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Modelling of a Lake Outburst as a Result of the Development of Piping

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Abstract: The retreat of mountain glaciers inevitably leads to an increase in the number of outburst moraine lakes. One of the possible mechanisms of moraine dam outburst along with overflow over the crest is the formation of a filtration channel in the body of the moraine dam (piping). An algorithm for calculating the outburst flood hydrograph, describing the development of a filtration channel in the body of a moraine dam and the subsequent formation of water overflow when the soil above the channel collapses, is proposed in this paper. Verification of proposed methodology was carried out on the basis of experimental data and published data of real outbursts. Satisfactory results verifying this methodology made it possible to use the proposed methodology for the calculation of the hydrograph of the outburst of Lake Bashkara in the Elbrus region, which occurred on 1 September 2017. It is shown that the simulation results are quantitatively comparable with the estimates obtained from field data: the time of water discharge through the channel was 16 min, the period of the outburst wave passage was 40 min, and the maximum discharge was 636 m^3/s. Thus, the possibility of applying the proposed methodology for calculating the destruction of natural moraine dams has been demonstrated.

Keywords: mathematical modelling; moraine dam outbursts; dangerous hydrological phenomena; filtration; piping

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1. Introduction

The retreat of mountain glaciers, currently occurring as a result of global climate change, leads to an increase in the area of mountain lakes and an increased risk of outburst of moraine dams [1,2]. Such processes have been observed in the Himalayas, the Altai, the Pamir, Tibet, the Cordillera Blanca region, Spitsbergen, and Iceland [1,3–7]. One of the possible mechanisms for the outburst of moraine dams, along with overflowing over the crest, is the formation of a filtration channel (piping). In some cases, this phenomenon serves as an additional trigger for dam outburst [2]. A similar process can develop in the case of artificial dams. It is believed that about 33% of cases of the destruction of artificial dams in the world occurred as a result of water filtration [8], including due to the development of internal erosion with the formation of filtration channels (piping). An example of this is the largest disaster—the accident at the Teton dam (Idaho, USA), with an outburst flood discharge of more than 57,000 m^3/s. The main reason for the outburst was the formation of an internal channel, the expansion of which in the permeable loess soil of the dam body caused its collapse. Natural dams (moraine and landslide dams) are susceptible to internal erosion largely due to the heterogeneity and decomposition of the soil combined with the lack of seepage protection. If the flow from a moraine lake is carried out by filtration or through underground channels, then the probability of its outburst increases [9–11]. Examples include the outburst of a moraine lake on the river Boku (China) in 1981, triggered by the formation of a channel which led to the sudden destruction of the dam and the formation of a catastrophic mudflow [12], and the destruction in 1988 of the terminal moraine dam on Lake Guangxieco (China) as a result of the influence of two mechanisms: overflow and the formation of a filtration channel [13].

In addition to moraine dams, the development of internal erosion with the formation of a filtration channel is also possible in landslide dams. There are two known cases of the destruction of such dams—the outburst of the Cerro Condor Seneca dam (the Western Cordillera, Argentina) and Lake Yaskinkul, in the Isfairamsay River basin (Kyrgyzstan). After the channel was formed, the upper arch of the dam collapsed, and water outflowed through the breach [14]. On the territory of the Altai Mountains, an example of an outburst of a landslide lake with underground drainage channels is the outburst of Lake Maashey, which occurred on 15 July 2012 [9]. Prolonged intense precipitation led to severe watering of the dam, which could lead to increased filtration of water through its body, the subsequent erosion of the dam, and an outburst of the lake.

Thus, the formation of a filtration channel may cause the destruction of dams, after which water would overflow. This process requires a close study, both based on field data and modelling results. However, there are few publications with the results of such studies. Let us note some works that present the results of a study of the formation of a filtration channel based on physical modelling (experiments on channel expansion in the body of small soil dams) [15,16] and mathematical modelling (based on the calculation of the erosion rate) [17,18]. The mentioned research only published the results of physical experiments describing the outburst process. In these works, the results of physical experiments are not compared with the results of calculations using mathematical models. Sometimes physical experiments are used to determine the dependence of the breach width on time [19]. In this article, physical experiments were performed not only to describe the outburst process in detail but also to obtain the observed outburst hydrograph and the necessary input data for mathematical modelling using the proposed calculation methodology. In this regard, the purpose of this work was to adapt existing algorithms for calculating the formation of a filtration channel in the dam body and verify them.

2. Materials and Methods

2.1. Study Object

Lake Bashkara was chosen as an object for testing the proposed methodology. Lake Bashkara is a periglacial lake located near the valley glacier Bashkara (valley of the Adyl-Su River, Elbrus region, Kabardino-Balkarian Republic, Russia) (Figure 1). According to [20], the lake was formed in the late 1930s–early 1940s. The lake is characterized by dynamism and instability, as lake outbursts occurred repeatedly: in August 1958 and 1959, in October 1960, and in September 2017.

The outburst of the lake system located near the Bashkara glacier is one of the most famous glaciological disasters in Russia in the 21st century, along with the Karmadon disaster in 2002 and the Tyrnauz tragedy in 2000. Quite a lot of scientific work has been devoted to this event, including efforts presenting different outburst scenarios and the results of the event's mathematical modelling [3,20–25].

