



# Article Waved-Shape Accumulation of Ice Jam—Analysis and Experimental Study

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Abstract: Ice jam is a unique hydrological phenomenon in rivers in cold regions. The appearance of an ice jam in a river results in an increase in the wetted perimeter of the flow cross-section, and thus an increase in flow resistance as well as water level. It may cause ice flooding sometimes. Similar to the "sand wave" phenomenon in riverbed, it has been observed in laboratory experiments that the waved-shape accumulation of ice particles (termed as "ice wave") under an ice jam occurred. In this study, an Equation for describing the relationship between the approaching flow Froude number (Fr) and the ratio of ice jam thickness to flow depth (t/H) has been proposed. Taking the inflection point value of the equation under different flow depths, a characteristic curve has been developed to judge whether ice waves under an ice jam occurs. When the flow Froude number in front of an ice jam is below the value at the inflection point of the curve, the ice jam can maintain a mechanical stability within the ice jam thickness in a range from the lower limiting value to the upper limiting value, which were close to the ice wave trough thickness and the ice wave crest thickness, respectively. An Equation for calculating the ice wavelength has been derived and verified by using results of laboratory experiments. The relationship between the migration speed of ice wave and the ratio of ice discharge to water flow rate (Qi/Q) has been also analyzed. At last, case studies have been conducted with respect to ice accumulation in the St. Lawrence River, the Beauharnois Canal and the La Grande River. Results of case studies show that the shoving and ice dam have been dominated by mechanical factors, which would be accompanied by the ice wave phenomenon during the ice jam accumulation process. Results of case studies about ice accumulation in natural rivers also show that the relative thickness of an ice jam (t/H) of 0.4 is the criterion for assessing whether an ice jam in a river belongs to an ice dam.

Keywords: ice jam; ice wave; experiment; wave crest; wave trough; wavelength; migration speed

# 1. Introduction

River ice is a hydrological phenomenon in cold regions. River ice has a great impact on the global hydrological system, especially in the northern hemisphere [1]. Ice jam and ice dam can be developed when ice floes transported by flowing water is arrested by obstacles such as stationary ice cover or congested [2]. Due to a large aggregate thickness and a high hydraulic resistance comparing to those of a sheet ice cover, ice jams tend to disturb the riverbed, and cause high water level. River ice has many repercussions for operation and design including determining the overturning moment on structures, controlling the severity of spring flooding and assessing the associated water level—frequency relationships, or of predicting riverbed scour due to surges associated with suddenly rebreak-up of ice jams [3–5]. The Yellow River in China is one of the rivers with serious ice flooding disaster [6,7]. During winter periods, flooding resulted from ice jams happens very often in some river reaches including the Inner Mongolia Reach, Hequ Reach, and the Lower reach of the Yellow River [2,4,7–9].



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**Copyright:** © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). A variety of research works regarding river ice hydraulics have been carried out based on prototype observations [2,4,7–9], laboratory experiments [10,11], theoretical analysis [12–14] and numerical simulation [15–18]. Up to date, most of the reported research is focused on the formation, evolution and mechanism of ice jams. According to field observations in natural rivers, many scholars believe that the development of an ice cover or jam will generally go through following stages, ice pans (floe) floating period, freeze-up period, ice-covered/jammed period and river break-up period.

During the ice-pans floating period, the surface layer of flowing water lost too much heat due to the decrease in air temperature [19]. On the other side, the turbulent mixing process of flowing water causes the heat exchange between the water surface layer and the main water body. The heat loss from flowing water in a river is carried out rapidly, especially in the main channel. Thus, with the decrease in the temperature, the production of frazil ice increases quickly and float on water surface. As a consequence, ice pans on water surface will be gradually developed and transported downstream.

During the period of river freeze-up, an ice cover is generally developed at cross section with relatively slow velocity, provided the coverage of ice pans on water surface is high enough [20]. Once an initial ice cover is formed on water surface, the incoming floating ice pans/floes will be either stopped in front of this initial ice cover if the flow velocity is low; or submerged under this initial ice cover if the flow velocity is high enough, and consequently an initial ice jam will be developed. The process of an ice jam formation and evolution is accompanied by the continuous advancement of the ice jam head toward upstream and the ice jam toe toward downstream. During the period of development of an ice jam, the ice jam thickness varies depending on the flow velocity under the ice jam and the amount of incoming ice pans/floes from upstream. When the flow condition in a river does not change much, the ice cover or ice jam formed in this river normally becomes stable during an ice-covered period.

Once a river is covered by an ice cover or ice jam, the distribution of flow velocity under an ice cover is completely different from that under an open flow condition. The presence of an ice cover/jam results in the complexity of the river hydraulics and sometimes significantly affects the hydrodynamic characteristics of a river [21], due to the increase in the wetted perimeter of a flow, and thus the resistance to the flow. Affected by the roughness coefficient of both ice cover/jam and riverbed, the maximum flow velocity under an ice-covered jammed condition is located between the channel bed and ice cover instead of at water surface under an open flow condition when the resistance caused by the air is neglected [22,23]. Furthermore, the increase in the resistance to the flow due to the appearance of an ice cover on water surface results in the decrease in flow velocity. For the same flow discharge, the presence of an ice cover/jam on water surface will cause a significantly increase in water level comparing to that under an open flow condition [24].

With the increase in temperature in spring, ice cover/jam in a river will be broken up. During the river breakup process, a lot of ice floes with different size will be generated. These ice floes may be congested at narrow sections of a river and form ice jams again [25]. The river ice jam during a river breakup period was termed by some scholars as the breakup ice jam, which is normally accompanied by ice flooding disaster.

The development and evolution of an ice jam continues during the entire ice-covered period. The migration process of vast amount of frazil ice particles at the bottom of an ice jam is similar to motion of sediment particles on a riverbed (as shown in Figure 1), which is commonly called as bed-load movement or "sand wave" [26]. Considering the similarity of sediment movement on a riverbed to that of frazil ice particles at the bottom of an ice jam, the movement of frazil ice particles should also have a "cover"-load layer (similar to the "bed-load" for sediment motion on a riverbed) and a suspended layer of frazil ice particles (similar to the "suspended load" of sediment over a riverbed). So, frazil ice particles at the bottom of an ice jam should be also delivered to downstream as the waved-shaped migration which is similar to the process of sand waves on a riverbed when certain conditions are met, that can be named as "ice wave" [27].



Figure 1. Sketch of the migration of sand waves.

The migration of ice waves at the bottom of an ice jam or cover is a special form of transportation of frazil ice particles. The migration of an ice wave can be described by various characteristic parameters such as wave crest, wave trough, wavelength, and migration speed (or wave speed). In this paper, Equations for describing ice jam thickness and flow condition during the migration of ice waves have been obtained. The criteria for the occurrence of an ice wave phenomenon have been proposed, and the equation for calculating the ice wavelength has been derived. By using equations mentioned above, calculation results of ice wave thickness at both wave crest and wave trough as well as wavelength are verified by results of laboratory experiments. The factors influencing the migration speed of ice waves have been also analyzed.

