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Single-Snapshot Direction of Arrival Estimation for Vehicle-Mounted Millimeter-Wave Radar via Fast Deterministic Maximum Likelihood Algorithm

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Abstract: As one of the fundamental vehicular perception technologies, millimeter-wave radar's accuracy in angle measurement affects the decision-making and control of vehicles. In order to enhance the accuracy and efficiency of the Direction of Arrival (DoA) estimation of radar systems, a super-resolution angle measurement strategy based on the Fast Deterministic Maximum Likelihood (FDML) algorithm is proposed in this paper. This strategy sequentially uses Digital Beamforming (DBF) and Deterministic Maximum Likelihood (DML) in the Field of View (FoV) to perform a rough search and precise search, respectively. In a simulation with a signal-to-noise ratio of 20 dB, FDML can accurately determine the target angle in just 16.8 ms, with a positioning error of less than 0.7010. DBF, the Iterative Adaptive Approach (IAA), DML, Fast Iterative Adaptive Approach (FIAA), and FDML are subjected to simulation with two targets, and their performance is compared in this paper. The results demonstrate that under the same angular resolution, FDML reduces computation time by 99.30% and angle measurement error by 87.17% compared with the angular measurement results of two targets. The FDML algorithm significantly improves computational efficiency while ensuring measurement performance. It provides more reliable technical support for autonomous vehicles and lays a solid foundation for the advancement of autonomous driving technology.

Keywords: DoA estimation; FDML; millimeter-wave radar; super-resolution



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1. Introduction

Millimeter-wave radar technology plays a crucial role in modern autonomous systems. Due to its high reliability in various weather conditions and its ability to detect small objects, millimeter-wave radar has become a key component of automotive safety systems. Compared to Light Detection and Ranging (LiDAR), it offers advantages like smaller size, lower cost, and higher angular resolution, making it ideal for autonomous vehicles.

Direction of arrival (DoA) estimation stands as a central task in millimeter-wave radar systems. Accurate DoA estimation is crucial for vehicles to achieve precise environmental perception, prevent collisions, and enhance road safety. Due to the dynamic changes and multi-path effects in the automotive environment, achieving high-precision DoA estimation is a significant challenge.

Initial methodologies for DoA estimation, named digital beamforming (DBF) [1], have angular resolution limited by the physical aperture of the array. Conventional DoA estimation algorithms, such as Multiple Signal Classification (MUSIC) [2,3] and Estimating Signal Parameters via Rotational Invariance Techniques (ESPRIT) [4], rely on multiple snapshots to improve estimation accuracy. These algorithms perform well in static or

low-dynamic environments, but in high-speed moving car scenarios, they may only obtain limited or single snapshots, which limits their application.

The scholarly community has proposed approaches based on the principles of Compressed Sensing (CS) [5,6] and the Orthogonal Matching Pursuit algorithm (OMP) [7], aiming to address the single-snapshot DoA estimation dilemma. However, these methodologies may exhibit elevated side-lobe levels under conditions characterized by coherent sources. In [8], the iterative adaptive approach (IAA) demonstrated robust performance in scenarios characterized by limited snapshots and the existence of coherent sources. However, its computational complexity is significant, requiring an iterative process across all observed directions to the DoA. In [9], the fast iterative adaptive approach (FIAA) was proposed with the aim of mitigating the computational inefficiencies inherent in IAA. Nevertheless, it does not provide assurances regarding the accuracy of angle estimation. The Deterministic Maximum Likelihood (DML) algorithm [10–12], on the other hand, adopts a model-based estimation approach. It determines parameters by maximizing the likelihood function of the signal, thereby eliminating the need for reliance on the number of snapshots. The strength of this methodology lies in its insensitivity to single-snapshot data, enabling reliable DoA estimation, even in scenarios with limited data acquisition. However, the DML algorithm needs to process a large amount of data and perform complex iterative calculations, and it becomes a bottleneck for real-time processing.

In recent developments, inspired by traditional machine learning paradigms, a neural network dubbed the Iterative Hard Thresholding network (IHT-Net) [13], predicated on the Iterative Hard Thresholding algorithm, has been introduced. This network has demonstrated rapid convergence and heightened accuracy in the realms of signal reconstruction and DOA estimation. Additionally, in the scholarly work reported in [14], a neural network approach based on Dimension Alternating Full Connection (DAFC) was proposed, which leverages a solitary instance of an NN to concurrently perform source enumeration and DOA estimation. The advent of deep learning technologies has also furnished novel perspectives for DOA estimation. The authors of [15] propounded Complex Value Deep Convolutional Networks (CV-DCNs), showcasing their formidable capacity to address complex patterns and nonlinear issues. However, the deployment of deep convolutional networks is often contingent upon access to substantial training datasets and computational resources, which may be impractical in scenarios where the costs associated with data collection are prohibitive.

In summary, the performance of DoA algorithms is crucial for Advanced Driver Assistance Systems (ADASs). However, this technology faces several challenges in practical applications, including the following:

1. Issues with signal processing precision: In single-snapshot signal processing, DoA algorithms are frequently constrained by the sampling time and processing capabilities, resulting in suboptimal estimation accuracy. This limitation may arise from inadequate time-frequency characteristics of the signal or the algorithm's susceptibility to noise and multipath effects.
2. Issues with computational complexity and real-time requirements: Super-resolution algorithms, such as IAA or DML optimization algorithms, although capable of providing higher resolution and precision, often come with heightened computational complexity. In the vehicular domain, ADASs impose strict demands for real-time performance. Consequently, the extended computational duration of these algorithms can pose a bottleneck in system capability.

