

Article

Scenario-Constrained Simplified Forward Modeling of Underwater UXO Transient Electromagnetic Secondary-Field Responses and Applicability of Symmetric-Point Differencing

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Abstract

This numerical study develops a scenario-specific response atlas for underwater unexploded ordnance (UXO) and evaluates the applicability of symmetric-point differencing to center-loop transient electromagnetic (TEM) profile data. The modeled configuration comprises a 5 m water layer; a highly conductive target 0.60 m long and 0.15 m in diameter; a surface center loop with a 2 m radius, 10 turns, and a 20 A current; and target-center burial depths of 0.5–2.0 m below the seabed. Target-only induced-voltage responses are calculated using a circular-loop excitation model and an orthogonal magnetic-dipole equivalent representation under the assumptions of a compact, isolated, weakly coupled target. At 0.5 ms after turn-off, inclination primarily controls peak topology, whereas azimuth controls the principal orientation of the anomaly. For this configuration, the normalized maximum response decreases from 1.00 to approximately 0.25 as burial depth increases from 0.5 to 2.0 m, yielding the empirical, model-specific coefficient $k = 0.93 \text{ m}^{-1}$. A separate idealized background model is used solely for a mechanistic analysis of symmetric-point differencing. Under the study-defined conservative requirements $\eta_{\text{res}} \leq 0.10$ (equivalently, $R_{\text{bg}} \geq 90\%$) and $C_{\text{tar}} \geq 2$, and with a sampling interval of $\Delta\varepsilon = 0.1$, $\varepsilon = 0.1$ is the largest tested heterogeneity level that satisfies both the background-control and target-visibility requirements; at $\varepsilon \geq 0.3$, the modeled residual becomes comparable to or larger than the target anomaly. A qualitative analysis of the idealized background over 0.5–5 ms further indicates that differencing performance must be assessed channel by channel. No tank or field observations are included. The attenuation coefficient and heterogeneity intervals are therefore theoretical outputs of this single numerical scenario rather than universal field-correction parameters.

Keywords: transient electromagnetic method; underwater UXO; orthogonal magnetic dipole model; symmetric-point differencing; shallow water; background heterogeneity

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1. Introduction

Unexploded ordnance (UXO) in shallow-water and seabed environments poses a safety risk to marine construction, cable and pipeline installation, dredging, and remediation activities [1]. Acoustic methods are widely used for underwater classification, but their records can be affected by scattering and multipath propagation [2]. Passive

magnetic surveys and magnetic-gradient techniques are also important, although discrimination can be difficult for weakly magnetic targets or in disturbed magnetic backgrounds [3–7]. Transient electromagnetic (TEM) systems actively excite conductive targets and are therefore well suited to UXO detection [8–13]. In marine settings, however, the target-induced response is superimposed on responses from the conductive water column and sediment, which can reduce target visibility and complicate interpretation [14–18].

Previous electromagnetic UXO studies have established time- and frequency-domain signatures, equivalent-dipole and intrinsic-polarizability descriptions, discrimination algorithms, and receiver or survey configurations [11,19–33]. Recent numerical and portable-system investigations have examined shallow-buried or multiple targets, target characterization, localization, attitude effects, and acquisition design [10,34–38]. More broadly, three-dimensional inversion, joint interpretation, and open-source simulation frameworks in applied electromagnetics provide tools for moving beyond reduced-order analyses [31,39–42], although these studies are not specific validations of underwater UXO detection.

Despite this progress, existing studies do not provide a unified, scenario-specific atlas that links shallow-water UXO inclination and azimuth to multi-component anomaly topology while simultaneously quantifying burial-depth attenuation [14–18,22,34,35]. In addition, although differential and gradient measurements can suppress common-mode responses [5–7,43], the model-specific boundary at which lateral background heterogeneity makes symmetric-point differencing unreliable for center-loop profile data has not been stated using the residual ratio, suppression rate, and target-to-residual contrast together.

Scope and Novel Contributions

This work provides a scenario-constrained quantitative benchmark rather than a universal underwater TEM solver. First, it constructs a multi-component induced-voltage response atlas for changes in the inclination and azimuth of a 0.60 m UXO in a 5 m shallow-water setting, thereby linking peak number, polarity pattern, symmetry, and principal-axis orientation to target attitude. Second, it summarizes burial-depth attenuation over 0.5–2.0 m using an empirical coefficient of $k = 0.93 \text{ m}^{-1}$ for the specified loop, current, and 0.5 ms channel. Third, it establishes a model-specific decision framework for symmetric-point differencing. The residual ratio η_{res} and suppression rate R_{bg} are complementary forms of the same background-control criterion, whereas C_{tar} independently evaluates target visibility. Under the study-defined conservative requirements $\eta_{\text{res}} \leq 0.10$ (equivalently, $R_{\text{bg}} \geq 90\%$) and $C_{\text{tar}} \geq 2$, and with the adopted sampling interval $\Delta\epsilon = 0.1$, $\epsilon = 0.1$ is the largest tested heterogeneity level that satisfies all criteria. The resulting applicability intervals are theoretical classifications for the present idealized background model rather than universal field thresholds.

All numerical values reported in this paper apply only to the parameter set defined in Section 2.4. The orthogonal magnetic-dipole representation does not resolve complex target geometry, higher-order moments, or strong multibody coupling, and the idealized background model is not a coupled air–seawater–sediment TEM forward solution. Accordingly, the derived coefficient and thresholds are intended for sensitivity analysis and survey-design guidance, not for the direct correction of field measurements.

2. Materials and Methods

2.1. Measurement Quantity and Response Decomposition

After transmitter-current turn-off, induced currents in the target and conductive environment generate a time-dependent secondary magnetic field. A receiver coil records an induced voltage proportional to the time derivative of magnetic flux [8,9,12,13]. In this manuscript, V_x , V_y , and V_z denote the three modeled induced-voltage components. The

target-only calculations in Sections 3.1 and 3.2 are distinct from the idealized background analysis in Sections 3.3 and 3.4; consequently, the target-only maps should not be interpreted as complete field measurements in a coupled marine environment [14–18].

2.2. Circular-Loop Excitation and Orthogonal Magnetic Dipole Approximation

The transmitter is represented by a circular loop discretized into finite current elements, and the primary field is evaluated by numerically summing the Biot–Savart integral [8,12,13,37]. This standard calculation is used only to determine target excitation and is not presented as a methodological innovation.

The UXO is represented by three mutually orthogonal equivalent magnetic dipoles whose amplitudes are governed by the target polarizability tensor [20,21,23–29,32,44]. This approximation is introduced a priori and is restricted to a compact, isolated, highly conductive target whose dimensions are small relative to the transmitter–target and receiver–target distances. It is intended to reproduce the dominant low-order response morphology at modest computational cost. It does not represent detailed current circulation in irregular geometries, higher-order magnetic moments, interacting targets, or strong target–host coupling. Errors are therefore expected to increase for clustered UXO, complex fragments, strongly coupled targets, or thick, highly conductive sediments that substantially alter the excitation and propagation fields [14–18,29]. These restrictions define the interpretation limits of the attitude and burial-depth results in Sections 3.1 and 3.2.

Under the compact-target approximation, the induced magnetic dipole moment of the UXO is expressed in terms of the magnetic polarizability tensor $M(t)$ and the primary magnetic field at the target center:

$$m(r_T, t) = M(t)B_p(r_T) \quad (1)$$

where r_T denotes the target-center position, $B_p(r_T)$ is the primary excitation field generated by the transmitter loop, and $M(t)$ is a symmetric magnetic polarizability tensor. The tensor depends on the target shape, dimensions, attitude, permeability, conductivity, and transient decay characteristics.

For a receiver located at r_R , the modeled three-component target response V can then be written as

$$V = -G(R) \frac{dM(t)}{dt} B_p(r_T) = G(R)L(t)B_p(r_T) \quad (2)$$

$$G(R) = \frac{3e'_R e_R - I}{4\pi R^3} \quad (3)$$

where $R = r_R - r_T$ is the target–receiver separation vector, and $L(t) = -\frac{dM(t)}{dt}$ is the time derivative of the polarizability tensor. The dyadic Green tensor $G(R)$ relates the target position to the receiver. The tensor $L(t)$ represents the characteristic target response [36].

