

## Article

# Linear Programming Formulation for Planning of Future Model-Year Mix of Electrified Powertrains <sup>†</sup>

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## Abstract

When looking towards the goal of reducing greenhouse gas (GHG) emissions, automotive manufacturers face several challenges when planning future vehicle offerings in different markets. The planned vehicle offerings must cope with uncertainties in the supply chains of critical materials and adhere to regulatory requirements in different regions, all while appealing to customer preferences and maintaining low cost. Regulatory requirements, which are often based on tailpipe GHG emissions, do not necessarily align with Lifecycle Analysis (LCA) of GHG emissions, which becomes yet another challenge towards attaining sustainability goals. Planning the future mix of vehicles to be manufactured under all such considerations can be a complex task, often relying on methods with poor transparency, unguaranteed optimality, or requiring difficult-to-predict a priori knowledge. This paper considers the special case of a short time window (one future model-year), which allows for modelling the future planning decisions as a linear programming (LP) problem, which in turn, can be solved to global optimality via well-established algorithms, such as Dual-Simplex. The proposed formulation is demonstrated via one simple example, as well as a scaled-up study with two regions, two vehicle size categories, and four powertrain configurations. A key insight that the proposed formulation is able to demonstrate in the scaled-up study is how the optimum (lowest) LCA GHG solution depends on the availability of battery materials, ranging from an increased share of hybrids under low battery supply to an increased share of electric vehicles for abundant battery supply.

**Keywords:** electric vehicles; plug-in hybrid vehicles; hybrid electric vehicles; supply and value chain; modeling and simulation



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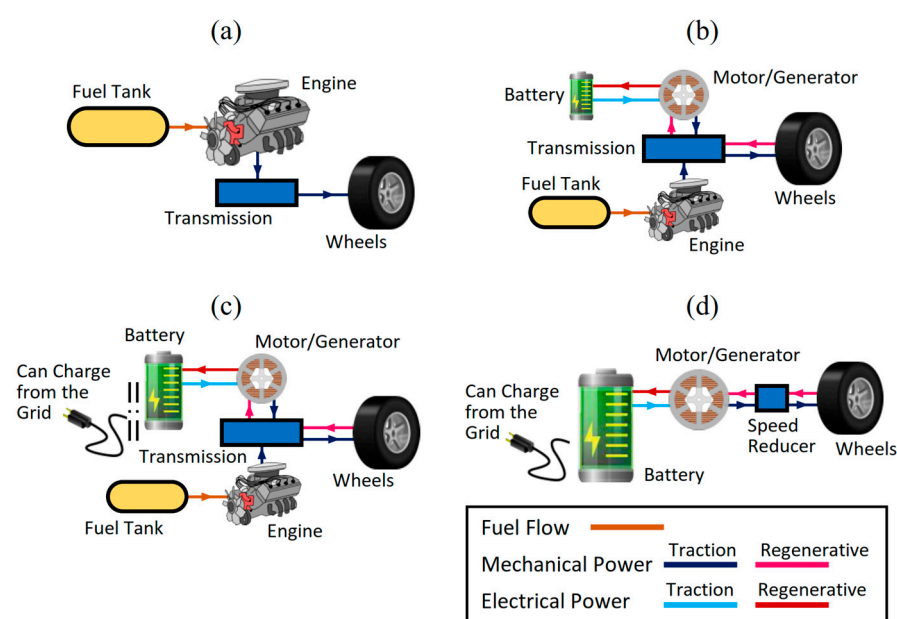
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## 1. Introduction

Vehicles with electrified powertrains [1], which include (non-plug-in) Hybrid Electric Vehicles (HEVs), Plug-in Hybrid Electric Vehicles (PHEVs), and Battery-only Electric Vehicles (BEVs), carry the promise of significant LCA GHG reductions compared to Conventional Internal Combustion Engine (CICE) vehicles. Electrified vehicles also include Hydrogen-powered Fuel Cell Electric Vehicles (FCEVs) [2] and Plug-in Fuel Cell Electric Vehicles (PFCEVs) [3]. However, fuel cells are a relatively new technology that not only face challenges from vehicle design and manufacturing side, but also from readiness and availability of Hydrogen refueling infrastructure, as well as cost and supply chains of Hydrogen as a fuel [4–6]. As such, further discussions of electrified powertrains in this paper will be limited to HEVs, PHEVs, and BEVs.

An illustrative diagram for the layout of the powertrain types considered in this paper is shown in Figure 1. HEVs may be considered as the “low end” or “entry level” electrified powertrain; having one or more electric motor(s), a “small capacity” traction battery, an internal combustion engine (ICE), and no means to charge the battery from the electric grid. Fuel consumption in the ICE of HEVs is the only source of energy, much like CICEs, and yet, compared to CICEs, HEVs have efficiency improvement features, including: (i) ability to recapture vehicle kinetic energy while decelerating (also known as “regenerative braking”) by operating the motor(s) as generator(s) and storing the recaptured energy in the battery for later re-use, and (ii) ability to take advantage of electric motors’ high efficiency across a broad range of torque and speed output requirements, which in turn allows the ICE to either operate close to its optimum conditions, or get briefly turned off, with the difference in traction power being compensated by the battery and electric motor(s). With those efficiency improvement features, it is estimated that HEVs are capable of achieving up to 25–35% reduction in tailpipe GHG emissions compared to equivalent CICEs [7,8]. HEVs also present the least compromise from the perspective of tentative new adopters, who only need to “trust that the new technology will not fail prematurely”, since HEVs are refueled and operated the same as CICEs, and do not require developing/managing a “new activity” such as charging of the vehicle. HEVs also require the least amount of battery materials since they utilize small-capacity batteries compared to PHEVs and BEVs.



**Figure 1.** Illustration of various powertrain type layouts: (a) CICE, (b) HEV, (c) PHEV, and (d) BEV.

At the other end of the spectrum of electrified powertrains, BEVs may be considered the “high end” or “advanced level” among electrified powertrains; with no ICE at all, higher powered electric motor(s) and a large-capacity traction battery that gets charged from the electric grid when the vehicle is parked at a charger. BEVs not only have the same energy efficiency features of HEVs (regenerative braking and high efficiency electric motors), but also have the capability for larger reduction in LCA GHG by relying on grid electricity as the energy source, which depending on the electricity generation mix in of the grid, is often of lower Carbon Intensity (CI) compared to Gasoline or other typical liquid fuels. However, in addition to requiring larger amounts of battery materials per manufactured vehicle, BEVs also face higher challenges in mass market adoption due to various reasons, including higher initial cost (primarily due to larger batteries), as well as concerns/perceptions about driving range, charging time, and availability of charging infrastructure [9–11]. As a plausible “middle

ground”, PHEVs combine traits of both HEVs and BEVs. Though the components in the powertrain of a PHEV may be similar to those of HEV, key distinctions include the ability of PHEVs to charge their batteries via electricity from the grid (similar to BEVs), as well as PHEVs generally having larger battery capacities compared to HEVs. Under favorable conditions of low CI electricity, consistent charging, and infrequent long trips, the LCA GHG of a PHEV could approach or even excel compared to that of an equivalent BEV [12,13]. PHEVs also relieve range anxiety in long-distance travel since they can operate like HEVs (consuming fuel in the ICE as the energy source) when the battery runs out. However, when conditions are unfavorable, such as little or no charging, the LCA GHG of a PHEV could become slightly worse than that of an equivalent HEV.

This paper focuses on optimization of a future mix of powertrains among CICEs, HEVs, PHEVs, and BEVs for minimization of both LCA GHG and cost, all while adhering to regulatory requirements, supply chain limitations (especially battery materials), and projected market demand. Though not the same type of future decisions as the focus of this paper, heuristic approaches have been applied towards the optimization of production processes in the automotive industry [14,15], and application to vehicle fleet size and mix was done in [16]. Despite many demonstrated successes, one downside to heuristic approaches, however, is the lack of guarantee of global optimality of the solutions. Other popular approaches for future production planning are approaches such as agent-based modeling [17,18] and dynamic programming [19]. Agent-based modeling tends to be popular for problems involving high uncertainty that becomes difficult to model via closed-form formulas. However, aside from non-guaranteed optimality similar to heuristic approaches, a drawback to agent-based models is the lack of transparency as to how the various agents in the model produce their outputs. In general, dynamic programming focuses on sequences of discrete decisions [20] and has the attractive quality of being able to guarantee reaching global optimality. However, dynamic programming has the limitation of not necessarily being capable of modeling the details of all real-world considerations, as well as being somewhat complex to implement.

