

Article

Hayward Boson Stars

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Abstract

We examined the Einstein–Klein–Gordon system coupled to a nonlinear electrodynamics framework that asymptotically supports a Hayward spacetime. We explored the solution space of a static, spherically symmetric, complex scalar field minimally coupled to the gravitational field, known as Hayward boson stars. We constructed families of Hayward boson stars in the ground state, for different values of the charge parameter Q and different values of the central scalar field. One of the main results of our analysis is the fact that the existence of boson stars is guaranteed once the condition $\frac{\sqrt{\beta}}{Q} > 1.49661$, where β is a parameter of the electrodynamic theory, is fulfilled. When this condition is not satisfied, the electrovacuum spacetime contains at least one horizon, and the spacetime can not support a scalar field configuration.

Keywords: boson stars; Hayward spacetime; regular black holes; compact objects

1. Introduction

Boson stars (BSs) are self-gravitating, regular, and asymptotically flat configurations of massive bosonic fields (either scalar or Proca fields), whose gravitational collapse is prevented by the Heisenberg uncertainty principle [1–3]. These gravitationally bound objects arise as solutions in General Relativity (GR) coupled to massive bosonic fields with continuous symmetry [4,5]. The existence of this symmetry allows us to build a stationary stress-energy tensor that, via Einstein’s equations, produces a stationary spacetime. The simplest model of boson stars is the one that considers spherical symmetry and a potential with a mass term; these are also called mini-boson stars [6–8].

Generalizations of BSs by including couplings to electromagnetic fields have been extensively studied; some examples are found in Refs. [9–14]. These studies enriched the range of possible solutions in scalar field theories, allowing for a more comprehensive understanding of how scalar fields interact with fundamental forces, particularly gravity and electromagnetism.

Despite lacking an event horizon, the external spacetime geometry of a BS can closely mimic that of black holes (BHs), particularly in terms of mass, compactness, and gravitational lensing properties [15,16]. As a result, BSs are capable of reproducing several observational signatures typically associated with black holes, such as the bending of light [17], quasi-normal mode spectra [18,19], and accretion disk behavior [20–22].

Although classical BHs are well described by GR, they present a singularity where the theory is unable to give a complete description [23,24]. Many efforts have been made in order to alleviate the problem of singularities in GR. The seminal work of Bardeen on



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a regular black hole [25–27] gave rise to several studies exploring nonsingular solutions within the GR framework, inspiring a broad class of models in which central singularities are avoided through modifications to the matter content. A common approach to describing regular BHs within GR has been to consider nonlinear electrodynamics (NLEDs) as the source of the spacetime [28–31].

The Hayward solution [32] introduces a carefully constructed mass function designed to eliminate the central singularity in such a way that the mass function interpolates smoothly between the Schwarzschild behavior at large distances and a de Sitter-like core near the origin. Later, Fan and Wang showed in Ref. [33] that the underlying physical mechanism responsible for this regularity can be modeled through an NLED incorporating a magnetic monopole source in Einstein’s field equations. In this context, magnetic monopole-like configurations arising from the NLED field provide the necessary energy conditions to support a non-singular geometry (see also [34]). In addition to the original Bardeen and Hayward constructions, the landscape of regular black hole solutions has significantly broadened in recent years. For instance, Lorentzian–Euclidean black hole models have been proposed, in which regularity is achieved through a change in spacetime signature connecting Lorentzian and Euclidean regions [35–37]. Moreover, black hole geometries that incorporate dark matter halo profiles have been explored as phenomenological extensions that modify the intermediate and large-scale structure while preserving regularity at the core [38–40].

Other well-known examples include a variety of regular solutions arising in modified gravity theories and effective quantum gravity-inspired scenarios. These developments highlight the diversity of mechanisms leading to nonsingular spacetimes and further motivate the study of matter fields in regular black hole backgrounds [41–46].

The Hayward metric has been used as a prototypical example of a regular BH while remaining asymptotically flat and maintaining some key features expected from a compact object [47].

The purpose of this paper is to describe the properties of non-charged boson stars in an NLED model inspired by Hayward spacetime, as shown in Ref. [48], where Hayward boson stars (HyBSs) were introduced for the first time.

HyBSs are obtained by coupling Einstein gravity to a massive, uncharged scalar field and incorporating the NLED framework proposed by Fan and Wang, originally developed to support the Hayward spacetime. These configurations represent self-consistent, regular solutions in which the interaction among gravitational, scalar, and electromagnetic fields gives rise to relevant structures in the context of modified compact objects. We consider a non-charged scalar field whose bosonic component interacts with the electromagnetic component only via the gravitational field. Although different couplings are possible, such a simple interaction allows us to extract some conclusions regarding the existence and properties of solutions of the system. In Ref. [49], other NLEDs inspired by the Bardeen spacetime have been used to model Bardeen stars. In that work, the emphasis has been on the limiting behavior of solutions whose frequency $\omega \rightarrow 0$, describing frozen stars [50].

In this work, we present sequences of HyBS solutions and describe some of their properties. We construct families of HyBSs, concentrating on the ground-state configurations for various values of the charge parameter Q , and different central amplitudes of the scalar field.

One of the key findings of our analysis is that HyBS solutions exist only when the ratio $\frac{\sqrt{\beta}}{Q}$, where β is the electrodynamic parameter described below, exceeds a certain critical threshold. This condition defines the region in the parameter space where regular, self-gravitating scalar field configurations can form.

