

Article

Modeling of a Jet Pipe Servovalve Considering Nonlinear Flow Forces Acting on the Spool

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Abstract: The design and analysis of jet pipe servovalves are mostly based on linear models. However, there are some nonlinear factors in this kind of electromechanical–hydraulic structure. The article deduces a linear rotation equation for the armature assembly and a linear flow equation for the control cavity. With the consideration of nonlinear hydraulic reaction forces in the second stage, the nonlinear dynamic equation of the main spool in an ideal jet pipe servovalve is derived. Based on the MATLAB (R2016a) software, the nonlinear model of a certain type of jet pipe servovalve is numerically investigated. The equilibrium points of the nonlinear system are calculated, the phase portraits are plotted, and the Hopf bifurcations caused by the flow-pressure coefficient as the control parameter and the period-doubling bifurcations caused by the variation of the input signal are analyzed. The vibration frequency of the time-domain response of the fifth-order system with a cosine signal as input is 242 Hz, which is similar to the experimental value of 233 Hz. The relative error between the two is 3.9%, verifying the validity of the nonlinear system model.

Keywords: jet pipe servovalve; nonlinear dynamic equation; steady-state flow force; transient flow force; bifurcation



Citation: Kang, S.; Kong, X.; Zhang, J.; Du, R. Modeling of a Jet Pipe Servovalve Considering Nonlinear Flow Forces Acting on the Spool. *Water* **2024**, *16*, 683. <https://doi.org/10.3390/w16050683>

Academic Editor: Alessandra Feo

Received: 30 January 2024

Revised: 18 February 2024

Accepted: 23 February 2024

Published: 26 February 2024



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1. Introduction

A two-stage jet pipe servovalve with force feedback is designed based on the jet principle, and its structure is shown in Figure 1. Fluid with a given pressure is delivered to the jet pipe through a filter screen. When current flows through the coil, the armature and jet pipe assembly rotates around the armature pivot point under the action of electromagnetic torque, causing unequal pressures in the two receiving channels, due to the unequal inflow from the jet pipe in the two receiving orifices. This causes the main spool to move from the pressure difference starting the servovalve operation. When the electrical signals are zero, the jet flows evenly into the two receiving channels because there is no differential pressure between the left and right control cavities of the spool, and the servovalve is stationary [1,2]. Compared with the dual flapper-nozzle servovalve, the jet pipe servovalve is characterized by low pollution potential, failure alignment, high-pressure recovery coefficient, and flow recovery coefficient. It is widely used in the control of actuators on various ships and airplanes. However, due to the complexity of the flow field within the servovalve and the difficulty in describing the nonlinear elements caused by fluid–solid coupling, its dynamic and static behavior is not easy to predict, so the design of jet pipe servovalves relies mainly on experience and experimental data [3,4].

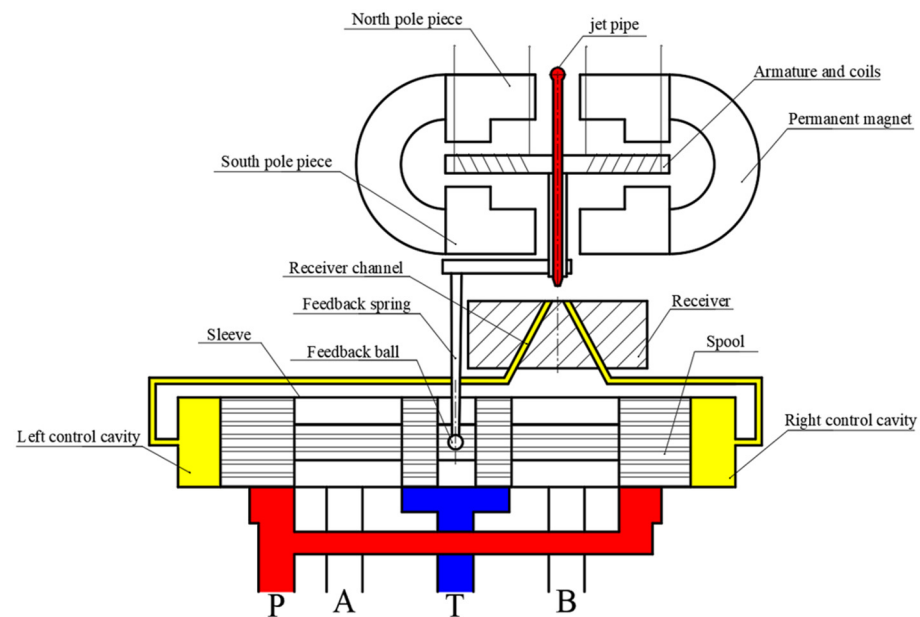


Figure 1. Schematic of the jet pipe servovalve. (The red color indicates high-pressure oil in the flow channel, the blue color means low-pressure oil in the flow channel, and the yellow color shows that the fuel in the flow channel is the control fluid).

With the development of CFD technology, more research is focused on numerical simulation to analyze the flow field characteristics of the pre-stage of the jet pipe servovalve [5–10]. Numerical simulation results of the pre-stage have been used to establish the mathematical models describing the behavior of the whole jet pipe servovalve, in order to analyze its characteristics more accurately. Somashekhar [11,12] developed a finite element model of the jet pipe servovalve with fluid–solid coupling and studied the pressure changes in the control chambers after the coil is energized, as well as the dynamic response of the jet pipe and the spool. Li et al. [13] established a numerical model of the jet pipe servovalve and analyzed the effects of the structural parameters of the jet pipe and the back pressure of the system on the characteristics of the flow field. Zhao et al. [14] built a structural model of the armature assembly, a magnetic field model of the torque motor, and a flow field model of the pre-stage based on Ansys. An overall model of the servovalve was derived by linking the above models together to obtain the dynamic characteristics. Li et al. [15] constructed a mathematical model of the force-feedback jet pipe servovalve, performed simulations of the response features in the time domain and frequency domain, analyzed the influence of key parameters on the performance of the valve, and proposed an optimization scheme accordingly. Yin et al. [16] suggested a mathematical model of the jet pipe servovalve considering the eddy current effect and investigated the effects of the main parameters in the torque motor on the dynamic behavior of the servovalve. The test shows that the difference between the theoretical and experimental results is about 5%. Chen et al. [17] used a mathematical model of the jet pipe servovalve and optimized the structural parameters based on multi-objective hierarchical particle swarm optimization genetic algorithm. The experimental results show that the model has a certain degree of accuracy.

However, all of the above literature studies are linear mathematical models of jet pipe servovalves. In fact, there are many nonlinear factors in servovalves, such as the hysteresis characteristics of the torque motor, the submerged jet characteristics of the jet hydraulic amplifier, the flow forces applied to the spool, and the dead zone caused by the overlap of the valve ports [18–20]. To analyze and design using classical control theory, these nonlinearities are usually ignored or the nonlinear equations are treated by the local linearization method. The resulting model loses some critical information. Models of hydraulic valves using nonlinear theory, on the other hand, have the potential

to accurately simulate and predict their excitation characteristics. Hayashi et al. [21,22] studied the instability and self-excited vibration of a poppet valve in a hydraulic system. They gave a nonlinear model depicting the pilot-type poppet valve and simulated the effect of pipe length on the stability of this circuit by numerical methods. The results show the possibility of chaos at the period-doubling bifurcation. Hős et al. [23–26] applied the theory of non-smooth dynamical systems to study the chattering behavior due to spool–seat collision during the opening and closing of a pressure-reducing valve. They pointed out that the Hopf bifurcations and grazing bifurcations may generate self-excited oscillations. Bouzidi et al. [27,28] investigated theoretically and experimentally the flow–acoustic–structural coupling of a spring-loaded valve. They concluded that the natural frequency of the combined valve–pipe system is the main factor affecting the vibration modal characteristics of the system. Awad and Parrondo [29] looked into the mechanism of self-excited vibration of an annular seal in a spherical valve in a hydroelectric power plant, derived the nonlinear governing equations, and solved them numerically using MATLAB code. They explained that the self-excited vibration arises due to the unstable coupling between the flow of water in the line and the vibratory motion of the seal.

However, to the best of the authors’ knowledge, there is very little literature on the nonlinear modeling of servovalves. As servovalve machining technology becomes more and more sophisticated, some neglected factors may cause the difference between theoretical analysis and practical use. Some papers point out that the values of flow forces calculated using linear formulas are inconsistent with the measured ones [30–33]. This indicates, to some extent, that there is some error in the analysis using a linear model. In addition, linear system theory suggests that when the spool is in the neutral position, the natural frequency is the lowest, the damping ratio is the smallest, and the stability is the worst, so more research is carried out to study the servovalve dynamic characteristics at the initial zero position [34]. But when the spool changes direction, it may also exhibit instability. When the system is initially in equilibrium, a small perturbation may increase the amplitude of flow and pressure oscillations [35]. This situation is difficult to explain by linear theory. In this paper, under the assumption of an ideal jet pipe servovalve, i.e., without manufacturing errors, a mathematical model of the machine–liquid part of the servovalve considering the combination of nonlinear flow forces acting on the spool is established.