## 2. Materials and Methods

## 2.1. Laboratory Experiment Setup

Laboratory experiments have been carried out in a flume at Hefei University of technology. As shown in Figure 2, the flume for this experimental study is 26.68 m long, 0.4 m width. The smooth bottom of the flume is made of concrete with the bottom slope of zero. In total, 22 cross sections (CS) for measurements have been setup. The spacing distance between adjacent cross sections is 1.2 m. The model frazil ice particles are made of polyethylene material, which is also commonly used to model frazil ice particles in relevant research work in the world. The mass density of model frazil ice particles is 0.917 g/cm<sup>3</sup>, which is nearly the same as that of natural ice. The model frazil ice particles have an ellipsoid shape. These particles are about 3.5 mm long with the length-to-thickness ratio of about 1.7. The static friction angle of ice particles is about 45. An ice feeder which is located between CS-4 and CS-5 was used to discharge ice particles to the channel with designated rates. One model ice cover made of Styro foam panel which is 1 m long and 0.4 m wide was placed between CS-20 and CS-21 in the downstream. This model ice cover was used to initiate an initial ice jam at the end of the flume.



Figure 2. Layout plan of the flume for experiments.

The initial hydraulic condition for each experimental run (including approaching flow velocity, flow depth and ice discharge from the ice feeder) at CS-4 is used as the control condition. After ensuring that the flow discharge is constant and uniform flow is developed in the flume for each experiment run, the ice feeder was turned on to discharge model frazil ice particles into the flume. For each experimental run, the ice discharge rate is kept as a constant. During each experimental run, both water level and ice jam thickness at all cross

sections were measured. The water level was measured by an external pressure measuring tube, and the ice jam thickness was read using a scale.

One can observe from experiments that, after model ice particles were discharged into the flume at CS-4, they will float on water surface and delivered to downstream but will be stopped and accumulated in front of model ice cover at the end of the flume. Gradually, an initial ice jam will develop and advance to the upstream section. During the development of an initial ice jam, if all ice particles are submerged at the leading edge of the ice jam, this initial ice jam can no longer advance to the upstream. The accumulation process of ice particles under an ice jam continues until it reaches an equilibrium state along the entire channel. The present experimental study belongs to a conceptual study, and does not consider the geometric similarity or geometric scale.

The real-time data such as pressure head and ice jam thickness at each cross section are recorded every 30 min. Each experiment lasted 20 h to make sure the evolution of the ice jam achieves an equilibrium state.

#### 2.2. Mechanical Analysis of an Equilibrium Ice Jam

An ice jam is resulted from the accumulation of a large amount of frazil ice particles and ice floes under an ice cover. During the development and evolution of an ice jam, frazil ice particles are normally transported at the bottom of the ice jam in the form of "cover-load" (similar to the "bed-load" on a riverbed). Similar to the movement of sediment particles on a rivered, the motion of frazil ice particles at the bottom surface of an ice cover/jam also has a cover-load particle layer and a suspended particle layer. In the present conceptual study, the 1 m long Styro foam panel was placed on water surface at the end of the flume to initiate the formation of an initial ice jam along the channel. This model ice cover (Styro foam panel) did not affect the development of ice waves during the experiments.

In a laboratory flume, the movement of frazil ice particles at the bottom surface of an ice cover clearly demonstrates a phenomenon similar to the sand wave phenomenon on a riverbed, which can be called as "ice wave phenomenon" (as shown in Figure 3). Different from the sand wave movement on a riverbed, the migration of ice waves is more complex and more easily changeable due to the less difference in mass density between water and ice comparing to that between sand and water. Additionally, the changes in the incoming ice discharge also affect the appearance of ice waves.



Figure 3. Schematic diagram of ice wave phenomenon.

During the evolution process of an ice jam, the cross-sectional area for flow changes depending on flow velocity and incoming ice discharge. When the cross-sectional area for flow becomes smaller (i.e., at the cross section where wave crest appears), the flow intensity will be increases, leading to an increase in the erosion process of ice particles at the wave crest and thus a decrease in the jam thickness. When the cross-sectional area for flow becomes larger, it will lead to less erosion activity of ice particles. More ice particles will be deposited in the zone of the wave trough. When the flow velocity under the ice jam reaches a certain value, ice waves at the bottom of an ice jam begins to migrate downstream. In the meantime, flow velocity varies accordingly during the migration process of ice waves due to changes in the cross-sectional area for flow.

With the increase in the length of an ice jam, its internal force will be also increased. Obviously, if the internal strength of an ice jam is less than the stress caused by the external force, the ice jam will partially collapse. As a consequence, the ice jam thickness downstream may be increased due to increased ice particles caused by the partially collapse of an ice jam in the upstream section until the ice jam can withstand a higher new stress. As pointed out by Beltaos [28], the theory for a wide river jam was first proposed by Pariset et al. [29]. By applying the limiting-stability relationship used for soil mechanics, the one-dimensional mechanical equilibrium differential Equation for an ice jam in river can be expressed as follows

$$\frac{d}{dx}(\overline{\sigma}_x t) + \frac{2k_0k_1}{B}(\overline{\sigma}_x t) = \rho_i gtJ + \tau_0 - \frac{2tC_i}{B}$$
(1)

According to the definition of Michel [30],  $\sigma_x$  is the normal stresses (including hydrostatic pressure) in *x* directions; *t* is the ice jam thickness; *B* is the channel width;  $k_1$  is the lateral thrust coefficient;  $C_i$  is the adhesion force of an ice jam;  $k_0 = \tan\varphi$ ,  $\varphi$  is the internal friction angle of an ice jam;  $\rho_i$  is the mass density of ice;  $\tau_0$  is the flow shear stress at the bottom of an ice jam; *J* is the hydraulic gradient. By applying the theory of mechanics and continuity equation, considering internal force and assuming various boundary conditions, Equation (1) has been solved by Beltaos [28].