Unlike the prior models that rely on simulation-based heuristics or behavioral adoption assumptions, this work formulates the planning of a future powertrain mix as a linear program (LP) [21], which leads to a convex optimization problem, enabling provably globally optimal solutions. While noting that LP has similar limitations as dynamic programming in not being capable of modeling all real-world considerations, this paper will demonstrate its applicability to several classes of objectives and constraints, including regulatory (such as the 2024 US EPA GHG standards [22] or California ZEV mandate [23]), materials supply, and forecasted vehicle demand. Other advantages of LP include the transparency between input–output, ease of implementation, high computational efficiency, and scalability to a large number of decision variables.

This paper is a revised and expanded version from the one presented at [24]. The paper started with an overview of electrified powertrains, leading to motivation for the work. The remainder of the manuscript is organized as follows: Section 2 presents the details of the generalized mathematical model; Section 3 presents a small-scale example study that considers only one region, one vehicle size category, and three powertrain alternatives (CICE, PHEV, and BEV). The small example allows for step-by-step visualization of various considerations in the LP optimization process. Section 4 then presents a larger scale example with two geographic regions, two vehicle size categories (mid-sized sedan and small SUV) and four powertrain alternatives (CICE, HEV, PHEV, and BEV) for a total of sixteen decision variables, where the LP optimization algorithm known as “Dual Simplex” [25] is shown to successfully find optimal solutions (mix of powertrains) across the two regions and the two size categories, not only for the minimization of LCA GHG, but also, a second

objective such as cost reduction, as well as generating Pareto-optimal solutions that explore the trade-off between two objectives. Summary of findings and future work are briefly discussed in the conclusion of the paper in Section 5.

## 2. Mathematical Model

### 2.1. Overview

The formulation adopted in this paper conforms with the standard LP form [21], recapped in Equations (1)–(3). This work further adopts the notation where boldface italic symbols denote vector quantities, capitalized boldface non-italic symbols denote matrix quantities, and non-boldface italic symbols denote scalar values. The optimization goal in linear programming is to maximize a reward function, denoted by the symbol  $f$  in Equation (1), by finding the optimal combination among a set of decision variables denoted by the vector  $\mathbf{x} = \{x_i\}$ , with  $i \in \{1, \dots, N\}$  being an index for the decision variables. A vector of reward coefficients  $\mathbf{c} = \{c_i\}$  in Equation (1), with  $c_i$  defining the expected amount of reward corresponding to one unit of the corresponding decision variable  $x_i$ . Thus, the reward function  $f$  can be simply calculated as the dot product of the vectors  $\mathbf{c}$  and  $\mathbf{x}$ . The matrix  $\mathbf{A}$  and vector  $\mathbf{b}$  in Equation (2), along with the condition in Equation (3) that all decision variables must be non-negative, define the “feasible space” for the decision variables. An index  $k \in \{1, \dots, K\}$  is utilized to denote the rows of the matrix  $\mathbf{A}$  (or elements of the vector  $\mathbf{b}$ ).

$$\text{Maximize} \quad f = \mathbf{c} \cdot \mathbf{x}, \quad (1)$$

$$\text{Subject to} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \quad (2)$$

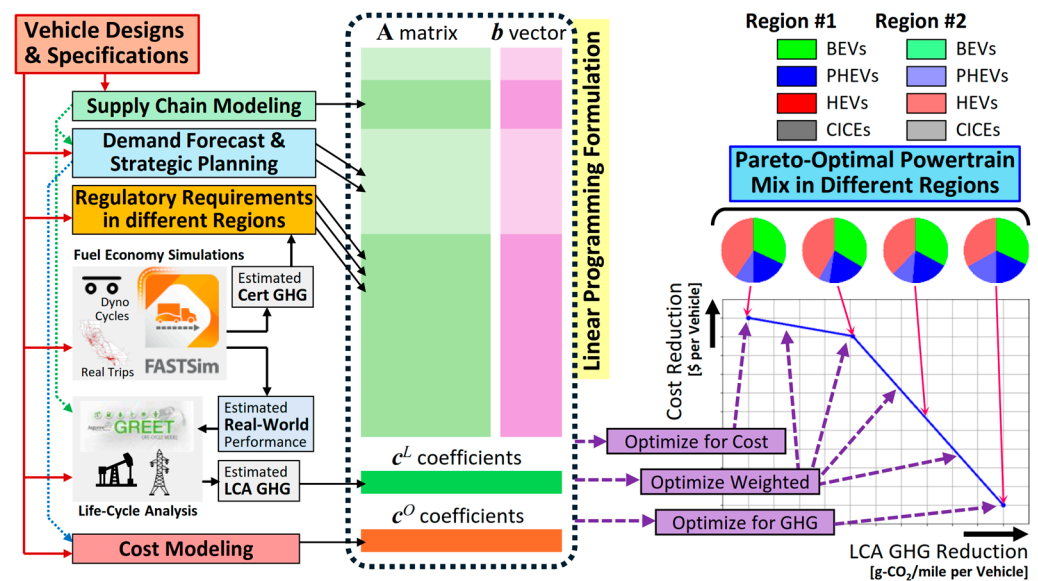
$$\mathbf{x} \geq \mathbf{0}. \quad (3)$$

A conceptual illustration of a framework for the implementation of the LP formulation is shown in Figure 2. The LP formulation in Equations (1)–(3) can be efficiently optimized via well-established algorithms, such as the Dual-Simplex algorithm [25] adopted in this work. However, the main challenge that highlights the contribution of this paper is that of translating the various planning considerations into the standard LP form. Towards that goal, we utilize a structured partitioning of the decision variables such that for a given total planned production of a number of vehicles  $M$ , there is a decision variable  $y_{srp}$  with three subscript indices  $s \in \{1, \dots, S\}$ ,  $r \in \{1, \dots, R\}$ ,  $p \in \{1, \dots, P\}$ , respectively defining the vehicle size category, region for deployment or sales, and the powertrain type or specification. In some of the examples, we adopt the notation of  $p = 1$  indicating BEVs,  $p = 2$  indicating PHEVs,  $p = 3$  indicating HEVs, and  $p = 4$  indicating CICEs, but this notation can easily be adopted to incorporate BEVs and PHEVs with different electric range offerings or additional powertrain types. It follows that there exists a direct correspondence between  $y_{srp}$  and the standard form of LP decision variables, which is expressed as:

$$y_{srp} = x_{(s-1) \times R \times P + (r-1) \times P + p}. \quad (4)$$

While elements of the matrix  $\mathbf{A} = [a_{ki}]$  are typically expressed via the row and column indices ( $k$  and  $i$ , respectively), the column index is replaced by the three indices of the corresponding decision variable  $y_{srp}$ , thus the elements of the matrix  $\mathbf{A}$  will be expressed as  $[a_{ksrp}]$ . For convenience, the row index (number of constraints in the matrix  $\mathbf{A}$ ) is further partitioned by constraint type into:

- Fleet consistency constraints  $k \in K^F$ ,
- Resource-bound constraints  $k \in K^E$ ,
- Pre-dictated fleet fraction constraints  $k \in K^D$ .



**Figure 2.** Illustration of a framework for implementation of the LP formulation, including pre-processing (for estimation of various coefficients) and post-processing for results interpretation.

Details for implementation of the different types of constraints are discussed in Sections 2.2–2.4. Section 2.5 discusses formulations for the objective function (elements of the vector  $c$  in Equation (1)), while Section 2.6 discusses the adopted approach for generating Pareto-optimal (trade-off) solutions among two different objectives.

## 2.2. Fleet Consistency Constraint(s)

The decision variables  $y_{srp}$  represent a fraction of the total number of vehicles ( $M$ ) that are to be manufactured (for a certain future model-year). As such, the sum of the decision variables must add up to exactly one, this would imply that the number of vehicles produced is less/greater than  $M$ . The fleet consistency constraint is expressed as:

$$\sum_{s=1}^S \sum_{r=1}^R \sum_{p=1}^P y_{srp} = 1. \quad (5)$$

The equality constraint from Equation (5) could be expressed in standard LP form as two inequality constraints where the sum must be  $\leq 1$  and  $\geq 1$ . It follows that the corresponding elements of the matrix  $A$  and vector  $b$  for the fleet consistency partition (two inequality constraints) are expressed as:

$$a_{k_1^F srp} = 1 \quad \forall s, r, p \text{ \& } b_{k_1^F} = 1, \quad (6)$$

$$a_{k_2^F srp} = -1 \quad \forall s, r, p \text{ \& } b_{k_2^F} = -1. \quad (7)$$

## 2.3. Resource-Bound Constraints

This category of constraints allows for modeling situations where each manufactured vehicle in the future model-year that is being planned would “consume” a certain amount of a limited “resource”. This type of constraint is typical in strategic planning problems where it is necessary to enforce that any “valid solution” must adhere to not exceeding the maximum allotted amount(s) of available resource(s). This category of constraints can be utilized to model considerations such as the total available materials for making vehicles or tailpipe-based regulation GHG, where the unit of resource “consumed” to produce a

vehicle is its GHG rating, while the net available “resource” is the target compliance for fleet average GHG.