2.3. Symmetric-Point Differencing Post-Processing

For a selected induced-voltage component, the total profile response is decomposed as

$$V(x, t) = V_{bg}(x, t) + V_{tar}(x, t) \quad (4)$$

In a center-loop profile, the transmitter and receiver move synchronously along the survey line. Symmetric-point differencing is therefore a post-processing operation on point-by-point measurements, not a simultaneous dual-receiver measurement. For two locations at equal offsets from a selected symmetry center, the difference is

$$\Delta V(x, t) = V(+x, t) - V(-x, t) \quad (5)$$

Approximate background equality at the paired locations can be expressed as

$$V_{bg}(+x,t) \approx V_{bg}(-x,t) \quad (6)$$

$$\Delta V(x,t) \approx V_{tar}(+x,t) - V_{tar}(-x,t) \quad (7)$$

The output is a spatial-difference feature rather than the absolute target voltage at either point. It can suppress a smooth common-mode component, but it also amplifies positioning errors and any lateral mismatch in the background (Figure 1). Its reliability therefore depends on the validity of the local-symmetry assumption, the selected symmetry center, sensor repeatability, and survey positioning [5–7,43].

Symmetric-Point Differencing for Center-Loop Profile Data

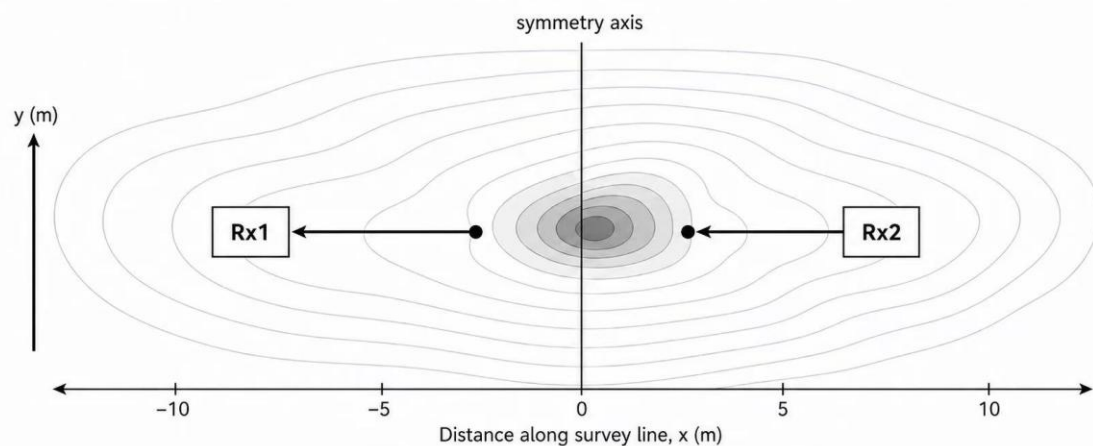


Figure 1. Geometry and notation for symmetric-point differencing of center-loop profile data.

2.4. Scenario Definition, Observation System, and Quantitative Metrics

The loop plane is defined as $z = 0$, and the seabed lies 5 m below this reference plane. The target burial depth d is the vertical distance from the sediment–water interface to the UXO center; the total loop-to-target-center separation is $H = 5 \text{ m} + d$. For the target-only attitude and burial-depth analyses in Sections 3.1 and 3.2, the target is located beneath the survey center. By contrast, the mechanistic illustration in Section 3.3 uses a single target offset from a selected background-symmetry axis so that the real target anomaly and its mirror differencing feature can be distinguished; this is a post-processing illustration rather than a second physical scenario. The transmitter–receiver system uses a center-loop configuration with a 2 m loop radius, 10 transmitter turns, and a 20 A current over a $20 \text{ m} \times 20 \text{ m}$ survey area. Target-attitude and burial-depth maps are evaluated at 0.5 ms after turn-off. This channel was selected as a representative early-to-intermediate delay with a strong modeled target response, but it does not define a universal optimum. Sections 3.3 and 3.5 address only the qualitative time dependence of idealized background mismatch over 0.5–5 ms; the numerical attenuation coefficient and ϵ intervals remain specific to the 0.5 ms channel (Table 1).

Table 1. Scenario-specific environment, target, observation-system, and time parameters.

Category	Parameter	Value
Environment	Water depth from loop plane to seabed	5 m
	Seawater conductivity	5 S/m
	Baseline sediment conductivity in idealized model	0.50 S/m
Target	UXO length/equivalent diameter	0.60 m/0.15 m

Observation	UXO conductivity	1×10^5 S/m
	Burial depth d (seabed to target center)	0.5, 1.0, 1.5, 2.0 m
	Loop radius/transmitter turns	2 m/10 turns
	Transmitter current	20 A
	Survey area	20 m $\times$ 20 m
Time	Target-response analysis channel	0.5 ms after turn-off
	Qualitative background time interval	0.5–5.0 ms

For burial-depth attenuation, $A_{\max}$ is the maximum absolute induced-voltage amplitude and A_{norm} is its value normalized to $d_0 = 0.5$ m. For the idealized background analysis, A_{bg} is the maximum background amplitude before differencing, $A_{\Delta\text{bg}}$ is the maximum residual after differencing, and $A_{\Delta\text{tar}}$ is the maximum target-related differential anomaly. The metrics are

$$R_{\text{bg}} = (1 - A_{\Delta\text{bg}}/A_{\text{bg}}) \times 100\% \quad (8)$$

$$\eta_{\text{res}} = A_{\Delta\text{bg}}/A_{\text{bg}} \quad (9)$$

$$C_{\text{tar}} = A_{\Delta\text{tar}}/A_{\Delta\text{bg}} \quad (10)$$

for the background, whereas C_{tar} evaluates whether the target differential anomaly exceeds the residual. Because $R_{\text{bg}} = (1 - \eta_{\text{res}}) \times 100\%$, R_{bg} and η_{res} are complementary expressions of a single background-control condition rather than independent constraints. To provide a transparent decision rule, a tested case is assigned to the conservative applicability region only when $\eta_{\text{res}} \leq 0.10$ (equivalently, $R_{\text{bg}} \geq 90\%$) and $C_{\text{tar}} \geq 2$. The first requirement limits the residual background to 10% of its pre-differencing amplitude, whereas the second requires the target differential anomaly to be at least twice the residual. $C_{\text{tar}} = 1$ is treated as the equal-amplitude boundary, below which the residual dominates. These dimensionless limits are study-defined, model-specific criteria rather than field-calibration standards.

2.5. Supplementary Parameter Tests

To evaluate the influence of selected observation-system and target-response parameters, three supplementary tests were performed while all remaining reference parameters were held constant. The results are summarized in Table 2.

Table 2. Representative effects of selected observation-system and target-response parameters.

Parameter Test	Test Values	Main Numerical Result	Effect on Interpretation
Transmitter current I	10, 20, and 30 A	Normalized peaks: 0.50, 1.00, and 1.50; normalized morphology unchanged.	Current mainly rescales amplitude and should be normalized when amplitude is used for depth estimation.
Profile spacing ΔX	0.05, 0.20, 0.50, and 1.00 m	At 1.00 m spacing, the peak decreased by 2.03% and the localization error increased to 0.35 m.	Coarser sampling preserves a broad anomaly but reduces localization accuracy and lobe-pattern resolution.
Target characteristic-response parameters L(t)	Representative Sets 1, 2, and 11	Relative 0.5 ms peaks: 0.37, 1.00, and 11.22; remaining amplitudes at 5 ms: 11.83%, 4.56%, and 11.07%.	Target properties affect both response strength and temporal decay, so the preferred delay channel may vary.

These tests distinguish three parameter roles in the present reduced-order model: transmitter current mainly controls signal level, observation spacing controls spatial resolution and localization, and the target characteristic-response parameters control both amplitude and decay. Independent sensitivities to seawater depth and sediment conductivity were not inferred because they require a coupled three-dimensional marine TEM model.

2.6. Numerical Accuracy and Convergence Assessment

The numerical accuracy of the circular-loop calculation and the symmetric-point differential response was evaluated before the modeled residuals were interpreted. The test used the configuration adopted in this manuscript: a 2 m radius transmitter loop with 10 turns and a 20 A current, a loop-to-target-center separation of 5.5 m, a target inclination of 45°, an azimuth of 0°, and the 0.5 ms response channel. The center-loop profile extended from −10 to 10 m with a 0.05 m sampling interval.

As an initial verification of the primary-field calculation, the numerical field on the loop axis was compared with the analytical circular-loop solution. With 1000 current elements, the absolute difference in B_z was 4.125×10^{-12} T and the relative difference was $1.645 \times 10^{-4}\%$, confirming the accuracy of the adopted circular-loop discretization.

The loop was then discretized using 500, 1000, and 2000 current elements, and the 2000-element differential response was treated as the reference solution. The maximum absolute and relative differences were evaluated as

$$E_{\text{abs}} = \max_i |\Delta V_{1000,i} - \Delta V_{2000,i}| \quad (11)$$

$$E_{\text{rel}} = \frac{\max_i |\Delta V_{1000,i} - \Delta V_{2000,i}|}{\max_i |\Delta V_{2000,i}|} \times 100\% \quad (12)$$

where i indexes all profile positions and modeled voltage components. The results are presented in Table 3.

Table 3. Numerical convergence of the symmetric-point differential response.

