

Article

The Schwarzschild Precession of S-Stars as a Probe for General Relativity: The Possible Presence of a Fifth Force at the Galactic Center

Predrag Jovanović ^{1,*} , Vesna Borka Jovanović ²  and Duško Borka ² ¹ Astronomical Observatory, Volgina 7, P.O. Box 74, 11060 Belgrade, Serbia² Department of Theoretical Physics and Condensed Matter Physics (020), Vinča Institute of Nuclear Sciences–National Institute of the Republic of Serbia, University of Belgrade, P.O. Box 522, 11001 Belgrade, Serbia; vborka@vinca.rs (V.B.J.); dusborka@vinca.rs (D.B.)

* Correspondence: pjovanovic@aob.rs

Abstract

In this paper, we investigated the capability of Yukawa gravity to explain the Schwarzschild precession of S-stars as a probe for General Relativity and to map the allowed parameter space of a potential fifth force. We simulated the S38 star orbit in a Yukawa gravity model and fitted it into the observed astronomical data of the S38 star using the Markov Chain Monte Carlo method in order to constrain the parameters (strength δ and range λ) of the Yukawa interaction. Comparing these findings with previous results for the S2 star reveals that the best-fit values for λ are remarkably close, while the magnitudes of δ are slightly smaller for S38. These results map the joint statistical boundaries of a fifth force at the Galactic Center. Although the General Relativity limit ($\delta = 0$) falls well within the reported 1σ uncertainties, rendering potential nominal deviations statistically insignificant, the interaction range λ exhibits a highly stable spatial scale across different stellar orbits. This cross-consistency between independent datasets, with stable clusters near 360, 1900, and 7000 AU, keeps a viable physical window open for a fifth force. This agreement demonstrates that the interaction scale is robust against individual single-orbit systematic errors, ensuring that this parameter domain remains a prime target for future high-precision astrometric observations. Ultimately, analyzing S-star kinematics within Yukawa gravity provides a powerful independent tool for testing General Relativity and bounding non-standard gravitational effects at the Galactic Center.

Keywords: theories of gravity; general relativity; fifth force; supermassive black holes; stellar dynamics



Academic Editors: Lorenzo Iorio and Øyvind Grøn

Received: 26 June 2026

Revised: 28 July 2026

Accepted: 3 August 2026

Published: 6 August 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

The standard theory of gravity, General Relativity (GR), or Einstein's theory of gravity, recognized as one of the most profound and beautiful physical theories, has nice theoretical properties, and great phenomenological achievements have been attained with this theory. GR has been tested in the Solar System, and it is known that it is well-confirmed therein. It has important astrophysical implications in predicting several significant phenomena that were confirmed: the perihelion precession of planetary orbits, the deflection of light in the gravitational field (gravitational lensing), gravitational redshift, Shapiro time delay, the precession of orbiting gyroscopes (Lense–Thirring effect), black holes and gravitational waves. One of the main features of GR is its capability to make many predictions

without new degrees of freedom and fine tuning of parameters. It has been considered the standard theory of gravity since its formulation, in 1915, until the present time and serves as the theoretical paradigm for gravitational investigations at all astrophysical and cosmological scales.

In cosmology, GR is the basis of the standard cosmological model (the Λ CDM model) which assumes the existence of the dark content of the Universe, comprising approximately 68% Dark Energy (DE), 27% Dark Matter (DM) and only 5% visible (standard) matter [1]. Nevertheless, one should not comprehend GR as the complete (i.e., the final) theory of gravity at all scales and all states of matter. As we know, all physical theories have constraints on their validity, for example, those depending on scale, the complexity of the physical system, physical constants and parameters. There is no a priori reason that GR is an exception, i.e., it is not without constraints in the description of gravitational phenomena. GR is not verified at the galactic and cosmological scales; hence, its application at these scales should be carried out with caution. The existence of DM and DE has not been confirmed by fundamental experiments seeking new particles; hence, DM and DE are hypothetical ingredients of the Universe, while their effects at a large scale are evident (clustering of structures and accelerated expansion of the cosmic flow). Therefore, there is a sense to look for extensions of GR that could give effects that are usually attributed to DM and DE. Presently, the state of the art is encouraging researchers to explore the effects of DM and DE that are, at least partially, in the modified geometrical sector of GR, instead of the matter sector. DM is introduced as a possible explanation of observational evidence that the velocity of stars and gas in the outer parts of spiral galaxies almost does not depend on the distance from the center of galaxy. Thus, DM should be the main source of the total mass of galaxies, with its main concentration in the outer regions of the galactic disks and also in the surrounding halos. In that sense, according to Λ CDM, the baryonic matter of spiral galaxies is mixed with and embedded into non-baryonic dark matter, causing in that way the rotation velocities of the visible matter within the disk to be constant or increasing with disk radius. Let us also mention that GR gives solutions with black hole and Big Bang singularities. When a physical theory contains a singularity, it should be modified in the vicinity of the singularity. GR has a serious problem of renormalizability when attempting to quantize it. However, some modifications of GR are encouraging with regards to the possibility of quantization.

New observational data have introduced unexpected challenges for GR even within the Solar System, a domain where GR validity was previously considered absolute. High-precision observational techniques together with theories of planetary motions enable one to investigate a variety of small effects in the Solar System. For example, these advanced high-precision observations reveal additional perihelion precession of the planets with respect to the prediction of GR. Currently, it is assumed that the additional perihelion precession can be explained by the presence of DM in the Solar System [2].

Stellar dynamics indeed has a special role, mostly because it allows us to investigate various gravitational potentials by use of stellar motions. Here, we want to pay attention to S-stars, the bright stars which move around the Center of our Galaxy [3–18], and in this center Sgr A* is located. One of the brightest among these S-stars, with a high eccentricity, is called S2 [6,8]. The GRAVITY Collaboration detected the orbital precession of the S2 star close to the corresponding GR prediction [17], $\Delta\varphi = 0^\circ.20$ per orbital period, and according to data analysis in the framework of the Yukawa gravity model [19], the orbital precessions of the S38 and S55 stars are very close to the corresponding prediction of GR for these stars, $0^\circ.12$ and $0^\circ.11$ per orbital period, respectively. In Ref. [20] we analyze the orbital precessions of the S2 star. In this paper we will analyze the orbital precession of the S38 star.

The Keck group [15] and the GRAVITY Collaboration [13,14] have evaluated relativistic redshifts of spectral lines for the S2 star near its periastris passage in May 2018, and those redshifts were consistent with theoretical estimates done up to the first post-Newtonian (PN) correction. The orbital precession can occur due to a prograde pericentre shift (caused by relativistic effects) and due to a retrograde shift (caused by an extended mass distribution) [21]. In both cases the results will be rosette-shaped orbits [22]. The prograde precession could be detectable if the astrometric precision reached a few tenths of mas [23].

Deviations in the orbit of the S2 star from the predictions of GR have not been definitively confirmed, but they are actively investigated within the context of potential new physics. Analyses indicate that the current observational data is fully consistent with GR, while any potential deviations are treated as upper bounds to constrain parameters of modified gravity or dark matter distributions. Significant lines of research exploring these possibilities include analyses of the following:

1. The periastron precession of the S2 star's orbit to evaluate potential deviations from GR using the Schwarzschild precession parameter (f_{SP}), which would deviate from 1 in modified gravity frameworks [13,17];
2. The Scalar Field Models where the motion of the S2 star is explored in space-times seeded by scalar fields where deviations arise due to a non-zero scalar charge [16,24,25];
3. The Modified Gravity/Scalar-Tensor-Vector Gravity (MOG/STVG) in order to test Scalar-Tensor-Vector Gravity (STVG) and to derive upper limits for deviations from GR based on precise astrometric measurements [19,20,26];
4. The wormhole models and curved geometries where the potential deviations are investigated using alternative compact object metrics such as wormholes or non-Schwarzschild geometries [27–32].

The present paper represents a continuation of our previous investigations in which we evaluated the parameters of Yukawa gravity by Schwarzschild precession of S2 star and their extension to S38 star orbit. Our main goal is to test the predictions of GR against a Yukawa-like modification and to place tight constraints on the parameters of such a potential fifth force using the observed orbit of the S38 star. Despite the fact that both S2 and S38 stars move around the same supermassive black hole (SMBH), Sgr A*, their orbits differ. S38 has lower eccentricity, larger semi-major axis, and a longer orbital period than S2 [10]. Hees et al. [11] showed that a combined orbital fit of the S2 and S38 stars could in principle result with tighter constraints on a fifth-force parameters. In their analysis, the orbital elements were specific to each individual star, while the SMBH and the gravitational potential parameters were fitted jointly. However, the obtained results showed that zero fifth-force strength is always within the 68% confidence interval, meaning that no significant deviation from Newtonian gravity was found. Moreover, it was demonstrated that in their combined analysis of the S2 and S38 stars, the limit on a fifth force strength was primarily driven by the S2 star for the median values of λ and that the S38 star helped only for small and large values of λ [11]. Therefore, the study of Yukawa gravity using an independent star represents a highly important and crucial test because it enables one to demonstrate that the resulting fifth-force parameters are not solely linked to the specific orbital geometry or systematic errors in the S2 star dataset. Moreover, analyzing the motion of S38 star allows us to track how the intrinsic mathematical degeneracy between the strength δ and range λ of a fifth force varies across different orbital configurations. Here we will demonstrate that utilizing the independent kinematics of S-stars to map Yukawa-like deviations also results with the values for λ which remain remarkably stable and consistent with the results for the S2 star. This fact confirms the physical reliability of the spatial scale under investigation.

