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Constraints on Metastable Dark Energy Decaying into Dark Matter

Jônathas S. T. de Souza [1], Gustavo S. Vicente [2] and Leila L. Graef [1,*]

[1] Physics Institute, Fluminense Federal University, Niterói 24210-346, RJ, Brazil; jonathas_sts@id.uff.br
[2] Faculty of Technology, Rio de Janeiro State University, Resende 27537-000, RJ, Brazil; gustavo@fat.uerj.br
* Correspondence: leilagraef@id.uff.br

Abstract: We revisit the proposal that an energy transfer from dark energy into dark matter can be described in field theory by a first order phase transition. We analyze a metastable dark energy model proposed in the literature, using updated constraints on the decay time of a metastable dark energy from recent data. The results of our analysis show no prospects for potentially observable signals that could distinguish this scenario from the ΛCDM. We analyze, for the first time, the process of bubble nucleation in this model, showing that such model would not drive a complete transition to a dark matter dominated phase even in a distant future. Nevertheless, the model is not excluded by the latest data and we confirm that the mass of the dark matter particle that would result from such a process corresponds to the mass of an axion-like particle, which is currently one of the best motivated dark matter candidates. We argue that extensions to this model, possibly with additional couplings, still deserve further attention as it could provide an interesting and viable description for an interacting dark sector scenario based in a single scalar field.

Keywords: metastable dark energy; dark matter; bubble nucleation

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1. Introduction

One of the main goals of physical cosmology has been to understand the nature of the constituents of our Universe, especially dark matter (DM) and dark energy (DE) which, together, constitute nearly 96% of the total density of the Universe. Many theoretical models have been developed to explain the nature of the dark Universe [1]. The DM component, due to its clustering properties, has been investigated in the context of several astrophysical and cosmological experiments. Moreover, unlike the case of DE, it has also been studied at particle physics level, being sought in direct detectors on Earth. In this context, one of the most interesting possibilities is that the DM can be an axion-like particle (a good review on axion-like dark matter can be found in [2], for instance), which is one of the main candidates for this component today[1]. On the other hand, the nature of the DE component remains yet very obscure, although there have been important advances in modeling its behavior beyond the simple cosmological constant scenario.

The explanation for a late-time accelerated phase in the Universe remains a topic of much debate, which is related to the well-known cosmological constant problem [12,13]. In a different framework, there are many challenges when trying to embed such models in a more fundamental quantum gravity proposal. As an example, we can mention the Swampland conjecture, which describes a whole inhabitable landscape of field theories that are inconsistent with string theory, including the stable de Sitter vacua [14–16]. In the process of trying to understand the DE properties, we are faced with the recurring discussion in the literature concerning the issue of the stability of a de Sitter phase. In the context of the late time Universe, a stable de Sitter phase has shown either to be hard to achieve from fundamental physics or even to be inconsistent in different theoretical contexts. For example, it has been subject of a long debate whether a de Sitter space is unstable due to infrared (IR) effects, as conjectured in [17–21], for instance. An instability of a de Sitter phase has also been obtained as a consequence of the backreaction effects of super-Hubble modes. As shown in several works [22–28], the backreaction of super-Hubble modes could give a negative contribution to the effective cosmological constant, causing the latter to relax.

From the observational point of view, one motivation for investigating possibilities beyond the cosmological constant solution is the current tension in H_0 measurements, which has shown to be alleviated in some quintessence models, as well as in models with dark sector interaction [29–32]. Concerning the latter, in the framework of field theory, it can be argued that it is rather natural to consider an interaction between DM and DE, given that they are fundamental fields of the theory. Interacting models can also be claimed as a proposal for alleviating the coincidence problem [33], while also being able to provide a good fit to current data [19,34–40].

In a related framework, when going beyond the cosmological constant scenario, it is natural to investigate the possibility of a metastable DE. As pointed out in [41], the remarkable qualitative similarity between the properties of the present DE and the component that supposedly drove inflation in the very early Universe makes it rather natural to put forward the hypothesis that the current DE can also be metastable [41–50]. In this context, in [51], and more recently in [41], metastable DE phenomenological models were analyzed, in which the DE decay rate does not depend on external parameters, being assumed to be a constant depending only on DE intrinsic properties. In the latter analysis, they considered data from Pantheon compilation [52] in combination with BAO data from 6dFGS [53], MGS [54], BOSS DR12 [55], eBOSS DR14 [56], Lyα [57], and CMB [58,59], and found that the typical decay time in these scenarios must be many times larger than the age of the Universe.

One possible theoretical model that can provide a field theory description for the class of phenomenological scenarios considered in the analysis of [41] is the model proposed in [48], hereafter referred to as the MDE (Metastable Dark Energy) model. In this MDE model, a positive “cosmological constant" is modeled by a nonzero scalar vacuum energy with a potential of the expected order $V \approx 10^{-47}\text{GeV}^4$ in the false vacuum. The potential of the scalar field in this model has a doubly degenerate energy minimum with small symmetry breaking terms that provide such a small energy difference[2]. In this scenario, the field at the false vacuum represents DE. After the field passes the potential barrier, decaying from the false to the true vacuum, its equation of state is no longer that of dark energy, as it acquires non-negligible kinetic energy[3]. Analogously to what happens in the old inflationary scenario, the transition to the true vacuum occurs through the formation of bubbles of a new vacuum [65,66]. Through this process, the energy released in the conversion of the false vacuum into the true can produce a new component, which has the properties of DM. In the MDE model, there is a single scalar field describing the dark sector of the Universe. Such a field is not responsible for inflation at early times and it has no significant coupling to the standard model sector (or to the inflation), except through the gravitational interaction. In addition to the absence of significant coupling to the standard model particles, despite there being coupling with gravity, we are in the small field regime. In this regime, where the field value is much smaller than the Planck scale, quantum corrections to the mass from gravity are expected to be small.