Loop Elements	Maximum $ \Delta V $ (V)	Absolute Difference	Relative Difference (%)
		from 2000 Elements (V)	
500	$2.3901238836 \times 10^{-15}$	$1.4743551838 \times 10^{-20}$	6.168492×10^{-4}
1000	$2.3901356784 \times 10^{-15}$	$2.9487147332 \times 10^{-21}$	1.233700×10^{-4}
2000	$2.3901386271 \times 10^{-15}$	—	—

With 1000 current elements, the maximum absolute difference from the 2000-element reference was 2.949×10^{-21} V, and the relative difference was $1.234 \times 10^{-4}\%$. The 500-element result also differed from the reference by less than 0.001%, demonstrating systematic convergence as the loop discretization was refined.

Because the theoretical difference in an exactly symmetric response approaches zero, a normalized symmetry residual was additionally evaluated using the V_z response of a centered vertical target:

$$E_{\text{sym}} = \frac{\max_i |\Delta V_{z,i}|}{\max_i |V_{z,i}|} \times 100\% \quad (13)$$

The absolute symmetry residual was approximately 4.49×10^{-28} V, and the normalized symmetry residual was approximately $9.61 \times 10^{-12}\%$. These values are at the level of floating-point cancelation and are many orders of magnitude smaller than the modeled

differential responses. The residual trends analyzed for the non-uniform cases are therefore not dominated by circular-loop discretization or symmetric-subtraction error.

All numerical modeling, data processing, and figure generation were performed using custom-written scripts in MATLAB R2024b.

3. Results

3.1. Scenario-Specific Effects of Target Attitude on Induced-Voltage Patterns

Figures 2 and 3 together constitute a scenario-specific multi-component induced-voltage response atlas for the investigated shallow-water configuration. The atlas links variations in target inclination and azimuth to changes in peak number, polarity distribution, symmetry, and principal-axis orientation.

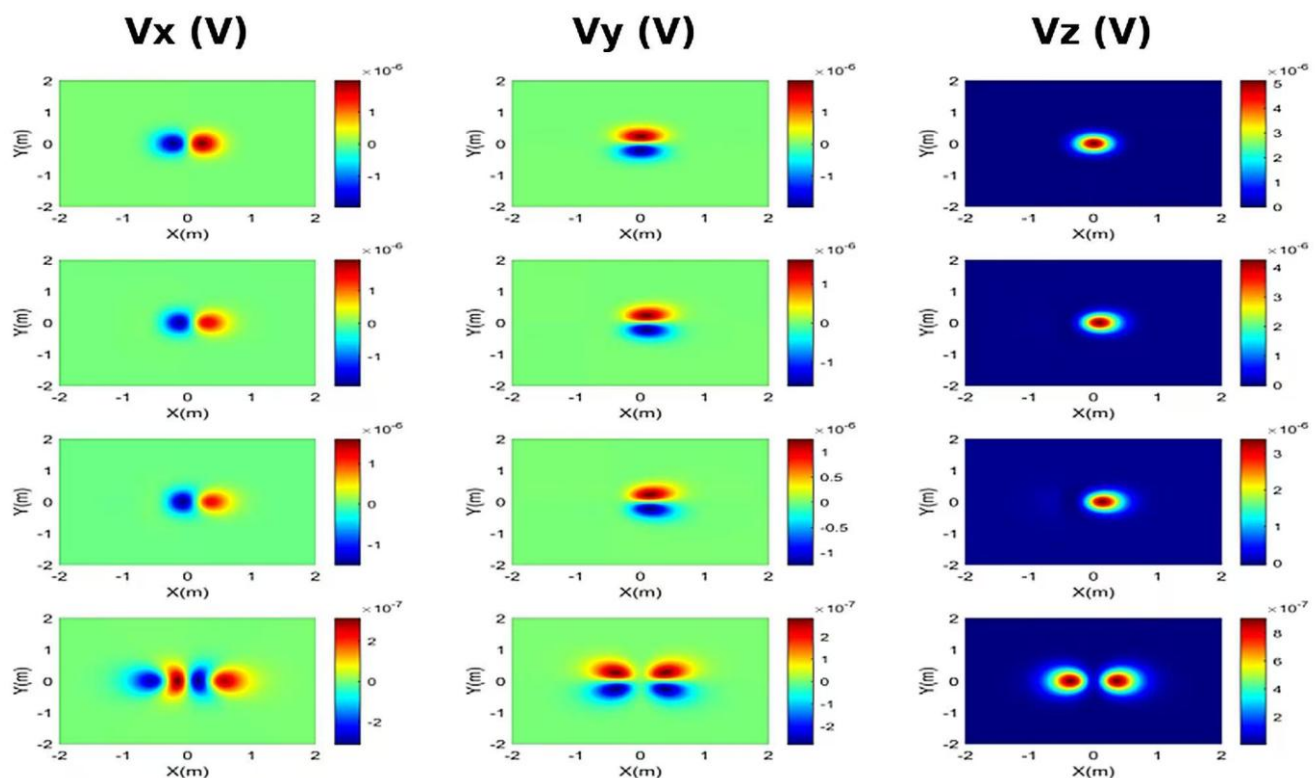


Figure 2. Inclination-dependent part of the scenario-specific multi-component response atlas.

Figure 2 presents the target-only induced-voltage components at 0.5 ms for azimuth $\varphi = 0^\circ$ and inclinations $\theta = 0^\circ, 30^\circ, 45^\circ$, and 90° . The columns correspond to V_x , V_y , and V_z . At $\theta = 0^\circ$, V_x and V_y exhibit orthogonal bipolar patterns, while V_z forms a positive, approximately axisymmetric peak. Increasing inclination progressively modifies the polarity distribution and peak topology. At $\theta = 90^\circ$, V_x develops four alternating lobes, and V_y and V_z form double-peak patterns. Within this model, V_z has the largest amplitude and provides the clearest inclination-related transition from a single peak to a double peak. These maps describe the dominant dipole response of an isolated target and should not be directly transferred to complex multi-object or strongly coupled seabed settings.

Figure 3 isolates the effect of azimuth by fixing $\theta = 90^\circ$ and rotating the target in the horizontal plane. As φ increases from 0° to 90° , the anomaly lobes rotate with the target principal axis. The peak count and polarity arrangement remain associated with the horizontal posture, whereas the orientation of the symmetry axis changes. Thus, inclination is primarily inferred from topology, including the peak number and polarity pattern, while

azimuth is inferred from the principal direction of the multi-component anomaly. This interpretation requires a known survey-coordinate system and adequate spatial sampling.