Pariset et al. [29] assumed that for the cohesionless an ice jam ( $C_i \approx 0$ ), a wide ice jam can occur under limiting conditions. From the viewpoint of mechanics, the internal resistance of an ice jam is equal to the sum of the external forces, also represents a complete mobilization of the passive resistance of an ice jam. The general expression for a wide ice jam proposed as follows:

$$\frac{BV^2}{C^2 H^2} = \mu \frac{\frac{\rho_i}{\rho} \left(1 - \frac{\rho_i}{\rho}\right) \frac{t^2}{H^2} \left(1 - \frac{\rho_i}{\rho} \frac{t}{H}\right)^3}{1 + \frac{\rho_i}{\rho} \frac{t}{H}}$$
(2)

where, *C* is the Chézy coefficient which represents the composite friction coefficient of the channel bed and the ice jam bottom. Results of calculation based on measured data showed that the Chézy coefficient value for both experimental study and field observations varies, indicating the Chézy coefficient is a comprehensive coefficient reflecting the roughness of a channel. Under an ice-covered/jammed flow condition, the wetted perimeter for a flow in a laboratory flume is 2(h + B); and the wetted perimeter for a flow in a wide natural channel can be approximated as 2*B*. As shown in Figure 4, *H* is the approaching flow depth and *V* is the approaching flow velocity in front of the ice jam and equal to the discharge divided by flow cross sectional area. According to Beltaos [28], coefficients  $k_0$ ,  $k_1$  and  $k_2$  have following relationship:  $\mu = k_0k_1k_2 = 1.28$ ,  $k_2$  is the passive soil pressure coefficient.



Figure 4. Schematic diagram of an ice jam.

According to Beltaos [28], to derive Equation (2), some assumptions have been introduced, such as the channel has been assumed as a prism channel with a rectangular cross-section. Convert the left end of Equation (2) to the flow Froude number through algebraic transformation, the following Equation can be obtained:

$$Fr = f\left(\frac{\mu C^2}{g}, \frac{H}{B}, \frac{t}{H}\right) = \sqrt{\frac{\mu C^2}{g} \left(\frac{H}{B}\right) \frac{\frac{\rho_i}{\rho} \left(1 - \frac{\rho_i}{\rho}\right) \frac{t^2}{H^2} \left(1 - \frac{\rho_i}{\rho} \frac{t}{H}\right)^3}{1 + \frac{\rho_i}{\rho} \frac{t}{H}}}$$
(3)

According to the stable flow depth under the condition of equilibrium ice jam measured from four experimental runs, the value of H/B is 0.30, 0.40, 0.54 and 0.64, respectively. The influence curves of the flow Froude number Fr in front of the leading edge of an ice jam depending on the relative thickness of an ice jam t/H are showed in Figure 5 (Note: The Chézy coefficient *C* is taken as 14 which is an average value of laboratory results, as showed in Table 1). It can be seen from this figure that the upstream flow Froude number Fr has a limiting value (maximum) once the evolution of an ice jam achieves a stable state under the condition of a certain ratio of flow depth to the channel width (H/B).



**Figure 5.** Dependence of flow Froude number Fr on the relative thickness of an ice jam t/H under condition of different H/B.

Table 1. Calculated C	hézy coefficients f	for 11 ex	perimental runs.
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	Initial Conditio	n		Equilibrium Ice Jam					
Flow Depth (cm)	Flow Rate (L/s)	Ice Discharge (L/s)	Fr	Flow Depth (cm)	Average Ice Thickness (cm)	Water Head (cm)	Ice Jam Length (cm)	Hydraulic Gradient	Chézy Coefficient
10	5.61	0.01299	0.109	11.9	5.25	2.71	7.08	$1.07  imes 10^{-3}$	14.10
15	9.00	0.02177	0.106	16.6	7.5	2.74	11.39	$9.77  imes 10^{-4}$	14.83
15	9.60	0.00993	0.121	15.9	5.55	3.35	9.83	$6.32  imes 10^{-4}$	16.22
15	10.81	0.0367	0.116	17.7	8.1	3.19	11.75	$1.52 \times 10^{-3}$	13.17
15	12.02	0.03699	0.133	17.3	6.95	4.07	10.14	$1.09  imes 10^{-3}$	15.45
20	12.02	0.03558	0.093	22.04	8.75	2.49	17.08	$1.38  imes 10^{-3}$	9.45
20	14.43	0.03694	0.114	21.75	8.65	3.25	15.49	$1.26 \times 10^{-3}$	12.10
20	15.20	0.04025	0.120	21.7	8.375	3.51	14.99	$1.01  imes 10^{-3}$	13.94
25	16.00	0.01599	0.096	26	9.8	2.78	20.44	$7.18  imes 10^{-4}$	12.95
25	18.01	0.01327	0.111	25.62	6.9	3.30	19.11	$3.69 imes10^{-4}$	16.36
25	20.00	0.02034	0.122	25.82	7.4	3.77	18.51	$5.54 imes10^{-4}$	15.20

One can see from Figure 5, if the upstream approaching flow Froude number Fr is below the limiting value (or maximum value), there are two relative thickness t/H ratios correspond to one Fr value. One can draw a horizontal line which is parallel to the X

axis with the flow Froude number Fr less than the limiting value. The intersection on the left limb of the curve is the minimum value for the thickness of an ice jam, and the intersection on the right limb of the curve is the maximum value for the thickness of an ice jam. Thus, one can say from Pariset's result (Equation (3)) that the ice jam thickness during the mechanical evolution process varies dynamically between the minimum thickness and the maximum thickness. These two thicknesses are approximately corresponding to the thickness of ice wave trough and ice wave crest, respectively. One can say from this finding that when the measured flow Fr value is lower than the limiting Fr value, ice wave phenomenon would occur at the bottom of an ice jam.

Considering different ratios of the flow depth to the channel width (H/B), the limiting value of flow Froude number for an ice jam to achieve an equilibrium state is changed with the value of H/B. One can also see from Figure 5, with a certain channel width in the laboratory tests, the limiting flow Froude number for an ice jam to achieve an equilibrium state only depends on the flow depth, the deeper the flow depth, the larger the limiting flow Froude number for an equilibrium state.

According to Figure 3, the following Figure 6 shows the schematic diagram of ice wave thickness and wavelength, where,  $t_{min}$  represents the thickness at wave trough and  $t_{max}$  represents the thickness at wave crest.



Figure 6. Schematic diagram of the thickness and length of an ice wave.

The periodic distance between the wave trough to the wave crest is the ice wavelength which is described by  $\lambda$  in Figure 6. The adhesion force *Ci* for a cohesionless ice jam in Equation (1) is relatively small and can be ignored, Equation (1) can be derived as follows:

$$\frac{df}{dx} + \frac{2k_0k_1f}{B} = \rho_i gtJ + \tau_0 \tag{4}$$

where,  $f = \sigma_x t$ . As pointed out by Pariset et al. [29], the internal resistance f can be expressed as follows based on the soil mechanics:

$$f = tg^2 \left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \rho_i \left(1 - \frac{\rho_i}{\rho}\right) \frac{gt^2}{2} = k_2 \rho_i \left(1 - \frac{\rho_i}{\rho}\right) \frac{gt^2}{2}$$
(5)

where, the coefficient  $k_2$  can be determined by the theory of granular mechanics.