In the previous work of [48], it was shown that the mass of the DM in this scenario would correspond to the mass of an axion-like particle for a decay time of DE on the order of the age of the Universe, as assumed in the work of [48]. The association of the equation of state of this real scalar field with the equation of state of a DM particle can be justified by the fact that after the phase transition, the oscillations about the quadratic minimum of the potential become the main important aspect [67,68]. Apart from providing a unified description of the dark sector, the fact that scalar field in the MDE model has the mass of an axion-like particle, which is considered to be among the main candidates for DM today [2–11,69,70], motivates us to further explore this model in the light of the new constraints [41].

It is important to emphasize that the field after the phase transition is not the QCD axion. Following this transition, the oscillations around the quadratic minimum of the potential become the primary factor determining its effective equation of state. Consequently, the field behaves in a manner consistent with a dark matter equation of state, with a mass comparable to that of an axion-like particle.

However, despite this effective behavior[4], the field may not possess all the characteristics of a standard axion dark matter candidate. In fact, it is debatable whether it can be classified within the broader category of axion-like particles, which encompasses a wide range of candidates [2]. Nonetheless, due to similarities in mass and equation of state, leading to similarities in the cosmological behavior, we will refer to this “axion mass-like field" as an “axion-like field" for simplicity.

Here, we revisit the MDE model using the new constraints from [41] in order to test if the model is still viable and whether the newly constrained decay time still predicts the same mass for the DM particle produced in this scenario. As discussed above, due to the similarities of the current DE and the primordial inflation, one of our main goals is also to check if this model inherits the same problems of old inflation [71,72], i.e, potentially observable inhomogeneities from the process of bubble nucleation and evolution.

In particular, we aim to address the following questions:

1. Is MDE still a viable model considering the latest cosmological constraints?
2. What is the mass of the DM resulting from this process?
3. Would the bubble nucleation process in this model lead to inhomogeneities that could plague the model?
4. Could this model leave observational imprints that could be searched for in future experiments?

In order to address these questions, we organize this paper as follows: In Section 2, we present the MDE model proposed in [48] to describe an interacting dark sector model based on field theory. In Section 3, we compute the mass of the DM particle of this model that results from the DE decay with a characteristic time compatible with the recent constraints from [41]. In Section 4, we analyze the bubble nucleation process and its subsequent dynamics in order to see if the model would predict observable inhomogeneities. In Section 5, we conclude and discuss some future prospects. In Appendix A, we demonstrate the validity of the approximations considered.

2. A Model for Dark Energy Decay

In this section, we present the MDE model proposed in [48] to describe an interacting dark energy model based on field theory. Since the model contemplates an energy exchange between dark energy and dark matter, none of these components is separately conserved. The equations describing the model are written as:

$$\dot{\rho}_{\rm DE} + 3H\rho_{\rm DE}(1+\omega_{\rm DE}) = Q_{\rm DE}, \tag{1}$$
$$\dot{\rho}_{\rm DM} + 3H\rho_{\rm DM}(1+\omega_{\rm DM}) = Q_{\rm DM}, \tag{2}$$

where dot denotes the derivative with respect to cosmic time, ω_i is the equation of state parameter (EoS), H is the Hubble parameter, Q_i is the interaction term, and the subscript i indicates DE or DM.

We consider that Q_i is linearly proportional to the energy density of DE [48], as follows:

$$Q_{\rm DM} = -Q_{\rm DE} = \Gamma\,\rho_{\rm DE}. \tag{3}$$

In the model we are going to consider, the DE decay rate, Γ, does not depend on external parameters such as the curvature of the Universe or the scale factor. Instead, the DE decay rate is assumed to be a constant (similar to the case of the radioactive decay of unstable particles and nuclei).

Once the interaction term is defined, we can write an effective EoS for DE

$$\omega_{\rm DE}^{\rm (eff)} = \omega_{\rm DE} + \frac{\Gamma}{3H}. \tag{4}$$

From Equation (4), we see that in a DE-DM interaction model, the effective EoS can have a phantom-like behavior ($\omega_{\rm DE}^{\rm (eff)} < -1$) when the decay rate is negative (energy flow DM $\rightarrow$ DE), or a quintessence-like behavior ($\omega_{\rm DE}^{\rm (eff)} > -1$) when the decay rate is positive (energy flow DE $\rightarrow$ DM), which is our focus.