In Section 2, the linear mathematical models of the armature assembly and the jet pipe amplifier are given, the nonlinear descriptions of steady-state flow force and transient flow force are presented, and the nonlinear dynamic equation of the spool is further derived. Section 3 is an example of nonlinear analysis based on MATLAB (R2016a) software for a type of jet pipe servovalve. In order to prove the accuracy of the nonlinear model, experimental validation is carried out in Section 4, and Section 5 is the conclusion.

2. Theoretical Model

2.1. Linear Model

2.1.1. Kinematic Analysis of Armature Assembly

It is assumed that when the jet pipe servo valve operates, the spool is located on the right side, and the jet pipe is in the neutral position under the action of the spring force and electromagnetic force. The electrical signal is reversed at the time of t_0 , subjecting the armature assembly in the torque motor to the electromagnetic torque Me , rotating counterclockwise around the pivot point C by an angle of θ , and the spool moves to the left under the operation of the pressure difference of the two control chambers as shown in Figure 2.

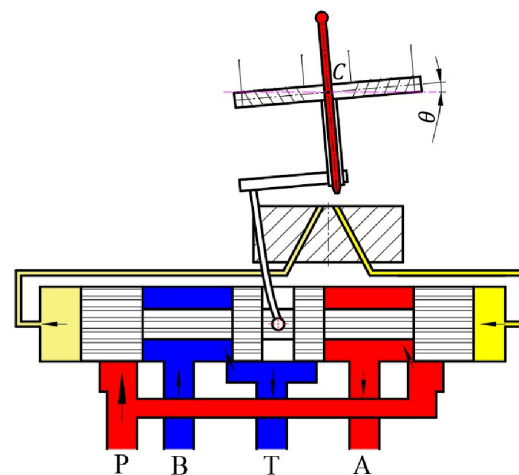


Figure 2. Schematic diagram of the mechanical-hydraulic part of the servovalve when reversed. (The red color indicates high-pressure oil in the flow channel, the blue color means low-pressure oil in the flow channel, and the yellow color shows that the fuel in the flow channel is the control fluid. The directions of the arrows indicate the direction of the flow.).

The force analysis of the armature assembly is shown in Figure 3. The oil ejected from the jet pipe exerts a jet counterforce F_j on the jet pipe. The feedback spring bends and deforms under the influence of the feedback ball, generating the force F_{fr} . Since the armature assembly is rotating around point C, there is almost no deformation of the jet pipe and its casing, so it is assumed that the elastic force of the jet pipe is zero. Neglecting the effect of the spilled flow from the receiving orifice on the jet pipe, applying the law of rotation to point C yields

$$J_{cj}\ddot{\theta} + B_{cj}\dot{\theta} = M_e - F_{jx}l_j - F_{fr}l_{fr} \quad (1)$$

where J_{cj} is the moment of inertia of the rotating parts, kg/m^2 ; θ is the rotation angle of the jet pipe, rad; B_{cj} is the damping coefficient for rotating parts, Nms/rad ; M_e is torque due to input electrical signal, Nm ; F_{jx} is component of jet counterforce F_j in x-direction, N; l_j is a vertical distance from the nozzle to C, m; F_{fr} is force acting at the extremity of the feedback spring, N; l_{fr} is a vertical distance from the feedback ball center to C, m.

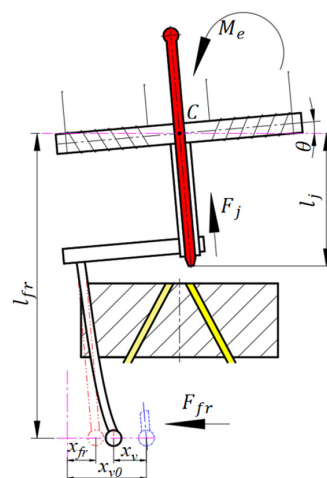


Figure 3. Force analysis diagram of armature assembly. (The red color indicates high-pressure oil in the flow channel, and the yellow color shows that the fuel in the flow channel is the control fluid. The directions of the arrows indicate the direction of the force or moment acting.).

According to the principle of underdeveloped impact jet [36], it is approximated that the velocity of the jet ejected in time interval dt is equal everywhere, so the equivalent mass is

$$dm_j = \rho \pi R_j^2 v_j \quad (2)$$

where ρ is the fluid density, kg/m^3 ; R_j is the nozzle radius, m ; v_j is the velocity of the oil at the nozzle outlet, m/s .

Based on the momentum theorem, the force of the jet on the jet pipe is found to be

$$F_j = \rho \pi R_j^2 v_j^2 \quad (3)$$

From the literature [10], the velocity of the oil at the nozzle outlet is

$$v_j = \sqrt{\frac{2}{\rho}(p_s - p_t) + 2g\Delta h} \quad (4)$$

where p_s is the inlet pressure, MPa ; p_t is the valve return pressure, MPa ; Δh is the vertical distance from the nozzle inlet to outlet, m .

Since the angle of declination θ is small, there is

$$F_{jx} \approx \rho \pi \theta R_j^2 v_j^2 \quad (5)$$

Assuming that the engagement of the feedback ball with the feedback slot in the spool is a clearance-free fit, it is approximated that whenever there is a relative displacement, it is constrained. Considering that after deflecting the jet pipe by an angle θ , the feedback spring is similarly deflected by an angle θ , as shown in Figure 3, the resulting deformation is

$$\Delta x_{fr} = x_{v0} - x_{fr} - x_v \quad (6)$$

where x_{v0} is the spool displacement before the changeover, m ; x_{fr} is the position of the end of the feedback spring without deformation, m ; x_v is the spool displacement, m .

Approximately,

$$x_{fr} = l_{fr}\theta \quad (7)$$

Resulting in a force

$$F_{fr} = k_{fr}(x_{v0} - l_{fr}\theta - x_v) \quad (8)$$

where k_{fr} is the stiffness of the feedback spring, N/m .

Substituting Equations (3)~(5) and Equation (8) into Equation (1), we obtain

$$J_c \ddot{\theta} + B_c \dot{\theta} + a\theta + b(x_{v0} - x_v) = M_e \quad (9)$$

where $a = \rho \pi l_j R_j^2 \left[\frac{2}{\rho}(p_s - p_t) + 2g(h_s - h_j) \right] - k_{fr} l_{fr}^2$; $b = k_{fr} l_{fr}$.

2.1.2. Flow Equations for the Control Chamber

From the literature [15], it is assumed that the pressure-flow equation of the jet pipe amplifier is

$$q_L = K_{qfj}\theta - K_{cpfj}\Delta p \quad (10)$$

where K_{qfj} is the flow gain; K_{cpfj} is the flow-pressure coefficient.

The movement of the main spool in the sleeve is shown in Figure 4.

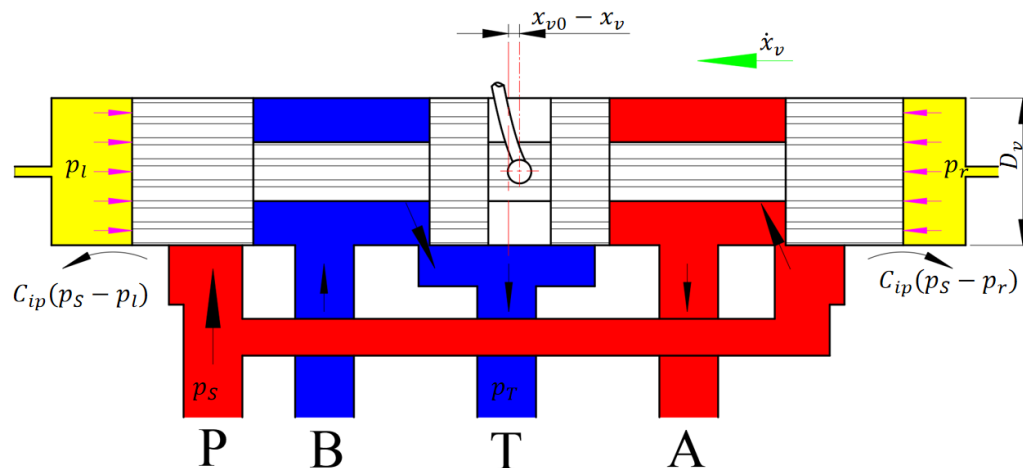


Figure 4. Schematic diagram of the main spool movement. (The red color indicates high-pressure oil in the flow channel, the blue color means low-pressure oil in the flow channel, and the yellow color shows that the fuel in the flow channel is the control fluid. The directions of black arrows indicate the direction of the flow. The directions of the magenta arrows indicate the direction of fluid pressure action. And the direction of the green arrow indicates the direction of spool movement).