The paper is organized as follows. In Section 2, we introduce the model of a scalar field in Einstein gravity coupled to an NLED and describe the requirements to construct a spherically symmetric object, including boundary conditions. In Section 3, we construct a variety of solutions of BSs and compare them with the known Hayward spacetime. We also revisit some of the properties of a Hayward spacetime with no horizons. By the end of the section, we describe the spacetime of the HyBs. Finally, in Section 4, we provide a brief discussion on the results and draw some conclusions.

2. Set Up

We consider the Einstein–electromagnetic–scalar model with the action given by

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi} - \mathcal{L}(F) - \nabla_\alpha \Psi^* \nabla^\alpha \Psi - U(|\Psi|) \right], \quad (1)$$

where R is the Ricci scalar associated with the spacetime metric $g_{\mu\nu}$, which has determinant g , $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the Maxwell 2-form, $F = F^{\alpha\beta} F_{\alpha\beta}$, A_α is the gauge 4-potential, Ψ is a complex scalar field, $*$ denotes complex conjugate and $U(|\Psi|)$ denotes the scalar potential, which in this work is taken as $U = \mu^2 |\Psi|^2$. The parameter μ is the associated mass of the scalar field particle [51]. The specific form of the nonlinear electrodynamics Lagrangian

$$\mathcal{L}(F) = \frac{3}{\beta\pi} \frac{(\beta F)^{3/2}}{[1 + (\beta F)^{3/4}]^2}, \quad (2)$$

corresponds to the nonlinear electromagnetic theory introduced in Ref. [33] by Fan and Wang, for which the Einstein–NLED field equations admit the Hayward spacetime as an exact magnetically charged solution. In this framework, the nonlinear electromagnetic field acts as an effective matter source that regularizes the geometry at the origin while preserving asymptotic flatness.

The Einstein–Maxwell–scalar field equations, obtained by varying (1) with respect to the metric, scalar field and electromagnetic field, are, respectively,

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi \left[T_{\alpha\beta}^{(\text{EM})} + T_{\alpha\beta}^{(\Psi)} \right], \quad (3)$$

$$\nabla_\alpha \nabla^\alpha \Psi = \frac{dU}{d|\Psi|^2} \Psi, \quad (4)$$

$$\nabla_\alpha (\mathcal{L}_F F^{\beta\alpha}) = 0, \quad (5)$$

where $\mathcal{L}_F = d\mathcal{L}/dF$. Notice that, since the scalar field is not charged, the Maxwell Equations, Equation (5), decouple from the Klein–Gordon equation, Equation (4), and this will be important for our further results.

The electromagnetic and scalar components of the energy-momentum tensor are

$$\begin{aligned} T_{\alpha\beta}^{(\text{EM})} &= \mathcal{L}_F F_\alpha{}^\delta F_{\beta\delta} - \frac{1}{4} \mathcal{L}(F) g_{\alpha\beta}, \\ T_{\alpha\beta}^{(\Psi)} &= \nabla_\alpha \Psi^* \nabla_\beta \Psi + \nabla_\beta \Psi^* \nabla_\alpha \Psi - g_{\alpha\beta} \left[\frac{1}{2} g^{\gamma\delta} (\nabla_\gamma \Psi^* \nabla_\delta \Psi + \nabla_\delta \Psi^* \nabla_\gamma \Psi) + U(|\Psi|) \right]. \end{aligned} \quad (6)$$

2.1. Spherically Symmetric Spacetime

Since our goal is to construct boson star configurations supported by a regular compact object background, we restrict our analysis to static and spherically symmetric spacetimes; thus, we take the metric as

$$ds^2 = -N(r)\sigma(r)^2 dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad N(r) = 1 - \frac{2m(r)}{r}, \quad (7)$$

where $N(r)$ and $\sigma(r)$ are functions of the areal radius r .

Furthermore, spherical symmetry provides the simplest setting for studying self-gravitating scalar field configurations and allows a direct comparison with previously known boson star and regular black hole solutions. Accordingly, the complex scalar field is described by the harmonic ansatz

$$\Psi = \psi(r)e^{-i\omega t}, \quad (8)$$

where the time dependence is encoded in a phase factor with frequency ω . This form ensures that physical observables, such as the energy-momentum tensor, remain time-independent despite the explicit time dependence of the scalar field.

For the electromagnetic potential, we assume the form

$$A = a(\theta) d\varphi. \quad (9)$$

This choice is motivated by both symmetry considerations and the requirement of recovering the Hayward electrovacuum solution in the limit of vanishing scalar field.

The nonvanishing components of the Faraday tensor are

$$F_{\theta\varphi} = -F_{\varphi\theta} = \partial_\theta a, \quad \Rightarrow \quad F = \frac{2(\partial_\theta a)^2}{r^4} \csc^2\theta. \quad (10)$$

The Maxwell Equations (5) with the ansatz for the four potentials (9) and the form of the spacetime metric (7) yield a differential equation for $a(\theta)$,

$$\partial_{\theta\theta} a - (\cot\theta)\partial_\theta a = 0, \quad (11)$$

that can be solved directly, giving the four potentials as

$$a(\theta) = Q \cos\theta, \quad (12)$$

where Q is a parameter that can be interpreted as the magnetic charge source.