2.2. Methods

This methodology is based on the following ideas about channel expansion in the body of a soil dam: the dam already has an initial small channel formed as a result of reverse erosion or enhanced filtration. Water passing through the channel creates shear stress. If the shear stress value is greater than the critical value, then the channel expands; if it is less, then erosion does not occur. Next, the erosion rate is calculated. The resulting increment is added to the channel diameter (Figure 2). The process of erosion of the filtration channel stops when the diameter of the channel reaches 1/5 of the dam height, since the formed arch cannot withstand the upper arch of the soil [26]. Next, water overflows through the resulting breach. In some cases, when the soil is waterlogged and has a high specific gravity, the collapse process does not occur. Water moves through the channel until the reservoir is completely empty [27].

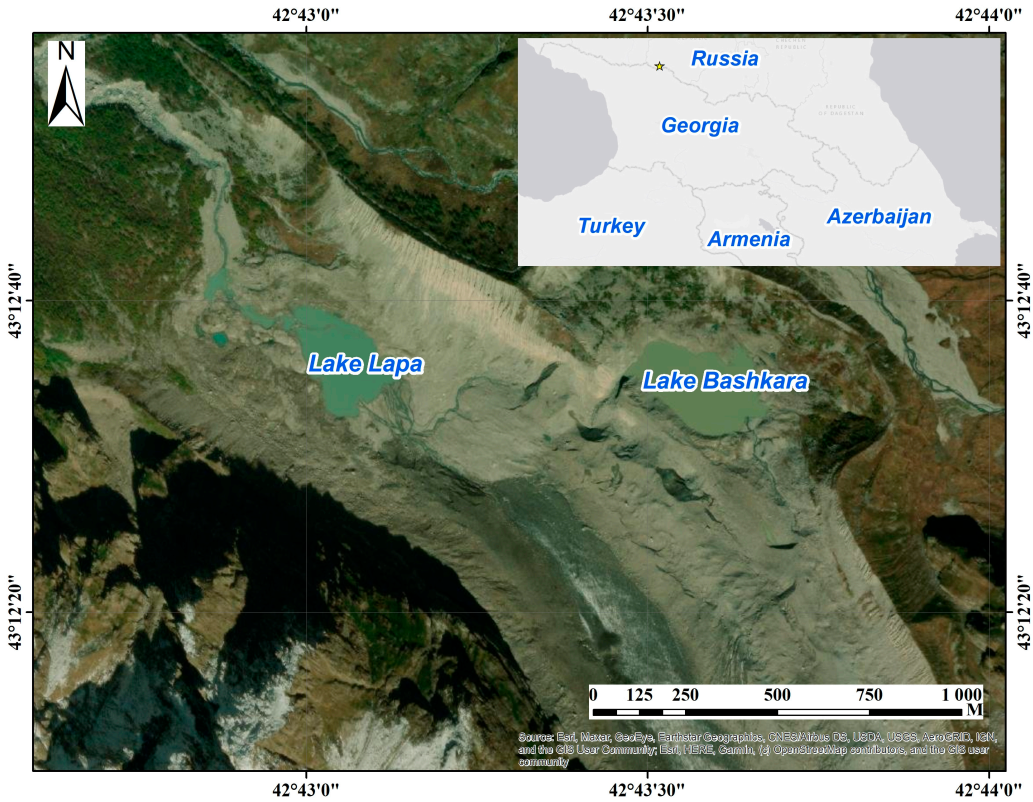

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Figure 1. Scheme of Lake Bashkara's location. The star marks the location of the lake.

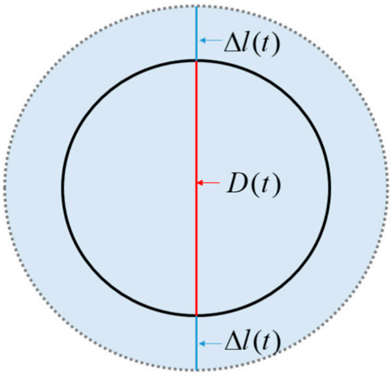

Figure 2. Scheme of the cross-section of the channel.

The change in the water volume in the reservoir $\Delta V(t)$ over time Δt will be

$$\Delta V = Q\Delta t$$

or, moving to infinitesimal quantities,

$$\frac{\partial V}{\partial t} = Q(t) \tag{1}$$

In this case, the discharge Q in the filtration channel is calculated using Formula (2), which is often used in similar models [2,28]:

$$Q(t) = \omega(t)\sqrt{\frac{2gH(t)}{h_f(t)}}, \tag{2}$$

where g is the acceleration of gravity (9.81 $\mathrm{m/s^2}$); $\omega(t)$ is the cross-sectional area of the channel, which is calculated as $\frac{\pi D(t)^2}{4}$, where $D(t)$ is the channel diameter (m); water pressure $H(t)$ indicates the difference between the water elevation in the flow (m), $z(t)$, and the expected elevation of the channel centre z_{pip}(m); and h_f is the pressure loss along the length of the channel, which is calculated as $h_f = \sqrt{1 + \frac{fL}{4R}}$, where L is the channel length (m), R is the hydraulic channel radius (m), and f is a friction-dependent parameter, which is calculated as $f = 0.2162 \times \left(\frac{D_{50}}{D(t)}\right)^{1/6}$, with D_{50} being the average diameter of the soil particles (m) [2,29].

