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Friction-Based Scaling of Streamwise Turbulence Intensity in Zero-Pressure-Gradient and Pipe Flows

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Abstract

We explore the analogy between the asymptotic (high Reynolds number) scaling of two canonical wall-bounded turbulent flows, namely zero-pressure-gradient (ZPG) and pipe flows. We find that the two flows can be characterised using similar scaling laws that relate the streamwise turbulence intensity to friction: the product of the dimensionless drag and the square root of a dimensionless boundary-layer thickness for ZPG flow plays the role of the streamwise turbulence intensity for pipe flow, and the two flows are linked through a Reynolds-number-dependent correction term derived from the logarithmic mean-velocity profile. As a consequence, the squared product scales as the friction factor. Establishing a common, friction-based turbulence intensity scaling for ZPG and pipe flows is of interest both for fundamental studies of canonical wall-bounded flows and for the specification of the turbulence intensity in computational fluid dynamics simulations.

Keywords: streamwise turbulence intensity; friction-based scaling; zero-pressure-gradient and pipe flows; correction term; computational fluid dynamics

1. Introduction

The streamwise turbulence intensity (TI) occupies a central position at the intersection of fundamental and industrial fluid mechanics. From a fundamental perspective, the TI quantifies the relative magnitude of the velocity fluctuations and is therefore a compact descriptor of the energetics of wall-bounded turbulence. From an industrial perspective, the TI is routinely required as an inlet boundary condition for computational fluid dynamics (CFD) simulations [1], and it governs practically important quantities such as flow-induced noise and the repeatability of flowmeter measurements [2]. A well-documented scaling of the TI with Reynolds number is consequently of considerable value, yet such expressions have historically been difficult to find in the literature; providing them has been the aim of a series of studies based on the Princeton Superpipe measurements [2–4].

Canonical wall-bounded flows—the zero-pressure-gradient (ZPG) flat-plate turbulent boundary layer, fully developed pipe and channel flows, and the plane Couette and Poiseuille flows—are studied precisely because their relatively simple boundary conditions make them tractable benchmarks for theory, simulation and experiment [5,6]. A recurring question is the extent to which these flows share universal features. At sufficiently high Reynolds number, the logarithmic mean-velocity profile is a robust and approximately universal element of the description [7], and Townsend’s attached-eddy hypothesis predicts a corresponding logarithmic variation of the streamwise turbulence intensity in the overlap region; this behaviour has been confirmed experimentally for friction Reynolds numbers $Re_\tau \gtrsim 2 \times 10^4$ [7,8]. Differences nevertheless persist between the flows, for instance in the



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value of the von Kármán constant κ and the additive constant A , which depend on the flow geometry and, for A , on wall roughness [7]. Identifying which scaling features are shared across canonical flows is a prerequisite for constructing improved common models [5].

For pipe flow, we have established—using publicly available Princeton Superpipe measurements [8,9]—that the global (radially averaged) streamwise TI scales with the square root of the friction factor, $I \sim \sqrt{\lambda}$ [2–4]. The radial averaging can be performed in several ways, and the resulting definitions and their interrelations have been catalogued in [4]. More recently, this scaling has been refined by the characterisation of a high Reynolds number transition at $Re_\tau \sim 11,000$ [10], by the inclusion of a viscous term that gives rise to a global peak in the fluctuating velocity [11], and by an extrapolation of the scaling beyond $Re_\tau \gg 10^5$, which confirmed that the squared TI is proportional to the friction factor [12]. The same body of measurements has motivated an algebraic non-equilibrium turbulence model that captures the increasing turbulence production-to-dissipation ratio observed in connection with the transition region [13]. These developments provide the pipe-flow backdrop against which the present comparison with ZPG flow is made.

A relatively recent paper [14] on ZPG flow has introduced an asymptotic (high Reynolds number) scaling law for a flat-plate turbulent boundary layer of thickness δ :

$$\tilde{U}_\tau \sim \frac{1}{\sqrt{\tilde{\delta}}}, \quad (1)$$

where

$$\tilde{U}_\tau = \frac{U_\tau \nu}{M} \sim \frac{\nu}{U_\tau \delta} = \frac{1}{Re_\tau} \quad (2)$$

is named the “dimensionless drag” and

$$\tilde{\delta} = \frac{\delta M}{\nu^2} \sim \frac{\delta^2 U_\tau^2}{\nu^2} = Re_\tau^2 \quad (3)$$

scales as the friction Reynolds number (Re_τ) squared. Note that we use $\sim$ to mean “scales as”. Here, U_τ is the friction velocity, $M = \int_0^\delta U(z)^2 dz$ is the kinematic momentum rate through the boundary layer, ν is the kinematic viscosity, δ is the boundary layer thickness, z is the distance from the wall and U is the mean velocity in the streamwise direction. Following [14], δ is the wall-normal distance at which the mean velocity reaches 99% of the free-stream velocity U_∞ , i.e., $\delta = \delta_{99}$. More generally, δ should be read as the outer length scale of the flow, that is, the wall-normal distance over which the mean velocity varies from zero at the wall to its maximum. Fully developed pipe flow does not possess a boundary layer that grows in the streamwise direction, but it does possess such an outer length scale, namely the pipe radius R ; the situation is the same as in fully developed open-channel (free-surface) flow, where the flow depth is the outer length scale once the boundary layer has grown to fill the depth and no further streamwise development takes place. The identification $\delta = R$ used in the remainder of this paper is therefore an equivalence of outer length scales and carries no implication that a boundary layer grows inside the pipe; see also Section 2. Note the asymptotic scaling $M \sim U_\tau^2 \delta$ has been proposed in [14] and applied in Equations (2) and (3).

In this paper we will show that the product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ scales as the global, i.e., radially averaged, turbulence intensity (TI) $I = \sqrt{\overline{u^2}}/U$, where u is the streamwise velocity fluctuation and overbar denotes time averaging [2–4]. As a consequence, the squared product ($\tilde{U}_\tau^2 \tilde{\delta}$) scales as the friction factor λ . We note that drag was addressed in [14]; the TI was not discussed.

A remark on the notation is required at this point, since Equation (1) states that the product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ tends to a constant, whereas the turbulence intensity and the friction factor

are both known to vary with Reynolds number. The two statements are compatible because Equation (1) is a leading-order asymptotic law in which a slowly varying logarithmic factor has been suppressed: the symbol $\sim$ retains the power-law dependence only. Restoring that factor—which is precisely what the correction term $\sqrt{f(Re_\tau)}$ of Section 3 does—gives $\tilde{U}_\tau \sqrt{\tilde{\delta}} \sim 1/\sqrt{f(Re_\tau)}$, which decays as $\kappa/\log(Re_\tau)$ and is consequently not constant. The residual, slowly varying factor carries the entire Reynolds-number dependence of the turbulence intensity, and its square carries that of the friction factor. Since a logarithmic decay is very nearly flat over any finite range of Reynolds numbers, it is invisible at the order retained in Equation (1) and is well represented over the measured range by the weak power law $Re_\tau^{-0.11}$ obtained in Section 3. The quantity that is genuinely constant is not the product itself but the corrected product $\tilde{U}_\tau \sqrt{\tilde{\delta} f(Re_\tau)}$, which is verified against the measurements in Equation (23).