In response to these challenges and opportunities, this paper explores an efficient DoA estimation algorithm strategy known as Fast Deterministic Maximum Likelihood (FDML). The algorithm optimizes the real-time performance and resolution of DoA. It notably reduces time complexity and facilitates precise DoA estimation within stringent time constraints. This algorithm, the focal point of this paper, has been meticulously optimized for real-time performance and high resolution. It notably reduces time complexity, facilitating precise DoA estimation within stringent time constraints. First, based on virtual

array elements, the target azimuth angle is roughly calculated using the DBF algorithm. Subsequently, utilizing the estimated azimuth angles, the research narrows down the region of interest. Finally, leveraging the DML algorithm and a virtual array model, we conduct a targeted search in the vicinity of potential angles of the target. This enhances angular resolution significantly and reduces time complexity greatly. In the analytical phase, the research assesses the efficacy of the algorithm strategy by examining two far-field, non-coherent sources within the angular resolution.

The results demonstrate that the FDML algorithm notably surpasses other DoA estimation algorithms in terms of angular estimation accuracy. By leveraging the statistical properties and structural information of the signal, the FDML algorithm achieves precise estimation of the target's direction. Furthermore, while maintaining angular resolution equivalent to that of other DoA estimation algorithms, the FDML algorithm has exhibited significant enhancements in computational efficiency, achieving a reduction in computation time of up to 99.30%. These improvements signify that the FDML algorithm ensures high-resolution angular estimation while significantly reducing the computational resources required in practical applications. This enhancement enhances the algorithm's utility and real-time capabilities.

The remainder of this paper is structured as follows. Section 2 delineates the virtual array of the radar and the echo signal model. Expounding on this framework, Section 3 elucidates the FDML algorithm. Section 4 outlines four simulation experiments designed to validate the effectiveness of the proposed algorithm. Finally, Section 5 concludes the paper.

2. Virtual Array System Model

In the domain of Multiple-Input–Multiple-Output (MIMO) millimeter-wave radar systems, estimating the horizontal azimuth angle depends on the utilization of virtual array elements within the horizontal plane. By analogy, the estimation of the elevation azimuth angle depends on virtual array elements oriented in the elevation direction, as illustrated in Figure 1. Assume a configuration consisting of M virtual array elements, each with an inter-element spacing designated as d , where d equals half the wavelength. The vector representation of the relative positions of the virtual array elements is expressed as follows [16]:

$$\mathbf{r} = [0, d, \dots, (M-1)d]. \quad (1)$$

The angular resolution based on virtual array elements is

$$\theta_{3\text{dB}} = \frac{\lambda}{d \times M \times \cos \theta_B} \times \frac{180}{\pi}, \quad (2)$$

where, θ_B is the antenna scanning angle.

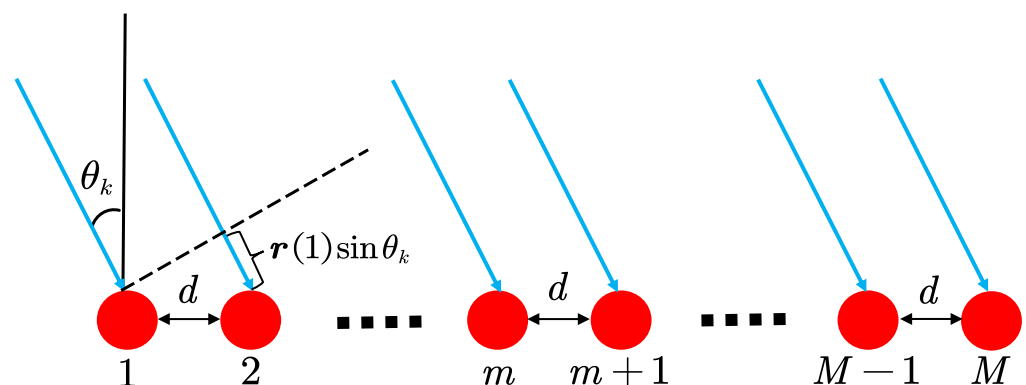


Figure 1. Schematic diagram of virtual array elements.

For simplicity, the model assumes that targets are coplanar with the array elements. Assuming the presence of K non-coherent targets within the detection envelope, with azimuth angles denoted by $(\theta_1, \theta_2, \dots, \theta_K)$, the distance between the k -th target (where $k = 1, 2, \dots, K$) and the m -th virtual array element (where $m = 1, 2, \dots, M$) is defined as $r_{k,m} = r(m) \sin \theta_k$. When millimeter-wave radar transmits a linear frequency-modulated (LFM) continuous wave (CW), the emitted signal can be mathematically expressed as follows:

$$s(t) = \exp(j2\pi f_c t + j\pi t^2 B/T), \quad (3)$$

where f_c represents the carrier frequency, B denotes the signal's bandwidth, and T signifies the temporal duration of the chirp.

