water

MDPI

Article

Comparison of Methods Predicting Advance Time in Furrow Irrigation

George Kargas [1], Dimitrios Koka [1], Paraskevi A. Londra [1,*] and Leonidas Mindrinos [2]

[1] Laboratory of Agricultural Hydraulics, Department of Natural Resources Development and Agricultural Engineering, Agricultural University of Athens, 11855 Athens, Greece; kargas@aua.gr (G.K.); stud616029@aua.gr (D.K.)

[2] Laboratory of Mathematics and Theoretical Mechanics, Department of Natural Resources Development and Agricultural Engineering, Agricultural University of Athens, 11855 Athens, Greece; leonidas.mindrinos@aua.gr

* Correspondence: v.londra@aua.gr

Abstract: In the design of furrow irrigation, and in general in surface irrigation, the reliable estimation of the advance time at the furrow end (t_L) is a key issue for improving the efficiency and uniformity of irrigation. In this study, three methods are used for estimating the t_L, and their results are compared with the experimental data of fifteen different furrows from the international literature. These methods are as follows: (a) the Valiantzas equation, (b) the method presented by Walker and Skogerboe, based on solving the volume balance equation by the Newton–Raphson iterative procedure and (c) the method of Philip and Farrell. The first two methods assume that the infiltration is described by the Lewis–Kostiakov equation and the extended Lewis–Kostiakov equation, respectively, while in the case of the Philip and Farrell method, the infiltration is described by the Philip equation and the Lewis-Kostiakov equation. The results showed that in most cases of the first two methods, the absolute relative error value of the predicted time t_L was less than 10%. The Philip and Farrell method using the Lewis–Kostiakov infiltration equation underestimates the time t_L and fails especially in the case where the volume of the surface water is not negligible compared to the total volume of water entering the system. The Valiantzas method is recommended because it was simpler and easier to use and showed greater prediction accuracy of t_L, resulting in better planning of irrigation systems and contributing to water saving, which is currently a big issue.

Keywords: furrow irrigation; advance time; infiltration; Lewis–Kostiakov equation

Citation: Kargas, G.; Koka, D.; Londra, P.A.; Mindrinos, L. Comparison of Methods Predicting Advance Time in Furrow Irrigation. *Water* **2024**, *16*, 1105. https://doi.org/10.3390/w16081105

Academic Editors: Haijun Liu and Jay Jabro

Received: 28 February 2024
Revised: 4 April 2024
Accepted: 10 April 2024
Published: 12 April 2024

1. Introduction

The main characteristic of surface irrigation is the simultaneous advance and infiltration of water from the inlet to the end of a field. Part of the total water moves on the surface of dry soil as a thin surface layer, while at the same time the other part infiltrates into the soil when a soil surface comes into contact with water [1]. The shorter the advance time along the system, the greater the uniformity of the water depth infiltrating along the system [2]. However, this is very difficult to achieve, because the advance phase and in particular its completion time (t_L) are affected by the water supply at the inlet of the system, the roughness coefficient, the longitudinal slope of the system and the infiltration rate.

In the design of surface irrigation systems, the accurate estimation of the time t_L, i.e., the time that the advancing water front has reached the system end, plays a decisive role, because conventionally the irrigation time is usually taken as the sum of the time t_L and the time that is required to infiltrate a water depth equal to the irrigation dose at the lower end of the field (t_a). Thus, in the case where a water depth equal to the net irrigation dose (Z_n) has been infiltrated at the lower end of the system, the degree of storage will be $E_s = 1$ and the degree of efficiency E_a will be equal to $E_a = \frac{Z_n L}{q_o t_\delta}$, where q_o is the inlet water rate, L is the furrow length and t_δ is the total irrigation time, which is equal to $t_\delta = t_L + t_a$.

To predict the advance phase, several models have been proposed based on numerical solutions of surface flow models (e.g., kinematic wave model, zero inertia model, etc.) or the volume balance equation [3,4]. The zero-inertia model and the kinematic wave model have been found to reliably predict the advance phase in furrow irrigation [5–7]. However, the application of these models is not always easy, due to the complexity and difficulties related to the various numerical optimization techniques as well as the large number of their parameters. Such techniques are not easily adopted for routine furrow design applications.

Philip and Farrell [8] showed that by using the Laplace transformation, the general analytical solution of the Lewis and Milne [9] equation, which describes the infiltration–advance of water in surface irrigation when there is a constant inflow, can be obtained. Furthermore, by applying various infiltration equations, such as the Lewis–Kostiakov [10,11], Philip [12] and Horton [13] equations, into the general analytical solution, they obtained specific analytical solutions for the advance phase x(t) for each infiltration equation when the amount of surface water is considered negligible. However, in most cases, the infiltration equations used are empirical, and therefore their parameters have no physical meaning. In addition, the two-term infiltration equation of Philip [12] is only appropriate for infiltration at short to medium infiltration times [14].

The U.S. Soil Conservation Service (SCS) [15] has proposed an empirical equation to predict the duration of the advance phase, the application of which requires the classification of the soil in the appropriate permeability group.

Walker and Skogerboe [3], using the two-point volume balance equation and assuming that the advance phase is described by an exponential form equation, proposed an iterative procedure (Newton–Raphson method) to estimate the time t_L from the furrow distance. Valiantzas [16] proposed algebraic equations to calculate the advance time as a function of inflow rate, without requiring iterative calculation procedures. A disadvantage of the method is perhaps that the time of advance versus distance relationship is described by three equations of different mathematical form [16].

Also, Valiantzas [17], based on the volume balance equation, proposed an equation to calculate the advance time as a function of the inflow rate and the parameters of the Lewis–Kostiakov infiltration equation, which gives similar results as the zero-inertia model. The equation was obtained by linear superposition of the two advance solutions for short times and the asymptotic solution for a longer time. This equation can also be applied to any form of the infiltration equation. Compared to the empirical SCS [14] equation, the Valiantzas [17] equation predicts the advance time much better.

Cook et al. [14] introduced into the Lewis and Milne [9] equation the two-parameter infiltration equation for linear soils presented by Philip [18]. This infiltration equation is based on physically meaningful parameters such as sorptivity (S) and saturated hydraulic conductivity (K_s) and is able to give adequate infiltration and advance behavior over all time scales.