As the spool moves to the left, the flow rate into the right control chamber is

$$q_{LR} = A_v \dot{x}_v + C_{ip}(p_s - p_r) + \frac{V_r}{\beta_e} \dot{p}_r \quad (11)$$

where A_v is the spool cross-sectional area, $A_v = \frac{\pi D_v^2}{4}$, m^2 ; C_{ip} is the internal leakage factor; V_r is the volume of the right control cavity, m^3 ; β_e is the bulk modulus of oil, Pa.

Similarly, the flow rate out of the left control oil chamber is obtained as follows:

$$q_{LL} = A_v \dot{x}_v + C_{ip}(p_s - p_l) - \frac{V_l}{\beta_e} \dot{p}_l \quad (12)$$

The volume of the right control chamber is as follows:

$$V_r = V_0 + A_v x_v \quad (13)$$

where V_0 is the initial volume of oil in the spool side chamber, m^3 .

The volume of the left control chamber is

$$V_l = V_0 - A_v x_v \quad (14)$$

Thus

$$q_L = \frac{q_{LR} + q_{LL}}{2} = A_v \dot{x}_v + \frac{C_{ip}}{2} (2p_s - p_l - p_r) + \frac{V_0}{2\theta_e} \Delta \dot{p} + \frac{A_v x_v}{2\theta_e} (\dot{p}_r + \dot{p}_l) \quad (15)$$

Neglecting the leak, Equation (15) simplifies to

$$q_L = A_v \dot{x}_v + \frac{V_0}{2\beta_e} \Delta \dot{p} + \frac{A_v x_v}{2\beta_e} (\dot{p}_r + \dot{p}_l) \quad (16)$$

when the main spool moves to the left, the pressure p_l in the left control chamber increases and the pressure p_r in the right control chamber decreases.

$$\dot{p}_r + \dot{p}_l = O(\epsilon) \quad (17)$$

where $O(\epsilon)$ is an infinitely small quantity.

Suppose that $A_v x_v \ll V_0$, Equation (16) becomes

$$q_L = A_v \dot{x}_v + \frac{V_0}{2\beta_e} \Delta \dot{p} \quad (18)$$

2.2. Nonlinear Dynamic Analysis of Spool

According to Newton's second law [37], the kinetic equation of the spool in the reverse direction is

$$m_c \ddot{x}_v = \Delta p A_v + F_s - F_t + F_{fr} \quad (19)$$

where m_c is the mass of spool and attendant fluid, kg; Δp is the differential pressure between two control chambers, Mpa; F_s is the steady-state flow force, N; F_t is the transient flow force, N.

When the spool has a small displacement x_v under differential pressure Δp , the mass of the spool and the accompanying fluid is

$$m_c = m_v + \rho A_v x_v \quad (20)$$

where m_v is the mass of the spool, kg.

Substituting Equation (19) into (20) yields

$$m_v \ddot{x}_v + \rho A_v x_v \ddot{x}_v = \Delta p A_v + F_s - F_t + F_{fr} \quad (21)$$

Drawing on the mathematical model of the spring [38–40], the nonlinear description of the steady-state hydrodynamic force is set as follows:

$$F_s = K_{s1}(x_{v0} - x_v) - K_{s2}(x_{v0} - x_v)^3 \quad (22)$$

where K_{s1} is the primary term coefficient; K_{s2} is the cubic term coefficient.

Consider the transient hydrodynamic formulation [41]:

$$F_t = \rho L \dot{Q} \quad (23)$$

where L is the damping length, m; Q is the flow rate through the orifice, m^3/s .

The damping length L affects the stability of the spool, as shown in Figure 5. In order to avoid the situation where the transient flow force acts as a negative damping force, it is determined during the design of the servovalve. The usual formula is

$$L = x_P - x_A - (x_B - x_T) \quad (24)$$

where x_i is the distance from each port to the spool center, $i = P, A, B, T, m$.

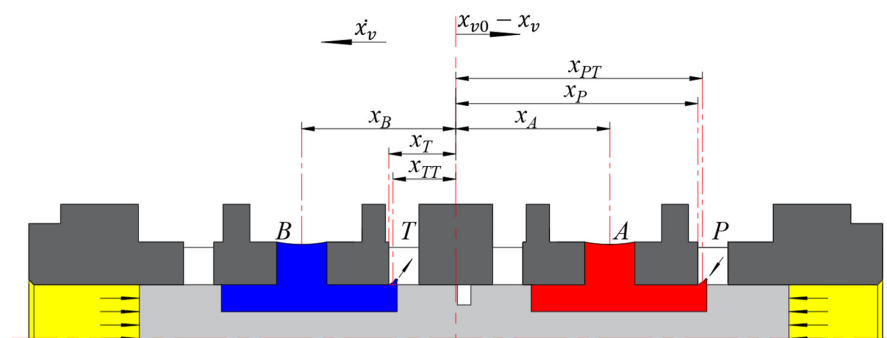


Figure 5. Schematic diagram to determine the damping length. (The red color indicates high-pressure oil in the flow channel, the blue color means low-pressure oil in the flow channel, and the yellow color shows that the fuel in the flow channel is the control fluid. The directions of the arrows indicate the direction of the flow.).

It is noted that the port centerline is considered when calculating the A-port coordinate x_A and the B-port coordinate x_B , but the T-port coordinate x_T and the P-port coordinate x_P are calculated with the port boundary. In the figure, x_{TT} and x_{PT} are the T-port and P-port coordinates considering the centerline position.

If the centerline of each port is used, then the transient hydrodynamic equation becomes

$$F_t = \rho(L + x_{v0} - x_v)\dot{Q} \quad (25)$$

Therefore, similar to the steady-state flow force, the nonlinear description is defined as follows:

$$F_t = [K_1(x_{v0} - x_v) + K_2](0 - \dot{x}_v) \quad (26)$$

where K_1 is the coupling term coefficient; K_2 is the velocity term coefficient.

For analytical simplicity, setting $L' = \frac{K_2}{K_1}$ yields

$$F_t = K_{t1}[L' + (x_{v0} - x_v)](0 - \dot{x}_v) \quad (27)$$

where K_{t1} is the equivalent transient flow force coefficient; L' is the equivalent damping length.

Substituting Equations (8), (22), and (27) into Equation (21), and letting $x_{vv} = x_{v0} - x_v$, we obtain

$$m_v \ddot{x}_{vv} + \rho A_v x_v \ddot{x}_{vv} = \Delta p A_v - K_{s2} x_{vv}^3 + K_{s1} x_{vv} - K_{t1}(L' + x_{vv})\dot{x}_{vv} + k_{fr}(x_{vv} - l_{fr}\theta) \quad (28)$$

Rearranging

$$(1 + \frac{\rho A_v}{m_v} x_v) \ddot{x}_{vv} = -\frac{K_{t1}}{m_v}(L' + x_{vv})\dot{x}_{vv} - \frac{K_{s2}}{m_v} x_{vv}^3 + \frac{K_{s1} + k_{fr}}{m_v} x_{vv} + \frac{\Delta p A_v - k_{fr} l_{fr} \theta}{m_v} \quad (29)$$

Considering that $\frac{\rho A_v}{m_v} x_v$ is on the order of 10^{-4} , which is much less than unit. So Equation (29) further reduces to

$$\ddot{x}_{vv} = -\eta(L' + x_{vv})\dot{x}_{vv} - \kappa \varepsilon^2 x_{vv}^3 + \kappa x_{vv} + F_u \quad (30)$$

where $\kappa = \frac{K_{s1} + k_{fr}}{m_v}$; $\varepsilon = \pm \sqrt{\frac{K_{s2}}{K_{s1} + k_{fr}}}$; $\eta = \frac{K_{t1}}{m_v}$; $L' = \frac{K_2}{K_1}$; $F_u = \frac{\Delta p A_v - k_{fr} l_{fr} \theta}{m_v}$.

2.3. Mathematical Modeling of Closed-Loop Systems

Equations (9), (10), (18), and (30) can be combined to form the block diagram in Figure 6. It can be seen that the jet pipe valve and the slide valve form a closed loop defined as Γ_1 . The armature assembly, the jet pipe valve, and the slide valve form another closed loop defined as Γ_2 .