The increment in channel diameter $\Delta l(t)$ (Figure 2) is determined by the erosion rate *E(t)*. To calculate the erosion of the moraine material by water flow, we used Equation (3), widely used in foreign mathematical models [16,30,31]:

$$E(t) = k_{er}[\tau(t) - \tau_c] \tag{3}$$

Water moving through the channel creates some shear stress τ, $\mathrm{N/m^2}$. When the critical value τ_c (Equation (7)) is exceeded, the process of erosion begins [29,30]. Since the time step is 1 s, the channel increment $\Delta l(t)$ is equal to the erosion rate $E(t)$, as follows:

$$\Delta l(t) = E(t) = k_{er}[\tau(t) - \tau_c]\Delta t \tag{4}$$

Here, k_{er} is the erosion coefficient, which is determined using the following formula, according to [30]:

$$k_{er} = \frac{10 \times \rho_w}{\rho_s} \times exp\left\{-0.121 \times \aleph^{0.406} \times \left(\frac{\rho_s}{\rho_w}\right)^{3.1}\right\}, \tag{5}$$

where ρ_s is the density of the dam material, ρ_w is the density of water, and $\aleph$ is the proportion of clay content in the dam material. The shear stress $\tau(t)$ initiated by the water flow is calculated as follows [31]:

$$\tau(t) = \rho_w g R(t) S(t), \tag{6}$$

where R is the hydraulic radius; $S(t)$ is the energy slope, with expression$S(t) = \overline{v}^2 n^2 [R(t)]^{-4/3}$, in which $\overline{v}$ is the average velocity of the water flow; and n is the Stickler coefficient, depending on the size of the soil particles, characterized by values in the range from 0.01 to 0.05 and determined by $n = \left(\frac{0.15}{\sqrt{g}}\right)k^{1/6}$, where k is the diameter of the soil particles.

To determine the critical shear stress, we used the formula proposed by [32]:

$$\tau_c = 6.8(\xi)^{1.68}\aleph^{-1.73}\varsigma^{-0.97}, \tag{7}$$

where ξ is the soil plasticity index (%) (a characteristic reflecting the soil's ability to hold water—for sandy loam, it varies within 1–7%; for loam, 7–17%; and for clay, more than 17%), and ς is the porosity of the soil.

As soon as the channel diameter reaches a critical value equal to 1/5 of the water height, the material located above the channel will collapse and be carried away by the water flow, after which the calculation will be carried out as for an overflow.

While calculating the water overflow, it was accepted as an assumption that the water flow through the breach was close in parameters to the water flow through the broad-crested weir (Figure 3), as is most often found in mathematical models for calculating outburst floods formed in soil artificial dams and during the destruction of moraine dams [32].

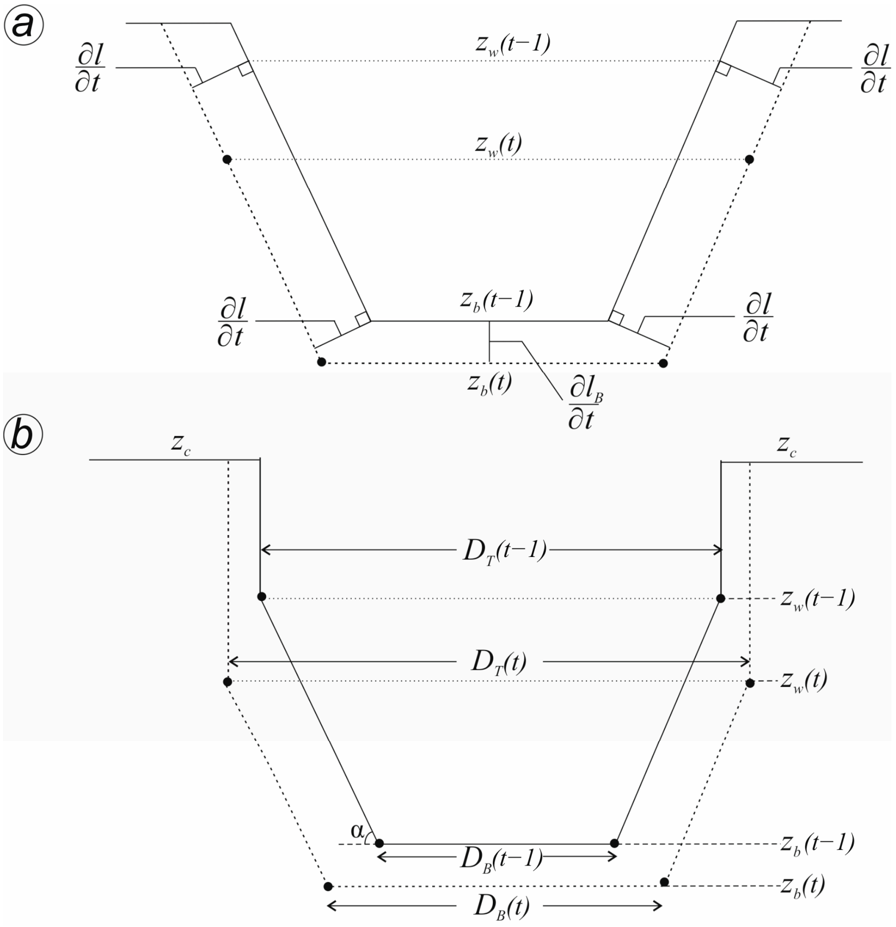

Figure 3. Scheme for calculating the cross-sectional area of flow (**a**) and breach area (**b**). *Designations*: $D_T(t)$ and $D_B(t)$ are the breach width in the upper and lower parts, respectively; $\frac{\partial l}{\partial t}$ is the denudation rate corresponding to the average flow velocity; $\frac{\partial l_B}{\partial t}$ is the denudation rate corresponding to the bottom flow velocity; $z_W(t)$ and $z_B(t)$ are the water surface elevation and the breach bottom elevation; and $z_C(t)$ is the dam crest elevation.