From the abovementioned information, it appears that if we exclude the various simulation models of the duration of the advance phase, several analytical solutions and methodologies have been proposed that may be sufficient for routine furrow irrigation design applications. However, these solutions and their different methods rely on different assumptions and use different infiltration equations. However, a comparative evaluation of them, as far as we know, is absent from the literature.

The purpose of this paper is to compare (1) the Newton–Raphson iterative process proposed by Walker and Skogerboe [3], (2) the Valiantzas [17] equation and (3) the Philip and Farrell [8] method in predicting the advance time t_L, as well as a comparison of their results with the experimental data from 15 experimental fields presented in the international literature. In the first two methods, it is assumed that the infiltration follows the Lewis–Kostiakov equation, while in the third method, both the infiltration equations described by Philip [12] and Lewis–Kostiakov are examined. The comparison of the three methods predicting the water advance in furrow irrigation could help irrigation system designers to choose the most suitable method each time. The choice of a reliable, easy and quick estimation of t_L will help, among other things, to save water, which is currently a big issue.

2. Materials and Methods

2.1. Philip and Farrell Method [8]

The equation of Lewis and Milne [9] can describe the advance of water along a furrow as

$$q_0 t = cx + \int_0^x i(t - t_s)ds \quad (1)$$

where q_0 (L^3/T) is the constant inflow rate per furrow, c (L^2) is the average section area of the stream flow, x (L) is the distance water has advanced along the field at time t, t_s (T) is the value of t when water has arrived at location s behind the advancing front and i (L^3/L) is the infiltration volume per unit length of furrow as a function of opportunity time $t - t_s$.

An equivalent equation of (1) is Equation (2):

$$q_0 t = cx + \int_0^t i(t - t_s)x^{/}(t_s)dt_s \quad (2)$$

where $x^{/} = \frac{ds}{dt_s}$ is the advance rate at time t_s corresponding to location s.

Philip and Farrell [8] presented a solution of Equation (2) in series form, which gives the advance equation x(t) for short and long times via the Laplace transformation without assuming a functional form of x(t) before the integration of Equation (2).

If the Philip [12] equation (Equation (3)) is used as the infiltration equation

$$i(t) = St^{0.5} + At, \quad (3)$$

where S ($L^2/T^{0.5}$) is the soil sorptivity and A (L^2/T) is related to saturated hydraulic conductivity K_s (L^2/T) and varied $1/3\,K_s < A < 2/3\,K_s$ [18], then the solution of Equation [2] for c = 0 is

$$x(t) = \frac{q_0}{A}\left[1 - \exp^{\frac{4A^2t}{\pi S^2}} \operatorname{erfc}\frac{2At^{0.5}}{\pi^{0.5}S}\right] \quad (4)$$

According to Philip and Farrell [8], the case c = 0 is of particular interest because it shows in a simple way the dependence of x(t) on i(t) when the surface water volume relative to cumulative infiltration is small.

It should be noted that Equation (3) is valid for short to medium infiltration times [19]. Knowing the values of q, S and A of Equation (4), the value of t_L can be calculated by using any generalized unconstrained technique in which $f(t_L)$ is minimized to zero. In this study, the t_L was estimated using Excel Solver provided with Microsoft Excel 365 [20,21]. Excel Solver is an easy-to-use tool because it requires no programming knowledge.

More specifically, if Equation (4) is applied at the furrow distance x=L, where L is the furrow length, and the infiltration parameters A and S are known, then the only unknown parameter is the advance time $t = t_L$. Equation (4) can be transformed into the following equation where the only unknown parameter is t_L:

$$f(t_L) = \frac{q_0}{A}\left[1 - e^{\frac{4A^2t_L}{\pi S^2}} \operatorname{erfc}\left(\frac{2At^{0.5}}{\pi^{0.5}S}\right)\right] - L = 0 \quad (5)$$

The following steps were taken to solve Equation (5) and to estimate the t_L by using the Solver tool:

Step 1: Enter the values of the parameters q_0, A, S and t_L into an Excel worksheet. The value $t_L = 5A_0L/q_0$ can be used as an initial value of t_L, where A_0 is the wetted cross-sectional area of a furrow.
Step 2: In a new cell, calculate the $f(t_L)$ using Equation (5).
Step 3: Go to the tools menu and click the Solver tool.

Step 4: In "set objective", set the cell created in step 2, then set it to receive the value zero according to Equation (5), and set the cell containing the value of t_L as the Solver optimization variable. GRG nonlinear is chosen as the solution method.
Step 5: Press OK and obtain an optimal value of t_L.

Accordingly, if the infiltration equation used in Equation (2) is the Lewis–Kostiakov equation

$$i = kt^{\alpha}, \tag{6}$$

where k (L^2/T^a) and α (-) are empirical coefficients, then the following analytical solution for c = 0 is obtained:

$$x(t) = \frac{q_0 t^{1-\alpha}}{k\Gamma(1+\alpha)\Gamma(2-\alpha)} \tag{7}$$

where Γ is the gamma function and k and α the parameters of the Lewis–Kostiakov equation. The calculation of time t_L from Equation (7) is easier compared to Equation (4) since the calculation of t_L can be performed explicitly from Equation (7) if the values of q_0, k and α are known.

2.2. Newton–Raphson Iterative Procedure

The volume balance equation is based on the law of conservation of mass and was first applied by Lewis and Milne [9] (Equation (2)). It shows that the total volume of water at the inlet of the furrow at time $t \le t_L$ is expressed as $q_0 t$ and is equal to the sum of the water volume flowing on the furrow surface and the water volume infiltrating into the soil according to the following relationship [3,22,23]:

$$q_0 t = \sigma_y A_0 x + \sigma_Z k t^{\alpha} x \tag{8}$$

where q_0 (L^3/T) is the inflow rate, x (L) is the distance of the advance water front at each time t (T), σ_y (-) is the surface profile shape factor, usually equal to 0.77 [24], σ_Z (-) is the subsurface shape factor, which ranges from 0.6 to 1, and the infiltration was assumed to follow the Lewis–Kostiakov equation, which is the most commonly used infiltration equation in surface irrigation models. A_0 is the cross-sectional area of the inlet flow and is calculated from the Manning equation [3]:

$$A_0 = \left(\frac{{q_0}^2 n^2}{3600\rho_1 S_0}\right)^{\frac{1}{\rho_2}} \tag{9}$$

where n (-) is the Manning roughness coefficient, ρ_1 and ρ_2 (-) are furrow shape parameters and S_0 (-) is the longitudinal slope of the system (m/m).