Our paper represents an expansion of the validity of TI scaling with friction factor, since we have focused exclusively on pipe flow in previous published work [2–4,10–13]. Having a well-defined TI for ZPG flow is important for, e.g., CFD simulations [1], which is the main motivation for the present work. Another aim is to search for shared features of canonical wall-bounded flows [5] which may lead to improved common models.

The paper is organized as follows: In Section 2, we briefly review results from asymptotic scaling of TI in pipe flows; these findings are related to ZPG flows in Section 3. We discuss the implications of the established equivalence in Section 4 and conclude in Section 5. During the preparation of this manuscript, the author used Claude Opus 5 (Anthropic) for editing, drafting assistance, analysis support, and the generation of updated figures; all AI-assisted output was reviewed and verified by the author against the underlying calculations and data, and the author takes full responsibility for the content of this publication.

2. Asymptotic Pipe Flow Scaling of the Streamwise Turbulence Intensity

The material in this section is a summary of research on pipe flow contained in [2–4]. The local (streamwise) TI is defined as:

$$I_{\text{local}}(r) = \frac{\sqrt{u^2(r)}}{U(r)}, \quad (4)$$

where r is the radial coordinate ($r = 0$ is the pipe axis and $r = R$ is the pipe wall), so that the wall distance is $z = R - r$. As set out in Section 1, the pipe radius R is the outer length scale of fully developed pipe flow and therefore plays the role that the boundary layer thickness δ plays in an open boundary layer, or that the flow depth plays in fully developed open-channel flow. What the three geometries have in common is a single wall-normal distance across which the mean velocity varies from zero at the wall to its maximum value, and it is this distance that enters the scaling; the identification $\delta = R$ used below is an equivalence of outer length scales, not a claim about a layer growing in the streamwise direction. The local TI can be used to define a global TI:

$$I_{\text{global}} = \langle I_{\text{local}}(r) \rangle, \quad (5)$$

where $\langle \cdot \rangle$ indicates radial averaging; see [4], where several definitions of radial averaging have been documented. In the remainder of this paper, we treat the global TI; for simplicity of notation, we drop the subscript “global” and refer to I instead of I_{global} .

For pipe flow, the streamwise turbulence intensity I_{pipe} scales roughly with the ratio of the friction and mean velocities [3,4]:

$$I_{\text{pipe}} \sim \frac{U_\tau}{U_m} = 2 \times \frac{Re_\tau}{Re_D}, \quad (6)$$

where U_m is the mean velocity and $Re_D = DU_m/\nu$ is the bulk Reynolds number based on the pipe diameter D . For pipe flow, $Re_\tau = RU_\tau/\nu$, where R is the pipe radius. The friction factor λ scales with the square of this ratio:

$$\lambda = 8 \times \frac{U_\tau^2}{U_m^2} = 32 \times \frac{Re_\tau^2}{Re_D^2}. \quad (7)$$

As a consequence, the streamwise turbulence intensity scales with the square root of the friction factor:

$$I_{\text{pipe}} \sim \sqrt{\lambda}. \quad (8)$$

An example of the scaling using Princeton Superpipe measurements [8,9] is Equation (23) in [4]:

$$I_{\text{pipe area, AM}} = \frac{1}{R} \int_0^R \frac{\sqrt{u^2(r)}}{U(r)} dr = 0.66 \times \lambda^{0.55}, \quad (9)$$

where AM is an abbreviation for the “arithmetic mean” radial averaging.

Since the radial and the wall-normal coordinates span the same interval ($z = R - r$, so that dr and dz both sweep $0 \leq z \leq R$), Equation (9) is an arithmetic mean of the local TI taken over the wall-normal coordinate. The equivalent averaging procedure for ZPG flow is therefore obtained by replacing the pipe radius with the boundary layer thickness:

$$I_{\text{ZPG}} = \frac{1}{\delta} \int_0^\delta \frac{\sqrt{u^2(z)}}{U(z)} dz, \quad (10)$$

which is the definition of the global ZPG turbulence intensity adopted in this paper. The two flows are thus averaged in exactly the same manner, each over its own outer length scale, so that the comparison made in Section 3 is between like quantities. Equation (10) cannot be evaluated for the ZPG dataset used here [14], because the streamwise velocity fluctuations are not reported; this is why I_{ZPG} is accessed through a proxy in Section 3, and the consequences are set out in Section 4.

3. Equivalence Between Zero-Pressure-Gradient and Pipe Flows

In [15], we have used the “log law” for the streamwise mean velocity [7] to derive a correction term $\sqrt{f_{\text{ZPG}}(Re_\tau)}$ for the asymptotic scaling of drag presented in Equation (1); to make the present paper self-contained, the full derivation of this correction term—valid for both ZPG and pipe flows with the appropriate constants—is reproduced in Appendix A. The result is:

$$\tilde{U}_\tau \times \sqrt{f_{\text{ZPG}}(Re_\tau)} = 1.23 \times \delta^{-0.51} \sim \frac{1}{\sqrt{\delta}}, \quad (11)$$

where

$$\begin{aligned} f_{\text{ZPG}}(Re_\tau) &= \frac{2}{\kappa_{\text{ZPG}}^2} - \frac{2A_{\text{ZPG}}}{\kappa_{\text{ZPG}}} + A_{\text{ZPG}}^2 \\ &\quad + \log(Re_\tau) \left(\frac{2A_{\text{ZPG}}}{\kappa_{\text{ZPG}}} - \frac{2}{\kappa_{\text{ZPG}}^2} \right) + \log(Re_\tau)^2 / \kappa_{\text{ZPG}}^2. \end{aligned} \quad (12)$$

Here, $\kappa_{\text{ZPG}} = 0.39$ (von Kármán constant) and $A_{\text{ZPG}} = 5.7$ are constants derived in [15] for a fit to ZPG measurements; see Figure 1. Throughout this paper, $\log$ denotes the natural logarithm.