Besides, combining two highly sensitive astrometric datasets with different systematic errors and observational cadences requires a specific dedicated statistical framework to prevent one dataset from introducing bias into the other, which is out of the scope of the present paper.

This paper is organized as follows: in Section 2 we briefly list some of the most important successes and shortcomings of General Relativity, while in Section 3 we describe our method of calculation based on the Markov Chain Monte Carlo (MCMC) analysis of the S38 star orbit in the extended Parametrized Post-Newtonian (PPN) formalism for Yukawa gravity. Our main results are presented and discussed in Section 4, and finally we point out our main conclusions in Section 5.

2. An Overview of General Relativity: Successes and Shortcomings

Before exploring a specific modification of GR, we will first offer its brief summary, highlighting its key outcomes and addressing both its successes and shortcomings—see, for example, Refs. [33–36] and their related references.

From a phenomenological point of view, any relativistic theory of gravity must meet certain minimal requirements. The first requirement is that it has to explain astronomical observations, like mapping planetary orbits and the gravitational potential of self-gravitating structures. This signifies that Newtonian dynamics must be achieved in the weak-field, slow-motion limit. In order to pass at the post-Newtonian (PN) level, the theory has to recover the Newtonian potential, to accurately reproduce the observed galactic dynamics accounting for the known baryonic constituents, and to successfully pass the classical Solar System tests. Furthermore, the theory of gravity must tackle the issue of the generation and evolution of the large-scale structure of the Universe, and cosmological dynamics has to be reproduced. In particular, the main cosmological parameters (like the deceleration parameter q_0 , the Hubble parameter H_0 , and density parameters Ω_M , Ω_Λ and Ω_κ) have to be predicted in a self-consistent way. With a certain degree of precision, the simplest theory that satisfies the previous requirements is GR. Formulated by Albert Einstein between 1909 and 1915, GR remains the best current description of gravitational interaction, conceived as the geometry of spacetime. In particular, GR is based on four fundamental assumptions:

1. The Principle of Relativity, stating that all observers must be equally valid to describe physics. Specifically, inertial frames are not a priori preferred.
2. The Equivalence Principle, stating that the dynamics of massive bodies in non-inertial systems, and that in inertial systems in the presence of the gravitational field, is the same. Essentially, it is possible to locally reduce any non-inertial frame to an inertial frame.
3. The Principle of General Covariance, stating that the field equations must be tensor equations that are “generally covariant”, with the same form in all coordinate systems.
4. The Principle of Causality, dictating that causality is preserved, i.e., each spacetime point must have a consistent notion of past, present and future, being the same for all physical observers.

3. Extended PPN Formalism for Yukawa Gravity Model

In this paper, we evaluate the parameters of Yukawa gravity using the orbital fitting of S38 star by numerically integrating the respective equation of motion. Among other things, this allowed us to estimate the Schwarzschild precession as a result. This estimate naturally emerges from the obtained best-fit integrated orbit. In our evaluation it is assumed that the precession per orbital period of S38 star is close to the GR prediction of $0^\circ.12$. At short ranges, we can say that gravity is relatively well-constrained [37], but when investigating it at long ranges, further tests are still needed (as an example for different ranges, see

Figures 9 and 10 from Ref. [37]). That is why we believe it is very important to investigate gravity in the vicinity of SMBHs, especially because the environment around these objects is substantially different from that in the Solar System framework.

A screening mechanism, such as the Vainshtein [38], chameleon [39] or symmetron screening [40], is important in modified gravity theories to suppress fifth forces in high-density environments like our Solar System, and in that way to ensure compatibility with local precision tests. However, the environment probed by the S-stars near the GC presents a very different physical setup compared with Solar System environments. It consists of a massive central compact object Sgr A*, surrounded by a bulk distribution of matter which includes a stellar cluster, interstellar gas distribution and dark matter (see, e.g., [41] and references therein). The activation of screening fields depends strictly on local mass-density gradients or specific curvature thresholds [42]. Because the spatial regime covered by S2 and S38 orbits consists mostly of empty space between the stars and the central object, the local ambient density is too low to trigger screening mechanisms. In that way, the environment remains largely “unscreened”, allowing a perturbative fifth force to potentially manifest and remain observationally accessible.

A mass distribution extending over a certain region induces a retrograde shift in the pericenter of orbiting stars [21]. This effect actively counteracts the prograde relativistic Schwarzschild precession. It creates an environment-induced systemic effect that could mimic or mask signals of modified gravity.

In this work, we investigated the isolated Yukawa correction as the sole source of deviation to cleanly map its available parameter space without introducing highly degenerate free parameters. While including an extended mass halo would modify the gravitational potential and alter the resulting $\delta - \lambda$ degeneracy curve (see, e.g., [41]), other relativistic effects like the black hole’s spin (Lense–Thirring precession) remain far too weak to visibly disrupt S38 at its current periapsis distance [43].

Yukawa-like potentials differ from the standard Newtonian gravitational potential due to the presence of decreasing exponential terms [44–51]. For short ranges, Yukawa-like potentials are presented in the paper by Adelberger et al. [37]. Such potentials for long ranges are given for clusters of galaxies [52,53] and for rotation curves of spiral galaxies [46]. Also, other studies of long-range Yukawa-like modifications of gravity can be found in the following papers [45,54–59].

Yukawa-like corrections naturally emerge in the weak field limit derived by analytic Taylor expansions of $f(R)$ gravity with respect to Ricci scalar curvature R [60]:

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} R^n = f_0 + f_1 R + \frac{f_2}{2} R^2 + \dots \quad (1)$$

Although Sgr A* is a SMBH, the S-stars orbit at distances of thousands of gravitational radii from its event horizon. At such large radial distances, the curvature R is exceptionally small ($R \rightarrow 0$), physically validating the use of a linearized Taylor expansion around $R = 0$, which naturally yields the Yukawa-like modification to the gravitational potential (see, e.g., [34,61]). Hence, from this expansion, it is possible to obtain [47,62]:

$$\Phi(r) = -\frac{GM}{(1+\delta)r} \left[1 + \delta e^{-\left(\frac{r}{\lambda}\right)} \right], \quad (2)$$

where $\Phi(r)$ represents the modified gravitational potential, M is the central mass, λ is the range of the Yukawa interaction, depending on the scale of the specific gravitating system and expressed in length units, δ is a dimensionless universal constant representing strength of the Yukawa interaction, while G is the local value of the gravitational constant which is

related to the corresponding value G_∞ measured at infinity by: $G_\infty = G/(1 + \delta)$. In the case $\delta = 0$, the Yukawa potential reduces to the Newtonian one.

The Yukawa-like correction coming from $f(R)$ gravity can be used to fix the coefficients in the expansion in (2) [63]. For the expansion up to the second order, we have two parameters (f_1, f_2) to fix [61]. The relations between f_1, f_2 and δ and λ parameters are: $f_1 = 1 + \delta, f_2 = -(1 + \delta)/(\lambda^2)$ [61].

In our previous studies of the Yukawa gravity [41,61,64–70], we constrained these two parameters using the orbit of the S2 star around the Galactic Center. In [64] we constrained the parameters δ and λ , and in [61] we constrained the values of f_1 and f_2 . In order to estimate f_1 and f_2 under the assumption that $f_0 = 0$, the simulated orbits were fitted into the astrometric observations by minimizing the reduced χ^2 , and as a result we were able to reconstruct the $f(R)$ models up to the second order [61,64].

We also derived the following approximate formula for the additional contribution of Yukawa gravity to the Schwarzschild precession in the case when $a \ll \lambda$ [66,67]:

$$\Delta\varphi_Y \approx \frac{\pi\delta\sqrt{1-e^2}}{1+\delta} \frac{a^2}{\lambda^2}, \quad (3)$$

where e is orbital eccentricity and a is the semi-major axis of the orbital ellipse. The total orbital precession $\Delta\varphi$ can then be obtained by adding the above expression to the formula for Schwarzschild precession $\Delta\varphi_{Schw}$ in GR:

$$\Delta\varphi \approx \Delta\varphi_{Schw} + \Delta\varphi_Y = \frac{6\pi GM}{c^2 a(1-e^2)} + \frac{\pi\delta\sqrt{1-e^2}}{1+\delta} \frac{a^2}{\lambda^2}. \quad (4)$$

These investigations indicate that it is not possible to get reliable constraints on δ based only on the requirement that the precession angle in Yukawa gravity is like in GR and that an additional independent astronomical dataset is then necessary. Besides, the influence of extended mass distribution around Sgr A* should also be taken into account. When supposing that bulk distribution of matter exists near SMBH at our Galactic Center, a lower value of range λ is obtained (see [41] for more details). As a final remark, it is worth noting that several Extended Theories of Gravity (ETGs) show Yukawa-like corrections, as reported in Table I of Ref. [71]. This means that the Newtonian potential is a special case in a wide class of models where exponential corrections naturally emerge. This feature is related to the fact that gravity could not be a scale-invariant interaction, as assumed in GR, but its strength could be related to given astrophysical and cosmological scales [72].