Factor σ_Z is calculated by the following equation [25]:

$$\sigma_Z = \frac{\alpha + r(1-\alpha) + 1}{(1+\alpha)(1+r)} \tag{10}$$

where the advance curve is described by a power function of the form

$$x = pt^r \tag{11}$$

where p and r are empirical adjustment coefficients. The introduction of this relationship over-conditions the problem by defining the advance relationship before the integration of Equation (2).

Equation (9) includes the furrow shape parameters ρ_1 and ρ_2, the advance distance x at time t, the longitudinal slope of the system (m/m) S_0, the Manning roughness coefficient n, the inflow rate q_0, the coefficients k and α of the Lewis–Kostiakov equation and the two unknown parameters t_L and r. The parameter r is included in the calculation equation of the parameter σ_z (Equation (10)). Equation (8) can be solved using the Newton–Raphson iterative process to estimate the advance time t_L, as well as the coefficient r. The steps followed are as follows:

(i) First, an initial value of the parameter r is entered, which varies between 0.3 and 0.9. The value ri = 0.5 is usually chosen.

(ii) Then, the value of the parameter σz is calculated, as mentioned in Equation (10). It should be noted that the value of σz is recalculated every time the value of r changes.

(iii) The Newton–Raphson iterative procedure is then applied to find the advance time t_L using the initial value r_i as follows:

1. An initial estimate of t_{L0} is created. The value $t_{L0} = 5A_0L/q_0$ is usually considered as an initial value.
2. A better estimate of t_L is t_{L1} given by the Newton–Raphson method using the relationship

$$t_{L1} = t_{L0} - \frac{f(t_{L0})}{f'(t_{L0})} = t_{L0} - \frac{c_1 {t_{L0}}^a + c_2 t_{L0} + c_3}{c_1 a {t_{L0}}^{a-1} + c_2} \quad (12)$$

where

$$f(t_L) = c_1 {t_L}^a + c_2 t_L + c_3 = 0 \quad (13)$$

and $c_1 = \sigma_Z kL$, $c_2 = -q_0$ and $c_3 = \sigma_y A_0 L$.

3. Equation (13) is a transformed form of Equation (8) when x = L and t = t_L. The initial estimate t_{L0} is compared with the value t_{L1}. If the values t_{L0} and t_{L1} do not differ greatly, the next step, step 4 is applied; otherwise, step 2 is repeated and the t_{L2} value is calculated using t_{L1}. The iterative process stops when two consecutive values converge. Empirically, three to four repetitions are sufficient.
4. The advance time at the distance x = L/2 is calculated accordingly for the initial value r_i as described in steps 2 and 3, and the volume balance equation is applied by using the value L/2 instead of L.

(iv) The value r_{i+1} is calculated using the advance times t_L and $t_{L/2}$ calculated from the previous steps 3 and 4 as follows:

$$r_{i+1} = \frac{\ln \frac{L}{L/2}}{\ln \frac{t_L}{t_{\frac{L}{2}}}} \quad (14)$$

or

$$r_{i+1} = \frac{\ln 2}{\ln\left(\frac{t_L}{t_{L/2}}\right)} \quad (15)$$

(v) The initial estimate r_i is compared with the value r_{i+1} (Equation (15)). If the values converge, then it is assumed that the time t_L is the estimated one. Otherwise, steps 2 to 4 are repeated using as a new initial value the value r_{i+1}.

All the abovementioned steps of the Newton–Raphson iterative process can also be presented by the following flowchart (Figure 1).

It should be noted that if the inflow rate q_0 in the furrow is too small and the length L is too long, then there is a failure to converge the values in the iterative process in step iii. In this case, when designing furrow irrigation, either q_0 must be increased or the furrow length L must be reduced. Also, if the value of the coefficient r is known from some other method, then only steps ii and iii are applied to calculate t_L.