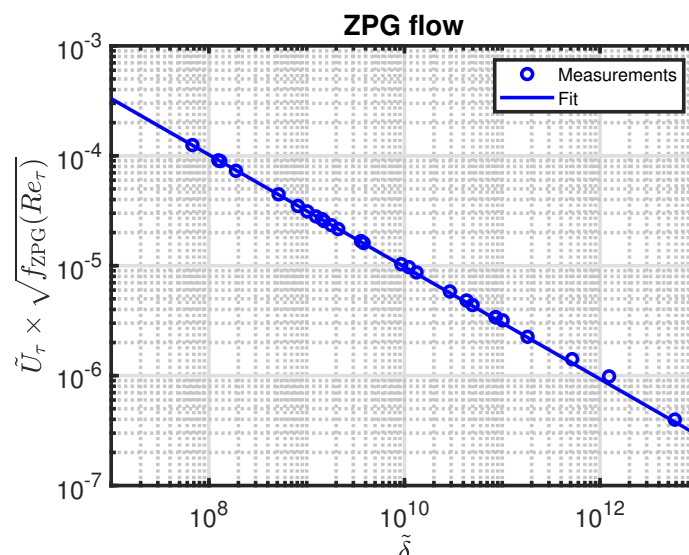


Figure 1. Correction term multiplied by dimensionless drag for ZPG flow as a function of $\tilde{\delta}$. Measurements from [14].

For pipe flow, we can define an equivalent correction term $\sqrt{f_{\text{pipe}}(Re_\tau)}$; above $Re_\tau \sim 11,000$ we find $\kappa_{\text{pipe}} = 0.34$ and $A_{\text{pipe}} = 1.0$ [10]. To relate this to the friction factor we perform a fit to the smooth-wall Superpipe data [8,9]:

$$\sqrt{f_{\text{pipe}}(Re_\tau)} = 2.24 \times \lambda^{-0.56} \sim \frac{1}{\sqrt{\lambda}} \sim \frac{1}{I_{\text{pipe}}}, \quad (13)$$

see Figure 2; the smooth-wall fit has a coefficient of determination $R^2 = 0.997$ ($n = 9$ data points), with prefactor 2.24 ± 0.11 and exponent -0.56 ± 0.01 (one standard error). Note that the correction term is different for smooth- and rough-wall flow since A depends on wall roughness [7]. Applying the same procedure to the rough-wall Superpipe data [8,9] yields a separate, steeper branch,

$$\sqrt{f_{\text{pipe, rough}}(Re_\tau)} = (0.41 \pm 0.20) \times \lambda^{-0.95 \pm 0.11}, \quad (14)$$

with $R^2 = 0.98$ ($n = 4$). The rough branch lies above the smooth branch in Figure 2 because the smaller additive constant A of the rough wall raises the correction term at a given friction factor. A fully quantitative rough-wall correction scheme would require the additive constant to be specified as a function of the roughness Reynolds number, $A = A(k_s^+)$ with $k_s^+ = k_s U_\tau / \nu$. The equivalent sand-grain roughness of the rough Superpipe is itself not unambiguous: the value $k_s = 8 \mu\text{m}$ obtained from the original roughness characterisation [3] does not reproduce the friction factors of the dataset used here, for which a fit of the Colebrook relation instead yields the more accurate value $k_s = 3 \mu\text{m}$, even though the same pipe is involved; the reason for the discrepancy is not clear, but the difference lies within the 5% experimental uncertainty quoted for the roughness measurement; see Section 5.1 of [4]. With only four rough-wall stations available here, and given this ambiguity in k_s , the extension is left for future work.

The link between the ZPG and pipe flows is their correction terms; see Figure 3. The correction terms increase monotonically with Reynolds number. To relate the two correction terms, we define their ratio Q and fit this to a logarithmic function:

$$Q(Re_\tau) = \frac{\sqrt{f_{\text{pipe}}(Re_\tau)}}{\sqrt{f_{\text{ZPG}}(Re_\tau)}} = 1.15 - 1.46 \times \log(Re_\tau)^{-0.89}. \quad (15)$$

The functional form of Equation (15) is not arbitrary. At large Reynolds number, each correction term is dominated by its quadratic-in- $\log(Re_\tau)$ contribution (Equation (12) and Appendix A), so that $\sqrt{f(Re_\tau)} \rightarrow \log(Re_\tau)/\kappa$ and the leading terms of the ratio cancel to leave $Q \rightarrow \kappa_{ZPG}/\kappa_{pipe}$; the additive constants A and the sub-leading $\log(Re_\tau)$ terms produce a positive, slowly decaying correction to this limit. The logarithmic fitting form of Equation (15) is thus the simplest expression that reproduces both the constant asymptote and this slow approach, and the constant $1.15 = 0.39/0.34 = \kappa_{ZPG}/\kappa_{pipe}$ is fixed by the two von Kármán constants rather than fitted. Physically, Q measures how much larger the pipe correction term is than the ZPG one at a given Reynolds number; its asymptotic value states that, once the additive constants have been outgrown, the two flows differ only through their von Kármán constants. Because this difference enters only logarithmically, the approach to the limit is extremely slow. Following the precedent of [12], where extrapolated turbulence intensity scaling was shown to be relevant to flows reaching bulk Reynolds numbers of 10^{15} – 10^{16} in accretion disks and early-universe cosmology, Figure 3 is drawn over the very wide range $Re_\tau = 10^2$ – 10^{200} solely to make the convergence to the asymptotic constant visible; the physically relevant behaviour for industrial and geophysical flows ($Re_\tau \lesssim 10^{10}$) is contained within the lower decades, where Q is still measurably below 1.15.

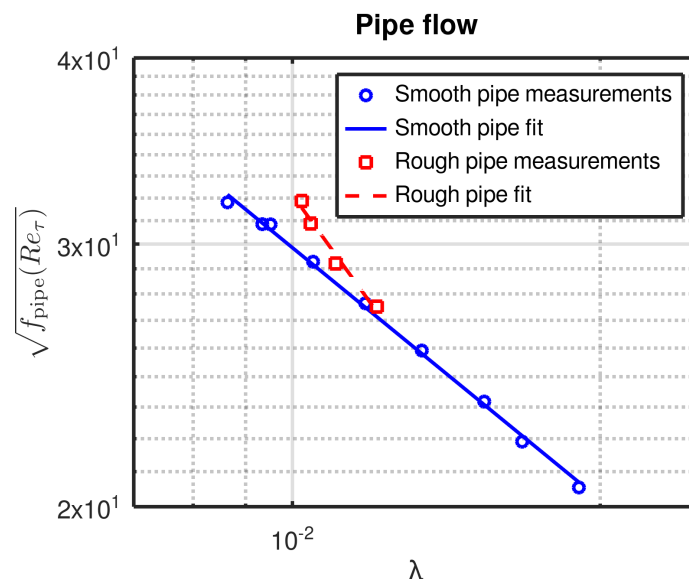


Figure 2. Correction term for pipe flow as a function of λ . Rough pipe measurements (root-mean-square roughness 5 μm , equivalent sand-grain roughness 8 μm [3]; a fit of the Colebrook relation to the friction factors of the present dataset gives the more accurate value 3 μm ; see Section 5.1 of [4]) and the corresponding rough-wall fit (Equation (14)) are shown for reference; the corresponding smooth pipe has a root-mean-square roughness of 0.15 μm (sand-grain roughness 0.45 μm). Measurements are from [8,9].