The previous ansatz for the scalar field and the solution (12) leave three radial functions to be determined: $\sigma(r)$, $m(r)$, $\psi(r)$. The corresponding field equations, resulting from (3) and (4) read, denoting radial derivatives by “primes”,

$$m' = 4\pi r^2 \left[N\psi'^2 + \mu^2\psi^2 + \frac{w^2}{N\sigma^2}\psi^2 \right] + \frac{3r^2}{\beta} \frac{(\beta F)^{3/2}}{[1 + (\beta F)^{3/4}]^2}, \quad (13)$$

$$\sigma' = 8\pi r\sigma \left[\psi'^2 + \frac{w^2\psi^2}{N^2\sigma^2} \right], \quad (14)$$

$$\psi'' + \left(\frac{2}{r} + \frac{N'}{N} + \frac{\sigma'}{\sigma} \right) \psi' + \frac{\omega^2\psi}{N^2\sigma^2} - \frac{\mu^2\psi}{N} = 0. \quad (15)$$

Alternatively, the equation for the mass, Equation (13), can be written as

$$m' = 4\pi r^2 (\rho_\Psi + \rho_{\text{EM}}), \quad (16)$$

where

$$\rho_{\Psi} = N\psi'^2 + \mu^2\psi^2 + \frac{\omega^2}{N\sigma^2}\psi^2, \quad \rho_{\text{EM}} = \frac{3}{4\pi\beta} \frac{(\beta F)^{3/2}}{[1 + (\beta F)^{3/4}]^2}, \quad F = \frac{2Q^2}{r^4}. \quad (17)$$

Furthermore, it is convenient for numerical purposes to write the Klein–Gordon equation, Equation (15), as

$$\psi'' + \frac{\psi'}{Nr} \left(1 + N - 8\pi r^2 \mu^2 \psi^2 - \frac{6r^2}{\beta} \frac{(\beta F)^{3/2}}{[1 + (\beta F)^{3/4}]^2} \right) + \frac{\omega^2 \psi}{N^2 \sigma^2} - \frac{\mu^2 \psi}{N} = 0, \quad (18)$$

where we have used Equation (14) and

$$N' + r \left[N \left(\frac{1}{r^2} + 8\pi \psi'^2 \right) + 8\pi \left(\mu^2 + \frac{\omega^2}{N\sigma^2} \right) \psi^2 - \frac{1}{r^2} + \frac{3}{\beta} \frac{(\beta F)^{3/2}}{[1 + (\beta F)^{3/4}]^2} \right] = 0 \quad (19)$$

that corresponds to the tt component of Einstein's equation.

2.2. Boundary Conditions

In order to numerically solve the field equations, it is necessary to specify suitable boundary conditions. These conditions are uniquely fixed by the requirement of obtaining regular and asymptotically localized configurations. Regularity at the origin requires the metric functions and scalar field to remain finite while the scalar field derivative vanishes. The boundary conditions near the origin are found by expanding the fields in a power series, substituting the power series into (13), (14), (18), and then equating coefficients of the same powers. The corresponding behavior is

$$m(0) \sim \left[\frac{4\pi}{3} \psi_0^2 \left(\frac{\omega^2}{\sigma_0^2} + 1 \right) + \frac{1}{\beta} \right] r^3, \quad (20)$$

$$\psi(0) \sim \psi_0, \quad (21)$$

$$\sigma(0) \sim \sigma_0 + 4\pi \frac{\omega^2}{\sigma_0^2} \psi_0^2 r^2. \quad (22)$$

which is parameterized in terms of the three unknown constants ω , σ_0 and the value of the scalar field at the origin ψ_0 .

At infinity, we require that the scalar field vanishes $\psi \rightarrow 0$, as $r \rightarrow \infty$, ensuring a localized boson star configuration with finite mass and that the spacetime is asymptotically flat and thus $\sigma \rightarrow 1$. Furthermore, the existence of decaying modes is given by the asymptotic behavior of the scalar field

$$\psi(r) \sim e^{-(\sqrt{\mu^2 - \omega^2})r} \quad \text{with} \quad \mu^2 > \omega^2. \quad (23)$$

To integrate the system of equations (13), (14), (18) outward from some small r_{min} , the three constants ω , σ_0 and ψ_0 must be provided. However, a look at the equations shows that these constants can be reduced to two by scaling σ and ω as

$$\sigma(r) \rightarrow \frac{\sigma(r)}{\sigma_0}, \quad \text{and} \quad \omega \rightarrow \frac{\omega}{\sigma_0}. \quad (24)$$

with this, the unknown constant σ_0 parametrizes the outer boundary condition. With this choice, only two constants, ψ_0 and ω , must be specified at the inner boundary.

Therefore, the numerical construction amounts to solving a boundary value problem for which the frequency ω is adjusted so that the asymptotic conditions are satisfied.

Although we employ a shooting method to solve the system, the same boundary value problem could alternatively be addressed using, for instance, a relaxation. The choice of the shooting method is motivated by its efficiency in constructing regular localized solutions satisfying the required boundary conditions.

3. Results

3.1. Hayward Spacetime

In the electrovacuum limit, that is, when the scalar field vanishes, $\psi(r) = 0$, throughout the domain, the equation for σ , Equation (14), and the equation for m , Equation (16), can be solved separately once the regularity condition at the origin, $r = 0$, is imposed and asymptotic flatness is ensured. Integration of (14) under these conditions gives $\sigma(r) = 1$.

A direct integration of Equation (16) with $\rho_\Psi = 0$ produces:

$$m(r) = \frac{2Q^2 r^3}{(2\beta Q^2)^{1/4} r^3 + 2\beta Q^2} = \frac{m_0 r^3}{r^3 + \ell^3}. \quad (25)$$

the second equality is used to compare with the known Hayward spacetime [32] and where the relation between the charge Q and the electrodynamics parameter β with the Hayward length-scale ℓ and mass m_0 , is

$$\ell^2 = Q(2\beta)^{1/2}, \quad m_0 = Q^{3/2} \left(\frac{8}{\beta} \right)^{1/4}. \quad (26)$$