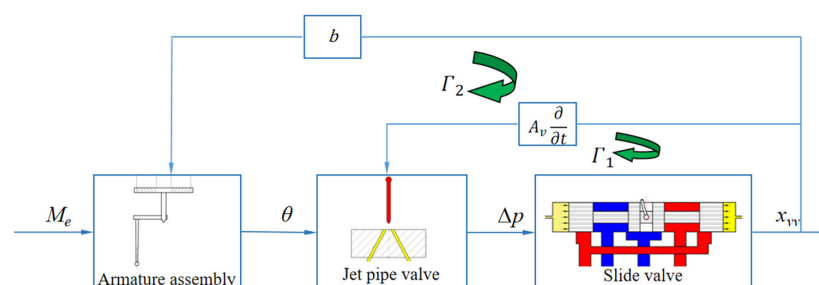


Figure 6. The block diagram of the closed-loop system. (The red color indicates high-pressure oil in the flow channel, the blue color means low-pressure oil in the flow channel, and the yellow color shows that the fuel in the flow channel is the control fluid. The directions of the blue arrows indicates the drection of signal flow. The block diagrams along the direction of the green arrows form a closed-loop system.).

Organizing Equations (10), (18), and (30) and setting $\Delta p = x_1$, $x_{vv} = x_2$, $\dot{x}_{vv} = x_3$, we obtain a set of nonlinear equations describing the third-order system Γ_1 :

$$\begin{cases} \dot{x}_1 = -\frac{2\beta_e K_{cpfj}}{V_0} x_1 - \frac{2\beta_e A_v}{V_0} x_3 + \frac{2\beta_e K_{qfj}}{V_0} \theta \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \frac{A_v}{m_v} x_1 - \kappa(\varepsilon^2 x_2^2 - 1)x_2 - \eta(L' + x_2)x_3 - \frac{b}{m_v} \theta \end{cases} \quad (31)$$

Organizing Equations (9), (10), (18), and (30) and setting $\Delta p = x_1$, $x_{vv} = x_2$, $\dot{x}_{vv} = x_3$, $\theta = x_4$, $\dot{\theta} = x_5$, we obtain a set of nonlinear equations describing the fifth-order system Γ_2 :

$$\begin{cases} \dot{x}_1 = -\frac{2\beta_e K_{cpfj}}{V_0} x_1 - \frac{2\beta_e A_v}{V_0} x_3 + \frac{2\beta_e K_{qfj}}{V_0} x_4 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \frac{A_v}{m_v} x_1 - \kappa(\varepsilon^2 x_2^2 - 1)x_2 - \eta(L' + x_2)x_3 - \frac{b}{m_v} x_4 \\ \dot{x}_4 = x_5 \\ \dot{x}_5 = -\frac{b}{J_{cj}} x_2 - \frac{a}{J_{cj}} x_4 - \frac{B_{cj}}{J_{cj}} x_5 + \frac{1}{J_{cj}} M_e \end{cases} \quad (32)$$

Due to space constraints, the transfer function of the fifth-order linear system proposed in the literature [15] is given here directly

$$X_v(s) = \frac{K_{qfj} l_j}{G_1(s) G_2(s) + K_{qfj} k_{fr} l_j^2} M_e(s) \quad (33)$$

where

$$G_1(s) = J_{cj} s^2 + B_{cj} s + k_j \quad (34)$$

$$G_2(s) = \frac{m_v V_0}{2\beta_e A_v} s^3 + \frac{K_{cpfj} m_v}{A_v} s^2 + [A_v + \frac{V_0(k_{fr} + 0.43w\Delta p_v)}{2\beta_e A_v}] s + \frac{K_{qfj}(k_{fr} + 0.43w\Delta p_v)}{A_v} \quad (35)$$

3. Numerical Analysis

The parameters used for numerical analysis are given in Table 1. Most of the parameters given in Table 1 are provided by the manufacturer. K_{qfj} and K_{cpfj} are calculated based on the data provided by the manufacturer. The coefficients related to the flow forces are fitted according to the CFD simulation results. All calculations and plots in this section are performed based on the MATLAB (R2016a) software [42–44].

Table 1. Simulation parameter list.

Symbol	Value	Unit	Symbol	Value	Unit
β_e	1.5×10^9	Pa	K_{t1}	-9.1245×10^4	-
V_0	1.97×10^{-7}	m ³	L'	1.788×10^{-4}	-
A_v	3.217×10^{-5}	m ²	K_{qfj}	2.232×10^{-5}	-
M_v	3.92×10^{-2}	kg	K_{cpfj}	9.8065×10^{-10}	-
k_{fr}	2.0×10^3	N/m	p_s	2.1×10^7	Pa
l_{fr}	2×10^{-2}	m	p_t	5×10^5	Pa
R_j	1.4×10^{-4}	m	Δh	2.865×10^{-2}	m
L_j	6×10^{-4}	m	J_{cj}	5.32×10^{-7}	kg m ²
K_{s1}	5.6713×10^4	-	B_{cj}	8.938×10^{-3}	Nms/rad
K_{s2}	1.5833×10^{11}	-	l_f	1.5×10^{-2}	m

3.1. Nonlinear Analysis of the Third-Order Closed-Loop Systems

3.1.1. The Calculation of Equilibrium Points

Let $\theta = 0$ in Equation (31), and consider the so-called unexcited equations of state:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} \\ 0 & 0 & 1 \\ \frac{A_v}{m_v} & \kappa(\varepsilon^2 x_2^2 - 1) & -\eta(L' + x_2) \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (36)$$

Let $\dot{x}_1 = 0$, $\dot{x}_2 = 0$, $\dot{x}_3 = 0$, and solve Equation (34) to obtain the equilibrium points $(0, 0, 0)$, $(0, \frac{-1}{\varepsilon}, 0)$, and $(0, \frac{1}{\varepsilon}, 0)$, which are denoted as points 0, 1, and 2. Considering that a positive or negative ε does not change the positions of the equilibrium points, only the case where ε is positive is analyzed.

The Jacobi matrix corresponding to the third-order system Γ_1 at point 0 is

$$J_0 = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} \\ 0 & 0 & 1 \\ \frac{A_v}{m_v} & -\kappa & -\eta L' \end{bmatrix} \quad (37)$$

Let $|J_0 - \lambda E| = 0$, the eigenvalues are found to be

$$\lambda_{01} = -5.003 \times 10^6, \lambda_{02} = 1.4338 \times 10^3, \lambda_{03} = -1.0446 \times 10^3.$$

Since there are two negative roots and a positive root, the point 0 is the saddle point. The Jacobi matrix corresponding to the third-order system Γ_1 at point 1 is

$$J_1 = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} \\ 0 & 0 & 1 \\ \frac{A_v}{m_v} & 2\kappa & -\eta L' + \frac{\eta}{\varepsilon} \end{bmatrix} \quad (38)$$

Let $|J_1 - \lambda E| = 0$, the eigenvalues are found to be

$$\lambda_{11} = -5.003 \times 10^6, \lambda_{12} = -514.11 + 1652.7i, \lambda_{13} = -514.11 - 1652.7i.$$

There exists a negative root and a pair of complex roots with negative real parts, so point 1 is a stable spiral.

The Jacobi matrix corresponding to the third-order system Γ_1 at point 2 is

$$J_2 = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} \\ 0 & 0 & 1 \\ \frac{A_v}{m_v} & 2\kappa & -\eta L' - \frac{\eta}{\varepsilon} \end{bmatrix} \quad (39)$$

Let $|J_2 - \lambda E| = 0$, the eigenvalues are found to be

$$\lambda_{21} = -5.003 \times 10^6, \lambda_{22} = 903.35 + 1476.3i, \lambda_{23} = 903.35 - 1476.3i.$$

There exists one negative root and a pair of complex roots with positive real parts, so point 2 is an unstable spiral. And the imaginary part of these two complex roots corresponds to the oscillation frequency, which is approximated to be 235 Hz.

3.1.2. Phase Portrait

The phase plane plot of Equation (34) is shown in Figure 7. The red * in the figures indicates the equilibrium point. Although point 1 is the stable spiral, the trajectories are repelled away from the equilibrium points because the other two equilibrium points are unstable.