For ZPG flow, we introduce Equation (11) from [15]:

$$\tilde{U}_\tau = 0.17 \times \tilde{\delta}^{-0.56}, \quad (16)$$

and combine it with Equation (11):

$$\sqrt{f_{ZPG}(Re_\tau)} = \frac{1.23 \times \tilde{\delta}^{-0.51}}{\tilde{U}_\tau} = 7.24 \times \tilde{\delta}^{0.05}. \quad (17)$$

For pipe flow, we combine Equations (9) and (13):

$$\sqrt{f_{pipe}(Re_\tau)} = 2.24 \times \lambda^{-0.56} = 1.47 \times I_{pipe \text{ area, AM}}^{-1.02}. \quad (18)$$

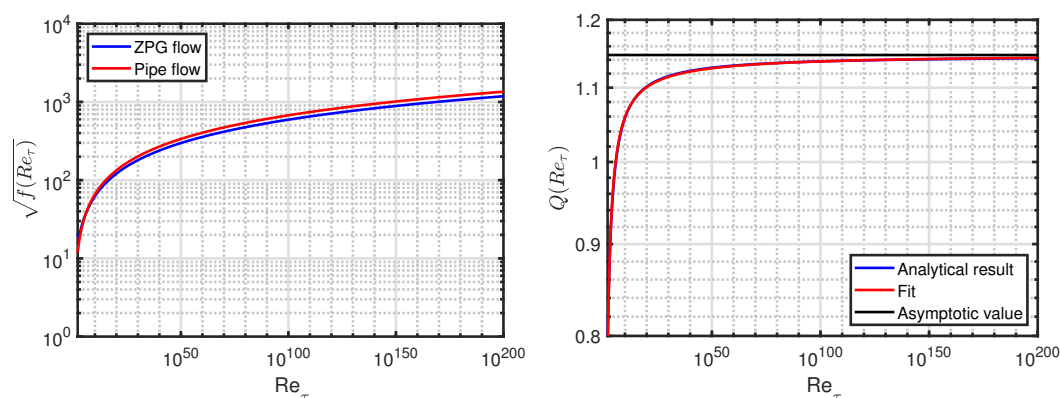


Figure 3. Note the extreme Reynolds number range, from 10^2 to 10^{200} . Left-hand plot: Correction terms for ZPG and pipe flows as a function of Re_τ . Right-hand plot: Ratio of correction terms as a function of Re_τ ; the horizontal black line indicates the asymptotic value of 1.15.

Since the correction terms in Equations (17) and (18) are related by Equation (15), we arrive at:

$$I_{\text{pipe area, AM}} = 1.19 \times \frac{\tilde{U}_\tau^{0.98} \tilde{\delta}^{0.50}}{Q^{0.98}}, \quad (19)$$

which can be approximated as:

$$I_{\text{pipe}} \sim \frac{\tilde{U}_\tau \sqrt{\tilde{\delta}}}{Q}. \quad (20)$$

By using Equations (3) and (16), we can express the product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ as a function of Re_τ :

$$\tilde{U}_\tau \sqrt{\tilde{\delta}} \sim \tilde{\delta}^{-0.06} \sim Re_\tau^{-0.12}, \quad (21)$$

which is scaling behaviour similar to what has been observed in pipe flow [2–4]. The exact fit to Equation (21) yields:

$$\tilde{U}_\tau \sqrt{\tilde{\delta}} = 0.10 \times Re_\tau^{-0.11}; \quad (22)$$

see Figure 4. The fitted exponent is -0.11 ± 0.004 with $R^2 = 0.96$ ($n = 27$ data points), and the prefactor is 0.101 ± 0.004 . The complete set of fitted parameters, with one-standard-error uncertainties, is collected in Appendix B.

Equation (22) settles the point of notation raised in Section 1. The product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ is measurably not constant—it decays as $Re_\tau^{-0.11}$ —although the leading-order law of Equation (1) would make it so. The quantity that is constant is the product corrected by the logarithmic factor:

$$\tilde{U}_\tau \sqrt{\tilde{\delta} f_{\text{ZPG}}(Re_\tau)} = 1.00 \pm 0.02, \quad (23)$$

where the quoted uncertainty is the standard deviation over the 27 measurement stations of [14]. Fitting a power law in Re_τ to the left-hand side of Equation (23) returns an exponent of -0.0008 ± 0.0037 , which is indistinguishable from zero, so that no residual Reynolds-number dependence is detectable. Equations (22) and (23) taken together state that the whole Reynolds-number dependence of the uncorrected product resides in the factor $1/\sqrt{f_{\text{ZPG}}(Re_\tau)}$, which is the quantity identified with the turbulence intensity in Equation (25) below. There is consequently no conflict between an asymptotic law in which the product is constant at leading order and a turbulence intensity and friction factor that vary with Reynolds number: the variation is logarithmic and is carried entirely by the correction term.

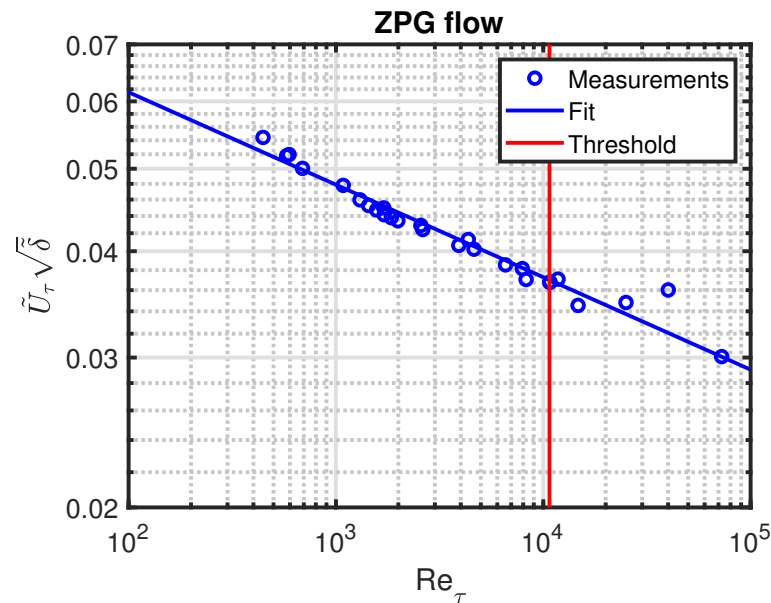


Figure 4. The product $\tilde{U}_\tau \sqrt{\delta}$ as a function of Re_τ . The vertical line at $Re_\tau \sim 11,000$ indicates the pipe flow transition found in [10]. Measurements from [14].