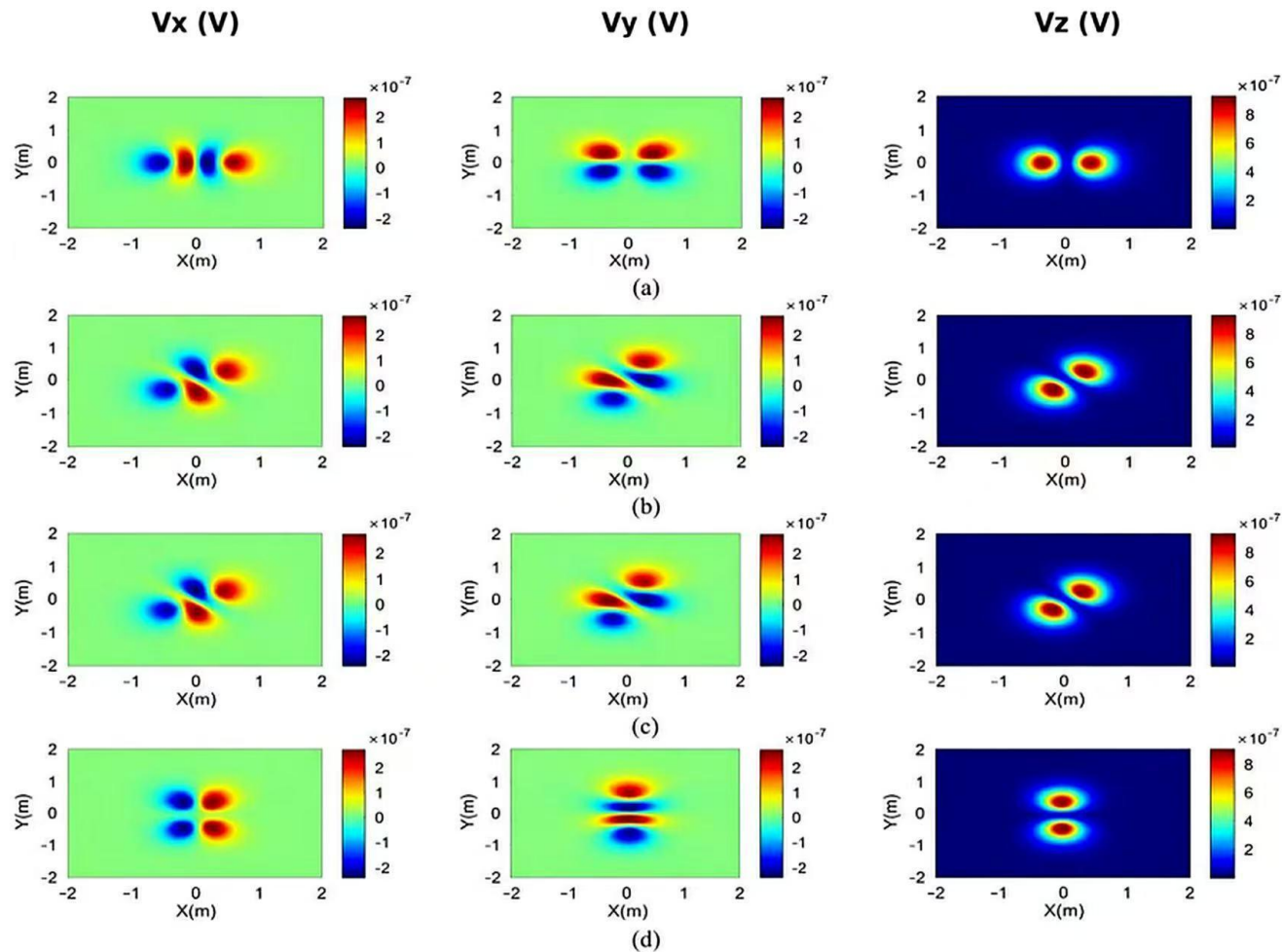


Figure 3. Azimuth-dependent part of the scenario-specific multi-component response atlas.

Table 4 summarizes the component-specific morphology and corresponding attitude criteria. Because these criteria are based on response topology rather than absolute amplitude, they are qualitative and model-specific.

Table 4. Summary of the scenario-specific multi-component attitude-response atlas and interpretation criteria at 0.5 ms.

Attitude Case	Vx Response Feature	Vy Response Feature	Vz Response Feature	Interpretation Criterion
Inclination $\theta = 0^\circ$ ($\varphi = 0^\circ$)	Bipolar; approximate y -axis symmetry.	Bipolar; approximate x -axis symmetry.	Single positive central peak.	Near-vertical posture: single-peaked Vz with orthogonal Vx–Vy bipolarity.
Inclination $\theta = 30\text{--}45^\circ$ ($\varphi = 0^\circ$)	Positive lobe expands; asymmetry increases.	Two lobes extend toward +x.	Central peak progressively deforms and shifts.	Intermediate inclination: progressive loss of axisymmetry across components.
Inclination $\theta = 90^\circ$ ($\varphi = 0^\circ$)	Four alternating lobes aligned with x.	Two peaks; approximate x -axis symmetry.	Two peaks; approximate y -axis symmetry.	Horizontal posture: four-lobe Vx together with split Vy and Vz responses.

Azimuth φ = 30–60° ($\theta = 90^\circ$)	Lobe system rotates obliquely.	Symmetry axis rotates with the target.	Double-peak axis rotates with the target.	Azimuth is estimated from the common principal-axis rotation.
Azimuth φ = 90° ($\theta = 90^\circ$)	Two peaks; approximate y -axis symmetry.	Four alternating lobes aligned with y .	Two peaks; approximate x -axis symmetry.	A ~90° rotation is indicated by interchange of the x - and y -oriented patterns.

Notes: The table applies to the target-only orthogonal-dipole model with a 5 m water depth, a 0.60 m target, a 2 m radius loop, a 20 A current, and a 0.5 ms channel. The criteria describe the dominant morphology and should not be used as stand-alone field-classification rules without multichannel and experimental validation.

These morphology-based criteria provide the attitude reference for the following depth analysis. Section 3.2 therefore examines amplitude attenuation while retaining the attitude-controlled pattern as the interpretive baseline.

3.2. Burial-Depth Attenuation and a Model-Specific Empirical Relation

After the attitude-related topology is established, burial depth is varied to quantify amplitude loss under the same observation configuration. Figure 4 shows V_x and V_z profiles along $y = 0$ at 0.5 ms for several inclinations. Increasing d reduces the amplitude while largely preserving the attitude-controlled pattern. Consequently, attitude and burial depth are not independent in inversion: posture controls which component and peak should be measured, whereas depth controls their amplitude and spatial width. The results are presented in Table 5.

Table 5. Normalized maximum induced-voltage amplitude for different target-center burial depths d .

Burial Depth D (Seabed to Target Center, m)	Loop-to-Target Center separation H (m)	Normalized Maximum Amplitude	Relative Amplitude	Attenuation Factor
0.5	5.5	1.00	100.00%	1.0
1.0	6.0	0.61	61.03%	1.64
1.5	6.5	0.39	38.61%	2.59
2.0	7.0	0.25	25.20%	3.97

Using the shallowest case, $d_0 = 0.5$ m, as the reference, the simulated maximum amplitudes were normalized as

$$A_{\text{norm}}(d) = A_{\text{max}}(d)/A_{\text{max}}(d_0) \quad (14)$$

The normalized amplitudes at $d = 0.5, 1.0, 1.5$, and 2.0 m are 1.00, 0.61, 0.39, and 0.25, respectively. The successive amplitude ratios for each 0.5 m increase in burial depth are therefore 0.61, 0.63, and 0.65. Their similarity indicates an approximately constant multiplicative attenuation over the investigated depth interval, which motivates the use of the empirical exponential relation:

$$A_{\text{norm}}(d) = \exp[-k(d - d_0)], \text{ with } k \approx 0.93 \text{ m}^{-1} \quad (15)$$

where $A_{\text{max}}(d)$ is the maximum absolute profile voltage at burial depth d , and k is an empirical attenuation coefficient.

For this scenario, which uses a 5 m water depth, a 2 m radius loop, a 20 A current, and a 0.5 ms channel, an additional meter of burial corresponds to a modeled peak-amplitude factor of $\exp(-0.93) \approx 0.39$. This coefficient is neither a material constant nor a universal

depth-inversion law. Changes in water depth, loop geometry or height, target dimensions and conductivity, attitude, sediment properties, or time channel will alter the fitted value.