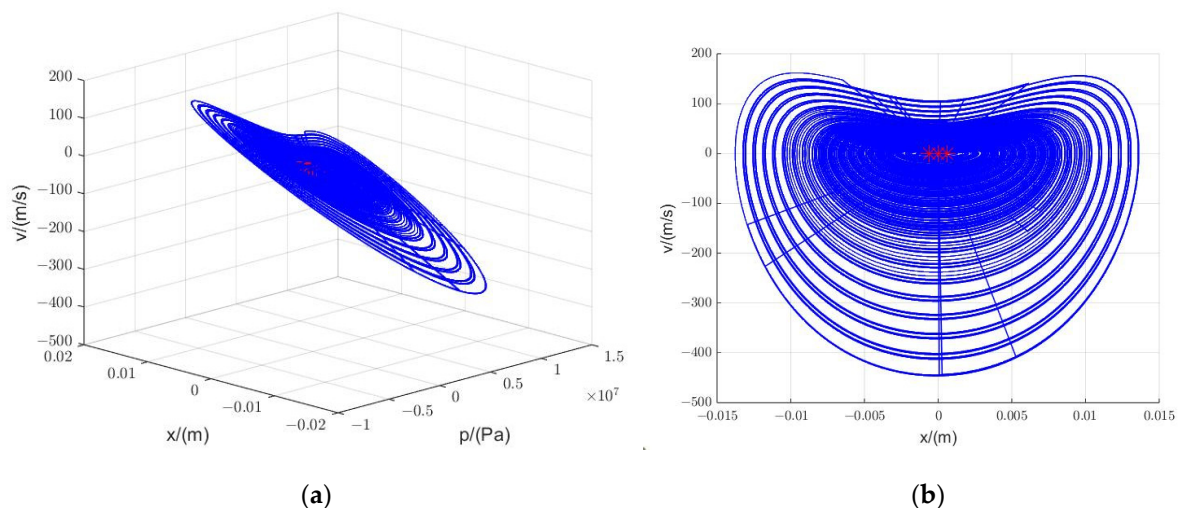


Figure 7. Phase plane plot of the third-order system: (a) pressure–displacement–velocity and (b) displacement–velocity. (The red * denotes equilibrium point).

3.1.3. Bifurcation Characteristics

- Hopf bifurcation

According to the Hurwitz criterion [45], it is known that the flow-pressure coefficient K_{cpfj} affects the stability of the nonlinear third-order system; this needs to be considered when the spool size is determined. Considering it as a bifurcation parameter, the eigenvalues of the Jacobi matrix 37 are computed to find that the real part of the eigenvalues of J2 is approximately zero when $K_{cpf} = 3.67590603 \times 10^{-13}$ and $K_{cpf} = 1.43899815 \times 10^{-11}$, and the system has Hopf bifurcations [46,47]. The effect on the system, when the bifurcation parameter K_{cpfj} is different, is shown in Figure 8, with a pressure–displacement–velocity phase trajectory starting from the initial value on the left and the displacement versus time curve on the right. The simulation time is set to 0.05 s, and the initial values (p_{v0} , x_{vv0} , v_{v0}) are chosen as $\{x, y, z | x = 1.5 \times 10^6, y = 6 \times 10^{-4}, z = 0\}$.

The system depicted in Figure 8c,d is stable based on the initial condition. Figure 8b,e depict spools with equal amplitude vibration, but the frequency and amplitude of the vibration are different from each other. Figure 8f depicts a spool motion that is unstable. Although Figure 8a appears to be stable, the result is oscillatory if the simulation time is increased to 1s. Moreover, it is worth noting that the phase and displacement diagrams may be different for different initial values.

Hopf bifurcation is closely related to the generation mechanism of self-excited oscillations. When the bifurcation parameter varies a little near the critical value, the stability characteristics of the system change. In jet pipe valves, K_{cpfj} is not a definite value, which is not only related to the valve operating point but also to the manufacturing error. Therefore, to avoid Hopf bifurcation, the effect of parameter K_{cpfj} needs to be considered in the design of the jet pipe valve.

- Period-doubling bifurcations

Based on the nonlinear system of Equation (31), the bifurcation due to the variations in the magnitude of the deflection angle θ and the frequency ω of the jet pipe is shown in Figure 9. Considering that the movement of the spool in the sleeve is bounded, an upper and lower limit of ± 6.1 mm (the maximum distance that the spool can move unilaterally between the sleeves) is given. The spool is essentially in period-doubling motion when the jet pipe inclination angle is varied within ± 0.175 rad (or $\pm 10^\circ$). Single-period motion exists only near 0 rad. And as the frequency increases, the inclination angle range of single-cycle motion increases.

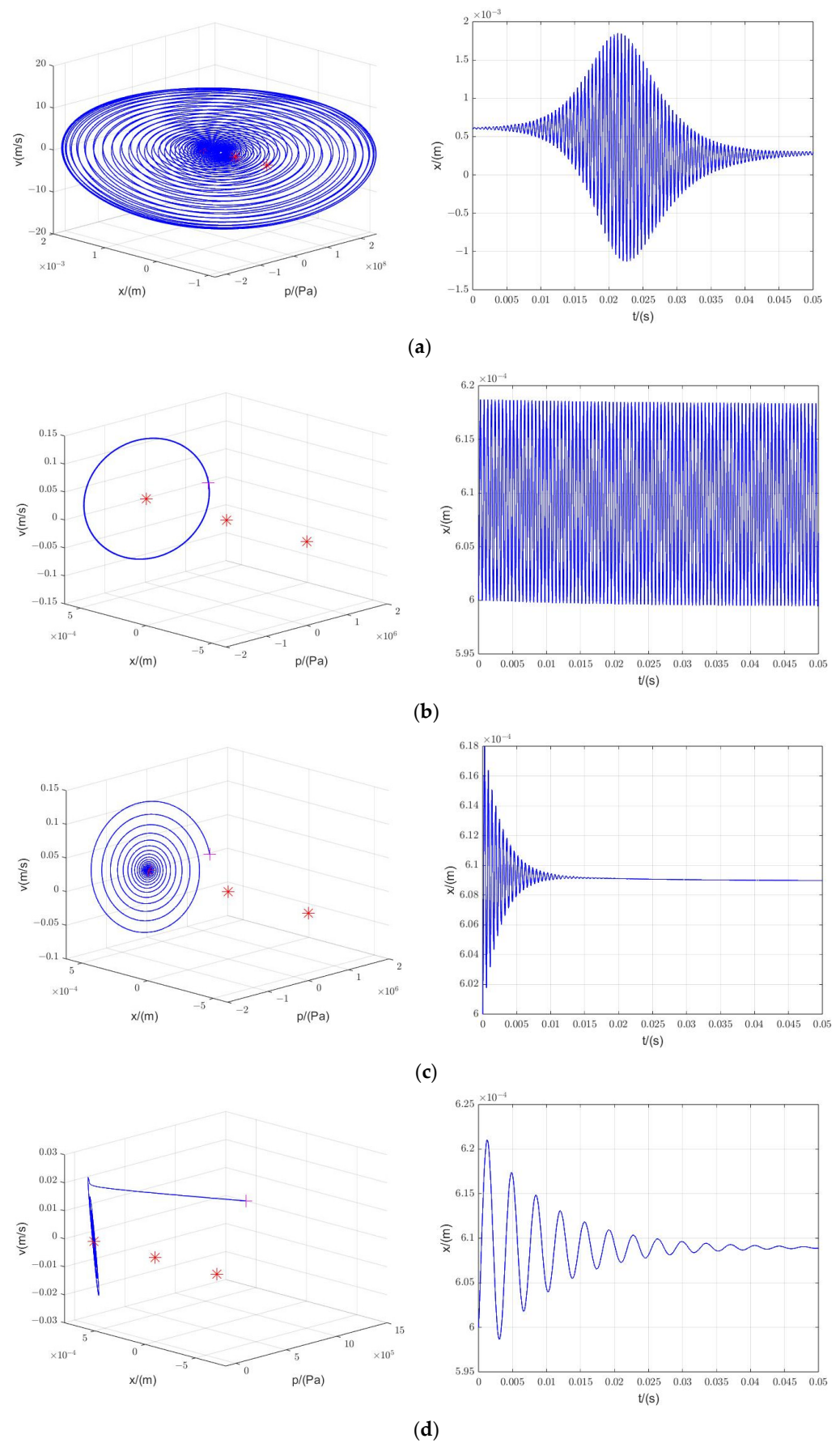


Figure 8. Cont.