At time t , the echo signal at the m -th virtual array element is characterized by the following expressions established based on the echo model of the system and considering targets moving with radial velocities (v_k):

$$\begin{aligned} r_{k,m}(t) &= r_{k,m}^c(t) r_{k,m}^b(t) + \omega_{k,m}(t). \\ r_{k,m}^c(t) &= \exp(j2\pi(f_c(t - \tau_{k,m}) + v_k t)), \\ r_{k,m}^b(t) &= \exp(j\pi(t - \tau_{k,m} + 2v_k t/c)^2 B/T), \end{aligned} \quad (4)$$

where $r_{k,m}^c(t)$ and $r_{k,m}^b(t)$ represent the carrier and base-band components of the echo signal, respectively, and $\omega_{k,m}(t)$ represents Gaussian white noise with zero mean and variance (σ^2). The time delay of the transmitted signal ($\tau_{k,m} = r_{k,m}/c$) depends on the range ($r_{k,m}$) and the speed of light (c). The Doppler shift associated with the k -th target, denoted as $v_k = 2v_k/\lambda$, is determined by the target's radial velocity (v_k) and the wavelength (λ) of the transmitted signal. This rigorous model is essential for accurately estimating target positions and velocities and critical for proficiently localization and tracking of targets in the radar system. Assuming the target narrow-band signal originates from the far field, the approximate is $\tau_{k,m} \approx \tau_k + r(m) \sin \theta_k / c$, where τ_k is the time delay of the k -th target, and $r(m) \sin \theta_k / c$ represents the time difference between the origin of the antenna array and the m -th virtual array element relative to target k . Since $r(m) \sin \theta_k / c$ is small compared to τ_k , the term $r(m) \sin \theta_k / c$ in $\tau_{k,m}$ can be neglected. Similarly, the term $2v_k t/c$ in $r_{k,m}^b(t)$ can also be neglected.

$$r_{k,m}^b(t) = \exp(j\pi(t - \tau_k)^2 B/T). \quad (5)$$

The backscattered signal after mixing, denoted as $y_{k,m}$, is expressed as follows:

$$y_{k,m}(t, \theta) = \exp(-j2\pi f_c r(m) \sin \theta_k / c) x(t, \theta) + \omega_{k,m}(t), \quad (6)$$

here,

$$x(t, \theta) = \exp(-j2\pi f_c \tau_k) r_k^b(t). \quad (7)$$

Thus, the backscattered signal of the m -th virtual array element can be represented as follows:

$$\mathbf{y}_m(t, \theta) = \mathbf{a}_m(\theta) x(t, \theta) + \boldsymbol{\omega}_m(t), \quad (8)$$

where $\mathbf{a}_m(\theta) = [1, e^{-j2\pi d \frac{\sin \theta}{\lambda}}, \dots, e^{-j2\pi(M-1)d \frac{\sin \theta}{\lambda}}]^T$ and $\boldsymbol{\omega}_m(t) = [\omega_1(t), \dots, \omega_M(t)]^T$.

By sampling $\mathbf{y}_m(t, \theta)$ to $\mathbf{y}_m(n, \theta)$, where n is the sample number ($n = 1, 2, \dots, N$) we have the following:

$$\mathbf{y}_m[n, \theta] = \mathbf{a}_m(\theta) x[n, \theta] + \boldsymbol{\omega}_m[n]. \quad (9)$$

Assuming the observation region is $[\theta_1, \theta_2, \dots, \theta_K]$, then the data vector received by the virtual array elements is represented as follows:

$$\mathbf{y}[n] = \mathbf{A}(\theta)\mathbf{x}[n] + \boldsymbol{\omega}[n], \quad n = 1, 2, \dots, N \quad (10)$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)]^T$ is the array manifold matrix with dimensions of $M \times K$. $\mathbf{x}[n] = [x[n, \theta_1], x[n, \theta_2], \dots, x[n, \theta_K]]^T$ is the signal matrix, and $\boldsymbol{\omega}[n]$ is the noise matrix.

3. Fast Deterministic Maximum Likelihood Algorithm

Maximum Likelihood Estimation (MLE) is a method employed to estimate parameters within a probabilistic model [17]. Operating under the assumption of a statistical model, MLE aims to maximize the likelihood function, thereby maximizing the confidence level of the observed data. Deterministic maximum likelihood (DML), a subset of MLE, is employed when random errors or noise within the model are deemed negligible or assumed to be zero. In DML, the likelihood function transforms into a deterministic form, eliminating stochastic elements and disregarding data uncertainty or randomness. Its core lies in optimizing the likelihood function within the conditional probability density function that includes angular parameters. This optimization leads to heightened computational complexity and diminished real-time performance.

To improve the real-time performance of the algorithm, reducing complexity is essential. Thus, building upon the DML algorithm, a novel strategy, namely FDML, is introduced to address the issue of high time complexity and to enhance the angular resolution, especially for closely spaced targets. The DBF algorithm approximates the target's coarse angle (θ_g) within the millimeter-wave radar's field of view of $([-\text{FOV}, \text{FOV}])$, as shown in Figure 2. Subsequently, the region of interest is narrowed down to $[\theta_g - \varphi, \theta_g + \varphi]$, where $\varphi = \theta_{3\text{dB}}$. Finally, the DML algorithm is applied to search for targets within the specified region, which aims to enhance target resolution and reduce the time required for DoA estimation.