We used the PPN equation of motion (see [73] for more details on PPN formalism) with the aim to simulate the orbits of S-stars around GC. As it is well-known that Yukawa-like potentials could not be entirely represented by the standard PPN formalism, these potentials require its extension/modification in the form of an additional Yukawa-like correction (see [67,73–75]). Although adding a Yukawa term to the PPN equations of motion represents a phenomenological test-bed rather than an exact, self-consistent black hole metric solution derived from a specific non-linear $f(R)$ theory, this framework is a standard and highly effective tool in the modern literature for setting model-independent boundaries on a fifth-force parameters [74–78]. In this so-called extended PPN formalism, the equation of motion (EoM) for Yukawa gravity is given by [79]

$$\ddot{\vec{r}} = -GM \frac{\vec{r}}{r^3} + \frac{GM}{c^2 r^3} \left[\left(4 \frac{GM}{r} - \ddot{\vec{r}} \cdot \ddot{\vec{r}} \right) \vec{r} + 4(\ddot{\vec{r}} \cdot \ddot{\vec{r}}) \ddot{\vec{r}} \right] + \frac{\delta \cdot GM}{1+\delta} \left[1 - \left(1 + \frac{r}{\lambda} \right) e^{-\frac{r}{\lambda}} \right] \frac{\vec{r}}{r^3}. \quad (5)$$

The last term in this Equation (5) is an addition to the standard PPN formalism for EoM in GR, and it vanishes when $\delta = 0$ or $\lambda \rightarrow \infty$, recovering in that way the standard PPN

equation of motion (one can see Ref. [73] for more details). As demonstrated in some of our recent papers (see, e.g., [67–70]), the above PPN EoM (5) could be used to directly calculate a simulated true orbit of an S-star in Yukawa gravity.

The PPN framework is fundamentally a weak-field, slow-motion expansion originally developed for systems such as the Solar System. The orbits of S-stars remain well outside the event horizon of Sgr A*, and they probe a gravitational field that is stronger than in the Solar System. For the Solar System, the dimensionless gravitational potential strength $\epsilon = GM/(rc^2)$ is $\epsilon < 10^{-5}$ [80], while for the S-stars at their pericenter, it is on the order of 10^{-4} to 10^{-3} [13,17,81]. This confirms that the motion of the S-stars is safely within the weak-field, slow-motion regime where $v/c \ll 1$, and fully justifies the usage of the first-order post-Newtonian (1PN) PPN equations as a valid approximation. Higher-order GR effects, such as 2PN corrections or frame-dragging (Lense–Thirring effect) due to the black hole’s spin, are suppressed by additional factors of 10^{-3} or more. It means that higher-order GR effects fall well below the current astrometric error margins of $\sim \text{mas}$.

The results given in this paper represent an extension of our previous results for the S2 star, with the aim to study the possible deviations from GR at GC, as well as to constrain the parameters of a potential fifth force by applying the MCMC analysis to the observed orbit of the S38 star. For this analysis we assumed the PPN EoM (5) as our model which we then fitted to the astrometric observations of S38 star using the MCMC method. This MCMC study of S38 star orbit using the extended PPN formalism is quite different and new regarding the standard PPN formalism commonly used for modeling stellar orbits in GR and some alternative theories of gravity.

With the aim to obtain the estimates of strength δ and range λ of a fifth force at GC, we first calculated the simulated orbits of S38 star in the extended PPN formalism using the EoM (5). Since orbital integration results with coordinates and velocity components in the orbital plane (i.e., along the true orbit), it is necessary to first project them onto the observer’s sky plane, i.e., calculate the corresponding coordinates along the apparent orbit (for the details, see our paper [68]). Then, we fitted these simulated orbits into the astrometric observations of S38 star from [10] using the MCMC method. For the orbital fitting we used the MCMC procedure <https://emcee.readthedocs.io/en/stable/> (accessed on 2 August 2026) (emcee) (the Python implementation of Goodman & Weare’s Affine Invariant MCMC Ensemble sampler [82]). More about MCMC analysis one can find in [70] and [83]. The MCMC fitting is performed by maximizing the likelihood $\mathcal{L} \propto \exp(-\chi^2/2)$, where the reduced χ^2 between the observed (x_i^o, y_i^o) and calculated (x_i^c, y_i^c) positions of S38 star along its apparent orbit is given by:

$$\chi^2 = \chi_{\text{pos}}^2 = \frac{\chi_x^2 + \chi_y^2}{2} = \frac{1}{2(N-\nu)} \sum_{i=1}^N \left[\left(\frac{x_i^o - x_i^c}{\sigma_{xi}} \right)^2 + \left(\frac{y_i^o - y_i^c}{\sigma_{yi}} \right)^2 \right], \quad (6)$$

with N —the number of observations, ν —the number of unknown parameters, $(\sigma_{xi}, \sigma_{yi})$ —the observed astrometric uncertainties. The fitted radial velocities were calculated from the positions and velocities along the resulting best-fit orbit, according to [64]:

$$v_{\text{rad}} = \frac{\sin i}{\sqrt{x^2 + y^2}} [\sin(\theta + \omega) \cdot (x\dot{x} + y\dot{y}) + \cos(\theta + \omega) \cdot (x\dot{y} - y\dot{x})], \quad \theta = \arctan \frac{y}{x}. \quad (7)$$

As a result of the above fitting procedure, we obtained the estimates of the posterior probability distributions of all fitted parameters, including S38 star orbital elements and a fifth force parameters δ and λ , as well as their best-fit values and uncertainties.

In this investigation we chose to fit the following minimal set of parameters which are necessary for constraining a fifth force at GC:

- SMBH mass M ,
- six orbital elements of S2 star: a [arcsec]—semi-major axis, e —eccentricity, i [°]—inclination, Ω [°]—longitude of the ascending node, ω [°]—pericenter argument and T [yr]—epoch of the pericenter passage,
- two parameters describing a fifth force: δ —its strength, and λ [AU]—its range.

For more references about MCMC analysis applied on S-star orbits, we refer the readers to these recent articles: [27,32,70,84–87].

4. Results and Discussion

To constrain the parameters of a potential fifth force by analyzing the orbit of the S38 star around Sgr A* and to compare the obtained results with those for the S2 star, we adopted the observed orbital elements of S38 star and their uncertainties from Table 3 of [10] and analyzed the following three cases for a fifth force range λ :

1. λ from 200 to 500 AU (i.e., much smaller than S38 star orbit),
2. λ from 500 to 2500 AU (i.e., on the order of S38 star orbit), and
3. λ from 1000 to 10000 AU (i.e., much larger than S38 star orbit).

We then performed MCMC analysis of S38 star orbit using the Gaussian priors for orbital parameters from Table 1, as well as the uniform priors for δ and λ from Table 2. The obtained results are then compared with the corresponding results for S2 star from [70].

Table 1. Gaussian priors for the MCMC analysis of S38 star orbit. The priors are common for all studied cases.

Parameter	Gaussian Priors	
	μ	σ
M ($\times 10^6 M_{\odot}$)	4.28	0.1
a (")	0.1416	0.0002
e	0.8201	0.0007
i (°)	171.1	2.1
Ω (°)	101.06	0.24
ω (°)	17.99	0.25
T (yr)	2003.19	0.01

Table 2. Specific uniform priors for range λ and strength δ of a fifth force. The priors are different for the three studied cases of S38 star.

	Parameter	Initial Value	Lower Limit	Upper Limit
Case 1:	λ (AU)	250	200	500
	δ	1×10^{-3}	−0.2	0.2
Case 2:	λ (AU)	2×10^3	500	2.5×10^3
	δ	1×10^{-3}	−0.3	0.3
Case 3:	λ (AU)	5×10^3	1×10^3	1×10^4
	δ	0.2	−0.95	0.95

Figure 1 presents an example of the best-fit orbit of S38 star obtained by our MCMC analysis in case 1 from Table 2. As can be seen from this figure, the best-fit orbit in the extended PPN formalism is in very good agreement with the observations. Orbital fits for the other two cases are omitted due to their visual similarity with the presented example.