Based on the findings in this paper we summarise the following TI analogies for ZPG and pipe flows:

$$I_{\text{pipe}} \sim \frac{1}{\sqrt{f_{\text{pipe}}(Re_\tau)}} \sim \frac{\tilde{U}_\tau \sqrt{\delta}}{Q} \sim \frac{I_{\text{ZPG}}}{Q}, \quad (24)$$

where we have used:

$$I_{\text{ZPG}} \sim \frac{1}{\sqrt{f_{\text{ZPG}}(Re_\tau)}} \sim \tilde{U}_\tau \sqrt{\delta}. \quad (25)$$

Note that it is only possible to present scaling properties of I_{ZPG} and not an explicit equation, since velocity fluctuation measurements are not available in [14]. We can reformulate Equation (24) to:

$$I_{\text{pipe}} \sqrt{f_{\text{pipe}}} \sim I_{\text{ZPG}} \sqrt{f_{\text{ZPG}}}. \quad (26)$$

Corresponding friction factor analogies can be found by taking the square of Equation (24):

$$\lambda \sim \frac{1}{f_{\text{pipe}}(Re_\tau)} \sim \frac{\tilde{U}_\tau^2 \delta}{Q^2} \sim \frac{1}{Q^2 f_{\text{ZPG}}(Re_\tau)}. \quad (27)$$

The correspondence established above is summarised in Table 1.

Table 1. Correspondence between the friction-based streamwise turbulence intensity scaling of zero-pressure-gradient (ZPG) and pipe flows. The constants κ and A are taken from [15] (ZPG) and [10] (pipe).

Quantity	ZPG Flow	Pipe Flow
Streamwise turbulence intensity	$\tilde{U}_\tau \sqrt{\delta}$ (proxy)	I_{pipe}
Friction variable	$\tilde{U}_\tau$ (dimensionless drag)	λ (friction factor)
Correction term	$\sqrt{f_{\text{ZPG}}(Re_\tau)}$	$\sqrt{f_{\text{pipe}}(Re_\tau)}$
von Kármán constant κ	0.39	0.34
Additive constant A	5.7	1.0
TI-correction relation	$I_{\text{ZPG}} \sim 1/\sqrt{f_{\text{ZPG}}}$	$I_{\text{pipe}} \sim 1/\sqrt{f_{\text{pipe}}}$
TI-friction relation	$I_{\text{ZPG}} \sim \tilde{U}_\tau \sqrt{\delta}$	$I_{\text{pipe}} \sim \sqrt{\lambda}$
Asymptotic limit ($Re_\tau \rightarrow \infty$)	$I_{\text{ZPG}} \rightarrow \kappa_{\text{ZPG}}/\log(Re_\tau)$	$I_{\text{pipe}} \rightarrow \kappa_{\text{pipe}}/\log(Re_\tau)$

4. Discussion

The central result of Section 3 is that the product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ for ZPG flow plays the same role as the global streamwise TI for pipe flow; see Equations (24) and (25). Because velocity fluctuation measurements are not reported in the ZPG dataset of [14], it is not possible to state an explicit equation for I_{ZPG} ; only its scaling can be inferred, through the proxy $I_{ZPG} \sim \tilde{U}_\tau \sqrt{\tilde{\delta}}$. This is the principal limitation of the present study and it is returned to, together with the remaining limitations, at the end of this section. The two flows are connected through their correction terms $\sqrt{f(Re_\tau)}$, whose ratio Q approaches the asymptotic value $\kappa_{ZPG}/\kappa_{pipe} = 1.15$, but only slowly with Reynolds number; see Figure 3. The correspondence is summarised in Table 1.

The equivalence has a simple physical basis. Using the definitions $\tilde{U}_\tau = U_\tau \nu / M$ and $\tilde{\delta} = \delta M / \nu^2$, the product reduces to $\tilde{U}_\tau \sqrt{\tilde{\delta}} = U_\tau \sqrt{\delta / M}$, which invites a velocity scale U_m defined through $M = U_m^2 \delta$. For pipe flow, the U_m appearing in Equation (6) is the cross-sectional average velocity, a well-defined global quantity at any fixed axial station. A ZPG boundary layer possesses no bulk mass-flux velocity of this kind, and the free-stream velocity U_∞ is not the relevant scale either, since it is attained only at the edge of the layer. The kinematic momentum rate supplies the appropriate definition directly:

$$U_{m,ZPG} \equiv \sqrt{\frac{M}{\delta}} = \left(\frac{1}{\delta} \int_0^\delta U(z)^2 dz \right)^{1/2}, \quad (28)$$

so that $U_{m,ZPG}$ is the root mean square of the mean-velocity profile across the layer, or equivalently the uniform velocity that would carry the same kinematic momentum rate M through a layer of thickness δ . With this definition the velocity-ratio relation becomes an identity rather than an approximation,

$$\tilde{U}_\tau \sqrt{\tilde{\delta}} = U_\tau \sqrt{\frac{\delta}{M}} = \frac{U_\tau}{U_{m,ZPG}}, \quad (29)$$

and we have verified that Equation (29) is satisfied to within 0.4% at every one of the 27 stations of [14], the residual being attributable to rounding of the tabulated values. The velocity scale so defined is bounded above by the free-stream velocity, taking the value $U_{m,ZPG}/U_\infty = 0.87 \pm 0.03$ across the 18 stations of [14] for which a dimensional free-stream velocity is tabulated. The ZPG product is therefore exactly the ratio of the friction velocity to the momentum-rate-equivalent mean velocity, which is the ratio that sets the pipe-flow turbulence intensity in Equation (6). The two definitions of U_m —cross-sectional average for the pipe and root mean square for the boundary layer—are not identical, since the root mean square of a profile exceeds its arithmetic mean, but they differ only by a profile shape factor of order unity: for a 1/7-power mean-velocity profile the difference is about 1%, which does not affect the scaling. The analogy is therefore mechanistic rather than coincidental: up to this shape factor, the ZPG product is the same velocity ratio, expressed through the dimensionless drag and the dimensionless boundary layer thickness.

The correction term originates from integrating the logarithmic mean-velocity profile across the layer, which replaces the leading-order approximation $M \sim U_\tau^2 \delta$ by $M \sim U_\tau^2 \delta f(Re_\tau)$ [15]. It should be regarded as a leading-order amendment, since the derivation applies the log law outside its strict range of validity [7,15]. The constants κ and A differ between ZPG and pipe flow, and A additionally depends on wall roughness [7]; this is why the pipe-flow correction term in Figure 2 separates into distinct smooth- and rough-wall branches. The fact that κ and A are flow-specific is consistent with the broader observation that, while the logarithmic region is approximately universal, several of its parameters and the behaviour of the large scales remain flow-dependent [5]. We

therefore use “universal” in a specific sense: it is the functional form of the friction-based turbulence intensity scaling—the inverse dependence on the correction term, the resulting $I \sim \sqrt{\lambda}$, and the common logarithmic asymptotic limit—that is shared by the two flows, whereas the numerical parameters (κ , A , and hence the prefactors) are geometry- and roughness-dependent. In this sense, the scaling is a shared structure carrying flow-specific coefficients rather than a single geometry-independent curve.