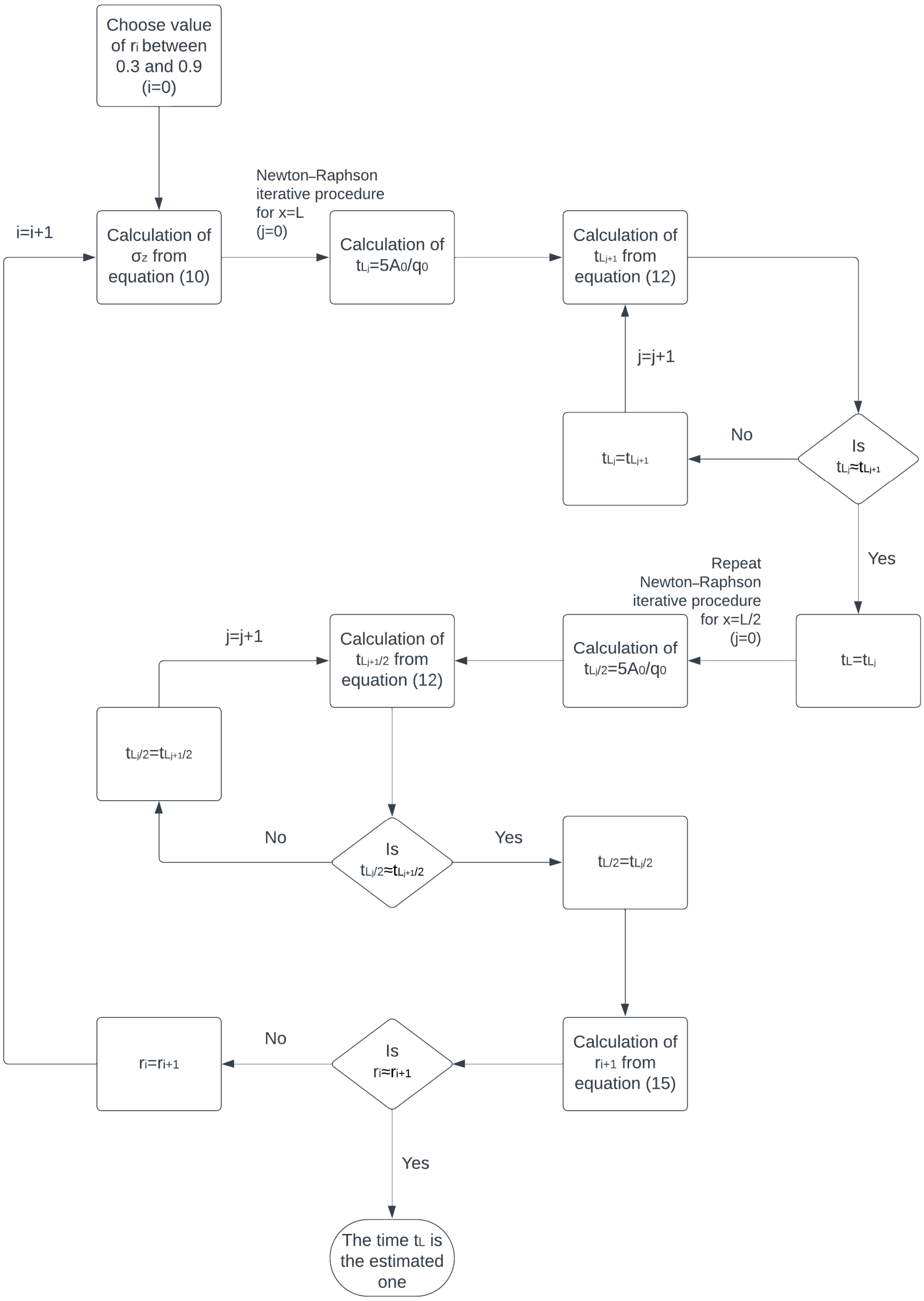

Figure 1. Graphical overview (flowchart) of the steps of the Newton–Raphson iterative process.

2.3. *Valiantzas Method [17]*

The equation of Valiantzas [17] calculates, directly and algebraically, the advance time, as long as the inflow rate, the cross-sectional area of the inlet flow and the parameters k and α of the Lewis–Kostiakov equation of the corresponding soil are known (Equation (16)). Equation (16) was obtained by linear superposition of the solutions of the volume balance equation in dimensionless form for short and long times.

$$t_L = (1 + 0.15\alpha)\frac{A_0 L}{q_0} + \left(\sigma_{ZF}\frac{kL}{q_0}\right)^{\frac{1}{1-\alpha}} \quad (16)$$

where k (L^2/T^a) and α (-) are parameters of the Lewis–Kostiakov equation, A_0 (L^2) is the inlet flow area, L (L) is the length of the furrow, q_0 (L^3/T) is the inflow rate and σ_{ZF} (-) is the value of parameter σ_Z (subsurface shape factor) at long times proposed by Hart et al. [26]:

$$\sigma_{ZF} = \frac{\alpha\pi(1-\alpha)}{\sin(\alpha\pi)} \tag{17}$$

The first term on the right-hand side of Equation (16) reflects the volume of surface water, while the second term reflects the volume of infiltrated water. At short times, the first term is important, while at the long times the first term becomes negligible, and the second term dominates.

Valiantzas [17] reported that the maximum error in the estimation of time t_L generally does not exceed ±7%, while in exceptional cases where the advance time is less than 30 min, the maximum error can exceed 10%.

2.4. Experimental Data

For the evaluation and comparison of the three methods, experimental data (data sets) from 15 different furrow irrigation experiments known from the international literature were used. More specifically, we used five series of experimental data derived from Walker and Busman [27], nine series from Wilson and Elliot [28] and one series from Camacho et al. [29]. The selected tests covered a wide range of soil infiltration parameters, inflow rates, furrow section shape parameters, field slopes and roughness coefficients. These data are presented in detail in Table 1. In all cases of experimental data, the authors present the parameters of k, α and f_0 of the extended Lewis–Kostiakov infiltration function:

$$i = kt^{\alpha} + f_0 t \tag{18}$$

where f_0 (L^2/T) is the steady infiltration rate.

The k and α parameters of the Lewis–Kostiakov formula, which are used in the three methodologies, were estimated using the Solver tool in Excel (Table 2). Excel Solver minimizes the objective function between the measured (extended Lewis–Kostiakov infiltration function) and predicted cumulative infiltration values (Lewis–Kostiakov infiltration function) at given times and then predicts the k and α parameters of the Lewis–Kostiakov formula [30]. As the initial values of k and α parameters, we consider the values of the k and α parameters from the extended Lewis–Kostiakov infiltration function.

To calculate the sorptivity S and the parameter A of Equation (3) to be introduced as input parameters in Equation (4), first a check was conducted to see if the experimental infiltration data (extended Lewis–Kostiakov infiltration function) of all tests can be described by the Philip [12] infiltration equation. With a suitable transformation of the experimental data of each case, it was examined whether the relationship $i/t^{0.5}(t^{0.5})$ is linear. In this linear relationship, the slope of the line is equal to A and the constant term is equal to S [31]. From this analysis, it appeared that only in six cases the above relationship was linear. Then, for these six cases, the values of S and A were calculated using the Solver tool. As the initial values of S and A, we considered the values $S = i_1/t_1^{0.5}$ and $A = (i_n - i_{n-1})/(t_n - t_{n-1})$, where n is the last value of the data. It should be noted that the values of S and A were almost the same with both the linearization method and the Excel Solver procedure.

Table 1. Furrow data.

Data Series		Walker and Busman [27]					Camacho et al. [29]
		Flowell Wheel	**Flowell Non-Wheel**	**Kimberly Wheel**	**Kimberly Non-Wheel**	**Greeley Wheel**	**Cordoba**
Inflow rate	q_0 (m^3/min)	0.12	0.12	0.09	0.048	0.114	0.09
Furrow slope	S_0(m/m)	0.008	0.008	0.0104	0.0104	0.008	0.003
Manning roughness coefficient	n	0.04	0.04	0.04	0.04	0.04	0.04
Furrow shape parameters	ρ_1	0.3269	0.3269	0.6644	0.6644	0.369	0.39
	ρ_2	2.734	2.734	2.8787	2.8787	2.81	2.797
Furrow length	L (m) = x_2	360	274	360	112	411	200
Advance time	t_L (min) = t_2	400	432	208	560	63	51.5
Advance distance and corresponding time	x_1 (m)	180	140	160	60	205.5	100
	t_1 (min)	41	40	48	120	26	20.25
Surface profile shape factor	σ_y	0.77	0.77	0.77	0.77	0.77	0.77
Extended Lewis–Kostiakov parameters	α	0.534	0.673	0.212	0.533	0.45	0.4550
	k (m^2/min^{α})	0.0028	0.0022	0.0088	0.007	0.0021	0.0033
	f_0 (m^2/min)	0.00022	0.00022	0.00017	0.00017	0.0000	0.0000