Recalling that the metric coefficient $N(r) = 1 - \frac{2m(r)}{r}$, the location of the horizons in the Hayward spacetime is determined by the condition $N(r) = 0$. The nature of the roots is determined by a limiting length ℓ_m , for which the spacetime may be a black hole with two horizons if $\ell < \ell_m$, an extreme black hole if $\ell = \ell_m$ and a spacetime without horizons if $\ell > \ell_m$ with

$$\frac{\ell_m}{m_0} = \frac{(32)^{1/3}}{3} = 1.05827 \dots \quad (27)$$

The outer r_+ and inner horizon r_- exist for values $0 \leq \ell \leq \ell_m$ [52] and are given by

$$r_+/m_0 = \frac{2}{3} + \frac{4}{3} \cos \left[\frac{1}{3} \cos^{-1} \left(1 - \frac{27\ell^3}{16m_0^3} \right) \right], \quad (28)$$

$$r_-/m_0 = \frac{2}{3} - \frac{4}{3} \cos \left[\frac{1}{3} \cos^{-1} \left(1 - \frac{27\ell^3}{16m_0^3} \right) + \frac{\pi}{3} \right]. \quad (29)$$

Figure 1 shows the metric coefficient $N(r)$ of the Hayward spacetime, for some values of ℓ/m_0 , representing the three possible cases for the spacetime: a black hole with two horizons, an extreme black hole with one horizon, or a spacetime with no horizons. For comparison, the Schwarzschild limit ($\ell = 0$) is also shown. Notice that the value of r_+ is always smaller than the Schwarzschild radius $r = 2m_0$.

Figure 2 shows the inner (Cauchy) and outer (event) horizons as a function of the ratio ℓ/m_0 . The figure also displays the radius r_{ext} for which the minimum of the metric function $N(r)$ is attained.

The event horizon is a decreasing function starting from $r_+ = 2m_0$, the Schwarzschild limit with $\ell = 0$, whereas the inner horizon is an increasing function. The three radii coincide in the extreme case $\ell_m/m_0 = 1.05827$, where $r_+ = r_- = r_{\text{ext,bh}} = 4m_0/3$. As the value of ℓ/m_0 increases, the roots of $N(r)$ become complex and the spacetime has no horizons. However, there is still a localized concentration of energy. We characterize the

size of the resulting object with the radius r_{ext} (which satisfies the equation $N'(r_{\text{ext}}) = 0$; $\Rightarrow r_{\text{ext}} = 2^{1/3}\ell/m_0$). This radius will become useful when comparing the size of this object with the effective size of boson stars presented in the following section. Figure 2 shows that for values $\ell > \ell_m$, the radius r_{ext} grows linearly with ℓ , as stated above. In other terms, the size of a horizonless Hayward spacetime is bounded from below by $r_{\text{ext,bh}}$.

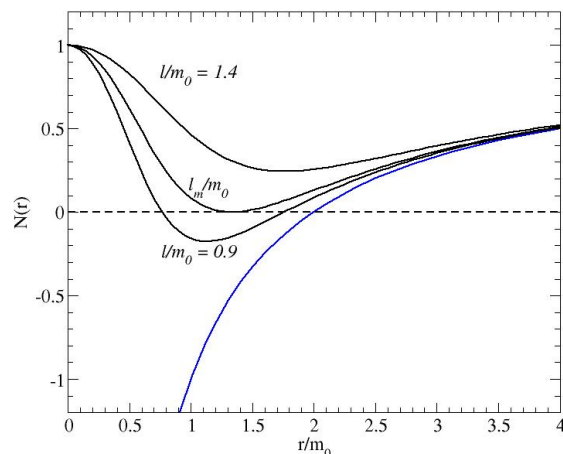


Figure 1. Metric coefficient $N(r)$ for the Hayward spacetime. Depending on the value of ℓ/m_0 , the spacetime may contain one, two, or no horizons. The location of the horizons is given by the roots of $N(r)$. The extreme value $\ell_m/m_0 = 1.05827$ produces the extreme black hole with only one horizon at $r_{\text{ext,bh}} = 4m_0/3$. The case $\ell/m_0 = 1.4$ corresponds to a spacetime with no horizons, whereas $\ell/m_0 = 0.9$ corresponds to a spacetime with two horizons. The blue line corresponds to Schwarzschild spacetime (with $\ell = 0$).

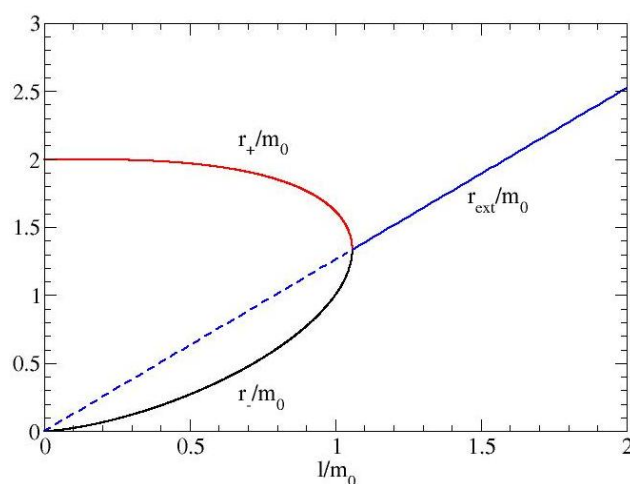


Figure 2. The radius of the event horizon r_+ and inner horizon r_- . The line $r_{\text{ext}} = 2^{1/3}\ell/m_0$ marks the location of the minimum of the metric function $N(r)$. For non-extreme black holes, the minimum is located between the horizons (dashed line). In the extreme case ($r_{\text{ext,bh}} = 4m_0/3$), the three radii coincide.

