Part 1 - Quantum Field with Time as a Dynamical Variable

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A Fundamental Question to Consider

In classical theory, the amplitude of a wave with vibrations in space is well defined. Since matter can have vibrations in space, can it also has vibrations in time, if space and time are to be treated on the same footing? In fact, we can in theory construct a plane wave with vibrations in time. Can this wave have something to do with our real physical world?

I. Vibrations in Time

Consider the background coordinates $(t, \mathbf{x})$ for the flat space-time as observed in an inertial frame O. Assume there exists a plane wave with matter that has vibrations in time relative to the background coordinate time, t. We will define a plane wave's amplitude for vibration in time, T, as the maximum difference between the time of matter inside the wave, t_f, and the external time, t. Therefore, if matter inside the plane wave carries a clock measuring its internal time, an inertial observer outside will see the matter's clock running at a different rate. The vibrations of matter in time can be written as

$$t_f = t + T\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = t + \mathrm{Re}(\zeta_t^+), \quad (1)$$

$$\zeta_t^+ = -iTe^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \quad (2)$$

$$\omega^2 = \omega_0^2 + |\mathbf{k}|^2. \quad (3)$$

We can further define a plane wave

$$\zeta^+ = \frac{T_0}{\omega_0}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \quad (4)$$

such that,

$$\zeta_t^+ = \partial_0\zeta^+. \quad (5)$$

$T_0 = (\omega_0/\omega)T$ is the amplitude for a plane wave with matter vibrating in proper time. Therefore, the vibrations of matter in time of a plane wave can also be described by ζ^+.

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II. Hamiltonian Density

Let us investigate the properties of a system in a cube with volume V that can have multiple particles with mass m vibrating in time. We make the following ansätze

$$\varphi^+ = \omega_0\sqrt{\frac{m}{2V}}\zeta^+ = T_0\sqrt{\frac{m}{2V}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \quad (6)$$

$$\varphi^- = \omega_0\sqrt{\frac{m}{2V}}\zeta^- = T_0^*\sqrt{\frac{m}{2V}}e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \quad (7)$$

The plane wave $\varphi^\pm$ satisfies the equation of motion:

$$\partial_u\partial^u\varphi^\pm + \omega_0^2\varphi^\pm = 0. \quad (8)$$

The corresponding Hamiltonian density is

$$H^\pm = (\partial_0\varphi^{\pm*})(\partial_0\varphi^\pm) + (\nabla\varphi^{\pm*})\cdot(\nabla\varphi^\pm) + \omega_0^2\varphi^{\pm*}\varphi^\pm. \quad (9)$$

The energy for a plane wave with vibrations of matter in proper time is of special importance in our study:

II. Hamiltonian Density (cont.)

$$\varphi_0^+ = T_0\sqrt{\frac{m}{2V}}e^{-i\omega_0 t}, \quad (10)$$

$$\varphi_0^- = T_0^*\sqrt{\frac{m}{2V}}e^{i\omega_0 t}. \quad (11)$$

Matter inside this plane wave $\varphi_0^\pm$ has vibrations in proper time only, i.e. $|\mathbf{k}| = 0$. The Hamiltonian density is

$$H_0^\pm = \frac{m\omega_0^2 T_0^* T_0}{V}. \quad (12)$$

The energy inside volume V is $E = m\omega_0^2T_0^*T_0$ of a simple harmonic oscillating system in proper time. Energy E shall correspond to certain energy intrinsic to matter. However, we have only consider matter with mass m in this simple harmonic oscillating system with no other force field. Here, we will consider this energy as the internal energy of mass.

Key Results

Allowing matter to have an additional degree of freedom to vibrate in time, we show that such system has the same properties of a bosonic field. The temporal vibrations are physical quantities introduced to restore symmetry between time and space in the matter field. The spacetime outside a particle with oscillation in time also satisfies the Schwarzschild field solution as shown in Part 2 of this poster.