Squaring the TI analogy yields the corresponding friction-factor relation of Equation (27), $\lambda \sim \tilde{U}_\tau^2 \delta / Q^2$. For pipe flow, this is consistent with the independently established proportionality between the squared TI and the friction factor, which has been confirmed for the area-averaged definition out to $Re_\tau \gg 10^5$ [12]. The present analysis therefore extends to the ZPG boundary layer a relation that is well documented for pipe flow, albeit in scaling form only.

The friction-based scaling also underlies the turbulence intensity correlation used to specify turbulent inlet conditions in CFD. Writing the friction factor as $\lambda = 8 U_\tau^2 / U_m^2$ and estimating the root-mean-square velocity fluctuation at the pipe centreline (CL) as $\sqrt{u_{CL}^2} \approx 0.8 U_\tau$ [16], the centreline turbulence intensity is $I_{CL} = \sqrt{u_{CL}^2} / U_m = 0.8 \sqrt{\lambda / 8}$. Inserting the Blasius friction law $\lambda = 0.3164 Re_D^{-1/4}$ [17], with $Re_D = DU_m / \nu$ the bulk Reynolds number, yields the widely used estimate [2]:

$$I_{CL} = 0.8 \times \sqrt{\frac{0.3164}{8}} \times Re_D^{-1/8} = 0.16 \times Re_D^{-1/8}. \quad (30)$$

The derivation of Equation (30), which traces the coefficient 0.16 to the centreline fluctuation estimate and the Blasius constant, was presented in [18]; to the best of our knowledge, it is published here in a paper for the first time. The global, area-averaged turbulence intensity studied here obeys the same proportionality $I \sim \sqrt{\lambda}$ (Equation (8)), with a different prefactor and a slightly weaker Reynolds-number dependence, $I \sim Re_D^{-0.11}$ [3]; the ZPG product $\tilde{U}_\tau \sqrt{\delta} \sim Re_\tau^{-0.11}$ is its ZPG counterpart. The present results therefore place this engineering rule of thumb on a friction-based footing and, through the equivalence established here, motivate an analogous estimate for ZPG flow.

We now make explicit what the unified scaling adds relative to existing, separate turbulence intensity models. Engineering practice typically specifies the pipe-flow inlet turbulence intensity through a stand-alone empirical correlation such as Equation (30) [2], and the boundary-layer and pipe literatures have largely treated the two geometries with independent fits. The present framework instead expresses both flows through a single friction-based relation, $I \sim 1/\sqrt{f(Re_\tau)} \sim \sqrt{\lambda}$, in which the only flow-specific inputs are the log-law constants κ and A ; the geometries are then linked quantitatively by the single ratio $Q(Re_\tau)$ with the fixed asymptote $\kappa_{ZPG} / \kappa_{pipe}$. Three practical advantages follow. First, the same expression covers ZPG and pipe flow, so a turbulence intensity estimate obtained for one geometry can be transferred to the other through Q rather than requiring a separate correlation. Second, because the scaling is tied to the friction factor, it inherits the documented $I^2 \sim \lambda$ proportionality and its extrapolation to $Re_\tau \gg 10^5$ [12], giving a controlled high-Reynolds-number behaviour that stand-alone power-law fits do not guarantee. Third, the framework localises the geometry dependence in κ , A and Q , which is exactly the information a common model needs.

A definitive statement can be made about the asymptotic limit. For large Reynolds number, the correction term in Equation (12) is dominated by its quadratic term, so that $\sqrt{f(Re_\tau)} \rightarrow \log(Re_\tau) / \kappa$. The turbulence intensity then decays logarithmically, $I \sim 1/\sqrt{f(Re_\tau)} \rightarrow \kappa / \log(Re_\tau)$, and the friction factor as the inverse square of the logarithm, $\lambda \sim 1/f(Re_\tau) \rightarrow \kappa^2 / \log(Re_\tau)^2$. Both ZPG and pipe flows share this same asymptotic form, differing only through their von Kármán constants; this is consistent with

the ratio Q tending to $\kappa_{ZPG}/\kappa_{\text{pipe}}$ (Equation (15)). The streamwise turbulence intensity and the friction factor therefore both vanish as $Re_\tau \rightarrow \infty$, but only logarithmically slowly.

We emphasise that the present turbulence intensity is a global, mean-velocity-normalised quantity, so that its decay with Reynolds number is governed by the friction factor. A distinct question is the Reynolds-number dependence of the inner-scaled streamwise variance peak $\overline{u^2}^+$ in the near-wall region, which remains debated: the attached-eddy framework [19] and a number of measurements indicate an unbounded, logarithmic growth with Re_τ , whereas other analyses argue that the peak is bounded and saturates as $Re_\tau \rightarrow \infty$ [20,21]. The averages defined in Equations (9) and (10) do extend over the near-wall region, so growth of the inner peak does contribute to the integral, and the relevant question is how much weight that contribution carries. Two properties bound it. First, the integrand is the local turbulence intensity $I_{\text{local}} = \sqrt{\overline{u^2}^+}/U^+$, in which a growing numerator is divided by the mean velocity at the same wall distance, and it is bounded throughout the near-wall region. At the wall itself, $\sqrt{\overline{u^2}^+}$ and U^+ both vanish linearly in z^+ , so that their ratio tends to a finite limit of order 0.4. At the location of the inner variance peak, $z^+ \approx 15$, the standard buffer-layer form of the mean-velocity profile gives a mean velocity of $U^+ \approx 10.5$; we stress that this is the mean velocity and not the variance $\overline{u^2}^+$, which happens to be of similar numerical magnitude at that wall distance. Taking $\overline{u^2}^+ \approx 11$ at $Re_\tau = 10^5$ accordingly gives $I_{\text{local}} \approx 0.3$ at the peak, and in the unbounded scenario this value grows only as $\sqrt{\log(Re_\tau)}$, reaching approximately 0.4 at $Re_\tau = 10^{10}$. The local turbulence intensity in the near-wall region is therefore of order 0.4 or below throughout the range of interest, and can grow at most logarithmically slowly. Second, the averaging is carried out in outer units, so the near-wall region occupies a vanishing fraction of the averaging interval: the layer $z^+ \leq 100$ spans $100/Re_\tau$ of the interval $0 \leq z \leq \delta$. The near-wall contribution to the average is consequently bounded by $100 \max(I_{\text{local}})/Re_\tau \lesssim 40/Re_\tau$, with a numerator growing at most as $\sqrt{\log(Re_\tau)}$, whereas the global turbulence intensity itself decays only logarithmically, $I \rightarrow \kappa/\log(Re_\tau)$. The near-wall share of the average therefore vanishes as $\log(Re_\tau)/Re_\tau$: it amounts to less than 1% at $Re_\tau = 10^5$ and is negligible beyond. The global average is thus least sensitive to the fate of the inner peak precisely at high Reynolds number, which is the regime in which the debate is framed. This is an argument about the weight of the near-wall region in an outer-scaled average and not a claim that the global turbulence intensity is decoupled from near-wall physics; on the contrary, the friction velocity that sets the entire scaling through λ is itself determined at the wall. The narrower statement established here, and the only one needed, is that the conclusions of the present paper do not depend on which side of the inner-peak debate proves to be correct.