Data Series		Wilson and Elliot [28]								
		Benson B-1	**Benson B-2**	**Benson B-3**	**Matchett M-1**	**Matchett M-2**	**Matchett M-3**	**Printz P-1**	**Printz P-2**	**Printz P-3**
Inflow rate	q_0 (m^3/min)	0.1668	0.0684	0.0702	0.051	0.0552	0.0264	0.2886	0.2094	0.1662
Furrow slope	S_0	0.0044	0.0044	0.0044	0.0092	0.0095	0.0095	0.0023	0.0025	0.0025
Manning roughness coefficient	n	0.03	0.02	0.02	0.03	0.02	0.02	0.03	0.02	0.02
Furrow shape parameters	ρ_1	0.46	0.58	0.34	0.3	1.35	2.12	0.92	0.615	0.73
	ρ_2	2.86	2.91	2.84	2.73	3	3.15	2.91	2.924	2.98
Furrow length	L (m) = x_2	500	500	500	400	400	400	200	300	300
Advance time	t_L (min) = t_2	175	344.5	247	124.3	232.2	213	178	45.5	73
Advance distance and corresponding time	x_1 (m)	300	300	300	200	200	200	100	100	200
	t_1 (min)	84.5	159	123.5	38	70.5	88.2	13.5	15	43
Surface profile shape factor	σ_y	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77
Extended Lewis–Kostiakov parameters	α	0.02	0.02	0.01	0.48	0.4	0.16	0.4	0.02	0.02
	k (m^2/min^{α})	0.0252	0.018	0.0173	0.0011	0.0033	0.0039	0.0078	0.013	0.0161
	f_0 (m^2/min)	0.00023	0.0001	0.00008	0.00003	0.00003	0.00002	0.00141	0.00049	0.0004

Table 2. Parameters of Lewis–Kostiakov equation (α and k) and Philip [12] equation (S and A) calculated with Solver tool.

Parameters	Walker and Busman [27]					Camacho et al. [29]
	Flowell Wheel	Flowell Non-Wheel	Kimberly Wheel	Kimberly Non-Wheel	Greeley Wheel	Cordoba
α	0.7534	0.7916	0.5143	0.6458	0.4500	0.4550
k (m^2/min^{α})	0.0017	0.0018	0.0039	0.0050	0.0021	0.0033
S ($m^2/min^{0.5}$)	0.003105	0.003547	0.003916	0.007826	-	-
A (m^2/min)	0.000237	0.000355	0.000022	0.000205	-	-

Parameters	Wilson and Elliot [28]								
	Benson B-1	Benson B-2	Benson B-3	Matchett M-1	Matchett M-2	Matchett M-3	Printz P-1	Printz P-2	Printz P-3
α	0.3533	0.3952	0.2767	0.5720	0.4731	0.2966	0.758	0.2929	0.3894
k (m^2/min^{α})	0.0103	0.0051	0.0077	0.0009	0.0027	0.0027	0.00576	0.0100	0.0083
S ($m^2/min^{0.5}$)	-	-	-	0.5720	-	-	0.006103	-	-
A (m^2/min)	-	-	-	0.000025	-	-	0.001296	-	-

3. Results and Discussion

Table 2 presents the values of the parameters k and α for all cases of furrows, which were calculated using Solver and are used to calculate the t_L values in Equations (7) and (16), as well as in the iterative procedure. Also, Table 2 presents the values of the parameters S and A for six cases of data sets (i.e., Flowell wheel, Flowell non-wheel, Kimberly wheel, Kimberly non-wheel, Matchett M-1 and Printz P-1). In these data sets, the relationship $i/t^{0.5}(t^{0.5})$ is strongly linear, and the calculated values of S and A from Solver are positive. In the remaining cases, the relationship was not linear while the Solver calculated values A = 0, which have no physical meaning. Thus, the remaining cases were not studied further by applying Equation (4).

Table 3 presents the calculated values of coefficient r, which were obtained from the experimental data of the advance phase presented by Wilson and Elliot [28], Walker and Busman [27] and Camacho et al. [29] at two points of the furrows by applying Equation (11). Usually, the two experimental advance points are at the middle (L/2) and the end (L) of a furrow [32]. Thus, applying Equation (11) for two points, the following equations are obtained:

$$L = pt_L^r \tag{19}$$

$$\frac{L}{2} = pt_{\frac{L}{2}}^r \tag{20}$$

If Equations (19) and (20) are divided by terms and the logarithm of the resulting new equation is calculated, then the value of the parameter r is obtained as follows:

$$r = \frac{\ln(2)}{\ln\left(\frac{t_L}{t_{\frac{L}{2}}}\right)} \tag{21}$$

Additionally, Table 3 presents the values of the coefficient r calculated from the Newton–Raphson iterative procedure.

From the results presented in Table 3, it can be seen that the values of the coefficient r calculated by the iterative procedure converge with the values calculated from the experimental data using Equation (11). More specifically, the difference between the values of r does not exceed 7.5%, with the exception of the data from the Printz P-2 furrow, where the difference is 25.5%.

Table 3. Calculated values of coefficient r from Equation (11) and the Newton–Raphson iterative procedure [3].

Experimental Furrows	Coefficient r	
	Equation (11)	**Newton–Raphson Iterative Procedure**
Flowell wheel	0.304	0.354
Flowell non-wheel	0.282	0.297
Kimberly wheel	0.553	0.574
Kimberly non-wheel	0.405	0.382
Greeley Wheel	0.783	0.763
Cordoba	0.743	0.734
Benson B-1	0.702	0.701
Benson B-2	0.661	0.665
Benson B-3	0.737	0.764
Matchett M-1	0.585	0.606
Matchett M-2	0.582	0.582
Matchett M-3	0.786	0.749
Printz P-1	0.269	0.309
Printz P-2	0.990	0.735
Printz P-3	0.766	0.691

Table 4 shows the predicted values of t_L from the three methods studied, as well as the measured values for all experimental furrows. The absolute values of relative errors (|RE|%) of the predicted values of t_L with respect to the measured ones are presented in Table 5.