### 2.3.1. Available Materials Constraint

When certain material resources are in limited supply, such as when an OEM already has limited contracts with battery suppliers, one could incorporate such a limitation in the powertrain mix planning via this type of constraint. If the total available number of materials for producing  $M$  vehicles is  $\Phi$ , then the average available number of materials for one vehicle is  $\phi = \Phi/M$ . Let  $\phi_{sp}$  be the number of materials needed to manufacture one vehicle of size category  $s$  with powertrain type  $p$ . Then, the limited resource material constraint can be expressed in linear form as:

$$\sum_{s=1}^S \sum_{r=1}^R \sum_{p=1}^P y_{srp} \phi_{sp} \leq \phi \quad (8)$$

Considering  $j$  as a rolling index for constraints within the partition (resource-bound constraints  $k \in K^E$  in this case), the corresponding elements of the matrix  $\mathbf{A}$  and vector  $\mathbf{b}$  in the standard form LP are:

$$a_{k_j^E srp} = \phi_{sp} \quad \forall s, r, p \text{ \& } b_{k_j^E} = \phi. \quad (9)$$

### 2.3.2. Regulatory GHG Constraint

Certain GHG regulations (such as [22]) are based on specific tailpipe lab-test GHG certification numbers rather than LCA GHG. Let  $\psi_{sp}$  be the GHG certification for a vehicle of size  $s$  and powertrain type  $p$ , with  $\psi$  being the fleet average GHG that the regulation allows for the target model-year. A constraint ensuring the powertrain mix complies with regulation GHG could be expressed as:

$$\sum_{s=1}^S \sum_{r=1}^R \sum_{p=1}^P y_{srp} \psi_{sp} \leq \psi. \quad (10)$$

However, in some GHG regulations (such as the US Federal [22]), the GHG allowance, which is the right-hand side of Equation (10) depends on the type of vehicle (“Car” or “Light Truck”) as well as its size, denoted by its floor area. Let  $\psi_s$  be the fleet average GHG allowance for the special case of all the produced vehicles being of size  $s$ . Then, for a mix of different vehicle sizes, the right-hand side of Equation (10) could be expressed as:

$$\psi = \sum_{s=1}^S \sum_{r=1}^R \sum_{p=1}^P y_{srp} \psi_s. \quad (11)$$

Substituting from Equation (11) into Equation (10) and rearranging:

$$\sum_{s=1}^S \sum_{r=1}^R \sum_{p=1}^P y_{srp} (\psi_{sp} - \psi_s) \leq 0. \quad (12)$$

Equation (12) can then be expressed in standard LP form for the values of elements of the matrix  $\mathbf{A}$  and vector  $\mathbf{b}$  as:

$$a_{k_j^E srp} = \psi_{sp} - \psi_s \quad \forall s, r, p \text{ \& } b_{k_j^E} = 0. \quad (13)$$

### 2.3.3. Region-Specific Regulatory GHG Constraint

Though not demonstrated via numerical examples in this paper, a more generalized form of Equations (10)–(13) could be derived for the case when the powertrain planning is being conducted for multiple regions that could have different GHG regulations in place. By simply utilizing the subscript  $r \in R^E$ , where  $R^E$  is a subset of the full set of region indices  $\{1, \dots, R\}$ ,  $\psi_{srp}$  being the GHG certification for a vehicle of size  $s$  and powertrain type  $p$  in region  $r$  with  $\psi_{sr}$  being the GHG allowance for size  $s$  in region  $r$ , the generalized version of Equation (12) could be expressed as:

$$\sum_{r \in R^E} \sum_{s=1}^S \sum_{p=1}^P y_{srp} (\psi_{srp} - \psi_{sr}) \leq 0. \quad (14)$$

### 2.4. Pre-Dictated Fleet Fraction Constraints

Due to certain considerations such as demand forecast or strategic planning, the task of powertrain mix planning could involve imposing upper bounds, lower bounds, or both upper and lower bounds on certain types of powertrains and/or vehicle sizes, for one or more geographic regions. Technology forcing type of regulations (such as the ZEV mandate [23]) could also be accommodated via constraints following this construction.

#### 2.4.1. Generalized Less-Than Fleet Fraction Type Constraint

Let  $R^{D1}$  be a subset of the region indices  $\{1, \dots, R\}$ ,  $S^{D1}$  be a subset of the vehicle size indices  $\{1, \dots, S\}$ , and  $P^{D1}$  be a subset of the powertrain type indices  $\{1, \dots, P\}$ . Let  $R^{D2}$  be another subset of the region indices (could be the same as  $R^{D1}$ , or a different subset of indices),  $S^{D2}$  be another subset of the vehicle size indices (could be the same as  $S^{D1}$ , or different),  $P^{D2}$  be another subset of powertrain type indices (could be the same as  $P^{D1}$ , or different) and let  $\alpha^{D1D2}$  be the target fraction not to be exceeded. The generalized form of the constraint could be expressed as:

$$\sum_{r \in R^{D1}} \sum_{s \in S^{D1}} \sum_{p \in P^{D1}} y_{srp} \leq \alpha^{D1D2} \sum_{r \in R^{D2}} \sum_{s \in S^{D2}} \sum_{p \in P^{D2}} y_{srp}. \quad (15)$$

Although this fully generalized form could be difficult to interpret, some of its special cases are more relatable and will be explored in Sections 2.4.3–2.4.5. Expressing Equation (15) in standard LP form for the values of elements of the matrix **A** and vector **b** can be done as:

$$a_{k_j^{D1D2}} = \begin{cases} 1 - \alpha^{D1D2} & \forall \{s, r, p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ 1 & \forall \{s, r, p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \notin \{S^{D2}, R^{D2}, P^{D2}\} \\ -\alpha^{D1D2} & \forall \{s, r, p\} \notin \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

$$b_{k_j^{D1D2}} = 0$$

#### 2.4.2. Generalized Greater-Than Fleet Fraction Type Constraint

Following the same notation as in Section 2.4.1, but considering  $\beta^{D1D2}$  as the target fraction not to be less than, the generalized form of the constraint could be expressed as:

$$\sum_{r \in R^{D1}} \sum_{s \in S^{D1}} \sum_{p \in P^{D1}} y_{srp} \geq \beta^{D1D2} \sum_{r \in R^{D2}} \sum_{s \in S^{D2}} \sum_{p \in P^{D2}} y_{srp}. \quad (17)$$

Noting that the standard LP form must be expressed as “less than”, rearranging Equation (17) for the values of elements of the matrix **A** and vector **b** can be done as:

$$a_{k_j^D srp} = \begin{cases} \beta^{D1D2} - 1 & \forall \{s, r, p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ -1 & \forall \{s, r, p\} \in \{S^{D1}, R^{D1}, P^{D1}\} \& \notin \{S^{D2}, R^{D2}, P^{D2}\} \\ \beta^{D1D2} & \forall \{s, r, p\} \notin \{S^{D1}, R^{D1}, P^{D1}\} \& \in \{S^{D2}, R^{D2}, P^{D2}\} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

$$b_{k_j^D} = 0$$

#### 2.4.3. Special Case: One Size Category Not Exceeding Fraction of Total

One example of a pre-dictated fleet fraction constraint could be the fraction of sedan vehicles, which generally have lower LCA GHG than larger-sized vehicles (such as SUVs, minivans, and Pickup Trucks) when comparing the same powertrain type, i.e., sedan HEV will have lower LCA GHG than SUV HEV, and the sedan BEV will have lower LCA GHG than SUV BEV. Though this would make “larger fraction of sedan” be a desirable direction for an optimization that seeks minimum LCA GHG, a counter-balancing market demand limit (with  $\alpha^{sedan}$  being the maximum fraction of the fleet) type of constraint could be formulated as a special case of Equation (15) as:

$$\sum_{r=1}^R \sum_{s=sedan} \sum_{p=1}^P y_{srp} \leq \alpha^{sedan} \sum_{r=1}^R \sum_{s=1}^S \sum_{p=1}^P y_{srp}. \quad (19)$$