In the MDE model, DE is represented by a scalar field with a potential energy endowed with doubly degenerate minima. When a small symmetry-breaking term is added, a small energy difference between the degenerate minima emerges, which we will denote by ϵ. The DE responsible for the current expansion of the Universe is represented by the scalar field at the metastable minima, where the potential energy is adjusted to $\epsilon \sim 10^{-47}\text{GeV}^4$. Such a potential can be written as

$$V(\varphi) = \left|2m\varphi - 3\lambda\varphi^2\right|^2 + Q(\varphi), \tag{5}$$

where φ is a real scalar field of mass m and coupling constant λ and $Q(\varphi)$ is the symmetry-breaking term, which is adjusted so that we have the value of cosmological constant at the metastable minimum[5]. Of course, this is a fine-tuned choice. This is the same fine tuning that exists in the standard ΛCDM model. Although the model analyzed here does not alleviate this fine tuning, it is an attempt to unify the dark sector by describing it through a single scalar field. In addition, this model can be viewed as a first step in obtaining a unified scenario that could describe a more dynamic dark energy. This possibility has become especially interesting after the recent BAO data released by the Dark Energy Spectroscopic Instrument (DESI) [73,74].

Equation (5) above is inspired in the Wess–Zumino model [60]. The potential is adjusted such that the stable minimum is the zero of the potential. The potential has minima at $\varphi = 0$ and $\varphi = \frac{2m}{3\lambda}$. The general shape of the potential is shown in Figure 1. Following the previous work [48], we will consider the value $\lambda = 10^{-2}$ for the coupling constant. Although this choice can seem fine tuned at first, we will show later that the results are not very sensitive to the value of λ. In addition, values for the coupling term with a magnitude not much bigger or much smaller than 1 can be viewed as natural choices in the context of Quantum Field Theory.

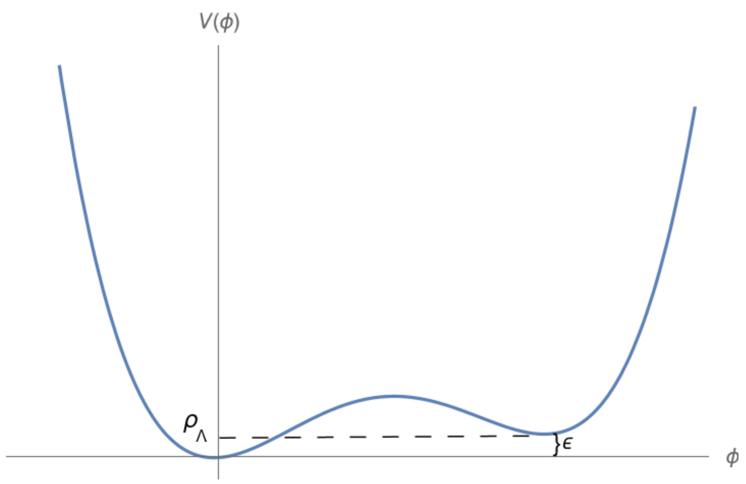

Figure 1. Shape of the potential $V(\varphi)$.

In the case we are interested in, with $\Gamma > 0$ and energy flowing from the cosmological constant into dark matter, it follows that the value of the dark matter density parameter extrapolated to high redshifts would be higher than the value predicted by ΛCDM, while the expansion rate would be lower than that in ΛCDM. If the energy transfer occurs at a non-negligible rate, this affects cosmological observables, which is why it is important to consider observational constraints in such a scenario.

In the work of [51], a generic model of metastable DE decaying into DM was analyzed and constraints on the value of the decay rate were obtained. Later in [41], a further analysis of this model was performed using more recent data. In the latter, they considered four combinations of datasets in their analysis: (1) Pantheon compilation [52] in combination with BAO data from 6dFGS [53], MGS [54], and BOSS DR12 [55]. (2) They add BAO data from eBOSS DR14 [56] to the first dataset. (3) They add high redshift BAO measurement from Lyα [57] to the second dataset. (4) They included the CMB distance prior [58,59] to the full combination of datasets. These four analyses constrained, respectively, the following values: (1) $\Gamma/H_0 = 0.47^{+0.5}_{-0.5}$ (mean with 1σ deviation); (2) $\Gamma/H_0 = 0.07^{+0.4}_{-0.3}$; (3) $\Gamma/H_0 = 0.51^{+0.27}_{-0.25}$; (4) $\Gamma/H_0 = -0.02^{+0.01}_{-0.01}$. More details on their analysis and methodology can be found in [41]. We can use these constraints in order to estimate an upper limit on the decay rate and to analyze the scenario that results from this limiting value. Although the constraints differ depending on the dataset used, we can consider $\Gamma/H_0 \leq \mathcal{O}(10^{-1})$ as a reasonable estimate for the upper limit. The analysis we are going to perform in the following sections will provide us with the mass of the resulting dark matter particle (Section 3), and the fraction of energy currently remaining in the false vacuum (Section 4).

3. Dark Matter from First-Order Phase Transition

The energy transfer in the MDE model occurs due to the tunneling from the metastable (false) to the stable (true) vacuum of the potential, Equation (5). This process can be described by a first-order phase transition according to the semi-classical method developed in [65,66]. Using this framework, we are going to compute the mass of the resulting DM particle as a function of Γ.