Assuming that the internal resistance f reaches the ultimate internal force at every cross section, taking the first derivative of Equation (5) into Equation (4), so:

$$dx = \frac{k_2 \rho_i \left(1 - \frac{\rho_i}{\rho}\right) gt dt}{\rho_i gt J + \tau_0 - \frac{2k_0 k_1 f}{B}}$$
(6)

The above formula can be solved discretely by means of the different method. Along an ice wave, *n* cross sections have been divided with an equal spacing distance, that is, the equation for calculating the wavelength can be expressed as follows:

$$\lambda = \sum_{i=1}^{n-1} (x_{i+1} - x_i) = \sum_{i=1}^{n-1} \frac{k_2 \rho_i \left(1 - \frac{\rho_i}{\rho}\right) g t_i (t_{i+1} - t_i)}{\rho_i g t_i J + \tau_0 - \frac{2k_0 k_1 f_i}{B}}$$
(7)

# 3. Results and Discussion

# 3.1. The Limiting Conditions for the Occurrence of Ice Wave Phenomenon

The formation, development, migration, and extinction of ice waves are accompanied by the continuous changes of various hydraulic factors in a river. Based on results of theoretical analysis expressed by Equation (3) and laboratory experiments, the limiting conditions for the occurrence of ice wave phenomenon have been studied.

The first derivative of the upstream flow Froude number to the ratio of the ice jam thickness to flow depth (t/H) could be obtained from Equation (3). Letting d(Fr)/d(t/H) = 0, one can obtain the limiting value for flow Froude number Fr under condition of different depth-width ratios (H/B). The calculated flow Froude number is the limiting value for the development of ice waves under an ice cover/jam. Results are presented in Figure 7.



**Figure 7.** Calculated limiting flow Froude number Fr for developing an ice wave compared to those of laboratory experiments.

In total, 21 laboratory experimental runs have been conducted to assess the migration process of ice waves during the accumulation process of ice jams in a flume. By means of the regression analysis, the Chézy coefficient under the condition of an ice-covered flow ranged from 9 to 16 with an average value of 14 (As shown in Table 1, this value may only be applicable to this experimental study. The resistance effect of ice waves on water may increase the roughness of the wet boundary of the flume). In Table 2, the measured data of 21 experimental runs are summarized including flow depth, flow and ice discharge, and upstream approaching flow Froude number Fr. The occurrence of ice wave phenomenon is also recorded. One can see from Table 2 and Figure 7 that if the flow Froude number is located above the limiting Fr-curve in Figure 7, an ice wave phenomenon will not occur. Obviously, ice waves will present at the bottom of an ice jam when the flow Froude number is located below the limiting Fr-curve. It should be recommended that the limiting Fr-curve can be used to assess whether an ice wave occurs during the accumulation process of an ice jam.

Table 2. Hydraulic conditions for 21 experiment runs with and without presence of ice waves.

Initial Flow Depth (cm)	Flow Discharge (L/s)	Ice Discharge (L/s)	Flow Depth during Equilibrium Ice Jam (cm)	Fr during Equilibrium Ice Jam	Location of Ice Jam Head at CS	Occurrence of Ice Wave
10	5.61	0.01299	11.90	0.109	5	Yes
10	6.40	0.00625	10.75	0.145	5	No
10	6.40	0.0105	10.75	0.145	5	No
10	7.20	0.01054	11.31	0.151	18	No
10	8.00	0.0274	11.08	0.173	17	No
15	9.00	0.02177	16.60	0.106	5	Yes

Initial Flow Depth (cm)	Flow Discharge (L/s)	Ice Discharge (L/s)	Flow Depth during Equilibrium Ice Jam (cm)	Fr during Equilibrium Ice Jam	Location of Ice Jam Head at CS	Occurrence of Ice Wave
15	9.60	0.00993	15.90	0.121	5	Yes
15	10.81	0.00613	16.17	0.132	15	Yes
15	10.81	0.0367	17.70	0.116	5	Yes
15	12.02	0.03699	17.30	0.133	5	Yes
15	13.20	0.02365	16.26	0.161	14	No
20	14.43	0.03694	21.75	0.114	5	Yes
20	15.20	0.04025	21.70	0.120	5	Yes
20	15.20	0.00743	21.28	0.124	17	Yes
20	16.03	0.0259	21.12	0.132	15	Yes
25	16.00	0.01599	26.00	0.096	5	Yes
25	18.01	0.01327	25.62	0.111	6	Yes
25	20.00	0.02034	25.82	0.122	5	Yes
25	20.50	0.01298	25.88	0.124	16	Yes
25	21.00	0.01327	26.16	0.125	18	Yes
25	22.00	0.02579	26.27	0.130	19	Yes

Table 2. Cont.

# 3.2. Ice Jam Thickness of Wave Crest and Wave Trough

Considering fully developed ice jams (developed to the upstream cross Section 4) with the occurrence and migration of ice waves, 11 out of these 21 experimental runs have been selected to study the thickness of wave crest and wave trough. By using Equation (3), both the lower limiting value and the upper limiting value of the ice jam thickness have been calculated, as shown in Table 3.

**Table 3.** Calculated results of ice jam thickness at wave crest and trough compared to those of laboratory experiments.

Ini	itial Cond	ition	Equilib Ja	orium Ice am	Calculate Thickn	ed Ice Jam ess (cm)	Measured Ice Jam Thickness I (cm)		Diff. Btw. Calculated and Measured Results		
Flow Depth (cm)	Flow Rate (L/s)	Ice Dis- charge (L/s)	Fr	Flow Depth (cm)	Lower Limiting (t <sub>LL</sub> )	Upper Limiting (t <sub>UL</sub> )	At Wave Trough (t <sub>WT</sub> )	At Wave Crest (t <sub>WC</sub> )	Ratio: t <sub>WC</sub> /t <sub>WT</sub>	(t <sub>LL</sub> - t <sub>WT</sub> ) (cm)	(t <sub>UL</sub> - t <sub>Wc</sub> ) (cm)
10	5.61	0.01299	0.109	11.9	2.71	7.08	4.0	6.5	1.63	-1.29	0.58
15	9.00	0.02177	0.106	16.6	2.74	11.39	5.5	9.5	1.73	-2.76	1.89
15	9.60	0.00993	0.121	15.9	3.35	9.83	4.8	6.3	1.31	-1.45	3.53
15	10.81	0.0367	0.116	17.7	3.19	11.75	5.0	11.2	2.24	-1.81	0.55
15	12.02	0.03699	0.133	17.3	4.07	10.14	5.0	8.9	1.78	-0.93	1.24
20	12.02	0.03558	0.093	22.04	2.49	17.08	4.2	13.3	3.17	-1.71	3.78
20	14.43	0.03694	0.114	21.75	3.25	15.49	5.5	11.8	2.15	-2.25	3.69
20	15.20	0.04025	0.120	21.7	3.51	14.99	4.25	12.5	2.94	-0.74	2.49
25	16.00	0.01599	0.096	26	2.78	20.44	6.6	13.0	1.97	-3.82	7.44
25	18.01	0.01327	0.111	25.62	3.30	19.11	4.0	9.8	2.45	-0.7	9.31
25	20.00	0.02034	0.122	25.82	3.77	18.51	5.0	9.8	1.96	-1.23	8.71