Based on this, to calculate the water discharge through the breach, Equation (8) was used:

$$Q_B = M\frac{D_T(t) + D_B(t)}{2}[z_w(t) - z_B(t)]^{3/2}, \tag{8}$$

where $z_W(t)$ is a function which describes the dependence of the water level elevation of a reservoir on its volume $\mathbb{F}(V(t))$, and $M \equiv \mu\sqrt{2g}$, where μ is the flow coefficient, depending on the type of weir and its operating conditions, varying over a wide range ($\mu = 0.3 \div 0.6$).

The increment in the linear dimensions of the breach, which are caused by bottom erosion $\Delta l_B(t)$, is less than that of its side parts $\Delta l(t)$, since the bottom velocity is lower than the average velocity value, which is used to calculate $\Delta l(t)$. To calculate the cross-sectional area of the flow $\omega(t)$ at an arbitrary moment of time t, $t > t_0$, its shape is approximated by a trapezoid, as the one most often encountered when describing the shapes of breaches (Figure 3a).

However, in the process of deepening the breach, the side walls are undermined, and their subsequent collapse occurs, as a result of which the breach cross-section takes on a more complex shape (Figure 3b). In practice, this means that the profile of the breach within the wetted perimeter remains trapezoidal and rectangular at the top.

The area of the breach Ω(t) at each moment of time can be represented as follows:

$$\Omega(t) = \omega(t) + D_T[z_C - z_W(t)] \tag{9}$$

The rate of increase in the linear dimensions of the breach is equal to the erosion rate *E*(*t*), determined by the shear stress on the eroded surface τ, which depends on the average flow velocity,

$$\Delta l(t) = E(t) = k_{er}[\tau(t) - \tau_c]\Delta t, \tag{10}$$

and the bottom velocity,

$$\Delta l_B(t) = E_B(t) = k_{er}(\tau_B(t) - \tau_c)\Delta t. \tag{11}$$

The calculation of $\tau(t)$ and $\tau_B(t)$ was carried out using Formulas (4) and (5). To calculate the bottom velocity v_B, the Karaushev equation was used: $v_B = \overline{v}\sqrt{1 - \frac{z}{R(t)}\left(0.57 + \frac{3.3}{C(t)}\right)}$ [33], where z is the immersion depth of the point, *C*(*t*) is the Chezy coefficient, and *R*(*t*) is the hydraulic radius of the flow.

The calculation algorithm for overflow is described in detail in [34].

The proposed methodology for calculating an outburst formed as a result of the formation of a filtration channel allows us to obtain an outburst flood hydrograph, the flow velocity, the changes in the water level, and the changes in the reservoir volume, as well as the channel diameter.

The described mathematical formulas were used as the basis for a computer program written in MatLab R2021b.

To test the proposed algorithm, physical experiments were carried out on the outburst of a soil dam resulting from the formation of a filtration channel, using an installation which is a container made of monolithic polycarbonate with dimensions of 1 × 1 × 1.5 m, consisting of two compartments. The dimensions of the first compartment, which was filled with water, were 1 × 1 × 0.6 m, and those of the second were 1 × 1 × 0.9 m. A partition with a rectangular hole (0.05 × 0.15 m) was installed between the compartments, through which water flowed from the first compartment into the second.

When filling the first compartment before the start of the experiment, the hole was closed with a shutter. To record the water level, a measuring scale was installed on the wall. The experimental dam had a trapezoidal shape. The material was sandy loam with rocky inclusions. The dam height was 30 cm, the dam width at the bottom was 73 cm, and the dam width at the crest was 10 cm. The dam length was 100 cm, the length of the slope on the upstream side was 34 cm, and the length of the slope on the downstream side was 50 cm.

To begin the process of expanding the channel, a pipe with a diameter of 1 cm was placed in the dam body, which was removed after water entered the compartment. A scale for measuring the water level was installed on the wall of the second compartment.

Physical experiments on the dam's destruction were carried out on the territory of the Priladozhskaya educational and scientific base of the St. Petersburg State University (Priozersky district of the Leningrad region, Russia) in April 2021. The process of the experiment is presented in Figure 4.

When the shutter is opened (Figure 4a), water enters the second compartment and immediately begins to move along the initiated channel. Next, the channel expands, and water discharge increases. A small amount of water during overflow forms an erosive incision on the dam slope. At 40 s (Figure 4b), the expansion of the channel and the deepening of the incision on the slope continue. Sixty seconds after the start of the experiment (Figure 4c), the filtration channel stops expanding, and a decrease in water discharge is observed. By 98 s (Figure 4d), the end of the experiment is recorded.