III. Proper Time Oscillator

Taking the energy of the harmonic oscillator as the internal mass-energy of matter, it can only be observed as the energy of mass m which is on shell. For a single particle system,

$$E = m = m\omega_0^2T_0^*T_0, \quad (13)$$

or

$$\omega_0^2T_0^*T_0 = 1. \quad (14)$$

Therefore, a particle with mass m has oscillation in proper time with amplitude $|\mathring{T}_0| = 1/\omega_0$. The amplitude of this oscillation is unique. The internal time $\mathring{t}_f^+$ of the point mass's internal clock is:

$$\mathring{t}_f^+(t) = t - \frac{\sin(\omega_0 t)}{\omega_0}. \quad (15)$$

III. Proper Time Oscillator (cont.)

The internal time rate is

$$\frac{\partial\mathring{t}_f^+}{\partial t} = 1 - \cos(\omega_0 t). \quad (16)$$

The average time rate is 1 bounded between 0 and 2 which is positive. The internal clock of this point mass moves only in the forward direction. Such particle travels along a time-like geodesic when averaged over time.

On the other hand, plane wave φ_0^+ with a particle traveling forward in time is mathematically equivalent to plane wave φ_0^- with a particle traveling backward in time. Plane wave φ_0^- describes antiparticles.

IV. Field Quantization

For a many-particle system,

$$\omega_0^2T_0^*T_0 = n. \quad (17)$$

Taking the point mass as a particle(antiparticle) with de Broglie's mass/energy ($m = \omega_0$),

$$H_0^\pm = \frac{n\omega_0}{V}. \quad (18)$$

The energy in the plane wave with proper time oscillations is quantized with $n = 0, 1, 2, ...$ Instead of considering $\varphi^\pm$, let us consider a plane wave $\varphi_n^\pm$ which is normalized in volume V when $n = 1$,

$$\varphi_n^\pm = \gamma^{-1/2}\varphi^\pm, \quad (19)$$

where $\gamma = (1 - |\mathbf{v}|^2)^{-1/2} = \omega/\omega_0$. Replace $\varphi^\pm$ with $\varphi_n^\pm$ in Eq. (9),

$$H_n^\pm = \gamma H_0^\pm = \frac{n\omega}{V}. \quad (20)$$

The energy in this plane wave is quantized with n particles (antiparticles) in volume V. We can obtain a real scalar field by superposition of plane waves,

$$\varphi(x) = \sum_{\mathbf{k}}\varphi_{n\mathbf{k}}^+(x) + \varphi_{n\mathbf{k}}^-(x) \quad (21)$$

$$= \sum_{\mathbf{k}}(2V\omega)^{-1/2}(\omega_0T_{0\mathbf{k}}e^{-ikx} + \omega_0T_{0\mathbf{k}}^*e^{ikx}). \quad (22)$$

It satisfies the Klein-Gordon equation. The transition to quantum field can be done via canonical quantization with $a_{\mathbf{k}} = \omega_0T_{0\mathbf{k}}$ as the annihilation operator and $a_{\mathbf{k}}^\dagger = \omega_0T_{0\mathbf{k}}^\dagger$ as the creation operator. $N_{\mathbf{k}} = \omega_0^2T_{0\mathbf{k}}^\dagger T_{0\mathbf{k}}$ is a particle number operator. **Therefore, the real scalar field has the same properties of a bosonic field. Although not shown in here, it is straight forward to show in non-relativistic limit that the system satisfies Schrodinger equation and has the properties of a quantum wave.**

Experimenting with Neutrino?

Neutrino is the lightest known elementary particle. The neutrino oscillation length is in the oder of kilometer. The energy in experiments are in the order of GeV. Neutrino can be an interesting candidate that can be used to study the possible effects of the temporal oscillation.