The decay rate (per unit volume) reads

$$\frac{\Gamma}{V} = \frac{S_E^2(\tilde{\varphi}(\rho))}{(2\pi\hbar)^2} \times \left[\frac{det'(-\partial_\mu\partial_\mu + V''(\tilde{\varphi}(\rho))}{det(-\partial_\mu\partial_\mu + V''(\varphi_+))}\right]^{-\frac{1}{2}} \times e^{-\left(\frac{S_E}{\hbar} - \frac{S_\Lambda}{\hbar}\right)}, \tag{6}$$

where φ_+ is the field amplitude at the false vacuum and $\tilde{\varphi}(\rho)$ is, in analogy with the case of particles, the classical field amplitude in Euclidean space crossing the potential $-V(\varphi)$ subject to boundary conditions $\varphi_{\text{initial}} = \varphi_{\text{final}} = \varphi_+$. S_E is the Euclidean action and S_Λ is the Euclidean action of a particle at the false vacuum. Looking for an analytical solution, we will consider the so-called thin wall approximation [65,66], as conducted in the previous work of [48]. In this approximation, the energy difference between the two minima of the potential, given by the parameter ϵ, is considered to be small, then perturbative results can be obtained in terms of ϵ.

The classical equation of motion in the Euclidean space for the field φ subject to the potential $V(\varphi)$ is obtained by the minimization of S_E, which reads

$$\frac{\delta S_E(\varphi(x))}{\delta\varphi} = -\partial_\mu\partial_\mu\varphi(x) + V'(\varphi) = 0. \tag{7}$$

We also suppose that

$$\lim_{\tau\to\pm\infty} \varphi(\vec{x},\tau) = \varphi_+, \tag{8}$$

where $\tau = -it$.

The solution is Euclidean invariant, which means that $\varphi(\vec{x},\tau) = \varphi((|\vec{x}|^2+\tau^2)^{\frac{1}{2}})$. Defining $\rho \equiv (|\vec{x}|^2+\tau^2)^{\frac{1}{2}}$, the equation of motion now reads

$$\frac{\partial^2\varphi}{\partial\rho^2} + \frac{3}{\rho}\frac{\partial}{\partial\rho}\varphi - V'(\varphi) = 0, \tag{9}$$

which is analogous to the equation of motion of a particle at position φ as a function of time ρ, subject to a potential $-V(\varphi)$ and a friction term.

The decay process occurs by the formation of bubbles of a true vacuum surrounded by the false vacuum outside. The field is at rest both inside and outside, and the friction-like term, $\frac{3}{\rho}\frac{\partial}{\partial\rho}\varphi$, is nonzero only at the bubble wall. Therefore, in the case the wall is thin, we can consider that $\rho = R$ at the wall, being R the bubble radius (see [65,66] for further details). Finally, for a small energy difference ϵ, the quantity $R = \rho$ is large and the friction coefficient $1/\rho \to 0$, then we can neglect the friction term also at the bubble wall. From these considerations, Equation (9) now reads

$$\frac{\partial^2\varphi}{\partial\rho^2} = V'(\varphi). \tag{10}$$

Then, considering the potential in Equation (5), the value of the field is each region is

$$\varphi = \begin{cases} 0 & \text{if } 0 < \rho \ll R\,, \\ \tilde{\varphi} & \text{if } R-\Delta < \rho < R+\Delta\,, \\ 2m/3\lambda & \text{if } \rho \gg R\,, \end{cases} \tag{11}$$

which corresponds to the regions inside the bubble ($0 < \rho \ll R$), at the thin wall ($\rho \approx R$) and outside the bubble ($\rho \gg R$).

Now, we compute the action by adding the separated contribution of each region, which reads

$$\begin{aligned} S \equiv S_E - S_\Lambda &\approx 2\pi^2\left(\int_0^{R-\Delta} d\rho\,\rho^3(-\epsilon) + \int_{R-\Delta}^{R+\Delta} d\rho\,\rho^3\left[\frac{1}{2}\left(\frac{d\tilde{\varphi}}{d\rho}\right)^2 + U\right] + \int_{R+\Delta}^{\infty} d\rho\,\rho^3\,0\,(\mathrm{GeV}^4)\right) \\ &= -2\pi^2\epsilon\frac{R^4}{4} + 2\pi^2R^3\int_{R-\Delta}^{R+\Delta} d\rho\left(\frac{1}{2}\left(\frac{d\tilde{\varphi}}{d\rho}\right)^2 + U\right) \\ &= -\frac{\pi^2R^4}{2}\epsilon + 2\pi^2R^3S_1, \end{aligned} \tag{12}$$

where Δ represents the width of the wall, and we defined

$$S_1 \equiv \int_{R-\Delta}^{R+\Delta} d\rho\left[\frac{1}{2}\left(\frac{d\tilde{\varphi}}{d\rho}\right)^2 + U\right]. \tag{13}$$