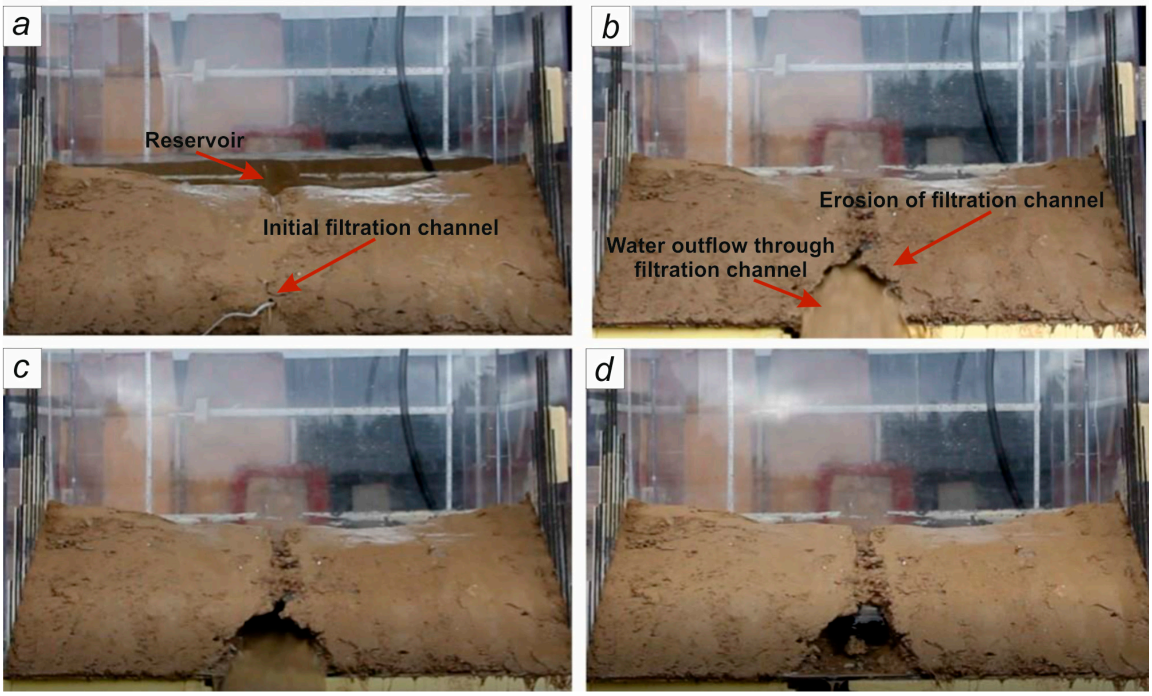

Figure 4. Process of the physical experiment: (**a**)—water begins to move along the initiated channel; (**b**)—the channel expands and water discharge increases; (**c**)—the filtration channel stops expanding; (**d**)—the end of the experiment.

Physical experiments made it possible to obtain an outburst flood hydrograph, which was then compared with the simulated hydrograph using the proposed calculation methodology. As a measure of the efficiency of the calculations, the Nash–Sutcliffe (NS) quality criterion was chosen [35], which is used in hydrological modelling to assess the convergence of calculated and observed series:

$$NS = 1 - \frac{\sum_{i=1}^{n}(Q_i - P_i)^2}{\sum_{i=1}^{n}(Q_i - \overline{Q})^2}, \tag{12}$$

where Q_i and P_i are, respectively, the observed and simulated discharges for the i time interval; $\overline{Q}$ is the observed discharge averaged over the entire modelling period; and n is the length of the row.

The range of criterion values in the general case is from $-\infty$ to 1. Modelling is considered satisfactory when NS > 0.5.

2.3. Numerical Experiments

One of the unresolved issues in the model is setting the initial diameter of the internal channel. In this regard, numerical experiments were carried out, where, with a constant dam geometry and identical soil characteristics, different channel diameters were set in increments of 1 mm (Figure 5); numerical experiments were performed on the destruction of a soil dam, which was built for the physical experiment described above. The soil dam was small in size. The described calculation methodology modelled the process of concentrated filtration, which can occur in the voids and pores of the soil. For the dimensions of the dam which was used in the numerical experiments, the selected step (1 mm) corresponded to the dimensions of the filtration channel, the erosion of which led to the outburst of the dam during the physical experiments. The calculation of outburst flood hydrographs (Figure 4) was carried out using the proposed calculation methodology.

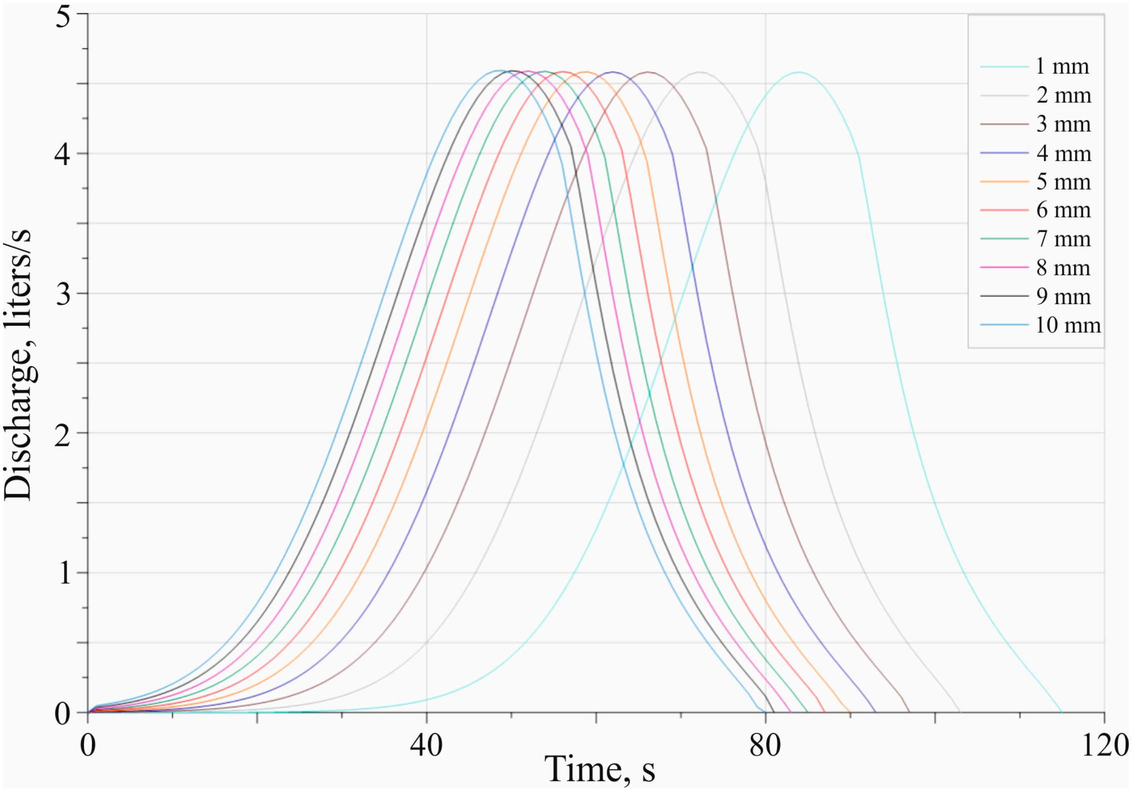

Figure 5. Hydrographs of outburst floods for different initial channel diameters.