#### 2.4.4. Special Case: One Region No Less than Some Fraction of Total

When planning the allocation of vehicles across multiple regions, a situation that may arise could be that one or some regions are “less favorable” be it due to higher operating costs, infrastructure situation, or stringency level of local regulations. To prevent the LP optimization from considering unrealistic solutions that may completely divest from one or more regions ( $r \in R^{D1}$ ), one may employ a strategic decision of maintaining some market share as a special case of Equation (17), expressed as:

$$\sum_{r \in R^{D1}} \sum_{s=1}^S \sum_{p=1}^P y_{srp} \geq \beta^{D1} \sum_{r=1}^R \sum_{s=1}^S \sum_{p=1}^P y_{srp}. \quad (20)$$

#### 2.4.5. ZEV Mandate Type Constraints

The ZEV mandate [23], for California and other states in the US that choose to follow California’s standard, dictates that new vehicle sales for model-year 2026 to 2035 will need to have a minimum fraction of ( $\beta^{ZEV}$ ) of BEVs or PHEVs (or fuel cell, which we are not considering in this work, but can be added in the formulation), with the value  $\beta^{ZEV}$  increasing each year until reaching 100% by 2035. The ZEV mandate also dictates that no less than 80% of  $\beta^{ZEV}$  must be BEVs (or fuel cell). Both of those constraints can be expressed as special cases of Equation (17) as:

$$\sum_{r \in \text{ZEV-state}} \sum_{s=1}^S \sum_{p \in \{BEV, PHEV\}} y_{srp} \geq \beta^{ZEV} \sum_{r \in \text{ZEV-state}} \sum_{s=1}^S \sum_{p=1}^P y_{srp}, \quad (21)$$

$$\sum_{r \in \text{ZEV-state}} \sum_{s=1}^S \sum_{p=BEV} y_{srp} \geq 0.8\beta^{ZEV} \sum_{r \in \text{ZEV-state}} \sum_{s=1}^S \sum_{p=1}^P y_{srp}. \quad (22)$$

## 2.5. Objective Functions

### 2.5.1. Minimization of LCA GHG

Estimation of the LCA GHG of various vehicle sizes, powertrain types, and the regions where they'll be driven (some regional aspects, such as the CI of the electric grid, do affect LCA GHG) is performed as a "preprocessing" step prior to LP optimization. Let  $\lambda_{srp}$  be the estimated LCA GHG (in g-CO<sub>2</sub>/mile) for a vehicle of size  $s$  and powertrain type  $p$  driven in region  $r$ ; we define a "minimization of LCA GHG" objective  $f^L$  for Equation (1) as the maximization of LCA GHG reduction compared to the worst performer, denoted by  $(\lambda^{\max})$  and identified as:

$$\lambda^{\max} = \max_{s,r,p}(\lambda_{srp}). \quad (23)$$

Values of elements of the vector  $c$  for standard form LP in Equation (1) can then be calculated as:

$$c_{srp}^L = \lambda^{\max} - \lambda_{srp} \forall s, r, p. \quad (24)$$

### 2.5.2. Cost Minimization

Similar to the formulation in Section 2.5.1, if there were a cost  $\mu_{srp}$  associated with choosing to allocate one vehicle of size  $s$  and powertrain type  $p$  into region  $r$ , then a cost minimization objective  $f^O$  for Equation (1) could be formulated as maximization of the cost reduction compared to the most costly choice denoted by  $(\mu^{\max})$  and identified as:

$$\mu^{\max} = \max_{s,r,p}(\mu_{srp}). \quad (25)$$

Values of elements of the vector  $c$  for the standard form LP in Equation (1) can then be calculated as:

$$c_{srp}^O = \mu^{\max} - \mu_{srp} \forall s, r, p. \quad (26)$$

## 2.6. Multi-Objective Optimization

Upon completion of the calculation of the various elements of the matrix  $\mathbf{A}$  and vector  $\mathbf{b}$  (defining constraints of the LP optimization problem), one may utilize the coefficients vector  $c^L$  (from Equation (24)) to seek a solution that minimizes LCA GHG, or the coefficients vector  $c^O$  (from Equation (26)) to seek a solution that minimizes cost. In order to generate additional solutions that potentially offer a good/balanced compromise between minimum LCA GHG and minimum cost, one may formulate a "weighted" objective as:

$$c^W = w c^L + (1 - w) c^O \quad (27)$$

where the scalar parameter ( $w$ ) in Equation (27) is a weighing factor that can be gradually changed (values between 0 and 1) in order to generate Pareto-optimal (trade-off) solutions. The procedure for parameter sweep to generate Pareto-optimal solutions is part of the illustration in Figure 2 and is summarized as follows:

- Start with setting ( $w = 1$ ), which (per Equation (27)) sets  $c^W = c^L$ . Run LP optimization, which will return the solution  $x^L$  that provides the maximum attainable LCA GHG reduction.
- Next, set ( $w = 0$ ), which (per Equation (27)) sets  $c^W = c^O$ . Run LP optimization, which will return the solution  $x^O$  that provides the maximum attainable cost reduction.
- If  $x^L$  is the same solution (in terms of values of the decision variables) as  $x^O$ , this implies that the best solution for LCA GHG is also the best solution for cost reduction. In other words, no trade-off exists, and the multi-objective optimization returns only one solution ( $x^L = x^O$ )

- If  $x^L$  is different from  $x^O$ , initiate a set of unique trade-off solutions  $T = \{x^L, x^O\}$  and begin parameter sweep in steps of  $(\Delta w)$  from  $(w = 1 - \Delta w)$ 
  - Calculate weighted objective coefficients per Equation (27) with  $1 > w > 0$
  - Run LP optimization, check the returned optimal solution  $x^w$  (for the current value of  $w$ ), if  $x^w$  is different from all the solutions in the set  $T$ , add  $x^w$  to the set  $T$
  - If the current solution  $x^w$  is the same as  $x^O$ , end the parameter sweep and return the set  $T$  of unique Pareto optimal solutions (since further adjustments to the value of  $w$  will keep yielding the same solution  $x^O$ ).
  - Set  $w = w - \Delta w$
  - If  $w = 0$ , end the parameter sweep and return the set  $T$  of unique Pareto optimal solutions, conduct the next iteration of parameter sweep with the adjusted value of  $w$

It is noted that for nonlinear problems, the adopted approach of weighing the objectives has a known limitation of being unable to discover Pareto-optimal solutions that are not on the convex portion of a Pareto-frontier [26]. However, for LP problems, the feasible domain is always a convex set [21], and with the objectives being linear in terms of the decision variables, the Pareto-frontier cannot have non-convex topology. As such, the weighted objectives approach has the capability to generate all Pareto-optimal solutions.

### 3. Example Study

Both studies considered in this paper (simple example in this section and the scaled-up study in Section 4) rely on the pre-processing work (to the left side of the LP block in Figure 2) having “already been done”, as listed in Table 1. However, we briefly examine some of those pre-processing elements, starting with the LCA GHG objective. Estimation of the LCA GHG given vehicle sizing and powertrain specifications is typically first split into two or three categories: i) Use-phase of the vehicle, in-which the GHG emissions primarily come from the vehicle consumption of fuel and/or electric energy, ii) Manufacturing of the vehicle, in-which the GHG emissions come from extraction, refinement, transportation and processing of all the materials for all components in the vehicle up till it is picked up by its first owner, and iii) disposal of the vehicle at the end of its useful life. In some cases, tools for LCA modeling, such as GREET [27], may lump manufacturing and disposal together via considering the end-of-life of the various materials (percentage recycling, energy, and emissions for material retrieval..., etc.) as part of the equivalent GHG in the materials used for manufacturing of the vehicle. Estimation of the use-phase GHG could be done via direct simulation methods that physically model the vehicle and its powertrain, as well as how the vehicle is utilized in the real world [28–30], or via meta-modeling-type approaches such as look-up databases [31] or expert systems [32]. The modeling platform for GHG estimates utilized in this paper is CarGHG [33], which is a free that brings together other open-source tools; primarily FASTSim [29] for fuel economy simulations in order to estimate fuel and electric energy amounts during the use phase of the vehicles, as well as GREET [27] for the equivalent GHG of traction batteries (scaled by kWh-battery) and everything else in the vehicle (via size-scale generic estimates. Detailed vehicle FASTSim models are available in Supplemental Material, though we briefly highlight the broadline inputs that result in values shown in Table 1, as:

- Target model-year estimates are for 2030, with Gasoline E10 blend at 10,680 g-CO<sub>2</sub>/gal (well to wheels), carbon intensity for electric energy generation (well-to-plug) at 120 g-CO<sub>2</sub>/kWh for the California electric grid and 180 g-CO<sub>2</sub>/kWh for the average grid elsewhere in the US.
- Traction battery sizing of 1.1 kWh, 12 kWh, and 75 kWh for mid-sized sedan HEV, PHEV with 40 miles range, and BEV with 300 miles range, respectively.