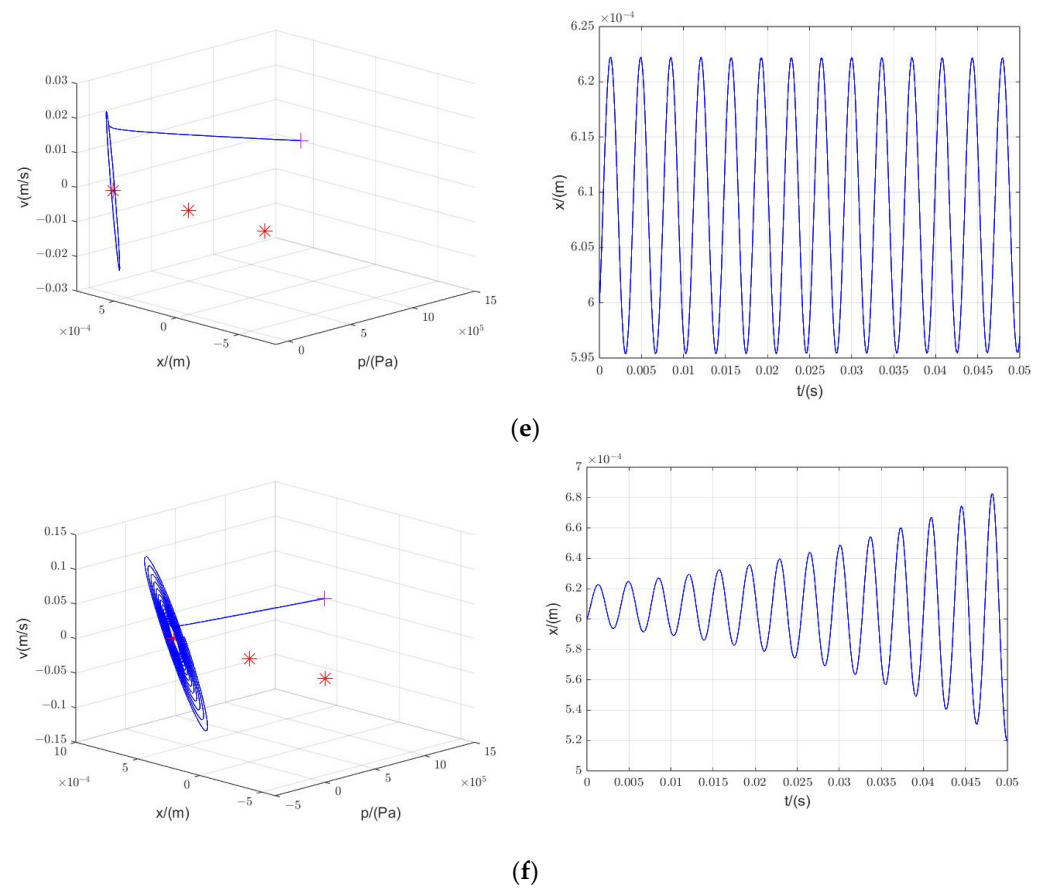


Figure 8. Schematic representation of the effect of parameter K_{cpfj} on the system. (a) $K_{cpfj} = 2.6 \times 10^{-13}$; (b) $K_{cpfj} = 3.67590603 \times 10^{-13}$; (c) $K_{cpfj} = 5.2 \times 10^{-13}$; (d) $K_{cpfj} = 1.3 \times 10^{-11}$; (e) $K_{cpfj} = 1.43899815 \times 10^{-11}$; (f) $K_{cpfj} = 1.5 \times 10^{-11}$. (The red * denotes equilibrium point, and the magenta + represents the starting point.).

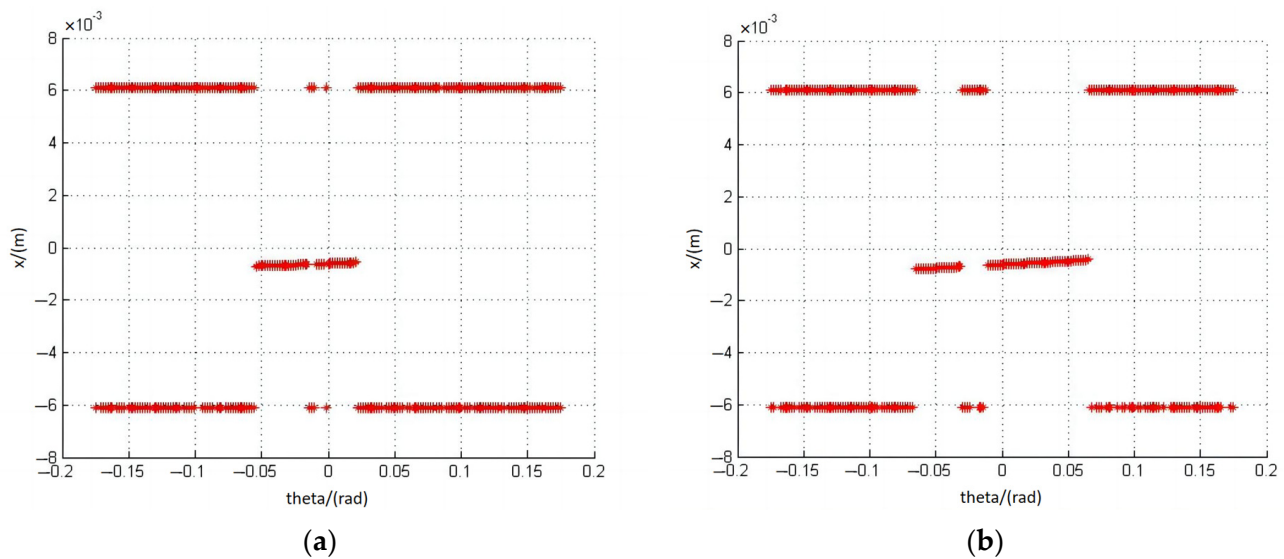


Figure 9. Bifurcation diagrams: (a) $\omega = 50$ and (b) $\omega = 1000$.

3.2. Nonlinear Analysis of the Fifth-Order Closed-Loop System

3.2.1. The Calculation of Equilibrium Points

Let $M_e = 0$ in Equation (32), and consider the so-called unexcited equations of state:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{pmatrix} = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} & \frac{2\beta_e K_{qfj}}{V_0} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{A_v}{m_v} & -\kappa(\varepsilon^2 x_2^2 - 1) & -\eta(L' + x_2) & -\frac{b}{m_v} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{b}{J_{cj}} & 0 & -\frac{a}{J_{cj}} & -\frac{B_{cj}}{J_{cj}} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \quad (40)$$

Let $\dot{x}_1 = 0, \dot{x}_2 = 0, \dot{x}_3 = 0, \dot{x}_4 = 0, \dot{x}_5 = 0$, and solve the Equation (38) to obtain the equilibrium points $(0, 0, 0, 0, 0)$, $(0, \frac{-1}{\varepsilon}, 0, 0, 0)$, and $(0, \frac{1}{\varepsilon}, 0, 0, 0)$, which are denoted as points I, II, and III.

The Jacobi matrix corresponding to the fifth-order system Γ_2 at point I is

$$J_{N0} = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} & \frac{2\beta_e K_{qfj}}{V_0} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{A_v}{m_v} & \kappa & -\eta L' & -\frac{b}{m_v} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{b}{J_{cj}} & 0 & -\frac{a}{J_{cj}} & -\frac{B_{cj}}{J_{cj}} \end{bmatrix} \quad (41)$$

Let $|J_{N0} - \lambda E| = 0$, the eigenvalues are found to be

$$\begin{aligned} \lambda_{N01} &= -5.003 \times 10^6, \lambda_{N02} = 1.4338 \times 10^3, \\ \lambda_{N03} &= -1.0446 \times 10^3, \lambda_{N04} = -154.63 + 2125.8i, \\ \lambda_{N05} &= -154.63 - 2125.8i. \end{aligned}$$

The Jacobi matrix corresponding to the fifth-order system Γ_2 at point II is

$$J_{N1} = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} & \frac{2\beta_e K_{qfj}}{V_0} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{A_v}{m_v} & -2\kappa & -\eta L' + \frac{\eta}{\varepsilon} & -\frac{b}{m_v} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{b}{J_{cj}} & 0 & -\frac{a}{J_{cj}} & -\frac{B_{cj}}{J_{cj}} \end{bmatrix} \quad (42)$$

Let $|J_{N1} - \lambda E| = 0$, the eigenvalues are found to be

$$\begin{aligned} \lambda_{N11} &= -5.003 \times 10^6, \lambda_{N12} = -580.3 + 1736.4i, \\ \lambda_{N13} &= -580.3 - 1736.4i, \lambda_{N14} = -80.42 + 2104.4i, \\ \lambda_{N15} &= -80.42 - 2104.4i. \end{aligned}$$

The Jacobi matrix corresponding to the fifth-order system Γ_2 at point III is

$$J_{N2} = \begin{bmatrix} -\frac{2\beta_e K_{cpfj}}{V_0} & 0 & -\frac{2\beta_e A_v}{V_0} & \frac{2\beta_e K_{qfj}}{V_0} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{A_v}{m_v} & -2\kappa & -\eta L' - \frac{\eta}{\varepsilon} & -\frac{b}{m_v} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{b}{J_{cj}} & 0 & -\frac{a}{J_{cj}} & -\frac{B_{cj}}{J_{cj}} \end{bmatrix} \quad (43)$$

Let $|J_{N2} - \lambda E| = 0$, the eigenvalues are found to be

$$\begin{aligned}\lambda_{N21} &= -5.003 \times 10^6, \lambda_{N22} = 943.16 + 1519.i, \\ \lambda_{N23} &= 943.16 - 1519.9i, \lambda_{N24} = -186.43 + 2147.4i, \\ \lambda_{N25} &= -186.43 + 2147.4i.\end{aligned}$$

Similarly, the imaginary part of these two complex roots with a positive real part corresponds to the oscillation frequency, which is approximated to be 242 Hz.