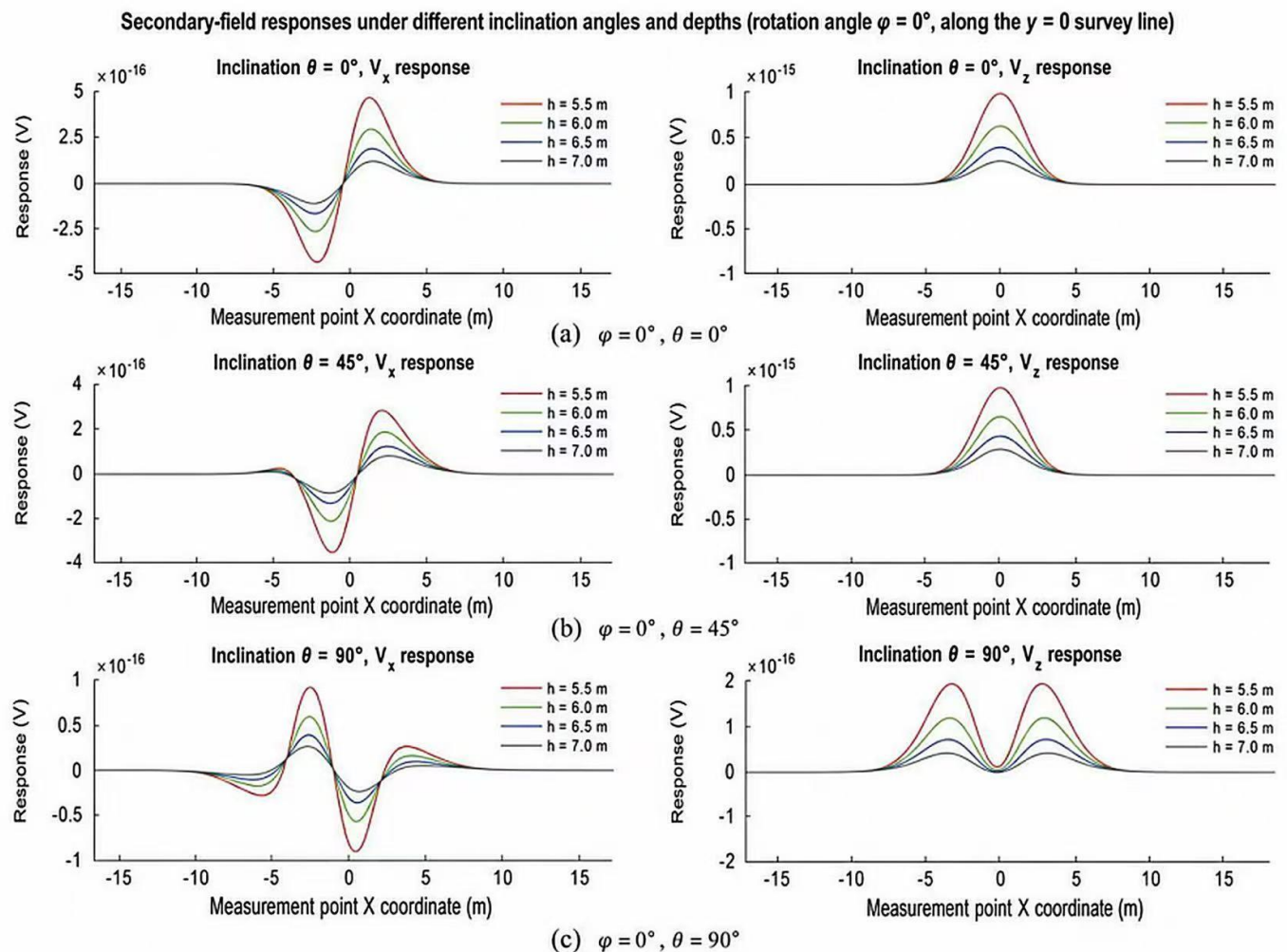


Figure 4. Target-only induced-voltage profiles V_x and V_z (V) along $y = 0$ at 0.5 ms for different inclinations and loop-to-target-center separations H .

3.3. Mechanism of Background Residuals in Symmetric-Point Differencing

The preceding maps contain only the reduced-order target response. Practical data also contain contributions from seawater, sediment, the transmitter, and instrumentation [14–18]. Figure 5 therefore uses normalized synthetic responses to illustrate the mechanism rather than a complete marine TEM solution. A single target is intentionally placed to the right of the selected background-symmetry axis. Under $\Delta V(x,y) = V(x,y) - V(-x,y)$, the real target appears as a positive anomaly, and its mirror differencing feature appears as a negative lobe; the negative lobe is not a second target. In the horizontally symmetric case, paired locations contain nearly the same smooth background, whereas lateral background mismatch leaves a residual that can distort the target anomaly or be mistaken for it.

Figure 6 extends this mechanism into the time domain. $V(r_1, t)$ and $V(r_2, t)$ denote the modeled background voltage-decay curves at the two symmetric observation positions. Their difference is $\Delta V(t) = V(r_1, t) - V(r_2, t)$.

For a uniform background, the normalized responses at the paired points remain nearly coincident over 0.5–5 ms, and ΔV remains close to zero. For a laterally varying background, the amplitudes and decay rates differ, producing a time-dependent residual.

The figure demonstrates that spatial symmetry at one channel does not guarantee identical decay behavior at every channel.

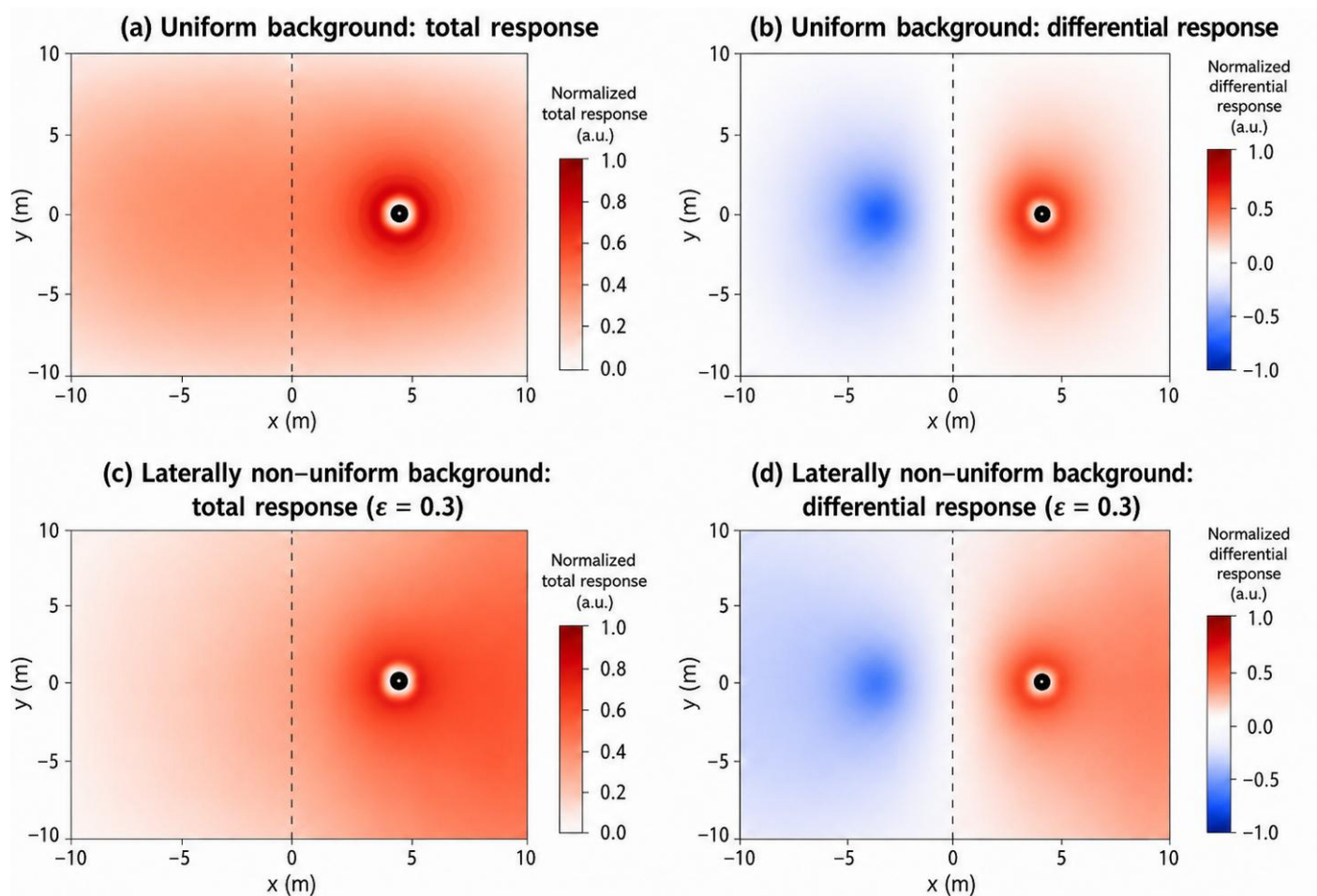


Figure 5. Normalized total and differenced responses (arbitrary units) for horizontally uniform and laterally non-uniform idealized backgrounds.

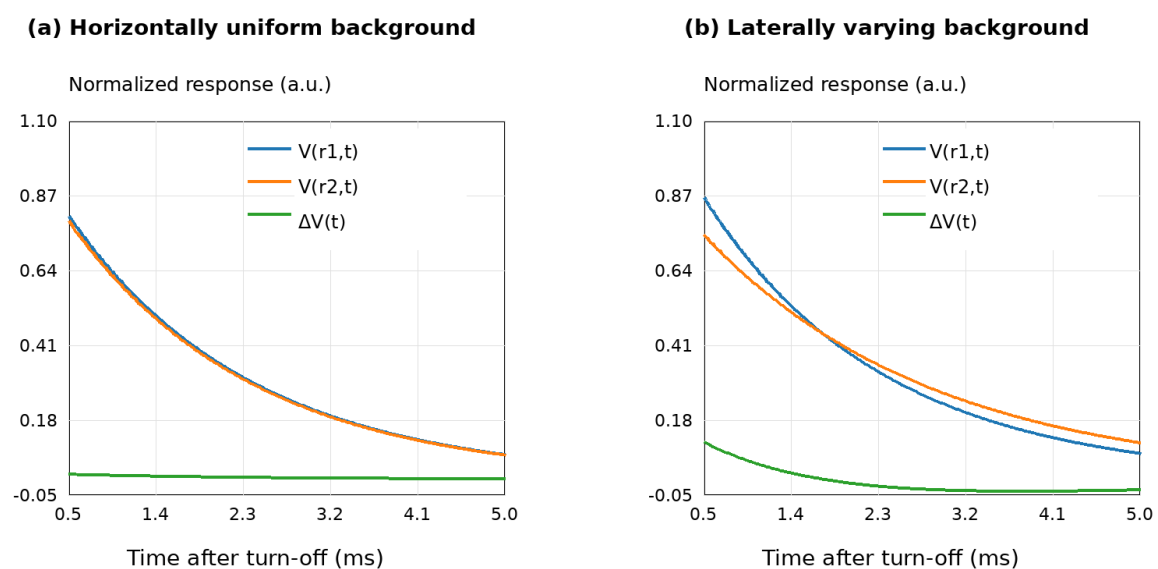


Figure 6. Normalized induced-voltage responses $V(r1,t)$, $V(r2,t)$, and $\Delta V(t)$ (arbitrary units) from 0.5 to 5 ms for uniform and laterally varying idealized backgrounds.