DBF is a virtual array element algorithm that assumes that the echo signal is in the θ direction and that the direction vector in that direction is

$$\mathbf{a}(\theta) = \left[1, e^{-j2\pi d \frac{\sin \theta}{\lambda}}, \dots, e^{-j2\pi(M-1)d \frac{\sin \theta}{\lambda}} \right]^T. \quad (11)$$

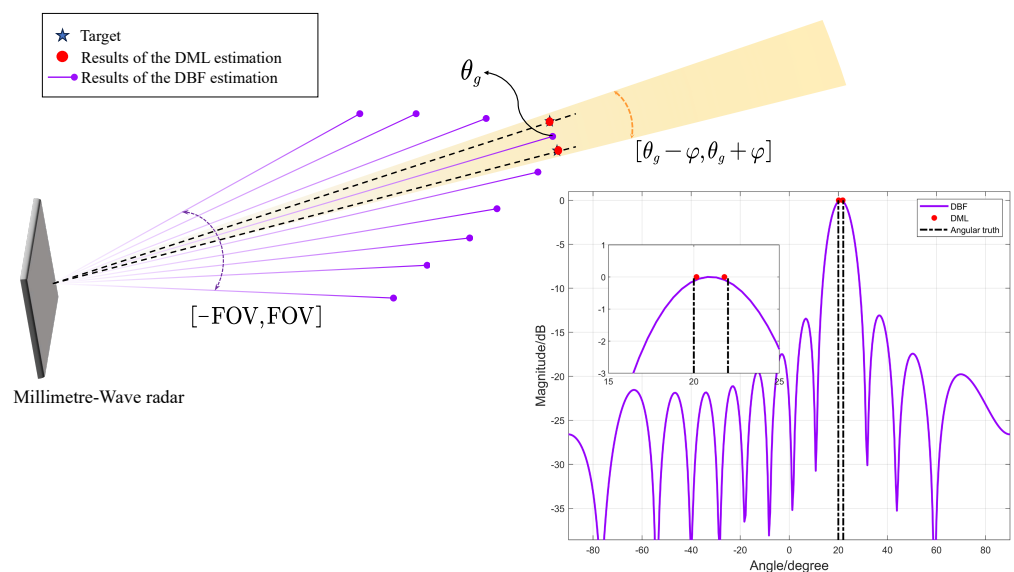


Figure 2. FDML algorithm model.

According to (10), the echo signal ($\mathbf{y}[n]$) needs to be weighted and summed as follows:

$$f[n] = \mathbf{W}^H \mathbf{y}[n] = x[n] \mathbf{W}^H \mathbf{a}(\theta) + \mathbf{W}^H \boldsymbol{\omega}[n], \quad (12)$$

where $\mathbf{W} = [W_1, W_2, \dots, W_M]^T$ is the weight vector. DBF uses the phase information of the weight vector to phase-compensate the components of the array signal such that the components in the direction of the signal of interest are in-phase additive and become the main flap of the direction map. In contrast, non-isotropic directions form a sub-flap or even exhibit zero energy.

The weight vector from θ_t is mathematically represented as follows:

$$\mathbf{W} = \mathbf{a}(\theta_t). \quad (13)$$

Up to this point, beamforming achieves the estimation of DoA.

Unfortunately, DBF cannot overcome the limitations of virtual aperture. It cannot resolve extremely close targets within the resolution or suffers from substantial estimation errors.

The purple line in Figure 2 shows the estimation of the DBF algorithm in the [-FOV, FOV] region. DBF can only estimate one angle (θ_g) with bias when dealing with two targets within the resolution range. Furthermore, the same result can be seen in the orientation map in Figure 2. Therefore, this paper considers DoA estimation of extremely close targets using the DML algorithm within the interval of $[\theta_g - \varphi, \theta_g + \varphi]$ near θ_g .

According to probability theory, the deterministic maximum likelihood joint probability density function of DoA is

$$p_{\hat{\theta}}(y_n) = \frac{1}{(\pi\sigma^2)^M} \exp\left(-\frac{1}{\sigma^2} |\mathbf{y}[n] - \mathbf{A}(\theta)x[n]|^2\right), \quad (14)$$

where $|\cdot|$ denotes the Frobenius paradigm of the matrix.

After mathematical derivation (see Appendix A for details of the derivation), the target parameter ($\mathbf{A}$) to be estimated can be obtained as follows:

$$\hat{\theta}_{DML} = \arg\left\{\max_{\theta} \text{tr}\left(\mathbf{P}_{\mathbf{A}(\theta)} \hat{\mathbf{R}}\right)\right\}, \quad (15)$$

where $\text{tr}(\cdot)$ represents the trace operation of the matrix, i.e., the sum of all diagonal elements. The $\hat{\theta}_{DML}$ of the DML estimate is the value of $\mathbf{y}[n]$ projected onto the model space orthogonal to the expected signal components according to (15).

The power of $\mathbf{y}[n]$ in this model space reaches its maximum when $\hat{\theta}_{DML} = \theta_k$ retains all the true signal components. The red dots in Figure 2 show the estimation results of DML, and the error between them and the real targets is minimal. So far, FDML has successfully discriminated two extremely close targets in less than $\Delta\theta$. The FDML algorithm can be summarized as Algorithm 1, assuming there are K targets.