From Tables 4 and 5, it can be seen that both the Valiantzas [17] method and the Newton–Raphson iterative procedure [3] satisfactorily approximate the measured values of t_L in most cases. In more than half of the experimental furrows, the relative error is less than 10%, indicating that both methods predict the time t_L fairly accurately. In general, the two methods converge on the t_L value, and thus their deviation from the measured t_L is approximately the same. The biggest differences in the two methods compared with the experimental data are observed in the Matchett data sets, where in all three cases (M1, M2 and M3) the RE values range from 11.96% to 25.81%. These deviations may be due to experimental errors related to inflow rate, roughness or furrow shape parameters. Possible problems with experimental measurements in these furrows were also reported by Valiantzas et al. [7].

Regarding the application of the Philip [12] method using the Lewis–Kostiakov infiltration equation in the data sets presented by Wilson and Elliot [28], Camacho et al. [29] and Walker and Busman [27] (Greeley Wheel data set), high RE values are shown, ranging from 18.19% to 59.24%, while in the remaining four data sets presented by Walker and Busman [27], the RE values are much smaller and closer to the values of the Valiantzas [17] method and the Newton–Raphson iterative procedure. In general, the method, in most cases, shows an underestimation of the time t_L. To explain these findings, it was investigated whether the amount of surface water is negligible. For this purpose, the index V_s, which is equal to the ratio of the amount of surface water at the end of the advance phase to the total amount of water applied, was estimated in the 15 data sets.

$$V_s = \frac{\sigma_y A_0 x}{q_0 t_L} \tag{22}$$

The calculated V_s values showed that the surface water was 5.1%, 3.6%, 9.7% and 1.4% of the total application water for the Flowell wheel, Flowell non-wheel, Kimberly wheel and Kimberly non-wheel furrows, respectively. That is, the surface water amounts are very small, with the exception of the Kimberly wheel case (9.7%) where the corresponding RE value is 16.55%. In the rest of the data sets, the values of the V_s index are quite large and range from 7.1% to 40.9%. The highest RE values are observed in the cases of Greeley wheel and Cordoba, 59.24% and 51.59%, respectively, where the corresponding V_s index values are 0.409 and 0.355. It may be assumed that the same causes play a role in the failure of the Philip and Farrell [8] method in combination with the Philip [12] infiltration equation.

Table 4. Measured and predicted values of advance time t_L from the (a) Valiantzas method [17], (b) Newton–Raphson iterative procedure [3] and (c) Philip and Farrell [8] method using the Lewis–Kostiakov and Philip [12] infiltration equations in all experimental furrows studied.

Experimental Furrows	Advance Time t_L (min)				
	Measured Values	Valiantzas Method	Newton–Raphson Iterative Procedure	Philip and Farrell Method	
				Lewis–Kostiakov Equation	Philip Equation
Flowell wheel	400	386.70	340.53	357.10	397.80
Flowell non-wheel	432	479.51	404.48	456.86	616.97
Kimberly wheel	208	201.87	198.13	173.57	175.31
Kimberly non-wheel	560	558.46	534.69	547.60	577.78
Greeley Wheel	63	61.47	61.04	25.67	-
Cordoba	51.5	50.33	50.40	24.93	-
Benson B-1	175	181.86	180.44	143.07	-
Benson B-2	344.5	310.00	305.30	271.13	-
Benson B-3	247	235.54	231.84	195.37	-
Matchett M-1	124.3	92.22	94.11	60.41	59.44
Matchett M-2	232.2	204.44	202.08	181.33	-
Matchett M-3	213	176.85	174.30	143.13	-
Printz P-1	178	160.31	156.95	145.62	533.71
Printz P-2	45.5	52.22	51.25	32.53	-
Printz P-3	73	79.35	79.08	57.29	-

Table 5. Absolute values of relative error (|RE|) between the measured and predicted values of advance time t_L from the (a) Valiantzas method [17], (b) Newton-Raphson iterative procedure [3] and (c) Philip and Farrell [8] method using the Lewis–Kostiakov and Philip [12] infiltration equations in all experimental furrows studied.

Experimental Furrows	|RE| (%)			
	Valiantzas Method	Newton–Raphson Iterative Procedure	Philip and Farrell Method	
			Lewis–Kostiakov Equation	Philip Equation
Flowell wheel	3.33	14.87	10.72	0.55
Flowell non-wheel	11.00	6.37	5.76	42.82
Kimberly wheel	2.95	4.74	16.55	15.72
Kimberly non-wheel	0.27	4.52	2.21	3.17
Greeley wheel	2.42	3.11	59.24	-
Cordoba	2.27	2.13	51.59	-
Benson B-1	3.92	3.11	18.25	-
Benson B-2	10.01	11.38	21.30	-
Benson B-3	4.64	6.14	20.90	-
Matchett M-1	25.81	24.29	51.39	30.83
Matchett M-2	11.96	12.97	21.91	-
Matchett M-3	16.97	18.17	32.80	-
Printz P-1	9.94	11.83	18.19	199.84
Printz P-2	14.76	12.63	28.50	-
Printz P-3	8.70	8.33	21.51	-

Figure 2 shows the relationships between the experimental values of time t_L ($t_{L,EXP}$) and the predicted ones ($t_{L,PRED}$) for the three methods. As can be seen, these relationships are linear with a very high value of determination coefficient (R^2 > 0.981). The Valiantzas [17] method gave the best results, since the slope of the linear relationship had the higher value (0.98), i.e., it is closer to the 1:1 line. In addition, it should be mentioned that this method is easy and simple to use, since the time t_L is directly calculated from the furrow length L.