3.4. Model-Specific Quantitative Applicability of Symmetric-Point Differencing

To quantify the effect of lateral background mismatch on symmetric-point differencing, a deliberately simplified, normalized background model is introduced while the basic symmetric spatial envelope is held fixed. This relation is not a coupled air–seawater–sediment TEM solution and is not intended to reproduce a unique physical marine background. Instead, it provides a controllable mechanistic model for examining how the progressive loss of lateral symmetry generates a differential residual. The idealized background voltage is expressed as

$$V_{bg}(x) = V_0 \exp\left[-\left(\frac{x}{a}\right)^2\right] \left(1 + \varepsilon \frac{x}{L}\right) \quad (16)$$

where V_0 is the common normalized background-amplitude scale, a controls the spatial width of the smooth background envelope, L is the half-width of the 20 m survey line, and ε is a dimensionless parameter controlling lateral amplitude mismatch.

The model contains two factors. The Gaussian term $V_0 \exp\left[-\left(\frac{x}{a}\right)^2\right]$ represents a smooth, even background centered at $x = 0$.

The multiplicative term $1 + \varepsilon \frac{x}{L}$ introduces a controlled left–right amplitude difference.

This factor is a first-order odd perturbation and can be regarded as the lowest-order approximation of a general smooth lateral perturbation near the survey center. When $\varepsilon = 0$, the model reduces to a perfectly symmetric Gaussian background. As ε increases, the background is enhanced on the $x > 0$ side and reduced on the $x < 0$ side, while its overall smooth envelope is retained.

The residual produced by symmetric-point subtraction is therefore

$$\Delta V_{bg}(x) = V_{bg}(x) - V_{bg}(-x) = 2V_0 \varepsilon \frac{x}{L} \exp\left[-\left(\frac{x}{a}\right)^2\right] \quad (17)$$

Equation (17) shows that a perfectly symmetric background, $\varepsilon = 0$, is completely removed by subtraction. For $\varepsilon > 0$, the residual is an odd function:

$$\Delta V_{bg}(-x) = -\Delta V_{bg}(x) \quad (18)$$

It therefore crosses zero at the symmetry center and has opposite signs on the two sides of the profile. The residual is small near $x = 0$ because of the factor x , and it also decreases far from the center because of the Gaussian decay. Differentiation of Equation (17) shows that its positive and negative extrema occur at $|x| = \frac{a}{\sqrt{2}}$. The corresponding maximum absolute residual is $\max_x |\Delta V_{bg}(x)| = V_0 \sqrt{\frac{2}{e}} \frac{a}{L} \varepsilon$.

Figure 7a shows that all background profiles retain a smooth Gaussian-like envelope, but increasing ε progressively strengthens one side and weakens the other. Figure 7b shows the corresponding bipolar residuals. All residual curves pass through zero at $x = 0$, have opposite signs on the two sides, and reach their extrema at approximately $x = \pm 4.95$ m. Because Equation (17) is linear in ε , increasing heterogeneity changes the residual amplitude but not the theoretical extremum positions when a remains fixed. The curve shapes in Figure 7 are therefore direct consequences of the adopted Gaussian-plus-linear mechanistic model and should not be interpreted as universal signatures of real marine backgrounds.

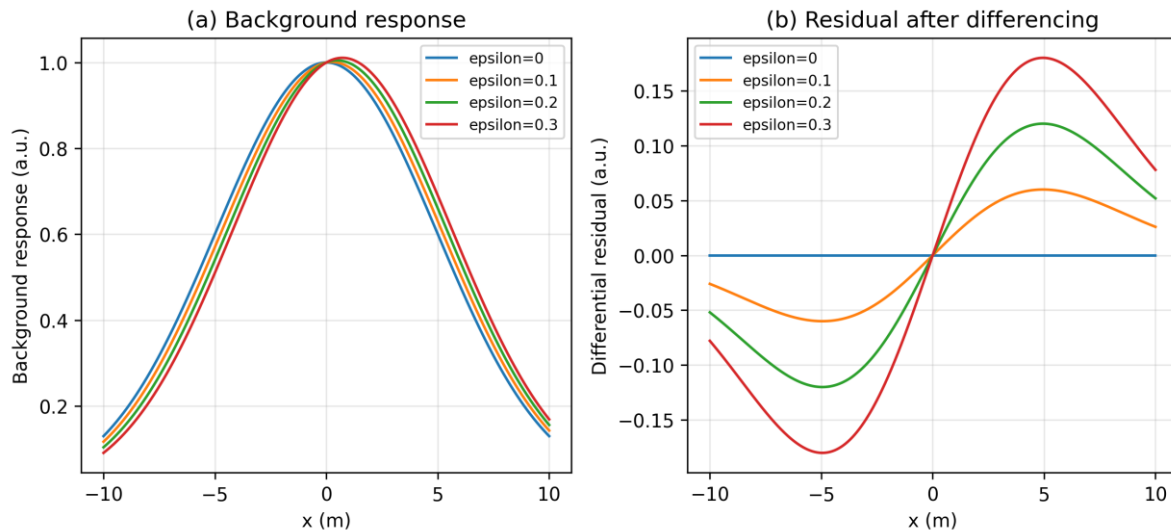


Figure 7. Normalized idealized background response and symmetric-point residual (arbitrary units) for different ϵ values.

The nominal sediment-conductivity ranges used to label the heterogeneity cases are illustrative mappings around the 0.50 S/m baseline. They are not outputs of coupled sea-water–sediment forward modeling and should not be used as field-calibration intervals without site-specific simulation or measurement. The results are presented in Table 6.

Table 6. Model-specific applicability metrics for symmetric-point differencing under the idealized background function.

ϵ	Background Type	Illustrative Sediment-Conductivity Range	η_{res}	R_{bg} (%)	C_{tar}
0.0	Horizontally layered	0.50–0.50 S/m	0.000	100.0	—
0.1	Weakly non-uniform	0.45–0.55 S/m	0.060	94.0	2.17
0.2	Moderately non-uniform	0.40–0.60 S/m	0.120	88.0	1.08
0.3	Strongly non-uniform	0.35–0.65 S/m	0.178	82.2	0.72

Notes: The conductivity ranges are illustrative mappings used to label ϵ ; they must not be used as field correction intervals without site-specific forward modeling or calibration. The conservative model-specific criteria are $\eta_{\text{res}} \leq 0.10$ (equivalently, $R_{\text{bg}} \geq 90\%$) and $C_{\text{tar}} \geq 2$; $C_{\text{tar}} = 1$ is the equal-amplitude boundary.

The applicability assessment follows the decision rule defined in Section 2.4. At $\epsilon = 0$, the residual vanishes by construction. At $\epsilon = 0.1$, $\eta_{\text{res}} = 0.060$ ($R_{\text{bg}} = 94.0\%$) and $C_{\text{tar}} = 2.17$; therefore, both the background-control and target-visibility requirements are satisfied. At $\epsilon = 0.2$, η_{res} increases to 0.120 ($R_{\text{bg}} = 88.0\%$) and C_{tar} decreases to 1.08, so the case no longer satisfies the conservative criteria and leaves little margin for noise, positioning error, or model mismatch. At $\epsilon = 0.3$, $C_{\text{tar}} = 0.72 < 1$, indicating that the modeled background residual exceeds the target anomaly. Consequently, $\epsilon = 0.1$ is the largest tested heterogeneity level in the conservative applicability region. Because the evaluated ϵ values are discrete, this result does not imply that the exact continuous critical point is $\epsilon = 0.1$.

For practical interpretation of the present model, the results are organized into three applicability intervals:

- (i) Conservative applicability region ($\varepsilon \leq 0.1$): The modeled residual remains limited and the target differential anomaly is at least twice the residual. Symmetric-point differencing can be used as a preliminary background-suppression step, subject to the assumptions of the idealized model.
- (ii) Transitional region ($0.1 < \varepsilon < 0.3$): The conservative criteria are no longer satisfied, and the target anomaly progressively approaches the residual level. Differencing should be combined with independent background estimation, repeat measurements, and uncertainty analysis.
- (iii) Residual-dominated region ($\varepsilon \geq 0.3$): The modeled residual is comparable to or larger than the target anomaly; symmetric-point differencing should not be used as the sole background-suppression method.

The continuous sweep in Figure 8 displays the background-control component of this classification: η_{res} increases monotonically and R_{bg} decreases in exact correspondence because $R_{bg} = (1 - \eta_{res}) \times 100\%$. The two curves are therefore complementary presentations of one criterion rather than independent evidence.