- Traction battery sizing of 1.2 kWh, 18 kWh, and 110 kWh for a small SUV HEV, PHEV with 40 miles range, and BEV with 300 miles range, respectively.

**Table 1.** Objective coefficients  $c^L$  and  $c^O$ .

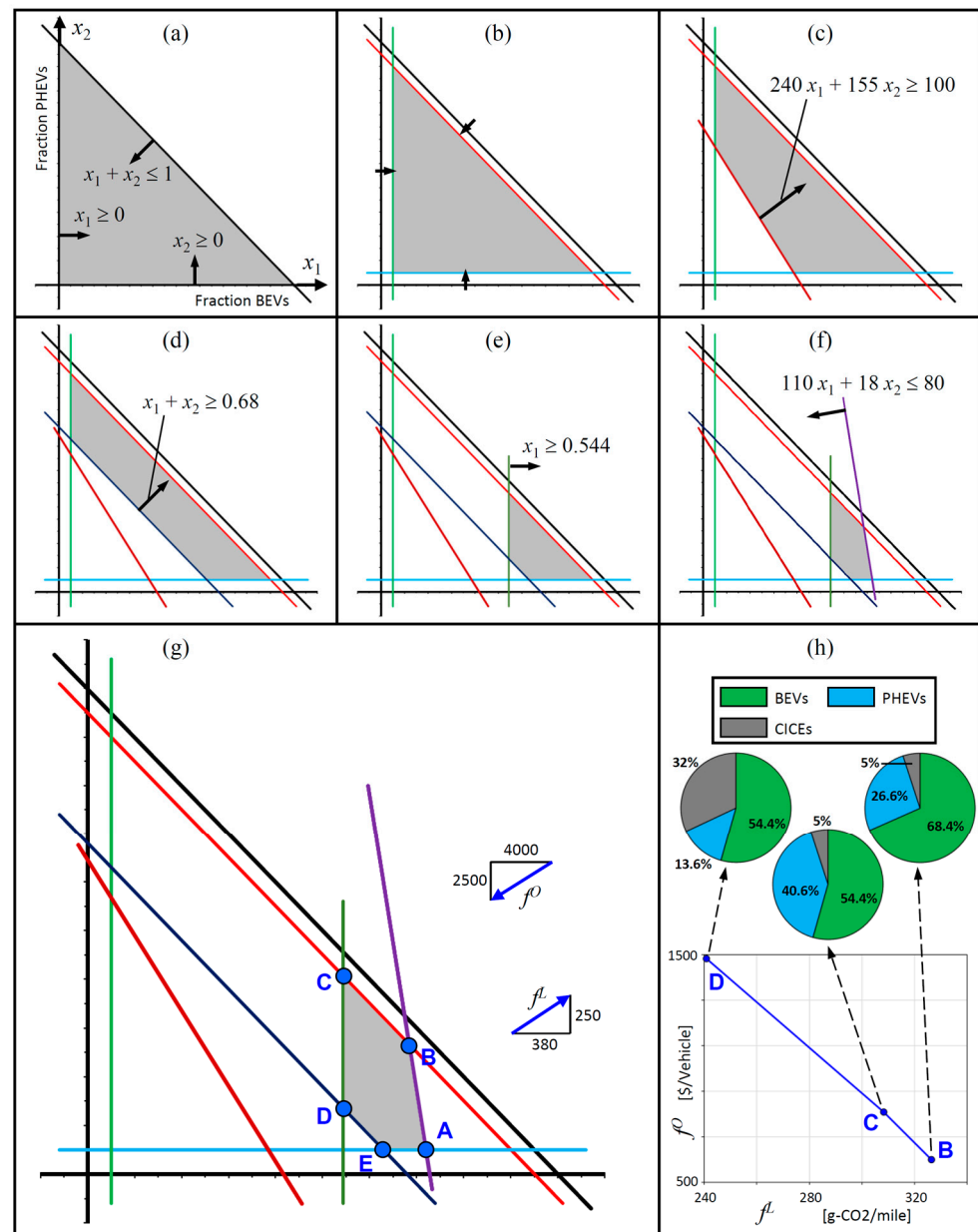
| Region $r$                  | Size $s$      | Powertrain $p$ | $c^L = \lambda^{\max} - \lambda_{\text{srp}}$ | $c^O = \mu^{\max} - \mu_{\text{srp}}$ | GHG Rating | GHG Allow. |
|-----------------------------|---------------|----------------|-----------------------------------------------|---------------------------------------|------------|------------|
| 1 = California & ZEV States | 1 = Sedan     | 1 = BEV        | 400                                           | 0                                     | 0          | 100        |
|                             |               | 2 = PHEV       | 285                                           | 1000                                  | 75         | 100        |
|                             |               | 3 = HEV        | 210                                           | 2000                                  | 150        | 100        |
|                             |               | 4 = CICE       | 90                                            | 3000                                  | 200        | 100        |
|                             | 2 = Small SUV | 1 = BEV        | 380                                           | 0                                     | 0          | 140        |
|                             |               | 2 = PHEV       | 250                                           | 1500                                  | 85         | 140        |
|                             |               | 3 = HEV        | 120                                           | 2500                                  | 175        | 140        |
|                             |               | 4 = CICE       | 0                                             | 4000                                  | 240        | 140        |
| 2 = Other US States         | 1 = Sedan     | 1 = BEV        | 350                                           | 0                                     | 0          | 100        |
|                             |               | 2 = PHEV       | 270                                           | 1000                                  | 75         | 100        |
|                             |               | 3 = HEV        | 210                                           | 2000                                  | 150        | 100        |
|                             |               | 4 = CICE       | 90                                            | 3000                                  | 200        | 100        |
|                             | 2 = Small SUV | 1 = BEV        | 320                                           | 0                                     | 0          | 140        |
|                             |               | 2 = PHEV       | 230                                           | 1500                                  | 85         | 140        |
|                             |               | 3 = HEV        | 120                                           | 2500                                  | 175        | 140        |
|                             |               | 4 = CICE       | 0                                             | 4000                                  | 240        | 140        |

GHG regulation standards in the US, as well as the enforceability of the ZEV mandate in California and other ZEV states, are currently subject to some uncertainty; thus, the regulatory constraints modeled in this paper represent those for the target model-year 2030 in the standards that were in place in 2024 [22,23], namely:

- Allowed fleet average GHG of 100 g-CO<sub>2</sub> per mile for vehicles that would be classified as light-duty cars [22] and fleet average GHG of 140 g-CO<sub>2</sub> per mile for vehicles that would be classified as light-duty trucks, with the mid-sized sedan classified as a car, while the small SUV is classified as a truck.
- GHG regulation considers only tailpipe GHG, and only the reference dynamometer test values are utilized [22]. As such, BEVs have regulatory GHG values of 0 g-CO<sub>2</sub> per mile. Typical 2024 GHG regulatory values for CICEs and HEVs are utilized for the respective GHG ratings in Table 1. For estimation of PHEVs' GHG regulation ratings, we utilize an approximate utility factor of 60% for a 40-mile range PHEV (implying that hybrid operation that results in tailpipe GHG emissions is for only 40% of the distance travelled), along with a typical 10% increase in fuel consumption (relative to an equivalent HEV) when the PHEV is operating in hybrid mode.
- ZEV mandate for 2030 requires 68% of new vehicle sales in California, and other ZEV states to be BEVs or PHEVs, with no less than 80% (of the 68%) being BEVs.