A comparison of the eigenvalues of the system Γ_2 and those of system Γ_1 shows that there is no change in all the real roots and a small change in the values of the real and imaginary parts of the complex roots. The real parts of the added eigenvalues are negative, so the stability properties of the equilibrium points of the fifth-order system are the same as those of the corresponding equilibrium points of the third-order system.

3.2.2. Phase Portrait

The phase plane plot of Equation (38) is shown in Figure 10.

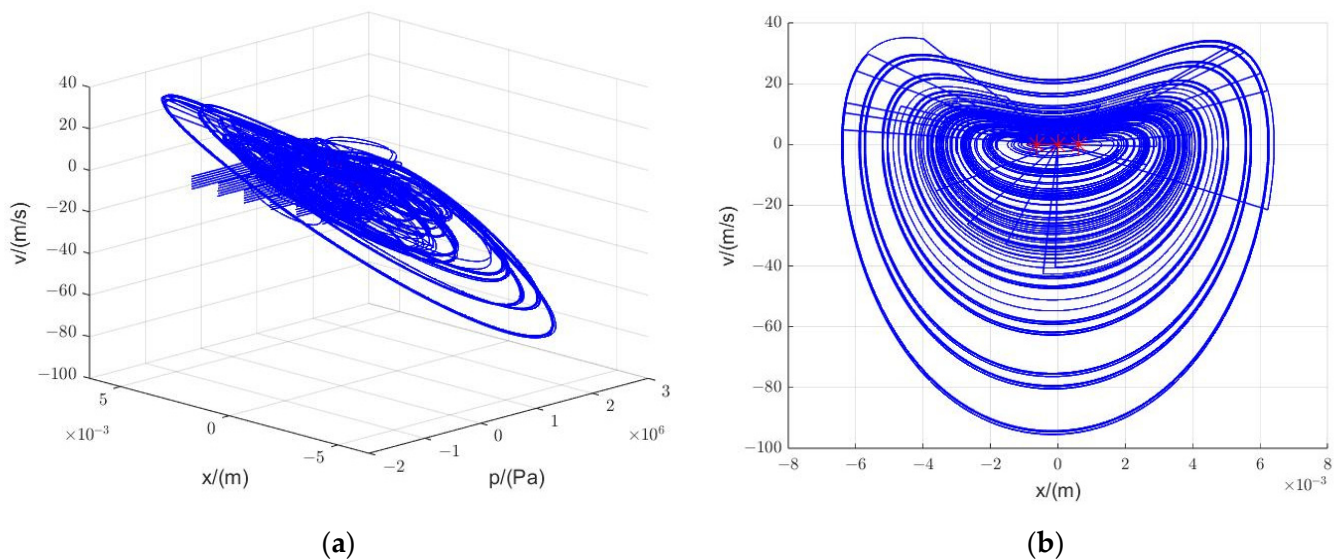


Figure 10. Phase plane plot of the fifth-order system: (a) pressure–displacement–velocity and (b) displacement–velocity. (The red * denotes equilibrium point.).

Compared to Figure 7, the phase portraits in the fifth-order system are nearly consistent with those of the third-order system. It shows that the addition of an armature assembly does not change the stability of the servovalve.

3.2.3. Simulation

Setting the initial condition as $(0, 0, 0)$ and the input signal as $0.1\cos 50t$, Equations (32) and (33) are solved. The simulation results are shown in Figure 11, where the left figures show the curve of the spool displacement versus time, and the right figures show the power spectrum of displacement.

The displacement curves in Figure 11 show that the spool vibrates around the neutral position by the cosine signal and that the vibration frequencies are different for the two. The plots of power spectral density in Figure 11 illustrate the difference between the linear and nonlinear mathematical models. In the power spectrum diagram in Figure 11a, the peak appears at 242 Hz, which is consistent with the previous calculation. And the frequency of 483 Hz, nearly twice that of 242 Hz, suggests that there is a period-doubling motion of the spool. Only a single frequency of 18 Hz exists in the power spectrum diagram in Figure 11b, indicating that the linear model cannot explain the multiply-periodic motion.

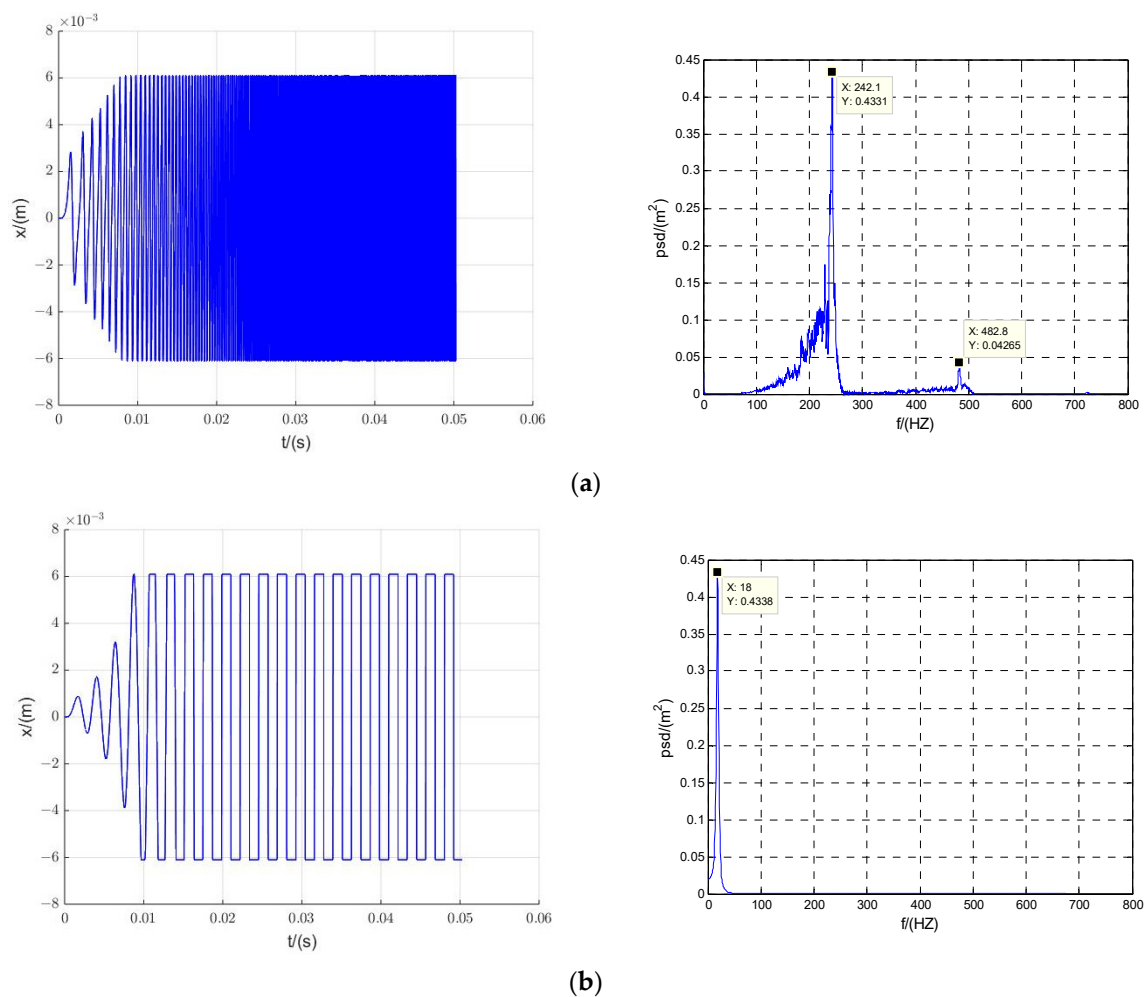


Figure 11. The simulation results: (a) nonlinear equations and (b) linear equations.

4. Experiment

A test platform was built to test the performance of the jet pipe servovalve, as shown in Figure 12. The system consists of the following parts: an electrohydraulic position servo control system and a data acquisition system.

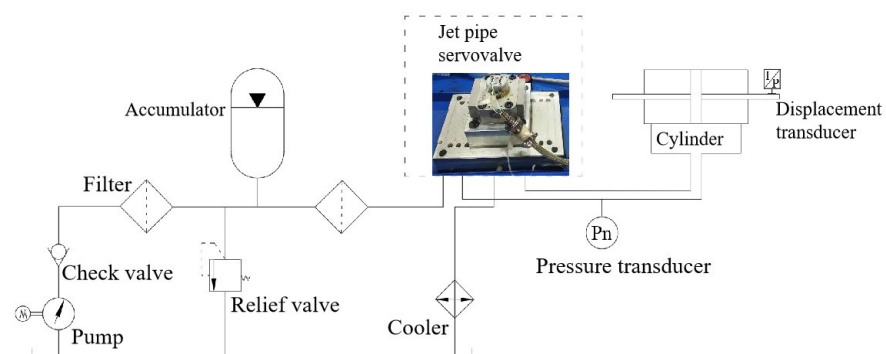


Figure 12. Hydraulic system schematic diagram.