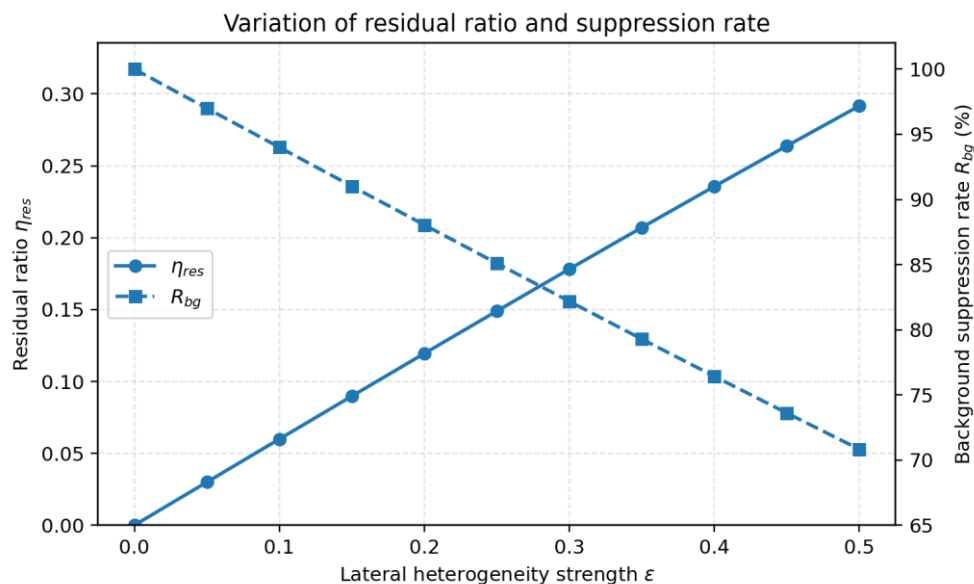


Figure 8. Model-specific variation in residual ratio η_{res} and background suppression rate R_{bg} with lateral heterogeneity ε .

Figure 9 provides the independent target-visibility component of the classification. The dashed $C_{tar} = 1$ line is the equal-amplitude boundary, below which the residual dominates. The more conservative requirement $C_{tar} \geq 2$ adopted in this study provides a safety margin for unmodeled noise and positioning error. The decrease from $C_{tar} = 2.17$ at $\varepsilon = 0.1$ to 1.08 at $\varepsilon = 0.2$ and 0.72 at $\varepsilon = 0.3$ explains why a high background-suppression percentage alone is insufficient to establish applicability.

Finally, Figure 10 compares symmetric-point differencing with subtraction of non-symmetric locations. Even for a smooth background, non-symmetric pairs retain a large residual because their background amplitudes are intrinsically different. This comparison reinforces the physical premise of the interval classification: the method is effective only when the selected pair has sufficiently similar background responses, rather than merely because two measurements are subtracted.

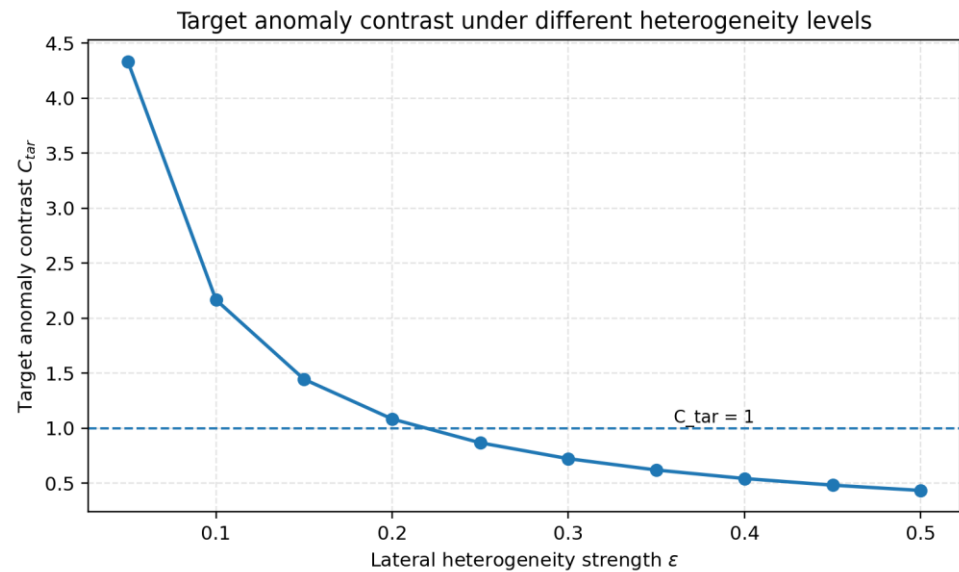


Figure 9. Model-specific target anomaly contrast C_{tar} as a function of lateral heterogeneity ϵ ; $C_{tar} = 1$ is the equal-amplitude boundary.

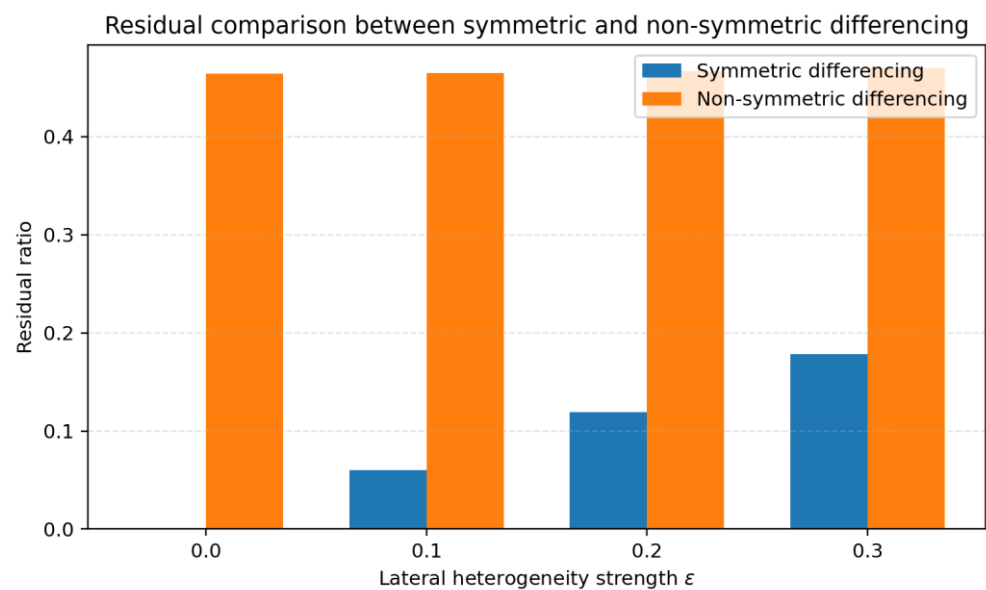


Figure 10. Residual-ratio comparison between symmetric-point and non-symmetric subtraction in the idealized background model.

3.5. Time-Window Dependence and Interpretation

The numerical attitude atlas and depth coefficient are evaluated at 0.5 ms, whereas Figure 6 illustrates idealized background decay over 0.5–5 ms. Near the earliest usable delay, the target amplitude can be high, but transmitter turn-off effects, primary-field residuals, seawater response, and timing errors can also be strongest. At intermediate delays, common-mode background amplitudes decrease, but differences in the decay rate can continue to generate a residual. At late delays, both target and background responses weaken, and instrument noise may dominate. A smaller absolute late-time residual therefore does not necessarily imply a larger target-to-residual contrast.

For multichannel data, $\eta_{res}(t)$, $R_{bg}(t)$, and $C_{tar}(t)$ should be calculated independently at each delay or over a predefined time gate, and the spatial position and principal direction of the anomaly should remain consistent across adjacent channels. The $\epsilon \leq 0.1$

recommendation in Section 3.4 applies only to the present 0.5 ms model. Figure 6 demonstrates time-dependent background mismatch; it does not simulate the full late-time eddy-current decay of a target coupled to seawater and sediment. A complete temporal assessment requires a coupled marine forward model and calibrated multichannel measurements, which are beyond the present numerical study.

4. Discussion

4.1. Relation to Previous Work and Engineering Interpretation

Previous UXO TEM research has established equivalent-polarizability models, discrimination algorithms, survey configurations, and underwater multilayer responses [11,14–30,32–35,44,45]. The present study complements rather than replaces those approaches. Its first added element is a compact response atlas that separates inclination-controlled topology from azimuth-controlled orientation in a fixed shallow-water geometry. Its second is a transparent, configuration-specific burial-depth coefficient. Its third is the joint use of η_{res} , R_{bg} , and C_{tar} to express when lateral background mismatch defeats symmetric-point differencing.