Though CarGHG includes cost models from the perspective of a vehicle buyer, it is difficult to identify reliable cost estimates from a vehicle planner's point of view. What is fairly accepted, however, is that at present, higher levels of electrification are more expensive to produce, and customer willingness to pay does make up for the difference in cost, resulting in slimmer margins or even losses [34,35]. While it may be unclear whether higher electrification levels will still have slimmer margins in 2030, we utilize placeholder cost numbers in Table 1 constructed in such a way to maintain, at least directionally, the notion of higher levels of electrification being more costly, hence preserving the "existence" of a trade-off between the LCA GHG objective and the cost objective.

The example in this section considers only three choices for the powertrain type (BEV, PHEV & CICE), only the small SUV size category, and only one region designation (California & ZEV states). It follows that the LP standard form has only three decision variables ( $x_1$ ,  $x_2$ ,  $x_3$ ), respectively corresponding to the fraction of BEVs, PHEVs, and CICEs. The fleet consistency constraint from Equation (5) could be rearranged so that  $x_3$  is calculated as a dependent variable ( $x_3 = 1 - x_1 - x_2$ ,  $x_1 + x_2 \leq 1$ ). Although the typical application of LP involves having all constraints and one objective simultaneously solved by the optimization algorithm [21], with only two independent decision variables ( $x_1$ ,  $x_2$ ), the problem can be visualized step-by-step, as shown in Figure 3.



**Figure 3.** Step by step example LP optimization for two independent decision variables: (a) initial feasible domain, (b–f) reduction of feasible domain as constraints are added, (g) final feasible domain and objective function vectors, (h) Pareto-optimal solutions.

Figure 3a shows the initial feasible domain with the fleet consistency constraint and simple bounds on the independent decision variables. Figure 3b shows the feasible domain becoming smaller as a result of adding a brief set of market forecasting constraints that every

powertrain type should retain at least 5% of the total, i.e., ( $x_1 \geq 0.05$ ,  $x_2 \geq 0.05$ ,  $x_3 \geq 0.05$ ). Though 5% is more of a demonstration value, this type of constraint prevents solutions that may otherwise completely eliminate one (or more) type(s) of powertrains from the mix. Such strategic considerations to maintain “some” (even if small) market foothold may not necessarily be reflected in the objective functions, but are necessary to observe so that the preferences of a diverse spectrum of vehicle buyers can be accommodated.

Figure 3c shows the effect of adding US federal GHG regulation, which corresponds to a constraint ( $85 x_2 + 240 x_3 \leq 140$ ), which in turn, after substituting for  $x_3$  in terms of  $x_1$ ,  $x_2$  becomes ( $240 x_1 + 155 x_2 \geq 100$ ), as shown in Figure 3c. For model year 2030, the ZEV mandate would require the sum of fractions of BEVs and PHEVs to be at least 68%, and that 80% of the 68% (i.e., 54.4%) must be BEVs, as shown in Figure 3d,e, respectively. Lastly, a constraint in Figure 3f on available materials for traction batteries (symbol  $\phi$  in Equation (8)) is 80 kWh per vehicle, with 110 kWh required to manufacture one BEV or 18 kWh required to manufacture one PHEV.

The remaining feasible domain (shaded area in Figure 3f) is shown at higher resolution in Figure 3g with its corner vertices labeled ‘A’ through ‘E’. Figure 3g also shows the direction vectors of the objective seeking to maximize LCA GHG reduction ( $f^L$ ), and the objective seeking to maximize cost reduction ( $f^O$ ). One notable observation from this example is that the optimum point for the objective  $f^L$  is the vertex ‘B’ (rather than vertex ‘A’). This observation exemplifies the notion that while BEVs may bring about larger GHG reductions than PHEVs for each retailed vehicle, an even higher priority is to displace as many CICEs as possible with either BEVs or PHEVs, per the available resources (battery materials in this case). Also observed from Figure 3g is that vertex ‘D’ represents a solution with the lowest cost (minimum amount of BEVs and PHEVs) yet is still feasible in terms of compliance with regulations for the target model-year.

When applying the approach discussed in Section 2.6 for the generation of Pareto-optimal solutions, starting with the objectives weighing scalar  $w$  at a value close to 1.0, the result of LP optimization will return the vertex ‘B’. As the weighing scalar  $w$  is gradually decreased, this corresponds to the direction vector of the weighed objective gradually “rotating” from being aligned with  $f^L$  towards becoming aligned with  $f^O$ , and at some intermediate value of  $w$ , the LP optimization will converge to the vertex ‘C’. And then with the continued decreasing of the value of  $w$ , the LP optimization will eventually converge to the vertex ‘D’, thereby generating the Pareto-optimal solutions shown in Figure 3h. Furthermore, from the properties of LP, when the direction vector of an objective is orthogonal to a facet (edge) of the feasible domain, all solutions along the facet are equally optimal. In other words, when a sequence of vertices ‘B’, ‘C’, ‘D’ is Pareto optimal, then all intermediate solutions on the edges BC and CD are also Pareto-optimal.

#### 4. Scaled-Up Study

In this section, we expand the simple study from Section 3 to all the combinations of region, size category, and powertrain type listed in Table 1. This includes California and ZEV states vs. other US states, models of mid-sized Sedan vs. Small SUV, and powertrain types {BEV, PHEV, HEV, CICE}, for a total of sixteen decision variables. In this case, the US federal GHG regulation constraint for model year 2030 per the 2024 rule-making [22] (via Equation (14), with GHG ratings and vehicle size GHG allowances from Table 1) is applied to the fleet average in all states. On the other hand, ZEV mandate constraints for the year 2030 (sum of BEVs and PHEV  $\geq 68\%$ , BEVs  $\geq 54.4\%$ ) are applied only within region 1. Market forecasting constraints were set that every powertrain type must maintain at least 5% of total vehicle sales in either size category segment, and to prevent region abandonment type solutions, additional constraints were imposed that at least 5% of California and ZEV

state vehicles would be either HEV or CICE, while other US states must have at least 5% of their vehicles be BEVs and another 5% be PHEVs.

Furthermore, recent data [36] indicates the share of total vehicle sales in California and ZEV states is approximately. 40% of total vehicle sales in the US, and the share of light-truck on a growing trend among light-duty vehicle sales in the US, reaching more than 60% in 2023 [37]. As such, bounds for the share of vehicles in region 1 were set between 35% and 45% (centered around the 40% value from [36]) and bounds for the share of small SUVs were set between 50% and 70% (centered around the 40% value from [37]). Such constraints are in place because the regional volume of sales and volume of sales by vehicle size categories are typically driven by consumer demand. Thus, even though a manufacturer may deem it beneficial to increase/decrease the sales fraction by region or size category, it is likely infeasible to attempt a significant change from current values.

The battery materials constraint was constructed per models for the vehicles in CarGHG [33] (as discussed in Section 3), with mid-sized sedans requiring 75 kWh, 12 kWh or 1.1 kWh to build one BEV, PHEV or HEV, respectively, and small SUVs requiring 110 kWh, 18 kWh or 1.2 kWh, respectively to build one BEV, PHEV or HEV. The average available battery materials per vehicle (symbol  $\phi$  in Equation (8)) may be difficult to predict for 2030, and as such, is treated as a study parameter (different values for different scenarios).

Figure 4a shows the Pareto plots (trade-off between  $f^L$  and  $f^O$ ) for all the considered scenarios. However, with the cost coefficients in Table 1 being placeholders, the only key message to infer from Figure 4a is that as long as higher levels of electrification remain more costly to the manufacturers (as is the case at present day [34,35]), then there will be a trade-off between  $f^L$  and  $f^O$ , and the framework in this paper can quantify such trade-off if the actual cost values (which may differ between different vehicle manufacturers, and are typically considered proprietary information) are assessed. As such, we focus further assessments in this section on only the extremum point (designated by an enlarged circle symbol at the end of the curve plots in Figure 4a) that represents the maximum LCA GHG reduction solution (at  $w = 1$  in Equation (27), or  $1 - w = 0$ ) which is the solution point not affected by choice of the cost coefficients. Figure 4b shows the optimum solution details (fraction of vehicles by region, size, and powertrain type shown as pie charts) that have the maximum LCA GHG reduction (enlarged circle symbol in Figure 4a) for each of the considered scenarios ( $\Theta^I$  to  $\Theta^{IV}$  and  $\Omega^I$  to  $\Omega^{IV}$ ). And for further higher-level insights, Figure 4c–e respectively, show summaries of the distribution of the vehicles by region, size category, and powertrain type corresponding to the respective pie charts in Figure 4b. Results of LP optimization for the problem are shown in Figure 4. The considered scenarios include different levels of average available battery materials per vehicle ( $\phi$ ), ranging from “abundant” battery supply to “less abundant”, with the scenarios designated by superscript Roman numbers {I, II, III, IV} for values of  $\phi$  at {80, 50, 30, 25} kWh, respectively. We also consider scenario symbols  $\Theta$  and  $\Omega$ , with  $\Theta$  representing the problem with all ZEV mandate constraints, while the symbol  $\Omega$  represents a scenario where the PHEVs cap (i.e., minimum BEVs constraint) is removed, thereby allowing more freedom in the decision-making about allocation of battery materials. For example,  $\Theta^I$  is a scenario where  $\phi = 80$  kWh and all the ZEV constraints are included, while  $\Omega^{IV}$  is a scenario where  $\phi = 25$  kWh and the constraint for the PHEV cap in the ZEV mandate is removed.