Minimizing the action S with respect to R, one obtains

$$\frac{dS}{dR} = -2\pi^2R^3\epsilon + 6\pi^2R^2S_1 = 0, \tag{14}$$

which gives the solution

$$R = \frac{3S_1}{\epsilon}. \tag{15}$$

For small ϵ, integrating Equation (10), we obtain $\frac{\partial}{\partial\rho}\varphi = \sqrt{2U}$. Using this expression and considering the potential from Equation (5), one obtains from Equation (13)

$$S_1 = \sqrt{2}\left(\frac{4m^3}{27\lambda^2}\right). \tag{16}$$

Finally, using Equation (16) in Equation (12), the action will result in

$$S \approx \frac{m^{12}}{\lambda^8 \epsilon^3}. \tag{17}$$

In the decay rate expression, Equation (6), the exponential term will dominate. We also know the pre-exponential term has the dimension of energy to the $4th$ power and that it will weight the overall value; then, we simply estimate it as $1\,\mathrm{GeV}^4$ in order to provide correct units. Therefore, Equation (6) can be written as

$$\frac{\Gamma}{V} = \exp\left\{-\frac{m^{12}}{\lambda^8 \epsilon^3}\right\} \mathrm{GeV}^4. \tag{18}$$

From the above equation, we can see that while λ is raised to the eighth power, the mass is raised to the twelfth power. This shows that the results we are going to obtain in this work are not very sensitive to the choice of λ.

Finally, by substituting the values of ϵ and λ, we obtain for the decay rate (per volume)

$$\frac{\Gamma}{V} = \exp\left\{-10^{157}\left(\frac{m}{\mathrm{GeV}}\right)^{12}\right\} \mathrm{GeV}^4. \tag{19}$$

One can verify that, within the validity of the semiclassical approach we are considering, maintaining the pre-exponential term in Equation (6) would have changed the result at most by a factor of 2. This factor of 2 would be multiplied by a factor of 10^{157} in the equation above. Therefore, it is really sufficient for our purposes to consider the pre-exponential term to be of order 1 GeV^4.

Due to the symmetry of our problem, we can invert Γ/V, take its fourth root, and interpret it as the decay time, t_{decay}. If we equate the decay time to the age of universe, $t_0 = 10^{17}$ s, as was conducted in [48], the value $m \sim 10^{-13}\,\mathrm{GeV}$ is obtained. This result implies in a mass of an axion-like DM particle [48]. The equation of state of this real scalar field can be associated with a DM particle since, after the phase transition, oscillations about the quadratic minimum of the potential are the important aspect to determine the equation of state [67,68].

We felt motivated to further explore this model in the light of the new cosmological data [41]. In order to do so, let us now consider the case when the inverse decay rate is not equated to the age of the Universe, but to an arbitrary decay time. We parameterize this case by introducing the parameter α, assuming arbitrary values smaller than 1. If we consider the quantity $(\Gamma/V)^{-1/4}$ to be related to the age of the universe t_0, the quantity $(\alpha\,\Gamma/V)^{-1/4}$ will imply in a decay time of the form $t_0/(\alpha)^{1/4}$. As a consequence, different choices of the parameter α (different choices of the decay rate/decay time) result in different values for the mass m of the DM particle.

In Figure 2, we illustrate the dependence of the mass of the resulting DM particle on α, with α ranging from 10^{-10} to 10^{-1}. From this figure, we notice that there is no change in the order of magnitude of the mass for this whole range of values of α.

The behavior shown in Figure 2 can be described by the analytical expression

$$m(\alpha) = 1.12246 \times 10^{-13}\,\sqrt[12]{94.8245 - \ln\,\alpha}, \tag{20}$$

which shows the weak dependence of the mass on α. The order of magnitude of the mass only changes for $\alpha \sim 10^{-107598}$, which corresponds to a decay time $10^{26900} t_0$. Obviously, such a case is not interesting for us. Therefore, for any non-negligible (and observationally allowed) decay rate, the prediction for the DM mass, $m \sim 10^{-13}\,\mathrm{GeV}$, remains consistent. In addition to that, one can verify that these results also do not change if one considers other similar values for the coupling constant, as $\lambda = 10^{-1}$ for instance.

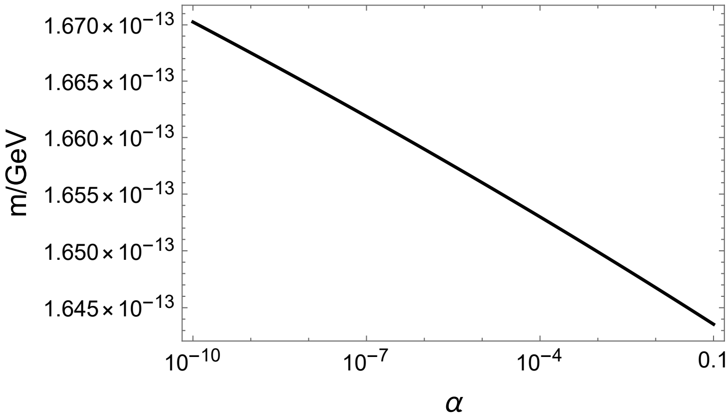

Figure 2. Dependence of the mass on the parameter α, with the latter ranging from 10^{-10} to 10^{-1}.

As is well known, first-order phase transitions occur through the random nucleation of bubbles. In this process, firstly the false vacuum energy is transferred to the kinetic energy of the bubbles wall, which asymptotically expands at the speed of light [66,75–77]. However, in some cases, at some point, the walls of these bubbles may percolate, and the energy of the walls is converted into particles (energy density) that eventually thermalize [72,78–80]. Let us analyze this process further in the following section.