For engineering interpretation, attitude should be inferred from multiple features rather than peak amplitude alone: peak number and polarity distribution provide inclination indicators, whereas the principal anomaly axis provides an azimuth indicator. The V_z component is particularly useful for identifying the transition from a single peak to a double peak in this model. Depth estimates based on $k = 0.93 \text{ m}^{-1}$ are only initial, scenario-specific estimates because amplitude also depends on target size, conductivity, attitude, loop height and radius, water depth, sediment conductivity, and time channel. Neither the response atlas nor k should be used as a stand-alone field-classification or depth-calibration rule; instead, they should provide initial hypotheses that are checked against multichannel spatial consistency and site-specific modeling.

4.2. Validity of the Reduced-Order Target Model

The orthogonal magnetic-dipole approximation is appropriate for the dominant low-order responses of a compact, isolated target when the observation scale exceeds the target dimensions and intertarget coupling is weak [32,44]. It cannot resolve detailed current paths in irregular casings, higher-order magnetic moments, multiple-object interactions, or strong coupling with thick, conductive sediments [17,18,33,45]. The response maps in Sections 3.1 and 3.2 should therefore be interpreted as morphology and sensitivity results. In complex seabed settings, full three-dimensional modeling is required before the atlas or attenuation relation is applied.

4.3. Idealized Background Model and Temporal Dependence

The Gaussian-plus-linear background function intentionally omits layered electromagnetic diffusion, bathymetric variation, sediment interfaces, system drift, and random noise. Consequently, the reported residual ratios and thresholds are theoretical, dimensionless outputs that explain a mechanism; they are unsuitable for the direct correction of field data. Figure 6 and Section 3.5 further show that background mismatch is time-dependent. Practical implementation should estimate local symmetry and the three metrics at each selected delay rather than transfer the 0.5 ms threshold to early or late channels.

4.4. Validation Status and Field Recommendations

No tank or field measurements are included in the present study; therefore, the reduced-order target model, idealized-background thresholds, and proposed engineering workflow have not been physically validated. The conclusions are stated as numerical hypotheses. A

staged validation should first compare the reduced-order calculations with coupled three-dimensional simulations, then with controlled tank measurements using known targets and backgrounds, and finally with shallow-water field profiles [16–18,30,31,38].

Figure 11 summarizes a conditional field-use workflow. Because ϵ is a model parameter rather than a directly measurable site quantity, background and repeat lines should be used to estimate $\eta_{\text{res}}(t)$, $R_{\text{bg}}(t)$, and $C_{\text{tar}}(t)$ at each selected channel. Differencing should be retained only as an auxiliary step when the local-symmetry assumption, repeatability, and target contrast are stable. If these checks fail or vary strongly with time, independent background estimation and site-specific three-dimensional modeling are required. The attitude atlas and k should then serve only as initial priors, followed by tank, known-target, or site-specific calibration.

Conditional Field-Use Workflow for Symmetric-Point Differencing

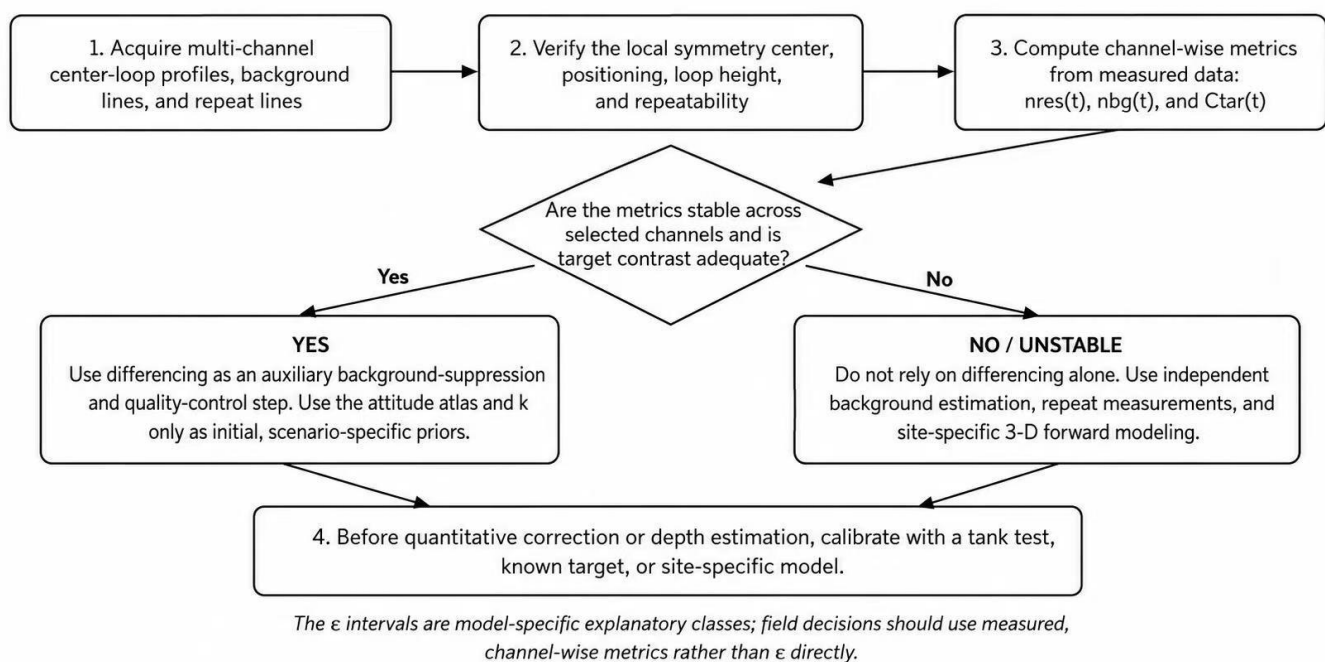


Figure 11. Conditional field-use workflow for symmetric-point differencing.

5. Conclusions

This numerical study provides a scenario-constrained benchmark for a 5 m shallow-water environment, a 0.60 m isolated UXO, a 2 m radius center loop with 10 turns and a 20 A current, and a 0.5 ms target-analysis channel. The principal conclusions and practical implications are as follows:

(1) Target attitude should be interpreted from the coordinated morphology of the three induced-voltage components rather than from peak amplitude alone. Under the investigated configuration, a near-vertical target is characterized by a single central V_z peak together with approximately orthogonal bipolar V_x and V_y patterns. Progressive loss of symmetry, peak displacement, and peak splitting indicate an intermediate inclination, whereas a horizontal target is characterized by a four-lobe horizontal-component pattern combined with split V_y and V_z anomalies. After the inclination-related topology has been identified, azimuth can be estimated from the common rotation of the principal anomaly axes across the components. For engineering interpretation, an attitude estimate should be accepted only when the dominant topology and principal direction are mutually consistent in at least two components and remain stable over adjacent measurement positions

or time channels. These criteria are morphology-based references for the present model rather than stand-alone field-classification rules.

(2) For target-center burial depths of 0.5–2.0 m, the normalized maximum response decreases from 1.00 to approximately 0.25 and is represented by the configuration-specific relation $A_{\text{norm}} = \exp[-k(d - d_0)]$, with $k = 0.93 \text{ m}^{-1}$. Because depth is inferred from response amplitude, attenuation caused by unfavorable target attitude, smaller size or lower conductivity, increased loop height or water depth, sediment effects, and background residuals may be misinterpreted as greater burial depth, whereas amplitude enhancement may lead to depth underestimation. The largest systematic errors are expected when the target attitude or response component is incorrectly identified, or when k is transferred to a substantially different survey configuration. Therefore, attitude should first be constrained using multi-component morphology, and the attenuation coefficient should be recalibrated before quantitative depth inversion.

(3) In the idealized background model, $\varepsilon = 0.1$ is the largest tested heterogeneity level satisfying the study-defined conservative requirements $\eta_{\text{res}} \leq 0.10$ ($R_{\text{bg}} \geq 90\%$) and $C_{\text{tar}} \geq 2$. Accordingly, $\varepsilon \leq 0.1$ is classified as the conservative applicability interval, $0.1 < \varepsilon < 0.3$ as a transitional interval, and $\varepsilon \geq 0.3$ as a residual-dominated interval. These ε intervals are mechanistic model classes, not field observables or universal thresholds. Field decisions should instead use measured, channel-wise $\eta_{\text{res}}(t)$, $R_{\text{bg}}(t)$, and $C_{\text{tar}}(t)$, together with repeatability and positioning checks; differencing should not be used alone when the criteria fail or vary strongly with time.

(4) The metrics and thresholds should be evaluated at each time channel. Field implementation requires accurate positioning, a verified local-symmetry center, repeat or background measurements, multichannel quality control, and site-specific validation. Symmetric-point differencing is therefore an auxiliary processing option rather than an automatic correction, whereas the attitude atlas and attenuation coefficient are initial interpretation priors rather than calibrated field relations.

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