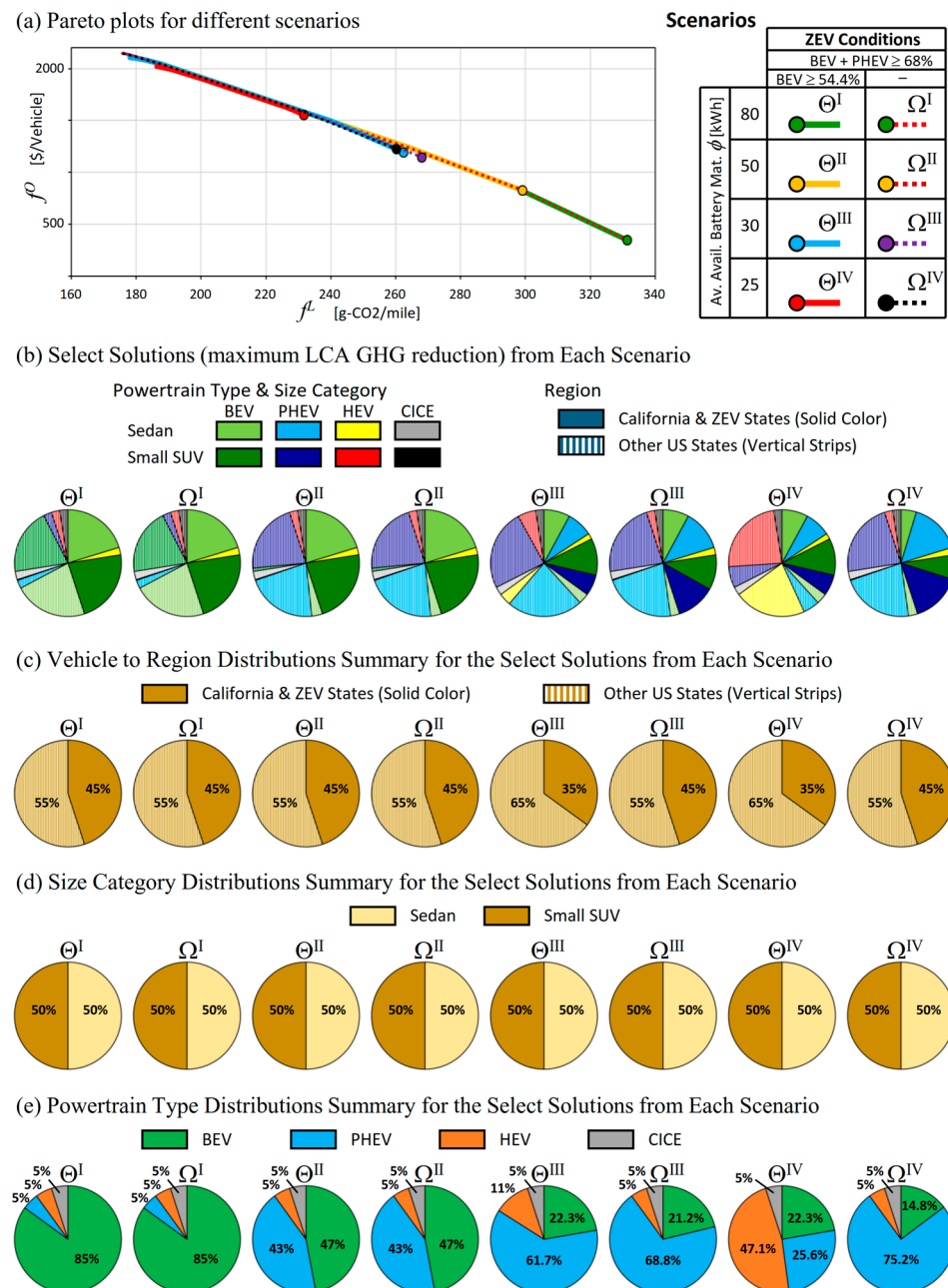


Figure 4. Results from a scaled-up study with sixteen decision variables.

While noting that the “scaled up” study in this section is still considerably simplified compared to a real powertrain planning task (which may involve dozens of different vehicle models), scaling up to larger studies is perceived as not being limited by LP optimization (which is known as capable of handling hundreds or even thousands of decision variables [21]). Rather, the challenge of handling larger problems may be more in compiling the necessary inputs (pre-processing stage) or in the interpretation of the results (post-processing stage). As such, the sizing of the study in this section is perceived as appropriate for the purpose of demonstrating how LP optimization makes “sensible decision” in different scenarios. Some of the notable observations include:

- When the supply of battery materials is abundant (scenarios  $\Theta^I$ ,  $\Omega^I$  in Figure 4), the maximum LCA GHG reduction solution involves maximizing BEVs across all regions and size categories (Figure 4b). The optimum solutions in these scenarios also involve maximizing the share of vehicles in California and ZEV states (reaching the 45%

maximum bound, as shown in Figure 4c for scenarios  $\Theta^I, \Omega^I$ , where the CI of the electric grid is lower. That type of solution (larger number of vehicles in California and ZEV states, with most of them being BEVs) takes advantage of BEVs having better marginal LCA GHG benefit per kWh of battery materials at lower grid CI.

- When the supply of battery materials becomes slightly less abundant (scenarios  $\Theta^{II}, \Omega^{II}$  in Figure 4), the optimum solution continues to prioritize placement of BEVs in California and ZEV states (Figure 4b), while displacing as many CICEs as possible with the “next best” to BEVs (i.e., PHEVs) in other US states.
- When the supply of battery materials is not highly constrained, the optimum solution is not affected by whether the PHEV cap in the ZEV mandate is considered or not (comparing  $\Theta^I$  vs  $\Omega^I$  and  $\Theta^{II}$  vs  $\Omega^{II}$  in Figure 4b). However, in scenarios  $\Theta^{III}$  and  $\Theta^{IV}$ , meeting the ZEV mandate while adhering to the PHEV cap is only feasible by reducing the share of vehicles in California and ZEV states (reaching the 35% minimum bound, as shown in Figure 4c for scenarios  $\Theta^{III}, \Theta^{IV}$ ) and an increasing share of HEVs in other US states, all of which result in less LCA GHG reduction compared to the respective scenarios of  $\Omega^{III}, \Omega^{IV}$  (where no PHEV cap is imposed) that allow shifting back the share of California and ZEV states to 45% (scenarios  $\Omega^{III}, \Omega^{IV}$  in Figure 4c) with maximum BEVs as allowed by available battery materials yet maximizing the displacement of CICEs via PHEVs.
- Though it might be challenging to achieve when considering the recent trend of consumer demand [37], all scenarios show that it would be beneficial to reduce the fraction of the larger sized vehicles (small SUV is larger than mid-sized sedan), with the optimum LCA GHG solution across all scenario keeping the share of small SUVs at the minimum bound of 50% in Figure 4d.