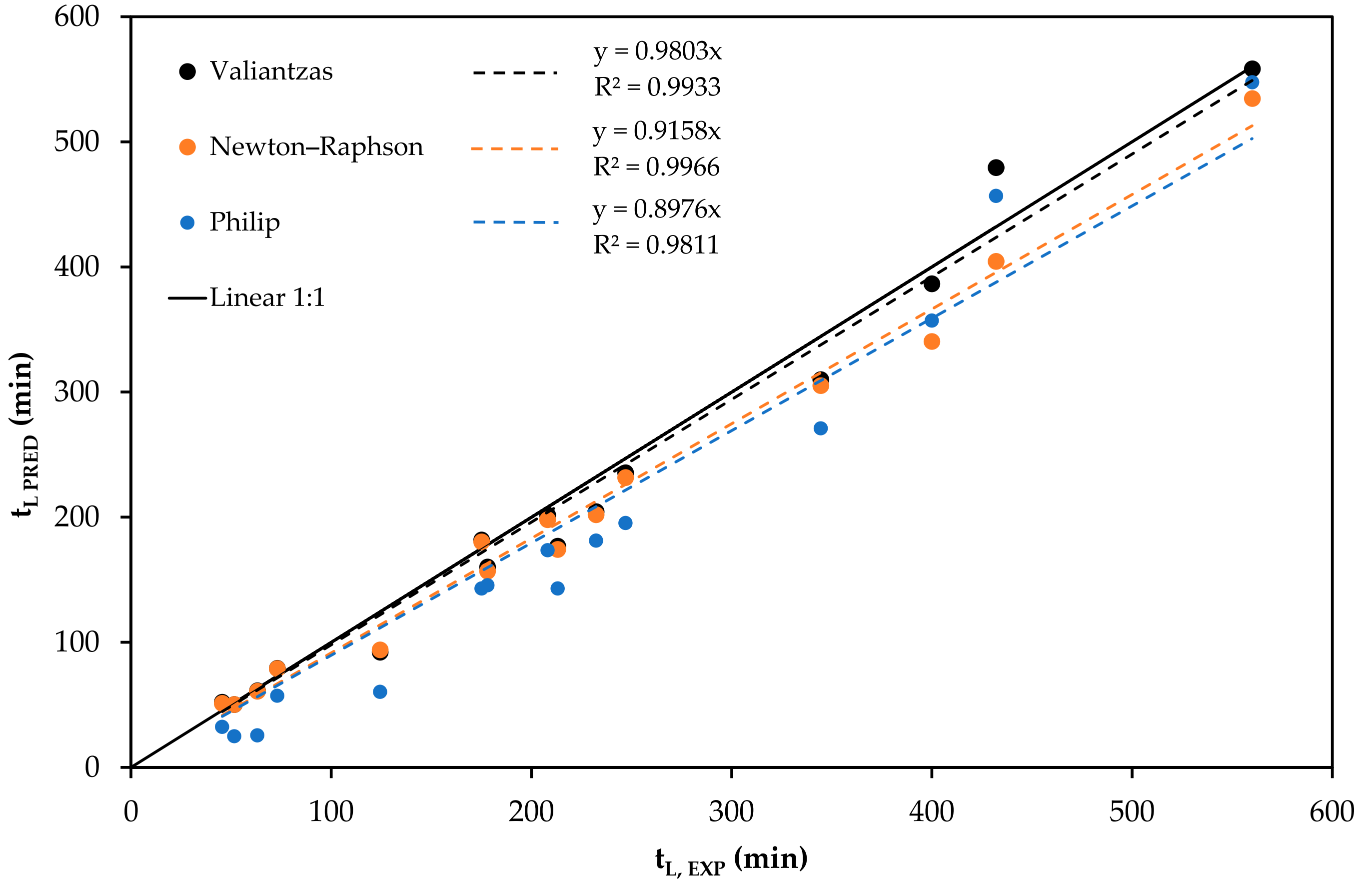

Figure 2. Comparative presentation between the measured values of advance time, $t_{L,EXP}$, and the predicted ones, $t_{L,PRED}$, obtained from the (a) Valiantzas method [17], (b) Newton–Raphson iterative procedure [3] and (c) Philip and Farrell [8] method using the Lewis–Kostiakov infiltration equation in all experimental furrows studied.

To obtain safer conclusions for the prediction of t_L, RMSE (Root Mean Square Error) values for the three methods were also calculated. The RMSE values for the Valiantzas [17] and Newton–Raphson iterative procedure [3] methods were 22.14 min and 26.66 min, respectively, while for the Philip and Farrell [8] method in combination with the Lewis–Kostiakov equation the value was 43.19 min, which is almost twice the value of the other two methods.

4. Conclusions

To achieve maximum irrigation uniformity and efficiency in the design of surface irrigation systems and especially furrow irrigation, the reliable prediction of advance time t_L is important. Especially currently, where water saving is a vital issue and the rational design of irrigation systems is required, the reliable and fast prediction of t_L under various irrigation scenarios can help to optimize irrigation design.

In this context, a comparison of the three methods (Valiantzas [17] method, Newton–Raphson iterative procedure [3] and Philip and Farrell ([8D] method) was conducted to predict the completion time of the advance phase, t_L, using experimental data of 15 different furrows from the international literature.

Among the three methods, the Valiantzas method was simpler and easier to use and showed greater prediction accuracy for experimental advance times from 45.5 to 560 min and for a wide range of values of the Lewis–Kostiakov equation parameters, i.e., under different soil types.

A similar performance was observed from the application of the volume balance equation in combination with the Newton–Raphson iterative procedure. Thus, this simple procedure can contribute to the design and evaluation of furrow irrigation systems. In most cases, the value of relative error for the Valiantzas method and the Newton–Raphson iterative procedure was less than 10%.

The Philip and Farrell method using the Kostiakov infiltration equation underestimated the time t_L and failed especially in the case where the volume of surface water is not negligible. This is expected since the corresponding equation is obtained by considering the amount of surface water as negligible. These results can be very beneficial for irrigation system designers to study the irrigation performance of systems that are already working and to propose optimal solutions for each area.

