the Fundamental category, but it would include for example technicolor, preon models, unparticles, etc., and for theories in the Fundamental category include symmetry details such as choice of compactification, brane type etc. The inflaton potential, which is often arbitrary in inflation models, I did not include as a separate attribute. This is partly accounted for in the model building beyond the SM attribute. Moreover from the symmetries in an inflation model, usually a scalar emerges that is identified as the inflaton. If the potential is nonrenormalizable with cutoff scale below m_p, it is also accounted for with the effective field theories with cutoff below m_p attribute and if the cutoff scale is at m_p it is accounted for with the quantum gravity attribute. Counting the number of speculations in a model mitigates the need for relying on individual opinions on this matter. Two categories are necessary since Fundamental attributes are a different degree of speculative to the Technical ones and it would be impossible to give any metric to compare between the two. Then within each category, a priori with no further information, the count treats all attributes equally and the degree of speculation in a model is simply down to how many of the attributes in the Table 1 it has.

One thing we can all agree on as theorists is a particle physics model of cosmology will require some degree of speculation, but we will never agree amongst us the degree to which each of the possibilities in the Table 1 is speculative. This speculation count provides a simple step, of at least counting each speculative attribute. What we will still not agree amongst us is what level of speculation is acceptable, since speculation might also be viewed by another word, insightful. However there are two limiting cases on which we can all agree. First, models involving quantum gravity are at the extreme end of speculation. We know so little about this thing we so conveniently call quantum gravity, that we do not even know if, near the Planck scale, physics behaves by the rules of quantum mechanics or whether at some scale, possibly much below the Planck scale, some entirely new type of physics kicks in that supercedes quantum mechanics just as that supercedes classical mechanics at around the atomic scale. Gravity is the only force which interacts directly, as far as we know, with all forms of energy and defies being encaged as a renormalizable point particle quantum field, so befuddles attempts at a unitary theory. These properties that separate gravity from all the other fundamental fields of Nature may be the earliest hints that at some high enough energy scale, its behavior goes beyond not four spacetime dimensions but rather beyond the rules of quantum mechanics. At present no one can exclude that possibility, thus there is little point in trying to argue that models requiring quantum gravity have any unique or urgent phenomenological relevance. In fact, we do not even have any definite idea how physics behaves just a couple of orders of magnitude in energy above the LHC scale, and so the Planck scale is well-beyond reach for meaningful application to phenomenology. Second, for any model at a low level of speculation, there is much less to be theoretically debated, so there is greater significance in testing how well it compares to data. In truth we do not even have a definite idea what is just around the corner at the next higher energy above the LHC scale. So even cosmological models with low speculation count should be treated with a great deal of caution. Nevertheless, for any model with a low level of speculation and especially with no Fundamental attributes, if it also fits well against data, then in a relative comparison it is amongst the best cosmological models.

The higher the speculation count, especially in the fundamental category for a model, the more it presses the question whether in order to obtain an adequate phase of quasi-exponential expansion, is the proposed model really a measured solution. For example, is adding six spacetime dimensions really a measured solution to just obtain a phase of inflation? The count forces a think about the purpose of a model. If one is developing string theory, it makes sense to see how well a string-based model can realize phenomenology relevant to real world data. However if one is interested in determining the most relevant models that agree with the data, too high a speculation count indicates those are not the primary models that should be tested. Cosmological observation basically fixes two data points, the scalar amplitude and the scalar index n_s, and gives one bound, on the tensor-to-scalar index r, and maybe a few more data points such as nongaussianity, running of n_s, isocurvature etc., As the speculation count rises, the assumptions going into a model swamp the limited data and nothing is really being tested.

Here we examine the Fundamental/Technical speculation count for a few models. We are not concerned here about how well the models compare to data, which we have already discussed in previous parts of the paper. Here the count is just assessing the theoretical aspects of these models. The potentials are written in terms of only the background inflaton field $\phi = \langle \Phi \rangle$.

D-Brane inflation model [98]: D-branes are solitonic solutions arising in string theories of type I, IIA and IIB. There is an interaction energy between two parallel brane and anti-branes, and this is the potential energy utilized to drive inflation. The inflaton field is a mode corresponding to a relative motion between two parallel branes. The model relies on the locality of the higher dimensional theory to allow for a sub-Hubble mass as necessary in cold inflation. For D-3 branes, the potential has the form,

$$V_{\mathrm{D-brane}}(\phi) = M^4(1 - \alpha/\phi^4)\ .$$

The counting of speculations from the Table 1 entering the D-brane inflation model is below. Here the square bracket indicates whether the attribute is Fundamental [F] or Technical [T], and the curved bracket gives the number of speculation counts for that attribute if it is larger than one:

- Quantum gravity [F]
- Dimensions beyond four (6) [F]
- Sub-Hubble mass inflaton [F]
- Supersymmetry [F]
- Symmetries not of type in SM—choice of compactification and D-p brane (2) [T]
- Symmetries included in the model (2) [T]
- Model building beyond the SM [T]

This gives a speculation count for Fundamental/Technical properties of 9/5. Here the choice of compactification and D-p brane I include in the technical category and not also as a fundamental attribute for new spacetime symmetries, since this model already has been penalized in the fundamental category for extra dimensions, which is sufficient.