## 5. Discussion and Limitations

The framework and formulation in this paper rely on LP, which has the attractive qualities of resulting in convex optimization problems with provably global optimal solutions [21] via well-established and relatively easy-to-implement algorithms, which in turn, provide good transparency between the inputs and outcome outputs. However, although several types of modeling considerations have been addressed in Sections 2.2–2.6, one must acknowledge that LP cannot model everything in the real-world. Among the considerations that have not been considered in this paper are:

- Cost of non-compliance with regulatory requirements. In the current formulation, compliance with regulatory requirements is treated as a hard constraint. In other words, only solutions that fully comply with regulatory requirements will be returned by LP; otherwise, if no fully compliant solutions exist (for example, if materials and parts supply are insufficient to produce enough highly electrified vehicles), the LP optimization fails to return any solutions. Alternative approaches in product planning may instead consider “cost penalties” upon non-compliance (be it considering the need to buy carbon and/or ZEV credits, or simply paying fines) that are incorporated into the cost objective (instead of being treated as a hard constraint). The structure of such penalty functions, however, may not be easily (or even feasibly) modelled via LP.
- The formulation and studies in this paper treated all quantities/coefficients as deterministically “known”, with uncertainty regarding one parameter only (supply of battery materials) considered via considering multiple scenarios. In reality, however, many of the constraints and/or objectives (projected costs, projected grid CI, projected manufacturing GHG, projected regulations) can be subject to uncertainty. Though the scenario-based analysis showed LP producing stable results, the generation of scenarios for many combinations of uncertain considerations may become infeasible.

Instead, approaches/frameworks such as stochastic LP [38] may be more appropriate and could be considered in future work.

- The formulation and studies in this paper consider planning decisions for only one future model-year. Considering only one model-year enables “static” treatment of a problem that is otherwise temporally dynamic. However, real-world planning considerations often need to consider a horizon of planning that extends through multiple years. To some extent, the proposed LP formulation could be extended in future work to multiple successive model years via chaining over time while imposing constraints on feasible changes year-to-year (e.g., it may be difficult to go from 10% of one powertrain in one year to 90% in the following year). However, other effects, such as “learning curve” (e.g., the cost coefficients for one powertrain in one year depending on the volume produced of that powertrain in the previous year), may be difficult to model via LP and may instead require other approaches, such as Dynamic Programming [20].
- Though not considered in this paper due to lack of sufficiently granular data, regional cost variations could readily be incorporated in the cost objective via the coefficients  $c_{s,p}^O$  having different values at same  $s, p$  but different  $r$ . Furthermore, choice of the place-holder cost coefficients echoed present-day status [33,34] of higher electrification levels being more costly to the vehicle manufacturers than CICEs or hybrids. In the event that higher electrification levels become less expensive to produce, Pareto plots such as in Figure 4a are likely to “collapse” into one point only per scenario since there will no longer be a trade-off between  $f^L$  and  $f^O$ .
- The formulation in this paper treats all powertrains equally adoptable subject to costs, demand, and materials supply. However, the formulation does not delve into other usability considerations. In the case of BEVs in particular, there have been reports showing dissatisfaction and/or intent to no longer continue adoption by at least some of present-day BEV owners [39,40]. Such considerations, however, are likely beyond the scope or capability of LP modeling.

## 6. Conclusions and Future Work

This paper presented a framework for formulating the decision-making process for future powertrain planning across multiple regions and vehicle sizes as a linear programming (LP) optimization problem. The LP formulation can incorporate generic limited resource type constraints and pre-dictated fleet fraction type constraints, which in turn, allow for practical modeling of many types of planning constraints, including regulatory GHG, ZEV mandate, materials supply, and forecasted future demand. The framework is also capable of simultaneously optimizing two different objectives, thereby generating Pareto trade-off solutions. The proposed framework was demonstrated via a simple three-decision variable example, which was further reduced to two independent variables, thereby allowing for step-by-step visualization. A scaled-up study with sixteen decision variables showcased LP optimization, making sensible choices for different scenarios considered for the problem.

Future extension of this work may incorporate further scaled-up studies with more vehicle size categories and/or further powertrain type options, such as BEVs and PHEVs with different electric ranges. Other extensions of this work, from a practical standpoint for future product planning, could include adding consideration of uncertainties (within bounds and/or probability distributions) for the coefficient values of the objectives and/or constraints, as well as the consideration of multi-model-year product planning across a range of future planning years.

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**Data Availability Statement:** Aside from dynamometer test GHG ratings, data that enable replication of results in this paper is available from public sources cited within the manuscript. Archival copy of dynamometer GHG ratings is available from the authors upon request. Java code that implements the LP mathematical formulations in this paper is available from the authors upon request.

**Conflicts of Interest:** Karim Hamza and Kenneth Laberteaux are employees of Toyota Motor North America. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

## List of Abbreviations and Symbols

The following abbreviations and symbols are used in this manuscript:

|            |                                                                                                                                                                                                                                                                        |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| BEV        | Battery Electric Vehicle                                                                                                                                                                                                                                               |
| CI         | Carbon Intensity (for the electric grid)                                                                                                                                                                                                                               |
| CICE       | Conventional Internal Combustion Engine                                                                                                                                                                                                                                |
| FCEV       | Fuel-Cell Electric Vehicle                                                                                                                                                                                                                                             |
| GHG        | Greenhouse Gas                                                                                                                                                                                                                                                         |
| HEV        | Hybrid Electric Vehicle                                                                                                                                                                                                                                                |
| LCA        | Lifecycle Analysis                                                                                                                                                                                                                                                     |
| LP         | Linear Programming                                                                                                                                                                                                                                                     |
| PFCEV      | Plug-in Fuel-Cell Electric Vehicle                                                                                                                                                                                                                                     |
| PHEV       | Plug-in Hybrid Electric Vehicle                                                                                                                                                                                                                                        |
| $A$ or $a$ | Constraint coefficients for standard form LP, boldface indicates a matrix, while regular type font along with subscripts (such as $k$ or $i$ ) indicates individual matrix elements                                                                                    |
| $b$        | Free term constraint coefficients for standard form LP, boldface indicates a vector, while regular type font along with subscript (such as $k$ ) indicates individual elements                                                                                         |
| $c$        | Reward coefficients for the objective function in standard form LP, boldface indicates a vector, while regular type font along with subscript (such as $i$ ) indicates individual elements, superscripts (such as $L$ or $O$ ) indicate type of the objective function |
| $f$        | Objective function for standard form LP, sought to be maximized, superscripts (such as $L$ or $O$ ) indicate type of the objective: LCA GHG reduction or cost reduction                                                                                                |
| $M$        | Total number of vehicles to be produced for a given model-year                                                                                                                                                                                                         |
| $N$        | Total number of decision variables                                                                                                                                                                                                                                     |
| $T$        | Set of unique Pareto trade-off solutions                                                                                                                                                                                                                               |
| $w$        | Weighing parameter for combining different objectives so that Pareto-optimal (trade-off) solutions can be generated                                                                                                                                                    |
| $x$        | Decision variables for standard form LP, boldface indicates vector, while regular type font along with subscript (such as $i$ ) indicates individual elements                                                                                                          |
| $y$        | Partitioned form of the decision variables, subscripts $r, s, p$ , respectively indicate the region, size category and powertrain type/specification                                                                                                                   |
| $\alpha$   | Constraint coefficient for prescribed less than or equal share of the total vehicles to be produced, subscripts $r, s, p$ are aligned with those of the partitioned form of the decision variables ( $y$ ), no subscript indicates the free term                       |
| $\beta$    | Constraint coefficient for prescribed greater than or equal share of the total vehicles to be produced, subscripts $r, s, p$ are aligned with those of the partitioned form of the decision variables ( $y$ ), no subscript indicates the free term                    |

|           |                                                                                                                                                                                                                                                                                                                           |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\phi$    | Constraint coefficient for a materials resource, subscripts $r, s, p$ are aligned with those of the partitioned form of the decision variables ( $y$ ), indicating amount of material used per vehicle, no subscript indicates the free term, indicating average amount of material available per vehicle                 |
| $\lambda$ | Estimated LCA GHG per vehicle, subscripts $r, s, p$ are aligned with those of the partitioned form of the decision variables ( $y$ )                                                                                                                                                                                      |
| $\mu$     | Expected revenue minus cost per vehicle, subscripts $r, s, p$ are aligned with those of the partitioned form of the decision variables ( $y$ )                                                                                                                                                                            |
| $\Phi$    | Total amount available of a materials resource                                                                                                                                                                                                                                                                            |
| $\Theta$  | Scenario indicator (along with superscripts I, II, III & IV) for different levels of available batter materials)                                                                                                                                                                                                          |
| $\Omega$  | Scenario indicator (along with superscripts I, II, III & IV) for different levels of available batter materials) as well as relaxed constraint on percentage of minimum BEVs                                                                                                                                              |
| $\psi$    | Constraint coefficient for GHG regulation modelled as a limited resource, subscripts $r, s, p$ are aligned with those of the partitioned form of the decision variables ( $y$ ), indicating the GHG rating per vehicle, no subscript indicates the free term, indicating the total average per vehicle not to be exceeded |

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