Part 2 - Gravitational Field of a Particle with Oscillation in Time

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I. Fourier Decomposition of the Proper Time Oscillation

From Part 1 poster, time at the location of a point mass is driven by its energy to oscillate. Taking this proper time oscillation as a part of the spacetime geometry, its geometry is different from the assumed flat spacetime at spatial infinity. The spacetime around it shall be curved. By neglecting all quantum effects and assuming the particle as a classical object that can remain stationary in space, the temporal vibrations at and around the particle are

$$\mathring{t}_f(t,\mathbf{x}) = t - \frac{\Pi(\mathbf{x})\sin(\omega_0 t)}{\omega_0} = t + \mathring{\zeta}_t(t,\mathbf{x}), \quad (1)$$

$$\mathring{\zeta}_t(t,\mathbf{x}) = -\frac{\Pi(\mathbf{x})}{\omega_0}\sin(\omega_0 t), \quad (2)$$

$$\Pi(\mathbf{x}) = 0 \text{ if } |\mathbf{x}| \geq \epsilon/2, \quad (3)$$

$$\Pi(\mathbf{x}) = 1 \text{ if } |\mathbf{x}| < \epsilon/2. \quad (4)$$

$\Pi(\mathbf{x})$ is a pulse at $\mathbf{x} = \mathbf{x}_0$ with width $\epsilon \to 0$. As a wave, the vibrations in time $\mathring{\zeta}_t$ can be decomposed into Fourier series of plane waves. This can be done with the superposition of plane waves $\zeta_{t\mathbf{k}} = -iT_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. However, in a relativistic theory, $\zeta_{t\mathbf{k}}$ can only be the 0-component of a relativistic plane wave. The spatial component of the relativistic plane wave with vibrations in space is $\zeta_{\mathbf{x}\mathbf{k}} = -i\mathbf{X}_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. After carrying out the superposition with the relativistic plane waves with vibrations in space and time, we find that there are additional oscillations in space around the proper time oscillator. Switching to a spherical coordinate system, the additional radial oscillations are,

$$\mathring{r}_f(t,r) = r + \mathring{\zeta}_r(t,r), \quad (5)$$

$$\mathring{\zeta}_r(t,r) = -\frac{\Pi'(r)}{\omega_0^2}\cos(\omega_0 t). \quad (6)$$

$\Pi'(r)$ denotes the derivative of $\Pi(r)$ with respect to r, such that

$$\Pi'(r) = 0 \text{ if } r \neq \epsilon/2, \quad (7)$$

$$\Pi'(r) = -\infty \text{ if } r = \epsilon/2. \quad (8)$$

The radial vibrations are on an infinitesimally thin spherical shell with radius $\breve{r} \to 0$. An observer $\breve{O}$ on this timelike hypersurface is stationary relative to an inertial observer O at spatial infinity.

II. Shell with Fictitious Oscillations

Instead of considering the shell with infinitesimal radius, we will first study a thin shell with finite radius $\breve{r}$. On this shell Σ, we introduce radial oscillations:

$$\mathring{t}_f(t,\breve{r}) = t, \quad (9)$$

$$\mathring{r}_f(t,\breve{r}) = \breve{r} + \breve{\Re}\cos(\omega_0 t), \quad (10)$$

$$\mathring{v}_f(t,\breve{r}) = \frac{\partial \mathring{r}_f(t,\breve{r})}{\partial t} = -\breve{\Re}\omega_0\sin(\omega_0 t), \quad (11)$$

where $\breve{\Re}\omega_0 < 1$. A fictitious observer $\underline{O}$ originally at $\breve{r}$ is displaced to $\mathring{r}_f$ at time t.

As shown in Eq. (9), it is the clock of fictitious observer $\underline{O}$ that synchronizes with the clock of observer O. In its fictitious frame, $\underline{O}$ is an inertial observer. From Eqs. (10) and (11), an observer $\breve{O}$ stationary at $r = \breve{r}$ has a fictitious displacement $\underline{r}_f$ and instantaneous velocity $\underline{v}_f$ relative to $\underline{O}$,

$$\underline{r}_f(t) = -\mathring{r}_f(t,\breve{r}) + \breve{r} = -\breve{\Re}\cos(\omega_0 t), \quad (12)$$

$$\underline{v}_f(t) = -\mathring{v}_f(t,\breve{r}) = \breve{\Re}\omega_0\sin(\omega_0 t). \quad (13)$$

Although $\breve{O}$ is stationary relative to O at spatial infinity, it is under the effects with $\breve{O}$ oscillating in the fictitious frame of $\underline{O}$. The fictitious oscillation is not a vibration that carry an observer through space. It is information that we will use to describe the geometrical structure of space-time at $r = \breve{r}$.