Comparing the calculated upper limiting values and lower limiting values of the ice jam thickness to those of measurements in laboratory, it can be seen that the measured thickness of ice wave crest and ice wave trough are all within the range of calculated ice jam thickness from the lower limiting values to the upper limiting values (Figure 5). This means that the measured results of ice wave crest and ice wave trough in laboratory experiments should be affected by some factors, and will be attenuated compared to the limiting value of thickness at wave crest to the thickness at wave trough ranges from 1.31. to 3.17. An ice jam which has a thickness within the range of calculated limiting values is obviously in the state of mechanical equilibrium.

In addition, the randomness and determinacy of the thickness of ice waves are verified from Table 3. Here, the randomness means that the thickness of ice waves at any designated cross section changes dynamically during the migration process of ice waves, which varies from the thickness of ice wave crest to that of ice wave trough. The determinacy of thickness of ice waves means that the thickness of ice wave crest and trough ranges from the lower limiting value to the upper limiting value obtained from Equation (3).

Under the same hydraulic condition, comparison has been conducted between the upper/lower limiting of ice jam thickness obtained from Equation (3) and the thickness at wave crest/trough measured from experiments. Results indicate that the measured thickness at ice wave crest from experiments approaches to the upper limiting value calculated from Equation (3), while the measured thickness at ice wave trough approaches to the lower limiting value calculated from Equation (3). As showed in Table 3, there is some difference between calculated results using Equation (3) and those of measurements from experiments in laboratory, this difference might be resulted from the approximately calculation of hydraulic radius which should be applicable to a wide and shallow river, namely, Equations (2) and (3) have been derived by assuming that the channel is a wide-and-shallow river. Therefore, the smaller the depth-width ratio H/B is, the higher accuracy of calculated results by using Equation (3) is.

### 3.3. Flow Froude Number under the Ice Jam

Besides the flow Froude number Fr, some scholars either use the flow Froude number under an ice jam ( $Fr_u$ ) or ice jam Froude number (Fi) to study the ice jam evolution characteristics, expressed as follows:

$$Fr_u = \frac{V}{\sqrt{gh}} \quad or \quad Fi = \frac{V}{\sqrt{gt}}$$
 (8)

Taking the flow Froude number ( $Fr_u$ ) as example, results of these 11 experimental runs showed the relative thickness of ice wave (t/H) increases with the flow Froude number under an ice jam  $Fr_u$ . It means, the larger the flow Froude number under an ice jam, the thicker the ice jam. One can see from Figure 8, for a larger thickness of an ice jam at cross section where a wave crest is located, the flow Froude number under the wave crest is clearly more than that under the ice wave trough. With a large flow Froude number under the wave crest, frazil ice particles at the wave crest will be eroded. Thus, the flow cross sectional area at the wave crest gradually increased. Thus, ice waves will be gradually migrated downstream. Considering the ice jam Froude number (Fi) as the variable, the large the ice jam Froude number, the thinner the ice jam. For a larger thickness of an ice jam at the wave crest, the ice jam Froude number is clearly less than that at the ice wave trough. This finding is reasonable, since the flow depth under an ice jam is  $h = H - (\rho_i / \rho)t$ , namely the thicker the ice jam (at the wave crest section), the shallower the flow under the jam, where, H is the approaching flow depth in front of an ice jam; h is the flow depth under an ice jam,  $\rho$  and  $\rho_i$  are mass density of water and ice, respectively.

The ice discharge rate from upstream river reach has an important influence on the ice jam thickness. The higher the ice discharge is, the thicker the ice jam will be. As showed in Figure 8 and Table 3, for a dynamic equilibrium ice jam, the thickness of an ice jam at both wave crest and wave trough will be affected. Thus, the migration process of ice waves will be affected by the incoming ice discharge rate from upstream.



**Figure 8.** Relationship between the relative thickness of ice wave (at wave crest and wave trough) and flow Froude number under an ice jam based on laboratory experiments.

#### 3.4. Wave Length and Migration Speed

Considering fully developed ice jams (covered the entire flume to the upstream cross Section 4) with the migration of ice waves, 11 out of those 22 experimental runs as mentioned above have been selected to study the wavelength and migration speed of ice waves. The wavelength has been determined using Equation (7), as shown in Table 4.

**Table 4.** Calculated ice wavelength compared to those of laboratory experiments and measured speed of ice wave migration.

]	Initial Condition		Equilibrium Ice Jam		Wave Length (cm)		Measured Results		
Flow Depth (cm)	Flow Rate (L/s)	Ice Discharge (L/s)	Fr	Flow Depth (cm)	Calculated Result	Measured Result	Ice Wave Periodic Migration Time (s)	Ice Wave Migration Speed (m/s)	
10	5.61	0.01299	0.109	11.90	33.11	29	109	0.00266	
15	9.00	0.02177	0.106	16.60	40.50	34	143	0.00238	
15	9.60	0.00993	0.121	15.90	23.98	25	137	0.00182	
15	10.81	0.03699	0.116	17.70	70.76	66	179	0.00369	
15	12.02	0.03699	0.133	17.30	69.93	64	151	0.00424	
20	12.02	0.03558	0.093	22.04	66.70	62	192	0.00323	
20	14.43	0.03699	0.114	21.75	68.23	69	242	0.00285	
20	15.20	0.04025	0.120	21.70	74.70	81	253	0.00320	
25	16.00	0.01599	0.096	26.00	38.07	54	428	0.00126	
25	18.01	0.01327	0.111	25.62	24.27	61	664	0.00092	
25	20.00	0.02034	0.122	25.82	24.80	50	330	0.00152	

It is found that when the flow depth is small (or H/B is small), compared to results of laboratory experiments, the difference between the calculated wavelength and that of measurement in laboratory experiment is very small. One can say that Equation (7) has certain accuracy. However, when the flow depth at the control section is 25 cm (or H/B is large), the calculated wavelength is smaller than the measured result from experiment, and there is a certain calculation error. This might be resulted from the approximated calculation of some integral expressions in the denominator of Equation (7). Further optimization of some parameters should be carried out to enhance the calculation accuracy.