A source of inaccuracy of our results is that the fits use measurements across the entire available Reynolds number range. For pipe flow, a transition has been identified at $Re_\tau \sim 11,000$, above and below which the global turbulence intensity scaling differs [10,11]. Above the transition, the turbulence production-to-dissipation ratio has been found to increase [13]. One proposed explanation of the transition region is an increase in large-scale structures in the wake region; whether such structures are active or inactive, i.e., whether they contribute to the turbulent shear stress, remains an open question [13]. It has furthermore been speculated that the transition is a counterpart of the “drag crisis”, i.e., a comparatively sudden change in the drag behaviour above a certain Reynolds number [12,13]. To quantify the impact of this transition on the present analysis, we refitted the smooth-wall pipe correction term of Equation (13) separately below and above the threshold: the exponent changes from -0.61 ± 0.02 ($Re_\tau \leq 11,000$, $n = 4$) to -0.51 ± 0.03 ($Re_\tau > 11,000$, $n = 5$), a difference larger than the combined uncertainty (Figure 5), whereas

the single power law of Equation (13) (-0.56 ± 0.01 , $R^2 = 0.997$) remains an adequate leading-order description across the whole range (Appendix B). The transition is therefore detectable but does not invalidate the unified scaling; a segmented treatment refines the exponent at the price of far fewer points per branch. The vertical red line in Figure 4 marks the same pipe-flow threshold on the ZPG axes, drawn purely for reference. Fitting the ZPG product on either side of it gives -0.12 ± 0.004 below ($n = 22$, $R^2 = 0.98$) but an ill-constrained -0.08 ± 0.04 above ($n = 5$, $R^2 = 0.56$); the boundary-layer data can therefore neither confirm nor exclude a counterpart of the pipe-flow transition, and settling this would require measurements that both resolve the velocity fluctuations and extend well beyond $Re_\tau \sim 10^4$.

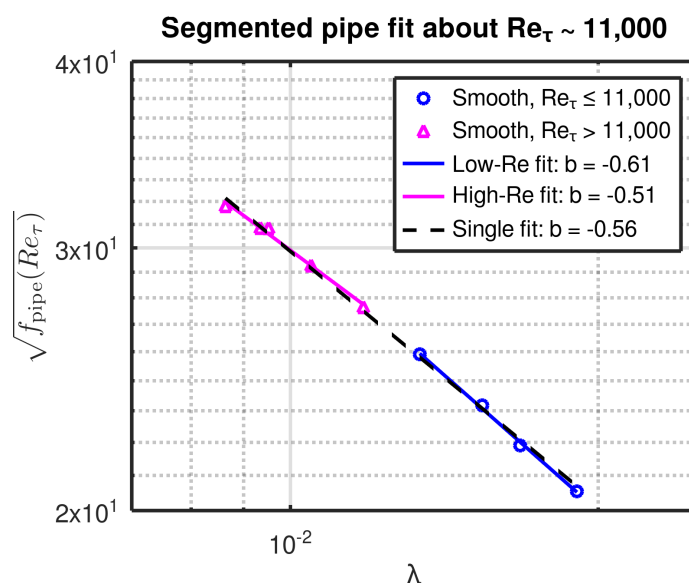


Figure 5. Segmented power-law fit of the smooth-wall pipe correction term $\sqrt{f_{\text{pipe}}(Re_\tau)}$ as a function of λ , split at the pipe-flow transition $Re_\tau \sim 11,000$ [10]. The exponents fitted below and above the threshold (-0.61 and -0.51) bracket the single-fit exponent (-0.56 , dashed), which remains an adequate leading-order description across the whole range. Measurements from [8,9].

For clarity, we collect the main limitations of the present study. (i) The principal limitation is that no direct streamwise velocity-fluctuation measurements are available for the ZPG dataset [14]. The averaging procedure of Equation (10) can therefore not be carried out for ZPG flow, and I_{ZPG} is accessible only through the proxy $\tilde{U}_\tau \sqrt{\delta}$, which is built entirely from mean-flow quantities, namely U_τ , M and δ . Identifying that proxy with a turbulence intensity rests on the mechanistic argument of Equations (28) and (29)—that the proxy is exactly the velocity ratio U_τ/U_m which sets the turbulence intensity in pipe flow—and on the agreement of its Reynolds-number scaling with the pipe-flow result, but not on any measured ZPG fluctuation. Only the scaling of I_{ZPG} can accordingly be stated, and neither an explicit closed-form expression nor an absolute level can be given; the ZPG side of the correspondence remains quantitatively unvalidated, and validating it requires a boundary-layer dataset reporting the mean-velocity profile and the streamwise fluctuations together over a comparable Reynolds number range. The mathematical framework developed above should be read with that restriction in mind. (ii) The correction term is a leading-order construction obtained by integrating the log law across the whole layer (Appendix A); the near-wall and wake regions are not represented, which is acceptable at high Reynolds number but limits accuracy at low Re_τ . (iii) The constants κ and A , and hence the correction terms and the ratio Q , are geometry- and roughness-dependent, so the scaling is universal in form but not in its coefficients. (iv) The main fits combine data across the $Re_\tau \sim 11,000$ pipe-flow transition; a segmented treatment detects a change in exponent but is limited

by the small number of points per branch. (v) The rough-wall branch rests on only four stations and does not yet incorporate an explicit dependence on the roughness Reynolds number. (vi) The extension to computational fluid dynamics inlet conditions is developed here at the level of scaling relations; a dedicated numerical validation of inlet turbulence intensity specification lies beyond the scope of the present, primarily analytical, study and is left for future work.

The broader motivation for relating canonical flows is the construction of improved common models [5]. The observation that the turbulence production-to-dissipation ratio increases through the transition region has already been incorporated into an algebraic non-equilibrium turbulence model calibrated to pipe flow [13], which can serve both as a self-contained model and as a source of inlet boundary conditions for CFD simulations. Establishing that ZPG and pipe flows share the same friction-based TI scaling, up to the geometry-dependent correction term, is a step towards extending such models across the wider family of canonical wall-bounded flows.

5. Conclusions

We have explored the correspondence between ZPG and pipe flows for asymptotic scaling of streamwise turbulence intensity with friction. It is demonstrated that similar scalings are valid for both types of flows; the product $\tilde{U}_\tau \sqrt{\delta^+}$ for ZPG flow is equivalent to the streamwise turbulence intensity for pipe flow I_{pipe} . The scaling of turbulence intensity with Reynolds number in ZPG flow closely matches the corresponding pipe flow scaling. In addition, we have shown that the turbulence intensity is inversely proportional to the correction term $\sqrt{f(Re_\tau)}$ and that κ and A for the correction term are different for ZPG and pipe flows.