The analysis of the hydrographs showed that a change in the channel's initial diameter affected the expansion time of this channel. The smaller the channel diameter, the slower the erosion rate. At the same time, the time of the outburst flood wave and the maximum discharge remained unchanged.

3. Results

3.1. Experimental Results

The data obtained during the physical experiment made it possible to obtain an outburst flood hydrograph and compare it with that calculated using the proposed methodology. Since the dam did not collapse completely during the experiment, the hydrograph was calculated only through the filtration channel. The characteristics of the dam material are presented in Table 1. The observed and simulated outburst flood hydrographs are shown in Figure 6.

Table 1. Dam material characteristics.

Parameter	Value
Channel diameter, m	0.01
Density of soil, kg/m^3	2610
Clay content, %	20
Plasticity index	8
Soil porosity, %	70
Average soil particle size, m	0.0002

During the physical experiments, spontaneous soil collapse occurred from the sides of the breach. In the calculation methodology described in this article, the spontaneous collapse of soil from the breach sides is not taken into account. The development of the breach occurred due to the erosion of its sides without the spontaneous collapse of the soil, which led to the smoother shape of the outburst flood hydrograph compared to the observed hydrograph. In addition, this led to a difference in the time of passage of the outburst flood. The maximum observed discharge was 4.82 L/s, and the simulated discharge was 4.67 L/s. The average observed discharge was 2.97 L/s, while the average simulated discharge was 3.05 L/s.

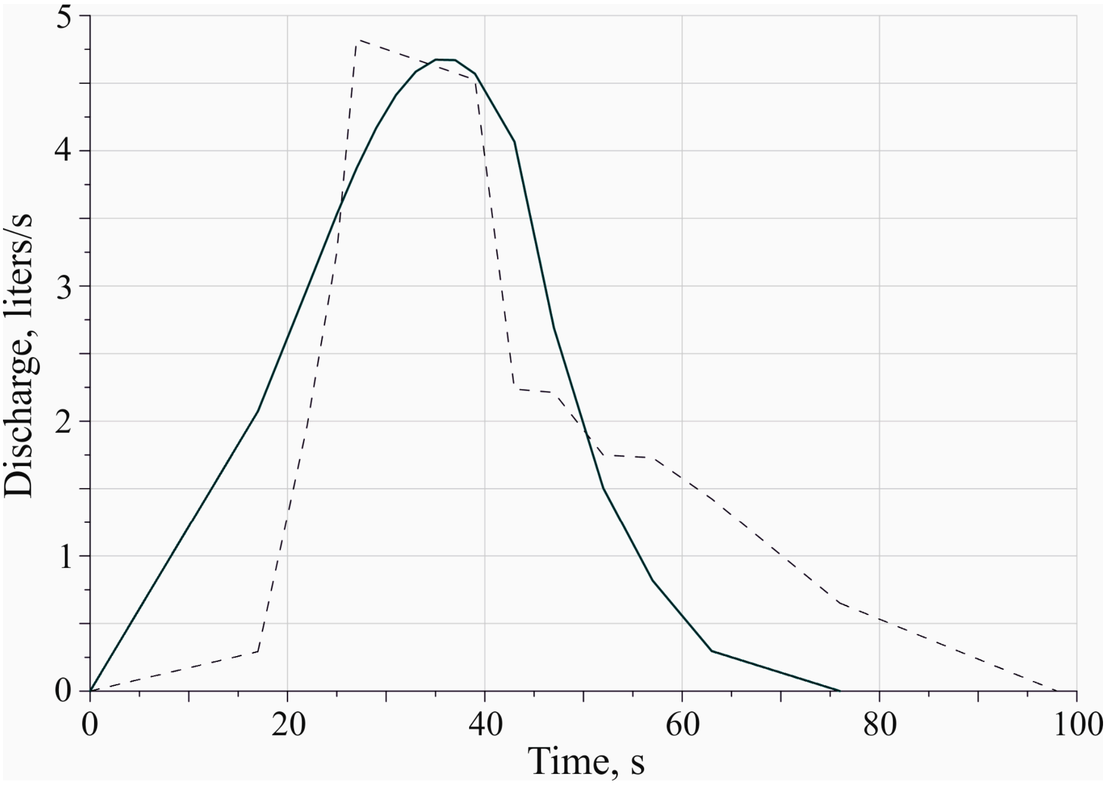

Figure 6. Observed (dashed line) and simulated (solid line) hydrographs of the outburst flood.

The convergence of the hydrographs was assessed using the Nash–Sutcliffe criterion. In our case, the Nash–Sutcliffe criterion value turned out to be 0.67. As the result lies in the range $0.65 < NS \leq 0.75$, it can be regarded as "good".

The satisfactory results verifying the developed calculation methodology, obtained during the physical experiments, made it possible to use the methodology for calculating the real case of outbursts of the Teton dam [36].