Results of migration speed of ice wave from laboratory experiments are also presented in Table 3. The periodic time of ice wave migration in Table 3 refers to the time required for an ice wave to migrate a distance equal to the wavelength to downstream. This means, the ratio of the wavelength to the periodic time of ice wave migration is the migration speed of an ice wave. Figure 9 shows the relationship between the migration speed of ice wave and the ice-water discharge ratio (Qi/Q) based on the 11 laboratory experimental runs. This linear relationship indicates that the migration speed of ice wave is mainly affected by the ice transport discharge (which is equal to the inlet ice discharge) and hydraulic condition of the experimental run. When the hydraulic condition does not change, the larger the ice discharge is, the faster the migration speed of ice waves during the evolution process of an ice jam is (see the test data of No.5 and No.6 in Table 4). When the ice discharge is (see the test data of No.4 and No.7 in Table 4).



**Figure 9.** The relationship between the migration speed of ice wave and the ice-water discharge ratio  $(Q_i/Q)$  from laboratory experiments.

It is worth mentioning that the above-mentioned migration speed of ice waves actually refers to the average velocity for an ice wave advancing toward downstream. From laboratory experiments, it has been observed that the thickness of the "cover-load" layer along different cross section of an ice wave is not the same. The average thickness of the "cover-load" layer can be obtained based on the ice discharge, the average migration speed and the channel width. However, the calculated result only represents an average thickness of the "cover-load" layer, and cannot preciously express actual thickness of the cover-load layer at each cross section.

## 3.5. Case Studies

The ice jam accumulation and evolution process are very complicated due to various size of ice floes in natural rivers as well as the combined effects of thermal and hydraulic factors. Based on filed observations of ice jams in natural rivers, following five types of ice jam accumulation may occur in natural rivers during the ice-covered periods.

- (a) Quasi-static ice jam: This kind of ice jam is also known as the continuous juxtaposition form. It is characterized by the continuous juxtaposition of a layer of ice floes with a specific thickness. The ice jam thickness completely depends on the initial ice thickness and the thermal effect during winter period, which is similar to but slightly different from the static ice cover in lakes.
- (b) Frontal progression: The kind of accumulation refers to an ice jam whose thickness exceeds its initial thickness after the submerged ice floes accumulate at the front of an ice jam (or the leading edge of an ice jam). A relatively stable ice jam head in a river is essential for the development of this kind of ice jam.
- (c) Packing ice jam: The front edge of an ice jam in rivers may advance and retreat during ice-jammed period, and when it advances upstream, local accumulation (packing) will often occur.

- (d) Shoving ice jam: In natural rivers, when the cohesive force of an ice jam is not enough to support the positive stress at the front edge of an ice jam, a long and thin ice cover is often broken suddenly and leads to a thick accumulation body at the downstream. During the period of the initial stage, this accumulation body is normally loose without cohesion, but it may form a solid accumulation ice jam later due to the influence of thermal factors.
- (e) Hanging dam: A hanging ice dam is referred to the development of an ice jam along a specific river section (such as the junction of rapid flow and slow flow), where the flow velocity is too high for ice floes to accumulate upstream. Under such a flow condition, the incoming ice floes will be entrained by water and accumulate under the bottom of the ice cover. The cross sectional area for flow under the ice accumulation decreases until the flow velocity is high enough to erode ice floes further to downstream.

In recent years, more hydraulic structures such as piers for bridges have been built in natural rivers. Many data of field measurements under the influence of natural conditions have been collected. Therefore, this study uses the valuable data of field observations collected in the St. Lawrence River, the Beauharnois Canal in the early 1950s and the La Grande River in the early 1970s. Data of field measurements provide excellent information including ice jam thicknesses at closely spaced cross sections, which measured sooner after the formation of ice cover/jams (so that there should be little impact from thermal aspect on the growth of the ice jam). Following field data are also available: cross sectional area of ice cover/jam, cross-sectional area for flow, flow discharge and river width. The flow velocity in front of ice cover was calculated using the flow discharge divided by total area for flow at time during the formation of an ice cover, and the velocity under the cover by using a similar calculation method. The ice cover/jam thickness is determined by total cross-sectional area of ice cover/jam divided by its surface width. The detailed data of field measurements are summarized in Michel's research [30].

It can be seen from those measurements data that the ratios of the flow depth to the channel width (H/B) of natural rivers are far less than those of laboratory experiments, the H/B values for the St. Lawrence River and Beauharnois Canal range from 0.002 to 0.015, and the H/B values for the La Grande River are from 0.005 to 0.087. Since it is difficult to accurately distinguish the frontal progression and packing ice jam in natural rivers, these two types of ice jam are considered as one category. Therefore, ice jams in natural rivers can be mainly divided into following four categories: quasi-static ice jam, frontal and packing ice jam, shoving accumulation, and ice dam. It should be noted that the ice dam phenomenon does not happen often in the above mentioned three field sites (only occurred in the La Grande River during winter periods from 1977 to 1978.

In addition, according to Michel [30], the Chézy coefficient *C* used for these field sites is ranged from 24 to 40. In the present study, the *C* value has taken the larger value. The value of  $\mu$  is ranged from 1.28 to1.3; *H*/*B* is the value of the actual measurement data, and the *t* value associated with the data point in the diagram is the actual measurement. The limiting *Fr*- curve for the natural rivers is drawn according to Equation (3). The calculated limiting flow Froude number *Fr* for natural rivers using Equation (3) is compared to those of field observations for different types of ice jam accumulation in the St. Lawrence River, Beauharnois Canal and the La Grande River, as showed in Figures 10 and 11.

It can be seen from Figures 10 and 11 that when either quasi-static or frontal/packing ice jams occur in natural rivers, all data points of field measurements are located either above or near the limiting *Fr*-curve. On the contrary, when ice jams in natural rivers appear as either shoving accumulation or ice dam, all data points of field measurements are located below the limiting *Fr*-curve. From the perspective of the mechanical mechanism of the development of an ice jam, hydraulic factors generally play a leading role in the accumulation process of ice jams that classified as either quasi-static or frontal/packing ice jams. The thickness of both quasi-static and frontal/packing ice jams is relatively small, and the value of flow Froude number corresponding to the open flow condition has a relatively large value. However, for ice jams appear as either the shoving accumulation or ice dam,

they generally reach an equilibrium state when the internal force of an ice jam is equal to the external force. The accumulation process for both shoving ice jam and ice dam is dominated by mechanical factors, and their thickness is relatively large. The value of the flow Froude number corresponding to the open flow is small (especially for the ice dam, the thickness often reaches a maximum value, with a minimum *Fr* value in front of the leading edge of an ice dam).



**Figure 10.** Calculated limiting flow Froude number *Fr* for natural rivers compared to those of field observations of the St. Lawrence River and Beauharnois Canal.