4. Bubble Nucleation

First-order phase transitions, as the one considered here, occur through the nucleation and growth of bubbles of the new phase[6]. This is similar to the process that happens in old inflationary models. In the case of old inflation, in order for the decay process to occur efficiently, it was necessary to have an inflation decay rate higher than a certain minimum value, in order to end inflation successfully. However, such a high decay rate would imply a bubble nucleation process, which leads to a highly inhomogeneous Universe. Given the similarities between the phase transition described here and the process that plagued the old inflationary model, we think it is important to discuss some implications of such a process in the context of the model considered here.

In our low-temperature late-time model for the dark sector, the bubble nucleation occurs through Coleman–Callan tunneling and the nucleation rate is essentially temperature-independent. Therefore, we consider the idealized model for the transition described in [71], in which the Universe is taken to be always at zero temperature, with negligible curvature. We can also approximately consider a de Sitter exponential expansion, $a(t) \propto e^{H\,t}$, in the late time Universe, H being the Hubble parameter assumed to be approximately constant. This is a sufficiently good approximation for our practical purposes. The de Sitter approximation can also be justified by the fact that the recent constraints that we are considering for the decay time implies a decay process that will only be effective in a distant future when the Universe is even more close to a de Sitter expansion.

We consider that bubble nucleation begins at a time t_B, and afterward occurs at a constant rate per unit of physical volume Γ/V_0. The decay rate per unit of coordinate volume is then given by $\Gamma(t)/V = (\Gamma/V_0)\mathrm{e}^{3Ht}$. We can consider the approximation in which a bubble starts expanding from a very small radius $r_0 \ll H^{-1}$. In order to show that this is the case, let us estimate the initial radius of the bubble from which it starts expanding. Using Equations (15) and (16), we can estimate this radius to be

$$R_0 = \sqrt{2}\left(\frac{12m^3}{27\lambda^2\epsilon}\right) \approx 10^{-2}\mathrm{cm}, \tag{21}$$

where in the last equality we used the particle mass determined in Section 3, $m \approx 10^{-13}$GeV, together with the values $\lambda = 10^{-2}$ and $\epsilon = 10^{-47}\mathrm{GeV}^4$.

In a successful first-order phase transition, most of the bubbles are nucleated and collide in a time interval comparable to or shorter than the Hubble time. This is the so-called fast transition, in which the nucleation rate is comparable to the expansion rate of the Universe [71,72]. After bubbles collide, in successful cases, the distribution of energy in the Universe becomes homogeneous as the relevant bubbles are sub-horizon sized when they collide. In the slow transitions, the nucleation rate is much smaller than the expansion rate of the Universe. We will show that in the case of the MDE model considered here, rare and very large bubbles are nucleated in a cosmological time.These bubbles can grow to astrophysical sizes and their dynamics cover a much longer period of time. After such a bubble is nucleated, it soon starts expanding at the speed of light [65,66]. For a bubble that emerges at a point $(t_B, \vec{x}_0)$ with a very small radius, at a later time t its radius will be given by

$$r(t, t_B) = |\vec{x} - \vec{x}_0| = \int_{t_B}^{t} dt' a(t')^{-1} \, . \tag{22}$$

Evaluating the integral above gives us the result $r(t, t_B) = H^{-1}(\mathrm{e}^{-Ht_B} - \mathrm{e}^{-Ht})$. We can see that the bubble radius will asymptotically assume the finite value

$$r_A \equiv \lim_{t\to\infty} r(t, t_B) = H^{-1}\mathrm{e}^{-Ht_B}, \tag{23}$$

hence, the volume of the Universe occupied by this bubble will asymptotically be

$$V_{\mathrm{asym}} = \frac{4\pi}{3H^3}\mathrm{e}^{-3Ht_B} \, . \tag{24}$$

Due to the approximately de Sitter expansion, the bubble will grow in comoving size for only about a H^{-1} time, and after that, being super-Hubble, it simply stretches with the scale factor as the universe expands. This suggests that independently from the time past after the nucleation, two bubbles that emerge simultaneously at points $(t_B, \vec{x}_0')$ and $(t_B, \vec{x}_0'')$ at a proper distance $D = a(t_B) \times \left|\vec{x}_0' - \vec{x}_0''\right| > 2H^{-1}$ will never percolate.

It is possible to analyze the evolution of the bubble distribution in the Universe by investigating the physical volume remaining in the false vacuum, given by $V_{phys} \propto a^3(t)p(t)$, where $p(t)$ is the probability that a given point in space is in the false vacuum at a time t. The quantity $p(t)$ can be written in the following form [71,72,85–87]

$$p(t) = \mathrm{e}^{-I(t)} \, , \tag{25}$$

where $I(t)$ is the expected volume of true-vacuum bubbles per unit volume of space at time t[7]. For a phase transition beginning at time t_0, we can expect this volume to be given, at time t, by the expression

$$I(t) = \frac{4\pi}{3}\int_{t_0}^{t} dt' (\Gamma/V_0) a^3(t') r^3(t, t') \, . \tag{26}$$

where a constant decay rate was considered Γ for the MDE model. Above, $r(t, t')$ is the coordinate radius at time t of a bubble that was nucleated at a time t', expressed in Equation (22). Note that we are integrating in the nucleation times. Above, we considered that the decay rate in the MDE model is a constant.