Figure 3 shows the plane ℓ vs. m_0 . The region of existence of black holes has been shaded. The separating line is the representation of Equation (27). For very small values of ℓ and larger masses m_0 , the resulting spacetime is regular with no horizons.

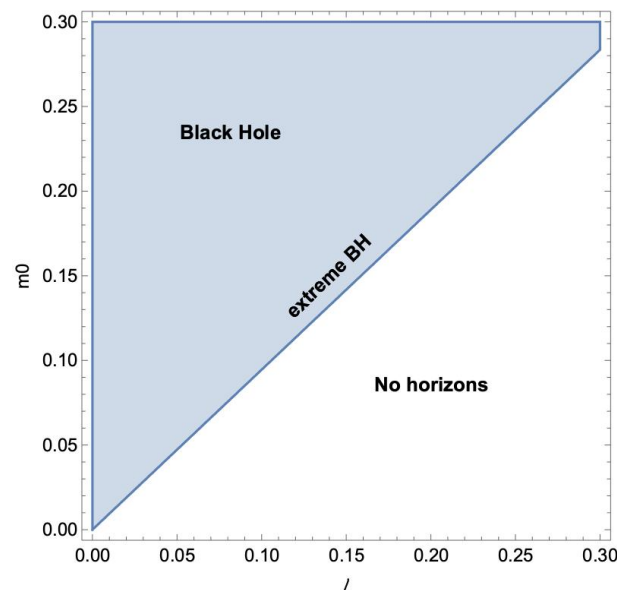


Figure 3. The existence region for black holes and a spacetime with no horizons. The diagonal line is given by expression $\ell/m_0 = 1.05827$, as indicated in Equation (27).

3.2. Boson Stars

In order to solve the system (13), (14) and (18) numerically, we use dimensionless quantities defined as

$$r \rightarrow \mu r, \quad \omega \rightarrow \frac{\omega}{\mu}, \quad m \rightarrow \mu m, \quad Q \rightarrow \mu Q, \quad \beta \rightarrow \mu^2 \beta. \quad (30)$$

This scaling leaves the equations invariant under the values of the mass parameter, allowing the solutions to be scaled with μ .

We solve the system for static solutions using a shooting method. We begin by choosing values for ψ_0 and ω . With these, we obtain the values of the fields at some small $r_{\min}$ through the conditions (20). We then numerically integrate the solution outward from $r_{\min}$ to some large $r_{\max}$. The equations are solved by using a standard Runge–Kutta ODE. In general, the integrated solution at large r will not satisfy the outer boundary condition $\psi \rightarrow 0$. The shooting method consists of varying the constant ω until the desired boundary condition is satisfied. Once the outer boundary conditions are satisfied, we have found a static solution.

In this work, we focus exclusively on ground-state solutions, which are characterized by the absence of nodes in the radial profile of ψ for $r > 0$ [53]. To construct Hayward boson star configurations, we began with known solutions for mini-boson stars and a fixed value of β and Q then gradually increasing the value of $\psi(0)$ to generate a family of solutions.

Given a static solution, the asymptotic value of $m(r)$ is the Arnowitt–Dessner–Misner (ADM) mass M , which is approximated by evaluating $m(r)$ at the last grid point r_{out} of the computational domain.

Although boson stars extend to infinity and hence do not have a surface, an effective size, R_{99} , is defined for them as the radius of the object within which 99% of the total mass is enclosed.

After obtaining solutions for larger values of ψ_0 , we also investigated configurations with larger values of Q , to ensure a consistent exploration of the solution space. However, as was pointed out in Ref. [48], the value of the charge can not grow without limit since there is a critical value at which solutions do not exist. In the following section, we will provide an explanation for the existence of this bound in the value of the charge.

Figure 4 displays the space of static solutions. The curves shown are constructed from a series of discrete points, each point representing a static configuration. Each of these configurations is characterized by a central value of the field ψ_0 , an associated frequency ω , a total mass M , and an effective radius R_{99} . In the figure, we present the equilibrium configurations for some representative values of $Q = \{0.10, 0.12, 0.15, 0.17\}$ and $\beta = 0.1$, showing the total mass as a function of ψ_0 . The mass increases monotonically with the central value of ψ up to a maximum point, after which it begins to decrease, mirroring the known behavior of standard mini-boson stars (with $Q = 0$). In the mini-boson star case, the maximum value of the mass, $M_{\max}$, separates stable from unstable configurations [53].

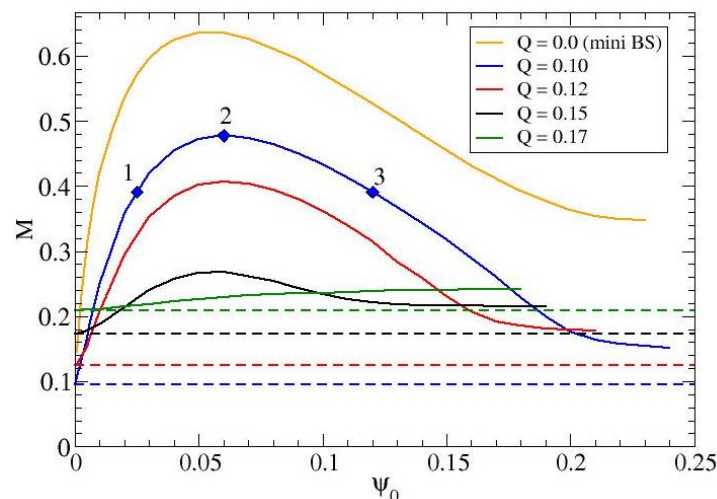


Figure 4. Mass M as a function of the central scalar field value ψ_0 for configurations with different values of Q . The horizontal dashed lines are the values of m_0 , the mass of the equivalent electrovacuum Hayward spacetime. Configurations marked with 1, 2, 3 will be analyzed below.