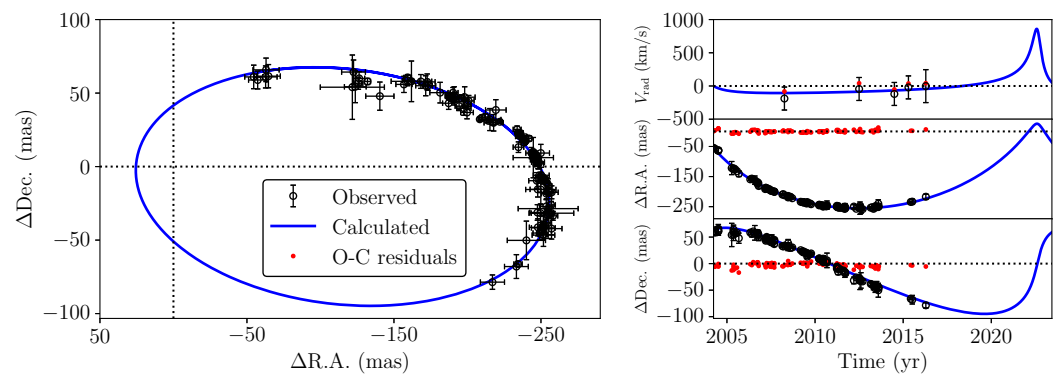


Figure 1. Left: The best-fit orbit determined from MCMC analysis of S38 star (blue solid line) in case 1 from Table 2, obtained using the corresponding astrometric observations from [10] (the observations are presented by black circles with error bars). Right: The calculated and observed radial velocity of the S38 star (top), as well as its R.A. (middle) and Dec. (bottom) offset relative to the position of Sgr A* at the coordinate origin. Red dots in all three right panels denote the corresponding O-C residuals.

The posterior probability distributions of the fitted parameters in all three studied cases, obtained by our MCMC analysis of S38 star orbit, are presented in Figures 2–4. The corresponding numerical results for the best-fit values and uncertainties of the parameters, obtained from the 16th, 50th and 84th percentiles of their posterior probability distributions, are given in Table 3.

Table 3. Results of the MCMC analysis of the S38 star orbit for all three cases from Table 2. The best-fit values of the parameters and their uncertainties are given. These values are obtained from the 16th, 50th and 84th percentiles of the posterior probability distributions which are presented in Figures 2–4.

Parameter	Case 1	Case 2	Case 3
M ($\times 10^6 M_{\odot}$)	$4.29^{+0.09}_{-0.10}$	$4.29^{+0.10}_{-0.09}$	$4.29^{+0.09}_{-0.09}$
a (")	$0.1416^{+0.0002}_{-0.0002}$	$0.1416^{+0.0002}_{-0.0002}$	$0.1416^{+0.0002}_{-0.0002}$
e	$0.8201^{+0.0007}_{-0.0007}$	$0.8201^{+0.0007}_{-0.0007}$	$0.8201^{+0.0007}_{-0.0007}$
i ($^{\circ}$)	$171.15^{+2.08}_{-2.10}$	$171.16^{+2.10}_{-2.09}$	$171.05^{+2.11}_{-2.04}$
Ω ($^{\circ}$)	$101.03^{+0.23}_{-0.23}$	$101.03^{+0.23}_{-0.22}$	$101.04^{+0.23}_{-0.23}$
ω ($^{\circ}$)	$18.02^{+0.24}_{-0.24}$	$18.01^{+0.24}_{-0.24}$	$18.02^{+0.24}_{-0.24}$
T (yr)	$2003.19^{+0.01}_{-0.01}$	$2003.19^{+0.01}_{-0.01}$	$2003.19^{+0.01}_{-0.01}$
δ	$-0.0012^{+0.0117}_{-0.0111}$	$-0.0003^{+0.0746}_{-0.0606}$	$0.1243^{+0.5092}_{-0.3639}$
λ (AU)	$362.54^{+96.59}_{-107.51}$	$1915.25^{+423.58}_{-685.61}$	$6990.08^{+2099.12}_{-2595.72}$

As it can be seen from the presented results, the best-fit values of a fifth force range λ obtained from the S38 star orbit in all three studied cases are $\lambda \approx 363$, 1915 and 6990 AU, respectively. These values are very close to our previous results of $\lambda \approx 360$, 1900 and 7400 AU, obtained in the case of S2 star [70].

The obtained results indicate that the strength δ of a fifth force increases and its relative error $\Delta\delta/\delta$ decreases with increase in the range λ . The best-fit values of the strength δ obtained from the S38 star orbit in all three studied cases are: $\delta \approx -0.0012$, -0.0003 and 0.1243 . The first two values are slightly smaller in magnitude, and the third one is very close to the corresponding results for S2 star, for which we obtained $\delta \approx -0.007$, -0.028 and -0.114 , respectively [70]. This difference is most likely caused by the slightly larger orbit of S38 star and somewhat worse statistics due to the smaller number of observations. The relative errors for δ are rather big, so that GR case cannot be excluded. The value $|\delta| \approx 0.0012$, obtained in the first case, is in good agreement with the recent constraints on the strength and range of a fifth force obtained by the GRAVITY Collaboration [85] for a

similar length scale, but for a slightly different model of Yukawa gravity. Moreover, the results obtained in the third case for both S2 and S38 stars are compatible with our earliest estimates for Yukawa gravity parameters given in [64].

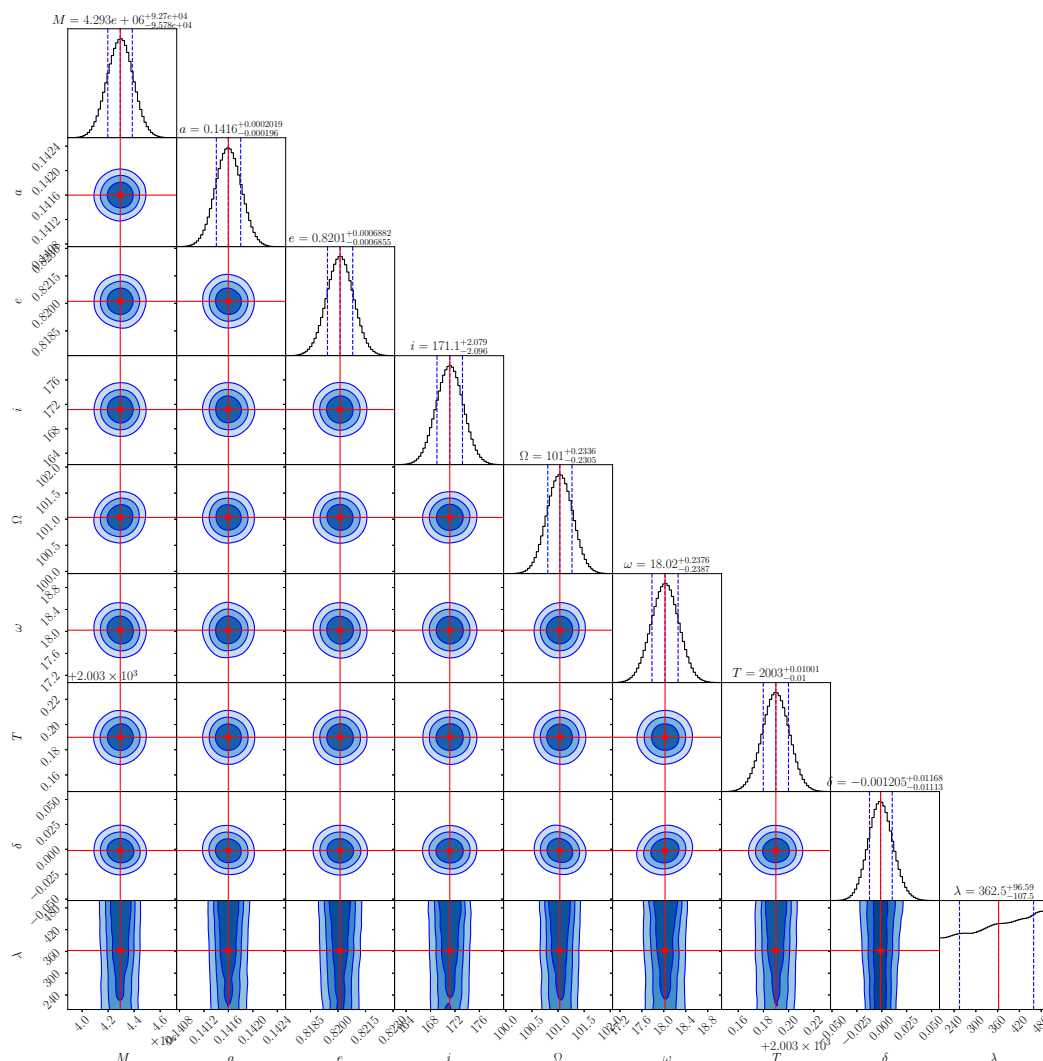


Figure 2. Posterior distribution of S38 star orbit fit determined from MCMC analysis in case when a fifth force range λ is in interval from 200 to 500 AU (case 1 from Table 2). The following parameters are fitted (see Section 3 for more details): SMBH mass M ($M_{\odot}$), semi-major axis a (arcsec), eccentricity e , inclination i ($^{\circ}$), longitude of the ascending node Ω ($^{\circ}$), pericenter argument ω ($^{\circ}$), epoch of the pericenter passage T (yr), strength of a fifth force δ and its range λ (AU). The 2D contours, shaded in blue, represent the confidence levels of 39.3%, 67.5% and 86.4% which correspond to the 1σ , 1.5σ and 2σ confidence regions for a 2D Gaussian distribution. The best-fit values of the parameters are presented with the red crosses over 2D contours. These values, and their uncertainties, were obtained from the 16th, 50th and 84th percentiles (vertical lines in 1D histograms) of the corresponding posterior probability distributions.