Algorithm 1 Fast Deterministic Maximum Likelihood**Input:** $y(n)$, K ;**Output:** $\hat{\theta}$;

1. Utilize the DBF algorithm to obtain the coarse angle of the target θ_g ;
2. Narrow the search range to $[\theta_g - \varphi, \theta_g + \varphi]$, $\varphi = \theta_{3\text{dB}}$, set $k = 1$;
3. Construct the orthogonal projection matrix in the array element space;

$$P_{A(\theta)} = A(\theta) \left(A^H(\theta) A(\theta) \right)^{-1} A^H(\theta);$$
4. Construct the signal covariance matrix;

$$\hat{R} = \frac{1}{N} \sum_{n=1}^N y(n) y^H(n);$$
5. Find the projection matrix F of $\hat{R}$ onto $P_{A(\theta)}$;

$$F = P_{A(\theta)} \hat{R};$$
6. Find the maximum value of the trace of F and calculate the estimated angle $\hat{\theta}_k$;

$$\hat{\theta}_k = \arg\{\max_{\theta} \text{tr}(F)\};$$
7. Update the observation vector $y(n)$;
8. Increment $k = k + 1$;

if $k < K$ **then**
 go to step 3;
else
 output $\hat{\theta}$ ($\hat{\theta} = [\hat{\theta}_1, \dots, \hat{\theta}_K]$);
 end;

4. Simulation and Analysis

In this section, to verify the effectiveness of the proposed algorithm, two coherent sources located in the far field are simulated. For simulations, the research considers a millimeter-wave radar system equipped with 12 virtual array elements, with an inter-element spacing of $d = \lambda/2$. Two targets are randomly generated within the millimeter-wave radar field of view (FOV), each with its arrival azimuth. This research assumes a zero-mean Gaussian distribution for the noise, with a variance of $\sigma^2 = 1$. The specific simulation parameters are summarized in Table 1. The simulation computer system is Windows 11, with an AMD Ryzen 77735H processor with Radeon Graphics, and the simulation software is MATLAB R2023b.

To evaluate the performance of the FDML algorithm, this simulation compares it against other angle estimation methods that are robust to variations in the number of snapshots, including DBF, DML, IAA, and FIAA. These algorithms are selected based on their relevance and effectiveness in similar scenarios.

To validate the effectiveness of the FDML algorithm, we conducted a simulation experiment for DoA estimation using the parameters outlined in Table 1.

Table 1. Parameters used in the simulation experiment.

Parameter	Symbol	Value
Speed of light	c_0	3×10^8 (m/s)
Carrier	f_0	77 GHz
Field of view	FOV	90 (degree)
Virtual array elements	$N_t N_r$	12 (3T4R)
Wavelength	λ	c_0 / f_c
Element spacing	d	$\lambda/2$
Bandwidth	B	300 MHz
Chirp period	T	40 μs
Noise variance	σ^2	1
Signal-to-noise ratio	SNR	[10,20,30] (dB)
Direction of arrival	DoA	[20,22] (degree)
Snapshot number	Snap	1
Angular resolution	$\theta_{3\text{dB}}$	8.467 (degree)

To evaluate the performance of the FDML algorithm, this simulation compares it against other angle estimation methods that are robust to variations in the number of snapshots, including DBF, DML, IAA, and FIAA. These algorithms are selected based on their relevance and effectiveness in similar scenarios. The results, as depicted in Figure 3, are summarized as follows. In this experiment, the DML and FDML algorithms estimate parameters by maximizing the likelihood function of the observed signal. The algorithm estimator can directly output the spatial coordinates of the target; thus, there is no output of the direction pattern.

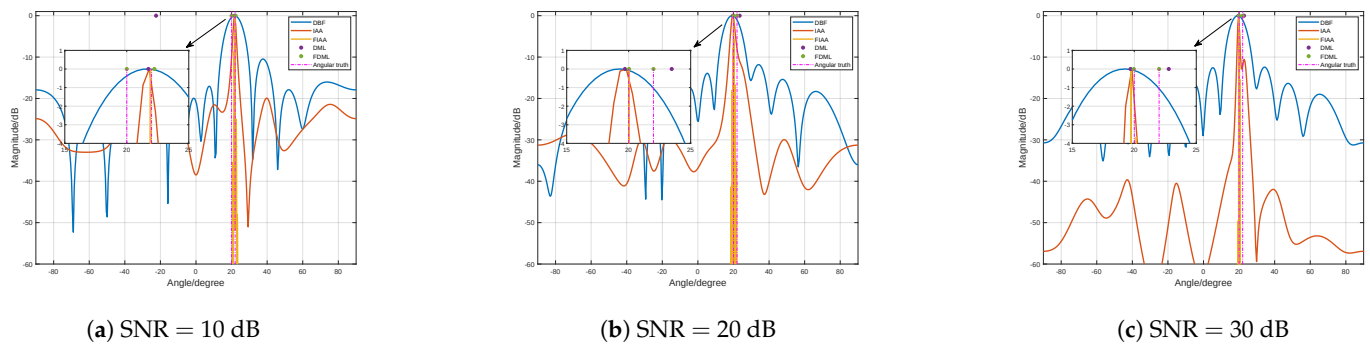


Figure 3. DoA directional angle plot.

From the simulation results, it is evident that at a signal-to-noise ratio (SNR) of 10 dB, the conventional DBF, IAA, FIAA, and DML algorithms accurately estimate the DoA of only one target entity, with significant errors in estimating the DoA of the other target entity. In contrast, the FDML algorithm not only accurately estimates the DoA of the first target entity but also maintains a smaller error range in estimating the DoA of the second target entity. As the SNR increases, the accuracy of other algorithms in estimating the DoA of two adjacent target entities also improves. This phenomenon suggests that, among the considered algorithms, the FDML algorithm is the least reliant on SNR and can maintain high estimation accuracy, even in low-SNR environments. This resilience primarily stems from the algorithm's noise suppression strategy, effectively mitigating the impact of noise on DoA estimation results.