α-attractor superconformal inflation model [99]: This is a supergravity model where the parameter α is inversely proportional to the curvature of the inflaton Kähler manifold. A common choice of potential is:

$$V_{\alpha-\mathrm{attractor}}(\phi) = \tanh^{2n}\left(\frac{\phi}{\sqrt{6\alpha}}\right),$$

for $n, \alpha > 0$. For large curvature, which corresponds to small α, the predictions agree well with CMB data. The counting of speculations entering this α-attractor model is:

- Quantum gravity [F]
- Sub-Hubble mass inflaton field [F]
- Supersymmetry [F]
- Symmetries not of type in SM—superconformal [T]
- Symmetries included in the model—three chiral multiplets and Kähler potential with superconformal and $SU(1,1)$ symmetries (5) [T]
- Model building beyond the Standard Model [T]

This gives a speculation count for Fundamental/Technical properties of 3/7.

***R*² Starobinsky model [39,100]:** This is a type of modified gravity model which has a curvature-squared $R^2/(6M^2)$ term added to the Einstein–Hilbert action, where R is the Ricci scalar and $M < m_p$. This action is transformed into the Einstein frame leading to an inflaton potential of the form,

$$V_{R^2}(\phi) = \Lambda^4 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{m_p}\right)\right]^2 .$$

The counting of speculations entering the R^2-Starobinksy model is:

- Quantum gravity [F]
- Modifications to gravity beyond GR [F]
- Sub-Hubble mass inflaton field [F]
- Symmetry not of type in SM—transform from the Jordan to Einstein frame [T]
- Symmetry included in the model [T]
- Model building beyond the Standard Model [T]

This gives a speculation count for Fundamental/Technical properties of 3/3. In the original paper by Starobinsky, he had viewed the R^2 term as dynamically generated as a self-consistent solution of the vacuum Einstein equations by one loop corrections due to quantized matter fields. The model can also have quantum gravity interactions treated semiclassically but these become subdominant for sufficient number of matter fields. Thus one could also count the assumptions from such a more first-principles approach, but that would need the details about the matter fields and interactions. Nevertheless in such a case the fundamental assumption added in our above list of modifications to gravity beyond GR would not be included, although assumptions about the underlying matter fields would need to be added.

Higgs Inflation model [101]: This model assumes there are no other fields in the universe aside from those in the Standard Model, and the Higgs field has a non-minimal coupling to gravity. In the initial Jordan frame the Higgs field, h, has a standard type of quartic symmetry breaking potential of the form $\sim\lambda(h^2 - v^2)^2$. To get rid of the non-minimal coupling to gravity, a conformal transformation is done to the Einstein frame. The Higgs field is then treated as the inflaton, which at high field value has the potential in the Einstein frame,

$$V_{\text{Higgs inflation}}(\phi) = \frac{\lambda m_p^4}{4\xi^2}\left[1 + \exp\left(-\frac{2\phi}{\sqrt{6}m_p}\right)\right]^{-2} ,$$

where ξ is a coupling constant between the Higgs field and the scalar curvature. As the authors' paper makes clear, it is not possible to have a rigorous discussion of quantum corrections due to the nonrenormalizable nature of gravity. The counting of speculations entering the Higgs inflation model is:

- Quantum gravity [F]
- Modification to gravity beyond GR [F]

- Sub-Hubble mass inflaton field [F]
- Symmetry not of type in SM—transform from the Jordan to Einstein frame [T]
- Symmetry included in the model [T]
- Model building beyond the Standard Model [T]

This gives a speculation count for Fundamental/Technical properties of 3/3.