As stated above, for each value of Q (with a fixed β), there are values of m_0 and ℓ that yield an electrovacuum Hayward spacetime. The horizontal dashed lines in Figure 4 correspond to the equivalent electrovacuum Hayward mass m_0 , for each value of Q , as given in Table 1, constructed with the help of Equation (26). Since the energy density of the scalar field is strictly positive [see Equation (17)], its presence contributes to adding mass to the system relative to the electrovacuum spacetime. As a result, the total mass of the HBS is always greater than that of its electrovacuum counterpart, and is in fact bounded from below by the corresponding value of m_0 .

Table 1. The maximum mass attained by the Hayward boson stars $M_{\max}$. For an extreme Hayward black hole $r_{\text{ext}} = 4/3m_0$ and thus $C_{\text{ext}} = m_0/r_{\text{ext}} = 3/4 = 0.75$.

Q	$M_{\max}$	R_{99}	$R_{99}/M_{\max}$	m_0	ℓ	ℓ/m_0	r_{ext}	r_{ext}/m_0
0	0.633	7.855	12.1409	—	—	—	—	—
0.1	0.477	8.602	17.9987	0.0945	0.2114	2.2360	2.8172	29.7903
0.12	0.407	8.598	21.1123	0.1243	0.2316	1.8633	2.3477	18.8843
0.15	0.267	7.499	28.0287	0.1737	0.2590	1.4907	1.8781	10.8101
0.17	0.241	1.216	4.6482	0.2096	0.2757	1.3153	1.6572	7.9056

The curves in Figure 5 are another presentation of the space of static solutions, showing the mass as a function of the frequency. We see that static solutions exist in the range $0 < \omega/\mu < 1$ and that as the value of Q increases, the frequency of the solutions drops to zero; this is the limit of *frozen boson stars* described in Refs. [49,50]. Furthermore, the characteristic spiral structure observed in mini-boson star solutions [54] gradually diminishes

as the charge Q increases, eventually disappearing for sufficiently large values of Q (up to a maximum value). The plot also reveals another important result: the maximum mass attainable by any HyBS is lower than that of mini-boson stars, whose maximum mass is $M_{\max} \simeq 0.633$ [55].

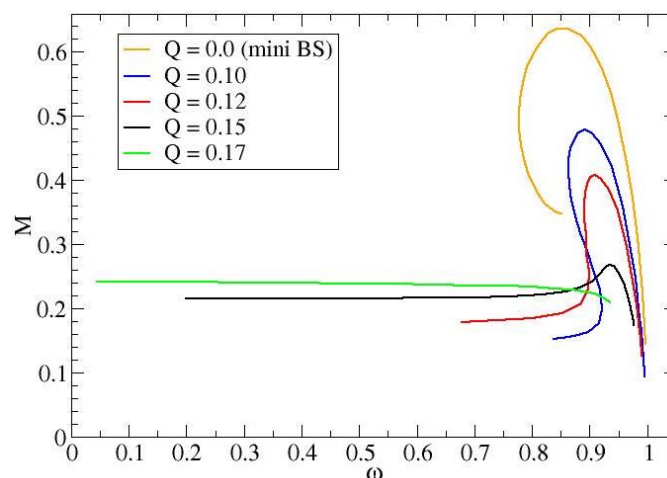


Figure 5. Mass M , as a function of the frequency ω , for the same configurations of Figure 4. The spiraling behavior of the mini-boson starts to be lost progressively for larger values of Q in the strong gravity regime.

In Figure 6, we show a plot for the total mass as a function of the effective radius R_{99} , the mass increases monotonically with R_{99} up to a maximum value, and then decreases. For mini-boson stars, M decreases monotonically with R_{99} , and beyond the maximum, configurations become diluted in the sense that they have smaller mass and larger size (this corresponds to the Newtonian gravity regime). For $Q \neq 0$, a couple of comments are in order. First, after the maximum is reached and as R_{99} increases, the curves turn towards smaller values of R_{99} ; thus, unlike miniboson stars, they can not attain an arbitrarily large size. Second, after a turn, the effective size of the HBS decreases to the value r_{ext} of the Hayward electrovacuum spacetime while its mass tends to the constant value m_0 , which is consistent with the fact that the limit of HyBS as $\psi \rightarrow 0$ is the Hayward spacetime, once the parameters Q and β are provided. Furthermore, as a result of this limited behavior, two distinct HyBS configurations may share the same effective radius R_{99} while possessing different masses.

Table 1 contains the mass $M_{\max}$, effective size R_{99} , and the inverse of compactness $R_{99}/M_{\max}$ of the maximum mass configurations for some representative values of Q . The table also shows the mass m_0 , effective length ℓ , and size r_{ext} of the equivalent electrovacuum Hayward spacetime obtained from relations (26). The ratios ℓ/m_0 and r_{ext}/m_0 are also provided.

One can see that, as the value of Q increases, the ratio ℓ/m_0 of the configurations decreases towards the value of the extreme Hayward BH with $\ell_0/m_0 = 1.05827$ [see Equation (27)]. This limit value would correspond to a configuration with $Q \simeq 0.21$. However, HyBS with this and higher charges does not exist. This expectation is confirmed by looking at the radius r_{ext} in Table 1. Moving to higher values of Q reveals that r_{ext} tends to the radius of the extreme Hayward BH, $r_{\text{ext,bh}} = 1.33m_0$.