The resulting values of λ presented in Table 3 and Figures 2–4 are bounded by the separately imposed prior intervals and should be viewed as boundary constraints rather than independent physical measurements. Since the GR value of $\delta = 0$ is completely enclosed within our reported 1σ uncertainties across all simulated cases, our study does not find statistically significant evidence for a non-zero fifth force. Instead, it establishes rigid upper bounds. Moreover, our additional theoretical considerations suggest that these two parameters could be correlated and that this correlation could be stronger in the case of weaker interactions [70]. However, in our MCMC analysis, the parameter space for the

interaction range λ is not arbitrary, rather, the best-fit values obtained independently from the S38 orbit ($\lambda \approx 363, 1915$, and 6990 AU) show a remarkable and highly stable consistency with our previous findings for the S2 star ($\lambda \approx 360, 1900$, and 7400 AU, respectively). While $\delta = 0$ is statistically allowed, the non-degenerate behavior and tight clustering of λ leave a well-defined physical window open for prospective fifth force scenarios at the GC.

Besides, the presented results of the orbital fitting are also in agreement with our constraints on the graviton Compton wavelength λ from Refs. [67–69], which were obtained from the values of the PPN parameter f_{SP} estimated by the GRAVITY Collaboration [17,84]. This parameter was introduced by the GRAVITY Collaboration [17] in order to parametrize the effect of the Schwarzschild metric and to study to which extent some gravitational model is relativistic. In theory, it is equal to $f_{SP} = (2 + 2\gamma - \beta)/3$, with β, γ being standard PPN parameters. In the GR case it is: $\beta = 1 = \gamma$, and thus $f_{SP} = 1$. Therefore, the parameter f_{SP} may be used for constraining a fifth force, as well as for studying possible deviations from GR and whether the simulated orbits of S-stars are compatible with the value of $f_{SP} = 1.10 \pm 0.19$ which was measured by GRAVITY Collaboration in the case of S2 star [17].

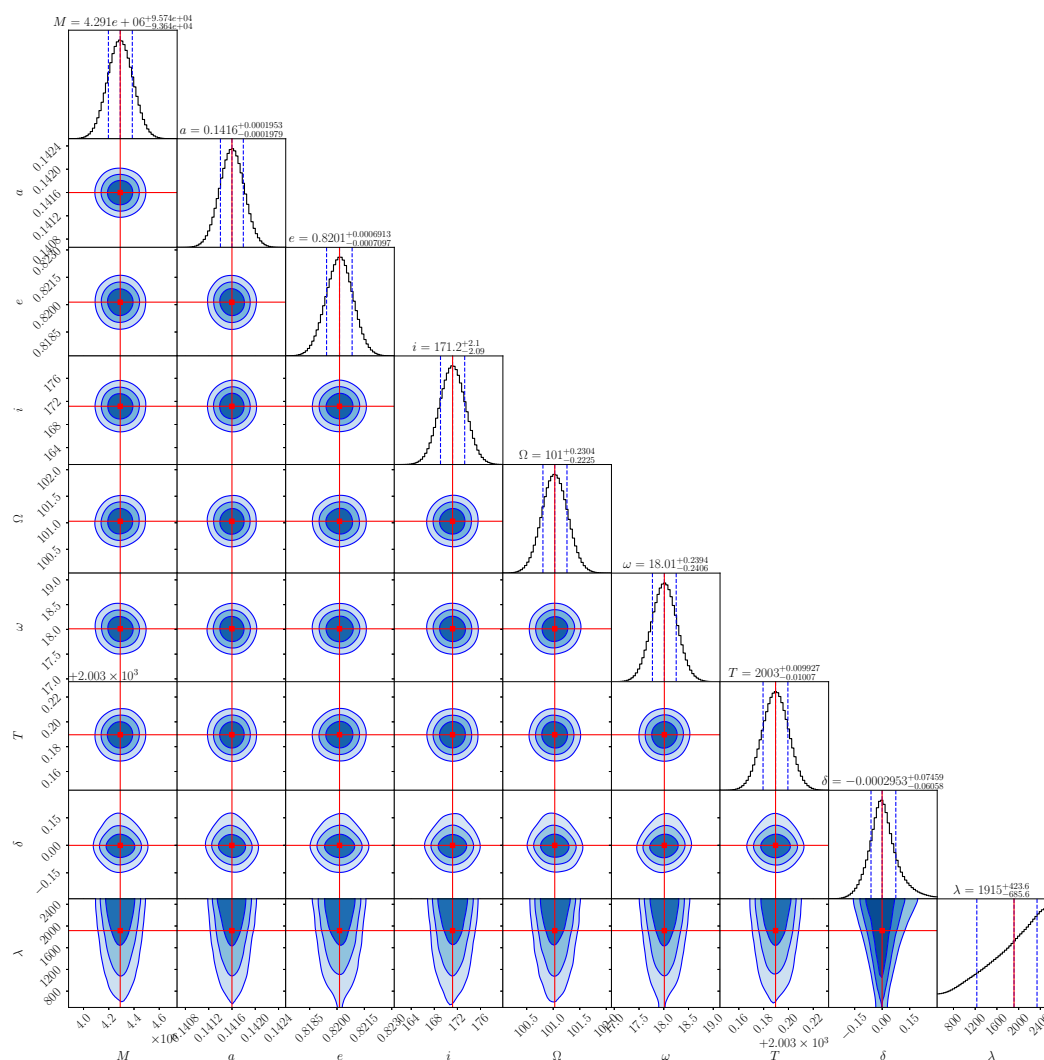


Figure 3. The same as in Figure 2, but for a fifth force range λ in interval from 500 to 2500 AU (case 2 from Table 2).

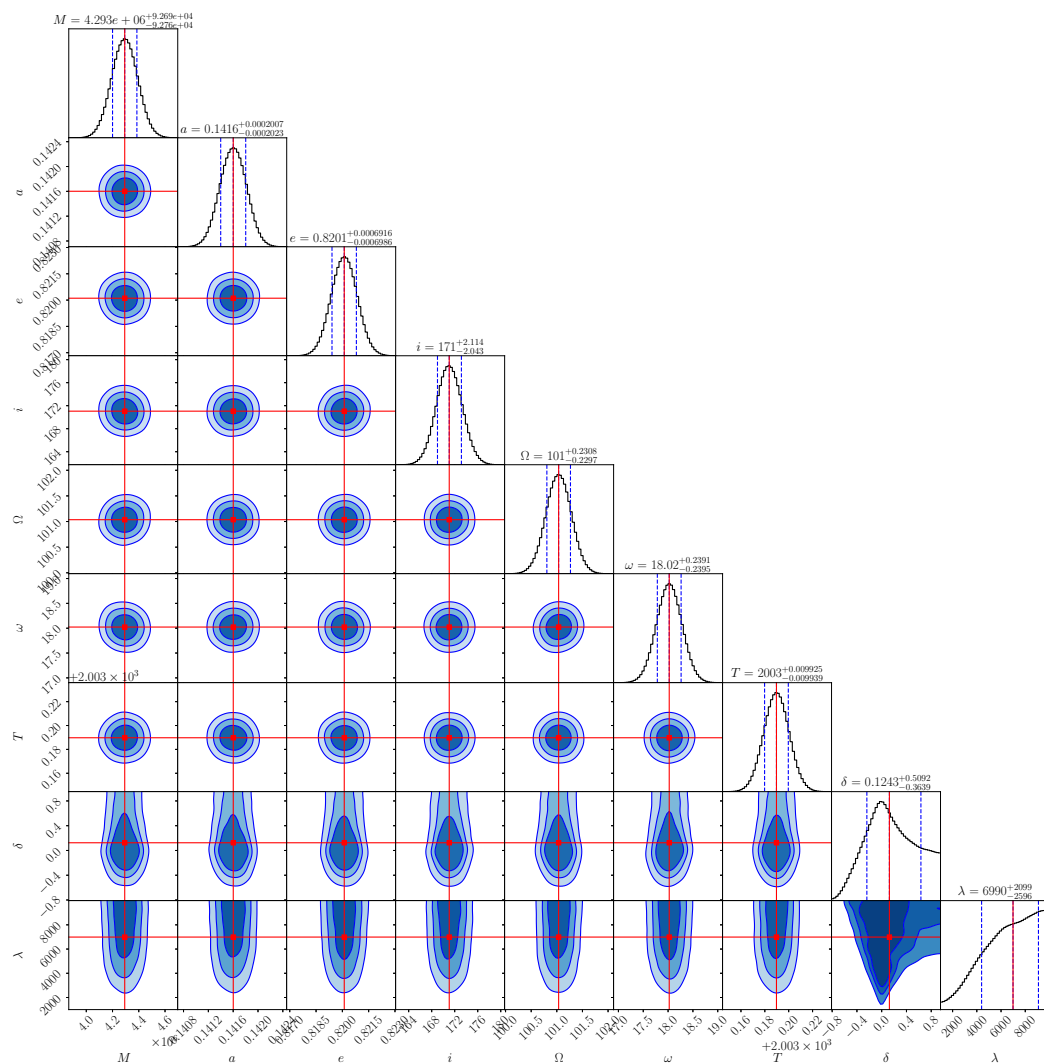


Figure 4. The same as in Figure 2, but for a fifth force range λ in interval from 1000 to 10,000 AU (case 3 from Table 2).