As a simple oscillating system, its total Hamiltonian is invariant over time. The system has a time translational symmetry as demanded by the Noether theorem. The fictitious displacement and its instantaneous velocity can have effects on $\breve{O}$. Their combined effects shall remain constant. Although the effects of the fictitious displacement are not yet defined, we can obtain the spacetime geometrical properties at $r = \breve{r}$ when there is only a fictitious velocity with $|\underline{v}_f| < 1$.

III. Measurements on the Shell with Fictitious Oscillations

At $t = t_m = \pi/(2\omega_0)$, the fictitious displacement and instantaneous velocity from Eqs. (12) and (13) are: $\underline{r}_f(t_m) = \underline{r}_{fm} = 0$ and $\underline{v}_f(t_m) = \underline{v}_{fm} = \breve{\Re}\omega_0$. Consider two events in frame $\breve{O}$ with infinitesimal separations $d\breve{t}$ and $d\breve{r}$ at $t = t_m$. They can be ....

III. Measurements on the Shell with Fictitious Oscillations (cont.)

... related to the increments dt and dr observed in frame O,

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} \Upsilon^t{}_{\breve{t}} & \Upsilon^t{}_{\breve{r}} \\ \Upsilon^r{}_{\breve{t}} & \Upsilon^r{}_{\breve{r}} \end{bmatrix} \begin{bmatrix} d\breve{t} \\ d\breve{r} \end{bmatrix}. \quad (14)$$

In the local frames of O and $\breve{O}$, the basis vectors in the temporal and radial directions are orthogonal, i.e. $\mathbf{e}_t \cdot \mathbf{e}_r = 0$, $\mathbf{e}_{\breve{t}} \cdot \mathbf{e}_{\breve{r}} = 0$, $\mathbf{e}_{\breve{t}} \parallel \mathbf{e}_t$, and $\mathbf{e}_{\breve{r}} \parallel \mathbf{e}_r$. The transformation matrix Υ is diagonal, $\Upsilon^t{}_{\breve{r}} = \Upsilon^r{}_{\breve{t}} = 0$. When $d\breve{r} = 0$, $d\breve{t}$ is a proper time measured by $\breve{O}$. Lorentz transform to the fictitious frame $\underline{O}$,

$$d\underline{t} = \gamma d\breve{t}, \quad (15)$$

$$d\underline{r} = \underline{\gamma v_{fm} d\breve{t}}, \quad (16)$$

where $\gamma = [1 - (\underline{v}_{fm})^2]^{-1/2}$. The clocks of O and $\underline{O}$ are synchronized. O shall measures the same time as $\underline{O}$,

$$dt = d\underline{t} = \gamma d\breve{t}. \quad (17)$$

However, O is physically stationary relative to $\breve{O}$, $dr = 0$. The underlined quantity in Eq. (16) is a fictitious displacement that appears only in the fictitious frame of $\underline{O}$. From Eq. (17),

$$\Upsilon^t{}_{\breve{t}} = \gamma = [1 - (\underline{v}_{fm})^2]^{-1/2} = (1 - \breve{\Re}^2\omega_0^2)^{-1/2}. \quad (18)$$

Next, consider a spacelike interval $d\breve{r}$ expressed as two events measured at endpoints of a rod simultaneously, $d\breve{t} = 0$. Lorentz transform to frame $\underline{O}$,

$$d\underline{t} = \underline{\gamma v_{fm} d\breve{r}}, \quad (19)$$

$$d\underline{r} = \gamma d\breve{r}. \quad (20)$$

The moving length of the rod is,

$$d\underline{l} = d\underline{r} - \underline{v_{fm} d\underline{t}} = \gamma^{-1} d\breve{r}. \quad (21)$$