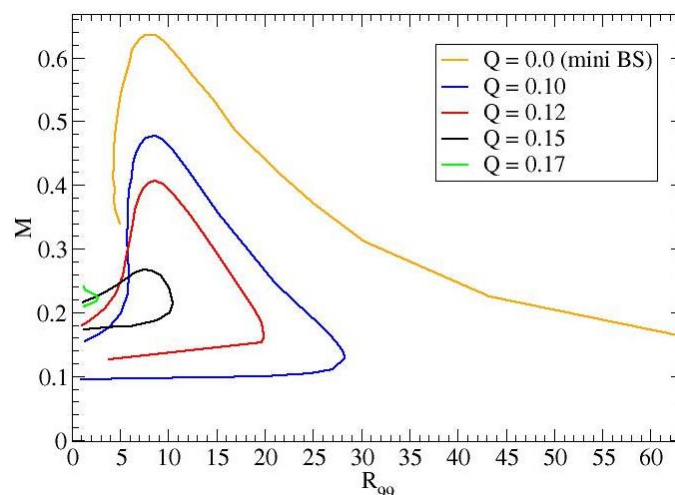


Figure 6. Mass M as a function of R_{99} for different values of Q . For $Q \neq 0$, configurations with fewer scalar fields start from the lower part of the curve with a small radius. As the contribution of the field increases, the mass grows slowly until a certain radius and then grows faster up to a maximum value, but with a smaller size.

Regarding properties of individual configurations, Figure 7 displays the scalar field profile $\psi(r)$ and the scalar energy density $\rho_\psi(r)$, for solutions marked with 1, 2 and 3 in Figure 4. The figure displays only the scalar field component of the energy density. Solution 2 corresponds to the one with maximum mass $M_{\max} \simeq 0.477$ for $Q = 0.1$. Configurations 1 and 3 have nearly the same total mass $M \simeq 0.391$. The maximum of the scalar field profile is at the origin and then decreases as r increases. Figure 7 also shows that the scalar profile becomes sharper in the near-origin region for larger values of ψ_0 .

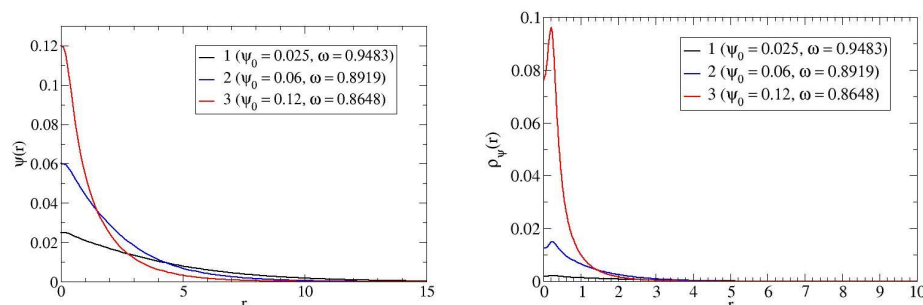


Figure 7. Scalar field profile ψ and scalar field density ρ_ψ for the configurations labeled with 1, 2 and 3 in Figure 4.

Remarkably, unlike mini-boson stars in spherical symmetry, the maximum of the scalar energy density for HyBS is not located at the origin. For the solution marked with 1, the density is more spread in space compared with solution 2, while the density for solution 3 has a larger value than the others. Since the maximum amplitude of the scalar field is located at the origin, but the maximum in the density is not there, we conclude that the shift in the location of the maximum is being caused (through gravity) by the presence of the electromagnetic contribution ρ_{EM} .

The metric coefficients σ and N as functions of r for the configurations 1, 2 and 3 in Figure 4 are shown in Figure 8. In all cases, the spacetime is asymptotically flat, that is, $\sigma \rightarrow 1$ and $N \rightarrow \text{const.}$ as $r \rightarrow \infty$.

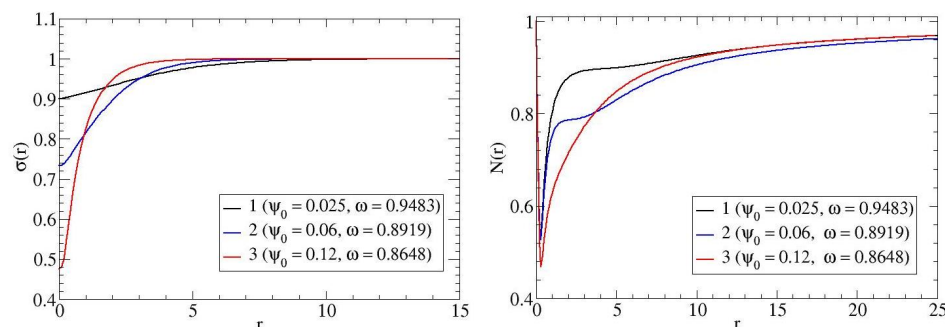


Figure 8. Metric coefficients σ and N , as a function of r for the configurations displayed with labels 1, 2 and 3 in Figure 4.

Figure 9 shows the mass function for configurations 1, 2, and 3. Asymptotically, curves 1 and 3 converge to the same value, that is, the ADM mass of the HyBS. Close to the origin, a slight bending is present in curves 1 and 2, being more noticeable in the first, where the contribution of the scalar field is smaller. This occurs because, in the region near the origin, the electromagnetic contribution to the spacetime is larger than that of the scalar field, and, as one moves outward, the scalar field contribution is dominant, producing a more massive object. As one moves in the solution space (curve Figure 4) towards larger values of ψ_0 , as for configuration 3, the contribution of the scalar fields dominates from the very beginning. Remarkably, the mass of the configurations does not grow without bound but reaches a maximum (configuration 2) and then decreases.