In the single-snapshot scenario (Snap = 1), the simulation utilizes the Root Mean Square Error (RMSE) metric to quantify the disparity between the actual and estimated DoA values produced by the DoA estimation algorithm. Notably, the generation of DoA in the experiment is highly stochastic.

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{m=1}^M (\theta_m - \hat{\theta}_m)^2}. \quad (16)$$

In this context, M denotes the number of iterations in the Monte Carlo method, θ_m represents the actual DoA value of the target, and $\hat{\theta}_m$ signifies the predicted DoA value of the target.

As shown in Figure 4, the angular estimation error of the FDML algorithm consistently decreases with increasing signal-to-noise ratio (SNR), indicating superior angular accuracy in low-SNR environments compared to other angular estimation algorithms. The FDML algorithm outperforms algorithms like DBF, IAA, DML, and FIAA in terms of measurement error. For instance, when compared to FIAA, the measurement error is reduced by 51.96%.

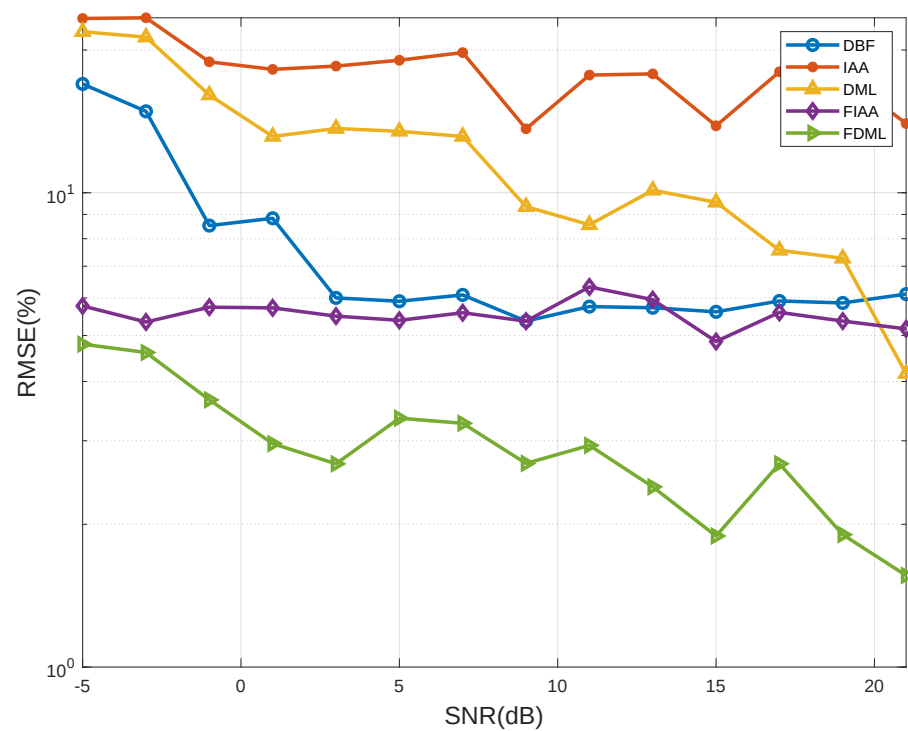


Figure 4. Comparison of algorithm performance (SNR–RMSE).

As outlined in [18], the operational SNR range of radar typically falls within [13 dB, 20 dB]. Within this range, the FDML algorithm reduces the measurement error by 61.26% compared to the FIAA algorithm, by 64.29% compared to the DBF algorithm, and by 73.01% compared to the DML algorithm. The IAA algorithm demonstrates the highest angular estimation error, with the FDML algorithm reducing the measurement error by 87.17% in comparison. Based on these observations, it is evident that the FDML algorithm attains minimal estimation error in low-SNR environments, significantly enhancing the accuracy of DoA estimation. With precise capture of target DoA in parameter estimation, the FDML algorithm consistently maintains minimized measurement error, even under low-SNR conditions.

Considering the algorithm's angular estimation performance, the simulation was followed by further analysis to examine the potential impact of angular discrepancy on DoA estimation error. The simulation results presented in Figure 5 demonstrate a decreasing trend in the estimation error of the FDML algorithm with the increase in DoA angle. This trend arises as the projection difference of the echo signal on the virtual array elements becomes more pronounced, consequently enhancing the accuracy of localization. In scenarios where two signal sources are closely positioned, particularly under a 20 dB signal-to-noise ratio (SNR), the FDML algorithm exhibits the lowest measurement error. This observation underscores FDML's ability to effectively discern neighboring signal sources, even amidst high noise levels, thereby mitigating localization inaccuracies. As the angle of the signal sources continues to increase, the measurement error of the FDML algorithm progressively converges towards that of the DML algorithm. This trend suggests that at sufficiently large signal source angles, the performance disparity between the two approaches diminishes. Notably, the superiority of the FDML algorithm becomes more pronounced when dealing with smaller angles. Consequently, the FDML algorithm offers the highest precision and resolution in estimating extremely proximate targets.