As inertial observers with their clocks synchronized, O measures the same length of the rod as $\underline{O}$,

$$dr = d\underline{l} = \gamma^{-1} d\breve{r}. \quad (22)$$

However, a rod carried by $\breve{O}$ is stationary relative to O. The underlined quantities in Eqs. (19) and (21) are fictitious displacements that only appear in the fictitious frame $\underline{O}$. The spacelike interval representing the rod length in frame O is measured simultaneously at endpoints, $dt = 0$. From Eq. (22),

$$\Upsilon^r{}_{\breve{r}} = \gamma^{-1} = [1 - (\underline{v}_{fm})^2]^{1/2} = (1 - \breve{\Re}^2\omega_0^2)^{1/2}. \quad (23)$$

IV. Schwarzschild field

As geometrical properties, both $\underline{r}_f$ and $\underline{v}_f$ from Eqs. (12) and (13) have effects on $\breve{O}$. Their summation is a constant under time translation as discussed in Section II. Therefore, we can define a constant,

$$\breve{I} = \omega_0^2(\underline{r}_f)^2 + (\underline{v}_f)^2 = \breve{\Re}^2\omega_0^2, \quad (24)$$

such that Eq. (14) becomes,

$$\begin{bmatrix} dt \\ dr \end{bmatrix} = \begin{bmatrix} (1-\breve{I})^{-1/2} & 0 \\ 0 & (1-\breve{I})^{1/2} \end{bmatrix} \begin{bmatrix} d\breve{t} \\ d\breve{r} \end{bmatrix}. \quad (25)$$

according to Eqs. (18) and (23). The measurements in the temporal and radial directions in frames O and $\breve{O}$ have different scales, i.e.

$$g_{tt}(\breve{r}) = \mathbf{e}_{\breve{t}} \cdot \mathbf{e}_{\breve{t}} = (1-\breve{I})\mathbf{e}_t \cdot \mathbf{e}_t = 1 - \breve{I}, \quad (26)$$

$$g_{rr}(\breve{r}) = \mathbf{e}_{\breve{r}} \cdot \mathbf{e}_{\breve{r}} = (1-\breve{I})^{-1}\mathbf{e}_r \cdot \mathbf{e}_r = -(1-\breve{I})^{-1}, \quad (27)$$

$$g_{tr}(\breve{r}) = g_{rt}(\breve{r}) = \mathbf{e}_{\breve{t}} \cdot \mathbf{e}_{\breve{r}} = \mathbf{e}_t \cdot \mathbf{e}_r = 0, \quad (28)$$

where $\mathbf{e}_t \cdot \mathbf{e}_t = 1$, $\mathbf{e}_r \cdot \mathbf{e}_r = -1$, and $\mathbf{e}_t \cdot \mathbf{e}_r = 0$. Therefore, the line element at $r = \breve{r}$ is,

$$ds^2 = [1 - \breve{I}]dt^2 - [1 - \breve{I}]^{-1}dr^2 - \breve{r}^2 d\Omega^2. \quad (29)$$

Setting $\breve{I} = 2m/\breve{r}$ or $m = \breve{r}\breve{\Re}^2\omega_0^2/2$, the vacuum spacetime υ^+ outside Σ is the Schwarzschild spacetime. Applying Birkhoff theorem, Σ can be contracted. As long as the equivalent mass m of the shell is remaining constant during this contraction, the metric of the external field will not be affected. The amplitude of the radial oscillation is,

$$\breve{\Re} = \sqrt{2/(\breve{r}m)}. \quad (30)$$

This amplitude $\breve{\Re}$ and the related curvature tensors derived are well defined until the shell is contracted to $\breve{r} \to 0$. At this point, the shell is infinitely small but has infinitely large amplitude, $\breve{\Re} \to \infty$. This is the same fictitious radial oscillations around the proper time oscillator with infinite amplitude as shown in Section I. We therefore arrive at the following result:

Key Result

By neglecting all quantum effects and assuming the particle as a classical object that can remain stationary in space, the space-time geometry resulting from the proper time oscillator can mimic the Schwarzschild gravitational field.