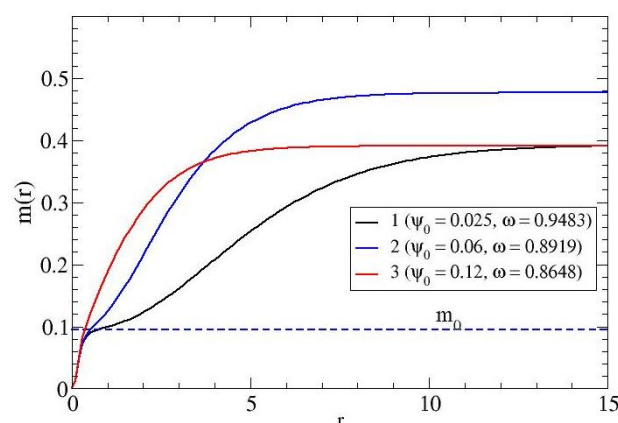


Figure 9. Mass profile as a function of r for configurations with labels 1, 2 and 3 in Figure 4. The horizontal dashed line marks the value of m_0 that corresponds to the Hayward spacetime with parameters $Q = 0.1$ and $\beta = 0.1$ ($\ell = 0.21147$ and $m_0 = 0.094574$, respectively.)

4. Discussion and Final Remarks

Once the parameter of the theory β and the charge Q are fixed, if they correspond in electrovacuum to a regular horizonless spacetime, it becomes possible to sustain a bounded, self-gravitating, and regular scalar field configuration or a HyBS. Conversely, when the chosen parameters in electrovacuum yield to horizons, self-gravitating scalar field configurations cannot be sustained.

The region of the parameter space defined by β and Q , where HyBS solutions exist, is shown in Figure 10. The condition of existence for HyBS can be found using the correspondence between the Hayward parameters and β and Q , and is given by [See (27)]

$$\ell/m_0 = \frac{1}{Q} \sqrt{\frac{\beta}{2}} > 1.05827, \quad \Rightarrow \quad \frac{\sqrt{\beta}}{Q} > 1.49661. \quad (31)$$

Different results in the literature establish the impossibility of having Schwarzschild or Reissner–Nordstrom BHs with scalar hair [56–59]. Some of these results can be derived from integral relations that can also be applied in the case of a Hayward Black Hole. In the following, we use one of these arguments to provide further insight into the existence of the BH solutions coupled with a scalar field.

One starts assuming a non-extremal Hayward BH. Then re-write the scalar field Equation (15) in the form

$$(Nr^2\sigma\psi')' = r^2\sigma\left(\mu^2 - \frac{\omega^2}{N\sigma}\right)\psi. \quad (32)$$

After integrating between the event horizon and infinity, one finds the following:

$$(Nr^2\sigma\psi')|_{r=r_+}^{\infty} = \int_{r_+}^{\infty} r^2\sigma\left(\mu^2 - \frac{\omega^2}{N\sigma}\right)\psi dr. \quad (33)$$

Since $N(r_+) = 0$ and $\psi'(\infty) = 0$ the *l.h.s.* of (33) vanishes. Furthermore, the term in parentheses in the *r.h.s.* of (33) is strictly positive outside the horizon; thus, solutions necessarily imply that the scalar field must vanish outside the horizon.

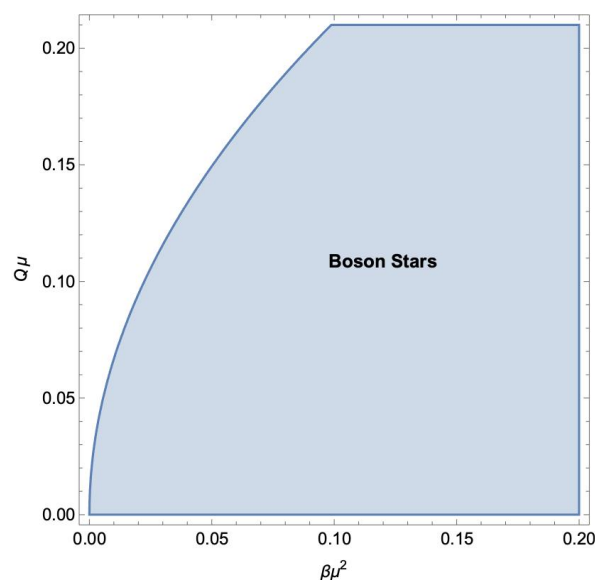


Figure 10. The shaded region marks the region where boson stars exist in the presence of an electromagnetic field sourcing a Hayward spacetime for the combination of parameters β and Q .

In conclusion, we have shown that the existence of HyBSs is guaranteed by a combination of the free parameter of the theory β and the magnetic charge that sources the spacetime Q . We found that the threshold values for the existence of HyBS are related to the existence of horizons in the equivalent electrovacuum Hayward spacetime. If the combination of the parameters of the Hayward spacetime, namely the ADM mass and length ℓ , is such that an event horizon appears, then the spacetime can not support a stationary scalar field configuration. This result suggests that, within the present setup, Hayward black holes cannot sustain self-gravitating, uncharged scalar field configurations.

The results obtained in this work show that the presence of the nonlinear electrodynamics sector plays an important role in determining the existence of HyBSs. Unlike mini-boson stars, the solutions described here approach a regular Hayward electrovacuum geometry, introducing an intrinsic length scale into the problem. This feature leads to a different structure of the solution space, including the appearance of turning points in the mass-radius relation and a nontrivial connection between the boson star

configurations and the underlying regular background. Understanding the dynamical stability of these stars is important for evaluating their viability and physical plausibility as alternative models to black holes. Although no dynamical or perturbative stability analysis has been performed in this work, this remains an open question that will be addressed in future work.

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