Warm little inflaton model [62,83]: This model has two complex Higgs fields with identical $U(1)$ charges. The fields have nonzero vacuum expectation values and the phases of both fields then yield two Nambu–Goldstone bosons. The relative phase of the two fields yields a singlet which is the inflaton. The Higgs fields are coupled to left-handed fermions with $U(1)$ charge and right-handed counterparts that are gauge singlets. There is an interchange symmetry between the two bosons and two fermions and they have identical couplings. There is an additional chiral fermion and singlet bosonic field to couple with the fermions for the particle creation decay width. The interaction Lagrangian for this model is given in Equation (22). The inflaton potential for this model is simply monomials,

$$V_{\text{warm little}}(\phi) = \frac{\lambda}{4!}\phi^4 \;\text{ or }\; \frac{1}{2}m_\phi^2\phi^2\,.$$

The counting of speculations entering the warm little inflaton inflation model in the strong dissipative regime is:

- Effective field theory methods with cutoff below m_p [T]
- Symmetries included in the model—two Nambu–Goldstone bosons, two $U(1)$, two interchange (6) [T]
- Extra fields beyond the SM not attributed to any symmetries (2) [T]
- Model building beyond the Standard Model [T]

This gives a speculation count for Fundamental/Technical properties of 0/10. This is the only model studied here with no speculation counts in the fundamental category. Note that in the weak dissipative regime the model would have at least one and up to two fundamental counts for sub-Hubble mass and quantum gravity, the latter because in cases there can be super-Planckian field excursion of the inflaton field. The speculation count highlights the importance of the strong dissipative regime of warm inflation. Nevertheless even in the weak dissipative regime there are less speculation counts in the fundamental category than all the other models examined here.

For the technical speculation points in each model, one can now look closer at the given model and see how well it can be parametrically constrained. The speculation count is only assessing the physical attributes of the model. One would need to look into the details of the inflation calculation in the given model to see how constrained it is for inflation. Alongside that one can also explore if other related cosmological phenomenon such as baryongensis, dark matter, dark energy, and cosmic magnetic fields can be explained by the model or extensions of it. From this one can assess the full parametric constraints on the model. Overall, from the models tested here, only the warm little inflaton model has no fundamental speculation points and so is in the best position for comparison to cosmological data. Of course this model still should be further scrutinized but also further model building can be explored to explain other cosmological phenomenon. Building inflation models involving ideas from higher theories such as quantum gravity, etc., can be interesting, but it is extremely challenging to build an inflation model based on just the standard QFT that we presently understand. The warm little inflaton has achieved this, so is a model of considerable interest.

The speculation count provides a measure beyond comparing to observational data to assess not just inflation models but cosmological models at large. Amongst the models that agree well with the data, the speculation count provides a semi-quantitative measure to disentangle the level of assumption going into each of them. If the fundamental count is low and so is the technical count, then there is a better chance comparing that model to data can provide some insight into the unknown fundamental attributes. However if the technical count is also high, then such a model can provide little such insight. If the fundamental count is high, then irrespective of the technical count, comparison to data will provide very little insight into the unknown fundamental attributes. If there is no fundamental count, then the model is in position to consider how well the data constrains the parameters of the model and whether the model can be used to provide other cosmological predictions.

With the proliferation of inflation models, with so many agreeing well enough with the limited cosmological early universe data that it may as well be an infinite number, more is needed to help separate out models in terms of their qualities. Theoretical cosmology is different from most other fields in science in that it requires a greater level of speculation to make any progress. A speculation count is necessary in theoretical cosmology, to keep cognizant of the levels of assumption entering into model building, thus helping to keep the field on an even scientific keel. This is the needed "doubt" I believe Landau's statement in jest was trying to convey that cosmologists lack.

This count should not be misunderstood as trying to make some simple-minded separation of models as good versus bad. From a different way of thinking, purely in a mathematical physics context, D-brane inflation is one of the most elegant models. However is it a physically relevant model or even close to that, based on what we currently know about Nature? In theoretical physics there is a notion that fundamental physical theories should be mathematically elegant, but this notion must be constrained first and foremost by the theory showing adequate connection with the physical world and being mathematically consistent. As detailed earlier in this Section, cosmology experiments intrinsically can provide very limited information about the early universe. For any theories with Fundamental attributes from the Table 1, that cosmological information on its own is not adequate to confirm the theory. It requires other independent empirical information, alongside of course the theory being consistent. Overstating the empirical relevance of string-based models or those based on other higher theories also does no justice to the core theoretical work being done in those fields. It just furthers the image of these subject areas as out of touch with physics, since they are unable to distinguish between pertinent empirical models versus those mainly for theoretical interest. Counting the number of physical attributes in a model that are speculative helps to assess which models are most relevant for testing against experimental cosmological data. The count provides a semi-quantitative measure on the degree of belief of a model given the present theoretical understanding. The higher a model's speculation count, especially in the fundamental category and to a lesser extent in the technical category, the less useful is the limited data able to select it out. This speculation count is time dependent. In the past 50 years, our confidence in the Standard Model has been fully confirmed and there are greater cosmological data, but amongst the fundamental attributes in the Table 1, not much has changed. Thus the timescale of significant change for the speculation count is around or larger than a career lifetime.