In order to study whether a fifth force could have a noticeable effect on the Schwarzschild precession of S38 star orbit, we used the results from Table 3 to estimate its total precession $\Delta\varphi$ per orbital period according to Equation (4). We made this estimation only for the third studied case, since only then the necessary condition $a \ll \lambda$ is satisfied. In this case we found that total orbital precession per orbital period is $\Delta\varphi \approx 0^\circ.44$ which is significantly larger than the Schwarzschild precession in GR of $\Delta\varphi_{Schw} \approx 0^\circ.12$. Therefore, provided that it exists, a fifth force could significantly affect the Schwarzschild precession of the S38 star orbit.

5. Conclusions

In this paper we investigated the capability of Yukawa gravity to explain the Schwarzschild precession of S-stars as a probe for GR and a possible presence of a fifth force. We simulated the S38 star orbit in the extended PPN formalism for Yukawa gravity that predicts the emergence of a fifth force. Using the MCMC method for fitting the astrometric observations of S38 star, we constrained the parameters δ and λ of Yukawa interaction and compared our results with the previously obtained results for the S2 star. We studied three cases for a fifth force range λ : (i) when it is a few times smaller than the orbit of S38 star, (ii) when it is on the order of S38 star orbit, and (iii) when it is much larger than the size of S38 star orbit. We also tested the compliance of S38 star orbit with GR, i.e., whether

it slightly deviates from the GR prediction. The results of this study can be summarized as follows:

1. Our analysis shows that a potential fifth force could significantly alter the Schwarzschild precession of the S38 star orbit, providing a sensitive probe for testing gravity;
2. The presented results for S38 star are in good agreement with the recent constraints on the strength and range of a fifth force obtained for S2 star by the GRAVITY Collaboration [85], as well as with our earliest estimates for Yukawa gravity parameters given in [64] and with our constraints on the graviton Compton wavelength λ obtained from the values of the PPN parameter f_{SP} [67–69];
3. The best-fit values of a fifth force range λ obtained from S38 star orbit in all three studied cases are $\lambda \approx 363, 1915$ and 6990 AU, respectively. These results are very close to our previous results for S2 star [70];
4. The best-fit values of a fifth force strength δ obtained from S38 star orbit in all three studied cases are $\delta \approx -0.0012, -0.0003$ and 0.1243 . These results are a little bit smaller in magnitude in respect to the corresponding values for S2 star [70], most likely due to the larger orbit of S38 star.
5. The strength of a fifth force δ increases and its relative error $\Delta\delta/\delta$ decreases with increase in the range λ ;
6. The obtained results outline the tight constraints on Yukawa-type modifications at the Galactic Center, defining the remaining allowed phase space for a potential fifth force. Since the GR case ($\delta = 0$) is enclosed well within the reported 1σ uncertainties, these boundaries do not constitute statistically significant evidence for deviations from GR. However, the remarkable cross-consistency between the λ values derived independently from the S38 and S2 orbits (exhibiting stable clusters around approximately 360, 1900, and 7000 AU) keeps a viable physical window open. This agreement demonstrates that the spatial scale of the interaction is robust against individual stellar orbit systematic errors, ensuring that the investigated parameter space remains a highly relevant target for future, higher-precision observations.

GR is one of the most profound and beautiful physical theories with great phenomenological achievements and nice theoretical properties. Nevertheless, GR is not a complete theory of gravity. While it very nicely describes the dynamics of the Solar System, at the galactic, extragalactic and cosmological scales, GR needs the inclusion of DM and DE, which are without experimental evidence yet. Hence, there is an ongoing work on modifications of GR with the aim of explaining a lot of astrophysical phenomena at different astronomical scales. Also, modified gravity models can address many problems at cosmological scales [88]. Here, we demonstrated the ability of Yukawa gravity to reproduce the stellar kinematics and dynamics at galactic scales, without need for introducing unknown forms of DM or DE. We hope that this could contribute to the efforts to find the gravitational theories capable of describing larger parts of the physical world than GR.

Author Contributions: All coauthors participated in writing, calculation and discussion of obtained results. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Ministry of Science, Technological Development and Innovations of the Republic of Serbia through the Project contracts Nos . 451-03-33/2026-03/200002 and 451-03-33/2026-03/200017.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DM	Dark Matter
DE	Dark Energy
ETGs	Extended Theories of Gravity
GC	Galactic Center
GR	General Relativity
GW	Gravitational Wave
MCMC simulation	Markov Chain Monte Carlo simulation
PPN formalism	Parameterized Post-Newtonian formalism
SMBH	Supermassive Black Hole

References

1. Aghanim, N. et al. [Planck Collaboration]. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.* **2020**, *641*, A6. [[CrossRef](#)]
2. Pitjeva, E.V.; Pitjev, N.P. Relativistic effects and dark matter in the Solar system from observations of planets and spacecraft. *Mon. Not. R. Astron. Soc.* **2013**, *432*, 3431–3437.
3. Ghez, A.M.; Morris, M.; Becklin, E.E.; Tanner, A.; Kremenek, T. The accelerations of stars orbiting the Milky Way’s central black hole. *Nature* **2000**, *407*, 349–351. [[CrossRef](#)] [[PubMed](#)]
4. Schödel, R.; Ott, T.; Genzel, R.; Hofmann, R.; Lehnert, M.; Eckart, A.; Mouawad, N.; Alexander, T.; Reid, M.J.; Lenzen, R.; et al. Closest star seen orbiting the supermassive black hole at the Centre of the Milky Way. *Nature* **2002**, *419*, 694. [[CrossRef](#)] [[PubMed](#)]
5. Genzel, R.; Eisenhauer, F.; Gillessen, S. The Galactic Center massive black hole and nuclear star cluster. *Rev. Mod. Phys.* **2010**, *82*, 3121–3195. [[CrossRef](#)]
6. Meyer, L.; Ghez, A.M.; Schödel, R.; Yelda, S.; Boehle, A.; Lu, J.R.; Do, T.; Morris, M.R.; Becklin, E.E.; Matthews, K. The Shortest-Known-Period Star Orbiting Our Galaxy’s Supermassive Black Hole. *Science* **2012**, *338*, 84–87. [[CrossRef](#)] [[PubMed](#)]
7. Ghez, A.M.; Salim, S.; Weinberg, N.N.; Lu, J.R.; Do, T.; Dunn, J.K.; Matthews, K.; Morris, M.R.; Yelda, S.; Becklin, E.E.; et al. Measuring distance and properties of the Milky Way’s central supermassive black hole with stellar orbits. *Astrophys. J.* **2008**, *689*, 1044–1062. [[CrossRef](#)] [[PubMed](#)]
8. Gillessen, S.; Eisenhauer, F.; Fritz, T.K.; Bartko, H.; Dodds-Eden, K.; Pfuhl, O.; Ott, T.; Genzel, R. The orbit of the star S2 around Sgr A* from very large telescope and Keck data. *Astrophys. J.* **2009**, *707*, L114–L117. [[CrossRef](#)]
9. Gillessen, S.; Eisenhauer, F.; Trippe, S.; Alexander, T.; Genzel, R.; Martins, F.; Ott, T. Monitoring stellar orbits around the massive black hole in the Galactic Center. *Astrophys. J.* **2009**, *692*, 1075–1109. [[CrossRef](#)]
10. Gillessen, S.; Plewa, P.M.; Eisenhauer, F.; Sari, R.E.; Waisberg, I.; Habibi, M.; Pfuhl, O.; George, E.; Dexter, J.; von Fellenberg, S.; et al. An Update on Monitoring Stellar Orbits in the Galactic Center. *Astrophys. J.* **2017**, *837*, 30. [[CrossRef](#)]
11. Hees, A.; Do, T.; Ghez, A.M.; Martinez, G.D.; Naoz, S.; Becklin, E.E.; Boehle, A.; Chappell, S.; Chu, D.; Dehghanfar, A.; et al. Testing General Relativity with Stellar Orbits around the Supermassive Black Hole in Our Galactic Center. *Phys. Rev. Lett.* **2017**, *118*, 211101. [[CrossRef](#)] [[PubMed](#)]
12. Chu, D.S.; Do, T.; Hees, A.; Ghez, A.; Naoz, S.; Witzel, G.; Sakai, S.; Chappell, S.; Gautam, A.K.; Lu, J.R.; et al. Investigating the Binarity of S0-2: Implications for Its Origins and Robustness as a Probe of the Laws of Gravity around a Supermassive Black Hole. *Astrophys. J.* **2018**, *854*, 12. [[CrossRef](#)]
13. Abuter, R. et al. [GRAVITY Collaboration]. Detection of the gravitational redshift in the orbit of the star S2 near the Galactic centre massive black hole. *Astron. Astrophys.* **2018**, *615*, L15. [[CrossRef](#)]
14. Abuter, R. et al. [GRAVITY Collaboration]. A geometric distance measurement to the Galactic center black hole with 0.3% uncertainty. *Astron. Astrophys.* **2019**, *625*, L10. [[CrossRef](#)]
15. Do, T.; Hees, A.; Ghez, A.; Martinez, G.D.; Chu, D.S.; Jia, S.; Sakai, S.; Lu, J.R.; Gautam, A.K.; O’Neil, K.K.; et al. Relativistic redshift of the star S0-2 orbiting the Galactic Center supermassive black hole. *Science* **2019**, *365*, 664–668. [[CrossRef](#)] [[PubMed](#)]
16. Amorim, A. et al. [GRAVITY Collaboration]. Scalar field effects on the orbit of S2 star. *Mon. Not. R. Astron. Soc.* **2019**, *489*, 4606–4621. [[CrossRef](#)]
17. Abuter, R. et al. [GRAVITY Collaboration]. Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic centre massive black hole. *Astron. Astrophys.* **2020**, *636*, L5. [[CrossRef](#)]
18. Hees, A.; Do, T.; Roberts, B.M.; Ghez, A.M.; Nishiyama, S.; Bentley, R.O.; Gautam, A.K.; Jia, S.; Kara, T.; Lu, J.R.; et al. Search for a Variation of the Fine Structure Constant around the Supermassive Black Hole in Our Galactic Center. *Phys. Rev. Lett.* **2020**, *124*, 081101. [[CrossRef](#)] [[PubMed](#)]