3.2. Verification of the Calculation Methodology on a Real Case of Outburst—The Teton Soil Dam

The Teton soil dam was built on the Teton River in Idaho, USA. Construction began in 1972 for flood protection and land irrigation purposes. On 5 June 1976, while filling the reservoir, the dam outburst. According to [36], in the period from 7:30 to 8:00 local time, the first signs of water outflow were recorded on the dam slope. By 9:30, a breach was discovered from which water was flowing at a discharge of about 0.5–0.85 m^3/s. Further, there was an increase in the diameter of the breach and an increase in water discharge. In the period from 11:15 to 11:30, the soil located above breach collapsed, followed by a process of water overflow with a maximum discharge of about 57,000 m^3/s.

Using the proposed methodology, the Teton dam outburst was simulated (Figure 7).

During the first 2 h, the outburst consisted of the water outflow through the filtration channel. In this case, the average water discharge was about 0.9 m^3/s. Then, the soil above the channel collapsed, and the water began to overflow. The maximum discharge during overflow was 57,360 m^3/s. The discrepancy between the simulated maximum discharge and the estimated discharge according to [36] was 0.6%. According to the report by [36], water discharge in the filtration channel was 0.5–0.85 m^3/s. The water flow in the filtration channel according to the calculation following the methodology was 0.8–0.9 m^3/s, which was also comparable to real data. The breach width calculated according to the methodology was 222 m. The estimated actual breach width was 650 ft (≈195 m), according to [37]. That is, the discrepancy between the simulated and the real value of the breach width was 14%.

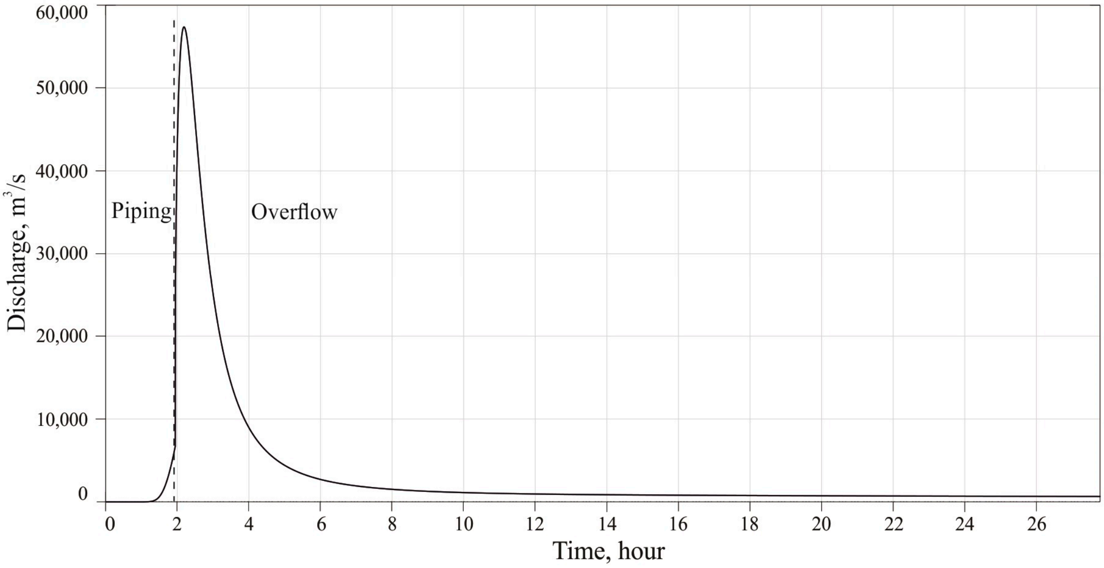

Figure 7. Simulated hydrograph of outburst flood formed due to Teton dam outburst.

Thus, the results verifying the methodology on published data of a real outburst also showed the adequacy and efficiency of the proposed calculation algorithm.

3.3. Modelling the Outburst of Lake Bashkara

The satisfactory results verifying the calculation methodology described in this study made it possible to use this method for the mathematical modelling of the Lake Bashkara outburst. The study by [22] describes the process of the destruction of the moraine dam damming Lake Bashkara. The extreme rainfall events resulted in a significant increase in water inflow to Lake Bashkara. In addition, landslide masses blocked the surface outflow, which led to a rise in the water level by 55 cm. In connection with this, increased filtration began to develop in the moraine dam body, which led to a shift of the lintel and then the formation of a breach with the water outflow through it. Next, the drainage channel was dammed with slid moraine sediments, as a result of which the outflow went directly along the tongue of the Bashkara glacier [24].

Developing this hypothesis, we assume that a filtration channel was formed in the moraine dam body, which was the first stage of the disaster. The water flow eroded the channel until the soil above the channel lost stability. After this, the soil collapsed, and a trapezoidal breach was formed. Then, water overflowed from the lake through the resulting breach.

Based on the described calculation methodology, an outburst flood hydrograph for Lake Bashkara was simulated (Figure 8).

The characteristics of the lake and the dam material are presented in Table 2. The lake volume prior to the outburst was obtained based on a bathymetric survey of the lake and a tacheometric survey of its coastal territory during fieldwork in July 2018 by employees of the Department of Land Hydrology of St. Petersburg State University. To determine the granulometric composition of the soil, samples of the moraine material were taken.