Author Contributions: Conceptualization, G.K.; methodology, G.K., D.K., P.A.L. and L.M.; software, D.K. and L.M.; investigation, G.K., D.K., P.A.L. and L.M.; data curation, G.K., D.K., P.A.L. and L.M.; writing—review and editing, G.K., D.K., P.A.L. and L.M.; supervision, G.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Li, Y.; Bai, M.; Zhang, S.; Wu, C.; Li, F. Development In Improved Surface Irrigation in China. *Irrig. Drain.* **2020**, *69*, 48–60. [CrossRef]
2. Or, D.; Silva, H.R. Prediction of Surface Irrigation Advance Trajectories Using Soil Intake Properties. *Irrig. Sci.* **1996**, *16*, 159–167. [CrossRef]
3. Walker, W.R.; Skogerboe, G.V. *Surface Irrigation: Theory and Practice*; Prentice-Hall, Inc.: Englewood Cliffs, NJ, USA, 1987.
4. Bautista, E.; Schlegel, J.L. *WinSRFR 5.1—Software and User Manual*; USDA-ARS Arid Land Agricultural Research Center: Maricopa, AZ, USA, 2019.
5. Strelkoff, T.; Katopodes, N.D. Border-irrigation hydraulics with zero inertia. *J. Irrig. Drain. Div.* **1977**, *103*, 325–342. [CrossRef]
6. Elliott, R.L.; Walker, W.R.; Skogerboe, G.V. Zero-inertia modeling of furrow irrigation advance. *J. Irrig. Drain. Div.* **1982**, *108*, 179–195. [CrossRef]
7. Valiantzas, J.D.; Aggelides, S.; Sassalou, A. Furrow infiltration estimation from time to a single advance point. *Agric. Water Manag.* **2001**, *52*, 17–32. [CrossRef]
8. Philip, J.R.; Farrell, D.A. General solution of the infiltration-advance problem in irrigation hydraulics. *J. Geophys. Res.* **1964**, *69*, 621–631. [CrossRef]
9. Lewis, M.R.; Milne, W.E. Analysis of border irrigation. *Agric. Eng.* **1938**, *19*, 267–272.
10. Kostiakov, A.N. On the dynamics of the coefficients of water percolation in soils and on the necessity of studying it from a dynamic point of view for purpose of amelioration. *Trans. 6th Commun. Int. Soc. Soil Sci. Russ. Part A* **1932**, 17–21. Available online: https://cir.nii.ac.jp/crid/1520853835465008512 (accessed on 27 February 2024).
11. Lewis, M.R. The rate of infiltration of water in irrigation practice. *Trans. Am. Geophys. Union* **1937**, *18*, 361–368. [CrossRef]
12. Philip, J.R. The theory of infiltration: 4. Sorptivity and algebraic infiltration equations. *Soil Sci.* **1957**, *84*, 257–264. [CrossRef]
13. Horton, R.E. An approach towards a physical interpretation of infiltration capacity. *Soil Sci. Soc. Am. J.* **1941**, *5*, 399–417. [CrossRef]
14. Cook, F.J.; Knight, J.H.; Doble, R.C.; Raine, S.R. An improved solution for the infiltration advance problem in irrigation hydraulics. *Irrig. Sci.* **2013**, *31*, 1113–1123. [CrossRef]
15. United States Department of Agriculture (USDA). *Furrow Irrigation*; Chapter 5, Section 15, Soil Conservation Service; USDA: Washington, DC, USA, 1983.
16. Valiantzas, J.D. Explicit time of advance formula for furrow design. *J. Irrig. Drain. Eng.* **1999**, *125*, 19–25. [CrossRef]
17. Valiantzas, J.D. Optimal furrow design. I: Time of advance equation. *J. Irrig. Drain. Eng.* **2001**, *127*, 201–208. [CrossRef]
18. Philip, J.R. Theory of infiltration. *Adv. Hydrosci.* **1969**, *5*, 215–296.
19. Parlange, J.-Y. Note on infiltration advance front from border irrigation. *Water Resour. Res.* **1973**, *9*, 1075–1078. [CrossRef]
20. Mailapalli, D.R.; Wallender, W.; Raghuwanshi, N.; Singh, R. Quick method for estimating furrow infiltration. *J. Irrig. Drain. Eng.* **2008**, *134*, 788–795. [CrossRef]
21. Rabbaniha, H.; Ebrahimian, H. Developing a simple method for estimating soil infiltration in furrow and border irrigation using advance, recession and runoff data. *Irrig. Drain.* **2024**, *73*, 50–63. [CrossRef]

22. Elliott, R.L.; Walker, W.R. Field evaluation of furrow infiltration and advance functions. *Trans. ASAE* **1982**, *25*, 0396–0400. [CrossRef]
23. Ebrahimian, H.; Ghaffari, P.; Ghameshlou, A.N.; Tabatabaei, S.-H.; Dizaj, A.A. Extensive comparison of various infiltration estimation methods for furrow irrigation under different field conditions. *Agric. Water Manag.* **2020**, *230*, 105960. [CrossRef]
24. Bautista, E.; Clemmens, A.J.; Strelkoff, T.S. Structured application of the two-point method for the estimation of infiltration parameters in surface irrigation. *J. Irrig. Drain. Eng.* **2009**, *135*, 566–578. [CrossRef]
25. Kiefer, F.W., Jr. *Average Depth of Absorbed Water in Surface Irrigation*; Special Publication; Department of Civil Engineering, Utah State University: Logan, UT, USA, 1965.
26. Hart, W.E.; Bassett, D.L.; Strelkoff, T. Surface irrigation hydraulics-kinematics. *J. Irrig. Drain. Div.* **1968**, *94*, 419–440. [CrossRef]
27. Walker, W.R.; Busman, J.D. Real time estimation of furrow infiltration. *J. Irrig. Drain. Eng.* **1990**, *116*, 299–318. [CrossRef]
28. Wilson, B.N.; Elliott, R.L. Furrow advance using simple routing models. *J. Irrig. Drain. Eng.* **1988**, *114*, 104–117. [CrossRef]
29. Camacho, E.; Perez-Lucena, C.; Roldan-Canas, J.; Alcaide, M. IPE: Model for management and control of furrow irrigation in real time. *J. Irrig. Drain. Eng.* **1997**, *123*, 264–269. [CrossRef]
30. Kargas, G.; Koka, D.; Londra, P.A. Determination of Soil Hydraulic Properties from Infiltration Data Using Various Methods. *Land* **2022**, *11*, 779. [CrossRef]
31. Smiles, D.E.; Knight, J.H. A note on the use of the Philip infiltration equation. *Aust. J. Soil Res.* **1976**, *14*, 103–108. [CrossRef]
32. Panahi, A.; Seyedzadeh, A.; Maroufpoor, E. Investigating the midpoint of a two-point method for predicting advance and infiltration in surface irrigation. *Irrig. Drain.* **2021**, *70*, 1095–1106. [CrossRef]