The equation above can be better understood if we view the quantity $I(t)$ as a function of the scale factor rather than t. In this case, we can rewrite the equation above as

$$I(t) = \frac{4\pi}{3}\int_{R_0}^{R(t)} da \frac{(\Gamma/V_0)}{H} a^2 \left(\int_{a}^{R(t)} da' \frac{1}{a'^2 H}\right)^3 \, . \tag{27}$$

Above, we used Equation (22) for $r(t, t')$. The integrals in this expression are dominated by the upper limit of the integration range. We can see that the quantity $(\Gamma/V_0)/H^4$ sets the magnitude of $I(t)$. Up to a numerical factor, this quantity measures the fraction of space occupied by large bubbles, which is given by $1 - p(t)$. Therefore, provided that this fraction is small, as it will be in all cases of interest, $1 - p(t) \approx I(t)$.

Even expanding at the speed of light, a bubble which nucleates at a time t_B can only grow to a finite comoving radius given by Equation (23). Then, as shown above, if the separation between two bubbles at time t is greater than $2r_A$, the bubbles will never meet. Therefore, the bubbles nucleated in a time interval of duration of H^{-1} can never fill space by themselves, but instead only occupy a fraction of order $(\Gamma/V_0)/H^4$ of the region which remained in the old phase at the time that they were nucleated. Although bubble nucleation continues indefinitely, and despite that the physical volume of the old phase region (proportional to $a(t)^3 p(t)$) is an increasing function of time, the bubbles produced have smaller and smaller comoving volume and so can fit in the remaining regions of old phase without overlapping.

Another simple way of understanding this is remembering that after a bubble is nucleated and grown to a size of about a Hubble radius, its size simply conformally stretches as the Universe expands. Then, from that point on, the volume fraction of the Universe that it occupies remains constant. The volume fraction occupied by the bubbles nucleated during the time interval $\Delta t = H^{-1}$ is roughly the volume of a bubble when it begins conformally stretching, which is around H^{-3}, times the number of such bubbles nucleated in this interval per unit volume given by $\Gamma/(V_0 H)$. Therefore the quantity $(\Gamma/V_0)/H^4$ indicates the volume fraction of space occupied by bubbles nucleated over a Hubble time at a given epoch. We will denote by ξ this important quantity,

$$\xi \equiv \frac{\Gamma/V_0}{H^4}. \tag{28}$$

A slow transition is considered to be the one in which the quantity above is much smaller than one, $\xi \ll 1$, and the Coleman–Callan tunneling is the only significant mechanism of bubble nucleation. When this is not the case, the transition is considered to be fast.

The decay time in our model is given by the quantity $t_{\text{decay}} = (\Gamma/V)^{-1/4}$. As discussed in Section 3, we can write this decay time as $t_{decay} \equiv \alpha^{-1/4} t_0$. Therefore, we can write the quantity ξ as

$$\xi = \frac{t_{\text{decay}}^{-4}}{t_0^{-4}} = \alpha. \tag{29}$$

We can consider in the above equation the absolute mean values for Γ/H_0 described in Section 2, which were obtained in the analysis of [41] (see Table 4 of this reference). For the values constrained with any of the four datasets used, Equation (29) gives us $\xi \ll 1$. Therefore, one can safely consider the slow transition regime.

We can hence conclude that such a model will not drive a complete transition to a dark matter-dominated phase. Even in the future, there will be a dominant region of the universe in the metastable phase, where we would still have an approximate de Sitter expansion. However, unlike old inflation, there is no observational restriction imposing that a complete transition from the dark energy state into dark matter must take place.

As shown in this section, the initial radius of the bubble from which it starts expanding is $R_0 \approx 10^{-2}$ cm. Although it expands very fast, the volume fraction of space occupied by bubbles nucleated over a Hubble time at a given epoch is much smaller than one, as shown above. Therefore, there are negligible cosmological consequences from the energy density associated with these bubbles. Since here we are only interested in the cosmological scales, we will not discuss further the issue of domain walls or gravitational waves.

5. Conclusions and Prospects

We considered the model proposed in [48], in which an energy transfer from dark energy into dark matter is described in field theory by a first-order phase transition. We further investigate this model in light of the recent cosmological data. We find that the model is not excluded by the data, although it is not distinguishable from ΛCDM. Since recent data constrain a decay time for metastable dark energy considerably larger than the age of the Universe, there is currently no prospect of observing the outcome of the DE decay process, which we show to be a dark matter particle with a mass corresponding to that of an axion-like particle, $m \sim 10^{-13}$GeV. We analyzed, for the first time, the process of bubble nucleation in this model, showing that such a model would not drive a complete transition to a dark matter-dominated phase even in a distant future.