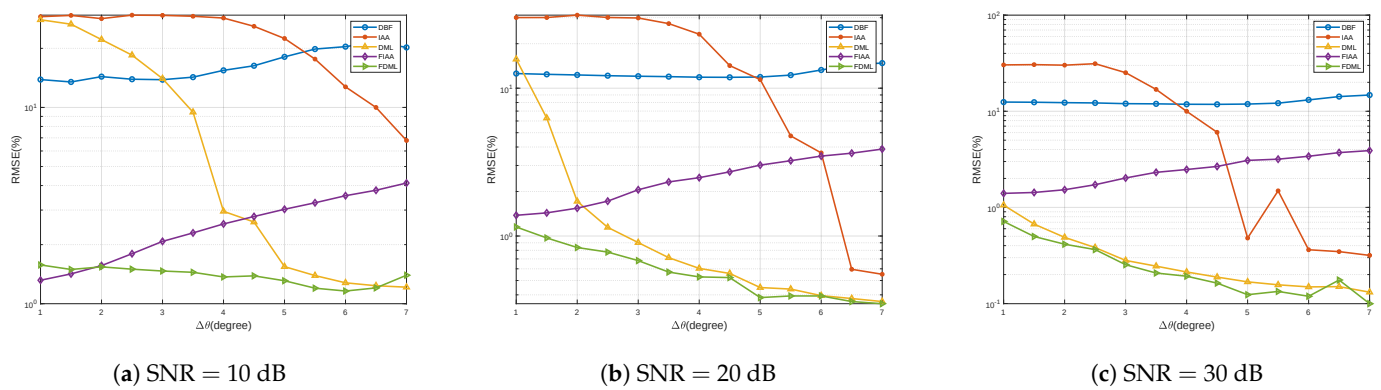


Figure 5. Comparison of algorithm performance ($\Delta\theta$ –RMSE).

Based on the outcomes of the three simulation experiments, it is evident that the FDML algorithm exhibits superior angular accuracy and resolution compared to alternative algorithms. Furthermore, this simulation evaluated the runtime performance of five DoA estimation algorithms, maintaining a scanning angular resolution of 0.25 (based on the simulation program runtime), as illustrated in Table 2. Here, $n = 720$ denotes the number of sampling points in the field of view (FOV) search range, $a = 50$ indicates the number of adaptive iterations, and $m = 24$ represents the number of sampling points in the secondary search range. The simulation environment corresponds to the parameters outlined in Table 1.

Table 2. Algorithm runtime and complexity.

Algorithm	Runtime (S)	Time Complexity
DBF	0.10644	$O(n)$
IAA	0.0415	$O(an)$
DML	2.4063	$O(n^2)$
FIAA	0.0390	$O(n + am)$
FDML	0.0168	$O(n + m^2)$

The table illustrates that at identical angular resolutions, the FDML algorithm's runtime is decreased by 84.20% compared to the DBF algorithm, by 59.52% compared to the IAA algorithm, by 99.30% compared to the DML algorithm, and by 56.85% compared to the FIAA algorithm. This reduction in computational time underscores the efficiency of the FDML algorithm relative to other methods.

5. Conclusions

The experimental results show that the accuracy of the FDML algorithm in angle estimation of super-resolution has been significantly improved compared to other DoA estimation algorithms. In the critical low- to medium-SNR range, FDML reduces angle measurement error by 61.26% compared to FIAA, 64.29% compared to DBF, 73.01% compared to DML, and 87.17% compared to IAA. These results demonstrate the advantages of the FDML algorithm in noise suppression and angle estimation accuracy.

Furthermore, comparative experiments demonstrated FDML's superior angular resolution robustness, particularly in scenarios with high resolution and low angular separation. Additionally, FDML significantly improved computational efficiency, reducing computation time by up to 99.30% while maintaining high-resolution performance.

In summary, the FDML algorithm demonstrates excellent performance and computational efficiency in the application of angle estimation for vehicle-mounted millimeter-wave radar systems. It has potential as a good solution for vehicle-mounted radar systems because of its high robustness in low-SNR environments and its ability to reduce the computational burden while maintaining high resolution. Future research will delve deeper into its applicability in complex environments and pursue optimization for even greater performance.

The FDML simulation reported in this paper was conducted for a single snap case with two target assumptions. Since FDML contains matrix-inverse operations, there is also a need to consider the space complexity issue in engineering implementation.

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Conflicts of Interest: Zhen Wang is an employee of China Automotive Innovation Corporation. This paper reflects the views of the scientists and not those of the company.

Appendix A. Mathematical Derivation of DML in FDML

In the context of MLE [19], assuming a distribution with a set of parameters (θ) and desiring to obtain an estimated value ($\hat{\theta}$) from samples, one may consider using relative entropy to evaluate the quality of the estimation.

$$D_{\text{KL}}(p_{\theta} \| p_{\hat{\theta}}) = p_{\theta} \log \frac{p_{\theta}}{p_{\hat{\theta}}}, \quad (\text{A1})$$

where p_{θ} denotes the actual target's joint probability density function, while $p_{\hat{\theta}}$ denotes the estimated target's joint probability density function.