The electrohydraulic position servo control system has three parts: an oil source, a control valve, and an actuator. The oil source includes a piston pump, check valve, accumulator, and relief valve to ensure the oil supply pressure is 21 MPa. Auxiliary components include an oil filter to ensure that the oil is clean and a cooler to ensure that the

oil temperature is controlled. Also, a 6-diameter jet pipe servovalve is adopted to control the servo actuator action, and a displacement sensor is used to transform the displacement of the servo cylinder piston into an electrical signal for feedback to the computer control system. The pressure sensor is placed near the outlet of the jet pipe servovalve to measure the change in oil pressure.

For the experiment, the piston pump is started to supply oil into the system. Enough oil is stored in the accumulator after the servo cylinder has been actuated a few times. The piston pump is turned off but still continues to control the opening and closing of the jet pipe servovalve using the accumulator to supply oil and eliminate the effect of piston pump vibration on the system. The resulting pressure change curve at the outlet of the servovalve is treated by the zero-mean method, and the obtained curve is shown in Figure 13a. The pressure signal is converted to the spool displacement curve according to the orifice equation, as shown in Figure 13b. The fast Fourier transform is performed on the displacement signal to obtain its power spectral density, as shown in Figure 13c.

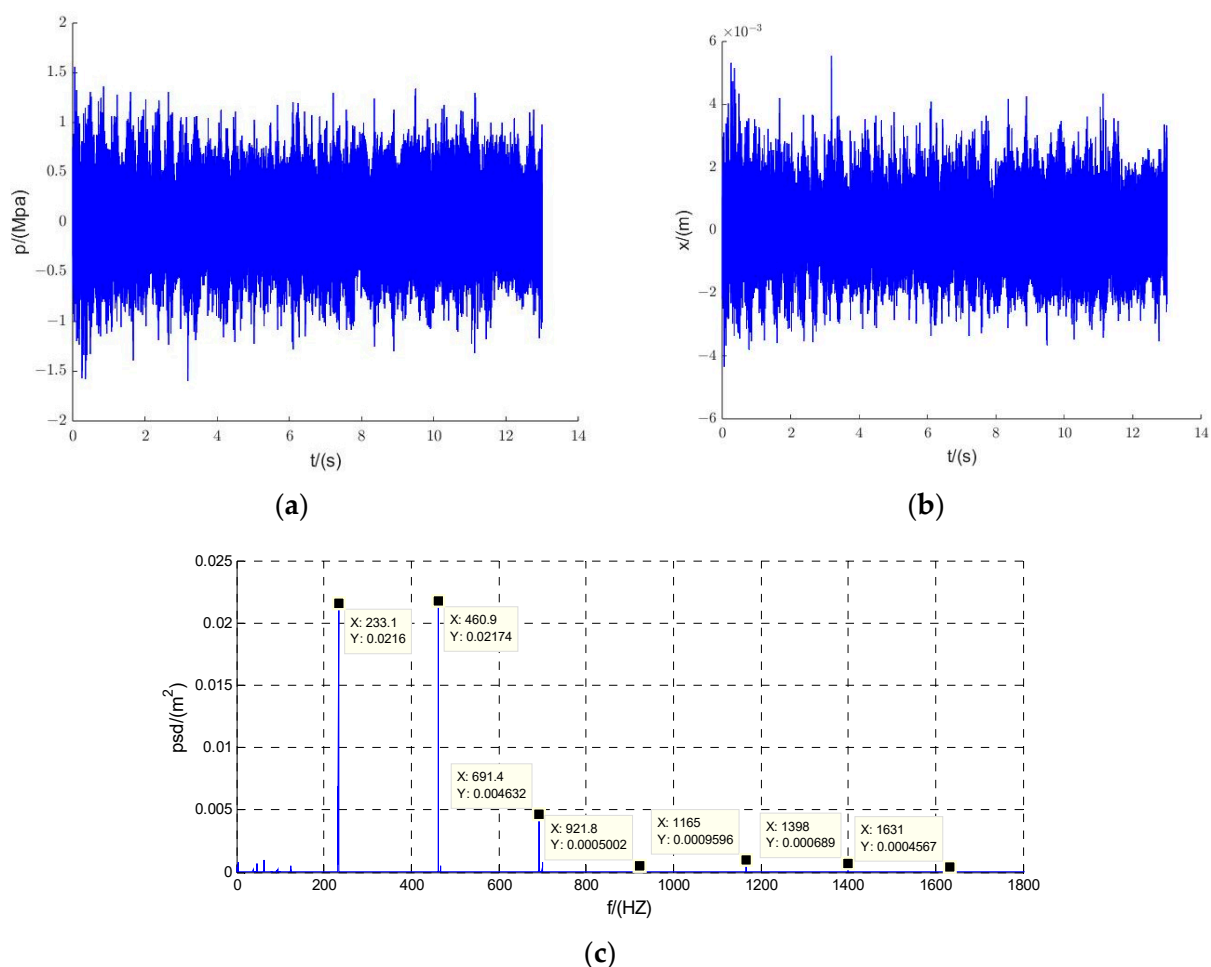


Figure 13. The experiment results: (a) measured pressure signal diagram, (b) converted spool displacement curve, and (c) the power spectrum diagram of displacement signal.

From Figure 13c, it is observed that the vibration frequencies of the system are 233, 461, 691, 921, 1165, 1398, and 1631 Hz. This indicates that the spool motion during the action of the jet pipe servovalve is a multi-frequency vibration, and its single-period frequency is 233 Hz. The energy of the single-period and period-doubling motions is larger, and the corresponding power spectral density decreases more with the period-tripling motion.

The experimentally measured fundamental frequency of the spool motion in the servovalve is basically consistent with the theoretical value, with a relative error of about

3.9%. However, the analysis considers the spool motion to be a period-doubling motion. The reason for this difference may be that the theoretical analysis only considers the mechanical–hydraulic structure of the servovalve, while the experimental data have the influence of components, such as torque motor, accumulators, piping, and even actuators.

In addition, the assumption that the spool limits are forced in the simulation analysis does not correspond to the actual situation, which may also lead to the discrepancy between the experiment and theory. In the flow field of the second stage in a jet pipe servovalve, the viscosity and elasticity of the fluid will inevitably prevent a “hard collision” between the spool and the sleeve under extreme conditions. When the spool end of the cavity is extremely compressed, the compressed fluid produces a large enough pressure and damping force to push the spool in the opposite direction. Obviously, the physical description function of this phenomenon is segmentally smooth, and its global model is discontinuous on the macroscopic time scale.

5. Conclusions

During servovalve action, we consider the fluid–solid coupling motion between turbulence and spool in the second stage to be nonlinear. Based on the characteristics of the steady-state flow force and transient flow force, the nonlinear descriptions of both under the condition of a small opening are proposed, and the nonlinear kinetic equation of the main spool is further established.

The equations of a third-order closed-loop nonlinear system consisting of a jet pipe valve and a spool valve are developed, the frequency of the system destabilizing oscillations is calculated to be 235 Hz, and the local bifurcation properties of the system are investigated. When the jet pipe inclination angle is varied within ± 0.175 rad, the system mostly undergoes period-doubling bifurcation. The study of the fifth-order closed-loop nonlinear system shows that the increase in armature assembly does not change the characteristics of the equilibrium points but slightly affects the distribution of the characteristic roots of instability, which increases the oscillation frequency to 242 Hz.

The vibration test of a jet pipe servovalve was carried out, and the results showed that the vibration frequencies with higher energy in the system were 233 and 461 Hz, which were similar to the theoretically calculated spool destabilization frequency of 242 Hz and the doubling period frequency of 484 Hz.

We try to establish a nonlinear mathematical model of servovalve and analyze the dynamic characteristics of the system based on nonlinear theory, providing a new perspective for the analysis and design of servovalves. However, in order to obtain a more accurate model, further theoretical work is needed, such as the description of the physical process of the reverse motion of the spool “collision” against the sleeve, the torque motor equations considering the asymmetry of the air gap, and the analysis of the global dynamics of the jet pipe servovalve.

Author Contributions: Methodology, X.K. and S.K.; software, S.K. and J.Z.; validation, J.Z. and R.D.; data curation, R.D.; resources, X.K. and J.Z.; formal analysis, J.Z.; writing—original draft preparation, S.K. and R.D.; writing—review and editing, X.K., S.K. and J.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant number 52275068; and the Key Project of Jiangsu Maritime Institute, China, grant number xjzd2020-5.

Data Availability Statement: Data are contained within the article.

Acknowledgments: This research would like to thank Zhu, J.X. and Wu, B.X. for their technical support.

Conflicts of Interest: The authors declare no conflicts of interest.

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