A source of inaccuracy of our results is that we have used measurements carried out at all Reynolds numbers. However, in [10] we have shown that a transition exists at $Re_\tau \sim 11,000$ for pipe flow; scaling is somewhat different below and above this threshold, and the transition has subsequently been examined in further detail [11–13]. Future research includes studies for higher Reynolds numbers to characterise scaling below and above the transition, ideally using ZPG datasets that resolve the streamwise velocity fluctuations, so that an explicit I_{ZPG} can be determined and tested quantitatively against the present scaling. A natural further step is to add channel flow as a third canonical case. Direct numerical simulation of channel flow currently reaches Re_τ of order 10^4 , below the $Re_\tau \sim 10^5$ attained by the Superpipe experiments, so a channel comparison is presently limited to a smaller Reynolds number range [5]; it is nonetheless attractive because simulation gives direct access to the velocity fluctuations that the ZPG dataset lacks, allowing an explicit—rather than proxy—turbulence intensity to be tested against the present scaling. Beyond the zero-pressure-gradient case, extending the correction-term construction to adverse-pressure-gradient boundary layers is an interesting but non-trivial prospect: a pressure gradient modifies the mean-velocity profile away from the simple log law integrated here, so the correction term would have to be generalised accordingly, and we regard this as a separate line of work. Establishing the same friction-based scaling across this wider family—channel flow, then the plane Couette and Poiseuille flows [6], and eventually pressure-gradient boundary layers—would substantiate the search for shared features and improved common models.

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Data Availability Statement: No new experimental data were created in this study; the analysis uses previously published pipe-flow [8,9] and zero-pressure-gradient boundary-layer [14] datasets. The

scripts used to reproduce the fits, uncertainties and figures of this revision are available from the author on reasonable request.

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Appendix A. Derivation of the Correction Term $f(Re_\tau)$

The correction term follows from integrating the logarithmic mean-velocity profile over the layer; throughout, $\log$ denotes the natural logarithm, consistent with the main text. Writing the log law as

$$\frac{U(z)}{U_\tau} = \frac{1}{\kappa} \log z^+ + A, \quad z^+ = \frac{zU_\tau}{\nu}, \quad (\text{A1})$$

the kinematic momentum rate $M = \int_0^\delta U(z)^2 dz$ becomes

$$M = U_\tau^2 \int_0^\delta \left[\frac{(\log z^+)^2}{\kappa^2} + \frac{2A}{\kappa} \log z^+ + A^2 \right] dz. \quad (\text{A2})$$

With $Re_\tau = \delta U_\tau / \nu$ and the value of z^+ at $z = \delta$, the three elementary integrals are $\int_0^\delta dz = \delta$, $\int_0^\delta \log z^+ dz = \delta(\log Re_\tau - 1)$ and $\int_0^\delta (\log z^+)^2 dz = \delta[(\log Re_\tau)^2 - 2 \log Re_\tau + 2]$, so that

$$M = U_\tau^2 \delta f(Re_\tau), \quad (\text{A3})$$

with

$$\begin{aligned} f(Re_\tau) &= \frac{2}{\kappa^2} - \frac{2A}{\kappa} + A^2 \\ &\quad + \log(Re_\tau) \left(\frac{2A}{\kappa} - \frac{2}{\kappa^2} \right) + \log(Re_\tau)^2 / \kappa^2. \end{aligned} \quad (\text{A4})$$

Equation (A4) is identical to Equation (12); the ZPG and pipe correction terms differ only through the constants (κ, A) , taken as $(\kappa_{\text{ZPG}}, A_{\text{ZPG}}) = (0.39, 5.7)$ [15] and $(\kappa_{\text{pipe}}, A_{\text{pipe}}) = (0.34, 1.0)$ [10], and the correction term itself is $\sqrt{f(Re_\tau)}$.

Two remarks concern the range of validity of this construction, addressing the fact that the log law holds strictly only in the overlap region while the integral (A2) extends across the whole layer. First, the integrand is dominated by the overlap and outer parts of the layer: near the wall $z(\log z^+)^2 \rightarrow 0$ as $z \rightarrow 0$, so despite the divergence of $\log z^+$, the boundary contribution vanishes and the integral is finite. The viscous sublayer and buffer region therefore contribute negligibly to M at high Reynolds number, and $f(Re_\tau)$ should be read as a leading-order momentum-rate correction rather than an exact result; the wake contribution is likewise not represented. Second, because the $\log(Re_\tau)^2 / \kappa^2$ term dominates at large Re_τ , $\sqrt{f(Re_\tau)} \rightarrow \log(Re_\tau) / \kappa$, which fixes the asymptotic turbulence intensity $I \sim \kappa / \log(Re_\tau)$ and the ratio $Q \rightarrow \kappa_{\text{ZPG}} / \kappa_{\text{pipe}}$ quoted in the main text.

Appendix B. Fitted Parameters and Their Uncertainties

Table A1 collects the power-law fits $y = ax^b$ used in this work, each with its one-standard-error uncertainty obtained from the fit covariance and its coefficient of determination R^2 (n data points). The pipe-flow segmented fits (below and above the $Re_\tau \sim 11,000$ transition) and the rough-wall branch are included. Per-station experimental uncertainties are not reported in the source datasets [8,9,14], so the uncertainties quoted are those

of the fitted parameters; these are the quantities relevant to the reproducibility of the scaling exponents.

Table A1. Power-law fits $y = ax^b$ used in this work, with one-standard-error uncertainties and coefficient of determination R^2 (n data points). The correction terms use $\kappa_{ZPG} = 0.39$, $A_{ZPG} = 5.7$ and $\kappa_{pipe} = 0.34$, $A_{pipe} = 1.0$.

Fit	x	a	b	R^2	n
Pipe smooth, Equation (13)	λ	2.24 ± 0.11	-0.56 ± 0.01	0.997	9
Pipe rough, Equation (14)	λ	0.41 ± 0.20	-0.95 ± 0.11	0.975	4
Pipe smooth, $Re_\tau \leq 11,000$	λ	1.87 ± 0.16	-0.61 ± 0.02	0.998	4
Pipe smooth, $Re_\tau > 11,000$	λ	2.86 ± 0.40	-0.51 ± 0.03	0.990	5
ZPG drag, Equation (16)	δ	0.173 ± 0.005	-0.564 ± 0.002	1.000	27
ZPG product, Equation (22)	Re_τ	0.101 ± 0.004	-0.109 ± 0.004	0.964	27

The asymptotic ratio is fixed by the two von Kármán constants, $Q_\infty = \kappa_{ZPG}/\kappa_{pipe} = 0.39/0.34 = 1.15$; propagating an illustrative uncertainty of ± 0.01 on each κ through $dQ_\infty = Q_\infty \sqrt{(d\kappa_{ZPG}/\kappa_{ZPG})^2 + (d\kappa_{pipe}/\kappa_{pipe})^2}$ gives $Q_\infty = 1.15 \pm 0.05$.

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