The evolution of a river ice jam in a natural river is a process of gradual transition from flow with a high Fr value to that with a low Fr value. The mechanical action gradually dominates the development/accumulation of an ice jam, and the impact of the hydraulic action gradually decreases. From Figures 10 and 11, one can see results of field observations compared to those of calculation. Note: the limiting Fr value mentioned in this study represents the maximum Fr value in front of the leading edge of an ice jam for an equilibrium ice jam under the mechanical action, which means that the ice jam accumulation can reach the

mechanical equilibrium state only when the Fr value under the open flow condition is less than or equal to the limiting Fr value. Figure 10 shows that almost all measured data of the two rivers during different winter periods are located near the limiting Fr-curve. One may conclude that the ice jam evolution achieves an optimal state when the value of flow Froude number Fr in front of the leading edge of an ice jam approaches the limiting Fr-curve.

As mentioned above, results of experiments showed that the limiting *Fr*-curve can be used to judge whether the ice wave phenomenon occurred. The main difference between the condition for laboratory experiments and the condition in natural rivers is whether there is a continuous supply of ice from the upstream. It should be noted that the formation of an ice dam phenomenon in a natural river generally requires the continuous incoming ice supply from the upstream. It can be concluded that when the ice jam has reached the mechanical equilibrium state, the incoming ice particles/floes floating on water surface would lead to the decrease in flow Froude number in the open section upstream of the ice jam and slowly move toward the front of the ice jam. However, under the ice jam, the flow velocity is faster than that in the open flow section since the cross section is partially occupied by ice. As a consequent, the incoming ice will be entrained by flowing water and submerged at the leading edge of ice jam and the accumulate under ice jam. Thus, the thickness of the ice jam will increase and the ice wave phenomenon may be in present under the bottom of the ice jam. Obviously, the incoming ice discharge affect the magnitude of flow Froude number. Under extreme condition with vast amount of incoming ice from upstream, the ice dam phenomenon may appear in a natural river with a sharply decrease in the cross sectional area for flow with high flow velocity under and ice jam in addition a significant increase in flow depth in front of the ice dam. Thus, the ice accumulate under the ice dam will be transported to the downstream. In such a case, the flow Froude number Fr in front of the leading edge of an ice dam decreases dramatically and approaches to zero (as shown in Figure 12). Thus, the ice dam could be considered as a special case for the ice wave phenomenon.



**Figure 12.** Variations of flow Froude number in front of the leading edge of ice jams with the relative thickness of an ice jam t/H—calculated results (curves) compared to those of field observations of the St. Lawrence River, Beauharnois Canal and La Grande River.

Figure 12 shows the variation of flow Froude number in front of the leading edge of ice jams with the relative thickness of an ice jam t/H for the shoving ice accumulation in the St. Lawrence River and the Beauharnois Canal (17 measured data in total), packing ice jam and ice dam in the La Grande River (14 measured data in total), compared to those of calculation using Equation (3) (curves). It can be seen from this figure that all data points of field measurements are within the range of the upper and lower limits of the ice thickness.

For the same flow Froude number in front of the leading edge of an ice jam, there will be some different relative thickness of an ice jam t/H in the natural rivers. Interestingly, when the relative thickness of an ice jam t/H < 0.4, all data points of field observations were for packing or shoving ice jams; while when t/H > 0.4, all data points of field observations are corresponding to the ice dam events. This means the relative thickness of an ice jam (t/H) of 0.4 is the limiting value for assessing whether an ice jam in a river belongs to an ice dam. This finding agrees well with result of Sui et al. (2002) [8] who carried out study regarding river ice accumulation in the Hequ Reach of the Yellow River in China. Based on many data of field observations along this 70 km-long river reach for six winters, Sui et al. found that the turning point of t/H is about 0.4 from an ice jam to an ice dam [8].

#### 4. Conclusions

The ice wave phenomenon will occur under certain conditions during the evolution process of an ice jam in a river. Through the mechanical analysis of wide jams theory, the relationship between the flow Froude number in front of an ice jam and the ratio of ice jam thickness to flow depth (t/H) as well as the ratio of flow depth to channel width (H/B) has been discussed. Results showed that the ice jam thickness during the process of dynamic evolution varies dynamically between the minimum jam thickness and the maximum jam thickness, depending on the flow Froude number.

When the approaching flow Froude number reaches a certain value, ice waves appear at the bottom of the ice jam and begin to migrate downstream. Otherwise, ice waves cannot be developed during the evolution process of an ice jam. One curve has been developed for assessing if ice waves will occur at the bottom surface of an ice jam. If the value of approaching flow Froude number is located above the limiting *Fr*-curve, the ice wave phenomenon will not occur. Ice waves will present provided the value of the approaching flow Froude number is located below the limiting *Fr*-curve.

Comparing results calculated for both upper and lower limiting values of the ice jam thickness to those of laboratory experiments, the measured jam thicknesses at the crest and trough of ice waves are all within the calculated range from the lower limiting values to the upper limiting values of ice jam thickness. The measured thicknesses at ice wave crest from experiments approaches to the upper limiting value calculated from Equation (3); while the measured thicknesses at ice wave trough approaches to the lower limiting value calculated from Equation (3).

The relative thickness of ice wave (t/H) increases with the flow Froude number under an ice jam  $Fr_u$ . Namely, the large the flow Froude number under an ice jam, the thicker the ice jam. This means, for a larger thickness of an ice jam at cross section where a wave crest is located, the flow Froude number under the wave crest is clearly more than that under the ice wave trough. With a large flow Froude number under the wave crest, frazil ice particles at the wave crest will be eroded. Thus, the flow cross sectional area at the wave crest gradually increased. This leads to a gradually migration of ice waves toward downstream.

An Equation for calculating the ice wavelength has been derived and verified by experimental data. The relationship between the ice wave migration speed and the ice-water flow ratio (Qi/Q) has been analyzed. Results showed that, the ice wave migration speed is mainly affected by the ice discharge and the hydraulic conditions of the flume. When the hydraulic condition does not change, the larger the ice discharge is, the faster the migration speed of ice waves during the evolution process of an ice jam is. When the ice discharge is constant, the smaller the flow rate is, the faster the migration speed of ice waves is.