19. D’Addio, A. S-star dynamics through a Yukawa-like gravitational potential. *Phys. Dark Universe* **2021**, *33*, 100871. [[CrossRef](#)]
20. Borka, D.; Borka Jovanović, V.; Capozziello, S.; Zakharov, A.F.; Jovanović, P. Estimating the Parameters of Extended Gravity Theories with the Schwarzschild Precession of S2 Star. *Universe* **2021**, *7*, 407. [[CrossRef](#)]
21. Rubilar, G.F.; Eckart, A. Periastron shifts of stellar orbits near the Galactic Center. *Astron. Astrophys.* **2001**, *374*, 95–104. [[CrossRef](#)]
22. Adkins, G.S.; McDonnell, J. Orbital precession due to central-force perturbations. *Phys. Rev. D* **2007**, *75*, 082001. [[CrossRef](#)]
23. Weinberg, N.N.; Milosavljević, M.; Ghez, A.M. Stellar dynamics at the Galactic Center with an extremely large telescope. *Astrophys. J.* **2005**, *622*, 878–891. [[CrossRef](#)]
24. Bambhaniya, P.; Joshi, A.B.; Dey, D.; Joshi, P.S.; Mazumdar, A.; Harada, T.; Nakao, K.I. Relativistic orbits of S2 star in the presence of scalar field. *Eur. Phys. J. C* **2024**, *84*, 124.
25. Yu, J.C.; Cao, Y.; Shao, L. Constraining Ultralight Scalar Dark Matter in the Galactic Center with S2 Star. *arXiv* **2026**, arXiv:2604.08053.
26. Della Monica, R.; De Martino, I.; De Laurentis, M. Constraining MODified Gravity with the S2 Star. *Universe* **2022**, *8*, 137.
27. Bambhaniya, P.; Dalal, P.; Vicentin, G.H.; Della Monica, R.; de Gouveia Dal Pino, E.M.; Patel, B. Testing the Spacetime Geometry of Sgr A* with the Relativistic Orbit of S2 star. *arXiv* **2026**, arXiv:2602.18994.
28. Jusufi, K.; Kumar, S.; Azreg-Aïnou, M.; Jamil, M.; Wu, Q.; Bambi, C. Constraining Wormhole Geometries using the Orbit of S2 Star and the Event Horizon Telescope. *Eur. Phys. J. C* **2022**, *82*, 633. [[CrossRef](#)]
29. Dai, D.C.; Stojkovic, D. Observing a Wormhole. *Phys. Rev. D* **2019**, *100*, 083513. [[CrossRef](#)]
30. Simon, J.H.; Kavic, M.J.; Minic, D.; Stojkovic, D.; Dai, D.C. Sensitive searches for wormholes. *Phys. Rev. D* **2021**, *104*, L081502. [[CrossRef](#)]
31. Bambi, C.; Stojkovic, D. Astrophysical Wormholes. *Universe* **2021**, *7*, 136. [[CrossRef](#)]
32. Navarret, C.; Vazquez-Chavez, F.; Cruz-Osorio, A.; Ortiz, N. Testing black hole space-times with the S2 star orbit: A Bayesian comparison. *Mon. Not. R. Astron. Soc.* **2026**, *546*, stag059. [[CrossRef](#)]
33. De Laurentis, M. The Newtonian and relativistic theory of orbits and the emission of gravitational waves. *Open Astron. J.* **2010**, *11*, 1874.
34. Capozziello, S.; De Laurentis, M. Extended Theories of Gravity. *Phys. Rep.* **2011**, *509*, 167–321. [[CrossRef](#)]
35. Poisson, E.; Will, C.M. *Gravity*; Cambridge University Press: Cambridge, UK, 2014.
36. Iorio, L. *General Post-Newtonian Orbital Effects: From Earth’s Satellites to the Galactic Centre*; Cambridge University Press: Cambridge, UK, 2024.
37. Adelberger, E.G.; Gundlach, J.H.; Heckel, B.R.; Hoedl, S.; Schlamminger, S. Torsion balance experiments: A low-energy frontier of particle physics. *Prog. Part. Nucl. Phys.* **2009**, *62*, 102–134. [[CrossRef](#)]
38. Vainshtein, A.I. To the problem of nonvanishing graviton mass. *Phys. Lett. B* **2002**, *529*, 394–400.
39. Khoury, J.; Weltman, A. Chameleon fields: Awaiting surprises for tests of gravity in space. *Phys. Rev. D* **2004**, *69*, 044026.
40. Burrage, C.; Copeland, E.J.; Millington, P. Radiative Screening of Fifth Forces. *Phys. Rev. Lett.* **2016**, *117*, 211102. [[CrossRef](#)] [[PubMed](#)]
41. Jovanović, P.; Borka, D.; Borka Jovanović, V.; Zakharov, A.F. Influence of bulk mass distribution on orbital precession of S2 star in Yukawa gravity. *Eur. Phys. J. D* **2021**, *75*, 145. [[CrossRef](#)]
42. Burrage, C.; Sakstein, J. Tests of chameleon gravity. *Living Rev. Relativ.* **2018**, *21*, 1. [[CrossRef](#)] [[PubMed](#)]
43. Iorio, L. Constraints on the Galactic Center supermassive black hole’s spin from the S2 star’s orbital precession. *Phys. Rev. D* **2011**, *84*, 124001. [[CrossRef](#)]
44. Talmadge, C.; Berthias, J.P.; Hellings, R.W.; Standish, E.M. Model-independent constraints on possible modifications of Newtonian gravity. *Phys. Rev. Lett.* **1988**, *61*, 1159–1162. [[CrossRef](#)] [[PubMed](#)]
45. Sereno, M.; Jetzer, P. Dark matter versus modifications of the gravitational inverse-square law: Results from planetary motion in the Solar system. *Mon. Not. R. Astron. Soc.* **2006**, *371*, 626–632. [[CrossRef](#)]
46. Cardone, V.F.; Capozziello, S. Systematic biases on galaxy haloes parameters from Yukawa-like gravitational potentials. *Mon. Not. R. Astron. Soc.* **2011**, *414*, 1301–1313. [[CrossRef](#)]
47. Sanders, R.H. Anti-gravity and galaxy rotation curves. *Astron. Astrophys.* **1984**, *136*, L21.
48. Sanders, R.H. Mass discrepancies in galaxies: Dark matter and alternatives. *Astron. Astrophys. Rev.* **1990**, *2*, 1–28. [[CrossRef](#)]
49. Iorio, L. Constraints on the range Λ of Yukawa-like modifications to the Newtonian inverse-square law of gravitation from Solar System planetary motions. *J. High Energy Phys.* **2007**, *10*, 041. [[CrossRef](#)]
50. Iorio, L. Putting Yukawa-like Modified Gravity (MOG) on the test in the Solar System. *Sch. Res. Exch.* **2008**, *2008*, 238385. [[CrossRef](#)]
51. Budhi, R.H.S. Solar System Constraints on the Yukawa Potential in $f(R)$ Gravity. *Indones. J. Appl. Phys.* **2025**, *15*, 280–292. [[CrossRef](#)]
52. Capozziello, S.; Stabile, A.; Troisi, A. Newtonian limit of $f(R)$ gravity. *Phys. Rev. D* **2007**, *76*, 104019.