The speculation count helps to avoid a lot of arguments and debates on nowhere issues, by quantifying the degree of theoretical uncertainty about a given model, and it can be obtained by a simple counting of assumptions entering in the model. This little input can help us view cosmological models a bit more objectively. The swampland development from a couple years ago is a good example where some objectivity is needed. Within theoretical cosmology that work caused considerable controversy, perhaps even a division in the field. However it has to be appreciated that our understanding of physics around the Planck scale is evolving to the extent that at present there is not even any experimental information, which typically is essential for any development in theoretical physics, at these extremely high energies. One should expect, not be surprised, at further such developments such as swampland in the future, since that is an obvious consequence of string theory and more generally quantum gravity being unsolved. Likewise the validity of models based on these ideas may also change with time.

In any event, superplanckian field excursions, which are not allowed by the swampland conditions, still imply, irrespective of the swampland conditions, that the model will be susceptible to higher dimensional operator corrections arising from quantum gravity.

These corrections are unknown and in such models one is forced to make an assumption that they do not affect the model. This is an uncontrolled assumption as it relates to quantum gravity, and once such an assumption enters model building, one may as well assume quantum gravity can sort everything out, never mind the model. Similar problems arise for models with an inflaton mass less than the Hubble scale. Thus, there is little point in venting one's ire on the swampland conditions, which has only magnified already known problems. String theorists are just doing their job in understanding a very difficult problem, and their views probably also will further evolve over time. It is not a simple-minded question of right or wrong in regards issues at and above the Planck scale. It is about how much clarity there is in the matter, and for the time being there is very little, which thus also applies to any models at this scale. This makes a speculation count useful in forcing one to accept from the onset that models with high counts, no matter how insightful, have greater chance to eventually be found theoretically incomplete or wrong. At the other end, models with low speculation count have less theoretical baggage, so more relevant to be tested against experiment. The ideal case is a speculation count of zero, so below the 10 TeV regime, where the Standard Model explains everything. We know for cosmology we have to go to a higher speculation count, but if inflation really is a viable theoretical idea, then in order to feel certain it is connected to the physical world, we need to find a model with low speculation count. We can not call some possible future tensor mode discovery in the CMB as inflation, if we do not even have a working, consistent, physically relatable, model of what this inflation is.

10. Discussion

For the many researchers who have and continue to study warm inflation, they work with confidence that data and theory indicate it is headed in the right direction. Today warm inflation is a strong contender in developing into a theory of the early universe. The strength of the underlying theoretical foundations of warm inflation provide at least some confidence that a tensor mode will eventually be detected. In this review I discussed primarily the early work done by me and the work I did with collaborators in developing warm inflation. Many researchers have made valuable contributions in developing warm inflation, including building interesting models [102–136], first-principles model building [80,137–160], as well as models that follow the basic ideas of warm inflation of particle production and associated dissipative effects during inflation [78,79,161–164]. Work on warm inflation has been done in understanding the underlying quantum field theory dynamics [3,12,14–18,26,165–167], examining density perturbations [58,59,168–175], studying non-gaussianity [176–184], and testing models to data [95,185–200]. There is work showing how warm inflation can realize cosmic magnetic fields [201–203], baryogenesis [204–206], primordial blackholes [207–211], and address the gravitino problem [212–214]. Studies have examined various aspects of the warm inflation scenario and dynamics [20,69,215–233], including for other cosmological problems [234–238], with application also of the warm inflation ideas of dissipation during vacuum energy driven expansion to dark energy [239–243] with a possible resolution to the Hubble tension [244]. There are various reviews of warm inflation [24,61,245–247].

Whether warm inflation, or inflation more generally, is the correct idea about the early universe will be decided by empirical data. Irrespective, warm inflation has introduced and substantiated two concepts that are broadly useful to early universe cosmology beyond just inflation and are here to stay. One, that particle production from quantum field interactions is possible at this early stage. Aside from filling the universe with particles, the other lesson to take from our work is the backreaction effects of this process on the source are equally important in having dynamical consequences for this early phase of the universe. Two, that the initial primordial fluctuations could be of classical rather than quantum origin, thus changing a way of thinking that had dominated for decades, even before the inflation idea. The warm inflation story is very much a part of the early universe cosmology story, and those who have ignored it, or the many other facets of this problem, have lost the plot.

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