In particular, in this work, we provided the following answers to some, until now, open questions:

1. Considering the recent cosmological data, the model proposed in [48] can still be considered, formally, a viable model for describing an unified dark sector.
2. The recent constraints in the decay time of the metastable dark energy imply in a resulting DM with a mass of an axion-like particle, although this resulting DM would only appear in the far future.
3. We do not expect this model to lead to observational imprints that could be searched for in future experiments, unless extra couplings are added to the Lagrangian of the model.
4. The bubble nucleation process was analyzed and we showed that the model considered, apart from not leading to current observable inhomogeneities, would not drive a complete transition to a dark matter-dominated phase, even in the far future.

We can conclude that in order for this model to successfully describe a transition to the true vacuum, the required value of the DE decay rate would need to be bigger than the range allowed by the current observations. Although this model does not currently inherit the characteristics of a typical interacting dark sector model due to the large decay time, it still presents some qualitative advantages, as it is able to describe a dark sector in a unified manner through a single scalar field. In addition, the fact that around the true vacuum, this field behaves as a DM with a mass consistent with the one of an axion-like particle, gives us an indication of the potential of the model. For these reasons, we believe that possible extensions of this model deserve further investigation, as they could lead to potentially observable signatures in case additional couplings are included in the model Lagrangian (in this context see for example the works of [44,45]). The model analyzed here can be viewed as a first step in obtaining a unified scenario that could describe a more dynamic dark energy. Furthermore, recently the new BAO data released by the Dark Energy Spectroscopic Instrument (DESI) [73,74] have encouraged the analysis of more dynamical DE models. In addition to that, there is evidence from different theoretical contexts that exact de Sitter solutions with a positive cosmological constant may not be suitable to describe the late-time universe. DE models based on scalar fields evolving in time are more promising in this regard, although in the context of the Swampland conjecture, for example, they still have to satisfy certain criteria [14–16]. Verifying whether extensions of the model here could satisfy these and other theoretical conjectures, and leave observable traces, is an issue left for a future work.

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Appendix A. Gravitational Effects

In this appendix, we will discuss whether it is necessary to include corrections in the action due to gravitational effects. In order to investigate these effects, let us work with the following action:

$$\bar{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) - \frac{\mathcal{R}}{16\pi G}\right), \tag{A1}$$

where $\mathcal{R}$ is the curvature scalar. Using the thin wall approximation, which is still valid for the action (A1) in the cases of interest, it is straightforward to show the following relation between the action $\bar{S}$ and the action S_0 without gravitational effects [88]

$$\bar{S} = \frac{S_0}{\left[1+\left(\frac{R_0}{2\Delta}\right)^2\right]^2}. \tag{A2}$$

Above R_0 is the initial radius of the bubble calculated in Equation (21) and Δ is the Schwarzschild radius associated with the bubble of the new vacuum, which is given by $\Delta = 2G\epsilon(4\pi R_0^3/3)$. The expression for Δ can be understood from the fact that in the decay process from the metastable vacuum with energy ϵ to the stable vacuum with zero energy, there exists liberation of energy proportional to the energy density of the metastable vacuum and the volume of the nucleated new vacuum bubble. From Equation (A2), we can see that if $R_0/\Delta \sim 1$, we need to consider gravitational effects in our calculations. The gravitational effects will be important when the radius of the new vacuum bubble is of the order of the Schwarzschild radius. We can obtain the radius of a nucleated bubble that would be equal to its Schwarzschild radius by equating $R_0 = \Delta$, which gives us

$$R_0 = \sqrt{\frac{3}{8\pi G\epsilon}}. \tag{A3}$$

Using Equations (15) and (16), we can see that such an initial radius would correspond to the decay of a particle with a mass of order $m \sim 10^{-2,8}$ GeV, which is much heavier than the particle we obtained. Therefore, the addition of gravitational effects would not significantly alter the results we obtained.

Notes

1 For constraints on DM mass in related contexts see for instance [3–11].

2 There are examples in which this configuration can naturally appear as, for example, in the case of the Wess–Zumino model [60], which has a double degenerate bosonic vacuum due to super-symmetry, presumably broken only non perturbatively.

3 Other interesting models which consider a unified dark sector through a single field can be found for instance in [61–64] and references therein.

4 We can think of any field theory in the context of cosmology as an effective field theory valid until some energy scale. The same is true for our model. We can think of our model as an effective model at low energies. Since this field in the metastable vacuum accounts for today's dark energy, having an energy density of the order of 10^{-47} GeV^4, it has a negligible cosmological contribution at earlier times. Any quantum correction at early times would be associated with a field with totally negligible contribution to the total energy, having no impact on cosmology, which is the reason why we do not explore these issues in the present work.

5 Another interesting possibility is to consider the scalar field with the even self-interactions up to sixth order, as analyzed in the work of [44].

6 Evolution of bubbles of new vacuum in de Sitter backgrounds has been studied in different context (see, for instance, [75–77,80–82]). In addition, first-order phase transitions have recently been considered as one of the possible explanations for the positive evidence of a low-frequency stochastic gravitational-wave background found in PTA experiments, see for example [83]. Another recent interesting application of first-order transitions is the New Early dark Energy models, see for instance [84].

7 As discussed in [72], the exponentiation of $I(t)$ corrects for some effects, like the fact that when calculating $I(t)$, regions in which bubbles overlap are counted twice. Furthermore, the virtual bubbles which would have nucleated and had their point of nucleation not already be in a true-vacuum region are also included.

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