The definition of expectation can be expressed by

$$D_{\text{KL}}(p_{\theta} \| p_{\hat{\theta}}) = \mathbb{E}_{\theta} \left[\log \left(\frac{p_{\theta}}{p_{\hat{\theta}}} \right) \right] = \mathbb{E}_{\theta} [\log p_{\theta}] - \mathbb{E}_{\theta} [\log p_{\hat{\theta}}]. \quad (\text{A2})$$

The larger the value of D_{KL} , the poorer the parameter estimation. To improve parameter estimation, it is essential to minimize the value of D_{KL} by adjusting the estimated parameters ($\hat{\theta}$) as follows:

$$\hat{\theta} = \arg \left\{ \min_{\hat{\theta}} D_{\text{KL}}(p_{\theta} \| p_{\hat{\theta}}) \right\}. \quad (\text{A3})$$

With N samples, according to the law of large numbers, (10) can be carried over to obtain the following:

$$\mathbb{E}_{\theta} [\log p_{\hat{\theta}}(y)] = \frac{1}{N} \sum_{n=1}^N \log p_{\hat{\theta}}(y_n). \quad (\text{A4})$$

Here, p_{θ} corresponds to $p_{\theta}(y_n)$, and $p_{\hat{\theta}}$ corresponds to $p_{\hat{\theta}}(y_n)$. Substituting these into (A2) yields the following:

$$D_{\text{KL}}(p_{\theta}(y) \| p_{\hat{\theta}}(y)) = \mathbb{E}_{\theta} [\log p_{\theta}(y)] - \frac{1}{N} \sum_{n=1}^N \log p_{\hat{\theta}}(y_i). \quad (\text{A5})$$

For a known distribution, the θ parameters are fixed. Therefore, $\mathbb{E}_{\theta} [\log p_{\theta}(y)]$ is a constant. The optimal estimated parameters can be obtained by minimizing the D_{KL} value as follows [20]:

$$\hat{\theta} = \arg \left\{ \min \left(\mathbb{E}_{\theta} [\log p_{\theta}(y)] - \frac{1}{n} \sum_{i=1}^n \log p_{\hat{\theta}}(y_i) \right) \right\}. \quad (\text{A6})$$

The above expression can be simplified as follows:

$$\hat{\theta} = \arg \left\{ \max_{\hat{\theta}} \prod_{n=1}^N p_{\hat{\theta}}(y_n) \right\}. \quad (\text{A7})$$

Then, the likelihood function is expressed as follows:

$$L = \prod_{n=1}^N p_{\hat{\theta}}(y_n). \quad (\text{A8})$$

According to probability theory, the deterministic maximum likelihood joint probability density function for the observed vector over N snapshots is

$$p_{\hat{\theta}}(y_n) = \frac{1}{(\pi\sigma^2)^M} \exp \left(-\frac{1}{\sigma^2} |\mathbf{y}[n] - \mathbf{A}(\theta)x[n]|^2 \right). \quad (\text{A9})$$

Thus, the likelihood function can be expressed as follows:

$$L = \prod_{n=1}^N \frac{1}{(\pi\sigma^2)^M} \exp \left(-\frac{1}{\sigma^2} |\mathbf{y}[n] - \mathbf{A}(\theta)x[n]|^2 \right). \quad (\text{A10})$$

To simplify the computation, take the logarithm of the likelihood function ($\ell(\theta)$), which shares the same maximum point with the likelihood function.

$$\ell(\theta) = \sum_{n=1}^N \log \left(\frac{1}{(\pi\sigma^2)^M} \exp \left(-\frac{1}{\sigma^2} |\mathbf{y}[n] - \mathbf{A}(\theta)x[n]|^2 \right) \right). \quad (\text{A11})$$

Here, M represents the number of virtual array elements. It can be simplified as follows:

$$\ell(\theta) = - \sum_{n=1}^N \frac{1}{\sigma^2} |\mathbf{y}[n] - \mathbf{A}(\theta)x[n]|^2 - MN \log(\pi\sigma^2). \quad (\text{A12})$$

To maximize $\ell(\theta)$, consider the inner product between $\mathbf{y}[n]$ and $\mathbf{A}(\theta)x[n]$. The orthogonal projection matrix ($\mathbf{P}_{\mathbf{A}(\theta)}$) is defined as follows:

$$\mathbf{P}_{\mathbf{A}(\theta)} = \mathbf{A}(\theta) \left(\mathbf{A}^H(\theta) \mathbf{A}(\theta) \right)^{-1} \mathbf{A}^H(\theta), \quad (\text{A13})$$

where $\mathbf{P}_{\mathbf{A}(\theta)}$ projects any vector onto the space spanned by $\mathbf{A}(\theta)$. In the context of MLE, by maximizing $\mathbf{y}[n]^H \mathbf{P}_{\mathbf{A}(\theta)} \mathbf{y}[n]$, (A6) yields θ such that the component of $\mathbf{y}[n]$ projected onto the signal space is maximized.

The signal covariance estimation equation for the array time-domain sampling space is defined as follows:

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{y}[n] \mathbf{y}^H[n], \quad (\text{A14})$$

where $\hat{\mathbf{R}}$ contains all the information of $\mathbf{y}[n]$; thus, the problem can be transformed by maximizing the product of traces as follows:

$$\hat{\theta} = \arg \left\{ \max_{\theta} \text{tr} \left(\mathbf{P}_{\mathbf{A}(\theta)} \hat{\mathbf{R}} \right) \right\}. \quad (\text{A15})$$

Here, $\text{tr}(\cdot)$ represents the trace of a matrix, that is, the sum of all diagonal elements. Maximizing $\text{tr}(\mathbf{P}_{\mathbf{A}(\theta)} \hat{\mathbf{R}})$ is equivalent to maximizing the sum of all $\mathbf{y}[n]^H \mathbf{P}_{\mathbf{A}(\theta)} \mathbf{y}[n]$.

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