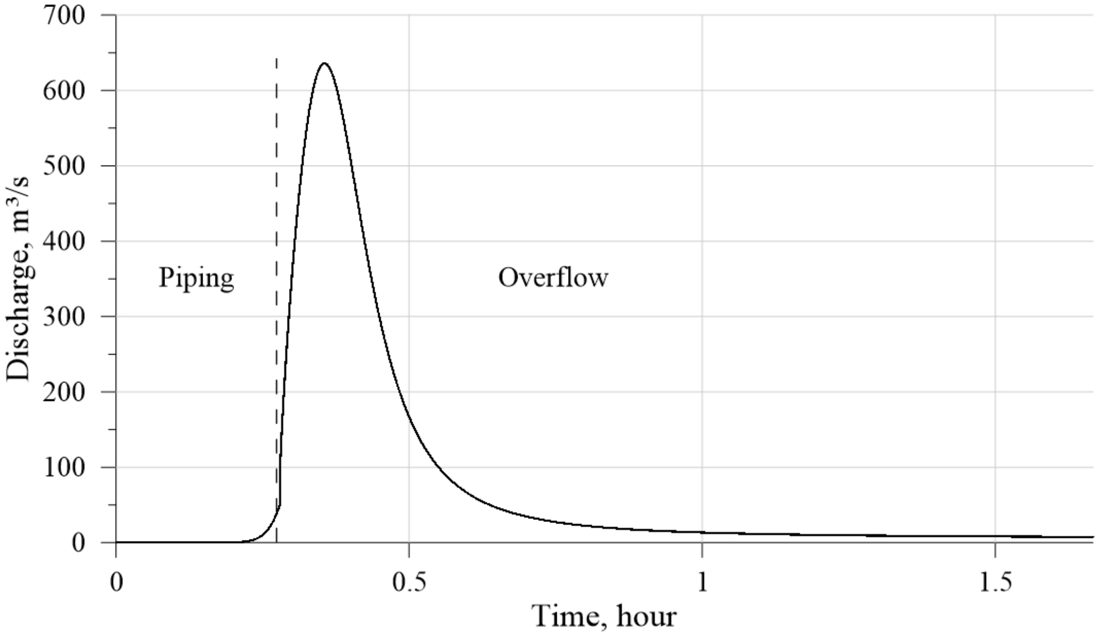

Figure 8. Simulated hydrograph of outburst flood during the outburst of Lake Bashkara.

Table 2. Characteristics of the lake and the dam material.

Parameter	Value
Channel diameter, m	0.01
Density of soil, kg/m^3	2760
Clay content, %	16
Plasticity index	13
Soil porosity, %	55
Average soil particle size, m	0.0002
Lake volume, m^3	800,000

The time step was set to 1 s. Calculations were carried out until the channel diameter reached 1/5 of the dam height, after which a calculation was carried out like that for an overflow. The calculated outburst flood hydrograph is presented in Figure 8.

According to the results obtained (Figure 8), the water discharge through the filtration channel occurred within 16 min, and an outburst flood wave was formed during the subsequent water overflow. The time period during which the outburst wave passed was about 40 min. The maximum discharge calculated using the proposed methodology was 636 m^3/s. The estimated breach area was 539 m^2.

4. Discussion

As part of the discussion, we note that one of the first calculations of the Lake Bashkara outburst was carried out using a mathematical model for calculating an outburst flood through an ice dam, developed by Yu.B. Vinogradov [38]. The results showed an underestimated value of the maximum discharge (123.5 m^3/s) and a fairly long time for the passage of the outburst flood (about 5 h), which do not correspond to the real situation of 1 September 2017, according to the actual data of the outburst given in the work by [19], where the values of the maximum discharge (600 m^3/s) and the time interval (40 min) during which the flood wave passed have been outlined. We did not find any other estimates of the outburst flood characteristics in the literature, and, most likely, there is no information obtained without the use of mathematical models that is more accurate. Thus, we can only compare the results obtained with the above: the outburst time period coincides with the actual value (about 40 min), and the maximum discharge is also comparable and amounts to 636 m^3/s. The hypothesis of the Lake Bashkara outburst presented in this research (the formation of a filtration channel at the initial stage of the outburst and then a water overflow) is supported by studies confirming the presence of filtration through a moraine dam [3,24,39].

5. Conclusions

The relevance of research into the processes and triggers of the outbursts of lakes of any origin becomes more obvious every year in connection with both climatic nonstationarity and the development of mountain territories by humans.

The presence of filtration through a moraine dam and, as a possible consequence of this, the development of an internal channel is a fairly common cause of the destruction of natural soil dams. The methodology presented in this research for calculating the outburst flood hydrograph to mathematically describe the outburst process makes it possible to obtain estimated discharges for further use in models of the formation of, for example, mudflows. The authors proposed a methodology consisting of two calculation blocks (erosion of the filtration channel in the dam body and/or water overflow through the dam crest). To more correctly display the process of the development of a breach, the model introduced the calculation of flow velocities in depth. The approximation of the cross-sectional shape of the breach proposed by the authors is more correct than those traditionally used (triangular and trapezoidal), describes the development of the breach over time, and better takes into account the change in shape by taking considering the unevenness of the erosion rate. Thus, the results of the numerical experiments and the verification of the methodology on the results of physical experiments and published data of real outbursts showed the adequacy and efficiency of the proposed calculation methodology and made it possible to use it to simulate the outburst of Lake Bashkara.

The outburst of Lake Bashkara is a complex and multifactorial disaster, one of the stages of which is the destruction of the moraine dam as a result of internal filtration. The results of modelling the outburst of Lake Bashkara on 1 September 2017 turned out to be comparable to the estimated data available in the literature (the time of water emptying through the channel was 16 min, the period of passage of the outburst flood wave was 40 min, and the maximum discharge was 636 m^3/s) and showed the possibility of using the proposed methodology for calculating the destruction of natural moraine dams. The disadvantage of models of this class is the lack of a description of the formation of the initial filtration channel, which will be further refined by the authors.

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