Ice wave is a microscopic phenomenon during the evolution of ice jams, which is observed in experimental study. However, it has not been reported in the natural rivers due to limitation in field observations. The continuous incoming ice particles during the laboratory experiments may also be one of the prerequisites for the development of ice waves. However, both the experimental study and prototype data reflect the periodic change of ice jam thickness when the hydraulic conditions are constant. The ice jam evolution achieves an optimal state when the value of flow Froude number Fr in front of the leading edge of an ice jam approaches the limiting Fr-curve. When the ice jam has reached the mechanical equilibrium state, the incoming ice particles/floes floating on water surface would lead to the decrease in flow Froude number in the open section upstream of the ice jam and slowly move toward the front of the ice jam. The incoming ice will be entrained by flowing water and submerged at the leading edge of ice jam and the accumulate under ice jam. Thus, the thickness of the ice jam will increase and the ice wave phenomenon may be in present under the bottom of the ice jam. Interesting results of case studies about ice accumulation in natural rivers show that the relative thickness of an ice jam (t/H) of 0.4 is the limiting value for assessing whether an ice jam in a river belongs to an ice dam. When the relative thickness of an ice jam t/H < 0.4, all data points of field observations were for packing or shoving ice jams; while when t/H > 0.4, all data points of field observations are corresponding to the ice dam events. This finding agrees well with result of Sui et al. (2002) [8] who carried out study regarding river ice accumulation in the Hequ Reach of the Yellow River in China.

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## References

- 1. Xiao, Y.; Pavelsky, T.M.; Allen, G.H. The past and future of global river ice. *Nature* 2020, 577, 69–73.
- Sui, J.; Karney, B.; Fang, D. Variation in water level under ice-jammed condition—Field investigation and experimental study. Hydrol. Res. 2005, 36, 65–84. [CrossRef]
- 3. Beltaos, S.; Burrell, B.C. Hydrotechnical Advances in Canadian River-Ice Science and Engineering during the Past 35 Years. *Can. J. Civ. Eng.* **2015**, *42*, 583–591. [CrossRef]
- 4. Sui, J.; Hicks, F.; Menounos, B. Observations of riverbed scour under a developing hanging ice dam. *Can. J. Civ. Eng.* **2006**, *33*, 214–218. [CrossRef]
- 5. White, K.D. Review of prediction methods for breakup ice jams. Can. J. Civ. Eng. 2003, 30, 89–100. [CrossRef]
- Lindenschmidt, K.E.; Das, A.; Rokaya, P.; Chu, T. Ice jam flood risk assessment and mapping. *Hydrol. Process.* 2016, 30, 3754–3769.
  [CrossRef]
- 7. Sui, J.; Wang, J.; Balachandar, R. Accumulation of frazil ice along a river bend. Can. J. Civ. Eng. 2008, 35, 158–169. [CrossRef]
- Sui, J.; Karney, B.W.; Sun, Z.; Wang, D. Field Investigation of Frazil Jam Evolution: A Case Study. J. Hydraul. Eng. 2002, 128, 781–787. [CrossRef]
- 9. Beltaos, S.; Carter, T. Field studies of ice breakup and jamming in lower Peace River, Canada. *Cold Reg. Sci. Technol.* 2009, 56, 102–114. [CrossRef]
- 10. Healy, D.; Hicks, F.E. Experimental study of ice jam formation dynamics. J. Cold Reg. Eng. 2006, 20, 117–139. [CrossRef]
- 11. Wang, J.; Hua, J.; Chen, P.; Sui, J.; Wu, P.; Whitcombe, T. Initiation of ice jam in front of bridge piers—An experimental study. *J. Hydrodyn.* **2019**, *31*, 117–123. [CrossRef]
- 12. Daly, S.F.; Axelson, K.D. Stability of floating and submerged blocks. J. Hydraul. Res. 1990, 28, 737–752. [CrossRef]
- 13. Wang, J.; Sui, J.Y.; Zhang, H.Y.; Chen, P.P.; Hirshfield, F. Mechanisms of ice accumulation in a river bend—An experimental study. *Int. J. Sediment Res.* **2012**, *27*, 521–537. [CrossRef]
- 14. Hicks, F.E.; Healy, D. Determining winter discharge based on hydraulic modeling. Can. J. Civ. Eng. 2003, 30, 101–112. [CrossRef]
- 15. Wang, J.; Chen, P.; Sui, J. Progress in studies on ice accumulation in river bends. J. Hydrodyn. 2011, 23, 737–744. [CrossRef]
- 16. Beltaos, S. Progress in the study and management of river ice jams. *Cold Reg. Sci. Technol.* **2008**, *51*, 2–19. [CrossRef]
- 17. Beltaos, S. Numerical prediction of ice-jam profiles in lower Athabasca River. Can. J. Civ. Eng. 2019, 46, 722–731. [CrossRef]
- Wang, J.; Sui, J.; Chen, P. Numerical simulations of ice accumulation under ice cover along a river bend. Int. J. Environ. Sci. Technol. 2009, 6, 1–12. [CrossRef]

- 19. Sola, D.; Scott, K.A. Efficient Shallow Network for River Ice Segmentation. *Remote Sens.* 2022, 14, 2378. [CrossRef]
- 20. Altena, B.; Kääb, A. Quantifying river ice movement through a combination of European satellite monitoring services. *Int. J. Appl. Earth Obs. Geoinf.* **2021**, *98*, 102315. [CrossRef]
- Sui, J.; Faruque, M.A.A.; Balachandar, R. Local scour caused by submerged square jets under ice cover. ASCE J. Hydraul. Eng. 2009, 135, 316–319. [CrossRef]
- 22. Sui, J.; Wang, J.; He, Y.; Krol, F. Velocity profiles and incipient motion of frazil particles under ice cover. *Int. J. Sediment Res.* 2010, 25, 39–51. [CrossRef]
- 23. Jafari, R.; Sui, J. Velocity field and turbulence structure around spur dikes with different angles of orientation under ice covered flow conditions. *Water* **2021**, *13*, 1844. [CrossRef]
- 24. Namaee, M.; Sui, J. Velocity profiles and turbulence intensities around side-by-side bridge piers under ice-covered flow condition. *J. Hydrol. Hydromech.* **2020**, *68*, 70–82. [CrossRef]
- 25. Cheng, T.; Wang, J.; Chen, P.; Sui, J. Simulation of ice accumulation around bridge piers during river breakup periods using a discrete element model. *J. Hydrodyn.* 2022, *34*, 94–105. [CrossRef]
- 26. Shen, H.T.; Wang, D.S. Under cover transport and accumulation of frazil granules. J. Hydraul. Eng. 1995, 121, 184–195. [CrossRef]
- 27. Wang, J.; Wu, Y.; Sui, J.; Karney, B. Formation and movement of ice accumulation waves under ice cover—An experimental study. *J. Hydrol. Hydromech.* **2019**, *67*, 171–178. [CrossRef]
- 28. Beltaos, S. River Ice Jams; Water Resources Publications: Littleton, CO, USA, 1995.
- 29. Pariset, E.; Hausser, R.; Gagnon, A. Formation of ice covers and ice jams in rivers. J. Hydraul. Div. 1966, 92, 1–24. [CrossRef]
- 30. Michel, B. Comparison of field data with theories on ice cover progression in large rivers. *Can. J. Civ. Eng.* **1984**, *11*, 798–814. [CrossRef]