53. Capozziello, S.; De Filippis, E.; Salzano, V. Modelling clusters of galaxies by $f(R)$ -gravity. *Mon. Not. R. Astron. Soc.* **2009**, *394*, 947–959. [[CrossRef](#)]
54. White, M.J.; Kochanek, C.S. Constraints on the long-range properties of gravity from weak gravitational lensing. *Astrophys. J.* **2001**, *560*, 539–543. [[CrossRef](#)] [[PubMed](#)]
55. Amendola, L.; Quercellini, C. Skewness as a test of the equivalence principle. *Phys. Rev. Lett.* **2004**, *92*, 181102. [[CrossRef](#)] [[PubMed](#)]
56. Reynaud, S.; Jaekel, M.T. Testing the Newton law at long distances. *Int. J. Mod. Phys. A* **2005**, *20*, 2294–2303. [[CrossRef](#)]
57. Sealfon, C.; Verde, L.; Jimenez, R. Limits on deviations from the inverse-square law on megaparsec scales. *Phys. Rev. D* **2005**, *71*, 083004. [[CrossRef](#)]
58. Moffat, J.W. Gravitational theory, galaxy rotation curves and cosmology without dark matter. *J. Cosmol. Astropart. Phys.* **2005**, *2005*, 022. [[CrossRef](#)]
59. Moffat, J.W. Scalar-tensor-vector gravity theory. *J. Cosmol. Astropart. Phys.* **2006**, *2006*, 004. [[CrossRef](#)]
60. Stabile, A.; Capozziello, S. Galaxy rotation curves in $f(R, \phi)$ gravity. *Phys. Rev. D* **2013**, *87*, 064002.
61. Capozziello, S.; Borka, D.; Jovanović, P.; Borka Jovanović, V. Constraining Extended Gravity Models by S2 star orbits around the Galactic Centre. *Phys. Rev. D* **2014**, *90*, 044052. [[CrossRef](#)]
62. Capozziello, S.; Faraoni, V. *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*; Fundamental Theories of Physics; Springer: Berlin/Heidelberg, Germany, 2011; Volume 170.
63. Capozziello, S.; Harko, T.; Koivisto, T.S.; Lobo, F.S.N.; Olmo, G.J. Wormholes supported by hybrid metric-Palatini gravity. *Phys. Rev. D* **2012**, *86*, 127504. [[CrossRef](#)]
64. Borka, D.; Jovanović, P.; Borka Jovanović, V.; Zakharov, A.F. Constraining the range of Yukawa gravity interaction from S2 star orbits. *J. Cosmol. Astropart. Phys.* **2013**, *2013*, 050. [[CrossRef](#)]
65. Zakharov, A.F.; Jovanović, P.; Borka, D.; Borka Jovanović, V. Constraining the range of Yukawa gravity interaction from S2 star orbits II: Bounds on graviton mass. *J. Cosmol. Astropart. Phys.* **2016**, *2016*, 045. [[CrossRef](#)]
66. Zakharov, A.F.; Jovanović, P.; Borka, D.; Borka Jovanović, V. Constraining the range of Yukawa gravity interaction from S2 star orbits III: Improvement expectations for graviton mass bounds. *J. Cosmol. Astropart. Phys.* **2018**, *2018*, 050. [[CrossRef](#)]
67. Jovanović, P.; Borka Jovanović, V.; Borka, D.; Zakharov, A.F. Constraints on Yukawa gravity parameters from observations of bright stars. *J. Cosmol. Astropart. Phys.* **2023**, *2023*, 056. [[CrossRef](#)]
68. Jovanović, P.; Borka Jovanović, V.; Borka, D.; Zakharov, A.F. Improvement of graviton mass constraints using GRAVITY's detection of Schwarzschild precession in the orbit of S2 star around the Galactic Center. *Phys. Rev. D* **2024**, *109*, 064046. [[CrossRef](#)]
69. Jovanović, P.; Borka Jovanović, V.; Borka, D.; Zakharov, A.F. Constraints on Graviton Mass from Schwarzschild Precession in the Orbits of S-Stars around the Galactic Center. *Symmetry* **2024**, *16*, 397. [[CrossRef](#)]
70. Jovanović, P.; Borka, D.; Borka Jovanović, V. Constraints on a Fifth Force from the Stellar Orbits Around the Central Supermassive Black Hole of the Milky Way. *Symmetry* **2026**, *18*, 557. [[CrossRef](#)]
71. Capozziello, S.; Borka Jovanović, V.; Borka, D.; Jovanović, P. Constraining theories of gravity by fundamental plane of elliptical galaxies. *Phys. Dark Universe* **2020**, *29*, 100573. [[CrossRef](#)]
72. Capozziello, S.; Jovanović, P.; Borka Jovanović, V.; Borka, D. Addressing the missing matter problem in galaxies through a new fundamental gravitational radius. *J. Cosmol. Astropart. Phys.* **2017**, *2017*, 044. [[CrossRef](#)]
73. Will, C.M. *Theory and Experiment in Gravitational Physics*, 2nd ed.; Cambridge University Press: Cambridge, UK, 2018; 360p.
74. Clifton, T. Parametrized post-Newtonian limit of fourth-order theories of gravity. *Phys. Rev. D* **2008**, *77*, 024041. [[CrossRef](#)]
75. Alsing, J.; Berti, E.; Will, C.M.; Zaglauer, H. Gravitational radiation from compact binary systems in the massive Brans-Dicke theory of gravity. *Phys. Rev. D* **2012**, *85*, 064041. [[CrossRef](#)]
76. Bernus, L.; Minazzoli, O.; Fienga, A.; Gastineau, M.; Laskar, J.; Deram, P. Constraining the Mass of the Graviton with the Planetary Ephemeris INPOP. *Phys. Rev. Lett.* **2019**, *123*, 161103. [[CrossRef](#)] [[PubMed](#)]
77. Gainutdinov, R.I. PPN Motion of S-Stars around Sgr A*. *Astrophysics* **2020**, *63*, 470–481. [[CrossRef](#)]
78. Gainutdinov, R.; Baryshev, Y. Relativistic Effects in Orbital Motion of the S-Stars at the Galactic Center. *Universe* **2020**, *6*, 177. [[CrossRef](#)]
79. Dong, Y.; Shao, L.; Hu, Z.; Miao, X.; Wang, Z. Prospects for constraining the Yukawa gravity with pulsars around Sagittarius A*. *J. Cosmol. Astropart. Phys.* **2022**, *2022*, 051. [[CrossRef](#)]
80. Will, C.M. The Confrontation between General Relativity and Experiment. *Living Rev. Relativ.* **2014**, *17*, 4. [[CrossRef](#)] [[PubMed](#)]
81. Parsa, M.; Eckart, A.; Shahzamanian, B.; Karas, V.; Zajaček, M.; Brito, L.; Sabha, N. Investigating the Relativistic Periastron Advance of Star S2 at the Galactic Center. *Astrophys. J.* **2017**, *845*, 22.
82. Foreman-Mackey, D.; Hogg, D.W.; Lang, D.; Goodman, J. emcee: The MCMC Hammer. *Publ. Astron. Soc. Pac.* **2013**, *125*, 306–312. [[CrossRef](#)]
83. Hogg, D.W.; Foreman-Mackey, D. Data Analysis Recipes: Using Markov Chain Monte Carlo. *Astrophys. J. Suppl. Ser.* **2018**, *236*, 11. [[CrossRef](#)]

84. Abd El Dayem, K. et al. [GRAVITY Collaboration]. Improving constraints on the extended mass distribution in the Galactic center with stellar orbits. *Astron. Astrophys.* **2024**, 692, A242. [[CrossRef](#)]
85. Abd El Dayem, K. et al. [GRAVITY Collaboration]. Exploring the presence of a fifth force at the Galactic Center. *Astron. Astrophys.* **2025**, 698, L15. [[CrossRef](#)]
86. Hassan, D.S.; Dio Danarianto, M.; Sulaksono, A. The Milky Way and M31 rotation curves in Yukawa gravity: Phenomenology and Bayesian analysis. *Eur. Phys. J. C* **2026**, 86, 198. [[CrossRef](#)]
87. Qi, Q.; Sang, Y.; Kuang, X.M. Probing a Lorentz-violating parameter from orbital precession of the S2 star around the galactic centre supermassive black hole. *arXiv* **2026**, arXiv:2601.06491.
88. Di Valentino, E.; Levi Said, J.; Riess, A.; Pollo, A.; Poulin, V.; Gómez-Valent, A.; Weltman, A.; Palmese, A.; Huang, C.D.; Bruck, C.; et al. The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics. *Phys. Dark Universe* **2025**, 49, 101965. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.